REDUCING COST OF IMBALANCE , IN SPOT MARKET , DUE TO WIND FORECASTING ERRORS USING SHORT GATE CLOSURE AND BATTERY ENERGY STORAGE SYSTEMS

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CERTIFICATE

This is to certify that the project report titled ***"REDUCING COST OF IMBALANCE , IN SPOT MARKET , DUE TO WIND FORECASTING ERRORS USING SHORT GATE CLOSURE AND BATTERY ENERGY STORAGE SYSTEMS"***, submitted by **JASKIRAT SINGH**, to the Indian Institute of Technology Madras, for the award of the degree of **Master of Technology**, is a bona fide record of the research work done by him under my supervision. The contents of this project report, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Place: Chennai

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Jaskirat Singh
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ABSTRACT

KEYWORDS: ARIMA, Balancing Market, Battery Sizing. Electricity Market , Forecasting Methods, Frequency Control, Gate Closure Lithuania, Wind Power.

The stochastic nature of renewable energy resources ,such as wind speed or solar radiation, presents a challenge for the grid integration of renewable power generation.Wind power is increasingly integrated into power systems through electricity markets. To make the market operation economically possible for wind power producers, the period between bids and delivery (gate closure time) is crucial. Thus, moving to shorter gate closure markets results in reduction of forecast errors and lowering of penalties to be paid by wind power producers .Further, the forecasting is done using the time series forecasting methods. These methods include Auto Regressive (AR), Moving Average (MA), Auto Regressive Integrated Moving Average (ARIMA), adoption of these methods lead to a further decrease in the forecasted errors. The imbalances between renewable power predictions and realised production are generally penalised by system operators since additional reserves are required to maintain the stability of the grid. The coupling of storage devices with renewable energy plants is one of the solutions studied to reduce those imbalances. In the present work , ARIMA based wind power forecasting method is used along with short gate closure to reduce forecasting error. Further, to reduce the penalty imposed by Independent System Operator (ISO) on wind power producers , due to imbalance caused by forecasting errors , Battery Energy Storage Systems (BESS) with proper rating is suggested.

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ABBREVIATIONS

MW	Megawatt
EU	European union
TWH	Terawatt Hours
CO2	Carbon Dioxide
RES	Renewable energy sources
PPA	Power purchase agreements
BESS	Battery Energy Storage System
NATO	North Atlantic Treaty Organisation
OECD	Organisation for Economic Co-operation and Development
NEIS	National Energy Integration Survey
GDP	Gross domestic product
ARIMA	Auto Regressive Integrated Moving Average
WPP	Wind Power Producers
ACF	Autocorrelation function
PACF	Partial Autocorrelation Function

NOTATION

Y_T	Time Series Value At Hour T
$\hat{Y}_{T+k}$	Time Series Value At Hour T + k
φ	Error Weights
$APEN$	Absorbed Penalty
x	Balancing Price
E	Prediction Error
ROI	Return on Investment
Rb	Rating of Battery
Cb	Cost of Battery
Xt	Time series value at time T
$L(x)$	Linearise function
$\mathrm{Cov}(x)$	Covariance Function
E(Yt)	Mean function
$\gamma_y(k)$	Covariance of k
μ	Mean
B	Back Step Operator
ε	White Noise
Θ	Error Weights
ϱy	Autocorrelation

CHAPTER 1

INTRODUCTION

1.1 Wind Energy

In recent years, rapid economic growth has been owed to the power production increment in different ways. Energy extracted from fossil fuels has many opponents because it leads to air pollution, ozone depletion and global warming. According to the Paris agreement, for the aim of limiting the global temperature rise under 2 ◦C, renewable energies have to supply two-thirds of the global energy demand up to 2050 [1]. Among all kinds of renewable energies like solar photovoltaic, tidal, waves and modern bioenergy, wind power has become extremely popular because it is highly efficient, cheap and beneficial for the environment [2]. Additionally, due to its abundance, wind energy plays a leading role in electricity production of the renewable energy sector [3]. It has the greatest demand and growth among all the renewable energy sources over the last decade [4]. At the EU level, Lithuania joined a coalition of five EU countries (Austria, Denmark, Ireland, Luxembourg and Spain) calling for the inclusion of a 100% renewable energy (electricity) scenario in the EU's long-term climate projections. Under the 2018 NEIS, about 3.8 terawatt hours (TWh) of annual onshore wind generation is envisaged by 2030, which requires a total capacity of 1300 megawatts (MW) (an additional 400 MW by 2025 and another 370 MW by 2030). In 2050, Lithuania will need around 18 terawatt hours (TWh) of annual electricity generation, more than half of which – about 10 TWh – is expected to be produced from wind both onshore and offshore [5**].**

1.2 Introduction to Wind Power Forecasting

The largest obstacle that suppresses the increase of wind power penetration within the power grid is uncertainty of wind speed. Therefore, accurate wind power forecasting is a challenging task, which can significantly impact the effective operation of power system. Wind power forecasting is also vital for planning unit commitment, maintenance scheduling and profit maximisation of power traders. The current development of cost-effective operation and maintenance methods for modern wind turbines benefits from the advancement of effective and accurate wind power forecasting approaches.

Also,Wind power prediction is extremely significant for evaluating future energy extraction from one or more wind turbines (referred to as a wind farm). However, the power generated by wind turbines varies rapidly due to the fluctuation of wind speed and wind direction. It is also dependent on terrain, humidity and time of the day [6]. This continuous change makes wind power management challenging for distribution networks, where a balance is highly desired between the power supply and demand. Therefore, one of the major reasons for wind power forecasting is to decrease the risk of uncertainties in wind, allowing higher penetration. It is also vital for better dispatch, maintenance planning, determination of required operating equipment, etc. Few published investigations, which have carried out wind power forecasting studies in recent years have been large enough to provide reliable estimates or guide for comparing different predictive methods.

1.3 Wind Prediction Error and Electricity Market

The initiation of wind power into power systems contributes to the reduction of CO2 emissions and the security of supply through the use of endogenous resources. However, the increased share of electricity produced by fluctuating and not completely predictable renewable generation is altering the traditional operations of power systems. Its effects can be seen in an increased unpredictability of electricity prices patterns, in an increase to the use of intraday adjustments and on the increased request for reserves. Furthermore, the reduced utilisation factor of renewable generators reduce the profitability of the farms, because of additional network reinforcements. However, the limited predictability of wind power production represents a constraint for large-scale wind power integration [7].

The presence of negative prices stems not only from low operating cost of RES, but also due to government subsidies, auctions, and other fixed price agreements such as power purchase agreements (PPA), which guarantees RES producers to make a profit despite the negative price biddings. On top of that, the significant cost associated with starting and shutting power plants also forces some plants to stay online during RES peak despite making a loss.

Economically, renewable energy sources (RES) with intermittent supply (such as solar and wind) differ greatly from traditional sources of energy (such as coal, oil, and gas) in the fundamental fact that they do not require raw material for electricity generation. Hence, the cost of thes RES are mostly fixed and predictable - during the initial setup, and the subsequent maintenance. This advantage of low operating costs grants them the ability to drive down electricity prices in the presence of strong supply .Most electricity markets of the world have a large intermittent RES mix. Gas power plants, which are cheaper to turn on and off compared to coal plants, are typically put on standby to make up for the shortage in energy production from these intermittent RES. Hence, during the time of low sun / wind, electricity prices will surge up to that of gas prices; while prices can even be driven down to negative during the periods of high renewable power output

In a competitive electricity market such as the Nordic and EU, accurate forecasting of wind speed can bring in great value as market prices are determined based on the cost of energy imbalances Besides that, it is also a critical aspect in developing a robust and well-functioning hour-ahead and day-ahead markets .

1.4 Gate Closure

In the day-head markets the day-ahead forecast horizon is too long for wind power forecasts and results in large forecast errors and imbalances that have to be settled through imbalance costs, thus complicating and artificially increasing the overall cost of energy. The impact of using more accurate forecasts ,due to short gate closure, from 2-hours-ahead, was quite large causing less forecast errors [8]. Thus, moving to shorter gate closure markets results in reduction of forecast errors and lowering of penalties to be paid by wind power producers(WPP).

1.5 Battery Energy Storage Integration

The stochastic nature of renewable energies resources such as wind speed or solar radiation and long gate closure time represents a challenge for the grid integration of renewable energy plants. The imbalances between renewable power predictions and realised production are generally penalised by system operators since additional reserves are required to maintain the stability of the grid. The coupling of storage devices with renewable energy plants is one of the solutions studied to reduce those imbalances.

A solution for part of these issues is the development of network storage capacity both associated and not associated with renewable producers. In particular, Storage associated with renewable power plants represents the most straightforward solution to mitigate renewables variability. As a flexible and adjustable power supply, the energy storage system provides a new idea to cope with the intermittent power integration . In various types of large-scale energy storage systems (such as pumped storage, compressed air storage, etc.), battery energy storage system (BESS) has the most promising broad in power applications benefiting from its high energy efficiency and weak requirement of geographical conditions [9]. Wind farm with BESS configuration will become a common model for large-scale wind power development in the future.

However, in addition to the high investment cost of BESS, how to properly size BESS size to balance the investment cost and the effect of levelling wind power fluctuation and uncertainty has been a research hotspot in recent years. Storage can be used to absorb excess electricity in hours of high renewable production and low demand and to re-inject it into the network later. The operating performance of such storage can be highly improved by considering simple charge-discharge plans based on short-term predictions of the renewable production. These predictions, generated by physical or statistical models using weather predictions and measurements as input are nowadays widely used by system operators in several countries. This increased performance can be translated directly into a reduced size of the storage.

1.6 Motivations & Objectives

Wind Power is a very popular but uncertain and less predictable renewable resource. The electricity markets do not have provision for bidding at short gate closure. This results in forecast error and higher imbalance costs [8].The implementation of better wind power forecasting techniques result in substantial reduction of forecast error [4]. There is a need for simultaneous adoption of better forecasting techniques, short gate closure and other relevant means to remove these shortcomings.

Wind power integration along with various sources such as pumped storage, flywheels , ultra-capacitors have been studied in recent past. These configurations were studied in order to provides a suitable means of reducing the uncertainty associated with wind power. Amongst, these configurations wind power along with BESS is most widely adopted. BESS can be used to absorb excess electricity in hours of high renewable production and low demand. Then, it can be supplied to the power system ,in times of low power generation and high demand.

The objective of this thesis is to study the impact of short gate closure, better forecasting techniques and BESS on reduction of forecast error and imbalance cost.

1.7 Organisation of Thesis

Chapter 2 gives an overview of the disadvantage, in terms of penalty , WPP face due to the large gate closure time. It states that the application of naïve forecast with short gate closure provides an incremental increase in the forecast accuracy. ARIMA based forecasting of wind power is explained. A method for choosing BESS size to reduce penalty in balancing markets is presented.

Chapter 3 provides an overview of the Lithuanian electricity sector . It provides an insight into the history of power generation , the energy generation mix , electricity reforms market carried out in the country. ARIMA based wind forecasting is implemented on Lithuania system and compared with naïve forecast. Sizing of BESS to reduce penalty and increase Return on Investment (ROI) is estimated.

Chapter 4 major conclusions, along with future scope are presented.

CHAPTER 2

ARIMA BASED WIND FORECASTING WITH SHORT GATE CLOSURE ALONG WITH BESS FOR REDUCING PENALTY ON WPP

2.1 Naive Wind Power Forecasting with Short Gate Closure

In a wholesale electricity market, gate closure refers to the time at which market participants must submit their final bids and offers for electricity. Following gate closure, no further trades may take place unless certain circumstances apply. Gate closure is typically presented as a minor feature of market design. Yet, its implementation could be pivotal to how power markets will operate under increasing penetrations of renewable energy in the generation mix.

At some point before real time, contracts (i.e., dispatch schedules) must be finalized for a predetermined upcoming delivery or settlement period. Gate closure is the point at which the finalization occurs. After gate closure, forward-looking data, such as physical information and contract volumes for the predetermined delivery period, are frozen. The system operator takes over the responsibility for balancing supply and demand through available reserves or ancillary services, thereby ensuring reliability, security, and the economic optimization of power system operations

In [8], the effect of short gate closure on wind power forecasting error in hour-ahead markets was studied. Naive forecast method was used for wind power forecasting, Battery energy storage systems for mitigating the penalty in hour-ahead markets due to forecast errors was not considered. In this chapter ARIMA based wind power forecasting is used with short gate closure. BESS is also used to reduce the penalty on WPP in hour-ahead markets.

2.1.1 Gate Closure

Gate closure time for bidding to day-ahead market is at noon of day D (Central European Time, CET) for all the 24 hours of the next day (D+1). This means a minimum of 12-36 hours forecast horizon. Taking into account that the latest meteorological forecasts arrive in the morning, the forecasting horizon is often much more. For instance, in the case of a Portuguese wind producer participating in the day-ahead market, the time horizon between the latest meteorological forecasts at 6 am to the first delivery hour is 18 hours. Longer forecast horizon results in larger forecast errors for the wind power producers and requires trading more energy in balance settlement.

The imbalances due to forecast errors need to be settled at either up or down balancing prices. In practice, the producer pays extra (in case of upward balancing is needed), or receives less (in case of downward balancing is needed) compared to spot price. The imbalance cost is therefore the amount that the producer needs to pay extra compared to getting spot price for all the generation. Significant reduction of imbalance is possible if short gate closure is sued. If closer to real time forecasts could be used in market bids, the imbalance costs could be reduced up to 50 % in Nordic countries (0.4-0.7 €/MWh),. In Portugal the imbalance costs were reduced by 1.7 €/MWh (by 30 %).

Wind power is increasingly integrated into power systems through electricity markets. In case where wind power producers participate in electricity markets then they are obliged to bid their generation to day-ahead (DA) markets. Intra-day markets can be used, if necessary, to correct for forecast errors that would cause large imbalance costs in some situations [10][11].

Wind power participating to system services, the so called Ancillary Services is studied with first pilots emerging in [12]. Balancing markets, one form of a manually operated frequency control, is the first option for wind power participation that is already happening to some extent in Denmark. In some areas, the participation of wind producers to balancing markets is not even possible (like Portugal) or is made technically and economically difficult by requiring that bidding be made more than a day in advance (Germany).

In [8], the imbalance costs are analyzed for a wind power producer acting in markets. The day-ahead forecast horizon is too long for wind power forecasts and results in large forecast errors and imbalances that have to be settled through imbalance costs, thus complicating and artificially increasing the overall cost of energy. Two very different market settings were used as case studies in [8]: Portugal in MIBEL and Finland and Denmark in Nordpool. The imbalance costs in Nordic countries were very low in the year 2014 case study, resulting in 1-1.5 €/MWh extra cost for the wind power produced. In Portugal this extra cost was more than 5 €/MWh. The impact of using more accurate forecasts, from 2-hours-ahead, was quite large, reducing the imbalance costs by 30-50 %. Moving to shorter gate closure markets would imply some costs both in transition and in work to trade in a more continuous basis.

2.2 Disadvantages of Naïve Forecast

NAIVE forecast, is the benchmarking model, is one of the simplest forecasting methods available [13]. Not only is it fast and easy to implement, the method also requires no assumption on the characteristics of the time series. Therefore, for the purpose of this study, the NAIVE model will also serve as the baseline performance by which other more computationally intensive forecasting methods are compared against. Having a benchmark model allows a clear indication on the gain/loss in accuracy performance with respect to the additional computational complexity from all other models. Besides that, it can also be used to provide a quantitative idea of how difficult the forecast problem at hand - forecasting of wind speed is. The NAIVE has the straightforward assumption that the next value is the same as the current value, based on the persistence algorithm in (2.1):

$$\hat{Y}_{T+k} \mid T = Y_T \qquad (2.1)$$

In [2.1] , the wind power at time T is given by YT. In naïve forecast, the forecasted wind power after “k” hours YT+k from T, is taken same as that of time T. In [8], a short gate closure is proposed where the WPPs can bid 2 hours before the actual supply .Therefore, with naïve forecast it is assumed that the wind power data available 2 hours before the actual supply will remain constant till the time of supply.The accuracy of this method can quickly deteriorate with the increment of prediction timescale [14].

However, wind speed at the ground level and other related atmospheric phenomena are clearly non stationary because of diurnal, seasonal, and inter-annual cycles. For wind applications, persistence models perform poorly for time horizons involving appreciable variations in the diurnal cycle, which limits their use .Hence ,advanced techniques like ARIMA are beneficial to be used .

2.3 ARIMA

2.3.1 Introduction

Forecasting methods based on naïve forecats are simplistic, inefficient and sometimes inappropriate because they do not take advantage of the serial dependence in the observations in the most effective way. To incorporate this dependent structure, a general class of models called Auto-Regressive Integrated Moving Average models or ARIMA models (also known as Box-Jenkins models) are widely used.

2.3.2 Linear Models for Stationary Time Series

The aim in forecasting is to find relationship between certain inputs and the output. These efforts usually result in models that are approximations of the relationship. A major assumption that often provides relief in modelling efforts is the linearity assumption. A linear filter, for example, is a linear operation from one time series~ Xt to another time series Yt.

$$y_I = L(x_t) = \sum_{i=-\infty}^{+\infty} \psi_i x_{l-i} \qquad (2.2)$$

with t = ... , -I, 0, 1, ... In that regard the linear filter is a "process·· that converts the input, Xt, into an output, Yt. and that conversion is not instantaneous but involves all (present, past, and future) values of the input in the form of a summation with different "weights", { ψ_i}, on each Xt. Furthermore, the linear filter in (2.2) is said to have the following properties:

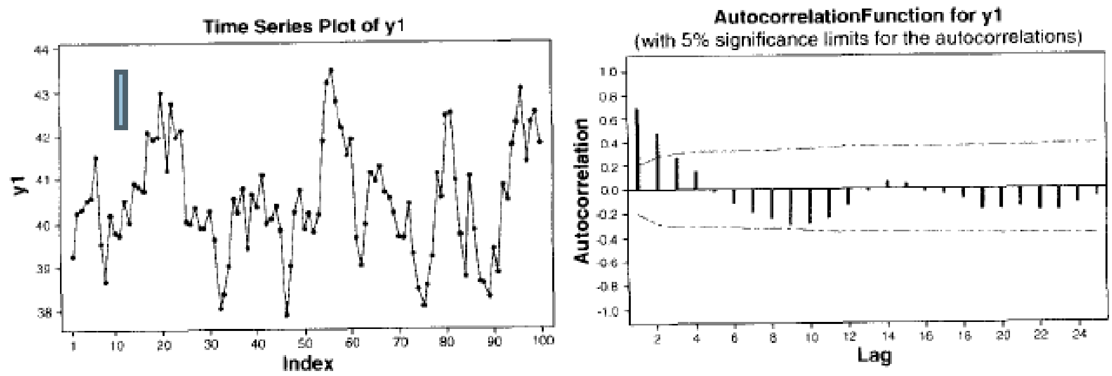
- Time Invariant
- Physically realisable
- Stable

In linear filters, under certain conditions. some properties such as stationarity of the input time series are also reflected in the output.

2.3.3 **Stationarity**

The stationarity of a time series is related to its statistical properties in time. That is., a stationary time series exhibits similar "statistical behavior" in time and this is often characterized as a constant probability distribution in time. However, it is usually satisfactory to consider the first two moments of the time series and define stationarity (or weak stationarity) as follows: (I) the expected value of the time series does not depend on time and (2) the autocovariance function defined as $\mathrm{Cov}(y_t, y_{t+k})$for any lag k is only a function of k and not time: that is. y (k) = Cov (Yt, Yt+k).

The stationarity of a time series can be determined by taking arbitrary snapshots of the process at different points in time and observing the general behaviour of the time series. If it exhibits similar behavior, one can then proceed with the modeling efforts under the assumption of stationarity. Further preliminary tests also involve observing the behavior of the autocorrelation function. A strong and slowly dying ACF will also suggest deviations from stationarity.Also, ther a number of other tests which can detect absence of stationarity. Figure 5.1 shows examples of stationary and nonstationary time series data.



(a) $y_{1,t} = 10 + 0.75y_{1,t-1} + \varepsilon_t$

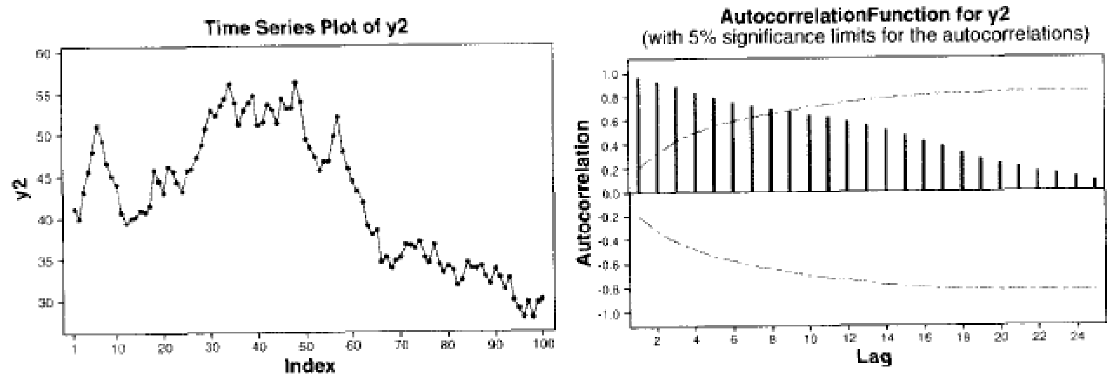


Figure 2.1 Realizations of (a) stationary, (b) non-stationary [31]

For a time-invariant and stable linear filter and a stationary input time series x 1 with $\mu_x = E(x_t)$ and $\gamma_x(k) = \mathrm{Cov}(x_i, x_{t+k})$, the output time series yt given in (2.2) is also a stationary time series with mean as in (2.3) and covariance as in (2.4)

$$y_I = L(x_t) = \sum_{i=-\infty}^{+\infty} \psi_i x_{l-i}$$

$$E(y_t) = \mu_y = \sum_{-\infty}^{\infty} \psi_i \mu_x \tag{2.3}$$

and

$$\begin{aligned} \text{Cov}(y_t, y_{t+k}) &= \gamma_y(k) \\ &= \sum_{i=-\infty}^{\infty} \sum_{j=-x}^{x} \psi_i \psi_j \gamma_x(i-j+k) \end{aligned} \tag{2.4}$$

It is then easy to show that the stable linear processs in (2.5) with white noise time series,$\in_t$ is also stationary:

$$y_t = \mu + \sum_{i=0}^{\infty} \psi_i \varepsilon_{t-i} \tag{2.5}$$

where $\in_t$ represents the independent random shocks with E ($\in_t$) = 0, and

$$\gamma_\varepsilon(h) = \begin{cases} \sigma^2 & \text{if } h = 0 \\ 0 & \text{if } h \neq 0 \end{cases} \tag{2.6}$$

So for the auto co-variance function of $y_\in$ for in (2.6), we have in 2.7

$$\begin{aligned} \gamma_y(k) &= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \psi_i \psi_j \gamma_\varepsilon(i-j+k) \\ &= \sigma^2 \sum_{i=0}^{\infty} \psi_i \psi_{i+k} \end{aligned} \tag{2.7}$$

The linear process is written in terms of the backshift operator, B, as in eqn 2.8.

$$\begin{aligned} y_t &= \mu + \psi_0\varepsilon_t + \psi_1\varepsilon_{t-1} + \psi_2\varepsilon_{t-2} + \cdots \\ &= \mu + \sum_{i=0}^{\infty} \psi_i B^i \varepsilon_t \\ &= \mu + \underbrace{\left(\sum_{i=0}^{\infty} \psi_i B^i\right)}_{=\Psi(B)} \varepsilon_t \\ &= \mu + \Psi(B)\varepsilon_t \end{aligned} \tag{2.8}$$

The (2.8) is also called the infinite moving average and serves as a general class of models for any stationary time series. The expression shows stationary time series can be seen as the weighted sum of the present and past values. Comparing (2.7) & (2.8) it is seen that relation between the weights and the auto covariance function.

In modeling a stationary time series as in (2.8), it is obviously impractical to attempt to estimate the infinitely many weights . Although very powerful in providing a general representation of any stationary time series, the infinite moving average model is useless in practice except for certain special cases:

- Finite order moving average (MA) models where, except for a finite number of

the weights , they are set to 0.

2. Finite order autoregressive (AR) models, where the weights are generated using only a finite number of parameters.

3. A mixture of finite order autoregressive and moving average models (ARMA).

These classes of models are studied in detail in subsequent sections.

2.3.4 Finite Order Moving Average (MA) Processes

In finite order moving average or MA models, conventionally 1/Jo is set to I and the weights that are not set to 0 are represented by the Greek letter (Θ) with a minus sign $\varphi_0$in front.

Hence a moving average process of order q (MA(q)) is given as in (2.9)

$$y_i = \mu + \varepsilon_t - \theta_1 \varepsilon_{l-1} - \cdots - \theta_q \varepsilon_{t-q} \tag{2.9}$$

where $\in_t$is white noise. Since (2.9) is a special case of Eq. (2.8) with only finite weights, a MA(q) process is always stationary regardless of values of the weights. In terms of the backward shift operator, the MA(q) process is given in (2.10).

$$\begin{aligned} y_I &= \mu + \left(1 - \theta_1 B - \cdots - \theta_q B^q\right)\varepsilon_l \\ &= \mu + \left(1 - \sum_{i=1}^{q} \theta_i B^i\right)\varepsilon_t \\ &= \mu + \Theta(B)\varepsilon_t \end{aligned} \tag{2.10}$$

Where $\Theta(B) = 1 - \sum_{i=1}^{4} \theta_i B^i$

Furthermore, since $\in_t$ is white noise, the expected value of the MA(q) process is as given in 2.11.

$$\begin{aligned} E(y_t) &= E\left(\mu + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \cdots - \theta_q \varepsilon_{t-q}\right) \\ &= \mu \end{aligned} \tag{2.11}$$

And its variance is (2.12)

$$\begin{aligned}\mathrm{Var}(y_t) &= \gamma_y(0) = \mathrm{Var}\big(\mu + \varepsilon_t - \theta_1\varepsilon_{t-1} - \cdots - \theta_q\varepsilon_{t-q}\big) \\ &= \sigma^2\big(1 + \theta_1^2 + \cdots + \theta_q^2\big)\end{aligned} \quad (2.12)$$

Similarly, the auto covariance at lag k can be calculated from eqn 2.13

$$\begin{aligned}\gamma_y(k) &= \mathrm{Cov}(y_t, y_{t+k}) \\ &= E\big[\big(\varepsilon_t - \theta_1\varepsilon_{t-1} - \cdots - \theta_q\varepsilon_{t-q}\big)\big(\varepsilon_{t+k} - \theta_1\varepsilon_{t+k-1} - \cdots - \theta_q\varepsilon_{t+k-q}\big)\big] \\ &= \begin{cases}\sigma^2\big(-\theta_k + \theta_1\theta_{k+1} + \cdots + \theta_{q-k}\theta_q\big), k = 1,2,\dots,q \\ 0, k > q\end{cases}\end{aligned} \quad (2.13)$$

From (2.12) and (2.13), the auto covariance function of the MA(q) process is (2.14).

$$\begin{aligned}\rho_y(k) &= \frac{\gamma_y(k)}{\gamma_y(0)} \\ &= \begin{cases}\dfrac{-\theta_k + \theta_1\theta_{k+1} + \cdots + \theta_{q-k}\theta_q}{1 + \theta_1^2 + \cdots + \theta_q^2} . k = 1,2 \dots \dots \\ 0, k > q\end{cases}\end{aligned} \quad (2.14)$$

This feature of the ACF is very helpful in identifying the MA model and its appropriate order as it "cuts off' after lag q. In real life applications, however, the sample ACF, will not necessarily be equal to zero after lag q. It is expected to become very small in absolute value after lag q. For a data set of N observations, this is often tested against ±2/√N limits, where 1/√N is the approximate value for the standard deviation of the ACF for any lag.

A special case would be white noise data for which Pj = 0 for all j's. Hence for a white noise process (i.e., no autocorrelation), a reasonable interval for the sample autocorrelation coefficients to fall in would be ±2/√N and any indication otherwise may be considered as evidence for serial dependence in the process.

The First-Order Moving Average Process, MA(l)

The simplest finite order MA model is obtained when q = I in. (2.15):

$$y_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1} \tag{2.15}$$

For the first-order moving average or MA(l) model, the auto covariance function as in (2.16)

$$\begin{aligned} \gamma_y(0) &= \sigma^2(1+\theta_1^2) \\ \gamma_y(1) &= -\theta_1\sigma^2 \\ \gamma_y(k) &= 0, k > 1 \end{aligned} \tag{2.16}$$

Similarly, we have the autocorrelation function as in (2.17)

$$\begin{aligned} \rho_y(1) &= \frac{-\theta_1}{1+\theta_1^2} \\ \rho_y(k) &= 0, k > 1 \end{aligned} \tag{2.17}$$

From in. (2.18), we can see that the first lag autocorrelation in MA(l) is bounded as

$$|\rho_y(1)| = \frac{|\theta_1|}{1+\theta_1^2} \leq \frac{1}{2} \tag{2.18}$$

and the autocorrelation function cuts off after lag 1.Consider in (2.19) for MA(l) model:

$$y_t = 40 + \varepsilon_t + 0.8\varepsilon_{t-1} \tag{2.19}$$

A realization of this model with its sample ACF is given in Figure 2.2. A visual inspection reveals that the mean and variance remain stable while there are some short runs where successive observations tend to follow each other for very brief durations, suggesting that there is indeed some positive autocorrelation in the data as revealed in the sample ACF plot.

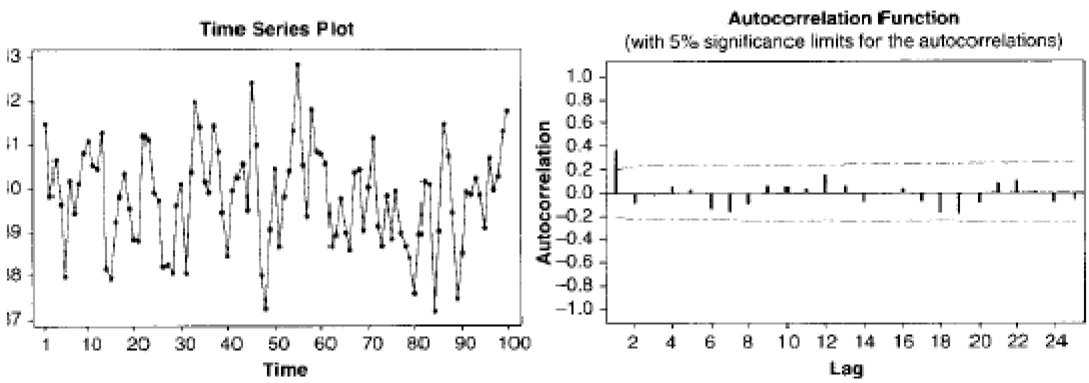


Figure 2.2 A realisation of MA(1) [31]

The Second-Order Moving Average Process, MA(2),

Another useful finite order moving average process is MA(2), given in (2.20)

$$\begin{aligned} y_t &= \mu + \varepsilon_t - \theta_1\varepsilon_{t-1} - \theta_2\varepsilon_{l-2} \\ &= \mu + (1 - \theta_1 B - \theta_2 B^2)\varepsilon_t \end{aligned} \tag{2.20}$$

The autocovariance and autocorrelation functions for the MA(2) model are given in (2.21) and (2.22).

$$\begin{aligned} \gamma_y(0) &= \sigma^2(1+\theta_1^2+\theta_2^2) \\ \gamma_y(1) &= \sigma^2(-\theta_1+\theta_1\theta_2) \\ \gamma_y(2) &= \sigma^2(-\theta_2) \\ \gamma_y(k) &= 0, k > 2 \end{aligned} \tag{2.21}$$

$$\begin{aligned} \rho_y(1) &= \frac{-\theta_1+\theta_1\theta_2}{1+\theta_1^2+\theta_2^2} \\ \rho_y(2) &= \frac{-\theta_2}{1+\theta_1^2+\theta_2^2} \\ \rho_y(k) &= 0, k > 2 \end{aligned} \tag{2.22}$$

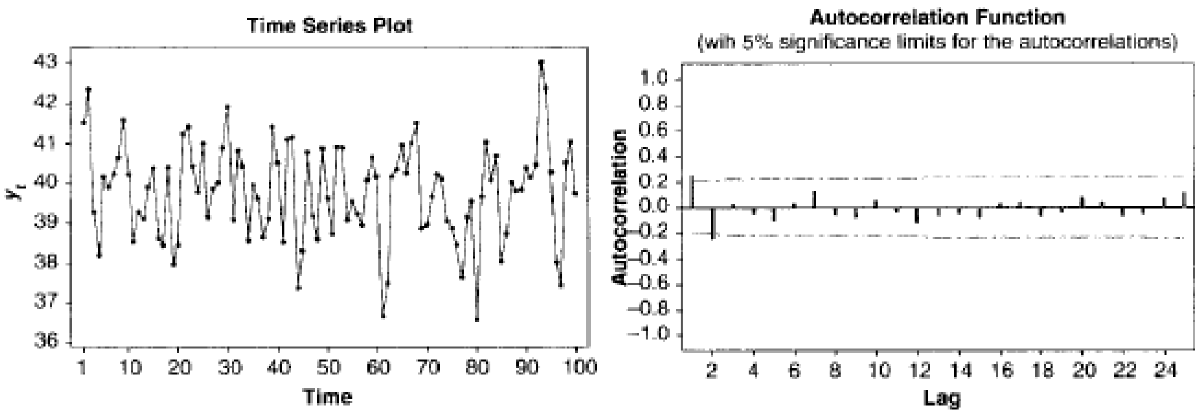


Figure 2.3 A realization of the MA(2) process [31]

MA(2) model, the sample ACF cuts off after lag 2.

$$y_t = 40 + \varepsilon_t + 0.7\varepsilon_{t-1} - 0.28\varepsilon_{t-2} \tag{2.23}$$

2.3.5 Finite Order Autoregressive Processes

Another interpretation of the finite order MA processes is that at any given time, of the infinitely many past disturbances, only a finite number of those disturbances "contribute" to the current value of the time series and that the time window of the contributors "moves" in time, making the oldest disturbance obsolete for the next observation. However, some processes might have these intrinsic dynamics. Also, for some others, it may be required to consider the lingering contributions of the disturbances that happened back in the past. This may result in estimating infinitely many weights. Another solution to this problem is through the autoregressive models in which the infinitely many weights are assumed to follow a distinct pattern and can be successfully represented with only a handful of parameters. It can be considered by some special cases of autoregressive processes.

First-Order Autoregressive Process, AR(l),

Considering time series from (2.8)

$$\begin{aligned} y_i &= \mu + \sum_{i=0}^{\infty} \psi_i \varepsilon_{i-i} \\ &= \mu + \sum_{i=0}^{\infty} \psi_i B^i \varepsilon_t \\ &= \mu + \Psi(B)\varepsilon_t \end{aligned} \tag{2.23}$$

As in the finite order MA processes, one approach to modeling this time series is to assume that the contributions of the disturbances that are way in the past should be small compared to the more recent disturbances that the process has experienced. Since the disturbances are independently and identically distributed random variables, we can simply assume a set of infinitely many weights in descending magnitudes reflecting the diminishing magnitudes of contributions of the disturbances in the past. A simple and yet intuitive set of such weights can be created following an exponential decay pattern. For that we will set $\psi_i = \phi^i$, where $|\phi| < 1$to guarantee the exponential "decay."

From (2.8), we then have

$$y_{t-1} = \mu + \varepsilon_{t-1} + \phi\varepsilon_{t-2} + \phi^2\varepsilon_{t-3} + \cdots \tag{2.24}$$

Also, from (2.24) we get (2.25)

$$\begin{aligned} y_t &= \mu + \varepsilon_t + \underbrace{\phi\varepsilon_{t-1} + \phi^2\varepsilon_{t-2} + \cdots}_{=\phi y_{t-1} - \phi\mu} \\ &= \underbrace{\mu - \phi\mu}_{=\delta} + \phi y_{t-1} + \varepsilon_t \\ &= \delta + \phi y_{i-1} + \varepsilon_t \end{aligned} \tag{2.25}$$

The process in. (2.25) is called a first-order autoregressive process, AR(l).

Hence an AR(1) process is stationary if φ< I. The mean of a stationary AR(l) process is given in (2.26).

$$E(y_t) = \mu = \frac{\delta}{1-\phi} \tag{2.26}$$

,The autocovariance function of a stationary AR(1) can be calculated as given in (2.27)

$$\gamma(k) = \sigma^2\phi^k \frac{1}{1-\phi^2} \text{ for } k = 0,1,2,\ldots \tag{2.27}$$

The variance can be given as in eqn 2.28

$$\gamma(0) = \sigma^2 \frac{1}{1-\phi^2} \tag{2.28}$$

Correspondingly, the autocorrelation function for a stationary AR(l) process is given as

$$\rho(k) = \frac{\gamma(k)}{\gamma(0)} = \phi^k \text{ for } k = 0,1,2,\ldots \qquad (2.29)$$

Hence the ACF for a stationary AR(l) process has an exponential decay form. A realization of the following AR(l) model of (2.30),

$$y_t = 8 + 0.8y_{t-1} + \varepsilon_t \qquad (2.30)$$

is shown in Figure 2.4. As in the MA(l) model we can observe some short runs during which observations tend to move in the upward or downward

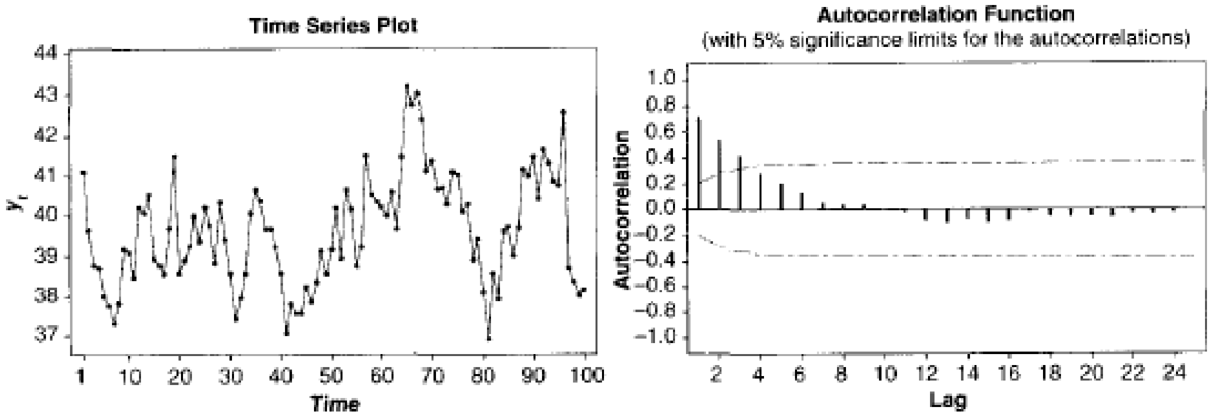


Figure 2.4 A realization of the AR(l) process [31]

Second-Order Autoregressive Process, AR(2),

For AR(2) , extension of eqn 2.25 can be writeen as [**eqn 2.31]**

$$y_t = \delta + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t \qquad (2.31)$$

The (2.31) can be represented in the infinite MA form and provide the conditions of stationarity for yt.

$$(1-\phi_1 B-\phi_2 B^2)y_i=\delta+\varepsilon_l$$

or

$$\Phi(B)y_t=\delta+\varepsilon_l$$

Furthermore, eqn 2.32 is obtained.

$$\begin{aligned} y_t &= \underbrace{\Phi(B)^{-1}\delta}_{=\mu}+\underbrace{\Phi(B)^{-1}}_{=\Psi(B)}\varepsilon_t \\ &= \mu+\Psi(B)\varepsilon_t \\ &= \mu+\sum_{i=0}^{\infty}\psi_i\varepsilon_{t-i} \\ &= \mu+\sum_{i=0}^{\infty}\psi_i B^i\varepsilon_{\mathrm{t}} \end{aligned} \tag{2.32}$$

(2.32) is used to get the weights in terms of ϕ_1 and ϕ_2

$$\Phi(B)\Psi(B)=1$$

$$(1-\phi_1 B-\phi_2 B^2)(\psi_0+\psi_1 B+\psi_2 B^2+\cdots)=1$$

$$\begin{aligned}\psi_0+(\psi_1-\phi_1\psi_0)B+(\psi_2-\phi_1\psi_1-\phi_2\psi_0)B^2 \\ +\cdots+\left(\psi_j-\phi_1\psi_{j-1}-\phi_2\psi_{j-2}\right)B^j+\cdots=1\end{aligned} \tag{2.33}$$

Right hand side of (2.33) does not have any backshift operators.

For $\Phi(B)\Psi(B)=1$, it should be eqn 2.34

$$\begin{aligned}\psi_0&=1 \\ (\psi_1-\phi_1\psi_0)&=0 \\ \left(\psi_j-\phi_1\psi_{j-1}-\phi_2\psi_{j-2}\right)&=0 \text{ for all } j=2,3,\ldots\end{aligned} \tag{2.34}$$

The equations in (2.34) can be solved for each $\psi_j$in an attempt to estimate infinitely many parameters. However, it should be noted that the in (2.34) satisfy the second-order linear difference equation and that they can be expressed as the solution to this equation in terms of the two roots m 1 and m2 of the associated polynomial.

$$m^2 - \phi_1 m - \phi_2 = 0 \qquad (2.35)$$

$$m_1, m_2 = \frac{\phi_1 \pm \sqrt{\phi_1^2 + 4\phi_2}}{2}$$

The eqn 2.35 the roots m1 and m2 are both less than I in absolute value, then the AR(2) model is stationary. Note that if the roots are complex conjugates of the form a ± i b, the condition for stationarity is that $\sqrt{a^2 + b^2} < 1$,the mean is given by (2.36)

.

$$\begin{aligned} E(y_t) &= \delta + \phi_1 E(y_{t-1}) + \phi_2 E(y_{t-2}) + 0 \\ \mu &= \delta + \phi_1 \mu + \phi_2 \mu \\ \Rightarrow \mu &= \frac{\delta}{1 - \phi_1 - \phi_2} \end{aligned} \qquad (2.36)$$

.

The auto co-variance of (2.31) is given by (2.37)

$$\begin{aligned} \gamma(k) &= \text{Cov}(y_i, y_{t-k}) \\ &= \text{Cov}(\delta + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t, y_{t-k}) \\ &= \phi_1 \text{Cov}(y_{t-1}, y_{t-k}) + \phi_2 \text{Cov}(y_{t-2}, y_{i-k}) + \text{Cov}(\varepsilon_t \cdot y_{i-k}) \\ &= \phi_1 \gamma(k-1) + \phi_2 \gamma(k-2) + \begin{cases} \sigma^2 & \text{if } k = 0 \\ 0 & \text{if } k > 0 \end{cases} \end{aligned} \qquad (2.37)$$

Thus $\gamma(0) = \phi_1\gamma(1) + \phi_2\gamma(2) + \sigma^2$ and

$$\gamma(k) = \phi_1\gamma(k-1) + \phi_2\gamma(k-2), k = 1,2,\ldots \qquad (2.38)$$

The (2.38) is called the **Yule-Walker** equations .Simillarly , the autocorrelation function is given by (2.39)

$$\rho(k) = \phi_1\rho(k-1) + \phi_2\rho(k-2). k = 1,/\ldots \qquad (2.39)$$

The Yule-Walker can be solved recursively as

$$\begin{aligned}\rho(1) &= \phi_1 \underbrace{\rho(0)}_{=1} + \phi_2 \underbrace{\rho(-1)}_{=\rho(1)} \\ &= \frac{\phi_1}{1-\phi_2} \\ \rho(2) &= \phi_1\rho(1) + \phi_2 \\ \rho(3) &= \phi_1\rho(2) + \phi_2\rho(1)\end{aligned}$$

A general solution can be obtained through the roots m 1 and m 2 of the associated polynomial $m^2 - \phi_1 m - \phi_2 = 0$. .There are three cases.

Case 1. If m 1 and m 2 are distinct, real roots, we then have eqn 2.35

$$\rho(k) = c_1 m_1^k + c_2 m_2^k, k = 0,1,2,\ldots \qquad (2.40)$$

where c1 and c2 are particular constants and can, for example, be obtained from p(O) and p(l). Moreover, since for stationarity, the autocorrelation function is a mixture of two exponential decay terms.

Case 2. If m 1 and m 2 are complex conjugates in the form of a ± i b, we then have (2.41)

$$\rho(k) = R^k[c_1\cos(\lambda k) + c_2\sin(\lambda k)], k = 0,1,2, \ldots \tag{2.41}$$

where $R = |m_i| = \sqrt{a^2 + b^2}$and A is determined by $\cos(\lambda) = a/R, \sin(\lambda) = b/R$ R. .Once again c 1 and c2 are particular constants. The ACF in this case has the form of a damped sinusoid, with damping factor R and frequency A; that is, the period is $2\pi/\lambda$.

Case 3. If there is one real root m0 , m 1 = m 2 = m 0 , we then have (2.42)

$$\rho(k) = (c_1 + c_2k)m_0^k k = 0,1,2, \ldots \tag{2.42}$$

In this case, the ACF will exhibit an exponential decay pattern.

In case I, for example, an AR(2) model can be seen as an "adjusted" AR(I) model for which a single exponential decay expression as in the AR(1) model is not enough to describe the pattern in the ACF, and hence an additional exponential decay expression is "added" by introducing the second lag term y_{t-2}.

General Autoregressive Process, AR(p),

A general, pth-order AR model is given as (2.43)

$$y_t = \delta + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t \tag{2.43}$$

Another representation of (2.43) , can be (2.44).

$$\Phi(B)y_t = \delta + \varepsilon_t \qquad (2.44)$$
$$\text{where } \Phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p$$

The AR(p) time series {y1) in Eq. (2.43) is stationary if the roots of the associated polynomial

$$m^p - \phi_1 m^{p-1} - \phi_2 m^{p-2} - \cdots - \phi_p = 0$$

are less than one in absolute value. Furthermore, under this condition, the AR(p) time series {y1) is also said to have an absolutely summable infinite MA representation

$$y_t = \mu + \Psi(B)\varepsilon_t = \mu + \sum_{i=0}^{\infty} \psi_i \varepsilon_{t-i} \qquad (2.45)$$

Whrere $\Psi(B) = \Phi(B)^{-1}$ with $\sum_{i=0}^{\infty} |\psi_i| < \infty$,

As in AR(2), the weights of the random shocks in (2.45) is given by (2.46)

$$\psi_j = 0, j < 0 \qquad (2.46)$$
$$\psi_0 = 1$$
$$\psi_j - \phi_1\psi_{j-1} - \phi_2\psi_{j-2} - \cdots - \phi_p\psi_{j-p} = 0 \text{ for all } j = 1,2, \ldots$$

It can be easily show that, for stationary AR(p),

$$\begin{aligned}\gamma(k) &= \text{Cov}(y_t, y_{t-k}) \\ &= \text{Cov}\big(\delta + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t, y_{t-k}\big) \\ &= \sum_{i=1}^{p} \phi_i \text{Cov}(y_{t-i}, y_{t-k}) + \text{Cov}(\varepsilon_t, y_{t-k}) \\ &= \sum_{i=1}^{p} \phi_i \gamma(k-i) + \begin{cases} \sigma^2 & \text{if } k = 0 \\ 0 & \text{if } k > 0 \end{cases}\end{aligned} \tag{2.47}$$

$$\gamma(0) = \sum_{i=1}^{p} \phi_i \gamma(i) + \sigma^2$$
$$\Rightarrow \gamma(0)\left[1 - \sum_{i=1}^{p} \phi_i \rho(i)\right] = \sigma^2$$

By dividing Eq. (2.47) by y (0) for k > 0, it can be observed that the ACF of an AR(p) process satisfies the Yule-Walker (2.48)

$$\rho(k) = \sum_{i=1}^{p} \phi_i \rho(k-i), k = 1.2 \ldots \tag{2.48}$$

The equations in (2.48) are pth-order linear difference equations implying that the ACF for an AR(p) model can be found through the p roots of the associated polynomial in Eq. (2.43). For example, if the roots are all distinct and real. we have (2.49)

$$\rho(k) = \sum_{i=1}^{p} \phi_i \rho(k-i), k = 1.2 \ldots \tag{2.49}$$

where c 1, c2 , ••. , c, are particular constants. However, in general, the roots may not all be distinct or real. Thus the ACF of an AR(p) process can be a mixture of exponential decay and damped sinusoid expressions depending on the roots of (2.49).

2.3.6 Mixed Autoregressive-Moving Average(ARMA) Processes

In an AR(1) process, the weights in the infinite sum are forced to follow an exponential decay form , But ,it may not be possible to approximate an exponential decay pattern.For that, we will need to increase the order of the AR model to approximate any pattern that these weights may in fact be exhibiting. On some occasions, however, it is possible to make simple adjustments to the exponential decay pattern by adding only a few terms and hence to have a more parsimonious model. Hence instead of increasing the order of the AR model to accommodate for this anomaly, we can add an MA(I) term that will simply adjust weight 1 while having no effect on the rate of exponential decay pattern of the rest of the weights. This results in a mixed autoregressive moving average or ARMA(l,l) model. In general, an ARMA(p, q) model is given as (2.50)

$$\begin{aligned} y_i &= \delta + \phi_1 y_{t-1} + \phi_2 y_{i-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-4} \qquad (2.50) \\ &= \delta + \sum_{i=1}^{p} \phi_i y_{t-i} + \varepsilon_t - \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \end{aligned}$$

Or

$$\Phi(B) y_t = \delta + \Theta(B) \varepsilon_t \qquad (2.51)$$

Stationarity of ARMA (p, q) Process

The stationarity of an ARMA process is related to the AR component in the model and can be checked through the roots of the associated polynomial given in (2.52).

$$m^p - \phi_1 m^{p-1} - \phi_2 m^{p-2} - \cdots - \phi_p = 0 \tag{2.52}$$

If all the roots of (2.52) are less than one in absolute value, then ARMA(p, q) is stationary. This also implies that, under this condition, ARMA(p, q) has an infinite MA representation as (2.53)

$$y_t = \mu + \sum_{i=0}^{\infty} \psi_i \varepsilon_{t-i} = \mu + \Psi(B)\varepsilon_t \tag{2.53}$$

with $\Psi(B) = \Phi(B)^{-1}\Theta(B)$. The coefficients in $\Psi(B)$ can be found from (2.54)

$$\psi_i - \phi_1 \psi_{i-1} - \phi_2 \psi_{i-2} - \cdots - \phi_p \psi_{i-p} = \begin{cases} -\theta_i, & i = 1 \ldots q \\ 0. & i > q \end{cases} \tag{2.54}$$

lnvertibility of ARMA (p, q) Process,

Similar to the stationarity condition. the invertibility of an ARMA process is related to the MA component and can be checked through the roots of the associated polynomial given in eqn 2.55

$$m^q - \theta_1 m^{4-1} - \theta_2 m^{q-2} - \cdots - \theta_q = 0 \tag{2.55}$$

If all the roots of (2.55) are less than one in absolute value, then ARMA(p, q) is said to be invertible and has an infinite AR representation,invertibility in (2.57).

$$\Pi(B)y_i = \alpha + \varepsilon_l$$

where $\alpha = \Theta(B)^{-1}\delta$ and $\Pi(B) = \Theta(B)^{-1}\Phi(B)$, The coefficients in $\Pi(B)$ can be found from (2.57)

$$\pi_i - \theta_1\pi_{i-1} - \theta_2\pi_{i-2} - \cdots - \theta_q\pi_{i-4} = \begin{cases} \phi_i, & i = 1 \ldots p \\ 0, & i > p \end{cases} \quad (2.57)$$

ACF and PACF of ARMA(p,q) Process,

As in the stationarity and invertibility conditions, the ACF and PACF of an ARMA process are determined by the AR and MA components. respectively. It is seen that the ACF and PACF of an ARMA(p, q) both exhibit exponential decay and/or damped sinusoid patterns, which makes the identification of the order of the ARMA(p, q) model relatively more difficult. The theoretical values of the ACF and PACF for stationary time series are summarized in Table 2.1

Table 2.1 Behavior of Theoretical ACF and PACF for Stationary Processes

Model		ACF	PACF
MA(q)		Cuts off after lag q	Exponential decay and/or damped sinusoid
AR (p)	Exponential decay and/or damped sinusoid	Cuts off after lag p	
ARMA(p, q)		Exponential decay and/or damped sinusoid	Exponential decay and/or damped sinusoid

2.3.7 Nonstationary Processes

A time series is homogeneous. nonstationary if it is not stationary but its first difference, or or higher-order differences. $w_l = (1 - B)^d y_l$.produce a stationary time series. Further call y_{f} an autoregressive integrated moving average (ARIMA) process of orders p,d,and q-that is, ARIMA(p, d, q)-fit s dth difference, denoted by $w_t = (1 - B)^d y_t$• produces a stationary ARMA(p. ql process. The term integrated is used since, for d = I, for example, we can write Yt as the sum (or "integral") of the w1 process as (2.58)

$$\begin{aligned} y_f &= w_i + y_{i-1} \\ &= w_t + w_{t-1} + y_{t-2} \\ &= w_t + w_{t-1} + \cdots + w_1 + y_0 \end{aligned} \tag{2.58}$$

Eqn 2.53

Hence an ARIMA(p, d, q) can be written as (2.59)

$$\Phi(B)(1 - B)^d y_f = \delta + \Theta(B)\varepsilon_r \tag{2.59}$$

Thus once the differencing is performed and a stationary time series is obtained, the methods provided by ARMA can be used to obtain the full model. In most applications first differencing (d = I) and occasionally second differencing (d = 2) is enough to achieve stationarity

Some Examples of ARIMA(p, d, q) Processes,

ARIMA(O, 1, 0) is the simplest nonstationary modeL It is given by (2.60)

$$(1 - B)y_i = \delta + \varepsilon_f \tag{2.60}$$

suggesting that first differencing eliminates all serial dependence and yields a white noise process.

The ARIMA(O, 1, 1) process is given by (2.61)

$$(1-B)y_t = \delta + (1-\theta B)\varepsilon_t \tag{2.61}$$

The infinite AR representation of Eq. (2.61),is derived and is given by (2.62)

$$\pi_i - \theta\pi_{i-1} = \begin{cases} 1, & i = 1 \\ 0, & i > 1 \end{cases} \tag{2.62}$$

vith $\pi_0 = -1$. Thus we have (2.63)

$$\begin{aligned} y_t &= \alpha + \sum_{i=1}^{\infty} \pi_i y_{t-i} + \varepsilon_t \\ &= \alpha + (1-\theta)(y_{t-1} + \theta y_{t-2} + \cdots) + \varepsilon_t \end{aligned} \tag{2.63}$$

This suggests that an ARIMA(O, I, I) can be written as an exponentially weighted moving average (EWMA) of all past values.Thus , it is clear that differencing leads to a stationary time series .In most of the applications the d=2 second differencing is sufficient for making the time series stationary.

2.3.8 Time Series Model Building

A three-step iterative procedure is used to build an ARIMA model. First, a tentative model of the ARIMA class is identified through analysis of historical data. Second, the unknown parameters of the model are estimated. Third, through residual analysis, diagnostic checks are performed to determine the adequacy of the model, or to indicate potential improvements.

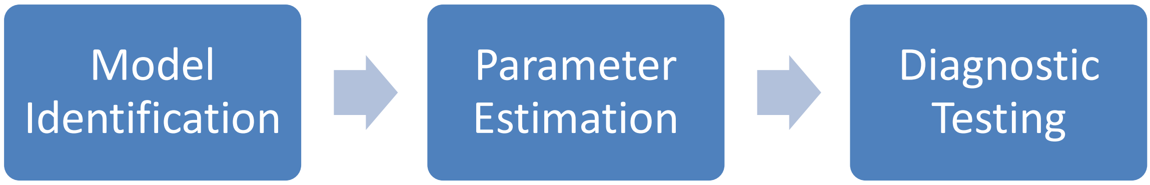


Figure 2.5 Time Series Model Building

- **Model Identification**, Simple time series plots should be used as the preliminary assessment tool for stationarity. The visual inspection of these plots should later be confirmed as described, If nonstationarity is suspected, the time series plot of the first (or dth) difference should also be considered. The unit root test by Dickey and Fuller can also be performed to make sure that the differencing is indeed needed.

The null hypothesis of the ADF test is that the time series is non-stationary. So, if the p-value of the test is less than the significance level (0.05) then you reject the null hypothesis can be rejected and it can be inferred that the time series is stationary.However, if p-value of ADF test is greater than 0.05 ,then first order differencing (d=1) is done and autocorrelation plot is examined.

If the autocorrelations are positive for many number of lags (10 or more), then the series needs further differencing. On the other hand, if the lag 1 autocorrelation itself is too negative, then the series is probably over-differenced. Generally, the series become stationary by maximum of second order differencing (d=2).

- **Estimation of Parameters** , the required number of AR terms or the p-value is found by inspecting the Partial Autocorrelation (PACF) plot .The PACF plot presents the correlation between the lag and the series. Initially take the order of AR term to be equal to as many lags that crosses the significance limit in the PACF plot.

MA term is the error of the lagged forecast.The order of the MA terms or q-value is decided by plotting the ACF plot ,The lags in the plot which exceed the significance limit are taken as the order of q-value. Intially , a lower order can be taken and corrected in case unsatisfactory results.

- **Diagnostic Testing,** This step the forecasted values are compared to the actual values in order to ascertain the correctness or accuracy of the model fit. In this dataset is divided into training and testing dataset by splitting the time series into 2 contiguous parts in approximately 50:50 ratio or a reasonable proportion based on the time series. The ARIMA model with p,d,q values (estimated in the parameter estimation step) is then trained on training dataset and the values forecasted. In the next step the actual values and the forecasted values are compared to gauge the accuracy of the model.The various accuracy metrics used to gauge the accuracy of the model are RMSE MAE MAPE etc. In case the accuracy metrics indicate a poor forecast , then the parameters can be adjusted to get a good fit.

Moreover, after the parameter estimation , train-test and implementation of model .If the accuracy metric indicates poor forecast ,then a loop is designed to run the ARIMA model for a range of (p,d,q) values. Thereafter, the accuracy results of the various combinations of p,d,q values is ccompared and the most accurate model is adopted.This approach has been used in this work for estimating a good fit.

2.4 Battery Energy Storage Systems

2.4.1 Battery Energy Storage Systems for reducing penalty on WPP in intra-day market

Energy Storage associated with renewable power plants represents the most straightforward solution to mitigate renewables variability. As a flexible and adjustable power supply, the energy storage system provides a new idea to cope with the intermittent power integration [24]. In various types of large-scale energy storage systems (such as pumped storage, compressed air storage, etc.), battery energy storage system (BESS) has the most promising broad in power applications benefiting from its high energy efficiency and weak requirement of geographical conditions.

2.4.2 Various Storage Solutions

Several solutions have been envisaged for the coordination between wind farms and storage plants. An example is the participation in electricity markets using pumped storage. Castronuovo and Lopes proposed in [17] the coordination between a wind farm and a pumped storage facility for increasing the controllability of the wind farm and maximize profits when participating in the Portuguese market. In [18] the same authors described new considerations about the optimal size of the pumped storage station. In [19] various methodologies are presented for coupling wind generation and storage for coordinated market participation. Reference [20] a method for sizing and operating an energy storage system coupled with a wind power plant under the Norwegian market conditions. [21] analysed the utilization of a generic energy storage device for balancing the differences between predicted and real productions in a wind farm located in Norway when acting in a market environment. Reference [22], analysed the combined operation of wind farms and a pumped storage facility for participating in the Spanish electricity market considering the uncertainties of both wind power generation and market prices.

All the above mentioned works showed that the optimal management of pumped storage coupled to wind farms results in an economic benefit and increase the controllability of the wind farm. However, for the specific case of isolated systems without hydropower potential, the use of DES is necessary. Several DES technologies such as battery storage [23], [24], ultra-capacitors [25] or flywheels [26], [27] are considered as an efficient way, together with accurate prediction models, to increase renewable energy penetration in islands without installing additional reserves (i.e. based on thermal generation), with the additional advantage of increasing energetic independence of these areas. DES can also be used to overcome network congestion problems or to allow wind farms to respect grid codes. The work in [28] showed that pumped storage can be also very useful in isolated systems, improving both the dynamic security and the economic operation of the grid. Another example is presented in [29] where the sizing of a battery for a grid connected wind farm in order to provide frequency support is described.

Wind farm with BESS configuration will become a common model for large-scale wind power development in the future. However, in addition to the high investment cost of BESS, how to properly size BESS to balance the investment cost and the effect of levelling wind power fluctuation and uncertainty has been a research hotspot in recent years. Storage can be used to absorb excess electricity in hours of high renewable production and low demand and to re-inject it into the network later. The operating performance of such storage can be highly improved by considering simple charge-discharge plans based on short-term predictions of the renewable production. These predictions, generated by physical or statistical models using weather predictions and measurements as input are nowadays widely used by system operators in several countries. This increased performance can be translated directly into a reduced size of the storage.

2.4.3 Methodology for battery sizing

Research on storage sizing for renewable energy integration is driven in part by the high capital cost of storage. In general storage sizing is studied as a minimisation problem of the fixed costs of the storage and and penalisation amount paid by the WPP . A complete analysis of the cash flow of a BESS used for integrating renewable power is studied and used to optimise the sizing of the storage. Then ,the return on investment (ROI) of the system is calculated taking into account the expected length of the battery life, calculated considering the known lifetime in cycles and the number of expected cycles of the storage. Furthermore, the desired lifetime is chosen for determining the optimal utilisation of the battery in the conditions chosen. In the sizing of a BESS for grid is calculated considering a risk based trade off method where the probability of not supplying the load is weighted against the cost of the necessary incremental capacity of the storage.

CHAPTER 3

SIMULATION RESULTS

3.1 Lithuania : An Overview

Lithuania imports around three-quarters of its electricity needs, as domestic electricity generation is fairly small, with only 3.6 Terawatt Hours (TWh) in 2019 .There has been 78% decrease in power generation ,since the end of 2009, when the country shut down its second (and last) nuclear reactor. In 2019, renewable energy dominated domestic electricity generation, accounting for 76% . Wind power makes up almost half of the total and has increased its role considerably. Net imports of 9 TWh play an important role to satisfy a consumption of 11 TWh, which is largely driven by the industry and services sectors.

The government has set ambitious targets for reaching 80% renewables in final energy demand by 2050. In the electricity mix, the country aims for a renewables share of 45% by 2030 and 100% by 2050 [30].

Lithuania has carried out a major restructuring of its electricity sector, in line with the EU(European Union). A range of critical reforms are being implemented based on the Lithuanian Electricity Market Development and Implementation. The government unbundled supply and transmission and market operation activities (Litgrid, AmberGas, EPSO-G, Ignitis), third-party access to transmission and distribution, and access to power trading within the Nord Pool market area .Lithuania balancing needs are met by balancing market of the Baltic countries.

3.2 Data

This section first familiarizes with data sets utilized in the thesis, followed by an in depth into explanation of the relevant features and compositions of the main data set - Lithuanian wind generation and forecast data. It aims to illustrate the stochastic nature of the wind speed through visualization and interpretation .The section then concludes with stationarity tests, to establish a generic idea on linear assumptions in the data, and necessary steps taken to prepare the data for further analysis.

Three data are used in this paper:

- Historical wind power generation data.
- Historical Day Ahead wind power forecast data
- Balancing prices

3.2.1 Lithuania Wind Power Generation Data

The main data set, Lithuanian Wind Power Generation data, is a complete data set containing hourly time series data wind farms of Lithuania. The duration of data spans across 1 year (2019), from 01.01.2019 00:00 to 31.12.2019 23:00.

3.2.2 Lithuanian Wind Power Forecast Data

This data set, Lithuanian Wind Power Forecast data from Nordpool, is a complete data set containing hourly time series data wind farms of Lithuania. The duration of data spans across 1 year (2019), from 01.01.2019 00:00 to 31.12.2019 23:00.

3.2.3 Lithuania Balancing Prices

This data set, Lithuanian balancing prices data from Baltic States Balancing Market, is a complete data set containing hourly time series market of Lithuania. The duration of data spans across 1 year(2019), from 01.01.2019 00:00 to 31.12.2019 23:00.

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3.3 Data Preparation

To formulate and evaluate forecasting models, all time series data was first split into train set and test set. Since 2019 is the complete year of data, all observations of data points from 01.01.2019 00:00:00 to 02.07.2019 23:00:00 were held out as test data. Also, besides the removal of duplicates and empty values were replaced by mean of the total data set.

The first step of ARMA model development is to determine the order of the AR and MA processes, p and q, respectively. This is known as the model identification phase, and it involves the analysis of the autocorrelation and partial autocorrelation factors.

After estimation, the model was checked with several diagnostics. It is not uncommon for a promising model identification to lead to a poorly performing model, so the diagnostic phase is important because it was used to remove the combination of orders that did not work well.

Simplistic persistence method is a primitive method of wind forecasting. This approach uses the past hour wind speed (or wind power) as the forecast for the next hour. As a forecasting technology, this method is not impressive, but it is nearly costless, and less complex. Therefore, other forecast methods are measured by the extent it can improve on persistence forecasts. That is the approach that was applied in this thesis. The persistence forecast can offer a range of forecasting accuracy, depending on the wind regime and the number of periods to be forecast. We calculated the root mean square error (RMSE) of each forecast over the relevant time period to compare our methods with the persistence model. A lower RMSE implies that the forecast is more accurate, whereas a high RMSE value implies less accuracy.

A similar set of p,d and q orders were applied to the time-series data The model specification, the ARIMA(1,2,4), did the best overall job. However, it is worth noting that the ARIMA(1,0,0) model and whereas the ARMA(0,0,3) models gave reasonable but not very accurate results.

3.4 Results

For Lithuania wind power generation data and day-ahead wind power forecasts were collected from Nord pool for the year 2019. The imbalance prices were taken from the Baltic countries balancing market for the year 2019.

A plot was drawn to give visual interpretation of the variation in the wind power generation and forecast. The plot of wind power generated and forecasted can be seen in Figure 3.6, forecasted value deviates from the other substantially, due to forecast error.

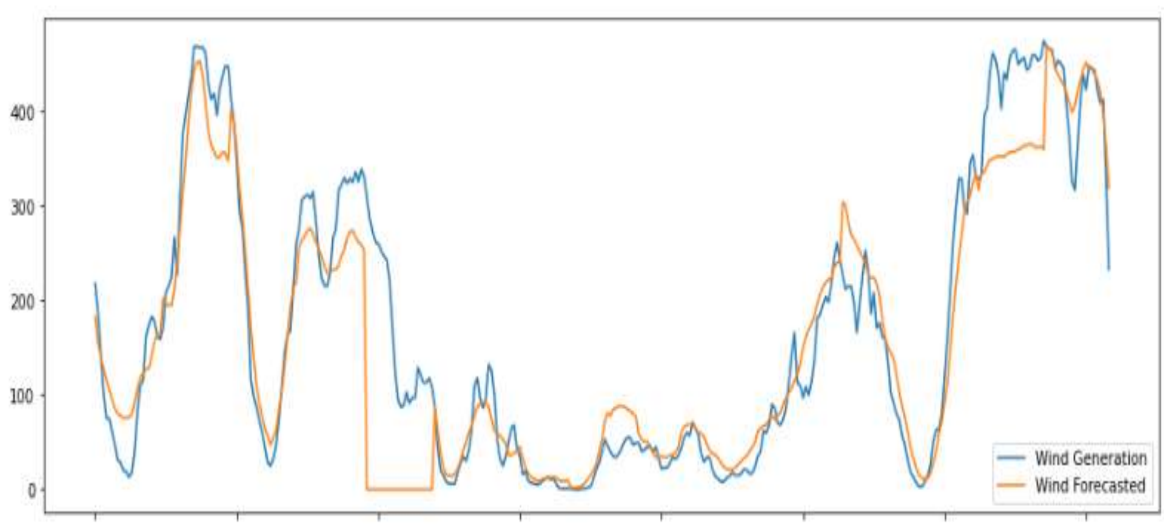

Figure 3.1 Plot of Wind Power Generated v/s Forecasted (Day Ahead)

3.5.1 Short term forecasts

A simple short term forecast (equation 3.1) was used for a short gate closure of markets. To simulate the available information 2 hours ahead, the forecast was made using a simple persistence / naïve forecast.

$$P(t) = P(t-2) \tag{3.1}$$

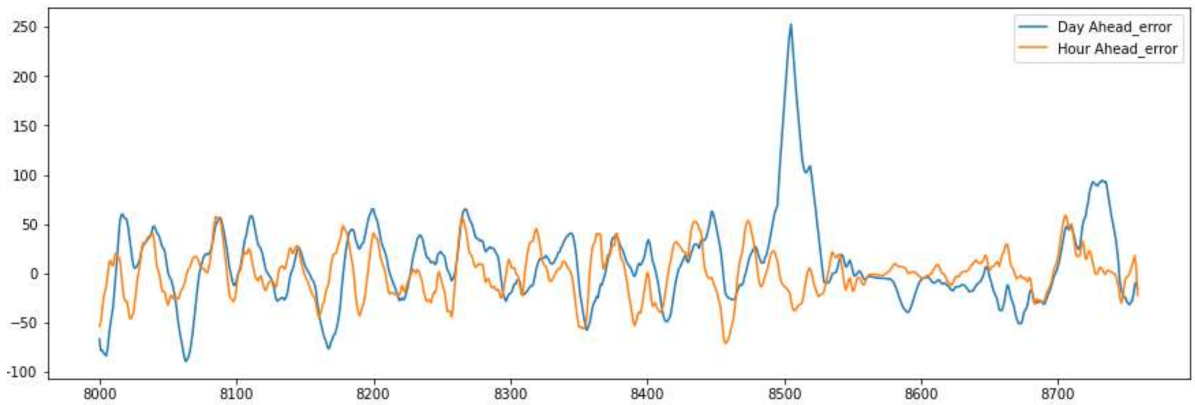

Figure 3.2 Plot for DA forecast & Naïve Forecast Error

The RMSE for 2-hours-ahead forecast was 40.42 %. The Figure 3.7 depicts the plot of the forecast error for Day Ahead &2 hour Ahead forecast It is evident from the plot that Day Ahead forecast error magnitude and variance is much more than 2 Hour Ahead Forecast.

Table 3.2 RMSE for DA forecast & Naïve Forecast

GATE CLOSURE	RMSE
Day Ahead	72.19 %
2 Hour Ahead	40.42 %

Also on comparison of RMSE for Day Ahead forecast & naïve forecast .It is clear from Table 3.2, when shorter gate closure of 2 Hour Ahead is implemented, the short-gate closure results in significant decrease of RMSE.

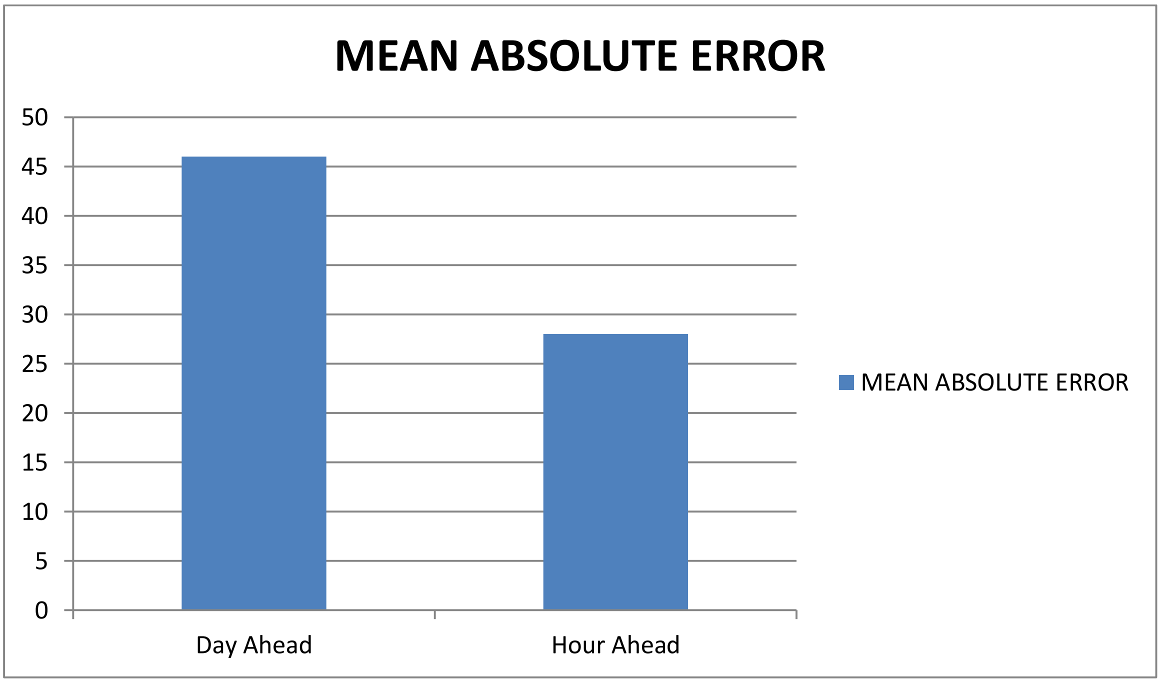

Figure 3.3 MAE for Day Ahead & Hour Ahead Forecast

The MAE for day ahead forecast and Hour ahead forecast (naïve forecast) can be seen in Fig 3.8..The bar chart clearly shows that the forecast error in case of Day Ahead is much larger than the hour Ahead forecast. Thus all the above mentioned results give credence to the fact that shorter gate closure has significant impact on wind forecast errors.

3.5.2 Implementation of ARIMA

The 2-hour-ahead forecast could be improved by using advanced statistical forecasting methods. Above, for this analysis a simplistic approach was taken as the aim is just to illustrate the difference between day-ahead and a more close to real time forecasting. In order to deal with imperfect wind power prediction, system operators have to face additional cost as a result of increasing reserve levels. The unexpected large forecast deviations would cause more operating costs, because it requires more balancing energy to balance the wind power forecast errors and the cost of balancing energy will be calculated with balancing energy price. A sophisticated forecasting strategy can be use of forecasting methods like AR, MA, ARIMA can avoid more cost caused by short-time forecast errors.

In order to implement the ARIMA, firstly the times series was checked for stationarity by performing ADF test .The p-value after performing the ADF test came as 0.124419, which is above the significance level of 0.05.This made it evident that the series is not stationary .Hence, in order to make the series stationary first order differencing (d=1) was applied .This resulted in lowering the p-value but still slightly above the significant limit. But, still the intial value of parameter d was taken as 1.

Thereafter, Auto-Regressive (AR) term the ACF was plotted to get the value of p-parameter. The plot was studied to find that the lag 1 is very significant. Hence, the tentative value of p-parameter was taken as 1.

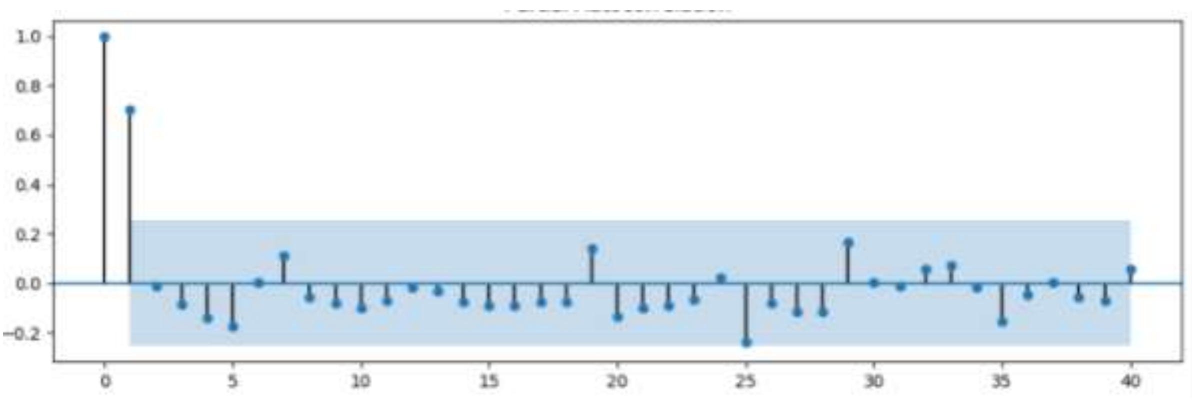

Figure 3.4 ACF Plot

The PACF plot was studied to find the value of q or the MA term.The PACF plot (Figure 3.5) shows the lags which crossed the significant limit was lag 4.Therefore the value of q was taken as 4.

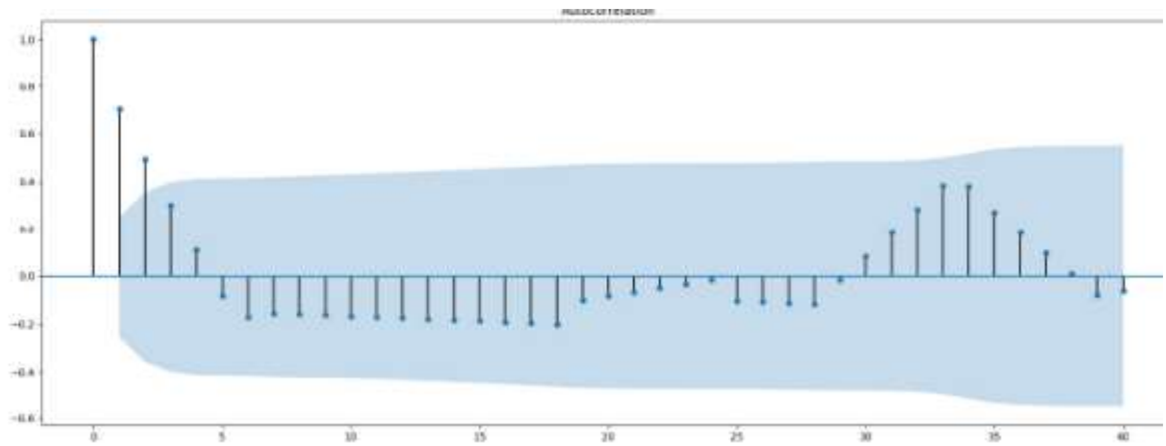

Figure 3.5 PACF plot

The parameter estimation gave an insight into the possible range of p,d,q values. Therefore, the various ARIMA models were made to run for the various combinations of p,d,q parameter values .The range of values was taken for p,q from 1 to 4 and for d from 0 to 2 .The RMSE for the various models of ARIMA were calculated to arrive at the best fit. It was found that ARIMA (1,2,4) gave the best RMSE value of 21.26 %

Also, the implementation of AR(1),MA(2),ARIMA gives the following MAEs 23.82% , 67.12% , 21.26 % respectively .

Since , RMSE for naive , ARIMA forecast are 72.19 % , 21.26 % . Hence, it is evident from the results that adoption ARIMA(1,2,4) method resulted in significant improvement of the forecast accuracy.

3.5.3 Storage

The sizing of the battery integrated with wind power is done by trial & error method where the characteristics of the storage, as well as its operation are taken into account. The battery is also characterised by its rating Rb [kWh] and its cost Cb [€/kWh].

The battery is used to filter the prediction error E, a time series of m elements representing the errors for each hour. The sizing of the battery is an maximisation of ROI problem, described by aimed at identifying the optimal power and energy rating of the storage.The objective function is considered to be equal to ROI, calculated as in Equation .The terms Rb [€] and Cb [€] represent the value of the rating of the battery and the cost of the battery respectively

$$APEN = x*E \quad (3.1)$$

$$ROI = \frac{APEN - (CbxRb)}{CbxRb} x100 \quad (3.2)$$

The cost of the battery (Cb) in (3.2) has been calculated by dividing the total cost of the battery by its lifetime (8 years). This approximation has been made, as the wind power data is considered only for one year. The constraint of the optimisation problem is represented by the necessity for the battery to avoided penalties that would be paid if the battery was not connected. This can be expressed analytically above The terms APEN (the avoided penalties), found in Equations where x [kWh] is the value of a prediction error and E[€/kWh] is the cost paid for each penalty.

After , the integration of BESS with the wind power. The APEN and the expenditure saved due to utilisation of BESS of 5 %, 10 % , 15 % was studied. It was found , refer table 3.3, that the APEN also increases ,with the increase in BESS size.

Table 3.3 APEN & Savings for various battery sizes

Battery Size (As percentage of total Wind Generation)	APEN(in MW)	Savings(in Euros)
5	6309	3,73,201
10	11621	4,60,000
15	16250	5,83,463

Table 3.4 Return on Investment for various battery sizes

Battery Size (As percentage of total Wind Generation)	ROI (%)
5	29.8
10	21.47
15	15.63

As shown in the Table 3.3, the size of the battery is taken as 5%,10%,15% of the total wind power generation capacity . The ROI for the above battery sizes are 29.8 % , 21.47 % , 15.63 % . The ROI at various battery capacities are depicted above .It is evident that based on the avoided penalties ,battery size of 5% of total wind power generation capacity provides the best ROI.

CHAPTER 4

CONCLUSION

4.1 Conclusion

In this work the Day Ahead error forecasts and hour ahead forecast using short gate closure time were studied Further, the forecasting was done by sophisticated the time series forecasting methods .These methods include AR,MA,ARIMA ,adoption of these methods lead to a significant decrease in the forecasted errors. As it can be seen that the RMSE for Day Ahead forecast and Hour Ahead forecast is 72.19 % and 23.93 %. The implementation of AR,MA,ARIMA gives the following RMSEs 23.82% , 67.12% , 21.26 % respectively, The various forecasting errors give credence to the fact that short gate closure times and sophisticated forecasting methods give less RMSE .

A method for sizing and operating an energy storage system coupled with a wind power plant under the Lithuanian market conditions was implemented .The utilization of a generic energy storage device for balancing the differences between predicted and real productions in a wind farm located in Lithuania when acting in a market environment was analysed. The analysis of combined operation of wind farms and battery storage facility for participating in the Lithuanian electricity market considering the uncertainties of both wind power generation and market prices was done.

The storage sizing for renewable energy integration is driven in part by the high capital cost of storage. In general storage sizing is studied as a maximisation problem of the ROI. The initial capital cost of a BESS is considered along with a complete analysis of the cash flow of a BESS used for integrating renewable power is studied and used to optimise the sizing of the storage.Furthermore, the desired lifetime is chosen for determining the optimal utilisation of the battery in the conditions chosen. In the sizing of a BESS calculated considering the avoided penalty (APEN) incurred due to not supplying the load as per forecast . The sizing of battery is done by trial and error method.The size of the battery is taken as 5%,10%,15% of the total wind power generation capacity.

The ROI at vaious battery capacities are computed .It is evident from the results that based on the avoided penalties , 5 % battery energy storage size provides the best ROI.

4.2 Future Scope

The Naïve,AR,MA,ARIMA model, also known as the Box-Jenkins model or methodology, has been analysed for forcasting wind power generation in this study. But, Artificial neural networks (ANNs) as a soft computing technique are the more accurate and widely used as forecasting models. Its wide usage is due to the several distinguishing features of ANNs that make them better than ARIMA. ANNs are data-driven, self-adaptive methods with few prior assumptions. They are also good predictor with the ability to make generalized observations from the results learnt from original data, thereby permitting correct inference .ANN-based approach is expected to demonstrate superior performance over the ARIMA. In future studies, ANN or hybrid of intelligent techniques using ARIMA- can be utilised to improve existing predictive models .

In this work, a balance has been found between the need to describe a realistic use case and an abstraction necessary to extract the fundamental behaviour of the phenomenon. the choice of reaching a specific rating of the battery namely 5%,10%15% of the total generation capacity is arbitrary.This should ideally be dictated by a result of an optimisation function

In real life application the storage system considered will be composed by an inverter .The inverter is characterised by its own rating, cost and effciency] which may be included in future studies.The simplified model used has not incorporated the change due to various battery technologies .Therefore, it can also be improved by modifying and if necessary adding new relations able to describe the behaviour of the specific battery technology. Also the results obtained are relative to a simplified storage with perfect efficiency and no losses from auxiliaries.

The objective function and constraints have also an impact on the results of the work, the choice of maximising the ROI is reasonable but could be coupled with the optimisation of other parameters and could be possibly further area of research.

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