CONSTRUCTION OF OPTIMIZED DETERMINISTIC MATRICES BASED ON MINIMIZATION OF MUTUAL COHERENCE

A Project Report

submitted by

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THESIS CERTIFICATE

This is to certify that the thesis titled **CONSTRUCTION OF OPTIMIZED DETERMINISTIC MATRICES BASED ON MINIMIZATION OF MUTUAL COHERENCE** , submitted by **Goda Sree Swathi**, to the Indian Institute of Technology, Madras, for the award of the degree of **MASTER OF TECHNOLOGY**, is a bona fide record of the research work done by him under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

KEYWORDS: Compressive Sensing; Measurement matrix; Restricted Isometry Property; Mutual Coherence.

Compressive Sensing (CS) is a signal acquisition paradigm to simultaneously acquire and reduce dimension of signals that admit sparse representation. This is achieved by collecting linear measurements of a signal, which can be formalized as a multiplying the signal with a "Measurement Matrix". If the measurement matrix satisfies the so-called Restricted Isometry Property (RIP), Then it will be appropriate for compressive sensing.

In this project, the focus is on Construction and Optimization of deterministic measurement matrices in compressed sensing. The construction of the deterministic matrix mainly based on the Optical Orthogonal Code (OOC) structure , This base matrix can be used to construct Gaussian, Binary, Hadamard, Discrete Cosine Transform (DCT) measurement matrices. The optimization is mainly to reduced Mutual Coherence value of the above mentioned measurement matrices based on Projection technique.

Contents

List of Figures

ABBREVIATIONS

CS	Compressive Sensing
RIP	Restricted Isometry Property
OMP	Orthogonal Matching Pursuit
OOC	Optical Orthogonal Code
SVD	Singular Value Decomposition
Bmat	Binary matrix
Hmat	Hadamard matrix
Gmat	Gaussian matrix
DCT	Discrete Cosine Transform
Bmat-P	Binary matrix with projection
Gmat-P	Gaussian matrix with projection
SNR	Signal to Noise Ratio

NOTATION

Ψ	Domain Transform Matrix
ϕ	Measurement Matrix
X	Input Matrix
Y	Output Matrix
D, A	Dictionary/ Measurement Matrix
G	Gram Matrix
B	Base Matrix
P	Projection Matrix
M	Number of rows in Measurement Matrix
N	Number of columns in Measurement Matrix
k	Sparsity
μ	Mutual Coherence
R	Residue
n	Basis Matrix dimension as n$\times$n
x	Down sampling factor
t	Threshold value

Chapter 1

INTRODUCTION

This chapter gives an overview on the traditional methods and their drawbacks. Introduction to the compressive sensing concept, scope and objective of the project.

1.1 Background

Signal acquisition is a main topic in signal processing. Sampling theorems provide the bridge between the continuous and the discrete-time worlds. With of advent of information technology such as information transfer, the demand for sampling rate increases hugely.As a benchmark for traditional signal processing shown in Fig. 1.1, the Nyquist sampling theorem requires that the rate of sampling at least two times faster than its bandwidth to prevent information from losing when uniformly sampling a signal. In general, if a signal can be approximated well by k large coefficients, then it can be defined as compressible signal. Without loss of generality, any signal in theory has compressibility, i.e., can be effectively compressed and sampled, as long as it can find its corresponding sparse representation space. Therefore,all types of signals can be considered with respect to sampling and compressing. However,two main dilemmas are caused by this traditional method.On the one hand,for many applications, including digital image,video cameras,a lot of sampling resources are waste associated with the process of compressing after sampling with high-speed. For example,only a few hundred Kbyte data is used to transform the image associated with a digital camera equipped with a megapixel image sensor.On the other hand,for other applications, including imaging systems (medical scanners, radars) and high-speed analog-to-digital converters,the requirement about the relationship between sampling rate and bandwidth is too strict to satisfy,which may give rise to huge consumption by increasing the sampling rate or density beyond the current state of the art.

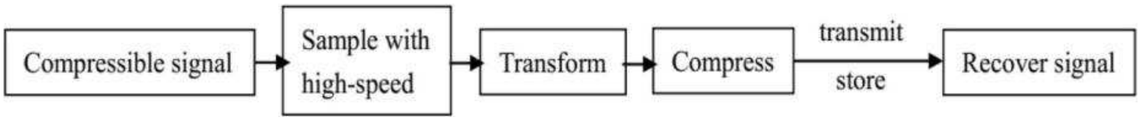

Figure 1.1: Process of traditional compression

1.2 Motivation

CS concept , (Candès and Wakin, 2008), a novel sampling paradigm that goes further than Shannon-Nyquist's theorem. CS allows to compress signals with sparse or compressible representation while they are sampled. It originates from the idea that it is not necessary to invest a lot of power into observing the entries of a sparse signal because most of them will be zero. Considering a sparse signal, it should be possible to collect only a small number of measurements that still allow for reconstruction. As it is explained in , (Donoho, 2006), most of the data acquired by modern systems and technologies can be thrown away with almost no perceptual lost. This phenomenon raises very natural questions: why to acquire all the data when most of that will be thrown away? why don't we try to just directly measure the part that will not be thrown away? Obviously, image/signal compressing and sampling can be processed simultaneously in terms of compressed sensing theory, which can reduce the cost greatly and avoid wasting sampling resources. Meanwhile,the recovery of signals is an optimization problem,which can be resolved by efficient algorithms. Therefore,this theory proposes an effective way to directly compress and sample signals into digital form, whose characteristic is direct sampling information. To sum up, the technology of CS will lead to the coming of new era with respect to information. In addition, the attraction of CS theory is that it has important influence and practical significance in many fields of applied science and engineering, such as statistics,information theory, coding, and so on.

1.3 Scope

From, Fig. 1.2 we can find that the signal processing with CS can be generally divided into two main parts. Firstly, a measurement matrix called (compressed) sensing matrix ϕ should be used to collect the information and simultaneously compress signals, which can be shown as the part associated with compressing and sampling with low-speed.

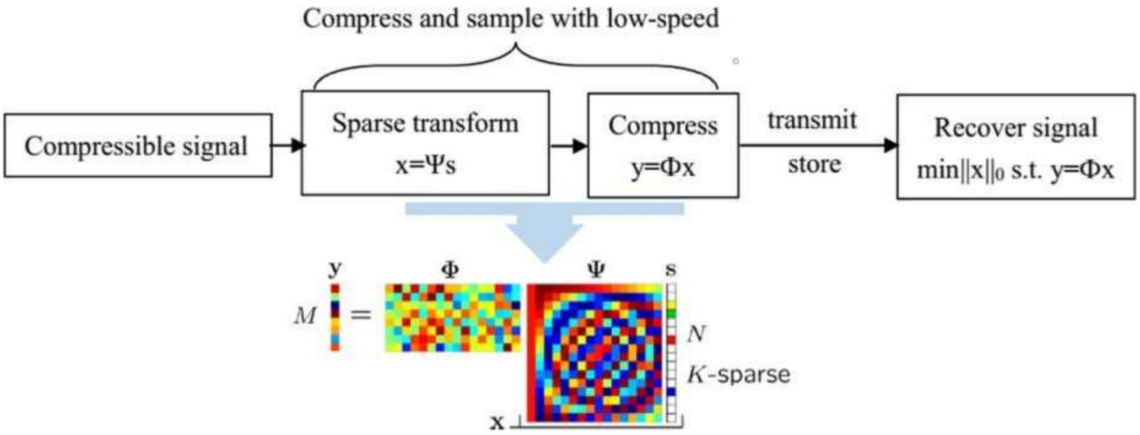

Figure 1.2: Compressive Sensing Technique

Secondly, the recovery of signals after transmitting and storage should be accomplished by solving an optimization problem with effective algorithms.

1.4 Objective

Objective is to construct a various determined measurement matrix using combinatorial designs and to compare those measurement matrices in Error rate and SNR (signal to noise ratio) perspective using OMP (orthogonal matching pursuit) as a reconstruction algorithm.

Optimization of constructed deterministic matrices based on minimization of mutual coherence value by projection technique.

1.5 Applications

1.5.1 Image Compression

A fundamental assumption in digital image processing is that natural images are piece wise smooth in the pixel basis. That is, there are very few edges in the image, and therefore, the differences between the values of adjacent pixels are usually zero or almost zero. The wavelet transform can be used to map images from the pixel domain to the wavelet domain in which they have sparse (or approximately sparse) representations.

Here, CS can be used as a new data acquisition framework, to overcome the inefficiencies of the classical image compression approach. In contrast to the classical approach, which involves sensing a high-resolution signal and then compressing it by throwing away part of the sensed data, CS attempts to develop methods to sense signals directly into compressed form.

1.5.2 Biomedical Imaging

Another important application for CS is in reducing the sampling rate in magnetic resonance imaging (MRI). Traditional MRI scanners sequentially sample Fourier coefficients of the human brain's image. Unfortunately, this traditional MRI approach is very costly, as the speed of data collection is limited by physical and physiological constraints However, most MRI images are sparse in the Fourier domain. As a result compressed sensing can be used to significantly decrease the number measurements without reducing the accuracy of the MRI image.

1.5.3 Digital Communications

The problem of wireless networks to enable network communication in the presence of inference is one of the major challenges facing communication research. One important case is managing inference in peer to peer networks and in an uplink where multiple sensors look to communicate with an access point. the interference mitigation for downlink communications in which a single transmitter (e.g. a cellular base station) communicates simultaneously with multiple (N) receivers.

The key idea connecting compressed sensing to wireless communication is that at each time only a small number of $K \times N$ receivers are active. The sender then maintains an $M \times N$ sensing matrix , such that the ith columns of is associated with the ith user. At each transmission time, the transmitted signal is constructed as the sum of individual signals, each intended to a different receiver. That is, the transmitted signal is a superposition of at most K columns of the matrix. With this strategy, each receiver can also invoke sparse reconstruction algorithms and decode its own information.

1.5.4 Quantum Computing

A major obstacle to engineer quantum devices such as quantum computers had been lack of an effective scheme for noise characterization in many component systems. The number of parameters required to represent the state of a quantum system grows exponentially with the number of its components in contrast to a classical system. As a result the number of measurements needed for full characterization of the noisy dynamics of a quantum system becomes astronomically large. So the CS technique effectively deal with this large data.

1.6 Organisation

In chapter 2, we will discuss some fundamental aspects in CS. In chapter 3, we will discuss construction of the different measurement matrices based on the combinatorial design. In chapter 4, we will discuss optimization of deterministic measurement matrices based on projection technique. In chapter 5, conclusions are made based on the observations drawn from the simulations specified in chapter 3 and chapter 4.

Chapter 2

FUNDAMENTAL ASPECTS

This chapter, gives overview on the fundamental aspects of the CS which are used in the project.

2.1 Measurement Principle

CS theory states that a sparsely re-presentable signal can be reconstructed using very few number of measurements compared to the signal dimension. So the measurement principle used in CS is the Dimensionality Reduction Technique. We are concerned with under sampled situations in which the number m of available measurements is much smaller than the dimension n of the signal f. This raises an important question about accurate reconstruction from $m \ll n$ measurements only. This can be achieved through the set of operations is shown in Fig. 2.1 .

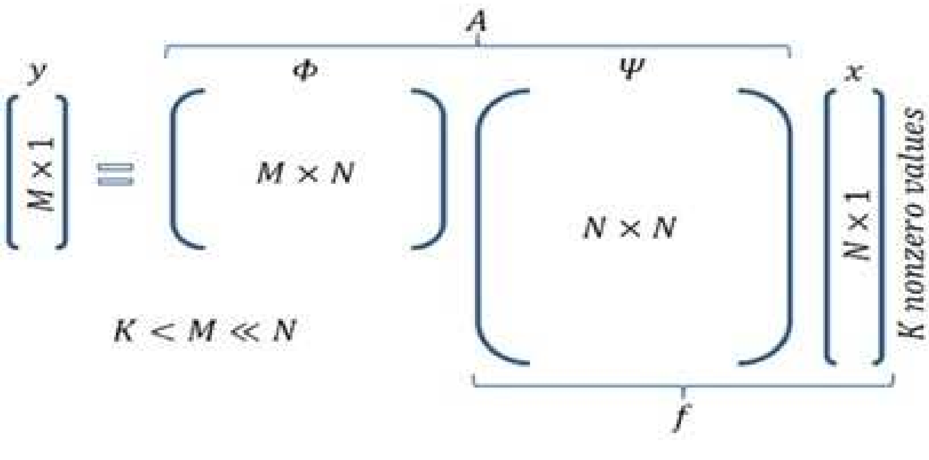

Figure 2.1: Dimensionality Reduction Technique

- A=Sensing Matrix
- X=N Dimensional Signal vector
- Y=M Dimensional Signal vector

2.2 Sparsity

Signals/images are sparse if it can have very few nonzero coefficients representation in certain subspace. The concept that most signals in our natural world are sparse. If it is not sparse in one domain it may sparse in another domain like if a signal is not sparse in time domain it may appear sparse in Fourier transform domain. So discarding the large number of zeroes with approximately reconstructs the original image/signal. Consider the following example shown in Fig. 2.2.

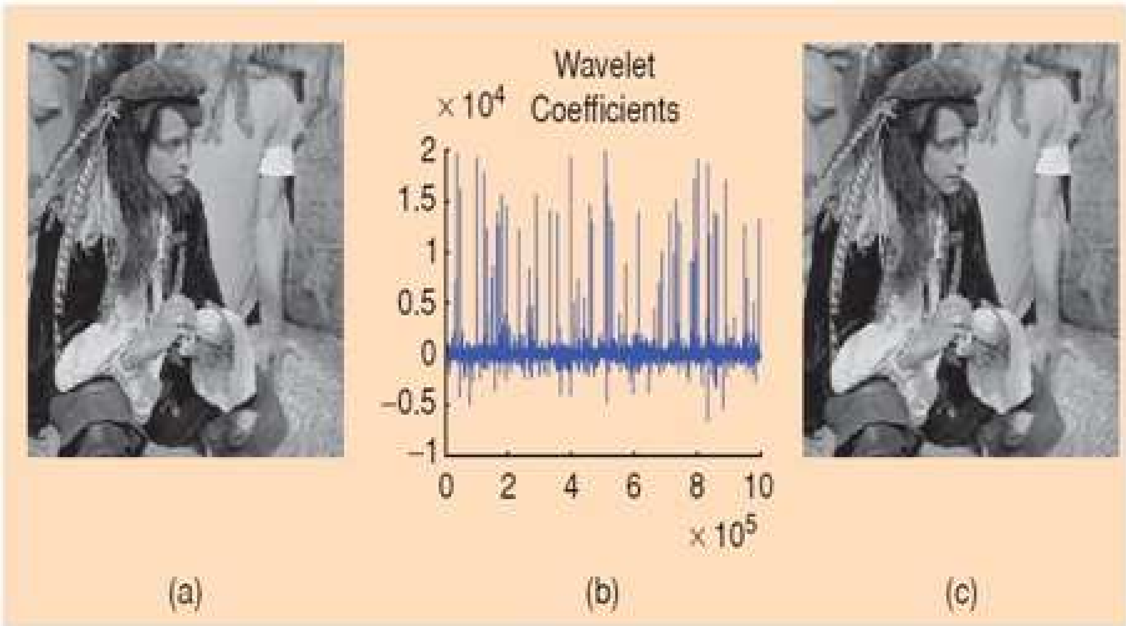

Figure 2.2: Role of Sparsity in the reconstruction of an image

(a) Original image

(b) Coefficients of image in wavelet domain

(c) Reconstructed image

2.3 Restricted Isometry Property

To decide what matrix is appropriate, we need some criteria. An insight into the geometry of sensing matrices leads to a widely used criterion named Restricted Isometry Property(RIP). A matrix ϕ is said to satisfy the RIP of order k if we have a constant $0 \leq \delta_k$ <1 such that

$$(1-\delta_k)||x||_2^2 \leq ||\phi x||_2^2 \leq (1+\delta_k)||x||_2^2 \tag{2.1}$$

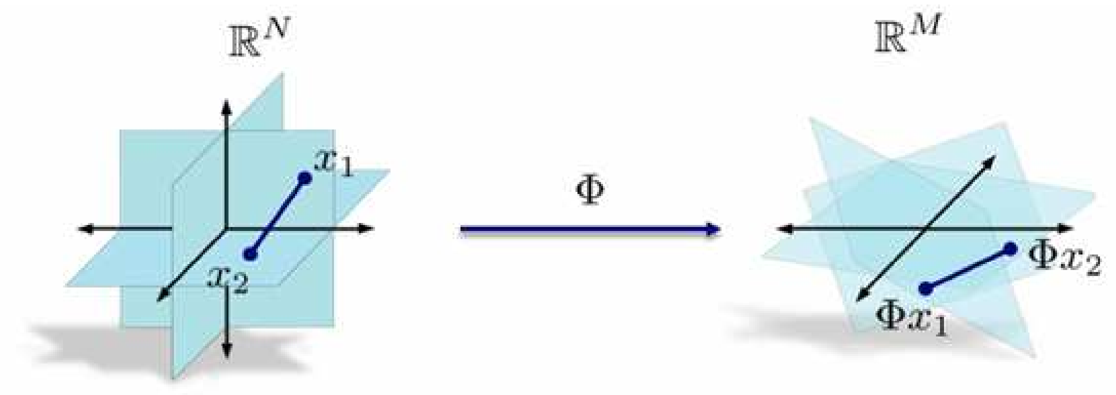

Figure 2.3: Schematic representation of how RIP can ensure measurement error does not blow up

such that holds for every k-sparse signal. In other words, if ϕ satisfies the RIP of order k, holds for any k-sparse signal x. δ_k is called Restricted Isometry constant(RIC) of order k, which is the smallest non negative number any k column vectors from behave like an almost orthogonal system. RIP is a sufficient condition which guarantees exact and robust reconstruction of sparse signals via l_1-minimization.

2.4 Coherence Factor

Coherence plays a central role in the deterministic constructions, because small coherence implies the RIP. The coherence of a matrix can be calculated by following formula (2.2)

$$\mu\{A\} = max\frac{|\langle a_{ij}\rangle|}{||a_i||_2.||a_j||_2} for, i \neq j, 1 \leq i, j \leq n \tag{2.2}$$

For a dictionary A, its mutual coherence is defined as the largest absolute and normalized inner product between different columns in the A.

2.4.1 Relation between Sparsity level and Mutual Coherence

$$||\alpha_0||_0 \leq \frac{1}{2}(+\frac{1}{\mu\{A\}}) \tag{2.3}$$

Where $||\alpha_0||_0$ is the Sparsity level

$\mu\{A\}$ is the Mutual Coherence value of a matrix A

2.4.2 Gram Matrix

The mutual coherence value can also be calculated using Gram matrix. μ is the maximum off diagonal value of a Gram matrix. The Gram matrix can be calculated by the equation (2.4)

$$G = A_q^T . A_q \tag{2.4}$$

A_q is the matrix obtained after normalizing columns of A matrix.

2.5 Reconstruction Algorithm

There are many recovery algorithms are there. Classification of Sparse Recovery Algorithms shown in Fig. 2.4 based on (Arjoune *et al.*, 2017).

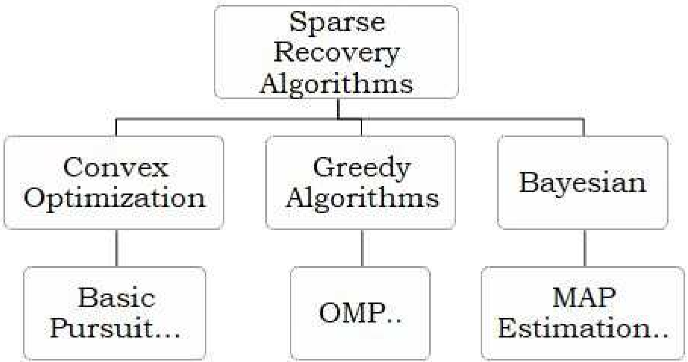

Figure 2.4: Classification of Sparse Recovery Algorithms

In this project we used OMP as the recovery algorithm to find the error rate of the constructed deterministic matrices and to compare their Performance. Fig. 2.5 describes the flow of OMP or working of the OMP.

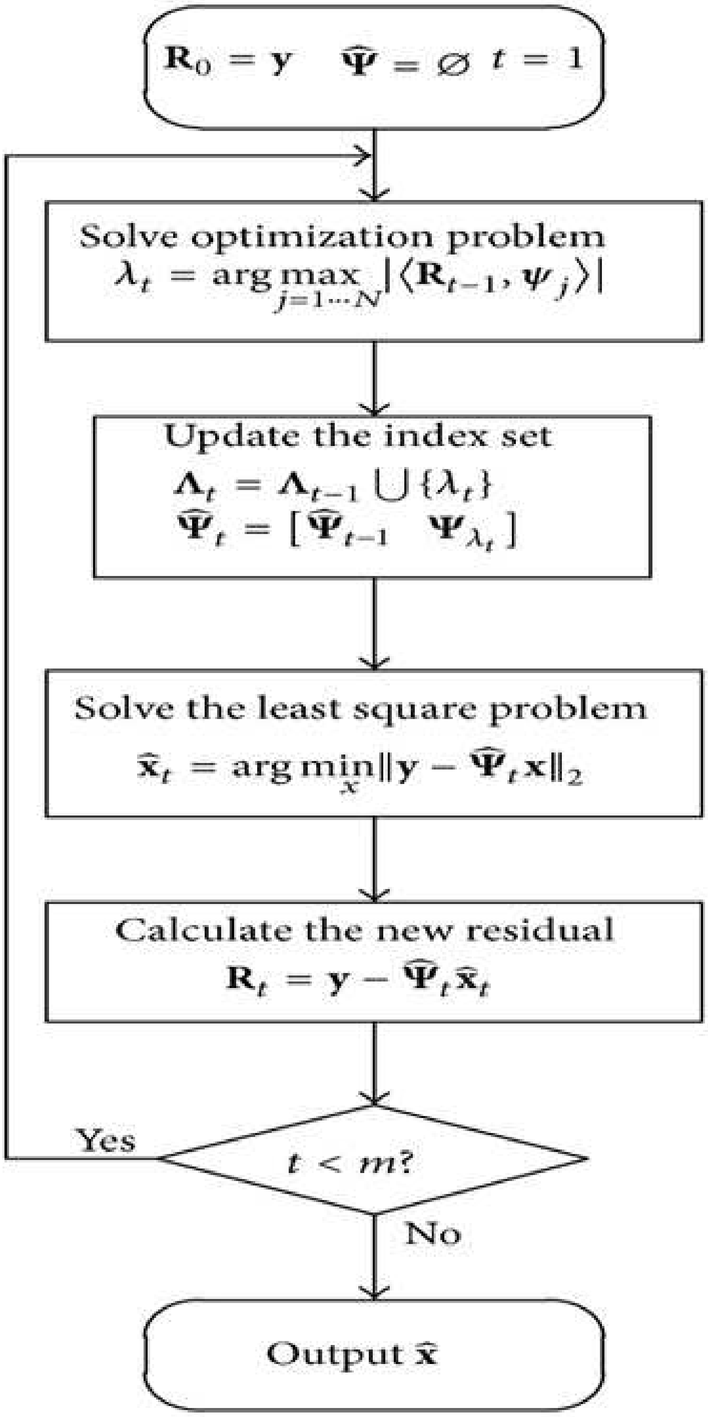

Figure 2.5: Flowchart of OMP

Chapter 3

Construction of Deterministic Measurement Matrices

In this chapter, we discuss how to construct determinstic measurement matrices based on combinatorial designs (Bryant *et al.*, 2017)and their performance.Deterministic matrices can be constructed in many ways, In this project the construction of measurement matrices having two main steps. First step is to construct a basis matrix based on OOC structure (Li and Ge, 2014). In the second step we construct the measurement matrices of different distributions using the basis matrix constructed in the first step (Bryant *et al.*, 2017), (Dias and Rane, 2013) .

3.1 Basis Matrix Design

Basis matrix is constructed by using OOC from structure of (n,ω,λ) which is equal to ($q^2 + q + 1$, $q + 1$, 1), where q is a integer values between $2 \leq q \leq 16$. By using the following look up table mentioned in Fig. 3.1 (Shen *et al.*, 2016) , we can construct a square matrix of size ($q^2 + q + 1$, $q^2 + q + 1$).

q	n	ω	Code
2	7	3	0 1 3
3	13	4	0 1 4 6
4	21	5	0 2 7 8 11
5	31	6	0 1 4 10 12 17
7	57	8	0 4 5 17 19 25 28 35
8	73	9	0 2 10 24 25 29 36 42 45
9	91	10	0 1 6 10 23 26 34 41 53 55
11	133	12	0 2 6 24 29 40 43 55 68 75 76 85
13	183	14	0 4 6 20 35 52 59 77 78 86 89 99 122 127
16	273	17	0 5 15 34 35 42 73 75 86 89 98 134 151 155 177 183 201

Figure 3.1: OOC structure look up table

3.2 Construction of Measurement Matrix

Consider matrix C1 as random square matrix of Gaussian or Binary or Hadamard or DCT matrices. Each column x of B determines r_x columns of measurement matrix, each zero in column x is replaced with the $1 \times r_x$ row vector (0, 0, . . . , 0), and each 1 in column x is replaced with a distinct row of C1 matrix which is shown in Fig. 3.2.

<u>Example</u>

N=3,q=4, B[7,7], OOC-B[7,28]

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \qquad C1 = \begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & 1 & -1 \end{bmatrix}$$

$$OOC\text{-}B1 = \left[\begin{array}{cccccccccccccccccccccccccccc} 1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ -1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 \end{array}\right]$$

Figure 3.2: Example of Construction of Measurement Matrix

- B=Basis matrix
- C1=Distribution matrix(Hadamard)
- OOC-B or A=Measurement matrix

3.3 Simulations

In this project the performance analysis can be categorized by firstly its error rate i.e,least square error between original matrix and reconstructed matrix. Secondly SNR value i.e, calculated by input as signal and noise as the difference between the input signal and reconstructed signal. So following graphs are drawn taking those two parameters as a reference with respect to the sparsity level by varying N value. The simulations are obtained using MATLAB tool.

3.3.1 Comparison of various Measurement Matrices

Here we are comparing Gaussian, Binary, Hadamard, DCT measurement matrices based on their error rate and SNR.

In Fig. 3.3 graph is drawn between sparsity versus error rate and sparsity versus SNR for the matrix dimensions are obtained as B[21,21],A[21,84] taking N=4,q=4. So, from the graph we can say that Gaussian and Binary are giving good results both in SNR and error parameters comparing to other two measurement matrices.

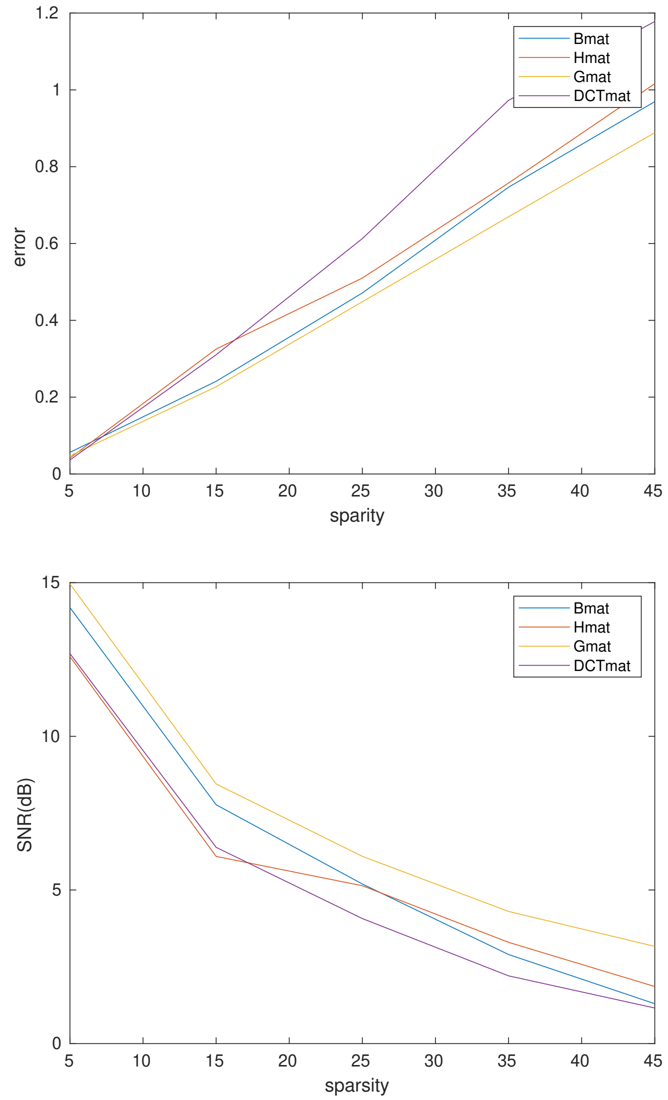

Figure 3.3: Comparison of error rate and SNR of various measurement matrices w.r.t sparsity having matrix size [21,84]

In Fig. 3.4 graph is drawn between sparsity versus error rate and sparsity versus SNR for the matrix dimensions are obtained as B[273,273],A[273,1092] taking

N=4,q=16.So,from the graph we can say that Gaussian and Binary are giving good results both in SNR and error parameters. Here Hadamard is giving less error rate but its SNR is low comparing to Gaussian and Binary and also Hadamard can only implement with matrix size is multiples of 4.

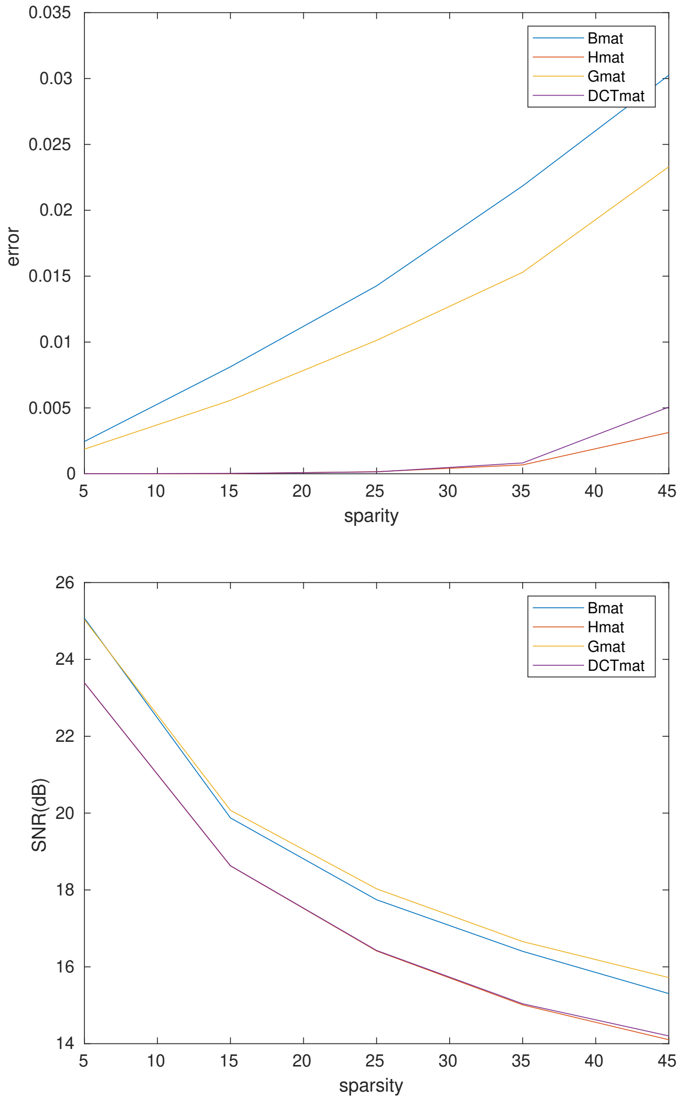

Figure 3.4: Comparison of error rate and SNR of various measurement matrices w.r.t sparsity having matrix size [273,1092]

In Fig. 3.5 graph is drawn between sparsity versus error rate and sparsity versus SNR for the matrix dimensions obtained as B[273,273],A[273,2184] taking N=8,q=16. Here matrix dimensions are higher compared to other two examples i have mentioned above. In this case also Gaussian and Binary giving good results.

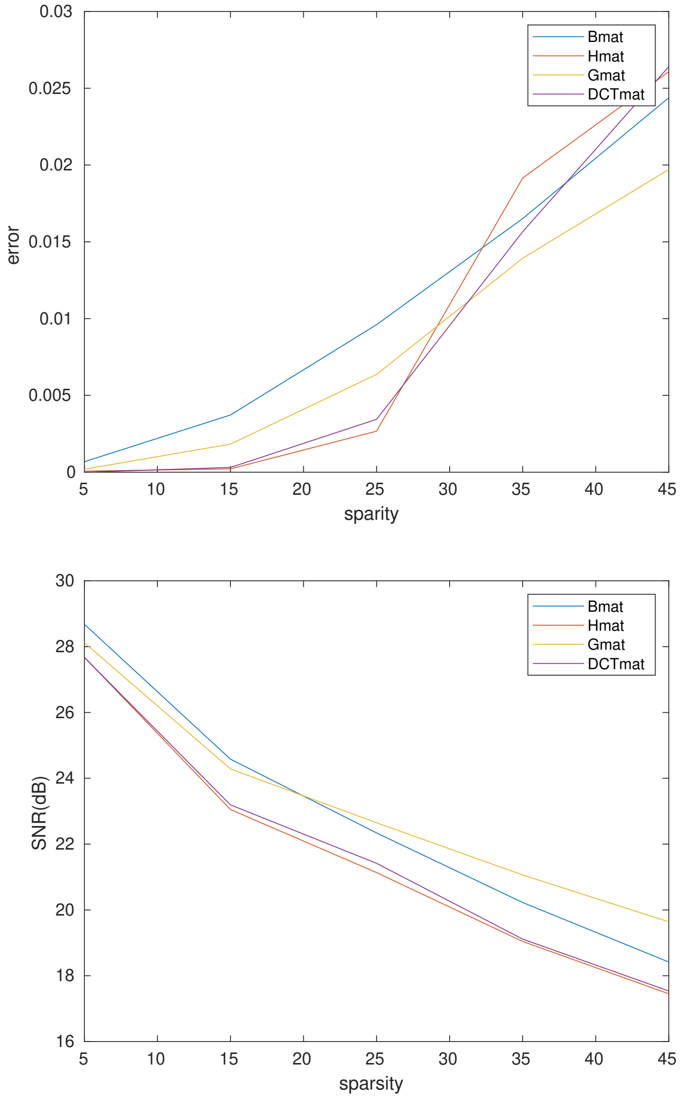

Figure 3.5: Comparison of error rate and SNR of various measurement matrices w.r.t sparsity having matrix size [273,2184]

3.3.2 Performance Analysis based on the Sparsity of input matrix

From the above we can see that the Gaussian and Binary are giving good results in maximum test cases. So,taking those two matrix as reference to get relation between sparsity and input matrix size in performance analysis.

In Fig. 3.6 graph is drawn between input matrix size versus error rate and input matrix size versus SNR for the matrix dimensions n $\times$ N*n (where n=21) for various Sparsity levels(k) ,Where size of input matrix = N*n taking Binary Matrix as reference measurement matrix.

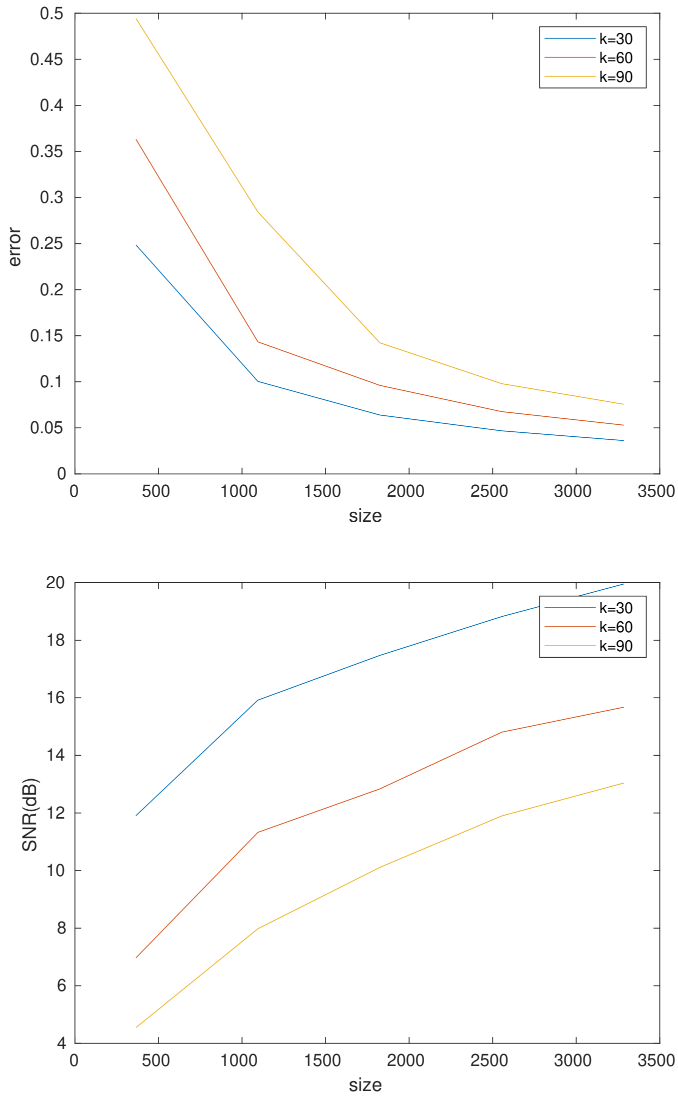

Figure 3.6: Comparison of error rate and SNR of various on sparsity w.r.t N value having matrix size [N,n*N] of a Binary matrix (where n=21)

In Fig. 3.7 graph is drawn between input matrix size versus error rate and input matrix size versus SNR for the matrix dimensions n $\times$ (N)*n (where n=21) for various sparsity levels(k), where size of input matrix = N*n taking Gaussian matrix as reference measurement matrix. Here sparsity(k) increases means no of non zero elements in input matrix increases. From the graph it is clear that sparsity increases performance decreases.

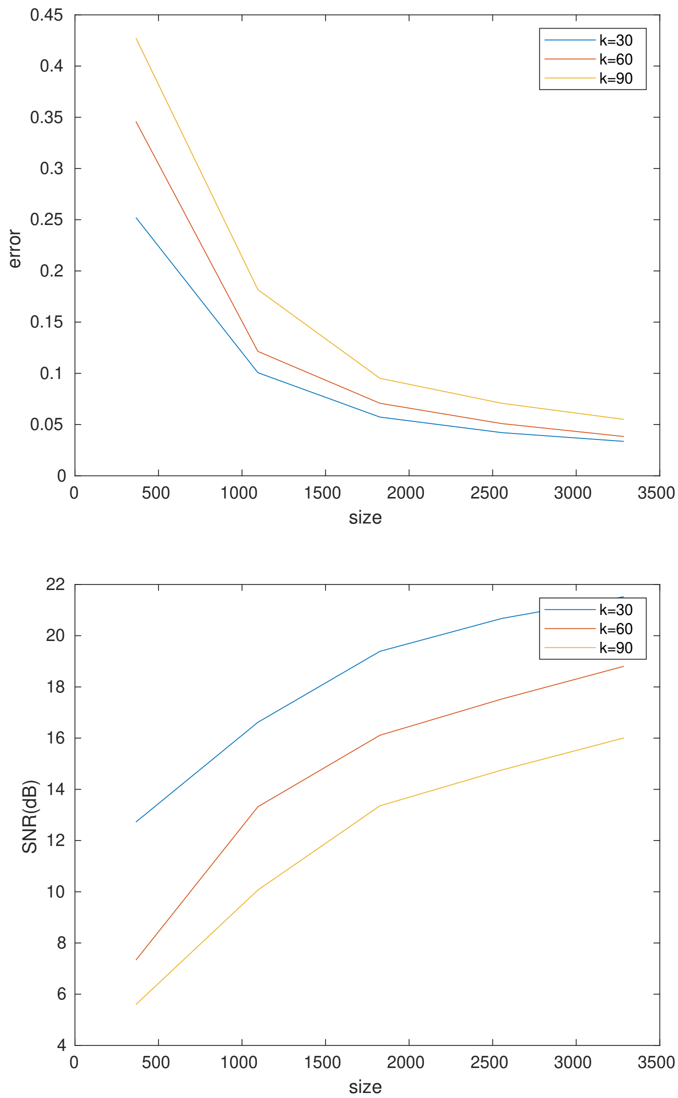

Figure 3.7: Comparison of error rate and SNR level of various on sparsity level w.r.t N value having matrix size [N,n*N] of a Gaussian matrix (where n=21)

3.4 Observations

- Gaussian and Binary measurement matrices performance is good as N value increases. Hadamard matrix is having one limitation the matrix should be 4 multiple size.
- As the sparsity level increases the error rate also increases since OMP gives better results for low sparse input signals.
- As N increases it is difficult to recover but if sparsity is constant number of non zero elements decreases since total matrix size is increasing so automatically error rate decreases.

Chapter 4

Optimization of Deterministic Measurement Matrices

In the previous chapter, we have observed that sparsity i.e, the number of non zero elements in the input matrix increases will decreases the performance. So, we are now concerned on how to get better performance at high sparsity. This can be achieved with reference to the equation (2.4) by reducing the mutual coherence value. The mutual coherence value can be minimized ref, (Elad, 2007), (Oey, 2014), (Abolghasemi *et al.*, 2010), (Xu *et al.*, 2010). In this chapter we will discuss how to construct the optimized projection matrix to reduce the mutual coherence value.

4.1 Projection Technique

As mentioned in the equation (2.4) the Sparsity level and the mutual coherence value are closely related. Suppose that the projection matrix P has been designed such that $\mu\{PD\}$ is as small as possible, this allows a wider set of candidate signals to reside under the umbrella of successful compressive sensing behaviour. Here $||\alpha_0||_0$ value increasing means number of non zero elements may present in a input matrix to get a good reconstruction using any recovery algorithm.

$$||\alpha_0||_0 \leq \frac{1}{2}(+\frac{1}{\mu\{PD\}}) \tag{4.1}$$

4.2 Projection matrix Construction

To minimize the mutual coherence value the optimized projection matrix construction is explained by the following algorithm.

Algorithm 1: How to construct a Projection matrix

Result: Optimized P matrix

Inputs

Dictionary/Measurement matrix(D), Down sampling Factor(x), Threshold of Coherence value(t), Number of iterations(I), Matrix dimensions of P,D.

Initialization

Let P matrix be an arbitrary random matrix with required size (taking Y and measurement matrix into consideration).

initialize q=0

while $q<I$ **do**

$\hat{D}$= P*D

D_q = Normalizing columns of $\hat{D}$

G_q = $\hat{D}_q.D_q$

Updation of G matrix

if $i==j$ **then**

$\hat{g}_{ij}$ = 1;

else

if $g_{ij}>t$ **then**

$\hat{g}_{ij}$ =g_{ij}*x;

else

$\hat{g}_{ij}$ =g_{ij};

end

end

Rank Reduction

Applying SVD to reduce rank of $\hat{G}_q$ to be equal to P matrix rank.

Square root

$\hat{G}_q$ = $S_q^T.S_q$ where S_q obtained should be having dimension equal to the P matrix.

P matrix updation

Updating the P matrix by minimizing the error of the $||S_q$ - PD$||^2$.

Set q=q+1

end

New Measurement matrix = PD

4.2.1 Initialization

Let D be $m \times n$ matrix. Here we will initialize P matrix be $p \times m$ of any arbitrary random matrix. To get the mutual coherence of a PD matrix first we will calculate the gram matrix(G) with reference to the equation (2.3).

4.2.2 Updation of G matrix

To decrease the mutual coherence we have to update G matrix off diagonal elements. The updation of the G matrix is the crucial part. In G matrix diagonal elements are updated as 1 to improve SNR and decreasing the off diagonal elements which are greater than the threshold value by multiplying with down sampling factor. Threshold value is our choice but we have to choose wisely i.e, it should be lower than the measurement matrix that we are going to be optimize.

4.2.3 Rank Reduction and Square Root

The rank reduction based on SVD technique. SVD decomposes any matrix into U,Δ,V. Let P matrix has the dimension of p rows. Here $\hat{G}_q$ is a square matrix here we will consider only p highest eigen values or p rows since P matrix has p rows.

From the knowledge of linear algebra $A^T A, AA^T$ are the symmetric matrices of any A matrix. For a symmetric matrix U and V of the SVD are same. Here $\hat{G}_q$ is the symmetric matrix so we can easily decompose matrix into $S_q^T S_q$. So,S_q be U$\sqrt{\Delta}$.

4.2.4 Minimization of error based on Least Square Approximation

Updating the P matrix by minimizing the error of the $||S_q - PD||^2$ based on least square approximation. We can minimize $S_q = PD$ with respect to P as $P = S_q D^T (DD^T)^- 1$.

4.3 Simulations

4.3.1 Explanation on how Projection improves performance

In Fig. 4.1 graph is drawn between sparsity versus error rate and sparsity versus SNR for the dimensions of the P matrix as [21 x 91] ,D matrix as [91 x 819], using above mentioned algorithm comparing with the dimensions of the D matrix as [21 x 819] without any Projection. Here taking Binary matrix as a reference measurement matrix, threshold value as 0.015 and down sampling factor as 0.4. From the graph it is evident that optimization based on the above algorithm decreases the error rate and increases the SNR.

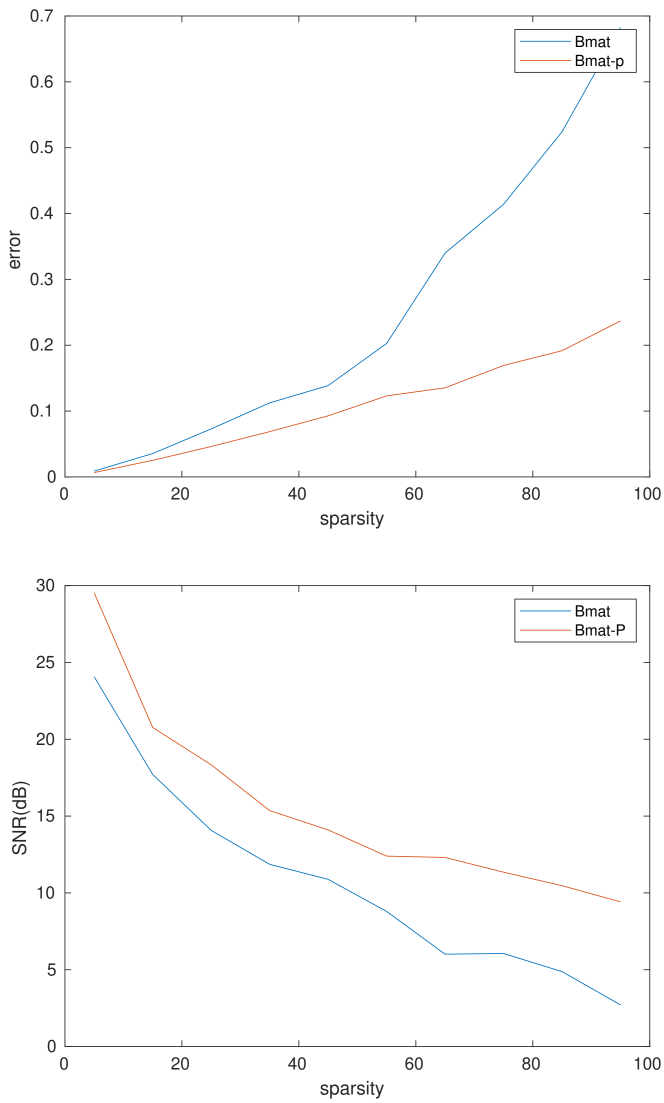

Figure 4.1: Comparison of error rate and SNR of projected and not projected Binary matrix w.r.t sparsity level having matrix size [21,819]

In Fig. 4.2 graph is drawn between sparsity versus error rate and sparsity versus SNR for the dimensions of the P matrix as [21 x 91] , D matrix as [91 x 819],Using above mentioned algorithm comparing with the dimensions of the D matrix as [21 x 819] without any Projection. Here taking Gaussian matrix as a reference measurement matrix, threshold value as 0.01 and down sampling factor as 0.2. Since Gaussian is having low mutual coherence value compared to Binary, so we have to consider a low threshold value and down sampling factor compared to Binary.From the graph it is evident that optimization based on the above algorithm decreases the error rate and increases the SNR.

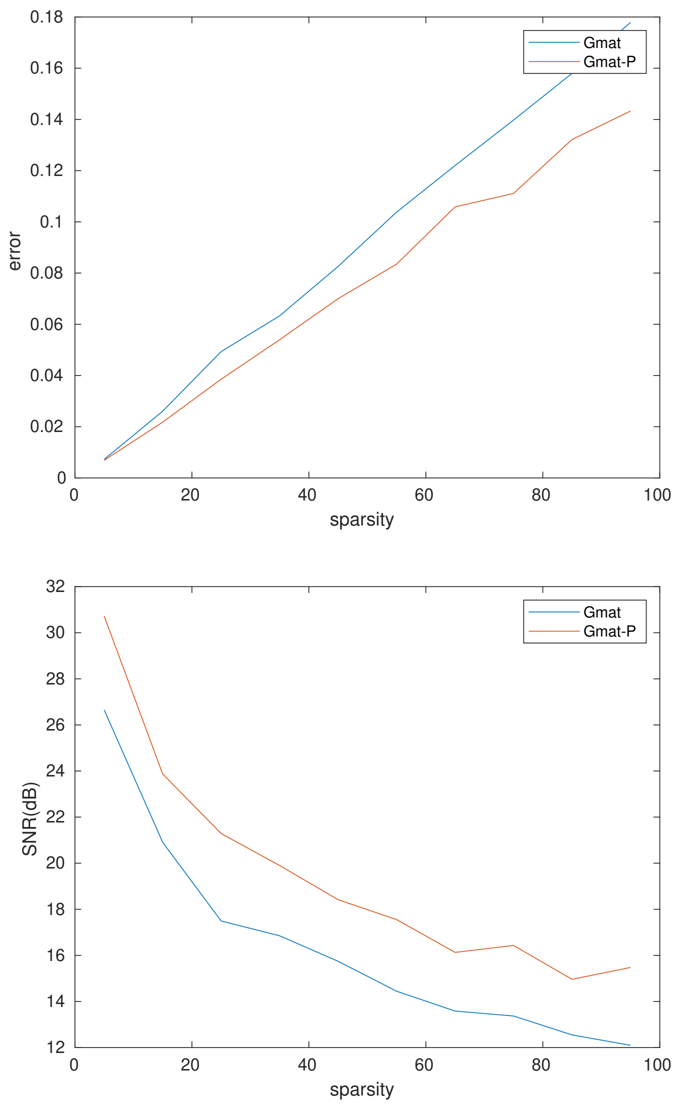

Figure 4.2: Comparison of error rate and SNR of projected and not projected Gaussian matrix w.r.t sparsity having matrix size [21,819]

In Fig. 4.3 graph is drawn between sparsity versus error rate and sparsity versus

SNR for the dimensions of the P matrix as [21 x 273] , D matrix as [273 x 2457],Using above mentioned algorithm comparing with the dimensions of the D matrix as [21 x 2457] without any projection. Here taking Binary matrix as a reference measurement matrix, threshold value as 0.015 and down sampling factor as 0.4. From the graph it is evident that optimization based on the above algorithm decreases the error rate and increases the SNR to the higher size of the matrix also.

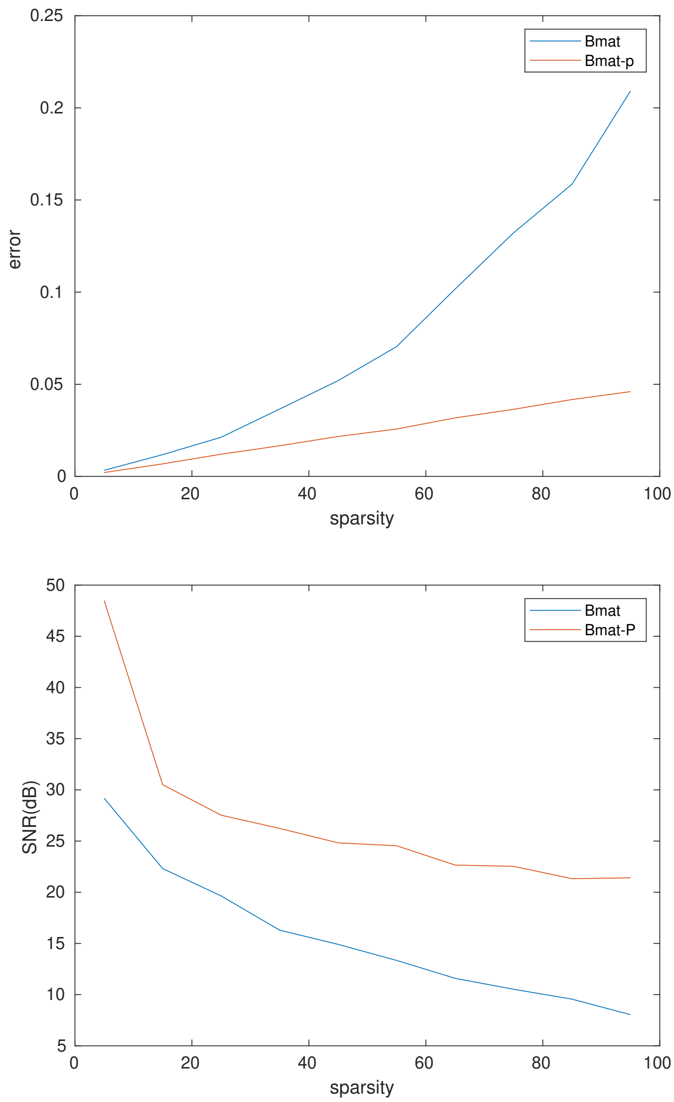

Figure 4.3: Comparison of error rate and SNR of projected and not projected Binary matrix w.r.t sparsity having matrix size [21,2457]

In Fig. 4.4 graph is drawn between sparsity versus error rate and sparsity versus SNR for the dimensions of the P matrix as [21 x 273] , D matrix as [91 x 2457], using above mentioned algorithm comparing with the dimensions of the D matrix as [21 x 2457] without any projection. Here taking Gaussian matrix as a reference measurement matrix, threshold value as 0.01 and down sampling factor as 0.2.Since Gaussian is having low mutual coherence value compared to Binary, so we have to consider a low threshold value and down sampling factor compared to Binary. From the graph it is evident that optimization based on the above algorithm decreases the error rate and increases the SNR.

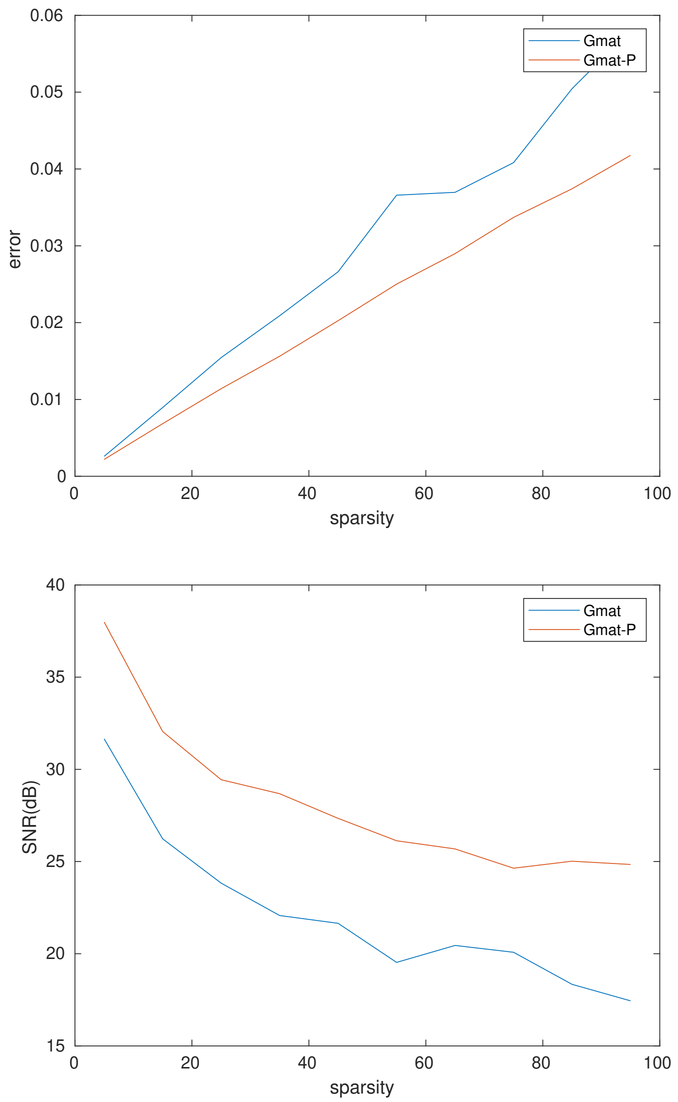

Figure 4.4: Comparison of error rate and SNR of projected and not projected Gaussian matrix w.r.t sparsity having matrix size [21,2457]

4.3.2 Performance Analysis of Random and Optimized Projections

Projecting the measurement matrix with certain dimension gives good results only when the projection matrix is constructed in an optimized way to decrease the coherence value if it is a random projection matrix then it will not give any improvement in performance besides, in some cases random projection matrix decreases performance of the original measurement matrix. The comparison between random projection and the optimized projection as shown Fig. 4.5 in having dimension of the matrix as P-[21 x 91] ,D-[91 x 819] taking Gaussian matrix as reference.

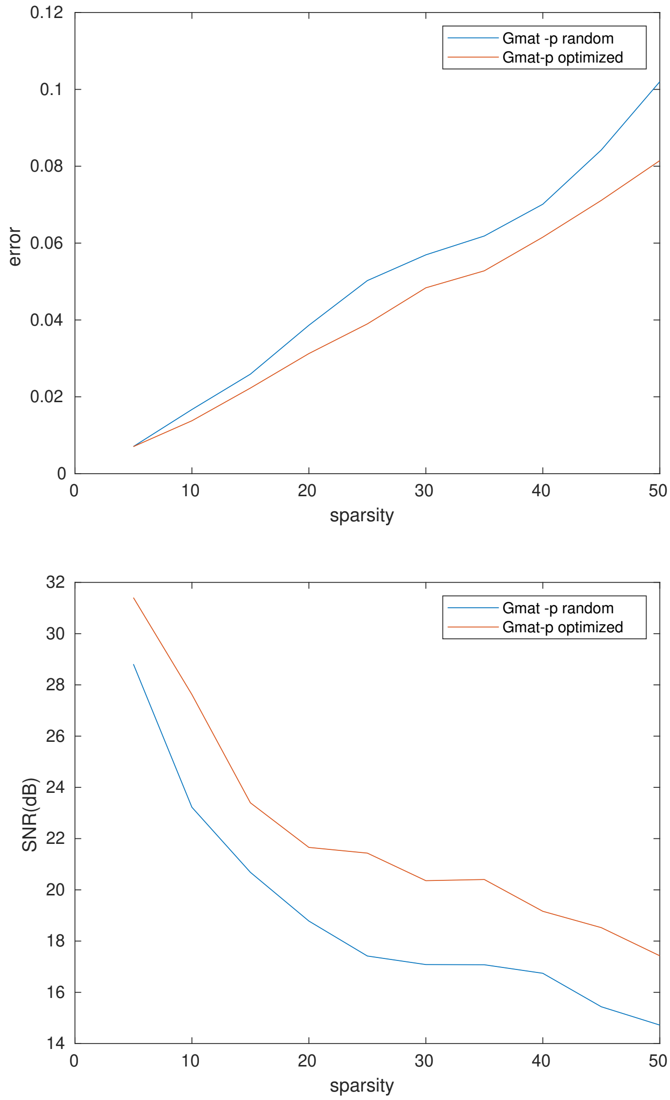

Figure 4.5: Comparing the error rate and SNR of random P and optimized P w.r.t sparsity of a Gaussian matrix

4.4 Observations

- Projection matrix P constructed using the algorithm discussed in this chapter gives better Performance.
- Only optimized P matrix gives better performance compared to random P matrix(optimized P is the projection matrix constructed using the algorithm discussed in this chapter).

Chapter 5

Conclusion

We studied compressive sensing basic principles and successfully implemented the OOC structure based deterministic matrix design to construct Gaussian, Binary, DCT, Hadamard measurement matrices and compared there performance based on the error rate and SNR. We further optimized the deterministic measurement matrix by projection technique to minimizing the mutual coherence value to get less error rate at high sparsity level.

The results obtained based on the algorithms discussed in the chapter 3 and chapter 4, it is evident that Gaussian and Binary are measurement matrices giving the good performance at maximum test conditions compared to other measurement matrices used in the project. Optimization improves their performance by reducing mutual coherence value there by having possibility to implement at high sparsity level with less error rate.

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