STABILIZATION OF CART-INVERTED PENDULUM USING POLE-PLACEMENT METHOD

A Project Report

submitted by:

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Prof.Dr. Arun D Mahindrakar

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THESIS CERTIFICATE

I hereby certify that the project titled **STABILIZATION OF CART-INVERTED PENDULUM USING POLE-PLACEMENT METHOD** which is submitted by **Katam Seva Shashank** for the fulfilment of the requirements for awarding of the degree of **Master of Technology (M.tech)** is a bonafide record of the project work carried out by the students under my guidance & supervision. To the best of my knowledge, this work has not been submitted in any part or fulfilment for any Degree or Diploma to this University or elsewhere.

Place : Chennai

Date : May 27, 2022

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Seva Shashank Katam

ABSTRACT

Keywords - stability, non-linearity,pole placement,feedback control

Stabilization of unstable systems is a significant area in the automatic control discipline. It is needed in many applications, some examples being missile guidance or stabilization of a biped robot. The main objective of this study is to research the single inverted pendulum to develop a controller for it. We aim to stabilize the pendulum's upright position while keeping the cart oscillating around the desired position. Full-state feedback controller, which can have its characteristics modified by changing the placement of the dominant poles, is considered. The results of the full-state feedback controller turned out to be very stable, precise and controllable.

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LIST OF NOTATIONS

- θ=pendulum angle from vertical
- ω = rate of change of angle
- x = cart position coordinate
- M = Mass of cart
- m = mass of pendulum
- I = moment of inertia of pendulum (neglected in calculations)
- b= friction of surface(b=0, in this case)
- l= length of pendulum
- g= acceleration due to gravity
- u= force applied on cart

Chapter 1

INTRODUCTION

1.1 Overview

The stability of an inverted pendulum has become a common engineering challenge for researchers. It is a classic problem in automatic control, which interests many other disciplines such as mechanics, physics, mathematics, etc. The inverted pendulum is an unstable and non-linear system. This system has two positions of equilibrium, one of them is stable while the other is unstable. The stable position corresponds to the pendulum pointing downright. Releasing the pendulum from any position other than exactly straight up makes it fall and oscillate around the stable position until it comes to rest due to friction forces. Therefore, the stable position requires no control, and it is not interesting from a control point of view.

1.2 Problem Formulation

The system in this example consists of an inverted pendulum on a cart; A pole of mass m and length l is attached to the cart, with θ in $[0,-\pi]$ for the left-hand plane and $[0,\pi]$ for the right-hand side. We suppose the cart moves on a rail, and the pole can go under it. **Inverted Pendulum** Our goal is to balance the pole above its supporting cart (θ=0) by displacing the cart left or right. Thus, two actions are possible.

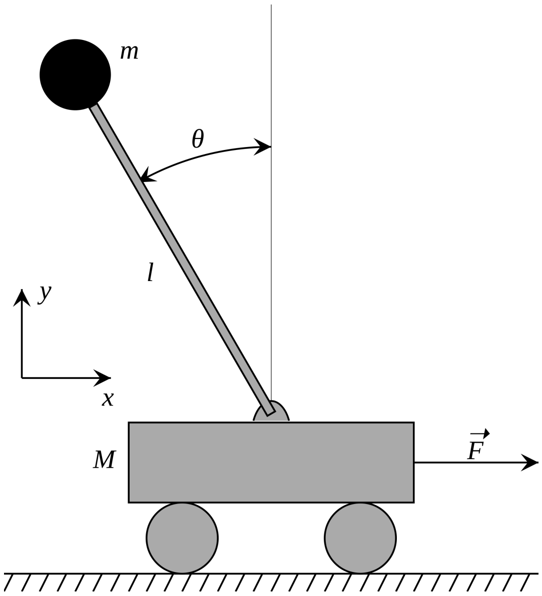

Figure 1.1: Inverted Pendulum Schematic

State vectors in this scenario are **position, speed, angle, angular speed**;

For this problem, the outputs are the cart's displacement (x in meters) and the pendulum angle (ϕ in radians), where ϕ represents the deviation of the pendulum's position from equilibrium, that is, $\theta = \pi + \phi$.

The design criteria for this system for a 0.2-m step in desired cart position x are as follows:

- Settling time for x and θ of less than 5 seconds
- Rise time for x of less than 0.5 seconds
- Pendulum angle θ never more than 20 degrees (0.35 radians) from the vertical
- Steady-state error of less than 2% for x and θ

We aimed to keep the pendulum vertical in the other situations in response to an impulsive disturbance force delivered to the cart. We did not attempt to direct the cart's movement. We're trying to keep the pendulum vertical while managing the cart's location to go 0.2 metres to the right.

We can use Full-state feedback to solve this problem. This type of control system's schematic is presented below,

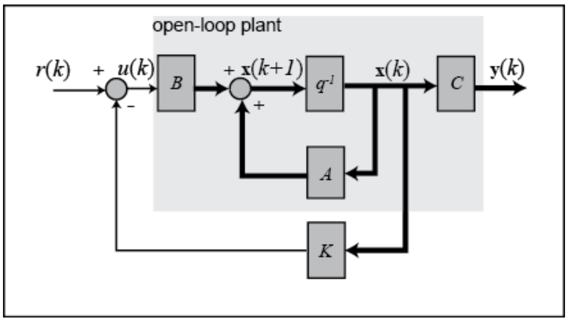

Figure 1.2: Full state feedback

Where K is a matrix of control gains, please take note that here we feedback all of the system's states rather than using the system's outputs for feedback.

1.3 Objectives

The scope of the project is to study and design the control system of an unstable system, in this case, *an inverted pendulum*. We will be simulating different alternatives on a computer with a modelled reality system. Our goal is to stabilize the pendulum's upright position while keeping the cart oscillating around the desired position to evaluate the alternatives and conclude.

1.4 Motivation

A cart Inverted pendulum system is one of the most common use to be considered for testing many control algorithms since it has some challenging problems associated with non-linearity, complexity and under-actuated system model. Non-linearity behaviour of the inverted pendulum can be observed easily. Different pendulum angle responses can be obtained by giving the same velocity in the cart. The cart inverted pendulum can be understood as an under actuated system since the system has a lower number of actuators than the degrees of freedom. One of the most convenient methods to model the inverted pendulum system is to use Lagrange's equation. However, many inverted pendulum models have been derived using a simplified physical model. This simplified model may lead to problems implementing the control algorithm in an actual physical inverted pendulum system. An inverted pendulum system model is presented in this report, where the mathematical model of the inverted pendulum was derived using Lagrange's equation. The determination of the pole-zero of the system is discussed. A simple method of **pole placement** is proposed to stabilize the pendulum at the desired position of the cart. Matlab simulation results show the effectiveness of the proposed method. And yet, this intuitive approach can lead to a better understanding of the control behaviours.

Chapter 2

BACKGROUND

We used different concepts and steps in this report to analyze the model of a cart inverted pendulum. These are:

1. Mathematical Modelling using (Lagrange equations)
2. Linearization of system
3. State-space modelling
4. Pole placement method

Let us discuss the background of some of the concepts I used in this report.

2.1 Inverted Pendulum

An inverted pendulum is a classic example of a model where the system's centre of mass is above its fulc which is similar to most real-life situations, such as humans, rocket ships and seismometers(old model).

The inverted pendulum is one of the most prominent and iconic examples in control theory. It is naturally unstable and non-linear and is frequently used as a model. As an academic example, a cart that can move freely beside a guide is frequently used to create an inverted pendulum. And a pendulum that can freely rotate around the cart's centre of rotation. The force applied to the cart is a controllable variable. The pendulum must be balanced.

For the pendulum to stay upright, the pendulum must be actively balanced. Stabilization can be accomplished in a variety of ways.

- Applying a torque at the pivot point.
- Moving the pivot point horizontally.
- Changing the rate of rotation of a mass mounted on the pendulum on an axis parallel to the pivot axis.
- Oscillating the pivot point vertically.

PID controllers, state-space representation, neural networks, fuzzy control, and other control systems are tested using the inverted pendulum as a benchmark. When considering the pendulum alone, it has two stable and unstable equilibrium positions. The goal is to use a control system to stabilize an unstable position, even if perturbed somehow. In a practical case, the guide along which the cart moves is finite; to simulate this, both the cart's location and the pendulum's position must be rendered stable.

In this report, We have used the **state-space model and Pole placement method** to control the pendulum position in space.

2.2 Linearization of the system

Linearization is a method for assessing the local stability of an equilibrium point of a system of non-linear differential equations or discrete dynamical systems. Linearization makes it possible to use tools for studying linear systems to analyze a non-linear system's behaviour near a given point of equilibrium. The inverted pendulum is a non-linear system. We must linearize the pendulum around the unstable equilibrium position, which is when the pendulum is in its upright position.

2.3 What is Pole Placement

One of the traditional control theories, the pole placement approach, has an advantage in system control for desired performance. To achieve the desired system behaviour, shift the pole location to that desired pole location once the system transfer function is specified mathematically. After that, each coefficient should be defined in the same order as the intended transfer function. A characteristic polynomial is compared to a transfer polynomial. This method of pole placement control produces the desired outcome. It is simple to control the gain numerically, but the precision of system transfer is critical. The function is highly critical and costly to implement in a high-order system. The Pole placement method is a controller design method in which we determine the places of the closed-loop system poles on the complex plane by setting a controller gain K.

Poles describe the behaviour of linear dynamical systems. Through feedback, you are attempting to change that behaviour more favourably. Thus we can decide where the closed-loop poles should be and then force them to be there by designing a feedback control system via the pole placement method where we choose a controller gain K that places the poles in their desired locations.

Let the following be the model of your system in state space:

$\dot{x}$=Ax+Bu.

The eigenvalues of A are the poles. Through the use of (for example) full state feedback, you set

u=-Kx,

with which the closed-loop system dynamics matrix becomes

Ac=A+BK .

Now the eigenvalues of Ac describe how your closed-loop system will behave. A and B are (usually constant) matrices that do not depend on K (they express a linear model of the physical system you want to control). Thus, Ac is a function of the controller gain K only; through modification, you can place the eigenvalues of Ac (i.e., the closed-loop poles) as desired (under certain conditions).

2.4 What is State-space model

State space is a mathematical modelling of systems in universe age old topic in control engineering . It represents a model of fixed inputs and outputs and a few sets of variables associated with first-order differential equations. State variables in this model are a form of the variable whose output changes over time and depends on the values given for the input variables. The value of the output variables relies upon the model of the state and input variables. Putting a model into state-space representation is the premise for many strategies on top of things evaluation and the dynamics technique.

Analysis of multi-input and multi-output structures is made smooth using the state-space modelling. Inside the case of the transfer function, while there's single input and a single output, we can take the Laplace model of input and the output and get the result. But, when there are multiple inputs and multiple outputs,but we can't perform the identical procedure on it. Because we've got a state vector in our state-space model, we can represent all our input and output variables in the vector form and perform to get the desired output for the device. The state-space model gives facts about controllability. We will say up to what extent the given model is controllable. It tells us how much lot quantity it can manipulate its functionality. The state-space model offers us information about the functionality of a specific model. The state-space modelling applies to all dynamic structures. We can examine all dynamic systems like linear, non-linear, time-variant, and time-invariant structures.

Chapter 3

Pole Placement Method - Working

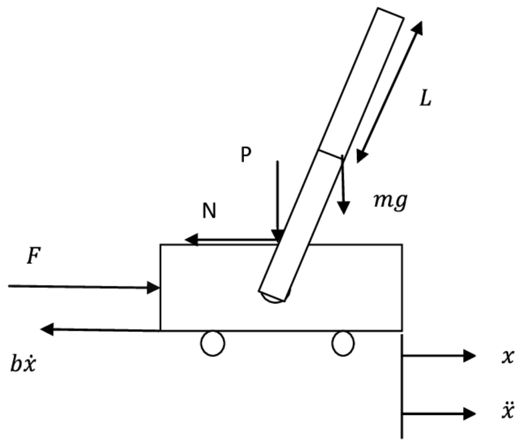

Figure 3.1: freebody Diagram

Above is the free body diagram for all forces acting on the inverted pendulum model.

$$(M+m)\ddot{x}+b\dot{x}+ml\ddot{\theta}\cos\theta-ml\dot{\theta}^2\sin\theta=u$$

$$(I+ml^2)\ddot{\theta}+mgl\sin\theta=-ml\ddot{x}\cos\theta$$

We first formulate the Newtonian equations and solve them using substitution and other mathematical methods. for simplification, let us assume

- friction-less surface (b=0)
- moment of inertia of pendulum is negligible. (I=0)
- air resistance is not applicable

We will be left with simplified equations of motion in terms of $\dot{x}$,$\ddot{x}$,$\dot{\theta}$,$\ddot{\theta}$

$$(M+m)\ddot{x}+ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = u$$

$$ml^2\ddot{\theta}+mgl\sin\theta = -ml\ddot{x}\cos\theta$$

for linearizing the equations assume small perturbations.
For small perturbations, $\theta \approx 0$ therefore, we have

- $sin\ \theta = \theta$
- $cos\ \theta = 1$
- $\dot{\theta}^2 = 0$

final equations used henceforth in this report are:

$$(M+m)\ddot{x}+ml\ddot{\theta} = u$$

$$l\ddot{\theta}+\ddot{x} = g\theta$$

On simplification and comparing with standard state space equations we will get matrices as:

$$\dot{x} = Ax+Bu$$

$$y = Cx+Du$$

where A= $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -mg/M & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & g(M+m)/Ml & 0 \end{bmatrix}$ B = $\begin{bmatrix} 0 \\ 1/M \\ 0 \\ -1/Ml \end{bmatrix}$ I calculated system response for open loop function and open loop poles' I considered a feedback loop with gain K and new equations will be

$$u = -Kx$$

$$\dot{x} = (A-BK)x$$

$$y = Cx+Du$$

I took the numeric values of masses and length accordingly for performing calculations. $M = 1kg$ $m = 0.1kg$ $l = 0.5m$ $g = 9.8m/s^2$

I calculated the system response of the closed-loop function. I assigned closed-loop poles by trial and error approach in Matlab, plotted graphs in each case, and selected poles such that the system response was good and satisfied our criterion. by using the Matlab command $K = place(A,B,P)$- I calculated values to the vector K and noted down the result of the closed-loop system function

To obtain a reasonable speed and damping in the response of the designed system (for example, the settling time of approximately 4∼5 sec and the maximum overshoot of 15%∼16% in the step response of the cart , let us choose the desired closed-loop poles at

$$P = [-8 - 5 - 2 - 0.8];$$

obtained from MATLAB iterations

I plotted graphs of state functions versus time for each state and verified the stability of the function.It is noted that, as in any design problem, if the speed and damping are not quite satisfactory,then we must modify the desired characteristic equation and determine a new matrix Computer simulations must be repeated until a satisfactory result is obtained.

Plots and observations of the Matlab report are discussed in the next chapter.

Chapter 4

EXPERIMENTAL RESULTS

4.1 Observations

I have plot state equations versus time in Matlab using plot function

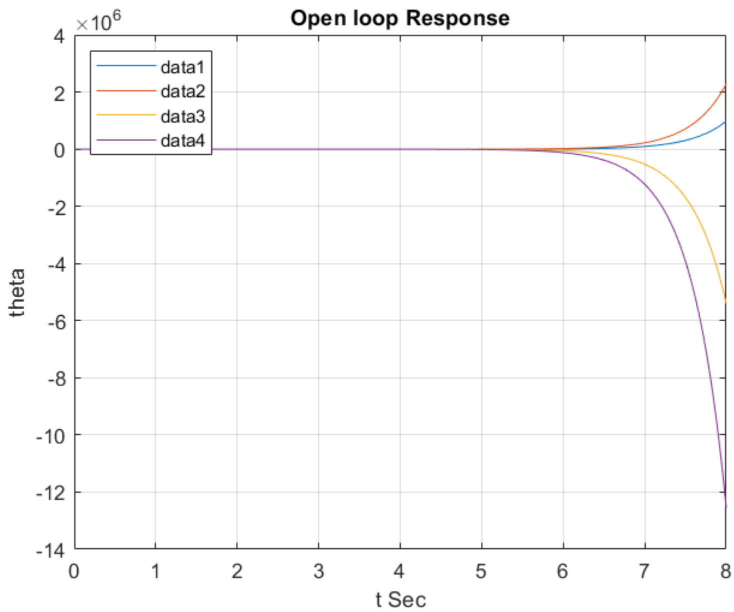

Figure 4.1: Open loop response

This is open loop plot of the all four states x,$\dot{x}$,θ,$\dot{\theta}$ we can see that these are diverging from stable point zero to infinity and very unstable.

We need to employ a negative feedback model to compensate for the instability in this graph. Below is a graph of all four states to time in the closed-loop model.

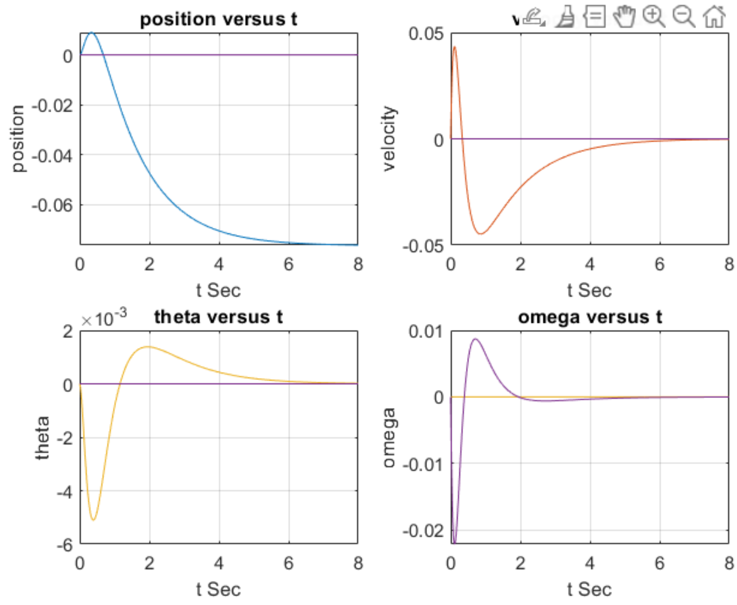

Figure 4.2: Closed loop response

Now, let us look individually at each graph and see how is equilibrium state achieved in each case.

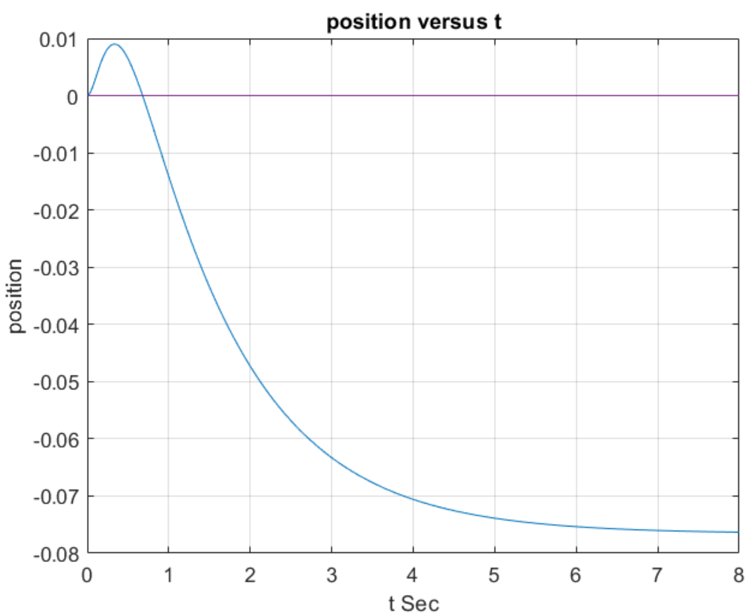

Figure 4.3: Position vs Time

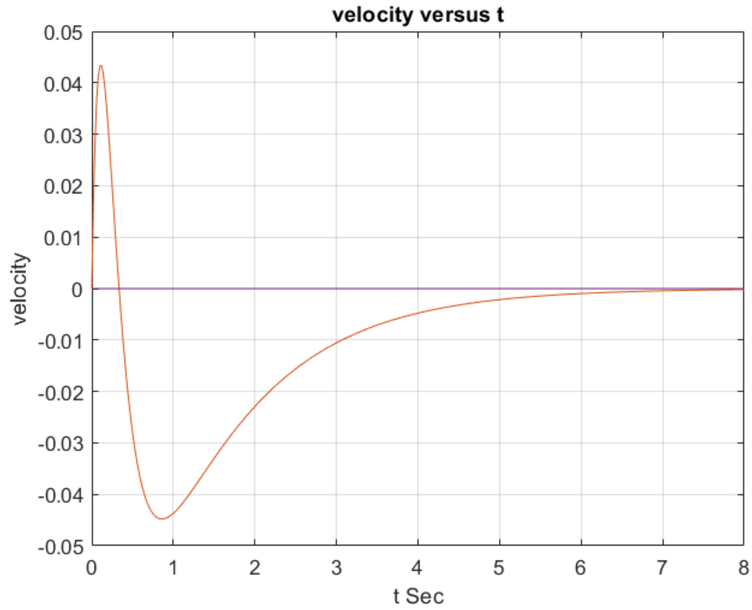

Figure 4.4: Velocity vs time

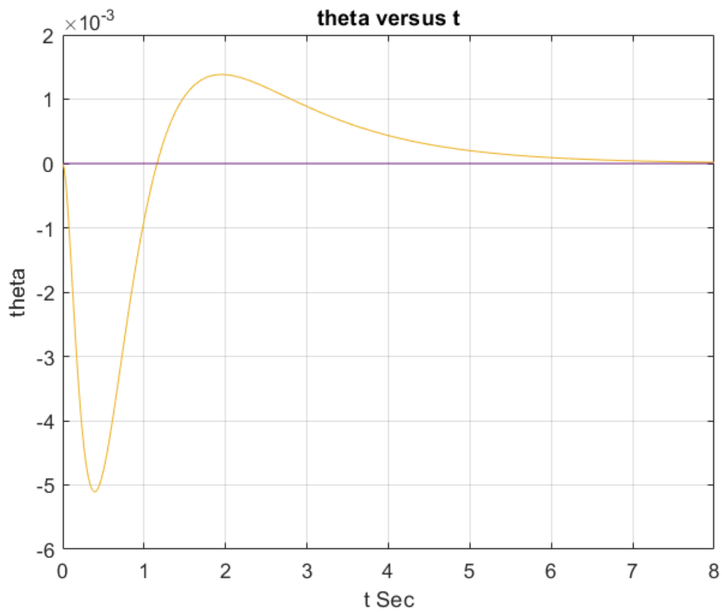

Figure 4.5: theta vs time

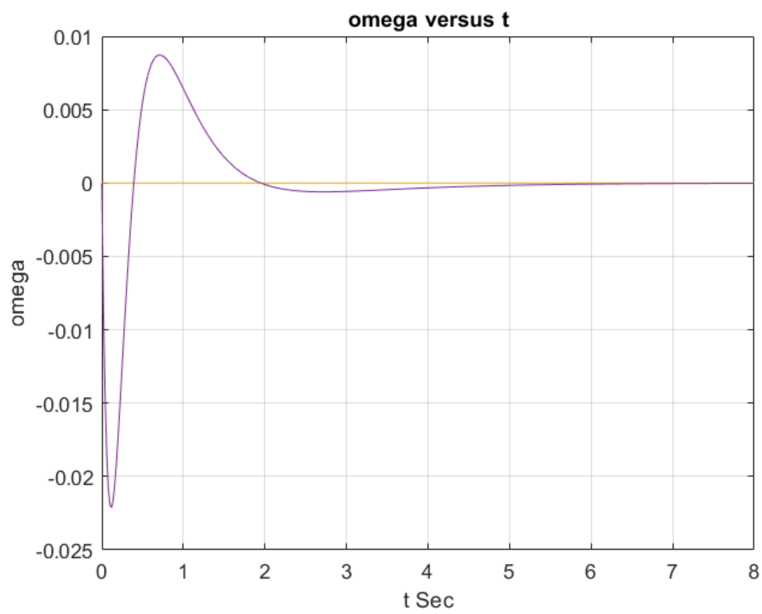

Figure 4.6: Omega vs time

We can notice that transient response is second order, and each plot (position approximately) reaches a steady state at zero point and attains equilibrium within 4 to 5 seconds.

Chapter 5

CONCLUSION AND APPLICATIONS

Stabilizing a cart inverted pendulum system using the pole placement method has been successfully conducted. The simulation result shows that the pole placement method can search feedback gain K. The simulation showed that the more negative the pole location makes, the pendulum's response will race toward point zero. Still, the more significant overshoot caused, and the response of the cart position is faster to reach the set point. But, the greater control output caused. Modelling of cart-inverted pendulum shows that system is inherently unstable as the pendulum swings to and fro without applying any external control forces with non-minimum phase zero. A full state feedback controller using a state variable feedback controller is proposed to stabilize a cart-inverted pendulum. In addition, the system has good robustness; it can overcome any external disturbances on the cart or pendulum rod.

5.1 Real Life Applications

There are quite a few examples in the real world of the inverted pendulum, both made by humans and in the natural world. The most prevalent example of the inverted pendulum is humans. People need to make constant adjustments to maintain the upright position of the body, whether standing, walking or running. Another simple example is balancing brooms with a hand or a finger.

The inverted pendulum has been employed in various devices. For instance, early seismometers' central component because any disturbance caused the pendulum to oscillate, and the response was measurable. Some two-wheeled personal transports that offer higher manoeuvrability are designed based on inverted pendulum models. The inverted pendulum is also related to rocket and missile guidance. The centre of gravity is behind the centre of drag, causing aerodynamic instability that needs to be corrected not to start spinning out of control. It is also related to the balance of a biped humanoid robot, which has similar characteristics as previously mentioned with humans.

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