# Linear Model for Prediction of RF Signal Attenuation in Ionospheric Plasma and Flight Vehicle Exhaust Plumes

***Project Report***

*Submitted by*

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*for the award of the degree*

*of*

***MASTER OF TECHNOLOGY***

**DEPARTMENT OF ELECTRICAL ENGINEERING**

**INDIAN INSTITUTE OF TECHNOLOGY MADRAS**

**CHENNAI-600036**

**JUNE 2020**

**"Expect the best, plan for the worst,**
**and prepare to be surprised."**

**~ Denis Waitley.**

**"Those who plan do better than those**
**who do not plan even though they rarely**
**stick to their plan."**

**~ Winston Churchill**

# DECLARATION

I am declaring that the thesis entitled **"LINEAR MODEL FOR PREDICTION OF RF SIGNAL ATTENUATION IN IONOSPHERIC PLASMA AND FLIGHT VEHICLE EXHAUST PLUMES"** submitted by me to the Indian Institute of Technology, Madras for the award of the degree of **Master of Technology** is the result of the research work carried out by me. The contents of this report, methods followed for development of mathematical model are not plagiarised from any other source. All the copy rights of the contents are reserved.

Place: Chennai

Date: June 2020

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# CERTIFICATE

This is to certify that the thesis entitled **"LINEAR MODEL FOR PREDICTION OF RF SIGNAL ATTENUATION IN IONOSPHERIC PLASMA AND FLIGHT VEHICLE EXHAUST PLUMES"** submitted by **A.NARESH** to the Indian Institute of Technology, Madras for the award of the degree of **Master of Technology** is a bonafide record of research work carried out by him under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Place: Chennai

Date: June 2020

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## ACKNOWLEDGEMENTS

*One looks back with appreciation to the brilliant teachers, but with gratitude to those who touched our human feelings. The curriculum is so much necessary raw material, but warmth is the vital element for the growing plant and for the soul of the child.*

— Carl Jung

I really feel very honoured for having come across brilliant teachers who not only taught me the academic subjects but also touched the deepest chords of my heart. I would like to convey my genuine appreciation and heart-felt gratitude to all of them. I would like to take this opportunity to thank one and all who have contributed to my learning process.

My first interaction with my M.Tech guide, Prof. Harishankar Ramachandran, was during advanced course on "Waveguides, Antennas and Microwave circuits" which helped me a lot in understanding electro magnetics and its application in detail ,I never forget his teaching method and the way he combines applications with mathematics.

I am extremely glad that I had a chance to do my M.Tech project work under someone who truly practices and appreciates deep thinking. I will be forever indebted to him for all the time he has spent with me in discussions. Thank you very much, Sir.

Study at IIT is always a great source of learning and inspiration. I would like to thank Director of IIT Madras, HOD of electrical engineering, all other faculty members, staff of the department who helped me directly and indirectly to complete my course at IIT. I would like to convey my sincere appreciation and heart-felt gratitude to one and all.

I am thankful to my organisation DRDO, Director DRDL, Hyderabad and TD,RSD for allowing me to complete post-graduation at IIT Madras. I would like to thank Director, ITR, Balasore and ITR Telemetry division for providing opportunity and helping me to complete the project.

**- A. Naresh**
**Chennai**
**June 2020**

# ABSTRACT

Keywords: Flight vehicle, Telemetry, Plume, Plasma, Re-entry, Blackout, Attenuation, Linear model.

The main aim of this project is to develop RF attenuation and behavioural model of flight vehicle telemetry signals when passing through Plasma and exhaust Plume in S-Band, C-band and Ka-band. Telemetry link between flight vehicle and ground station is vital in knowing missile parameters, but this link suffers from two types of plasma attenuations. First one is plume attenuation occurring mostly during launch phase and whenever ground station is behind the flight vehicle. RF blackout occurs in ballistic flight vehicle during re-entry phase, while passing through ionospheric plasma region and due to plasma created around nose cone because of high kinetic energy in re-entry.

The exhaust plume can be approximated to weak plasma but is not uniform in properties. It can be divided into different layers with uniform properties in each layer. Calculations can be made by finding transfer function of each layer and overall transfer function of combined layers. The total loss in plume is the incident power minus transmitted power.

The plasma attenuation and behaviour of RF signal through it can be modelled similar to plume model by dividing into more number of layers as plasma is more dense and properties are highly non uniform within plasma. The total power loss in plasma and behaviour of signal at different frequencies is modelled.

Simplified mathematical model is developed for both plasma and plume attenuation by dividing medium into piece wise linear isotropic layers and mathematical formulations are done such that this model can be easily simulated in computers or can be easily implemented in hardware for observing behaviour of RF signal in plasma , for predicting RF attenuation in plasma.

The developed MATLAB code accepts medium profile and other parameters as input files and calculates plasma and plume losses at each instant of time and outputs data file of losses with time stamp for integration with flight vehicle telemetry simulation software. After completing evaluation with real time flight data, it will be major breakthrough in plume, plasma loss calculations as this type of linear model is not attempted till now.

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# CHAPTER 1.0
# INTRODUCTION

In this chapter importance of telemetry link between flight vehicle and ground station, problems leading to communication failure, Plume attenuation, plasma RF blackout and RF attenuation measurement model are discussed

## 1.1 TELEMETRY IMPORTANCE

Flight telemetry is the RF communication link between the flight vehicle and ground station. It is used for monitoring of vehicle's critical parameters and giving commands to vehicle such as self-destroy etc. Telemetry is vital during the development phase of flight vehicles as we get performance evaluation only through the telemetry data analysis [1][2][3].

This communication link undergoes varied attenuation processes such as ground reflections, rain, clouds, range, line of sight issues, missile plumes, ionospheric plasma [1].

All the remaining attenuations can be estimated and measured somewhat accurately, and transmission power or receiver sensitivity can be adjusted accordingly for healthy communication link. But plume attenuation and ionospheric attenuation are highly uncertain in measurement and leads to RF blackout [1][4][5].

## 1.2 IONOSPHERIC PLASMA ATTENUATION

There are two main problems in measuring plasma attenuation. The first one is the problem of accurate estimation of plasma parameters i.e electron and ion density, collision frequency of electrons with ions and neutral atoms, thickness of various density layers around the flight vehicle[1][2][6]. The second one is the problem of dividing inhomogeneous, anisotropic plasma layer into several layers of homogenous and isotropic plasma(approximately) and simple mathematical formulation of attenuation equations based on plasma parameters for simulation on computers and implementation in digital hardware.

During the re-entry phase of space shuttles or ballistic missiles, flight vehicle passes through ionospheric layer, and due to high kinetic energy of vehicle, there will be formation of high density plasma around nose cone, this plasma completely reflects the RF signal passing through it leading to RF blackout and communication failure with ground station. This phenomenon continues until missile crosses ionospheric region. Based on velocity, mass, structure of vehicle the altitude range and time of blackout varies[4][5].

If a proper mathematical model is developed to study the RF signal reflection and refraction properties in the plasma ,we can remodel or re design the transmitters at required frequencies ,transmit power levels so that we can avoid RF block out during re-entry of the vehicle.

## 1.3 PLUME ATTENUATION

Plume attenuation is another major issue in telemetry link. Plume attenuation level will be high during launch phase and whenever ground station is behind the missile. Due to high temperature in exhaust, propellent components gets ionised and creates low density plasma around the exhaust. When RF signal passes through the exhaust, it gets attenuated based on the density and width of low-density plasma layer. Similar to ionospheric plasma attenuation, there are two main problems in measuring plume attenuation[1][2][3][5][8].

First one is estimation of plasma parameters accurately i.e. electron and ion density, collision frequency of electrons with ions and neutral atoms, thickness of various density layers in the exhaust plume. Second one is dividing inhomogeneous, anisotropic plasma layer into several layers of homogeneous, isotropic(approximately) and simple mathematical formulation of attenuation equations based on plasma parameters for simulation on computers and implementation in digital hardware.

Aspect angle is the angle between missile axis and ground station look up line. It decides angle of incidence of RF signal on plume and refraction angle out of the plume. So, aspect angle also decides the amount of attenuation in the plume exhaust [1][2].

The following figures 1.1 and 1.2 show the plasma attenuation variations with altitude of the flight vehicle. The density of plasma around the flight vehicle changes randomly because of change in composition and change in density of components present in the atmosphere and temperature of that altitude etc. Figure 1.1 shows the plume attenuation of RF signal and aspect angle.

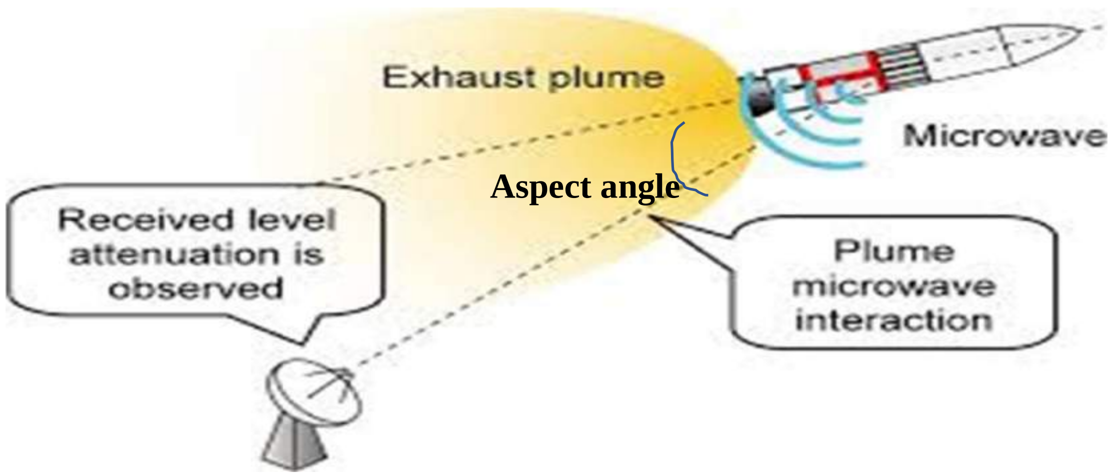



Figure 1.1: Plume attenuation with aspect angle

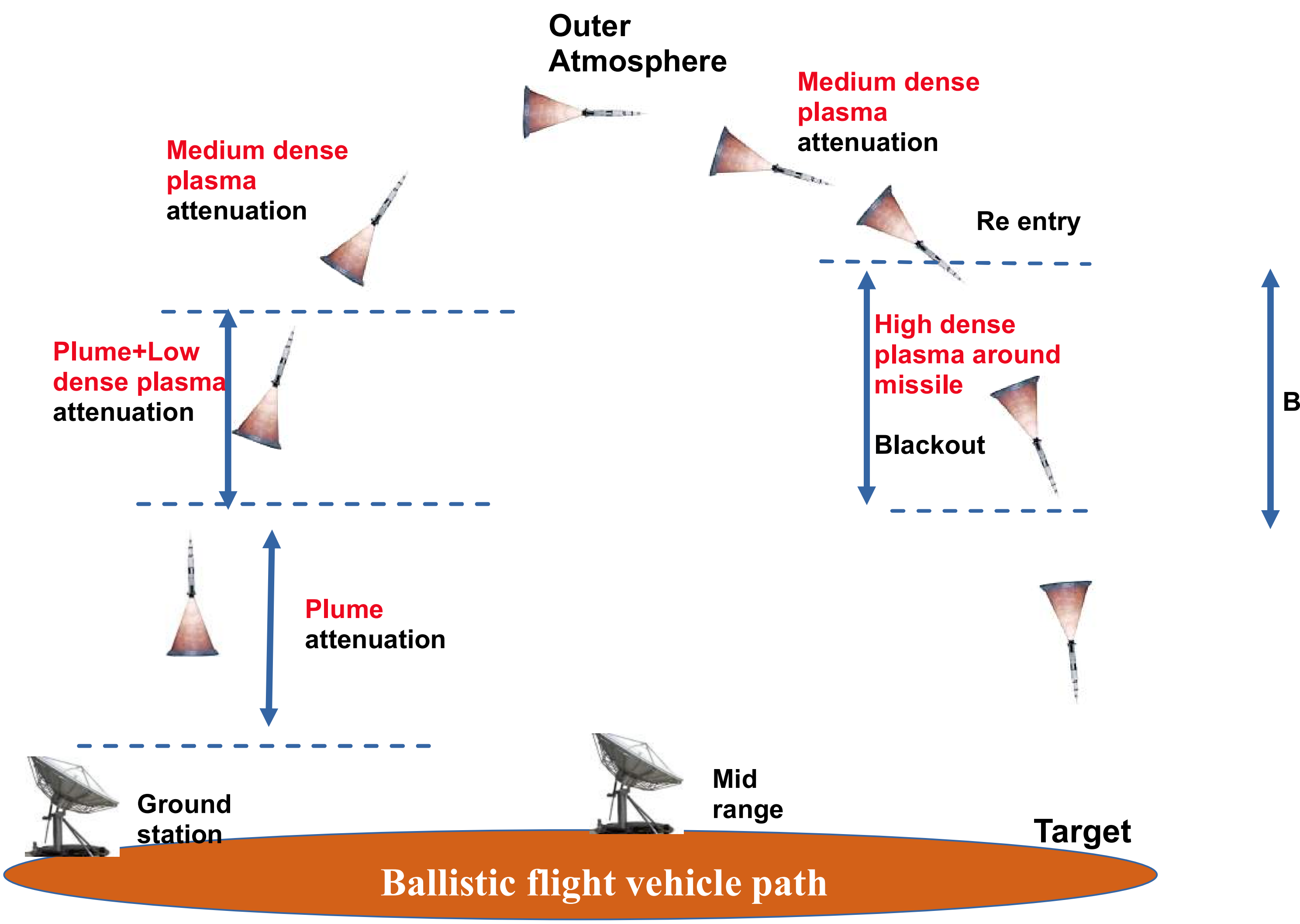



**Figure 1.2: Flight vehicle path with plume and plasma attenuation and RF blackout zones [1]**

Figure 1.3.1 : High dense plasma around flight vehicle during re-entry

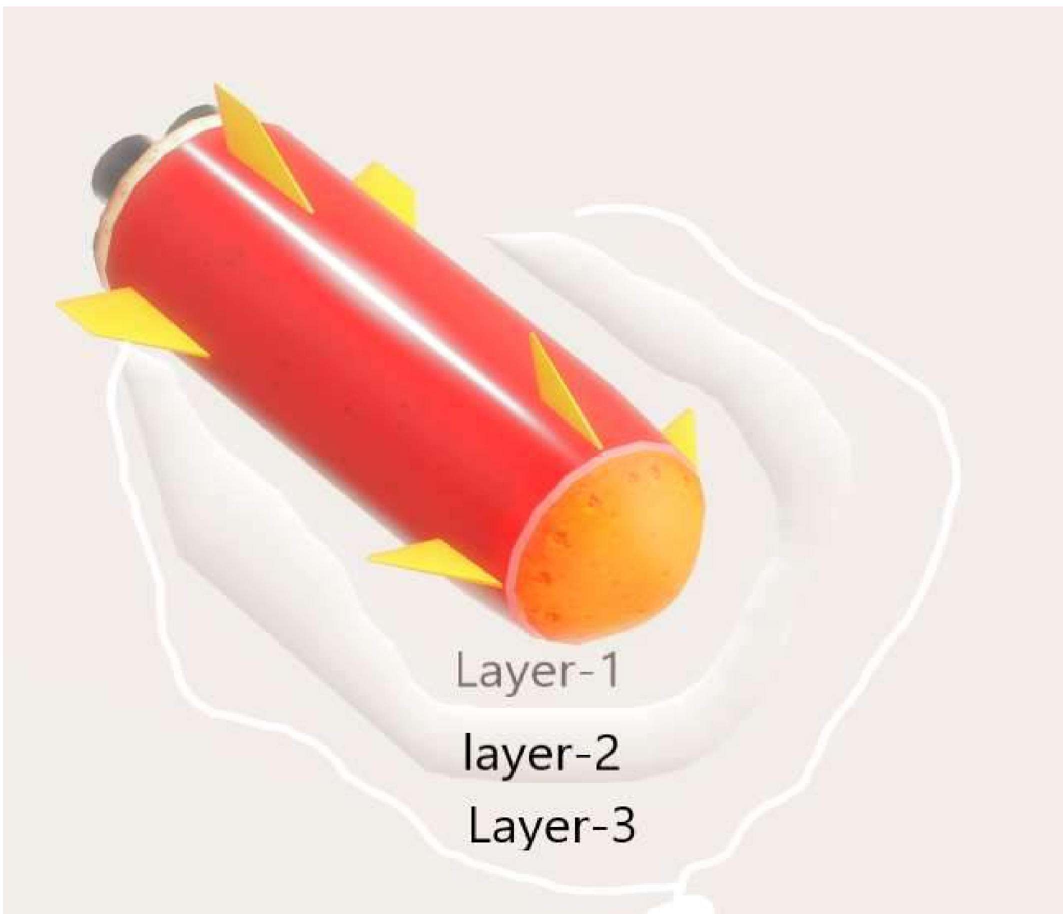



Figure 1.3.2 : different density plasma layers around flight vehicle during re-entry [1]

## 1.4 MODEL REQUIREMENT AND DESCRIPTION

Scope of the project involves development of simple linear model so that it can be implemented in digital hardware. As plasma is inhomogeneous and anisotropic, we cannot develop simple and linear model, but it can be divided into number of layers with almost same properties within each layer i.e each layer is assumed as homogenous and isotropic. By applying reflection and refraction properties of EM waves on multi layers, we can find the overall reflection and transmission coefficients, transmitted power. Matching and propagation matrix method is chosen to find transfer matrix of multilayer. The resulting equations are implemented in MATLAB for simulation of EM wave propagation in plasma.

### 1.4.1 The inputs to the model :

1.Number of plasma layers (n)
2.width of each layer ($l_d$)
3.components density in each layer
4.Collision frequency in each layer(v)
5.Incident angle
6. Aspect angle
7.Input frequency /frequency range
8.Polarization of wave

### 1.4.2 The outputs of the model:

1.Transmitted power spectrum
2.Power loss Vs Time stamp
3.Power loss Vs Altitude

## 1.5 CHALLENGES

In summary this model will be highly useful in establishing good telemetry link between ground station and flight vehicle and it helps in simulating expected loss calculation of channel before flight vehicle launch. Even though plume and plasma calculation models are developed, linear model is a challenging task as no such attempts are made in India till now. After Studying the problem thoroughly and going through the literature available ,we have arrived at the solution of dividing plasma region of plume or ionospheric plasma into small cascaded layers of almost equal temperature so that it can be approximately assumed as isotropic medium. Then next challenging task is relating RF signal loss equations with available plasma regions. After careful study it is observed that the main parameters which influence RF signal power loss in plasma are plasma density and collision frequency. From these two parameters we can find propagation constant, relative permittivity, intrinsic impedance and hence power loss in the plasma [6]. But next problem observed in this task is plasma density calculation in the given plasma region. Based on literature survey, arrived at the solution of using Saha equation [9] in estimating plasma density of the medium. Therefore, this project main input is plume or plasma medium profile with temperature, components and component densities. If these inputs are provided along with other required inputs such as RF signal frequency, polarisation, aspect angle of flight vehicle with respect to ground station etc., this model calculates both plasma and plume loss.

Plume temperature is generally high ranging approximately from 500K to 3000K[1]based on the type of flight vehicle, whereas temperature of re-entry plasma during blackout will be from 2000K to 4500K[1] or even more based on design of flight vehicle. The gas composition and densities of plume depends on flight vehicle propellants. The medium profile considered during CFD analysis of plume, nose cone of of flight vehicle during re-entry will be the main input for our model[1]. Therefore, the study of ionosphere, its composition, temperature variations are not given much thrust in this thesis. All the calculations will be based on the medium profile data provided by the user only.

# CHAPTER 2.0

# PLASMA RF PROPERTIES

## 2.1 PLASMA

Plasma is a special kind of ionised gas consisting of positive ions, electrons and neutrals(atoms, molecules and radicals).We can call ionised gas as plasma if it is quasi neutral and properties are dominated by electric and/or magnetic forces rather than hydro dynamic forces. Plasma is a state of matter in which an ionized gaseous substance becomes highly electrically conductive to the point that long-range electric and magnetic fields dominates[2][10].

### 2.1.1 Ionospheric Plasma

As we rise above the earth's surface, we encounter a partially ionized gaseous layer from around 50Km altitude to 1000km altitude, this ionized layer is called Ionosphere [6]. The ionosphere is a medium which consists of free electrons, ions and neutral molecules. The electron density gradually increases with altitude, becomes maximum and then gradually decreases as atmosphere density decreases with altitude. So the altitude range of 200Km to 50Km [1] is the main concern of plasma for re-entry flight vehicles.

When flight vehicle passes through the Ionosphere, Due to high kinetic energy of vehicle, there will be creation of very high temperature around the nose cone ranging from 2500 to 4000 degree centigrade[1], which creates very high dense plasma even in low dense atmospheric plasma leading to attenuation of telemetry signal.[1]

### 2.1.2 Exhaust plumes plasma

The exhaust of flight vehicle will be at high temperature leading to ionization of exhaust propellent materials leading to low dense plasma. It will have low density of electrons and collision frequency compared to Ionospheric plasma around the missile [3].

Plasma frequency and collision frequency are also less in plume plasma, with small layer width. Therefore, complete attenuation will not be there but high attenuation with fluctuation in power loss [1][3].

## 2.2 KEY PLASMA PARAMETERS

The key properties of the plasma which effect the propagation of electromagnetic waves inside the plasma are electron density, collision frequency of electrons with ions and neutrals and width of plasma. These three parameters of plasma decide electromagnetic wave attenuation inside the plasma layer. Plasma frequency depends on plasma density [3][4][8].

### 2.2.1 Electron density ($n_e$)

Because of the presence of free charge carriers and electrons present in the plasma, plasma reacts to the electromagnetic fields, conducts electrical current and possesses well defined potential.

For a plasma containing only singly charged ions, the ion population is adequately described by the ion density $n_i$ which is almost equal to electron density $n_e$ . The neutral density is indicated by $n_a$ [2][3].

Electron density = No of free electrons/volume

= No of electrons per cubic metre

### 2.2.2 Collision frequency ($\upsilon$ )

The frequency with which electrons collides with ions and neutral molecules within the plasma is collision frequency [6]. It is important parameter which decides attenuation level of electromagnetic wave inside the plasma [3][4].

### 2.2.3 Plasma frequency ($\omega_p$)

The natural frequency of electron oscillations within the plasma is called plasma frequency. It is function of electron density in the plasma. Electromagnetic waves above the plasma frequency can pass through the plasma [10][3].

### 2.2.4 Plasma width

Plasma layer width decides amount of attenuation in plasma. But plasma is anisotropic and inhomogeneous [3][4]. To develop model for attenuation, it is divided into number of layers such that properties are almost same within each layer. Then each layer is assumed as isotropic so that attenuation can be calculated easily.

## 2.3 Plasma as Dielectric medium

Maxwell's equations [10] are

$\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$..................(2.3.1)

$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$..............(2.3.2)

$\nabla \cdot \vec{D} = \rho$.......................(2.3.3)

$\nabla \cdot \vec{B} = 0$.........................(2.3.4)

For a perfect dielectric σ=0 (J=0) and ρ = 0

and equations 2.3.2 and 2.3.4 change into

$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$ and $\nabla \cdot \vec{D} = 0$

substituting time and space transform

$\nabla = -jk$ and $\frac{\partial}{\partial t} = j\omega$, $D=\varepsilon_0 \varepsilon_r E$ and $B=\mu H$

where $\varepsilon_r$ is relative permittivity of the dielectric and μ is permeability
The above equations will be

$\nabla \times \vec{E} = -j\omega\mu\vec{H}$................2.3.1

$\nabla \times \vec{H} = j\omega\varepsilon_0\varepsilon_r\vec{E}$...............2.3.2

$\nabla \cdot \varepsilon_0\varepsilon_r\vec{E} = 0$.......................2.3.3

$\nabla \cdot \vec{B} = 0$..............................2.3.4

But for plasma σ and ρ are not zero and it is a lossy medium
From equation of continuity

$\nabla \cdot \vec{J} = \frac{-\partial \rho}{\partial t}$

$\nabla \cdot \vec{J} = -j\omega\rho$

$\nabla \cdot \sigma\vec{E} = -j\omega\rho$

$\frac{\nabla \cdot \sigma\vec{E}}{-j\omega} = \rho$................................2.3.5

But
$\nabla \cdot \vec{D} = \rho$

$\nabla \cdot \varepsilon_0 \vec{E} = \rho$ substituting in equation 2.3.5

$$\nabla \cdot \varepsilon_0 \vec{E} = \frac{\nabla \cdot \sigma \vec{E}}{-j\omega}$$

$\nabla \cdot \varepsilon_0 \left(1 + \frac{\sigma}{j\omega\varepsilon_0}\right)\vec{E} = 0$ .......................2.3.6

comparing with dielectric equation 2.3.3

Plasma can be considered as perfect dielectric with $\varepsilon_r = 1 + \frac{\sigma}{j\omega\varepsilon_0}$ .......2.3.7

The conductivity of plasma [10] is given by

$$\sigma = \frac{n_e q^2}{m_e j \cdot (\omega + j\upsilon)}$$

where $n_e$ is electron density in plasma

$m_e$ is mass of electron

q is charge of electron

$\upsilon$ is collision frequency

and ω is incident wave frequency

substituting this in equation 2.3.7

we get $\varepsilon_r = 1 - \frac{n_e q^2}{m_e \varepsilon_0 \omega^2 \cdot \left(1 + \frac{j\upsilon}{\omega}\right)}$

But plasma frequency $\omega_p = \sqrt{\frac{n_e q^2}{m_e \varepsilon_0}}$

Therefore, plasma can be assumed as dielectric with relative permittivity

$\varepsilon_r = 1 - \frac{\omega_p^2}{\omega^2 \cdot \left(1 + \frac{j\upsilon}{\omega}\right)}$...................2.3.8

for plasma with high incident wave frequency, we can ignore collision frequency (frequency of interest to the user is 2.3Ghz).

Then relative permittivity, then can be approximated to

$\varepsilon_r = 1 - \frac{\omega_p^2}{\omega^2}$ .....................2.3.9

By substituting the values of $m_e$, q and $\varepsilon_0$ we get plasma frequency as

$f_p = 8.9\sqrt{n_e}$ …………..2.3.10

If ω changes or $n_e$ changes, there will be change in relative permittivity of plasma effecting wave propagation in plasma.

# CHAPTER 3.0

# SAHA EQUATION

## 3.1 SIGNIFICANCE

For a gas medium at a high temperature ( measured in energy units keV or Joules) and or density, the thermal collisions of the atoms will ionize some of the atoms, making an ionized gas. When several or more of the electrons that are normally bound to the atom in orbits around the atomic nucleus are freed, they form an independent electron gas cloud co-existing with the surrounding gas of atomic ions and neutral atoms. In turn, this generates an electric field, where the motion of charges generates currents, making a localised magnetic field, and creates the state of matter called plasma[11].

The Saha equation describes the degree of ionization for any gas in thermal equilibrium as a function of the temperature, density, and ionization energies of the atoms[9][12] i.e the Saha equation is a mathematical formula that allows us to find the relative concentrations of electron and ion densities in a medium if densities of individual components and temperature are known along with ionisation potentials of components. This expression is developed by Meghnad saha in 1920.

## 3.2 SAHA EQUATION

For a gas composed of a single atomic species, the Saha equation is written as[9]

$$\frac{n_{i+1}}{n_i} = \frac{2}{n_e}\frac{g_{i+1}}{g_i} \cdot \frac{(2\Pi m_e KT)^{\frac{3}{2}}}{h^3} \cdot e^{\frac{-\chi_i}{KT}} \qquad \text{3.2.1}$$

where

$n_{i+1}$ = density of atoms in ionization state i+1(per cubic metre)

$n_i$ = density of atoms in ionization state i (per cubic metre)

$n_e$ = density of electrons (per cubic metre)

$g_{i+1}$ = partition function of ionization state i+1

$g_i$ = partition function of ionization state i

$m_e$ = mass of electrons(Kg)

k = Boltzmann constant (Joules/Kelvin)

T = Temperature ( kelvin)

h = Planck constant (Joule sec)

For plasma 1st ionization is significant for KT<1ev only. When ions are ionized only once then density of ions($n_{ei}$), is equal to electron density ($n_e = n_{ei}$). If n is the density of neutral atoms before ionization, then $n-n_{ei}$ represents density of neutral atoms after ionization.

The above saha equation 3.2.1 then can be represented as

$$\frac{n_{ei}}{\mathrm{n}-n_{ei}} = \frac{2}{n_e}\frac{g_1}{g_0} \cdot \frac{(2\Pi m_e KT)^{\frac{3}{2}}}{h^3} \cdot \mathrm{e}^{\frac{-\chi_i}{KT}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.2.2$$

But $n_{ei} = n_e$ therefore above equation changes to

$$\frac{n_e}{n-n_e} = \frac{2}{n_e}\frac{g_1}{g_0} \cdot \frac{(2\Pi m_e KT)^{\frac{3}{2}}}{h^3} \cdot \mathrm{e}^{\frac{-\chi_i}{KT}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.2.3$$

For a given component $g_1$ , $g_0$ , $\chi_1$ are constants ,$m_e$ , k , h are constants. If we know the neutral density n of the component and temperature T of the gas, we can find the only unknown quantity, electron density $n_e$ .

In the above equation we can observe that electron density $n_e$ is depends on temperature T, as the temperature of the gas increases electron density hence plasma density increases.

**3.3 Hydrogen Plasma**

The usage of Saha equation can be easily explained with hydrogen plasma. If we consider ionization of hydrogen gas at high temperature, then the density of electrons due to ionization can calculated from 3.2.2 as

$$\frac{n_e}{n-n_e} = \frac{2}{n_e}\frac{g_1}{g_0} \cdot \frac{(2\Pi m_e KT)^{\frac{3}{2}}}{h^3} \cdot \mathrm{e}^{\frac{-\chi_i}{KT}}$$

where

n = density of neutral hydrogen atoms (per cubic metre)

$n_e$ = density of electrons (per cubic metre)

$g_1$ = Partition function of ionization state 1 (1)

$g_0$ = Partition function of neutral hydrogen (2)

$m_e$ = mass of electron(Kg)

k = Boltzmann constant(Joule/Kelvin)

T = Temperature in kelvin

h = Planks constant

$\chi_1$ = First ionization potential of hydrogen

$$\frac{n_e \cdot n_e}{n - n_e} = \frac{1}{1} \cdot \frac{(2\Pi m_e KT)^{\frac{3}{2}}}{h^3} \cdot e^{\frac{-13.6}{KT}} \text{ ...........3.3.1}$$

If we know the density of neutral atoms n at given temperature T , we can estimate the electron density $n_e$ of heated hydrogen gas as remaining parameters are constants and only unknown quantity is $n_e$. From this plasma density and hence electromagnetic wave propagation characteristics inside this hydrogen plasma can be calculated.

**3.4 General gas medium**

Consider a gas with more than one component. Specifically, let us consider two components.

Let $n_1$, $n_2$ be the neutral state densities of the three components and $\chi_1, \chi_2$ are their first ionisation potentials. Let $n_{e1}$ , $n_{e2}$ be the electron densities and $n_1$ , $n_2$ be the ion densities at the given temperature T due to ionisation. The density of ions is equal to the density of electrons.

Considering first ionisation.

$$\frac{n_{e1} \cdot n_e}{n_1 - n_{e1}} = \frac{2g_1}{g_0} \cdot \frac{(2\Pi m_e KT)^{\frac{3}{2}}}{h^3} \cdot e^{\frac{-\chi_i}{KT}} \text{ .....................3.4.1}$$

At given temperature the right-hand side of the Saha equation will be constant for each component. Let these constants are $C_1$ , $C_2$ for the two components.

The total electron density due to ionization of two components is

$n_e = n_{e1} + n_{e2}$ .

The Saha equation for the two components will be

component 1:

$$\frac{n_{e1} \cdot n_e}{n_1 - n_{e1}} = C_1 \text{ ............................................. 3.4.2}$$

where

$n_{e1}$ = The density of electrons due to ionisation

$n_1 - n_{e1}$ = The density of neutral components of 1 after ionisation

$n_e = n_{e1} + n_{e2}$ is total electron density.

Similarly for the component-2 :

$$\frac{n_{e2} \cdot n_e}{n_2 - n_{e2}} = C_2 \text{ ....................................3.4.3}$$

by substituting $n_e$ , we get two equations with two unknowns $n_{e1}$ , $n_{e2}$

**Solution-1:**

Let $x = n_{e1}$ and $y=n_{e2}$ then $n_e = x+y$

Equation 3.3.1 and 3.3.2 changes to

$x^2 + xy + C_1 x = n_1.C_1$…………….3.4.4

$y^2 + xy + C_2 y = n_2.C_2$…………...3.4.3

but $n_1$ , $n_2$ are known and $C_1$ and $C_2$ are constants.

Therefore n1.c1 is a constant, let $K_1 = n_1.C_1$ and $K_2 = n_2.C_2$ . The equations change as

$x^2 + xy + C_1 x = K_1$……………3.4.5

$y^2 + xy + C_2 y = K_2$…………...3.4.6

by using substitution method to solve above nonlinear equations.

Value of y from equation 3.4.5 is $y= (K_1 – C_1 x – x^2 )/x$…….3.4.7

Substituting this in equation 3.4.6 and simplifying, we get

$(C_1 - C_2 ) x^3 + ( C_1^2 – C_1 C_2 - K_1 K_2 ) x^2 + (C_2 K_1 - 2K_1C_1 ) x + K_1^2 = 0$ ………………3.4.8

It is 3rd order equation with only one unknown variable x, by solving this we get x.

then we get y from equation 3.4.7

total electron density is x+y.

**Solution-2:**

By solving the equations in MATLAB using the function *fsolve*() (used to find solution of nonlinear set of equations) , we can get the values of electron densities of two components hence total electron density in the gas medium.

The same equation can be extended to any number of components in the gas medium but only problem is, as order of nonlinear equations increases, complexity of solution also increases.

### 3.5 PLASMA DENSITY VARIATION OF HYDROGEN GAS MEDIUM

To observe the variation of plasma density in the gas medium with temperature, let us consider hydrogen gas. Plasma density depends on two variables for given component, they are molecular density and temperature.

**Case -1 :** Fixed density , variable Temperature(300K to 10000K)

Let the density of hydrogen is $n = 5x10^{22}$ molecules/$m^3$

Let the electron density is $n_e$

We know the remaining constants from literature

Ionisation potential of hydrogen, $\chi_1$=13.6ev,

Mass of electron is $m_e$=9.1x$10^{-31}$ Kg,

Planck constant, h=6.626 x$10^{-34}$ Joule-sec

Boltzmann constant K=1.38x$10^{-23}$ Joules/Kelvin

Hydrogen partition function $g_1/g_0$=1/2

The saha equation for hydrogen (3.3.1)is

$$\frac{n_e \cdot n_e}{n - n_e} = \frac{2g_1}{g_0} \cdot \frac{(2\Pi m_e KT)^{\frac{3}{2}}}{h^3} \cdot \mathrm{e}^{\frac{-\chi_1}{KT}}$$

$$\frac{n_e \cdot n_e}{n - n_e} = \frac{2g_1}{g_0} \cdot \frac{(2\Pi m_e K)^{\frac{3}{2}}}{h^3} \cdot T^{1.5} . \mathrm{e}^{\frac{-\chi_1}{KT}}$$

Substituting the constants in the right-hand side of the equation

$$\frac{n_e \cdot n_e}{n - n_e} = \frac{2g_1}{g_0} \cdot \frac{(2\Pi m_e K)^{\frac{3}{2}}}{h^3} \cdot T^{1.5} . \mathrm{e}^{\frac{-\chi_1}{KT}}$$

$$\frac{n_e \cdot n_e}{n - n_e} = 2.4093x10^{21} \cdot T^{1.5} . \mathrm{e}^{\frac{-1.579*10^5}{T}}$$

Here $n_1 = 5x10^{22}$ atoms/$m^3$ . At given temperature, the above equation is quadratic, by solving we can get $n_e$ value

If T=2000K then

$$\frac{n_e \cdot n_e}{n_1 - n_e} = 2.4093x10^{21} \cdot T^{1.5}.\mathrm{e}^{\frac{-1.579*10^5}{T}}$$

$$\frac{n_e \cdot n_e}{5 * 10^{22} - n_e} = 1.1208 * 10^{-8}$$

It is quadratic equation in $n_e$ upon solving we get $n_e = 2.3673x10^{07}$ electrons/$m^3$

This can be simulated in MATLAB by varying T . Variations are plotted and discussed in section 3.7

**Case -2 :** Fixed Temperature, variable density

Let T= 2000K. from above data

$$\frac{n_e \cdot n_e}{n - n_e} = \frac{2g_1}{g_0} \cdot \frac{(2\Pi m_e K)^{\frac{3}{2}}}{h^3} \cdot T^{1.5}.\mathrm{e}^{\frac{-\chi_1}{KT}}$$

$$\frac{n_e \cdot n_e}{n - n_e} = 2.4093x10^{21} \cdot T^{1.5}.\mathrm{e}^{\frac{-1.579*10^5}{T}}$$

At T=2000K

$$\frac{n_e \cdot n_e}{n - n_e} = 2.4093x10^{21} \cdot T^{1.5}.\mathrm{e}^{\frac{-1.579*10^5}{T}}$$

$$\frac{n_e \cdot n_e}{n - n_e} = 1.1208 * 10^{-8}$$

The above function is quadratic and depends on $n_1$.

If $n = 4x10^{20}$ , on solving above equation, we get $n_e$ =2..11x$10^{07}$

By varying n , we can observe the variation of plasma density. The variation of plasma frequency and

**3.6 PLASMA DENSITY OF HYDROGEN-NITROGEN GAS**

Let us consider a gas medium with two components hydrogen and nitrogen at the temperature of 3000 Kelvin. Let the density of hydrogen is $5x10^{22}$ atoms/$m^3$ and density of nitrogen is $4x10^{21}$ atoms/$m^3$ .

The first ionisation potential of nitrogen is 14.6 ev and first ionisation potential of hydrogen is 13.6ev.

Given information is $n_1 = 5x10^{22}$ atoms/$m^3$ and $n_2 = 4x10^{21}$ atoms/$m^3$

Temperature T=3000 K ,

We know the remaining constants from literature

Ionisation potential of hydrogen, $\chi_1$=13.6ev,

Ionisation potential of nitrogen, $\chi_2$=14.6ev,

Mass of electron is $m_e$=9.1x10$^{-31 Kg}$,

Planks constant, h=6.626 x10$^{-34}$ Joule-Sec

Boltzmann constant K=1.38x10$^{-23}$ Joules/Kelvin

Hydrogen $g_1/g_2$=1/2, Nitrogen $g_1/g_2$=3/4

Let $n_{e1}$ and $n_{e2}$ are electron densities of hydrogen and nitrogen respectively and total electron density is $n_e = n_{e1} + n_{e2}$

Applying saha equation for hydrogen:

$$\frac{n_{e1} \cdot n_e}{n_1 - n_{e1}} = \frac{2g_1}{g_0} \cdot \frac{(2\Pi m_e KT)^{\frac{3}{2}}}{h^3} \cdot e^{\frac{-\chi_1}{KT}}$$

Substituting values ,the right hand side of the equation is $C_1 = \frac{2g_1}{g_0} \cdot \frac{(2\Pi m_e KT)^{\frac{3}{2}}}{h^3} \cdot e^{\frac{-\chi_1}{KT}}$

$C_1 = 5.5162x10^3$

Substituting  in the equation , we get

$\frac{n_{e1} \cdot (n_{e1} + n_{e1})}{5x10^{22} - n_{e1}} = 5.5162x10^{03}$ ………………..(1)

Applying saha equation for nitrogen:

$$\frac{n_{e2} \cdot n_e}{n_2 - n_{e2}} = \frac{2g_1}{g_0} \cdot \frac{(2\Pi m_e KT)^{\frac{3}{2}}}{h^3} \cdot e^{\frac{-\chi_2}{KT}}$$

Substituting values ,the right hand side of the equation is $C_2 = \frac{2g_1}{g_0} \cdot \frac{(2\Pi m_e KT)^{\frac{3}{2}}}{h^3} \cdot e^{\frac{-\chi_2}{KT}}$

$C_2 = 172.6410$

$$\frac{n_{e2} \cdot n_e}{n_2 - n_{e2}} = C_2$$

Substituting in the equation , we get

$\frac{n_{e2} \cdot (n_{e1} + n_{e1})}{4x10^{21} - n_{e2}} = 172.6410$ ………………..(2)

Solving the equations, we get (1) and (2) we can get the electron densities $n_{e1}$ and $n_{e2}$ and hence total electron density of the plasma medium. Using $n_e$ ,we can estimate the plasma RF properties.

### 3.7 VARIATION OF PLASMA DENSITY AND PLASMA FREQUENCY WITH TEMPERATURE AND NEUTRAL GAS DENSITY

Plume temperature is generally extremely high ranging approximately from 500K to 3000K[1] based on the type of flight vehicle, whereas re-entry plasma during blackout will be from 2000K to 4500K [1]or even more based on design of flight vehicle. The effect of neutral concentration on plasma frequency and plasma density is observed at different values of temperature. The effect of temperature on plasma frequency and plasma density is observed at different values of neutral concentration.

Figures 3.7.1 to 3.7.3 shows the variation of plasma frequency and plasma density with variation of temperature from 2500K to 5000K for the hydrogen gas at different densities of $5x10^{20}$ molecules/$m^3$ , $5x10^{22}$ molecules/$m^3$ and $5x10^{24}$ molecules/$m^3$ .It is observed that plasma frequency and Plasma density are increasing continuously with rise in temperature. As the neutral density increases, the effect of temperature is more on plasma frequency and plasma density.

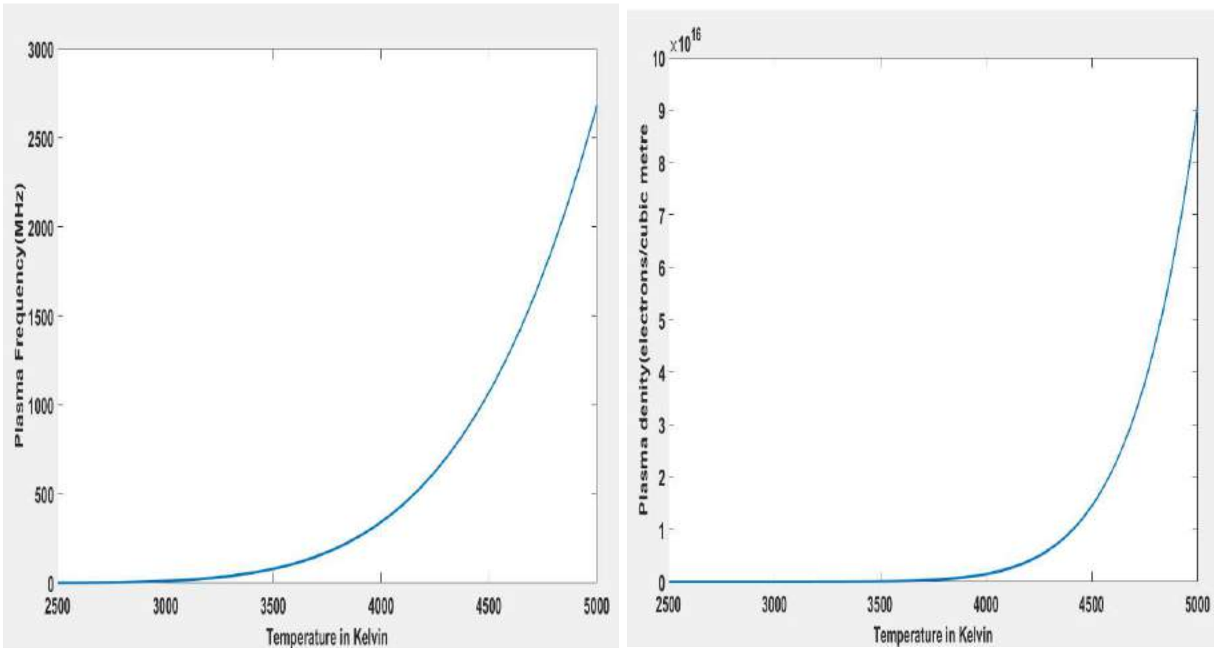



Figure 3.7.1 : Variation of plasma frequency and density with temperature at n=5x$10^{20}$ atoms/$m^3$

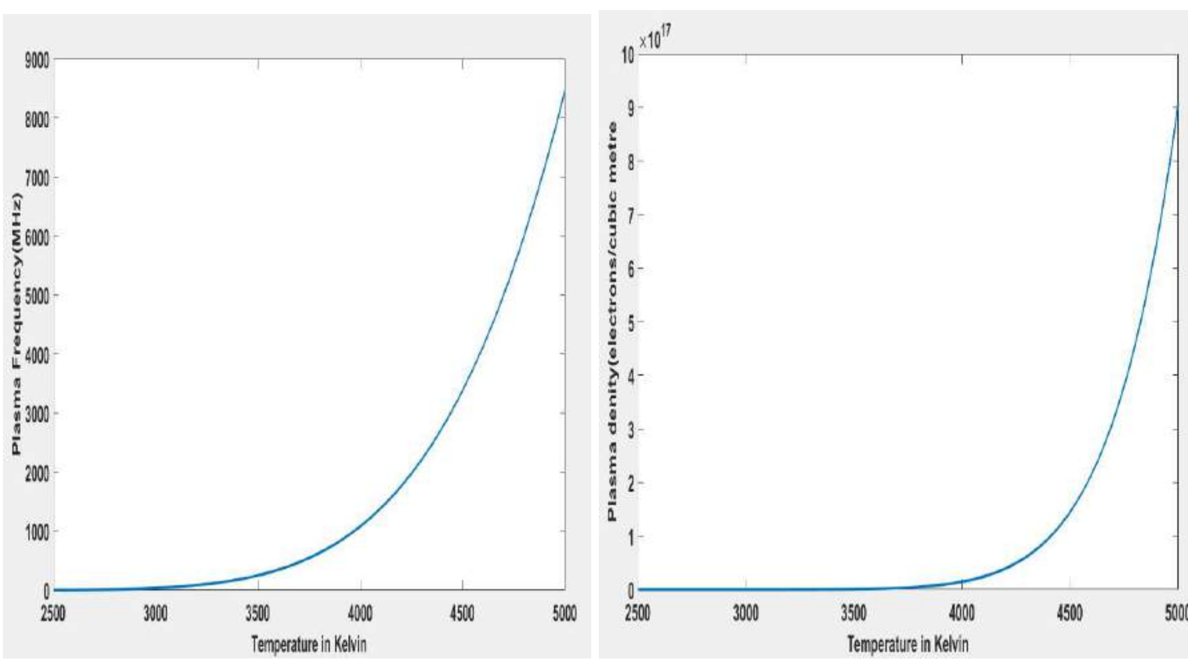



Figure 3.7.2 : Variation of plasma frequency and density with temperature at n=5x$10^{22}$ atoms/$m^3$

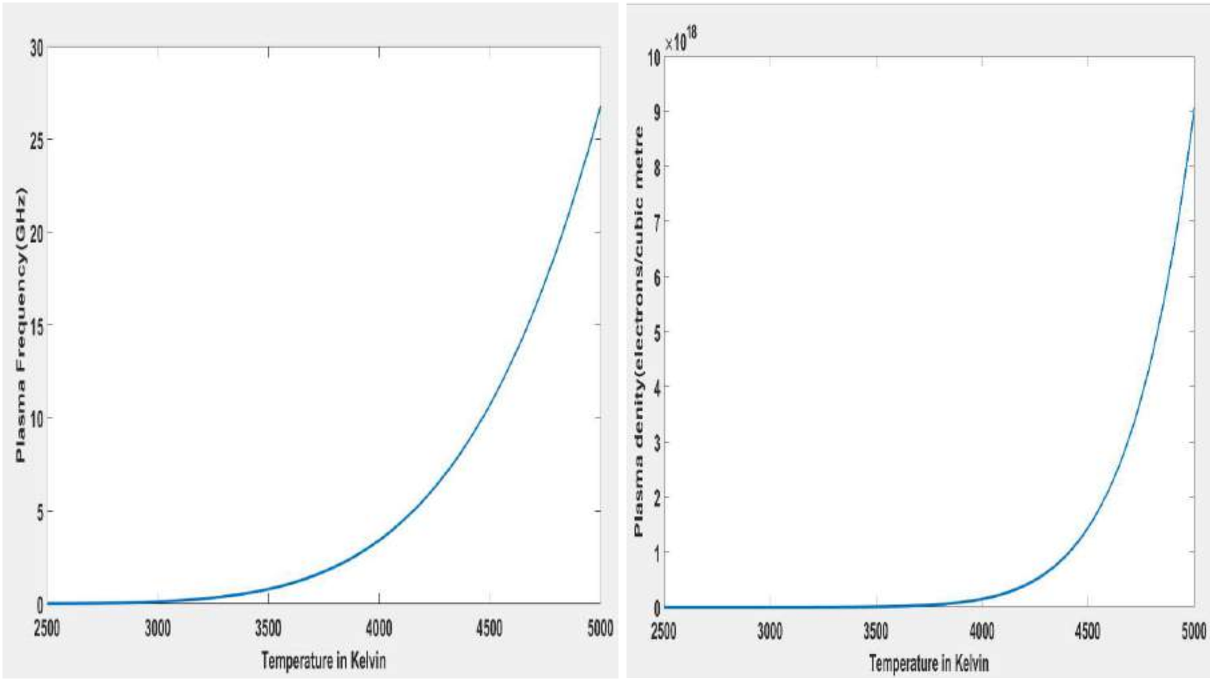



Figure 3.7.3 : Variation of plasma frequency and density with temperature at $n=5x10^{24}$ atoms/$m^3$

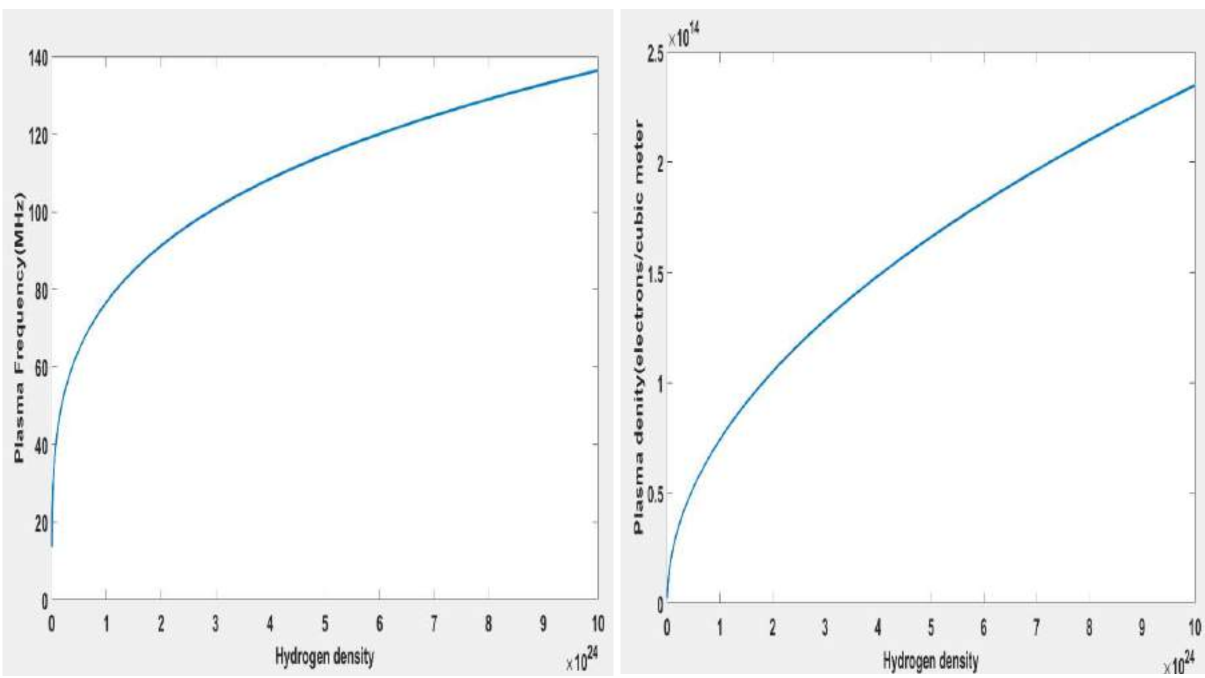



Figure 3.7.4 : Variation of plasma frequency and plasma density with n at T=3000K

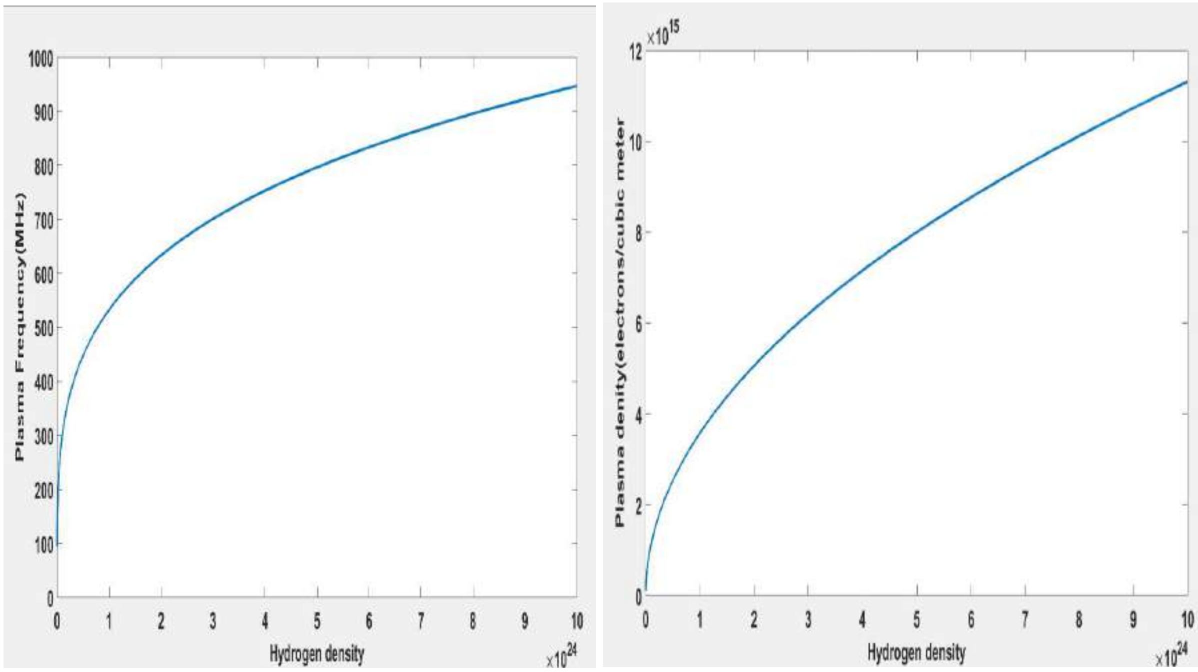



Figure 3.7.5 : Variation of plasma frequency and plasma density with n at T=3500K

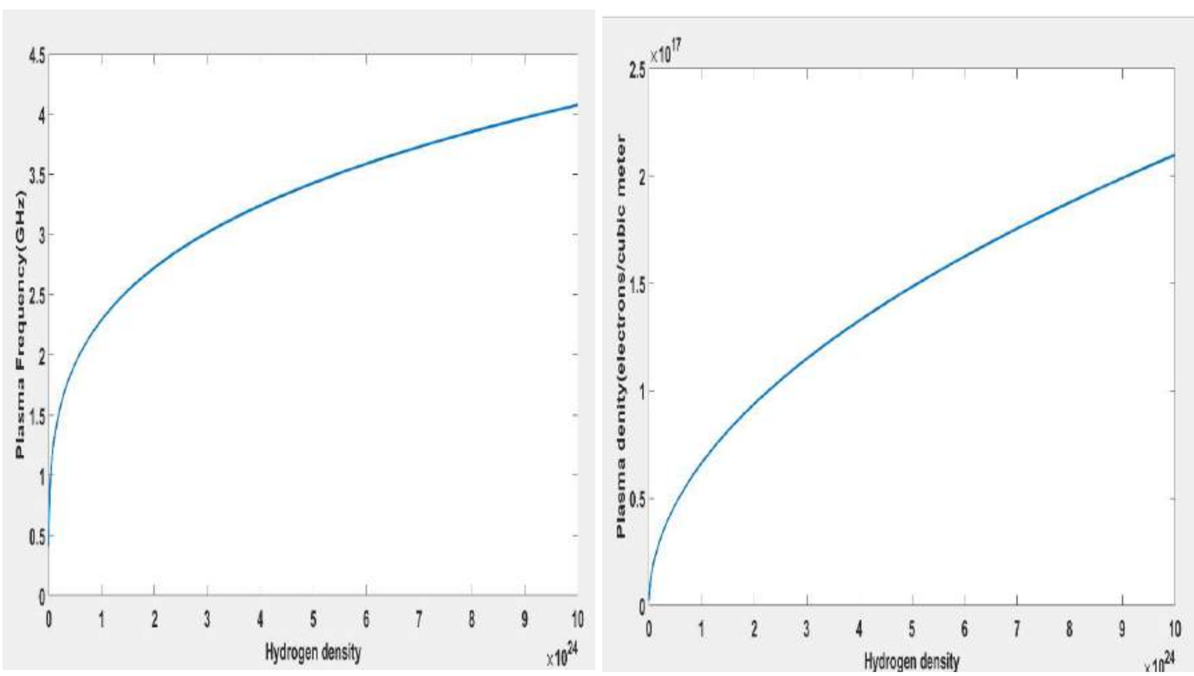



Figure 3.7.6 : Variation of plasma frequency and plasma density with n at T=4000K

Figures 3.7.4 to 3.7.6 shows the variation of plasma frequency and plasma density with variation of neutral density from $2x10^{20}$ molecules/$m^3$ to $1x10^{25}$ molecules/$m^3$ for the hydrogen gas at different Temperatures of 3000K, 3500K, 4000K .It is observed that plasma frequency and Plasma density are increasing continuously with increase in density. As the temperature increases, the effect of neutral density is more on plasma frequency and plasma density. i.e as the temperature increases plasma density increases more compared to increase in plasma density at constant temperature with increased neutral gas density

**Conclusion:**

If gas composition is known with density of components and its temperature, we can find the plasma density of the gas composition, which is the main input to the plasma loss calculations. The sensitivity of plasma frequency to temperature is much higher than the sensitivity to gas density.

# CHAPTER-4.

# PROPAGATION OF EM WAVE THROUGH PLASMA OR PLUME LAYER

## 4.1 INTRODUCTION

In this chapter, we apply the plasma formulae and the Saha equation to obtain dielectric constant profile near the flight vehicle. We assume that antenna is omni directional (see fig 1.1). The actual direction of the wave that matters is the direction towards the ground station. For this project flight vehicle trajectory and plasma profile are given ,The flight vehicle is oriented along its trajectory. Thus, desired direction of radiation is known at every point on the trajectory and is input to the code.

This means the radiation could be normal to the plasma or plume layer, or it could be oblique. Using this input, and using plasma density, the task is to compute the fraction of power that is transmitted towards ground station.

One problem that is ignored in this analysis is the impedance of the plasma medium. The design of antenna is based on the assumption that it is in vacuum. So when plasma is present there are two effects. First, the plasma represents a load impedance and $S_{11}$ is no longer unity. Second, the plasma being a very good conductor (better than copper) currents are induced by the waves. These induced currents are very close to the antenna and belong in the near field rather than far field. So effectively part of the antenna is in the plasma, Both these effects are ignored in the treatment if this M.Tech thesis ,

So this model considers an omnidirectional source (the antenna) and plasma layer at a small but sufficient distance from it. We treat plasma layer as one dimensional with the wave entering normally or obliquely. The layer of plasma has an inhomogeneous density and is modelled as a number of layers each with its own electron density. The problem therefore reduces to that of the propagation of the source wave through a multi-layer plasma.

Density of plasma ($n_e$) is not given but components and density of components in the layer and temperature of the layer are available. So, plasma density of each layer has to be derived from available data. Saha equation is used to find plasma density from the available data as discussed in chapter 3. From plasma density we can find plasma frequency, from plasma frequency we can find relative dielectric constant as discussed from the equations of chapter 2.

Plasma frequency, $\omega_p = \sqrt{\frac{n_e q^2}{m_e \varepsilon_0}}$ ………………4.1.1

where $n_e$ is electron density in plasma

$m_e$ is mass of electron

$q$ is charge of electron

$\upsilon$ is collision frequency

and ω is incident wave frequency

Relative permittivity, $\varepsilon_r = 1 - \frac{\omega_p^2}{\omega^2 \cdot \left(1 + \frac{j\upsilon}{\omega}\right)}$ …………………4.1.2

Assuming that the permeability that of vacuum, the propagation constant and impedance of the plasma are

Propagation constant is $k = \frac{2\pi f}{c} \cdot \sqrt{\varepsilon_r}$ or $k = \frac{2\pi f}{c} \cdot \sqrt{1 - \frac{\omega_p^2}{\omega^2 \cdot \left(1 + \frac{j\upsilon}{\omega}\right)}}$……………..4.1.3

Where f= signal frequency in Hz and c=velocity of light in vacuum.

Intrinsic impedance is $\eta = \frac{\eta 0}{\sqrt{\varepsilon_r}}$ …………………..4.1.4

For each plasma layer intrinsic impedance and propagation constants are calculated from medium composition and densities, temperature using Saha equation[13] as, other equations as discussed above. These values are fed to the simple optical sandwich model for power loss calculations as discussed in this chapter.

**4.2 MATCHING MATRIX**

Propagation matrix [13] relates the fields within the same medium at different locations.

Propagation matrix in a plasma layer of length l is

$$[P] = \begin{bmatrix} e^{+jkl} & 0 \\ 0 & e^{-jkl} \end{bmatrix}$$ --------------------4.2.1

Where k is calculated from equation 4.1.3

Matching matrix [13]relates the fields at the boundary of two different media.

Consider a planar interface (xy-plane at some location z) separating two dielectric/conducting media with characteristic impedance $\eta_1$ and $\eta_2$

as shown in figure

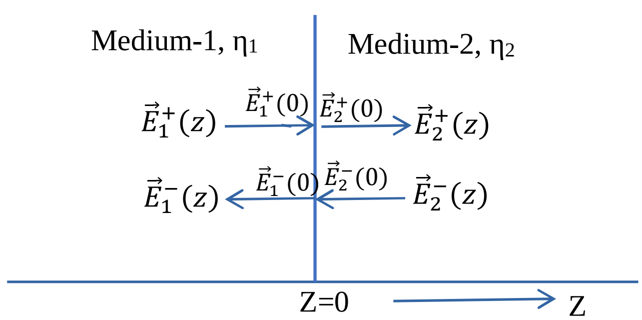



Figure 4.2 : Relating fields at boundary using matching matrix

As the normally incident fields are tangential to the interface plane, the boundary conditions require that the total electric and magnetic fields be continuous across the two sides of the interface

At boundary

In medium-1 total electric field at the boundary is $\vec{E}_1 = \vec{E}_1^+(0) + \vec{E}_1^-(0)$

In medium-2 total electric field at the boundary is $\vec{E}_2 = \vec{E}_2^+(0) + \vec{E}_2^-(0)$

Applying boundary condition $\vec{E}_1 = \vec{E}_2$

$\vec{E}_1^+(0) + \vec{E}_1^-(0)$ - $= \vec{E}_2^+(0) + \vec{E}_2^-(0)$ .................. (4.2.2)

similarly applying boundary condition for magnetic field

$$\frac{\vec{E}_1^+(0)}{\eta_1} - \frac{\vec{E}_1^-(0)}{\eta_1} = \frac{\vec{E}_2^+(0)}{\eta_2} - \frac{\vec{E}_2^-(0)}{\eta_2} \quad \text{.................(4.2.3)}$$

$$\vec{E}_1^+(0) - \vec{E}_1^-(0) = \frac{\eta_1 . \vec{E}_2^+(0)}{\eta_2} - \frac{\eta_1 . \vec{E}_2^-(0)}{\eta_2} \quad \text{.................(4.2.3)}$$

writing equations (4.3.1) and (4.3.2) in matrix form,

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} \vec{E}_1^+(0) \\ \vec{E}_1^-(0) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \frac{\eta_1}{\eta_2} & \frac{-\eta_1}{\eta_2} \end{bmatrix} \cdot \begin{bmatrix} \vec{E}_2^+(0) \\ \vec{E}_2^-(0) \end{bmatrix}$$

$$\Longrightarrow \quad \begin{bmatrix} \vec{E}_1^+(0) \\ \vec{E}_1^-(0) \end{bmatrix} = \frac{1}{2} \cdot \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ \frac{\eta_1}{\eta_2} & \frac{-\eta_1}{\eta_2} \end{bmatrix} \cdot \begin{bmatrix} \vec{E}_2^+(0) \\ \vec{E}_2^-(0) \end{bmatrix}$$

$$\Longrightarrow \quad \begin{bmatrix} \vec{E}_1^+(0) \\ \vec{E}_1^-(0) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 + \frac{\eta_1}{\eta_2} & 1 - \frac{\eta_1}{\eta_2} \\ 1 - \frac{\eta_1}{\eta_2} & 1 + \frac{\eta_1}{\eta_2} \end{bmatrix} \cdot \begin{bmatrix} \vec{E}_2^+(0) \\ \vec{E}_2^-(0) \end{bmatrix}$$

$$\Longrightarrow \quad \begin{bmatrix} \vec{E}_1^+(0) \\ \vec{E}_1^-(0) \end{bmatrix} = \begin{bmatrix} \frac{\eta_2 + \eta_1}{2\eta_2} & \frac{\eta_2 - \eta_1}{2\eta_2} \\ \frac{\eta_2 - \eta_1}{2\eta_2} & \frac{\eta_2 + \eta_1}{2\eta_2} \end{bmatrix} \cdot \begin{bmatrix} \vec{E}_2^+(0) \\ \vec{E}_2^-(0) \end{bmatrix}$$

$$\begin{bmatrix} \vec{E}_1^+(0) \\ \vec{E}_1^-(0) \end{bmatrix} = [M] \cdot \begin{bmatrix} \vec{E}_2^+(0) \\ \vec{E}_2^-(0) \end{bmatrix} \quad \text{.....................4.2.4}$$

where $[M] = \begin{bmatrix} \frac{\eta_2+\eta_1}{2\eta_2} & \frac{\eta_2-\eta_1}{2\eta_2} \\ \frac{\eta_2-\eta_1}{2\eta_2} & \frac{\eta_2+\eta_1}{2\eta_2} \end{bmatrix}$ is called matching matrix.

Therefore, Matching matrix is $[M] = \begin{bmatrix} \frac{\eta_2+\eta_1}{2\eta_2} & \frac{\eta_2-\eta_1}{2\eta_2} \\ \frac{\eta_2-\eta_1}{2\eta_2} & \frac{\eta_2+\eta_1}{2\eta_2} \end{bmatrix}$.........4.2.5

In summary, by using propagation and matching matrices, we can relate the fields of any number of layers. This makes computation easy in any mathematical model.

### 4.3 SINGLE PLASMA LAYER IN FREE SPACE

Consider hydrogen plasma having thickness $l_1$, present in the free space. Let an EM wave is incident normally on it. To find power loss in this layer, First we need to represent it with its electrical properties. Let the temperature of the gas is 3500K and hydrogen density is $4x10^{23}$ atoms/m$^3$ .Let the collision frequency is 2 MHz .

Applying the equations as discussed in section 3.5 of chapter 3

we get plasma density $n_e = 2.2627x10^{15}$ electrons/m$^3$

Now applying equations as discussed in chapter 2,

the plasma frequency is $\omega_p = \sqrt{\frac{n_e q^2}{m_e \varepsilon_0}}$

substituting $n_e$ obtained from Saha equation and other constants in the above equation, we get plasma frequency $f_p = 4.2335x10^8$ Hz (or) 423.35 MHz

The equation of relative permittivity is $\varepsilon_r = 1 - \frac{\omega_p^2}{\omega^2 \cdot \left(1+\frac{j\upsilon}{\omega}\right)}$ .If $\upsilon << \omega$ ,without much loss , we can ignore collision frequency for simple solution, then $\varepsilon_r = 1 - \frac{\omega_p^2}{\omega^2}$

If the frequency of the RF signal incident on the hydrogen gas is, f= 500Mhz, then

$\varepsilon_r = 1 - \frac{\omega_p^2}{\omega^2}$ or $\varepsilon_r = 1 - \frac{f_p^2}{f^2} = 0.2831$

Intrinsic impedance of the plasma layer is $\eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{377}{\sqrt{0.2831}}$ =708.5515 ohms

The propagation constant in the plasma is $k_1 = \frac{2\pi f}{c} \cdot \sqrt{\varepsilon_r}$

$= 278.617$

The values of $k_1$ and $\eta_1$ are used in power loss calculations as given below.

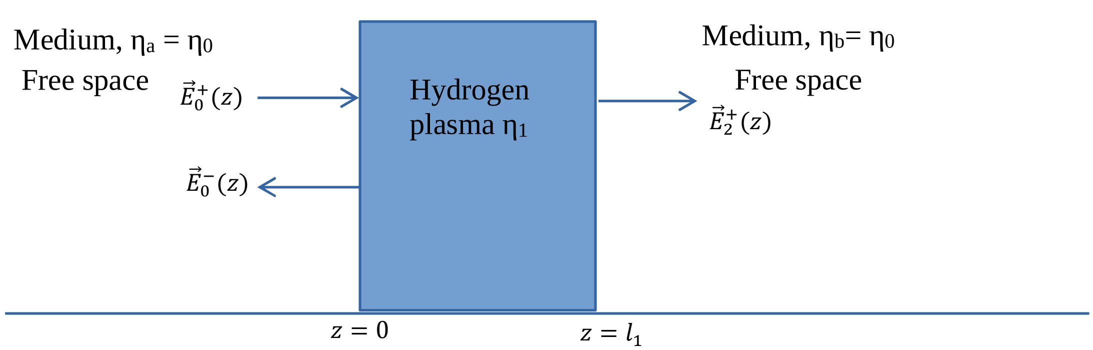



Figure 4.3 : Single plasma layer in free space

Let $n_1$ is the refractive index of the layer. $n_1 = \sqrt{\varepsilon_r}$

We know that $\eta_0 = 377$ ohms.

$\vec{E}_0^+(0)$ is the incident wave on the medium $\eta_1$ , $\vec{E}_0^-(0)$ is the reflected wave and $\vec{E}_2^+(l_1)$ is the transmitted wave . Assume the incident field is from the left medium $\eta_a$, and thus, in semi-infinite medium $\eta_b$ there is only a forward wave i.e. $\vec{E}_2^-(l_1) = 0$ .

The matching matrix at the boundary-1[13] ,to find the fields in medium,$\eta_1$ is

$$[M_0] = \begin{bmatrix} \frac{\eta_1+\eta_0}{2\eta_1} & \frac{\eta_1-\eta_0}{2\eta_1} \\ \frac{\eta_1-\eta_0}{2\eta_1} & \frac{\eta_1+\eta_0}{2\eta_1} \end{bmatrix}$$

$M_0$ relates the quantities to the left of boundary -1 to the quantities right of boundary -1

The propagation matrix in the medium[13], $\eta_1$ for the propagation length $l_1$ with propagation constant $k_1$

is $$[P_1] = \begin{bmatrix} e^{+jk_1l_1} & 0 \\ 0 & e^{-jk_1l_1} \end{bmatrix}$$

$P_1$ relates the quantities to the right of interface-1 to the quantities left of interface-2

Where $k_1 = \frac{2\pi f}{c} \cdot \sqrt{\varepsilon_r} = 278.617$

The matching matrix at the boundary-2 ,to find the fields in medium, $\eta_b$ is

$$[M_1] = \begin{bmatrix} \frac{\eta_0+\eta_1}{2\eta_0} & \frac{\eta_0-\eta_1}{2\eta_0} \\ \frac{\eta_0-\eta_1}{2\eta_0} & \frac{\eta_0+\eta_1}{2\eta_0} \end{bmatrix}$$

$M_1$ relates the quantities to the left of boundary-2 to the quantities to the right of boundary -2

The product $M_0 * P_1 * M_1$ relates the quantities to the left of boundary -1 to the quantities right of boundary 2,thus it represents overall transfer function of transmission and reflection the fields from medium, $\eta_a$ to medium $\eta_b$ are related by

$$\begin{bmatrix} \vec{E}_0^+(0) \\ \vec{E}_0^-(0) \end{bmatrix} = [M_0] * [P_1] * [M_1] \cdot \begin{bmatrix} \vec{E}_2^+(l_1) \\ 0 \end{bmatrix} \qquad \text{……………..4.3.1}$$

The number of matching matrices is equal to number of boundaries whereas the number of propagation matrices is equal to number of layers.

From the above equation we can determine the overall reflection coefficient Γ and transmission coefficient τ

$$\Gamma = \frac{\vec{E}_0^-(0)}{\vec{E}_0^+(0)} \text{ and } \tau = \frac{\vec{E}_2^+(l_1)}{\vec{E}_0^+(0)} \qquad \text{………..4.3.2}$$

from the above equations, for single slab we get the following equations for overall reflection and transmission coefficients.

There are two boundaries. Let $\Gamma_0$ , $\tau_0$ are reflection and transmission coefficients at boundary-1(free space to medium) and $\Gamma_1$ , $\tau_1$ are reflection and transmission coefficients at boundary-2 ( medium to free space).

$$\Gamma_0 = \frac{\eta_1-\eta_0}{\eta_1+\eta_0} \quad , \quad \tau_0 = \frac{2\eta_1}{\eta_1+\eta_0} \quad \text{and} \quad \Gamma_1 = \frac{\eta_0-\eta_1}{\eta_1+\eta_0} \quad , \quad \tau_1 = \frac{2\eta_0}{\eta_1+\eta_0}$$

On substituting these in equation 4.3.1, we get

$$\Gamma = \frac{\Gamma_0+\Gamma_1\cdot e^{-j2\cdot k_1\cdot l_1}}{1+\Gamma_0\cdot\Gamma_1\cdot e^{-j2\cdot k_1\cdot l_1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots 4.3.3$$

and $$\tau = \frac{\tau_0\cdot\tau_1\cdot e^{-j\cdot k_1\cdot l_1}}{1+\Gamma_0\cdot\Gamma_1\cdot e^{-j2\cdot k_1\cdot l_1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots 4.3.4$$

As incident and transmitted media both are same (free space $\eta_a = \eta_b$ ), $\Gamma_0 = -\Gamma_1$

The transmission response has an overall delay factor of $e^{-jk_1l_1}$ representing one-way travel time delay through the medium $\eta_1$.

Let delta, $\delta=k_1.l_1$ is propagation length in the medium $\eta_1$.

But $k_1 = \frac{2\pi}{c}\cdot\sqrt{\varepsilon_{r1}}$

where c=velocity of light in free space

f=frequency of wave in free space

$\varepsilon_{r1}$ = relative permittivity of medium η1

therefore $\delta = k_1\cdot l_1 = \frac{2\pi f}{c}\cdot\sqrt{\varepsilon_{r1}}\cdot l_1 = \frac{\pi\cdot f}{f_1}$

where $f_1 = \frac{c}{2\cdot\sqrt{\varepsilon_{r1}}\cdot l_1} = \frac{c}{2\cdot\sqrt{1-\frac{\omega_p^2}{\omega^2\cdot\left(1+\frac{j\upsilon}{\omega}\right)}}\cdot l_1}$

We can observe important phenomenon when frequency is integral multiples of $f_1$.

When 2 $\delta$ is multiples of $2\pi$ , equation 4.3.2 says reflection goes to zero, therefore periodic nulls are observed at different frequencies

## 4.4 Transmitted power and power loss

The power transmitted into the medium $\eta_b$ is

$$P_{transmitted} = \frac{\left|\vec{E}_2^{+}(l_1)\right|^2}{2\eta_b}$$

and the incident power is

$$P_{incident} = \frac{\left|\vec{E}_0^{+}(0)\right|^2}{2\eta_a}$$

therefore $\frac{P_{transmitted}}{P_{incident}} = \frac{\frac{\eta_a}{\eta_b}.\left|\vec{E}_2^+(l_1)\right|^2}{\left|\vec{E}_0^+(0)\right|^2} = |\tau^2|$ ……….4.4.1

$P_{reflected} = P_{incident} .(1- |\tau^2|)$…………….4.4.2

The variation of transmission coefficient, reflection coefficient and variation of power in plasma are plotted and discussed in chapter 7 analysis

**4.5 NORMAL INCIDENCE ON MULTI LAYERED PLASMA**

For multi-layered plasma, first we need to find intrinsic impedance and propagation constant of each layer from the plasma medium profile available as discussed in section 4.3.

The transfer matrix method can be extended to multi layers by representing each layer by matching and propagating matrix.

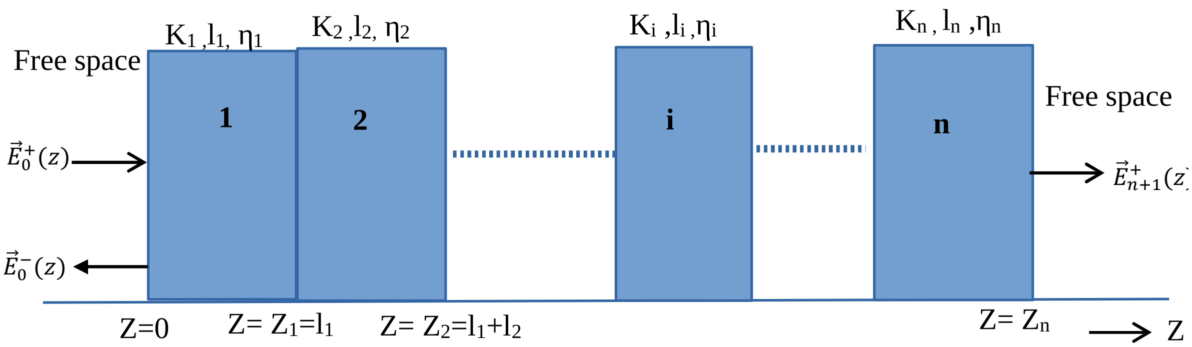



Figure 4.5 : Normal incidence on multi-layered plasma structure

Let there are n layers with i[th] layer represented by length $l_i$, impedance $\eta_i$ and wave number $k_i$. As there are n layers, there will be n+1 boundaries. Therefore, the number of matching matrices is equal to n+1 and number of propagation matrices will be n.

$\vec{E}_0^+(0)$ is the incident wave on the first boundary and $\vec{E}_0^-(0)$ is the reflected wave from the first boundary.

$\vec{E}_{n+1}^+(Z_n)$ is the transmitted wave out of the last boundary.

Each layer will have matching matrix and propagation matrix given by

$$[M_i] = \begin{bmatrix} \frac{\eta_{i+1}+\eta_i}{2\eta_{i+1}} & \frac{\eta_{i+1}-\eta_i}{2\eta_{i+1}} \\ \frac{\eta_{i+1}-\eta_i}{2\eta_{i+1}} & \frac{\eta_{i+1}+\eta_i}{2\eta_{i+1}} \end{bmatrix}$$

The propagation matrix given by

$$[P_i] = \begin{bmatrix} e^{+jk_il_i} & 0 \\ 0 & e^{-jk_il_i} \end{bmatrix}$$

where i=1,2,...,n , for propagation matrices,i=0,2,.....,n for matching matrices

Transmission, reflection coefficients and $P_{transmitted}$ , $P_{reflected}$ can be obtained by the equation

$$\begin{bmatrix} \vec{E}_0^{+}(0) \\ \vec{E}_0^{-}(0) \end{bmatrix} = M_0 \cdot \left(\prod_{i=1}^{n} P_i \cdot M_i\right) \begin{bmatrix} \vec{E}_{n+1}^{+}(Z_n) \\ 0 \end{bmatrix} \text{.........4.5.1}$$

The transfer matrix method can be extended to any number of layers

From the above equation we can find the combined transmission and reflection coefficient as

$$\Gamma = \frac{\vec{E}_0^{-}(0)}{\vec{E}_0^{+}(0)} \text{ and } \tau = \frac{\vec{E}_{n+1}^{+}(Z_n)}{\vec{E}_0^{+}(0)} \text{ ............4.5.2}$$

$$\frac{P_{transmitted}}{P_{incident}} = |\tau^2| \text{..........4.5.3}$$

## 4.6 OBLIQUE INCIDENCE ON MULTI LAYERED PLASMA

For multi-layered plasma, first we need to find intrinsic impedance and propagation constant of each layer from the plasma medium profile available as discussed in section 4.3

With slight modifications the transfer matrix method of normal incidence can be easily converted into oblique incidence. If we separate the fields into transverse components and longitudinal components with respect to the direction of propagation. If impedance η is replaced with transverse impedance $\eta_T$ then we can apply the same equations of normal incidence to oblique incidence.

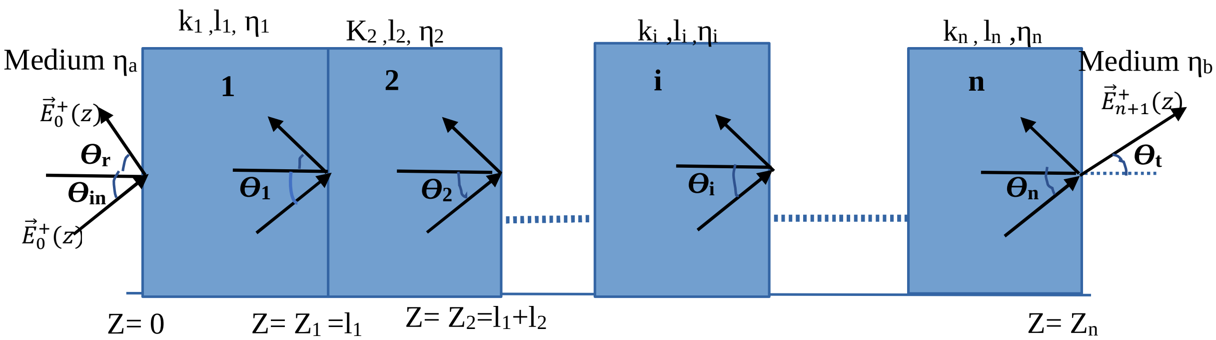



Figure 4.5 : Oblique incidence on multi layer plasma structure

Let there are n layers with lengths $l_1$, $l_2$, $l_3$ ,.......,$l_n$ and impedances $\eta_1$,$\eta_2$ ,.......$\eta_n$.

Let $\Gamma_i$ , $\tau_i$ are the reflection and transmission coefficients at the exit point of $i^{th}$ layer

Let θin is the incident angle on the 1st layer ,θr is the reflected angle and θt is the angle of transmitted wave out of nth layer.

Let $\vec{E}_0^+(z)$ is the incident wave on the 1st layer , $\vec{E}_0^-(0)$ is the reflected wave and $\vec{E}_{n+1}^+(Z_n)$ is the transmitted wave out of nth layer.

Here $\eta_a = \eta_b = \eta_0$ (free space).

Let $n_i = \sqrt{\varepsilon_{ri}}$ is refractive index of the ith layer.

Impedance of ith layer is $\eta_i = \eta_0 / n_i$ ;$= \frac{\eta 0}{\sqrt{\varepsilon_{ri}}}$

From snells law [16][17]

$$n_a \sin \theta_{in} = n_i \sin \theta_i = n_b \sin \theta_t$$

$$n_0 \sin \theta_{in} = n_i \sin \theta_i = n_0 \sin \theta_t$$

from the above equation, we can see that $\theta_{in} = \theta_t$ i.e That is transmitted angle is equal to incident angle .

$n_0 \sin \theta_{in} = n_i \sin \theta_i$

For Perpendicular polarization/TE case at the second interface of layers of $\eta_1$ , $\eta_2$ the reflection coefficient $\Gamma_1$ is

$$\Gamma_1 = \frac{\eta_2 \cdot \cos(\theta_1) - \eta_1 \cdot \cos(\theta_2)}{\eta_2 \cdot \cos(\theta_1) + \eta_1 \cdot \cos(\theta_{2t})} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.6.1$$

(transmitted angle out of one interface is same as incident angle in the next interface.

i.e $\cos(\theta_{1t}) = \cos(\theta_2)$

we can re write the reflection coefficient at boundary 2 as

$$\Gamma_1 == \frac{\eta_2/\cos(\theta_2) - \eta_1/\cos(\theta_1)}{\eta_2/\cos(\theta_2) + \eta_1/\cos(\theta_1)} \ldots\ldots\ldots\ldots\ldots\ldots 4.6.2$$

Therefore, we can define transverse intrinsic impedance of ith layer for Perpendicular polarization/TE.

$$\eta_{Ti} = \eta_i / \cos(\theta_i) \ \ldots\ldots\ldots\ldots\ldots 4.6.3$$

Re writing the equation 4.7.2

$$\Gamma_1 == \frac{\eta_{T2} - \eta_{T1}}{\eta_{T2} + \eta_{T1}} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.6.4$$

Based on the equation 4.7.2 ,the general equation for reflection coefficient $\Gamma_i$

$$\Gamma_i == \frac{\eta_{T(i+1)} - \eta_{Ti}}{\eta_{T(i+1)} + \eta_{Ti}} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.6.5$$

and $$\tau_i = \frac{2\,\eta_{T(i+1)}}{\eta_{T(i+1)} + \eta_{Ti}}$$

Similarly, we can write for parallel polarization/TM ,The transverse impedance is

$$\eta_{Ti} = \eta_i \cos(\theta_i) \ \ldots\ldots 4.6.6$$

for each layer $n_i \sin(\theta_i) = n_a \sin(\theta_{in})$ i.e $n_i \sin(\theta_i) = n_0 \sin(\theta_{in})$

but $\cos(\theta_i) = \text{sqrt}(1 - \sin^2(\theta_i))$

therefore, for i^th layer

$$\cos(\theta_i) = \mathrm{sqrt}(1 - (n_0^2/n_i^2 . \sin^2\theta_{in}) \ldots\ldots\ldots\ldots 4.6.7$$

Therefore, by replacing impedance by its transverse value, we can treat oblique incidence as normal incidence in transfer matrix method of implementation.

The matching matrix for i^th boundary boundary is $[M_i] = \begin{bmatrix} \frac{\eta_{Ti+1}+\eta_{Ti}}{2\eta_{Ti+1}} & \frac{\eta_{Ti+1}-\eta_{Ti}}{2\eta_{Ti+1}} \\ \frac{\eta_{Ti+1}-\eta_{Ti}}{2\eta_{Ti+1}} & \frac{\eta_{Ti+1}+\eta_{Ti}}{2\eta_{Ti+1}} \end{bmatrix}$

The phase propagation length in each layer for oblique incidence will be

$\delta_i = k_i l_i \cos\theta_i$

But $k_i = \frac{2\pi f}{c} \cdot \sqrt{\varepsilon_{ri}}$

where c=velocity of light in free space

f=frequency of wave in free space

$\varepsilon_{ri}$ = relative permittivity of medium $\eta_i$

The propagation matrix for i^th layer is $[P_i] = \begin{bmatrix} e^{+j\delta i} & 0 \\ 0 & e^{-j\delta i} \end{bmatrix}$

by using above equations, we can relate incident, reflected and transmitted fields

$$\begin{bmatrix} \vec{E}_0^{+}(0) \\ \vec{E}_0^{-}(0) \end{bmatrix} = M_0 \cdot \left(\prod_{i=1}^{n} P_i \cdot M_i\right) \begin{bmatrix} \vec{E}_{n+1}^{+}(Z_n) \\ 0 \end{bmatrix} \ldots\ldots\ldots 4.6.8$$

The transfer matrix method can be extended to any number of layers

From the above equation we can find the combined transmission and reflection coefficient as

$$\Gamma = \frac{\vec{E}_0^{-}(0)}{\vec{E}_0^{+}(0)} \text{ and } \tau = \frac{\vec{E}_{n+1}^{+}(Z_n)}{\vec{E}_0^{+}(0)} \ldots\ldots\ldots\ldots 4.6.9$$

$$\frac{P_{transmitted}}{P_{incident}} = |\tau^2| \ldots\ldots\ldots\ldots 4.6.10$$

From the above equations we can find transmitted power in terms of incident power and power loss in multi-layer plasma structure

# CHAPTER 5.0

# MEDIUM DATA

Capturing medium information in the form of data file is an important task for mathematical models. The captured data should reflect complete information of the medium and it should be accurate to reflect real time scenario. This data must be correlated with time and altitude so that easily it can used in software integration without much difficulty.

The plasma is inhomogeneous, anisotropic medium with uneven properties in different directions. So it is difficult to represent it by mathematical equations[3][6][7]. To convert this non linearity into linearity, we can divide plasma into number of layers such that the properties of each layer are almost constant within its width. Therefore by dividing plasma layer into multiple layers , we are converting non linearity into linearity for easy development of mathematical model.

## 5.1 MEDIUM PARAMETERS FOR MATHEMATICAL MODEL

The identified parameters which are useful in reflecting complete medium properties are explained below[1][2][3][6][7]

### 5.1.1.Number of plasma layers (n):

Based on approximately constant temperature zones observed on CFD analysis, we can divide the medium into number of layers with constant properties. Then each zone is considered as homogeneous, isotropic and linear medium. All the mathematical equation of linear medium can be applied easily so that computer simulations will be easy in the analysis of complete plasma medium

### 2.width of each layer:

Width of each layer is the input to the model for deciding propagation length and propagation delay in each layer. It also decides maxima and minima of reflection coefficient at different frequencies.

**3.Plasma density($n_e$) in each layer:**

Plasma properties are dominated by free electron concentration in it. Electron concentration along with collision frequency decides conductivity, relative permittivity and Plasma frequency. The RF blackout is the result of high electron density.

Measuring plasma density by sensors or other instruments is difficult activity on re entry flight vehicles[1]. In this model plasma density is estimated by using Saha equation. Each layer is specified by

i. Temperature

ii. Composition of the medium

iii. Density of each component

Parameters like ionisation potential, partition function values of each component are taken from literature. Saha equation is applied on each layer at given instant of time and plasma density in that layer is calculated

**4.Collision frequency in each layer(v):**

In plasma oscillating electrons collide with neutral atoms and ions, these collisions per second is collision frequency. It decides permittivity and conductivity of the plasma layer. Collision frequency can be neglected in calculation of permittivity and conductivity if the operational frequency is far greater than collision frequency [.

**5.Aspect angle:**

For plume loss calculations Aspect angle is most important parameter. It is angle between flight vehicle axis and telemetry station. If the signal passes through the plume, then only there will be plume loss otherwise it is zero. For a given flight vehicle there will be threshold aspect angle, below which signal will pass through the plume and above which signal avoids the plume. Aspect angle has no role in plasma loss calculations [1][7].

**6.Incident angle:**

It is Derived from Aspect angle, plume dimensions and exhaust nozzle dimensions. For aspect angle greater than threshold there will be no plume interference, therefore without any calculations plume loss is taken as zero [1][2][7]. If aspect angle is less than threshold specified, incident angle on the plume is used in plume loss calculations.

For plasma loss calculations it is independent of aspect angle [1].

**7.Input frequency /frequency range:**

This is the frequency of RF signal. We can observe attenuation in plasma or plume at fixed operational frequency or vary the frequency in the region of interest. All the simulations can be observed in S-band, C-band and Ka-band as defined in the scope [1].

**8.Polarization**

Polarization of incident wave may be parallel or perpendicular or circularly polarized. Output polarization can be estimated based on effective impedance phase and amount of attenuation in perpendicular and parallel polarizations [1][13]

The above parameters are captured as data file for plume and plasma explained below.

### 5.2 Plume data

The exhaust of flight vehicle will be at high temperature leading to ionization of exhaust propellent materials leading to low dense plasma. It will have low density of electrons and collision frequency compared to Ionospheric plasma around the flight vehicle during re entry.

Estimation of signal loss through the plume requires proper estimation of plume medium characteristics. But this medium is non homogeneous with uneven properties in all the directions which makes it highly difficult to estimate the wave propagation characteristics in the medium. If we can identify or separate into the regions in which properties are almost constant then we can assume medium as combination of many such linear isotropic layers.

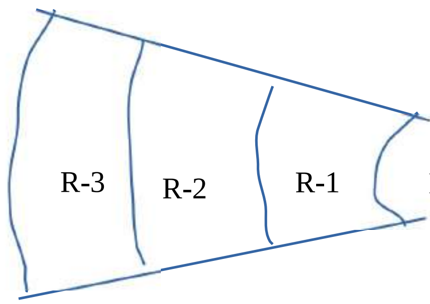



Figure 5.1: Plume as combination of different layers

The above figure shows separation of plume into three temperature regions R-1 , R-2 and R-3 . Each region is assumed to have constant temperature and other parameters in all directions. From composition of plume exhaust and by knowing the density of components in each region, we can estimate wave propagation characteristics.

As the plume exhaust is weekly ionised plasma, we need to find plasma density in each region. For finding plasma density in each region we can use Saha equation as discussed in chapter 4. After finding plasma density we can estimate wave propagation loss out of the plume using the methods discussed in chapter 2 and chapter 3.

### 5.2.1 Data file of plume medium

As the project aim is to find the loss in dB at instant of time (sampling time) from the start of the flight to the end of the flight, we need to have complete plume medium information at each instant of time.

The main parameters required for power loss estimation are

1.Layer width
2.Temperature in layer
3.Number of components in each layer
4.Density of each component

The above information is required for each region and at each and every instant of time. To co relate with altitude of flight vehicle, we can generate data file with altitude information also.

The data file of plume medium which is separated into three temperature regions contains data columns with following parameters

1.Time instant

2. Layer-1 width

3. Layer-1 temperature

4. Layer-2 width

5. Layer-2 temperature

6. Layer-3 width

7. Layer-3 temperature

9. Component-1 density

10. Component-2 density

11. Component-3 density

12.Incident angle

13.Aspect angle

14.Altitude

Altitude information is not required in power loss calculations but it taken only for correlation of different losses like path loss, rain loss etc in ACTS software data for estimation of cumulative loss at particular time and altitude.

Based on exhaust plume dimensions, aspect angle decides EM wave incident angle on the plume, which decides loss in plume medium. Plume medium information is different for different flight vehicles.

The sample data file of the plume medium is as shown in figure 5.2.1

PlumeMedium_Datafile - Notepad

File Edit Format View Help

| Time | Altitude | Asp_Angle | Inc_Angle | layer-1 | Temp-1 | layer-2 | Temp-2 | layer-3 | Temp-3 | Comp1_Densty | Comp2_Densty | Comp3_Densty |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.000000 | -0.041749 | 12.13 | 3.42 | 0.60 | 300.00 | 0.70 | 290.00 | 1.00 | 280.0000 | 5.000e+24 | 4.500e+22 | 4.000e+21 |
| 0.001000 | -0.039022 | 12.13 | 3.42 | 0.60 | 300.00 | 0.70 | 290.00 | 1.00 | 280.0010 | 5.000e+24 | 4.500e+22 | 4.000e+21 |
| 0.002000 | -0.036295 | 12.13 | 3.42 | 0.60 | 300.00 | 0.70 | 290.00 | 1.00 | 280.0020 | 5.000e+24 | 4.500e+22 | 4.000e+21 |
| 0.003000 | -0.033567 | 12.13 | 3.42 | 0.60 | 300.00 | 0.70 | 290.00 | 1.00 | 280.0030 | 5.000e+24 | 4.500e+22 | 4.000e+21 |
| 0.004000 | -0.030840 | 12.12 | 3.42 | 0.60 | 300.00 | 0.70 | 290.00 | 1.00 | 280.0040 | 5.000e+24 | 4.500e+22 | 4.000e+21 |
| 0.005000 | -0.028113 | 12.12 | 3.42 | 0.60 | 300.00 | 0.70 | 290.00 | 1.00 | 280.0050 | 5.000e+24 | 4.500e+22 | 4.000e+21 |
| 0.006000 | -0.025386 | 12.12 | 3.42 | 0.60 | 300.01 | 0.70 | 290.01 | 1.00 | 280.0060 | 5.000e+24 | 4.500e+22 | 4.000e+21 |
| 0.007000 | -0.022658 | 12.11 | 3.42 | 0.60 | 300.01 | 0.70 | 290.01 | 1.00 | 280.0070 | 5.000e+24 | 4.500e+22 | 4.000e+21 |
| 0.008000 | -0.019931 | 12.11 | 3.42 | 0.60 | 300.01 | 0.70 | 290.01 | 1.00 | 280.0080 | 5.000e+24 | 4.500e+22 | 4.000e+21 |
| 0.009000 | -0.017204 | 12.11 | 3.42 | 0.60 | 300.01 | 0.70 | 290.01 | 1.00 | 280.0090 | 5.000e+24 | 4.500e+22 | 4.000e+21 |
| 0.010000 | -0.014476 | 12.11 | 3.42 | 0.60 | 300.01 | 0.70 | 290.01 | 1.00 | 280.0100 | 5.000e+24 | 4.500e+22 | 4.000e+21 |

Figure 5.2 : Plume medium sample data file

### 5.3 PLASMA MEDIUM DATA

There are two main problems in measuring plasma attenuation. First one is estimation of plasma parameters accurately i.e electron and ion density, collision frequency of electrons with ions and neutral atoms, thickness of various density layers around the missile. Second one is dividing inhomogeneous, anisotropic plasma layer into several layers of homogenous and isotropic(approximately) .

During the re-entry phase of space shuttles or ballistic missiles, flight vehicle passes through ionospheric layer, and due to high kinetic energy of vehicle, there will be formation of high density plasma around nose cone, this plasma completely reflects the RF signal passing through it leading to RF block out and communication failure with ground station. This phenomenon continues until missile crosses ionospheric region.

The composition of medium remains same for all flight vehicles but plasma density around flight vehicle changes based on velocity, mass, structure, altitude of the vehicle.

### 5.3.1 Plasma medium data file

As the project aim is to find the loss in dB at every instant of time (sampling time) from the start of the flight to the end of the flight, we need to have complete plasma medium information at each instant of time.

The main parameters required for power loss estimation are

1. Plasma layer width around the vehicle
2.Temperature in each layer
3.Composition of medium
4.Density of each component

The above information is required for each region and at each and every instant of time. To correlate with altitude of flight vehicle , we can generate data file with altitude information also.

The data file of plasma medium which is separated into three temperature regions around the nose cone of flight vehicle contains data columns with following parameters

1.Time instant
2. Layer-1 width
3. Layer-1 temperature
4. Layer-2 width
5. Layer-2 temperature
6. Layer-3 width
7. Layer-3 temperature
9. Component-1 density
10. Component-2 density
11. Component-3 density
12.Incident angle
13.Altitude

Plume attenuation is completely independent of altitude but depends on aspect angle where as plasma attenuation depends on the altitude also. At different altitudes medium density , composition and gravitational pull on the flight vehicle changes which effects plasma density at that altitude. But this effect is taken into account with density and temperature parameters.

Altitude information is not required in power loss calculations, but it is taken only for correlation of different losses like path loss, rain loss etc in ACTS software data for estimation of cumulative loss at particular time and altitude. The sample data file of the plasma medium will be as shown in figure 5.3

PlasmaMedium_Datafile - Notepad

File Edit Format View Help

| Time | Altitude | Inc_Angle | layer-1 | Temp-1 | layer-2 | Temp-2 | layer-3 | Temp-3 | H2_Densty | N2_Densty | O2_Densty |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.000000 | -0.041749 | 3.42 | 0.60 | 300.00 | 0.70 | 290.00 | 1.00 | 280.0000 | 5.000e+24 | 4.500e+22 | 4.000e+21 |
| 0.001000 | -0.039022 | 3.42 | 0.60 | 300.00 | 0.70 | 290.00 | 1.00 | 280.0010 | 5.000e+24 | 4.500e+22 | 4.000e+21 |
| 0.002000 | -0.036295 | 3.42 | 0.60 | 300.00 | 0.70 | 290.00 | 1.00 | 280.0020 | 5.000e+24 | 4.500e+22 | 4.000e+21 |
| 0.003000 | -0.033567 | 3.42 | 0.60 | 300.00 | 0.70 | 290.00 | 1.00 | 280.0030 | 5.000e+24 | 4.500e+22 | 4.000e+21 |
| 0.004000 | -0.030840 | 3.42 | 0.60 | 300.00 | 0.70 | 290.00 | 1.00 | 280.0040 | 5.000e+24 | 4.500e+22 | 4.000e+21 |
| 0.005000 | -0.028113 | 3.42 | 0.60 | 300.00 | 0.70 | 290.00 | 1.00 | 280.0050 | 5.000e+24 | 4.500e+22 | 4.000e+21 |
| 0.006000 | -0.025386 | 3.42 | 0.60 | 300.01 | 0.70 | 290.01 | 1.00 | 280.0060 | 5.000e+24 | 4.500e+22 | 4.000e+21 |
| 0.007000 | -0.022658 | 3.42 | 0.60 | 300.01 | 0.70 | 290.01 | 1.00 | 280.0070 | 5.000e+24 | 4.500e+22 | 4.000e+21 |
| 0.008000 | -0.019931 | 3.42 | 0.60 | 300.01 | 0.70 | 290.01 | 1.00 | 280.0080 | 5.000e+24 | 4.500e+22 | 4.000e+21 |
| 0.009000 | -0.017204 | 3.42 | 0.60 | 300.01 | 0.70 | 290.01 | 1.00 | 280.0090 | 5.000e+24 | 4.500e+22 | 4.000e+21 |
| 0.010000 | -0.014476 | 3.42 | 0.60 | 300.01 | 0.70 | 290.01 | 1.00 | 280.0100 | 5.000e+24 | 4.500e+22 | 4.000e+21 |

Figure 5.3: Plasma medium sample data file

## 5.4 FIXED DATA INPUT FILE

Other parameters required for calculations which are having fixed values in the entire process are fed to MATLAB through another data file. This file contains frequency of RF signal, Aspect angle threshold value, polarisation information etc.

The sample input file for plasma calculations is

| Frequency (GHz) | Polarization(1-perp,0-parallel |
|---|---|
| 2.3 | 0 |

Table: 5.1 : Plasma input file format

The sample  format of input file for plume calculations is

| Frequency (GHz) | Incident_Angle_Threshold(deg) | Polarization(1-perp,0-parallel |
|---|---|---|
| 2.3 | 14 | 0 |

Table: 5.2 : Plume input file format

Once data files are generated , they will be accessed in MATLAB  for calculating loss and output file will be generated for offline usage.

# CHAPTER 6.0

# MATLAB SIMULATION

MATLAB code is developed separately for plume and plasma calculations. Matching and propagation matrix method discussed in chapter 4 is chosen to find transfer function and power loss of multi-layered plasma. The resulting equations are implemented in MATLAB for EM wave propagation loss calculation in both plume and plasma. This code accepts medium data file, fixed parameter input file and generates output file with loss data at each instant of time. The output file generated can be integrated with ACTE software (Channel simulation software of user) for simulation of plume and plasma channel losses of flight vehicle.

## 6.1 PLUME MODEL DESCRIPTION

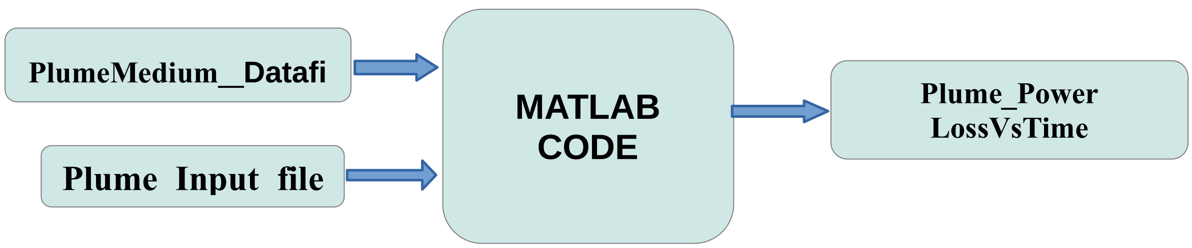



Figure 6.1.1 : Plume MATLAB code model

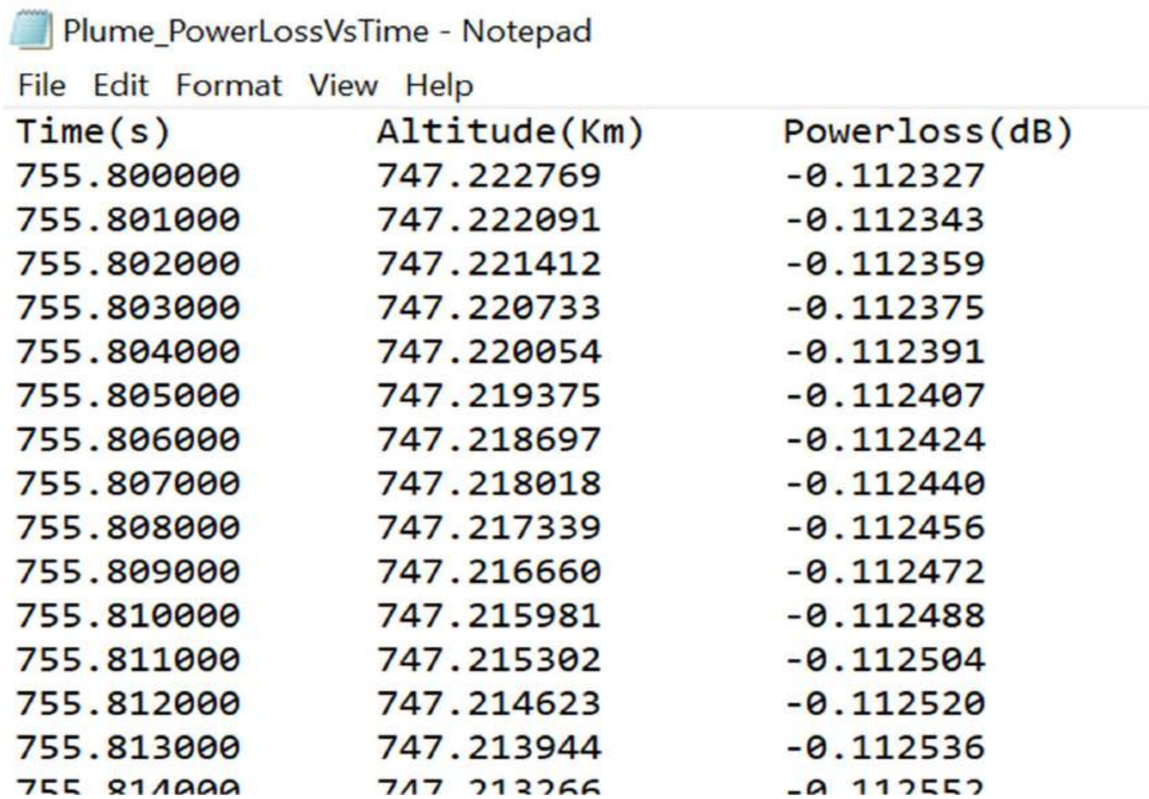
MATLAB code accepts two data files as inputs for loss calculations.

i. PlumeMedium_Datafile

ii.Plume_Input_file

PlumeMedium_Datafile contains medium information at each instant of time as discussed on section 6.3.1 and Plume_Input_file contains fixed parameter inputs required for calculations. At a particular time instant data of layers is accessed and processed for finding loss at that instant. Obtained loss in decibels is written into the output file along with time stamp and altitude information for offline access to simulation software.

The format of plume loss output file will be as given below



Plume_PowerLossVsTime - Notepad

File Edit Format View Help

| Time(s) | Altitude(Km) | Powerloss(dB) |
|---|---|---|
| 755.800000 | 747.222769 | -0.112327 |
| 755.801000 | 747.222091 | -0.112343 |
| 755.802000 | 747.221412 | -0.112359 |
| 755.803000 | 747.220733 | -0.112375 |
| 755.804000 | 747.220054 | -0.112391 |
| 755.805000 | 747.219375 | -0.112407 |
| 755.806000 | 747.218697 | -0.112424 |
| 755.807000 | 747.218018 | -0.112440 |
| 755.808000 | 747.217339 | -0.112456 |
| 755.809000 | 747.216660 | -0.112472 |
| 755.810000 | 747.215981 | -0.112488 |
| 755.811000 | 747.215302 | -0.112504 |
| 755.812000 | 747.214623 | -0.112520 |
| 755.813000 | 747.213944 | -0.112536 |

Figure 6.1.2: PlumelossVsTimeAltitude file format

## 6.2 PLASMA MODEL DESCRIPTION

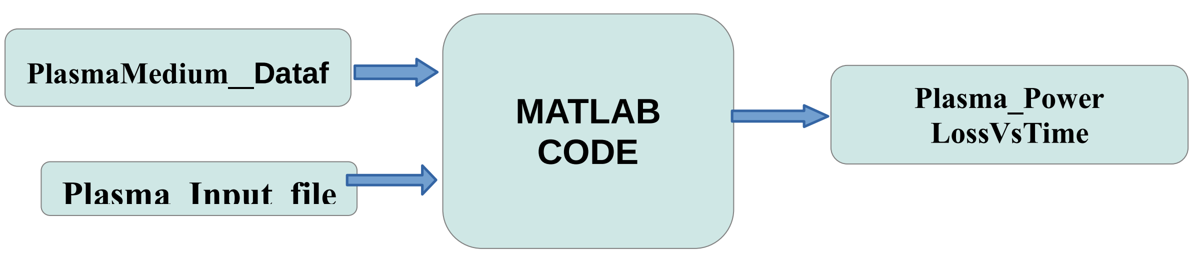



**Figure 6.2.1 : Plasma MATLAB code model**

MATLAB code accepts two data files as inputs for loss calculations.

i. PlasmaMedium_Datafile

ii.Plasma_Input_file

PlasmaMedium_Datafile contains medium information at each instant of time as discussed on section 6.4.1 and Plasma_Input_file contains fixed parameter inputs required for calculations. At a particular time instant data of all layers is accessed and processed for finding loss at that instant.

Obtained loss in decibels is written into the output file along with time stamp and altitude information for offline access to simulation software.

The format of plasma loss output file will be as given below

Plasma_PowerLossVsTime - Notepad

File Edit Format View Help

| Time(s) | Altitude(Km) | Powerloss(dB) |
|---|---|---|
| 755.800000 | 747.222769 | -1.112327 |
| 755.801000 | 747.222091 | -1.112343 |
| 755.802000 | 747.221412 | -1.112359 |
| 755.803000 | 747.220733 | -1.112375 |
| 755.804000 | 747.220054 | -1.112391 |
| 755.805000 | 747.219375 | -1.112407 |
| 755.806000 | 747.218697 | -1.112424 |
| 755.807000 | 747.218018 | -1.112440 |
| 755.808000 | 747.217339 | -1.112456 |
| 755.809000 | 747.216660 | -1.112472 |
| 755.810000 | 747.215981 | -1.112488 |
| 755.811000 | 747.215302 | -1.112504 |
| 755.812000 | 747.214623 | -1.112520 |
| 755.813000 | 747.213944 | -1.112536 |
| 755.814000 | 747.213266 | -1.112552 |
| 755.815000 | 747.212587 | -1.112568 |
| 755.816000 | 747.211908 | -1.112584 |
| 755.817000 | 747.211229 | -1.112600 |
| 755.818000 | 747.210550 | -1.112616 |
| 755.819000 | 747.209871 | -1.112632 |

**Figure 6.2.2: Plasma_PowerlossVsTime file format**

### 6.3 MATLAB OUTPUT

To reduce the complexity in calculations, power absorbed in the medium is not calculated. Incident power minus transmitted power is treated as power reflected (this includes power absorbed in plasma) because plasma is assumed as perfect dielectric with relative permittivity to simplify the calculations. In reality relative permittivity is function of frequency and plasma properties making it lossy medium, these losses are accounted in reflected power. So power reflected and power absorbed in the medium together is treated as power loss in the medium. This power loss is written into output file for offline access in simulation software.

# CHAPTER 7.0

# RESULTS AND ANALYSIS

Different simulation results are observed by varying density, thickness, and temperature of plasma layers along with frequency range and incident angle. All the results are satisfying with theoretical calculations.

## 7.1 MATLAB SIMULATION RESULTS FOR GENERAL OBSERVATION OF RF PROPAGATION THROUGH PLASMA

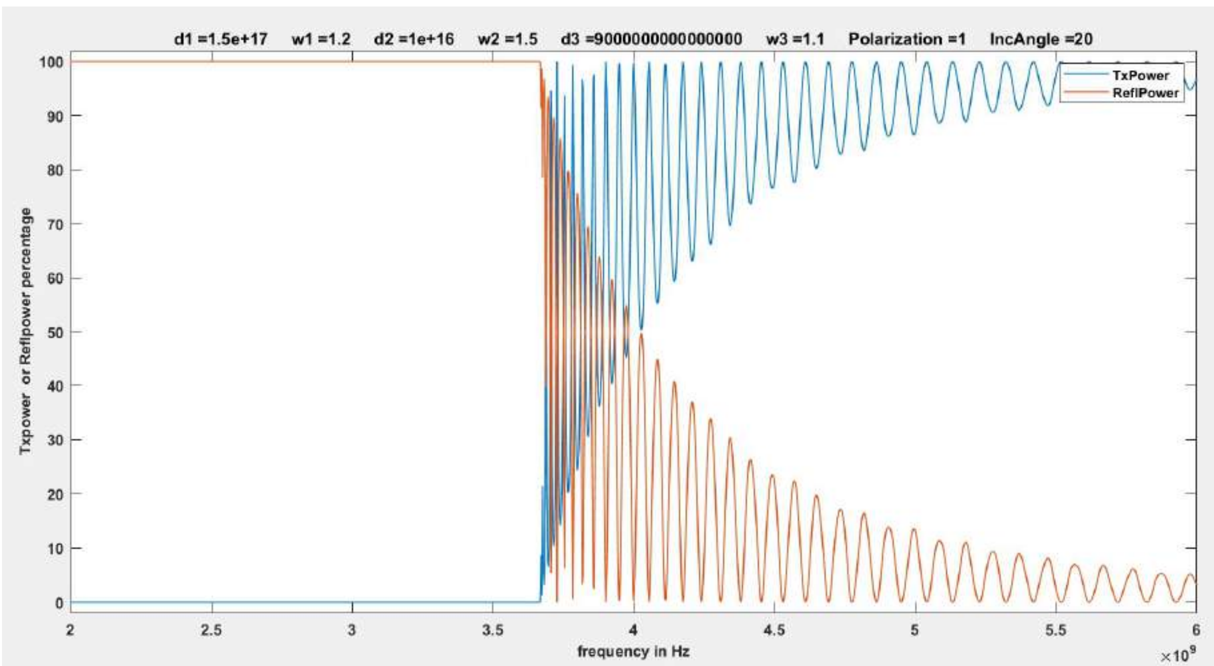



Figure 7.1: Transmitted and reflected power with respect to frequency

The above plot (*x-axis: frequency in Hz(x10$^9$ ), Y-axis: Percentage (0-100%)of Transmitted power(blue) and Reflected power(red) with respect to the incident power*) shows variation of transmitted and reflected power over the frequency range 2GHz to 6GHz in the medium with plasma frequency of 3.44 GHz. It can be observed that there is no transmission below plasma frequency, while beyond plasma frequency transmitted power is oscillating and increasing continuously.

This variation is in accordance with theoretical calculation as explained in section 4.4. For the same parameters the power in dB in given frequency range and power loss in dB with variation of incident angle from zero degrees to 90 degrees is given below.

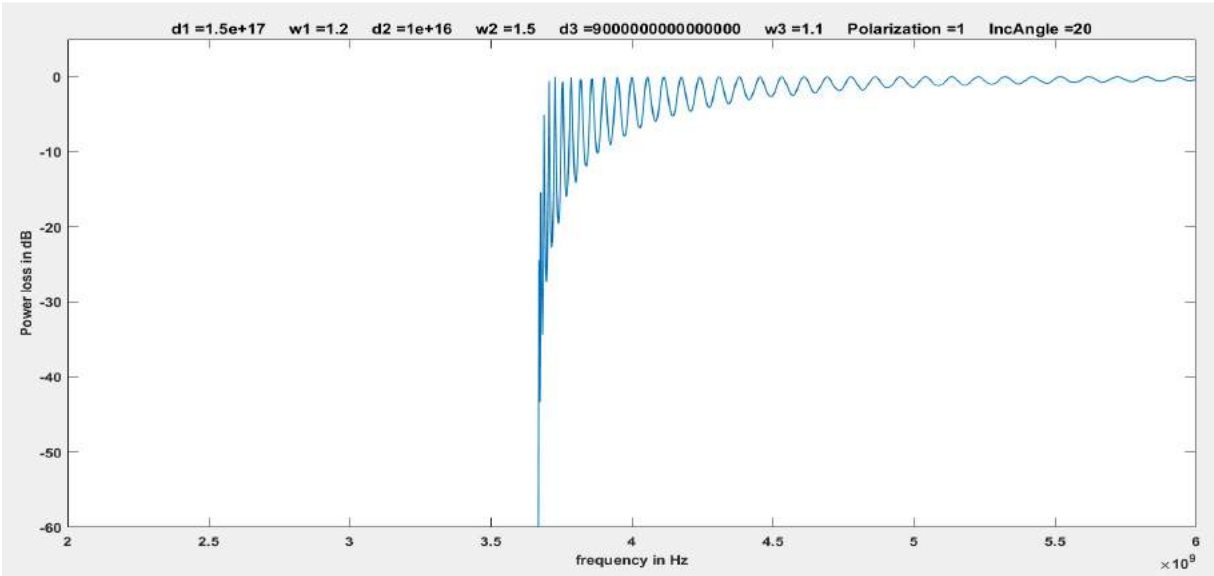



Figure 7.2 : Transmitted power loss in dB with frequency

The above plot (*x-axis: frequency in Hz(x10*$^{9}$ *), Y-axis: Transmitted Power loss in dB) with respect to the incident power*) shows variation of transmitted power with respect frequency. We can observe that as the frequency increases power transmitted is increasing.

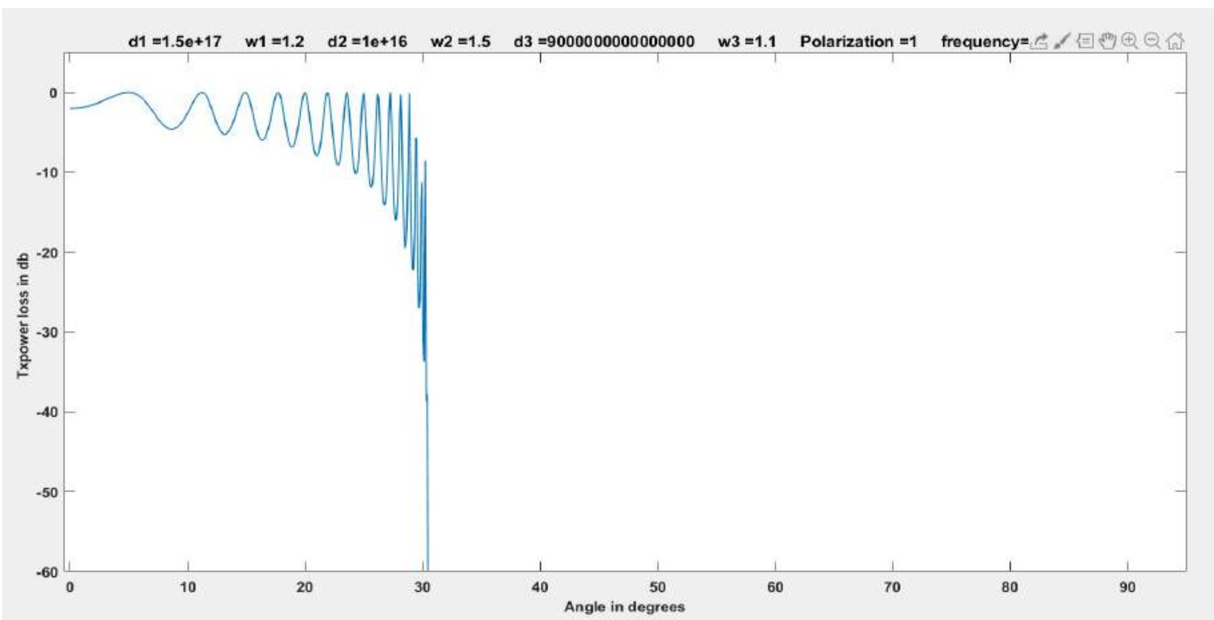



Figure 7.3 : Power Loss with variable Incident Angle at 4Ghz

The above plot (*x-axis: Incident angle in degrees(0-90 degrees), Y-axis: Transmitted Power loss in dB) with respect to the incident power*) shows the variation of transmitted power loss in dB with at different incident angles ,It is clear that as the incident angle increases losses will increase and if the angle is above some threshold angle, it completely reflects back without any transmission

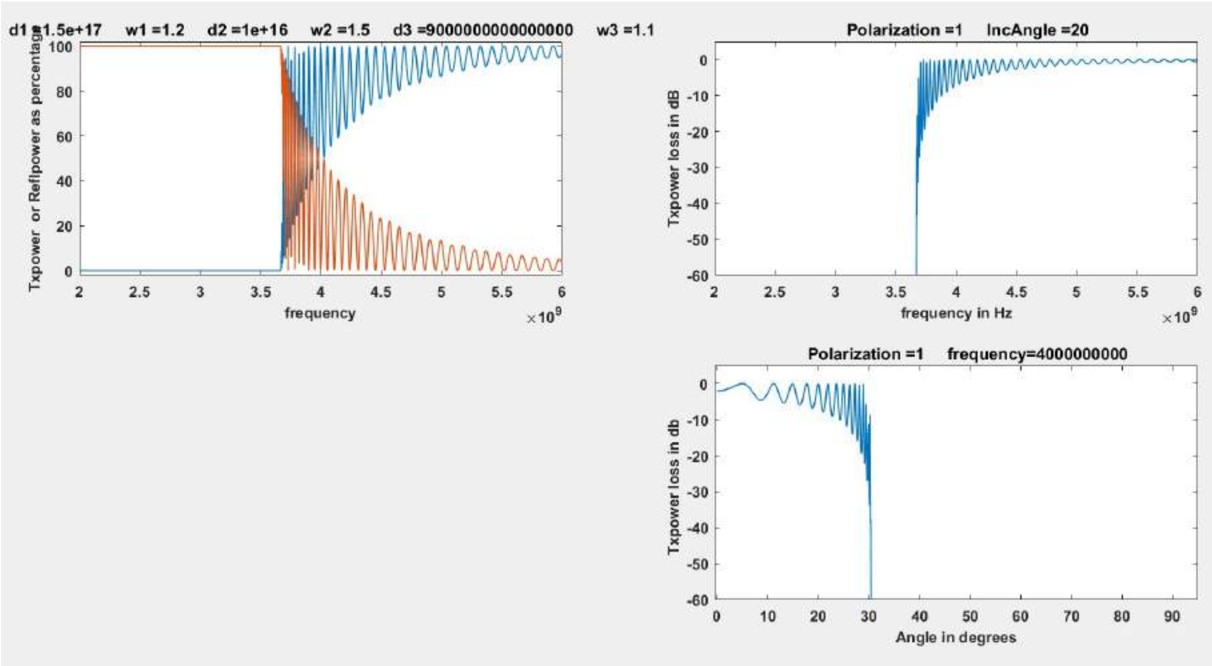



Figure 7.4 : Plots showing variation of power loss with frequency and incident angle

Figure 7.4 subplot (1x1) is same as plot shown if figure 7.1 at incident angle of 20 degrees.

Figure 7.4 subplot (1x2) shows variation of transmission and reflection coefficients in the plasma with respect to frequency variation 2GHz to 6GHz at 20 degrees incident angle

Figure 7.4 subplot (2x1) shows variation of power loss in dB in the plasma with respect incident angle variation from 0-90 degrees at 4GHz frequency

The following plots shows simulated output plots when layer width, frequency range, plasma density are varied

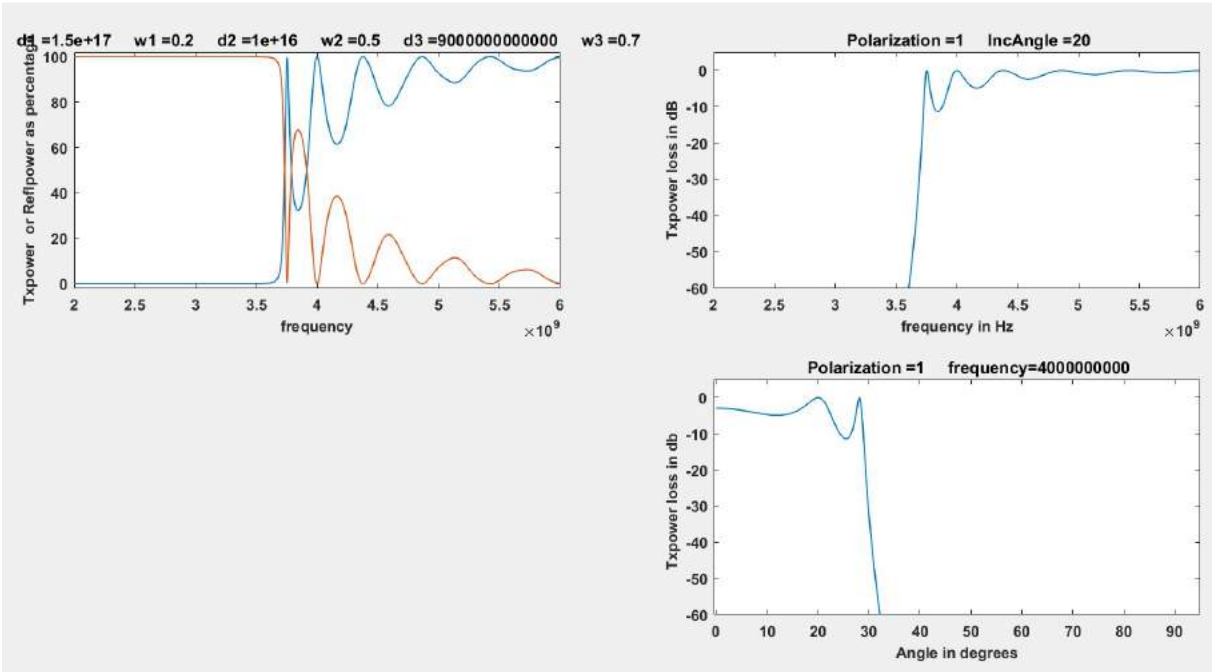



Figure 7.5 : Multi plot for 3 layers at different width and density

Assumed data:

Layer-1 width is 0.2m, plasma density is $1.5 \times 10^{17}$ electrons/$m^3$

Layer-2 width is 0.5m, plasma density is $1.0 \times 10^{16}$ electrons/$m^3$

Layer-3 width is 0.7m, plasma density is $9.0 \times 10^{12}$ electrons/$m^3$

Figure 7.5 subplot (1x1) (*x-axis: frequency in Hz(x10$^9$ ), Y-axis: Percentage (0-100%)of Transmitted power(blue) and Reflected power(red) with respect to the incident power*) shows variation of transmitted and reflected power over the frequency range 2GHz to 6GHz in the medium at an incident angle of 20 degrees with plasma frequency of 3.44 GHz. It can be observed that there is no transmission if the signal frequency is below plasma frequency, beyond plasma frequency transmitted power is oscillating and increasing continuously.

Figure 7.5 subplot (2x2) plot (*x-axis: Incident angle in degrees(0-90 degrees), Y-axis: Transmitted Power loss in dB) with respect to the incident power*) shows the variation of transmitted power loss in dB at different incident angles for 4GHz signal ,It is clear that as the incident angle increases losses will increase and if the angle is above some threshold angle, it completely reflects back without any transmission

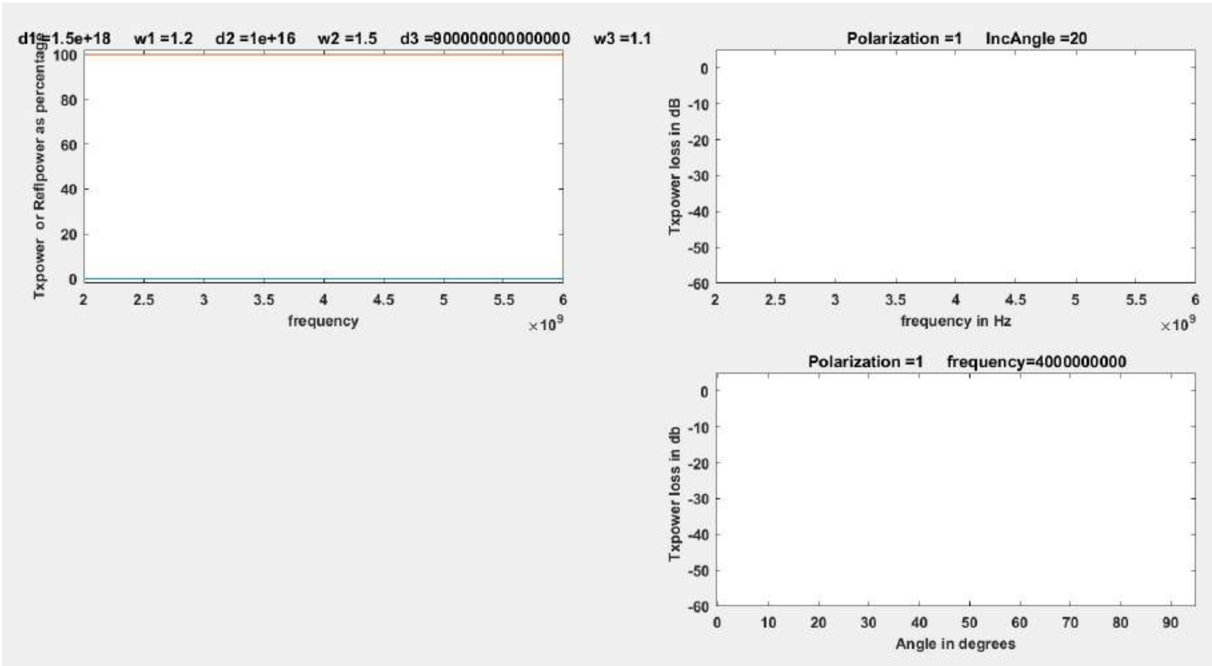



Figure 7.6 : Multi plot for 3 layers at different width and density

Assumed data:

Layer-1 width is 1.2m, plasma density is $1.5x10^{18}$ electrons/$m^3$

Layer-2 width is 1.5m, plasma density is $1.0x10^{16}$ electrons/$m^3$

Layer-3 width is 1.1m, plasma density is $9.0x10^{14}$ electrons/$m^3$

Figure 7.6 subplot (1x1) (*x-axis: frequency in Hz(x$10^{10}$ ), Y-axis: Percentage (0-100%)of Transmitted power(blue) and Reflected power(red) with respect to the incident power*) shows variation of transmitted and reflected power over the frequency range 2GHz to 6GHz in the medium at an incident angle of 27 degrees. It can be observed that there is no transmission as the signal frequency is below plasma frequency because density of plasma is increased by 10 times in the first layer.

Figure 7.6 subplot (2x2) plot (*x-axis: Incident angle in degrees(0-90 degrees), Y-axis: Transmitted Power loss in dB) with respect to the incident power*) shows the variation of transmitted power loss in dB at different incident angles for 4GHz signal ,It is clear that there no transmission of signal at all angles from 0-90 degrees because plasma frequency is above 4GHz.Signal it completely reflects back without any transmission

The oscillations in power loss spectrum are due to variation of frequency, length of plasma layer and incident angle at integral , integral+0.5 multiples of $f_{1,}$ as discussed in Chapter 4 maxima and minima of reflection coefficient

### 7.2 PLASMA LOSS CALCULATIONS

Plasma loss is simulated by assuming hydrogen plasma medium of 3 layers with widths 0.6m ,0.7m and 1.0 m. Temperature is varied from 280Kelvin to 6400 kelvin .RF signal frequency is assumed as 2.3GHz . Assumed hydrogen density is $5x10^{24}$ molecules per cubic meter. Time instant, altitude and incident angle, aspect angle are taken from the file provided by Captronics pvt ltd. This is fictitious data to simulate medium and helps in completing software integration.

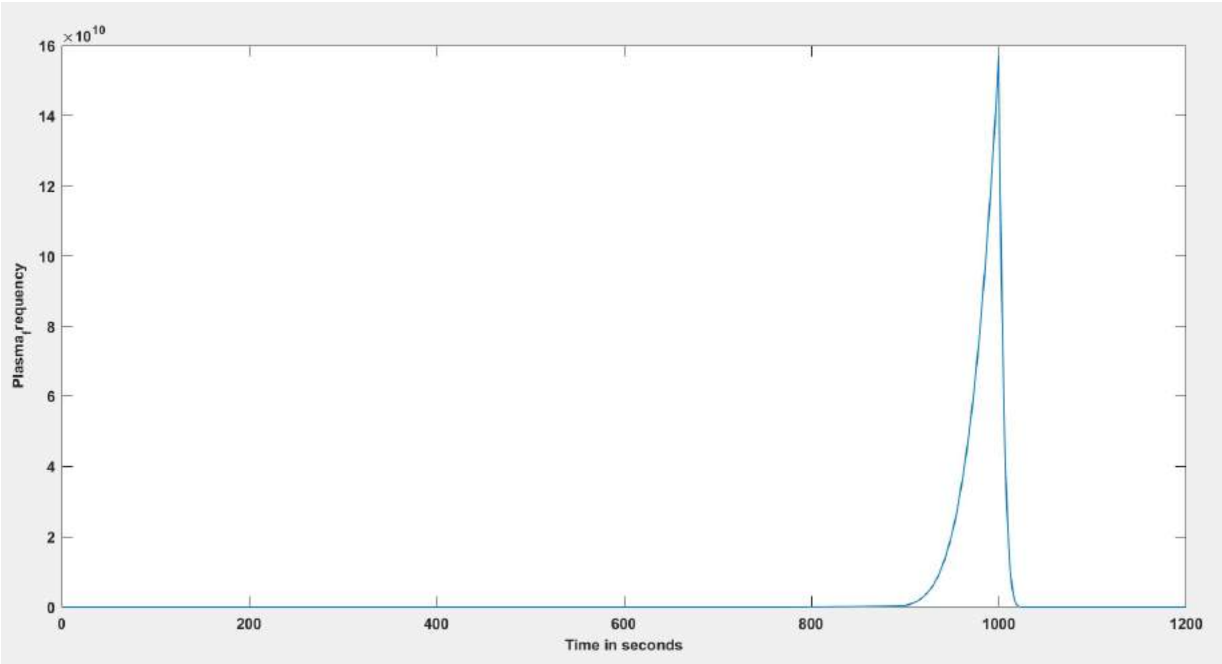



Figure 7.7: Plasma frequency variation with time

Figure 7.7 shows (x-axis :time in seconds ,Y-axis :plasma frequency in Hz($x10^{10}$ ) the variation of plasma frequency of the medium with respect to time .As the temperature of plasma is changing with respect to flight time ,we can observe that plasma frequency changing with time. It increases to very high value of almost 160GHz at 6400 kelvins at around 1000 seconds leading to blackout. Blackout occurs when the plasma frequency is increasing beyond 2.3GHz until it comes back to below 2.3GHz.

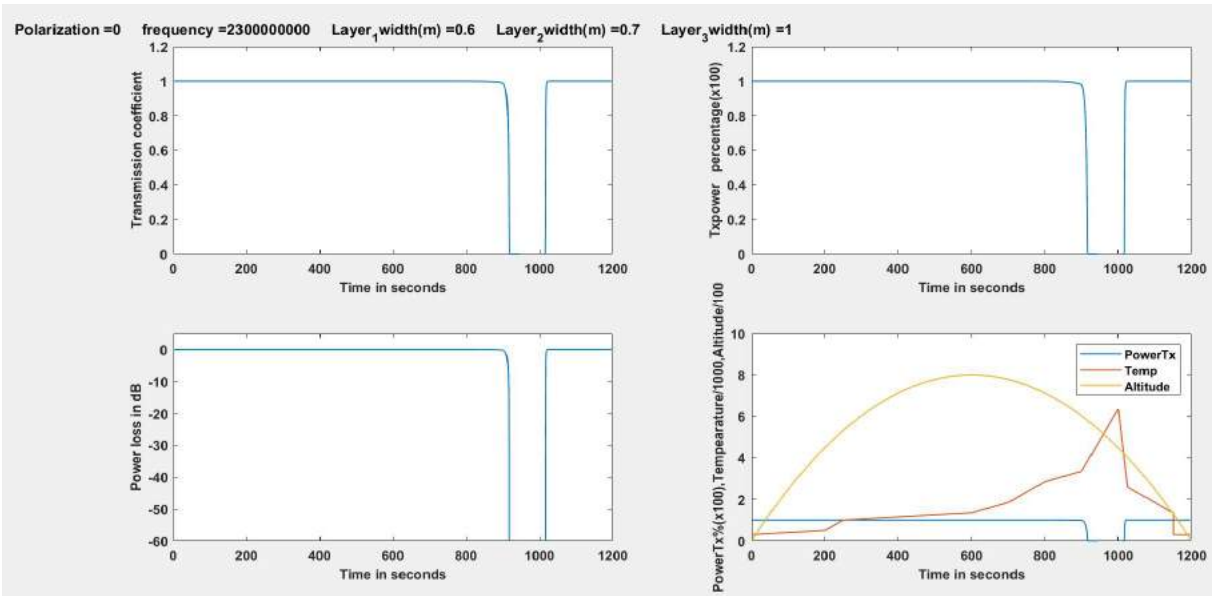



Figure 7.8: Plasma loss in dB versus time , altitude and temperature

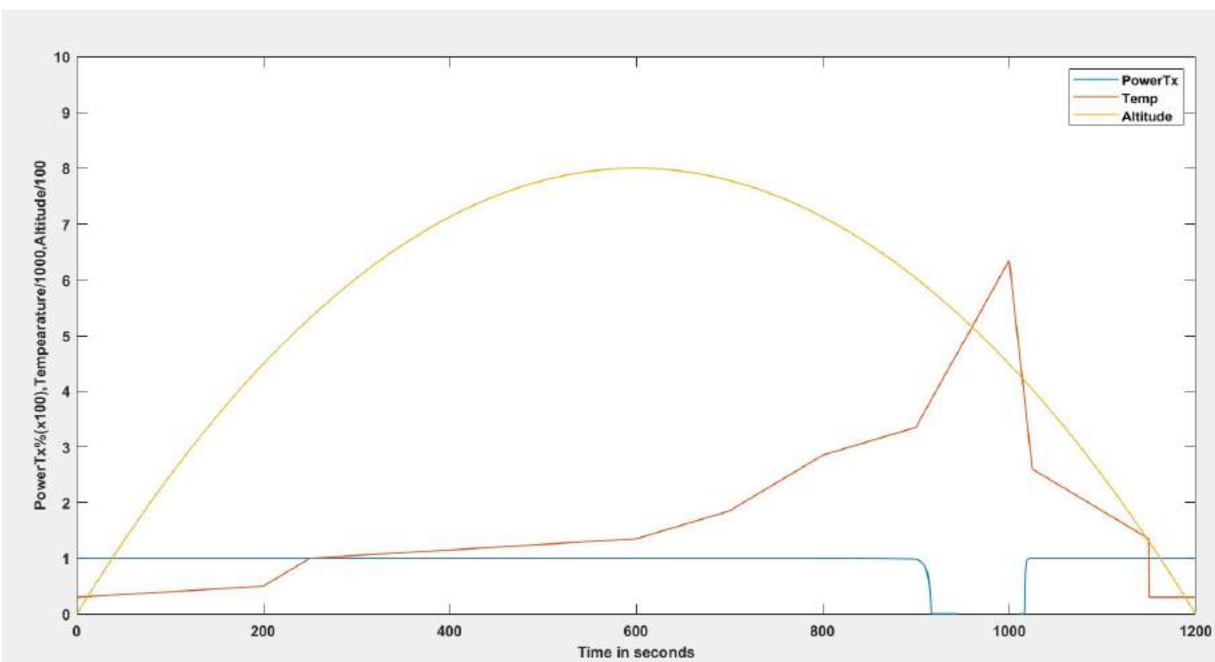



Figure 7.9: Plasma loss percentage with respect to incident power vesrus time , altitude and temperature

Figure 7.9 shows the plot (X-axis:Time,Y-axis :Altitude , Temperature ,Power transmitted percentage with respect to incident power).This plot clearly shows the blackout occurrence with variation of altitude .Power transmitted is at zero during blackout(blackout is starting at the threshold of around 4400K. when temperature decreases ,losses decrease and signal strength comes back to normal.

Plasma_PowerLossVsTime - Notepad

File Edit Format View Help

| | | |
|---|---|---|
| 1017.689000 | 416.881785 | -2.789122 |
| 1017.690000 | 416.879930 | -2.777985 |
| 1017.691000 | 416.878075 | -2.766468 |
| 1017.692000 | 416.876220 | -2.754592 |
| 1017.693000 | 416.874365 | -2.742380 |
| 1017.694000 | 416.872510 | -2.729856 |
| 1017.695000 | 416.870655 | -2.717043 |
| 1017.696000 | 416.868800 | -2.703965 |
| 1017.697000 | 416.866945 | -2.690647 |
| 1017.698000 | 416.865090 | -2.677115 |
| 1017.699000 | 416.863235 | -2.663394 |
| 1017.700000 | 416.861380 | -2.649510 |
| 1017.701000 | 416.859525 | -2.635489 |
| 1017.702000 | 416.857670 | -2.621357 |
| 1017.703000 | 416.855815 | -2.607141 |
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| 1017.705000 | 416.852105 | -2.578563 |
| 1017.706000 | 416.850250 | -2.564253 |
| 1017.707000 | 416.848395 | -2.549964 |
| 1017.708000 | 416.846540 | -2.535722 |
| 1017.709000 | 416.844685 | -2.521552 |
| 1017.710000 | 416.842830 | -2.507481 |
| 1017.711000 | 416.840975 | -2.493533 |
| 1017.712000 | 416.839120 | -2.479733 |
| 1017.713000 | 416.837265 | -2.466104 |
| 1017.714000 | 416.835410 | -2.452671 |
| 1017.715000 | 416.833555 | -2.439456 |
| 1017.716000 | 416.831700 | -2.426481 |
| 1017.717000 | 416.829845 | -2.413768 |
| 1017.718000 | 416.827990 | -2.401337 |
| 1017.719000 | 416.826135 | -2.389209 |
| 1017.720000 | 416.824280 | -2.377403 |
| 1017.721000 | 416.822425 | -2.365936 |
| 1017.722000 | 416.820570 | -2.354826 |

Figure 7.10: Plasma medium losses versus time and altitude

The above figure is the output file of MATLAB mathematical model .This file gives power loss data with respect to altitude and time instant. This file will be integrated with simulation software.

### 7.3 PLUME LOSS SIMULATION

Plume loss is simulated by assuming hydrogen component in the exhaust. 3 layers with widths 0.6m ,0.7m and 1.0 m. Temperature is varied from 280Kelvin to 6400 Kelvin. RF signal frequency is assumed as 2.3GHz . Assumed hydrogen density is $5x10^{24}$ molecules per cubic meter. Time instant, altitude and incident angle, aspect angle are taken from the file provided by Captronics pvt ltd. This is fictitious data to simulate medium and helps in completing software integration.
As real data is not available, plasma medium data is assumed to be plume data for software integration.

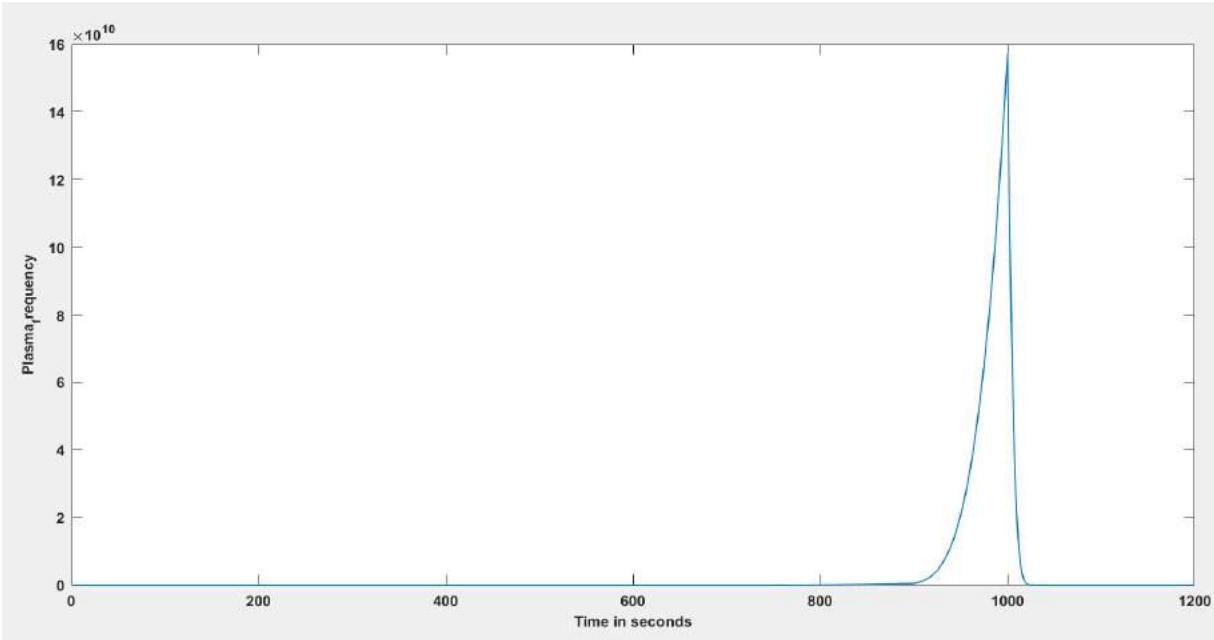


Figure 7.11: Plasma frequency variation with temperature

Figure 7.11 shows (x-axis :time in seconds ,Y-axis :plasma frequency in Hz(x$10^{10}$ ) the variation of plasma frequency of the medium with respect to time .As the temperature of plasma is changing with respect to flight time ,we can observe that plasma frequency changing with time. It increases to very high value of almost 160GHz at 6400 kelvins at around 1000 seconds leading to blackout. Blackout occurs when the plasma frequency is increasing beyond 2.3GHz until it comes back to below 2.3GHz.

Plume_PowerLossVsTime - Notepad

File Edit Format View Help

| | | |
|---|---|---|
| 911.952000 | 587.356200 | -2.630240 |
| 911.953000 | 587.354828 | -2.631721 |
| 911.954000 | 587.353456 | -2.633236 |
| 911.955000 | 587.352084 | -2.634784 |
| 911.956000 | 587.350712 | -2.636367 |
| 911.957000 | 587.349340 | -2.637983 |
| 911.958000 | 587.347968 | -2.639632 |
| 911.959000 | 587.346597 | -2.641316 |
| 911.960000 | 587.345225 | -2.643033 |
| 911.961000 | 587.343853 | -2.644783 |
| 911.962000 | 587.342481 | -2.646567 |
| 911.963000 | 587.341109 | -2.648384 |
| 911.964000 | 587.339737 | -2.650235 |
| 911.965000 | 587.338365 | -2.652119 |
| 911.966000 | 587.336993 | -2.654036 |
| 911.967000 | 587.335621 | -2.655986 |
| 911.968000 | 587.334249 | -2.657969 |
| 911.969000 | 587.332877 | -2.659985 |
| 911.970000 | 587.331505 | -2.662034 |
| 911.971000 | 587.330133 | -2.664115 |
| 911.972000 | 587.328761 | -2.666229 |
| 911.973000 | 587.327389 | -2.668376 |
| 911.974000 | 587.326017 | -2.670555 |
| 911.975000 | 587.324645 | -2.672766 |
| 911.976000 | 587.323273 | -2.675009 |
| 911.977000 | 587.321901 | -2.677284 |
| 911.978000 | 587.320529 | -2.679592 |
| 911.979000 | 587.319157 | -2.681930 |
| 911.980000 | 587.317785 | -2.684301 |
| 911.981000 | 587.316413 | -2.686703 |
| 911.982000 | 587.315041 | -2.689136 |
| 911.983000 | 587.313669 | -2.691600 |
| 911.984000 | 587.312297 | -2.694096 |
| 911.985000 | 587.310925 | -2.696622 |

Figure 7.12: Plume medium losses versus time and altitude

Plume loss data is simulated with assumed medium data. In general, there will be no blackout phenomenon but continuous high-low fluctuations in the plume. Here temperature is assumed to very high also to simulate all the conditions of losses and observe behaviour of model.

Figure 7.12 is the output data file of mathematical model showing the variation of plume loss in dB with respect to time, altitude.

## CHAPTER 8.0

## CONCLUSION

Simple linear mathematical model is developed by using transfer matrix method for reflection and transmission coefficients, computer simulation code is developed based on derived model.This Code can be easily implemented on any digital hardware or it can be simulated on computer software.

It is observed that for ionospheric plasma, S-band and C-band signals Signal suffers complete attenuation with typical plasma density in ionosphere, whereas Ka-band can pass through the plasma with less attenuation. Therefore, best solution to overcome ionospheric plasma Rf block out is choosing RF signal frequency in Ka band and designing onboard and ground stations as per the frequency requirement.

For plume attenuation as plasma is less dense, S-band , C-band signals also pass through it with varied attenuation based on thickness of plume and density variation within the plume, operational frequency etc. To overcome plume attenuation, we need to increase on board transmit power, adjust ground station sensitivity accordingly as per simulated calculations.

This model will help in the design of onboard RF transmitter and ground stations to overcome RF block out and plume attenuation problems in telemetry links of flight vehicles and space re-entry vehicles. The developed code generates plume and plasma loss data for given medium, so this model can be used in flight simulation soft wares to reflect real time losses during flight

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