Multi-Echelon Inventory Location Optimization by Pooling Safety Stock

A Dual Degree Project Thesis

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THESIS CERTIFICATE

This is to certify that the project thesis titled **Multi-Echelon Inventory Optimization for Safety Stock Calculation**, submitted by **S Viknesh**, to the Indian Institute of Technology, Madras, for the award of the degree of **Bachelor of Technology** and **Master of Technology**, is a bona fide record of the research work done by him under my supervision. The contents of this thesis, in full or in part, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

KEYWORDS: Network, Echelon, Material, Site, Inventory, Demand, Lead-time, Lead-time Demand, Reorder Point, Safety Stock, Pooling

This thesis aims to develop a pooling strategy that would minimize the cost of inventory for each product by optimizing combined safety stock value across warehouses. We develop various functions and models to compute safety stock values for all pooling combinations that determine flow of inventory to various warehouses. We have created measures to assess a site's eligibility for pooling based on the findings of our experiment. We tested the metrics only on the lowest echelon in the supply chain. However, it is possible to use similar metrics for other echelons in the network.

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ABBREVIATIONS

CDF	Cumulative Distribution Function
IITM	Indian Institute of Technology, Madras
ISS	Individual Safety Stock
ITP	Inventory Transportation Problem
LIP	Location Inventory Problem
LIRP	Location Inventory Routing Problem
MEIO	Multi Echelon Inventory Optimization
NP	Non-deterministic Polynomial-time
PDF	Probability Distribution Function
PSP	Pooling Site Preference
SEIO	Single Echelon Inventory Optimization

NOTATIONS

$P_X(x)$	PDF of variable X
$E(x)$	Expected value of variable X
$f : A \to B$	Function from elements in set A to elements in set B
Z^+	Positive Integers
D	Demand
LT	Lead-time
LTD	Lead-time Demand
s	Service Level
r	Reorder Point
SS	Safety Stock
PSS	Pooled Safety Stock
CSS	Combined Safety Stock
PSS_opt	Optimal Pooled Safety Stock
Site_opt	Optimal Pooling Site
CSS_opt	Optimal Combined Safety Stock
Comb_opt	Optimal Pooling Combination
μ_D	Mean of Demand distribution
σ_D	Standard Deviation of Demand distribution
μ_{LT}	Mean of Lead-Time distribution
σ_{LT}	Standard Deviation of Lead-Time distribution

CHAPTER 1

INTRODUCTION

In the conventional retail industry, inventory is sometimes one of the largest asset investments. Reduced inventory levels can free up not just much-needed cash for the company, but also premium storage space and associated investment expenditures. As a result, a firm's financial strength can be bolstered by a constant focus on inventory efficiency, which can set it apart from its competitors.

Company XYZ (anonymous) is a huge multinational corporation and an industry leader. XYZ has a global network of retail, distribution centres, and warehouses. XYZ discovered that they are keeping sub-optimal safety stocks, which typically results in surplus inventory. In this thesis, we will discuss the how multi-echelon inventory optimization can be used by XYZ to maintain lower inventory without increasing stock-outs.

1.1 Multi-echelon Inventory Optimization (MEIO)

An effective MEIO is crucial for smooth supply chain performance as maintaining optimal levels of stock at the right location at the right time helps in meeting supply chain demands efficiently. MEIO enables to keep the right levels of stock in accordance with multi-echelon planning throughout the supply chain distribution networks. Adopting a strategic and mathematical approach to calculating the right levels of inventory at each stage in the supply chain can help in solving the real-world challenges in inventory optimization.

A lot of supply chains are maintained through single-echelon inventory optimization (SEIO), i.e. each echelon is still managed separately, often based on simple rules of thumb. MEIO should then bring added value by performing a global rather than local optimization. Let's first discuss the issues of pursuing SEIO before discussing the benefits of MEIO

Disadvantages of SEIO: [3]

- Each warehouse can choose (optimize) its own inventory policy without any alignment with the other echelons. They could even change their inventory policies without letting the others know.
- The slightest change in the external demand might drastically impact the supply chain.
- Different nodes can even start to pursue rogue behaviors to secure some (profitable) stock, to the detriment of the supply chain as a whole.
- In many cases, supply chains that are stuck in local inventory optimization face a bigger issue than simple misalignment. They face the risk of a massive bullwhip effect as explained below:

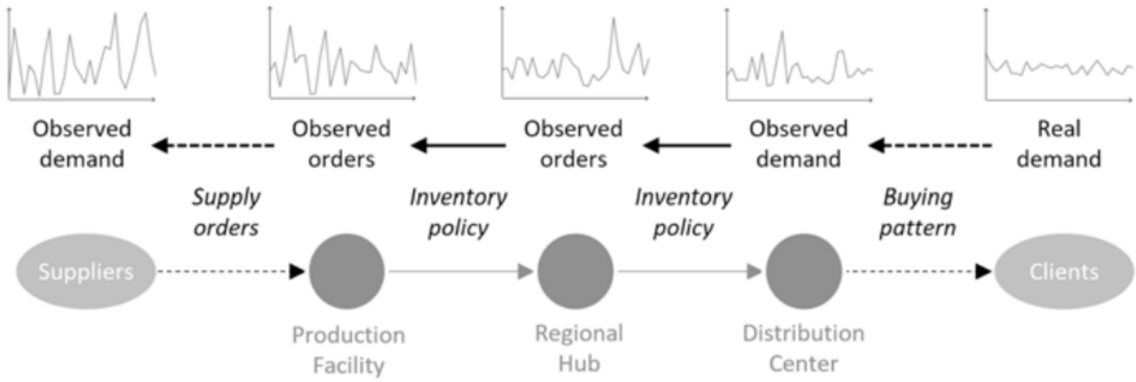

Figure 1.1: **Bullwhip Effect** [3]

Key advantages of MEIO: [3]

- Maintains the right levels of inventory across multiple warehouses (globally). This helps avoid scenarios leading to Bullwhip Effect.
- Enables well-informed supply chain decision-making through centralization of inventory policy making.
- Reduces global inventory handling and storage costs.

We will now discuss topics that describe a multi-echelon model's network design.

1.1.1 Echelon

Echelon is defined as the number of inventory levels between the supplier and the final customer in the supply chain, i.e. an n-echelon inventory model will have n levels of inventory storing sites between the supplier and end-customer.

1.1.2 Supply Chain Structures

The diversity of supply chains has lead to the establishment of different structures in the literature. We present the 5 basic types: [2]

- **Serial**
  A serial system is a system of warehouses where each node in the network may only have one predecessor and one successor. An example is shown below:

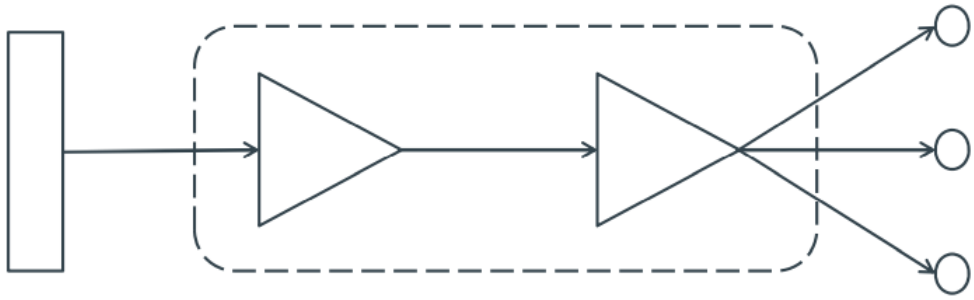

Figure 1.2: **Serial System with 2 echelons** [1]

  The supplier (rectangle) and the customers (circles) are outside of the planners control.

- **Divergent**
  A divergent system (also known as distribution system [2]) is a system of warehouses where each node in the network may only have one predecessor but many successors. Divergent structures usually refer to a replenishment or distribution system. Starting from the entry node inventory is distributed along the network. This is the classical system of spare parts inventory management. An example is shown below:

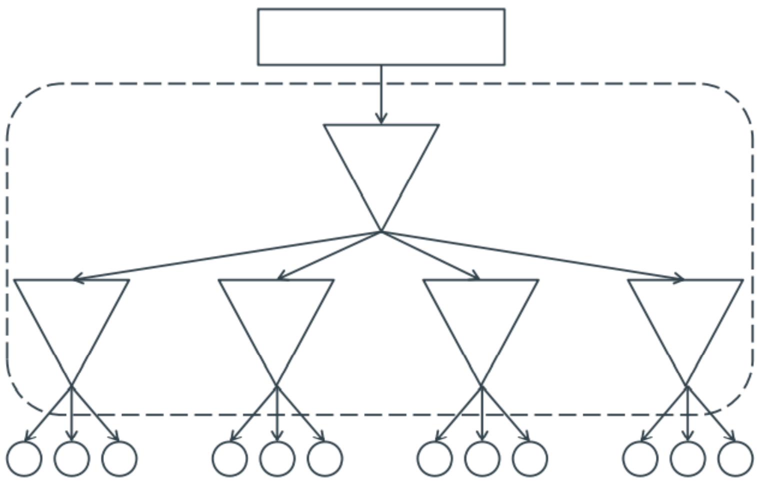

Figure 1.3: **Divergent System with 2 echelons** [1]

- **Convergent**
  A convergent system (also known as assembly system [2]) is a system of warehouses where each node in the network may have many predecessors but only one successor. Convergent systems have the contrary definition compared to divergent systems. This type of structures is mainly used for manufacturing supply chains where certain materials are processed through several echelons until the end product is manufactured. An example is shown below:

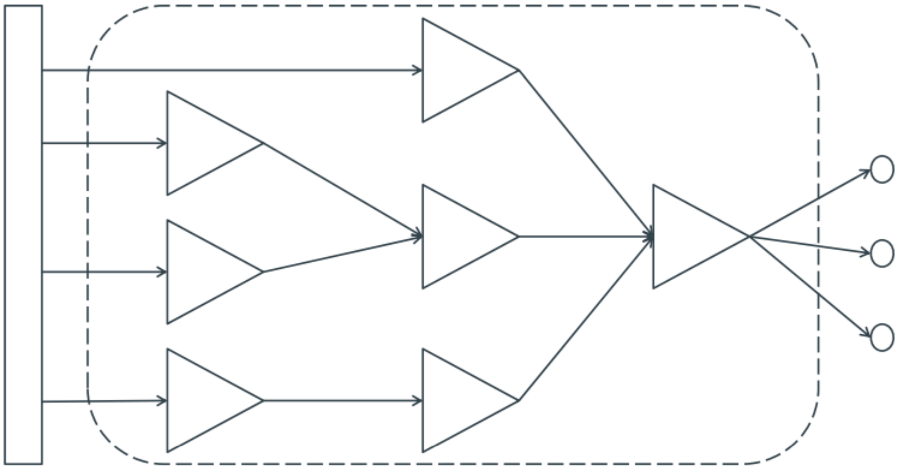

Figure 1.4: **Convergent System with 3 echelons** [1]

- **Tree**
  A tree system is a hybrids of assembly and distribution systems—each stage may have multiple predecessors and successors—but tree systems may contain no undirected cycles. (A cycle, in graph theory, is a portion of the graph whose links allow one to move from a starting node, through a sequence of other nodes, and back to the starting node, without repeating any other nodes links. An undirected cycle is a cycle in the graph that results from removing all of the arrows from the links so that movement can go in either direction.) An example is shown below:

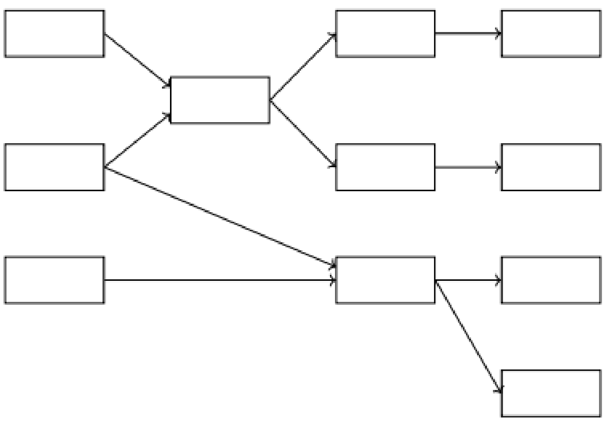

Figure 1.5: **Tree System (without suppliers and customers) with 4 echelons** [2]

- **General**
  A general system is a system of warehouses with no restrictions on the number of predecessors and successors. A general system allows for any kind of relationship and has no restrictions whatsoever. Examples for this are a generalization of distribution systems which permit lateral transshipment or assembly systems where a node has more than one successor, i.e. a material that is processed in one node is needed in more than one node in the next echelon. An example is shown below:

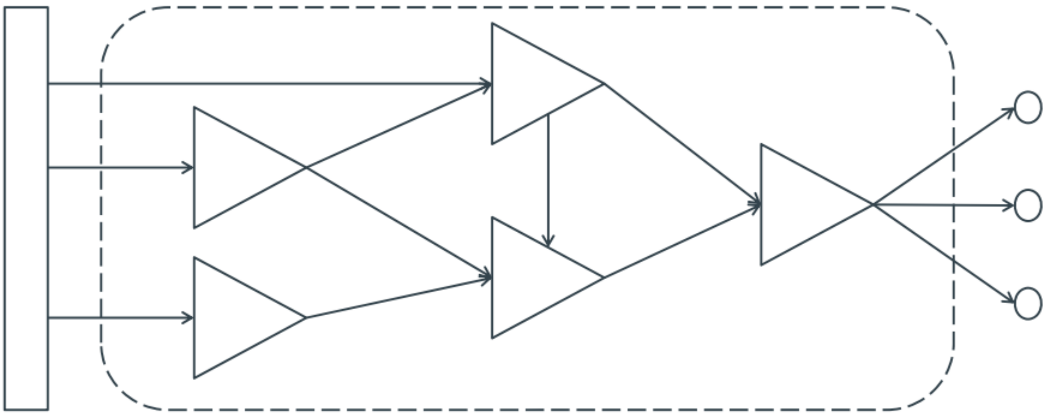

Figure 1.6: **General System with 3 echelons** [1]

  In this thesis, we will be dealing with a 3-echelon general system

Multi-echelon optimization can be performed both within an echelon as well as across echelons. In this thesis, we will focus on implementing a strategy for optimal inventory distribution within a single echelon. The echelon under discussion is shown below:

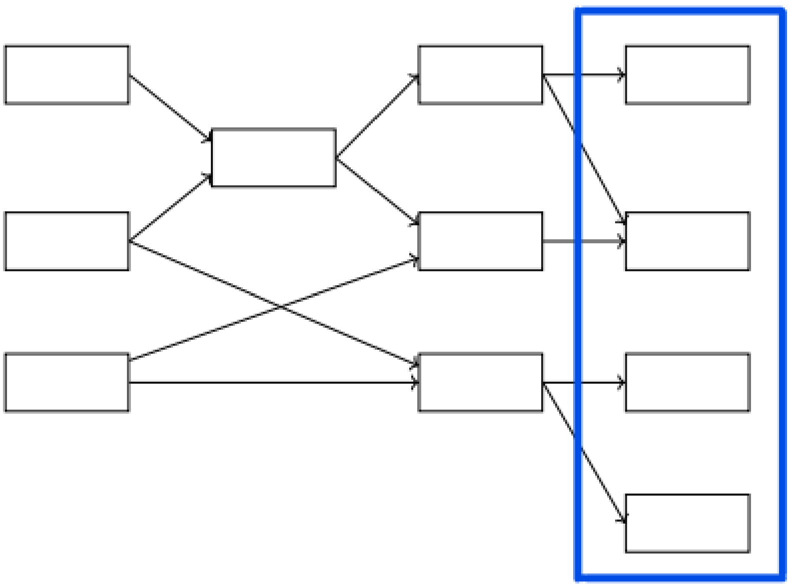

Figure 1.7: **Echelon highlighted in blue**

We now define the problem to be solved and describe the nature of data available

1.2 Problem Statement

XYZ wants to reduce its total cost of ownership by lowering inventory levels. XYZ thinks that, given the scale and complexity of its supply chain, there is a considerable opportunity and need to optimise its inventory in order to achieve a competitive advantage.

Our goal is to optimise the level of safety stock maintained for each material at numerous warehouses across continents. We recommend lowering the level of safety stock by implementing an inventory pooling technique, which would minimise inventory holding costs while also potentially lowering transportation expenses.

1.3 Available Data

1.3.1 Network Design

XYZ has a 3-Echelon general supply chain structure , i.e. the product flows as follows:

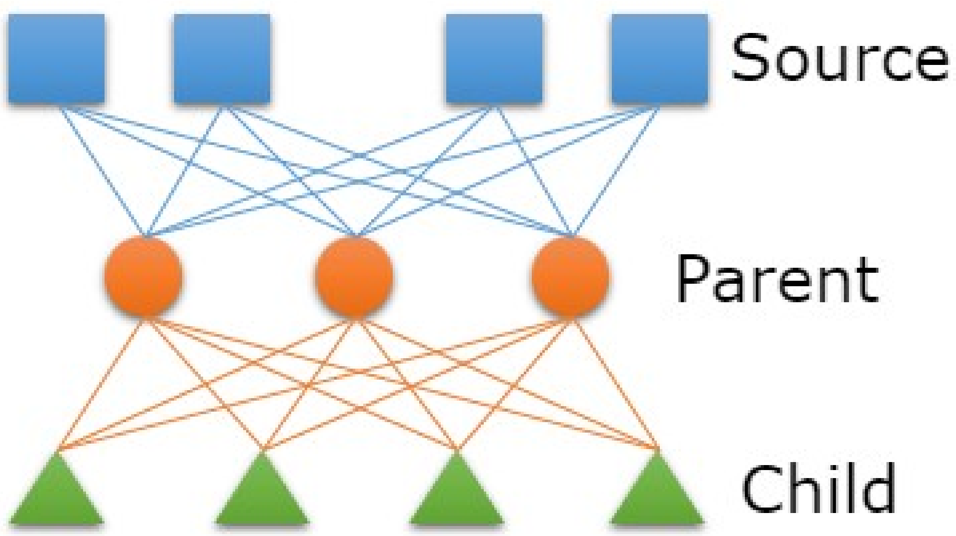

Figure 1.8: **XYZ Supply Chain Structure**

NOTE: This is the general network for any product. For a few exceptions, the network is only 2-Echelon

1.3.2 Sites and Materials

1.3.2.1 Site

Each node (warehouse) of the network is called a site. XYZ has 16 source sites, 26 parent sites and 59 child sites across 6 continents

1.3.2.2 Material

Every product manufactured and sold by the company is referred to as a material. XYZ has 1875 unique materials

1.3.3 Purchase and Sales orders

The data obtained from purchase and sales orders together provides complete knowledge of all the transactions that occurred within the time period of interest.

1.3.3.1 Demand

The demand distribution for each material at each site is obtained based on the sales order data for the first 180 days of 2021

1.3.3.2 Lead-time

The lead-time distribution for each material at each site is obtained based on the purchase order data for the first 180 days of 2021

Demand and Lead-time will be discussed in detail later. The following is a breakdown of the thesis's structure. In Chapter 2, we go over some key terms and the optimization strategy we employed to get to our target. We present our technique in Chapter 3, which includes data acquisition and analysis, as well as mathematical model construction and execution. We review the findings in Chapter 4 to show how safety stock pooling lowers holding costs. In Chapter 5, we wrap things up with some managerial takeaways and a look ahead.

CHAPTER 2

CONCEPTS INVOLVED

2.1 Theoretical Background

2.1.1 Inventory

The term inventory refers to both the raw materials utilised in production and the finished goods that are ready to sell. Inventory is one of a company's most valuable assets because inventory turnover is one of the key sources of revenue production and, as a result, earnings for the company's shareholders. Raw materials, work-in-progress, and finished goods are the three forms of inventory. On a company's balance sheet, it's classified as a current asset. [4]

The table below presents a few factors that influence inventory decisions

Table 2.1: **Factors Influencing Inventory Decisions** [2]

Why hold inventory?	Why avoid inventory?
Lead Times	Cost of capital
Economies of scale	Shelf space
Service levels	Perishability
Prevent Stock-out	Risks at warehouse

In this thesis, we will only discuss about finished goods

2.1.2 Lead-time Demand

The lead time demand (also called lead demand) is the total demand between now and the anticipated time for the delivery after the next one if a reorder is made now to replenish the inventory. This delay is named the lead time. Since lead demand is a future demand (not yet observed), this value is typically calculated using time series analysis. [4]

2.1.2.1 Demand distribution

Demand distribution is calculated based on the demand data obtained from the sales order. The table below shows the sales order for a 1 material at 1 site.

Table 2.2: **Sales order data for a random material at a random site**

Site	Material	Sales date	Quantity
A2	M13	2021-01-08	12
A2	M13	2021-01-13	1
A2	M13	2021-01-25	2
A2	M13	2021-01-31	6
A2	M13	2021-02-06	17
A2	M13	2021-02-10	37
A2	M13	2021-02-19	31
A2	M13	2021-03-10	3
A2	M13	2021-03-21	2
A2	M13	2021-03-31	54
A2	M13	2021-04-14	8
A2	M13	2021-04-27	2
A2	M13	2021-05-16	4
A2	M13	2021-05-23	3
A2	M13	2021-06-02	3
A2	M13	2021-06-13	2
A2	M13	2021-06-28	13

On the days when sales orders were placed, we will consider the demand equals the quantity sold. On the remaining days, demand equals zero. This will result in a discrete demand distribution over 180 days. The demand distribution obtained from the data in Table [2.2] is shown below.

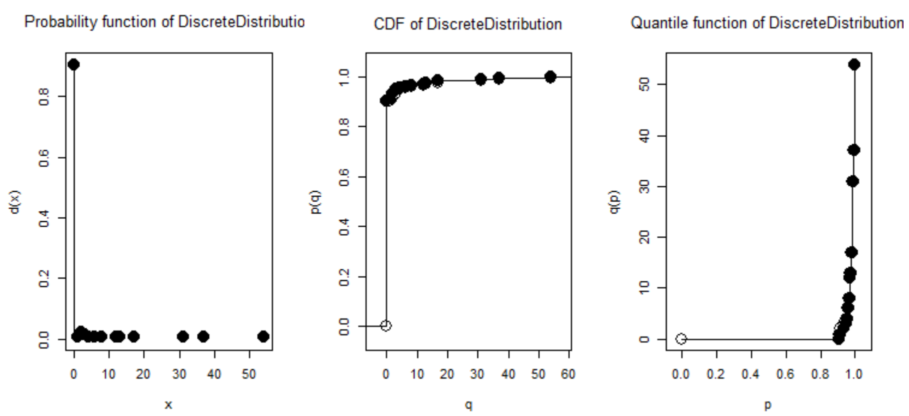

Figure 2.1: **Distribution of Demand data from Table [2.2]**

Limitations

Using a histogram to estimate the demand distribution comes with a few issues: [3]

- Non-zero probability is only given to a few values. In the above mentioned example, more than 90% of the demand values corresponds to 0
- We do not know what the histogram limits should be (left and right edges). Choosing another set of limits will also change the demand distribution representation.
- The resulting demand distribution is not continuous (in other words, smooth). As you can see in Figure [2.1], the variation between probability of demand = 0 and demand = 1 is too high. This is an abrupt change and it would be more realistic to see a smooth decrease. But, as stated earlier, with only a few possible demand values, a smooth variation is difficult

Kernel Density Estimation can help us overcome some of these inefficiencies. [3]. But for our experiments, we will continue using discrete demand distribution for calculation purposes.

2.1.2.2 Lead-time distribution

The time it takes for a warehouse order to be fulfilled, or from the moment the order is placed until the items arrive at the warehouse and are ready to meet demand, is known as lead time. As a result, lead time is made up of transportation time plus some additional time owing to stock-outs at the supplying unit, referred to as wait time.

It's worth noting that the transportation time includes more than just the time it takes to get from source to warehouse. It is the period of time from when the stock is available to fulfil the order at the supplier to when it is available for customer orders at the warehouse. Hence, it includes time spent inspecting products at the warehouse or time spent storing goods in the warehouse when they arrive, among other things. [1]

Lead-time = No. of days from Order date to Delivery date (both inclusive)

Lead-time distribution is calculated based on the Lead-time data obtained from the purchase order. The following table shows the purchase order for a 1 material at 1 site.

Table 2.3: **Purchase order data for a random material at a random site**

Site	Material	Order date	Delivery date	Quantity	Lead-time
A1	M18	2021-03-08	2021-04-06	12	30
A1	M18	2021-03-31	2021-05-17	17	48
A1	M18	2021-01-25	2021-04-06	29	72
A1	M18	2021-03-25	2021-05-25	6	62
A1	M18	2021-01-06	2021-03-13	10	67
A1	M18	2021-02-10	2021-04-19	32	69

The lead-time distribution obtained from the data in Table [2.3] is shown below.

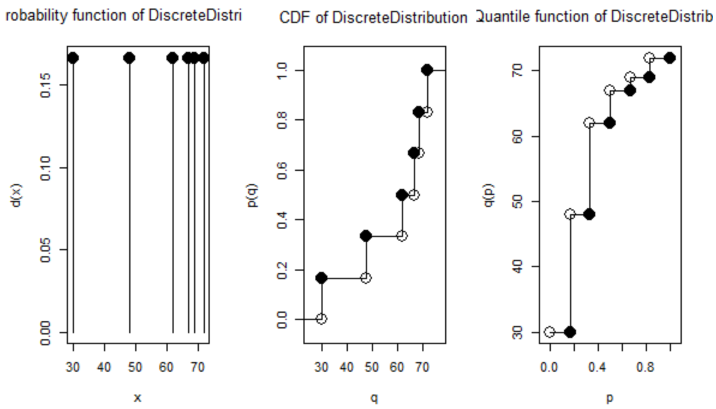

Figure 2.2: **Distribution of Lead-time data from Table [2.3]**

Lead-time Demand distribution is a compound probability distribution obtained from Demand distribution and Lead-time distribution as demonstrated below

2.1.2.3 Lead-time Demand distribution

D: Demand LT: Lead-time LTD: Lead-time Demand

$$LTD = \sum_{i=1}^{LT} D_i$$

D_i: Demand corresponding to day i

Let's consider the following example to understand how LTD distribution is calculated

$$P_D(d) = \begin{cases} 0.5 & d = 1 \\ 0.5 & d = 2 \\ 0 & otherwise \end{cases} \qquad P_{LT}(t) = \begin{cases} 0.5 & t = 1 \\ 0.5 & t = 4 \\ 0 & otherwise \end{cases}$$

The Lead-time Demand Distribution is calculated as shown in the table below

Table 2.4: **Lead-time Demand Distribution demonstration**

LT	$P_{LT}(t)$	**Demand Profile**	**P(Demand Profile)**	**LTD**	$P(LTD)$
1	0.5	1	0.5	1	$0.5 \times 0.5 = 0.25$
1	0.5	2	0.5	2	$0.5 \times 0.5 = 0.25$
4	0.5	1,1,1,1	$\binom{4}{0} \times 0.5^4 = 0.0625$	4	$0.5 \times 0.0625 = 0.03125$
4	0.5	1,1,1,2	$\binom{4}{1} \times 0.5^4 = 0.25$	5	$0.5 \times 0.25 = 0.125$
4	0.5	1,1,2,2	$\binom{4}{2} \times 0.5^4 = 0.375$	6	$0.5 \times 0.375 = 0.1875$
4	0.5	1,2,2,2	$\binom{4}{3} \times 0.5^4 = 0.25$	7	$0.5 \times 0.25 = 0.125$
4	0.5	2,2,2,2	$\binom{4}{4} \times 0.5^4 = 0.0625$	8	$0.5 \times 0.0625 = 0.03125$

The Lead-time Demand Distribution presented in Table [2.4] is shown below

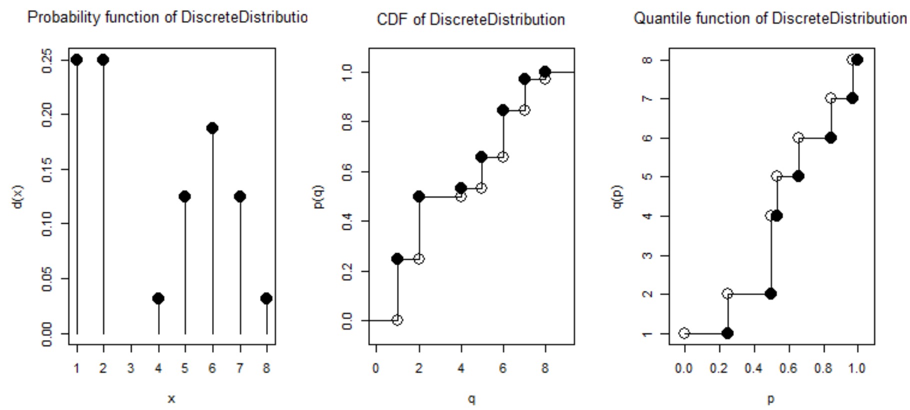

Figure 2.3: **Lead-time Demand Distribution from Table [2.4]**

In the upcoming sub-sections, we will explain how Lead-time Demand Distribution is used to eventually calculate safety stock value

2.1.3 Safety Stock

Safety stock is an inventory optimization method that indicates how much inventory need to be kept beyond the expected demand in order to achieve a given service level target. The extra stock acts as a "safety" buffer - hence the name - to protect the company against expected future fluctuations. The safety stock formula depends on both the expected future demand and the expected future lead time. [4]

2.1.3.1 Service level

Service level represents the expected probability of not hitting a stock-out. This percentage is required to compute the safety stock. Intuitively, the service level represents a trade-off between the cost of inventory and the cost of stock-outs. Companies usually prefer surplus inventory over stock-out. Hence, they always strive to reach high service levels. [4]

2.1.3.2 Reorder Point

The reorder point is the inventory level which signals the need for a replenishment order. The reorder point is classically viewed as the sum of the lead demand plus the safety stock. [4]

Let the service level (lies between 0 and 1) be represented by s and Reorder Point be represented by r. Then

$$\sum_{x=0}^{r} P_{LTD}(x) = s \qquad \Longrightarrow \qquad r = CDF_{LTD}^{-1}(s)$$

To understand this clearly, we will go back to the Lead-time Demand distribution presented in Table [2.4]. The table below contains the CDF of the same

Table 2.5: **CDF of Lead-time Demand presented in Table [2.4]**

x	1	2	4	5	6	7	8
$\boldsymbol{CDF_{LTD}(x)}$	0.25	0.5	0.53125	0.65625	0.84375	0.95875	1

Suppose service level equals 95%, then the reorder point will be 7

2.1.3.3 Safety Stock Calculation

Lead Demand = Average Demand $\times$ Average Lead-time = $E(D) \times E(LT)$

Safety Stock (SS) = Reorder Point - Lead Demand = $r - E(D) \times E(LT)$

For the example discussed in Table [2.5]:

$r = 7 \qquad [s = 95\%] \qquad E[D] = 1.5 \qquad E[LT] = 2.5$

$SS = 7 - 1.5 \times 2.5 = 7 - 3.75 = 3.25 \qquad$ Safety Stock = $\lceil 3.25 \rceil = 4$

2.1.4 Pooling

Inventory pooling refers to the consolidation of multiple inventory locations into a single one. Inventory locations may be associated with different geographical sites, different products, or different customers. Inventory pooling differs from centralization (centralized order decisions) where the managers still decide the order quantity for their own store (retail), whereas their inventories are pooled together.

Benefits of pooling inventory: [2]

- Total inventory and Aggregate demand uncertainty decreases
- Service level and Product availability increases
- Facilitates a more demand-driven replenishment

2.1.4.1 Pooling Rules

In our experiments, we will pool sites for a given material as explained below
Let X_1, X_2, ..., X_n be the n sites that will be pooled at site X_i $[1 \leq i \leq n]$
$D(X_j)$: Demand at site X_j $LT(X_j)$: Lead-time at site X_j

Table 2.6: **Pooling rules for Demand and Lead-time**

Pooled Demand	**Pooled Lead-time**
$\sum_{j=1}^{n} D(X_j)$	$LT(X_i)$
Sum of demand at all sites	Lead-time at pooling site

We will use the procedures mentioned earlier to compute Pooled Safety Stock value after we get pooled demand and lead-time.

2.1.4.2 Pooling Style

Pooling style is represented as sum of numbers that describes how sites are pooled.
Example: if pooling style is $1 + 1 + 2$, then 2 out of the 4 sites will be pooled and the remaining sites will not be pooled. All possible pooling styles for 4 sites are explained in the following table

Table 2.7: **Pooling styles for 4 sites**

Pooling Style	Description	
$1+1+1+1$	none of the sites will be pooled	
$1+1+2$	2 sites will be pooled together	remaining 2 will not be pooled
$1+3$	3 sites will be pooled together	remaining 1 will not be pooled
$2+2$	2 sites will be pooled together	remaining 2 will be pooled together at another site
4	all sites will be pooled together	

In general, if pooling style is $\sum_{j=1}^{k} m_j$, then each of the m_j sites will be pooled together at one of the m_j sites $\quad [\forall\, 1 \leq j \leq k]$

2.1.4.3 Pooling combination

For a given pooling style, a pooling combination refers to a way in which the given sites can be pooled. For example, let's consider 3 sites X, Y and Z. Suppose pooling style equals 1+2, the there are 3 pooling combinations:

- X and Y pooled together and Z is not pooled
- X and Z pooled together and Y is not pooled
- Y and z pooled together and X is not pooled

The number of pooling combinations for all possible pooling styles for 4 sites are given in the following table

Table 2.8: **Number of Pooling combinations for each Pooling style for 4 sites**

Pooling Style	Number of Pooling Combinations
$1+1+1+1$	1
$1+1+2$	$\binom{4}{2}=6$
$1+3$	$\binom{4}{3}=4$
$2+2$	$\binom{4}{2}/2=3$
4	$\binom{4}{4}=1$

NOTE: In pooling combination, we will only consider combination of sites that can be pooled. We will not permute the possible sites at which the combination can be pooled.

We will discuss how Optimal Pooling Combination is obtained while explaining the models used in experiments.

2.2 Location Inventory Problem (LIP)

After calculating the optimal Pooled Safety Stock values for each possibility, finding the combination with least total safety stock can be implemented using an LIP. Broadly, LIP can be classified into four types:

- Basic LIP
- Dynamic LIP
- Location Inventory Routing Problem (LIRP)
- Inventory Transportation Problem (ITP)

These types of problems focus on strategic decisions such as capacity planning, allocation decisions that, in concert with a location problem can compound supply chain network design problem. [5]

This study deals with Basic LIP as explained below

2.2.1 Problem Definition

Let's consider n sites that are to be pooled. Let set M contain all proper subsets of these n sites. $|M| = 2^n - 1$
Example: $n = 3 \implies M = \{\{1\};\{2\};\{3\};\{1,2\};\{1,3\};\{2,3\};\{1,2,3\}\}$

Let function $PSS : M \to Z^+$ compute the Optimal Pooled Safety Stock value (and pooling site which will be stored separately) of the input sites.
$PSS(m) = \min_{i \in m} \big(\text{safety stock when sites in } m \text{ are pooled at } i\big) \qquad [\forall m \in M]$

LIP would search for the optimal combination of mutually exclusive elements from M that cover the set while producing least combined safety stock value. Hence, the problem should be solved using an exclusive set covering approach. In a usual set-covering problem, it is not necessary that the selected sets should be mutually exclusive. Hence, our problem will have a higher time complexity due to the additional constraint.

O(Exclusive Set Covering) > O(Set Covering) ≈ NP Complete [6]

2.2.2 Mathematical Representation

m_i: i^{th} element of M

Exclusive Set Covering:

$$\min \sum_{i=1}^{|M|} c_i \times SS(m_i)$$

Subject to:

$$c_i \in \{0, 1\}$$
$$c_i + c_j \leq 1 \text{ if } m_i \cap m_j \neq \emptyset \ \forall i \neq j$$
$$i, j \in (1, |M|) \text{ and } i, j \in Z^+$$

2.2.3 Implementation

We will apply the Exclusive set covering approach without any approximation as not many sites will be pooled for practical purposes.

To simplify calculations, we will create a set U that will store all pooling combinations segregated based on pooling style. The table below contains set U for $n = 4$

Table 2.9: **Pooling combinations for each Pooling style for 4 sites**

Pooling Style	Pooling Combinations
$1+1+1+1$	{{1}, {2}, {3}, {4}}
$1+1+2$	{{1}, {2}, {3, 4}};{{1}, {2, 4}, {3}};{{1}, {2, 3}, {4}}; {{1, 4}, {2}, {3}};{{1, 3}, {2}, {4}};{{1, 2}, {3}, {4}}
$1+3$	{{1}, {2, 3, 4}};{{1, 3, 4}, {2}};{{1, 2, 4}, {3}};{{1, 2, 3}, {4}}
$2+2$	{{1, 2}, {3, 4}};{{1, 3}, {2, 4}};{{1, 4}, {2, 3}}
4	{{1, 2, 3, 4}}

In the next chapter, we will present the algorithm used to calculate the Combined Safety Stock value for each pooling combination and find the Optimal Pooling Combination for each pooling style.

CHAPTER 3

EXPERIMENT DETAILS

3.1 Project Scope

3.1.1 Associated costs

It's crucial to know more than just holding costs when analysing across echelons. Due to the lack of essential charges such as transportation and ordering costs, which will bear larger effect while optimizing across echelons, we have resorted to MEIO within a single echelon.

3.1.2 Network Complexity

Pooling inventory at child level is least dependent on transportation and ordering costs. Hence, we will implement our location optimization algorithm only across the child sites [1.7].

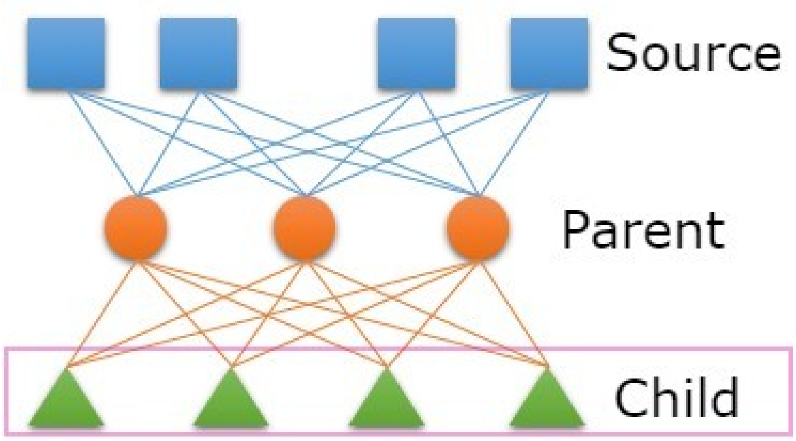

Figure 3.1: **Network Complexity**

Although we are implementing the optimizer only within an echelon, the method can be extended and used across echelons by modifying the cost function with the inputs from other costs.

3.2 Data generation

For our experiments performed here, we generated data that closely resembles the child site data of company XYZ. (Actual data remains undisclosed)

3.2.1 Sites and Materials

We will generate demand and lead-time data data for

- 30 child sites (across 5 continents)
- 5 prominent (high inventory levels) materials

This sample is large enough to obtain general insights for all child sites

3.2.2 Demand and Lead-time

For each site and material mentioned above, we will generate

- demand for 180 days
- corresponding lead-time

To keep the generated data as close to reality as possible, the distribution parameters (mean and variance) are taken from the actual data set.

3.3 Model Formulation

We address 3 important questions in our experiment

- The following questions are answered by Algorithm [1]
  - Which sites should be pooled?
  - At which location should these sites be pooled?
- The following question is answered by Algorithm [2]
  - Which pooling combination will produce optimal Combined Safety Stock?

Following segments present the algorithms used to obtain the desired results

3.3.1 Safety Stock Calculation

This algorithm will take the demand and lead-time data for a particular material from the sites to be pooled, and the service level as input. The Pooled Safety Stock value along with the pooling site will be the output

Input

x: list of sites to be pooled D: Demand at the sites   ($D[i]$: Demand at site i)

LT: Lead-time at the sites   ($LT[i]$: Lead-time at site i) s: service level

Algorithm 1: SS(x, D, LT, s)   Time Complexity: O(|x|)

```
Calculate pooled demand   [2.6]
Compute demand distribution   [2.1]
PSS_opt = ∞;  Site_opt = ∅
while i ∈ x do
    Site_opt = i
    Calculate pooled lead-time   [2.6]
    Compute lead-time distribution   [2.2]
    Compute lead-time demand distribution   [2.4]
    Calculate Reorder Point using s   [2.5]
    Calculate Pooled Safety Stock from Reorder Point
    if Pooled Safety Stock < PSS_opt then
        PSS_opt = Pooled Safety Stock
        Site_opt = i
    Move to next i
return PSS_opt , Site_opt
```

Output

PSS_opt: Optimal Pooled Safety Stock   Site_opt: Pooling Site

3.3.2 Optimal Pooling Combination

This algorithm will take the demand and lead-time data for a particular material from all the sites, and the service level as input. The Combined Safety Stock value along with the Optimal Pooling Combination for each Pooling Style will be the output

Input

N: list of all sites D: Demand at the sites   ($D[i]$: Demand at site i)

LT: Lead-time at the sites   ($LT[i]$: Lead-Time at site i) s: service level

Algorithm 2: OPC(N, D, LT, s) Time Complexity: NP Complete

```
Generate all possible pooling combinations and store them in set U    [7]
Segregate based on pooling style    [2.9]
while parsing through each pooling style do
    CSS_opt = ∞;  Comb_opt = ∅
    while C ∈ U with corresponding pooling style do
        CSS = 0;  Comb = ∅
        while i ∈ C do
            CSS += SS[i]    [1]
            Store pooling site in Comb
            Move to next i
        if CSS < CSS_opt then
            CSS_opt = CSS
            Comb_opt = Comb
        Move to next C
    Output: Pooling Style , CSS_opt , Comb_opt
    Move to next pooling style
```

Output

CSS_opt: Optimal Combined Safety Stock value for corresponding pooling style

Comb_opt: Optimal Pooling Combination for corresponding pooling style

3.3.3 Evaluation Metrics

3.3.3.1 Individual Safety Stock Order

The site with higher Individual Safety Stock (ISS) value, i.e. the safety stock at the chosen site before pooling, is given higher preference in ISS Order. ISS value of each site is computed using Algorithm [1]. Sorting these values will produce ISS Order.

3.3.3.2 Pooling Site Preference Order

The site with lower All Pooled Safety Stock value, i.e. the Pooled Safety Stock value [2.6] when all the sites are pooled at the chosen site, is given higher preference in Pooling Site Preference (PSP) Order. PSP Order is computed by inserting a sort function in Algorithm [1].

ISS Order: O(n^2) PSP Order: O(n^2) [n: number of sites]

CHAPTER 4

RESULTS AND ANALYSIS

4.1 Results

We will only pool sites within the same country since pooling cross border will require data regarding distance and transportation modes. In XYZ's network, the maximum no. of sites that can be pooled in the same echelon is 5 since no country has more than 5 sites in the same echelon. We will run our experiment on the sites in 2 countries that have maximum number of sites to ensure that the insights can be used across others countries.

<u>List of sites</u>: A1, A2, A3, A4 (country A) and B1, B2, B3, B4, B5 (country B)

For each country, we will run the experiment on 5 materials. The materials are chosen such that each one of them produce a unique ISS Order and PSP Order. This would increase the variety of pooling combinations obtained as output and improve the scope for generalized observations. We will later discuss how these 2 key metrics will determine Optimal Pooling Combination

<u>List of Materials</u>: M1, M2, M3, M4, M5

Hence, the results from these sites and materials will form a strong basis for managerial decisions. Every output displayed here has been generated at a service level of 98%. We observed the same trends at service levels of 99% and 99.5%

4.1.1 Country A

We will calculate Combined Safety Stock values for all possible pooling combinations of A1, A2, A3 and A4 using Algorithms [1] and [2]. Results obtained from a similar experiment on B1, B2, B3, B4 and B5 are presented in the next sub-section

4.1.1.1 Material M1

Table 4.1: **Key parameters for material M1 in country A**

	μ_D	σ_D	μ_{LT}	σ_{LT}
A1	1.23	2.50	57.22	32.67
A2	1.22	2.53	15.83	0.98
A3	1.18	2.36	86.50	5.80
A4	1.25	2.53	37.33	28.36

The following table contains the optimal pooled safety stock values for various pooling permutations

Table 4.2: **Optimal Pooling Site and Pooled Safety Stock for each pooling permutations**

Sites Pooled	Optimal Pooling Site	Pooled Safety Stock
A1	A1	238
A2	A2	62
A3	A3	132
A4	A4	208
A1 + A2	A2	42
A1 + A3	A3	98
A1 + A4	A4	139
A2 + A3	A2	41
A2 + A4	A2	43
A3 + A4	A3	96
A1 + A2 + A3	A2	60
A1 + A2 + A4	A2	62
A1 + A3 + A4	A3	141
A2 + A3 + A4	A2	60
A1 + A2 + A3 + A4	A2	80

The following table contains the optimal combined safety stock values for each pooling style

Table 4.3: **Optimal Pooling Combination and Combined Safety Stock for each pooling style. The pooling site is highlighted in bold**

Pooling Style	Optimal Pooling Combination	Combined Safety Stock
1+1+1+1	**A1** | **A2** | **A3** | **A4**	640
1+1+2	A1 + **A2** | **A3** | **A4**	382
1+3	**A3** | A1 + **A2** + A4	194
2+2	A1 + **A2** | **A3** + A4	138
4	A1 + **A2** + A3 + A4	80

ISS Order: A1 (238) > A4 (208) > A3 (132) > A2 (62)

PSP Order: A2 (80) > A3 (184) > A4 (278) > A1 (316)

4.1.1.2 Material M2

Table 4.4: **Key parameters for material M2 in country A**

	μ_D	σ_D	μ_{LT}	σ_{LT}
A1	1.19	2.34	16.86	15.18
A2	1.33	2.62	25.00	23.85
A3	1.36	2.75	47.33	27.59
A4	1.28	2.53	37.80	21.12

The following table contains the optimal pooled safety stock values for various pooling permutations

Table 4.5: **Optimal Pooling Site and Pooled Safety Stock for each pooling permutations**

Sites Pooled	Optimal Pooling Site	Pooled Safety Stock
A1	A1	155
A2	A2	168
A3	A3	176
A4	A4	194
A1 + A2	A1	104
A1 + A3	A1	103
A1 + A4	A1	106
A2 + A3	A2	115
A2 + A4	A2	119
A3 + A4	A3	124
A1 + A2 + A3	A1	152
A1 + A2 + A4	A1	149
A1 + A3 + A4	A1	151
A2 + A3 + A4	A2	173
A1 + A2 + A3 + A4	A1	201

The following table contains the optimal combined safety stock values for each pooling style

Table 4.6: **Optimal Pooling Combination and Combined Safety Stock for each pooling style. The pooling site is highlighted in bold**

Pooling Style	Optimal Pooling Combination	Combined Safety Stock
1+1+1+1	**A1** / **A2** / **A3** / **A4**	693
1+1+2	**A2** / **A3** / **A1** + A4	450
1+3	**A2** / **A1** + A3 + A4	319
2+2	**A1** + A4 / **A2** + A3	221
4	**A1** + A2 + A3 + A4	201

ISS Order: A4 (194) > A3 (176) > A2 (168) > A1 (155)

PSP Order: A1 (201) > A2 (224) > A3 (237) > A4 (257)

4.1.1.3 Material M3

Table 4.7: **Key parameters for material M3 in country A**

	μ_D	σ_D	μ_{LT}	σ_{LT}
A1	1.29	2.63	58.50	35.46
A2	1.26	2.62	62.50	18.91
A3	1.26	2.47	65.88	26.51
A4	1.18	2.44	4.60	3.65

The following table contains the optimal pooled safety stock values for various pooling permutations

Table 4.8: **Optimal Pooling Site and Pooled Safety Stock for each pooling permutations**

Sites Pooled	Optimal Pooling Site	Pooled Safety Stock
A1	A1	218
A2	A2	159
A3	A3	198
A4	A4	52
A1 + A2	A2	115
A1 + A3	A3	132
A1 + A4	A4	35
A2 + A3	A2	108
A2 + A4	A4	35
A3 + A4	A4	34
A1 + A2 + A3	A2	160
A1 + A2 + A4	A4	52
A1 + A3 + A4	A4	52
A2 + A3 + A4	A4	51
A1 + A2 + A3 + A4	A4	67

The following table contains the optimal combined safety stock values for each pooling style

Table 4.9: **Optimal Pooling Combination and Combined Safety Stock for each pooling style. The pooling site is highlighted in bold**

Pooling Style	Optimal Pooling Combination	Combined Safety Stock
1+1+1+1	**A1** | **A2** | **A3** | **A4**	627
1+1+2	**A2** | **A3** | A1 + **A4**	392
1+3	**A2** | A1 + A3 + **A4**	211
2+2	A1 + **A4** | **A2** + A3	143
4	A1 + A2 + A3 + **A4**	67

ISS Order: A1 (218) > A3 (198) > A2 (159) > A4 (52)

PSP Order: A4 (67) > A2 (209) > A3 (261) > A1 (293)

4.1.1.4 Material M4

Table 4.10: **Key parameters for material M4 in country A**

	μ_D	σ_D	μ_{LT}	σ_{LT}
A1	1.10	2.41	56.86	28.06
A2	1.44	2.88	60.00	38.34
A3	1.06	2.21	50.33	35.23
A4	1.22	2.36	58.50	25.95

The following table contains the optimal pooled safety stock values for various pooling permutations

Table 4.11: **Optimal Pooling Site and Pooled Safety Stock for each pooling permutations**

Sites Pooled	Optimal Pooling Site	Pooled Safety Stock
A1	A1	227
A2	A2	209
A3	A3	259
A4	A4	257
A1 + A2	A1	140
A1 + A3	A1	164
A1 + A4	A4	136
A2 + A3	A2	159
A2 + A4	A4	142
A3 + A4	A4	122
A1 + A2 + A3	A1	198
A1 + A2 + A4	A4	199
A1 + A3 + A4	A4	178
A2 + A3 + A4	A4	197
A1 + A2 + A3 + A4	A4	252

The following table contains the optimal combined safety stock values for each pooling style

Table 4.12: **Optimal Pooling Combination and Combined Safety Stock for each pooling style. The pooling site is highlighted in bold**

Pooling Style	Optimal Pooling Combination	Combined Safety Stock
1+1+1+1	**A1** | **A2** | **A3** | **A4**	952
1+1+2	**A1** | **A2** | A3 + **A4**	558
1+3	**A2** | A1 + A3 + **A4**	387
2+2	**A1** + A2 | A3 + **A4**	262
4	A1 + A2 + A3 + **A4**	252

<u>**ISS Order**</u>: A3 (259) > A4 (257) > A1 (227) > A2 (209)

<u>**PSP Order**</u>: A4 (252) > A1 (292) > A2 (295) > A3 (327)

4.1.1.5 Material M5

Table 4.13: **Key parameters for material M5 in country A**

	μ_D	σ_D	μ_{LT}	σ_{LT}
A1	0.33	4.46	35.50	0.71
A2	0.88	6.61	11.00	4.24
A3	0.77	3.87	41.00	15.19
A4	0.77	4.88	51.50	51.25

The following table contains the optimal pooled safety stock values for various pooling permutations

Table 4.14: **Optimal Pooling Site and Pooled Safety Stock for each pooling permutations**

Sites Pooled	Optimal Pooling Site	Pooled Safety Stock
A1	A1	77
A2	A2	60
A3	A3	69
A4	A4	129
A1 + A2	A2	71
A1 + A3	A1	91
A1 + A4	A1	102
A2 + A3	A2	71
A2 + A4	A2	79
A3 + A4	A3	109
A1 + A2 + A3	A2	84
A1 + A2 + A4	A2	89
A1 + A3 + A4	A1	110
A2 + A3 + A4	A2	83
A1 + A2 + A3 + A4	A2	96

The following table contains the optimal combined safety stock values for each pooling style

Table 4.15: **Optimal Pooling Combination and Combined Safety Stock for each pooling style. The pooling site is highlighted in bold**

<table>
<tr><th>Pooling Style</th><th colspan="4">Optimal Pooling Combination</th><th>Combined Safety Stock</th></tr>
<tr><td>1+1+1+1</td><td>A1</td><td>A2</td><td>A3</td><td>A4</td><td>335</td></tr>
<tr><td>1+1+2</td><td>A1</td><td>A3</td><td colspan="2">A2 + A4</td><td>225</td></tr>
<tr><td>1+3</td><td>A3</td><td colspan="3">A1 + A2 + A4</td><td>158</td></tr>
<tr><td>2+2</td><td colspan="2">A1 + A3</td><td colspan="2">A2 + A4</td><td>170</td></tr>
<tr><td>4</td><td colspan="4">A1 + A2 + A3 + A4</td><td>96</td></tr>
</table>

ISS Order: A4 (129) > A1 (77) > A3 (69) > A2 (60)

PSP Order: A2 (96) > A1 (141) > A3 (180) > A4 (308)

4.1.2 Country B

4.1.2.1 Material M1

Table 4.16: **Key parameters for material M1 in country B**

	μ_D	σ_D	μ_{LT}	σ_{LT}
B1	1.19	2.54	57.33	33.29
B2	1.25	2.49	55.67	25.09
B3	1.07	2.23	42.60	23.34
B4	1.12	2.33	51.83	27.97
B5	1.30	2.57	64.75	23.27

Table 4.17: **Optimal Pooling Site and Pooled Safety Stock for each pooling permutations**

Sites Pooled	Optimal Pooling Site	Pooled Safety Stock
B1	B1	68
B2	B2	73
B3	B3	48
B4	B4	70
B5	B5	69
B1 + B2	B1	127
B1 + B3	B3	94
B1 + B4	B1	123
B1 + B5	B5	131
B2 + B3	B3	94
B2 + B4	B2	135
B2 + B5	B5	130
B3 + B4	B3	89
B3 + B5	B3	86
B4 + B5	B5	124
B1 + B2 + B3	B3	141
B1 + B2 + B4	B1	183
B1 + B2 + B5	B5	190
B1 + B3 + B4	B3	136
B1 + B3 + B5	B3	144
B1 + B4 + B5	B5	185
B2 + B3 + B4	B3	137
B2 + B3 + B5	B3	143
B2 + B4 + B5	B5	185
B3 + B4 + B5	B3	138
B1 + B2 + B3 + B4	B3	183
B1 + B2 + B3 + B5	B3	191
B1 + B2 + B4 + B5	B5	245
B1 + B3 + B4 + B5	B3	186
B2 + B3 + B4 + B5	B3	185
B1 + B2 + B3 + B4 + B5	B3	232

Table 4.18: **Optimal Pooling Combination and Combined Safety Stock for each pooling style. The pooling site is highlighted in bold**

Pooling Style	Optimal Pooling Combination	Combined Safety Stock
1+1+1+1+1	**B1** | **B2** | **B3** | **B4** | **B5**	328
1+1+1+2	**B1** | **B2** | **B5** | B2 + **B3**	301
1+1+3	**B1** | **B5** | B2 + **B3** + B4	274
1+2+2	**B1** | B2 + **B3** | B4 + **B5**	286
1+4	**B1** | B2 + **B3** + B4 + B5	253
2+3	B1 + **B5** | B2 + **B3** + B4	268
5	B1 + B2 + **B3** + B4 + B5	232

ISS Order: B2 (73) > B4 (70) > B5 (69) > B1 (68) > B3 (48)

PSP Order: B3 (232) > B5 (296) > B1 (298) > B2 (328) > B4 (343)

4.1.2.2 Material M2

Table 4.19: **Key parameters for material M2 in country B**

	μ_D	σ_D	μ_{LT}	σ_{LT}
B1	1.40	2.80	52.38	19.21
B2	1.31	2.71	38.43	24.96
B3	1.33	2.67	59.67	33.24
B4	1.30	2.64	25.80	7.79
B5	1.28	2.55	31.17	24.09

The following table contains the optimal pooled safety stock values for various pooling permutations

Table 4.20: **Optimal Pooling Site and Pooled Safety Stock for each pooling permutations**

Sites Pooled	Optimal Pooling Site	Pooled Safety Stock
B1	B1	63
B2	B2	90
B3	B3	87
B4	B4	37
B5	B5	79
B1 + B2	B1	118
B1 + B3	B1	116
B1 + B4	B4	72
B1 + B5	B1	113
B2 + B3	B3	166
B2 + B4	B4	69
B2 + B5	B5	155
B3 + B4	B4	69
B3 + B5	B5	157
B4 + B5	B4	70
B1 + B2 + B3	B1	170
B1 + B2 + B4	B4	104
B1 + B2 + B5	B1	167
B1 + B3 + B4	B4	104
B1 + B3 + B5	B1	168
B1 + B4 + B5	B4	103
B2 + B3 + B4	B4	101
B2 + B3 + B5	B5	233
B2 + B4 + B5	B4	101
B3 + B4 + B5	B4	102
B1 + B2 + B3 + B4	B4	136
B1 + B2 + B3 + B5	B1	221
B1 + B2 + B4 + B5	B4	135
B1 + B3 + B4 + B5	B4	135
B2 + B3 + B4 + B5	B4	134
B1 + B2 + B3 + B4 + B5	B4	168

The following table contains the optimal combined safety stock values for each pooling style

Table 4.21: **Optimal Pooling Combination and Combined Safety Stock for each pooling style. The pooling site is highlighted in bold**

Pooling Style	Optimal Pooling Combination	Combined Safety Stock
1+1+1+1+1	**B1** | **B2** | **B3** | **B4** | **B5**	356
1+1+1+2	**B1** | **B3** | **B5** | B2 + **B4**	298
1+1+3	**B1** | **B5** | B2 + B3 + **B4**	243
1+2+2	**B5** | **B1** + B3 | B2 + **B4**	264
1+4	**B1** | B2 + B3 + **B4** + B5	197
2+3	**B1** + B5 | B2 + B3 + **B4**	214
5	B1 + B2 + B3 + **B4** + B5	168

ISS Order: B2 (90) > B3 (87) > B5 (79) > B1 (63) > B4 (37)

PSP Order: B4 (168) > B1 (275) > B5 (390) > B3 (405) > B2 (435)

4.1.2.3 Material M3

Table 4.22: **Key parameters for material M3 in country B**

	μ_D	σ_D	μ_{LT}	σ_{LT}
B1	1.37	2.69	53.44	29.51
B2	1.28	2.60	54.88	29.15
B3	1.24	2.50	38.33	11.15
B4	1.20	2.45	25.00	19.15
B5	1.26	2.54	45.20	33.68

The following table contains the optimal pooled safety stock values for various pooling permutations

Table 4.23: **Optimal Pooling Site and Pooled Safety Stock for each pooling permutations**

Sites Pooled	Optimal Pooling Site	Pooled Safety Stock
B1	B1	86
B2	B2	82
B3	B3	45
B4	B4	58
B5	B5	86
B1 + B2	B2	162
B1 + B3	B3	89
B1 + B4	B4	120
B1 + B5	B1	162
B2 + B3	B3	87
B2 + B4	B4	116
B2 + B5	B2	156
B3 + B4	B3	84
B3 + B5	B3	87
B4 + B5	B4	115
B1 + B2 + B3	B3	131
B1 + B2 + B4	B4	177
B1 + B2 + B5	B2	236
B1 + B3 + B4	B3	128
B1 + B3 + B5	B3	130
B1 + B4 + B5	B4	176
B2 + B3 + B4	B3	126
B2 + B3 + B5	B3	128
B2 + B4 + B5	B4	172
B3 + B4 + B5	B3	125
B1 + B2 + B3 + B4	B3	169
B1 + B2 + B3 + B5	B3	172
B1 + B2 + B4 + B5	B4	234
B1 + B3 + B4 + B5	B3	169
B2 + B3 + B4 + B5	B3	167
B1 + B2 + B3 + B4 + B5	B3	210

The following table contains the optimal combined safety stock values for each pooling style

Table 4.24: **Optimal Pooling Combination and Combined Safety Stock for each pooling style. The pooling site is highlighted in bold**

Pooling Style	Optimal Pooling Combination	Combined Safety Stock
1+1+1+1+1	**B1**	**B2**	**B3**	**B4**	**B5**	357
1+1+1+2	**B2**	**B4**	**B5**	B1 + **B3**	315
1+1+3	**B2**	**B4**	B1 + **B3** + B5	270
1+2+2	**B2**	B1 + **B3**	**B4** + B5	286
1+4	**B4**	B1 + B2 + **B3** + B5	230
2+3	B2 + **B4**	B1 + **B3** + B5	246
5	B1 + B2 + **B3** + B4 + B5	210

ISS Order: B1 (86) > B5 (86) > B2 (82) > B4 (58) > B3 (45)

PSP Order: B3 (210) > B4 (289) > B2 (380) > B1 (381) > B5 (410)

4.1.2.4 Material M4

Table 4.25: **Key parameters for material M4 in country B**

	μ_D	σ_D	μ_{LT}	σ_{LT}
B1	1.24	2.51	60.25	34.86
B2	0.99	2.10	53.89	28.73
B3	1.23	2.46	52.33	32.01
B4	1.33	2.61	44.00	41.54
B5	1.30	2.62	69.86	25.67

The following table contains the optimal pooled safety stock values for various pooling permutations

Table 4.26: **Optimal Pooling Site and Pooled Safety Stock for each pooling permutations**

Sites Pooled	Optimal Pooling Site	Pooled Safety Stock
B1	B1	76
B2	B2	64
B3	B3	83
B4	B4	100
B5	B5	70
B1 + B2	B1	132
B1 + B3	B1	145
B1 + B4	B1	150
B1 + B5	B5	129
B2 + B3	B2	134
B2 + B4	B2	141
B2 + B5	B5	116
B3 + B4	B3	166
B3 + B5	B5	130
B4 + B5	B5	135
B1 + B2 + B3	B1	200
B1 + B2 + B4	B1	207
B1 + B2 + B5	B5	176
B1 + B3 + B4	B1	219
B1 + B3 + B5	B5	189
B1 + B4 + B5	B5	194
B2 + B3 + B4	B2	212
B2 + B3 + B5	B5	177
B2 + B4 + B5	B5	182
B3 + B4 + B5	B5	194
B1 + B2 + B3 + B4	B1	275
B1 + B2 + B3 + B5	B5	236
B1 + B2 + B4 + B5	B5	242
B1 + B3 + B4 + B5	B5	253
B2 + B3 + B4 + B5	B5	242
B1 + B2 + B3 + B4 + B5	B5	301

The following table contains the optimal combined safety stock values for each pooling style

Table 4.27: **Optimal Pooling Combination and Combined Safety Stock for each pooling style. The pooling site is highlighted in bold**

Pooling Style	**Optimal Pooling Combination**	**Combined Safety Stock**
1+1+1+1+1	**B1** | **B2** | **B3** | **B4** | **B5**	393
1+1+1+2	**B1** | **B2** | **B3** | B4 + **B5**	358
1+1+3	**B1** | **B2** | B3 + B4 + **B5**	334
1+2+2	**B2** | **B1** + B3 | B4 + **B5**	344
1+4	**B2** | B1 + B3 + B4 + **B5**	317
2+3	**B1** + B2 | B3 + B4 + **B5**	326
5	B1 + B2 + B3 + B4 + **B5**	301

ISS Order: B4 (100) > B3 (83) > B1 (76) > B5 (70) > B2 (64)

PSP Order: B5 (301) > B1 (349) > B2 (359) > B3 (389) > B4 (444)

4.1.2.5 Material M5

Table 4.28: **Key parameters for material M5 in country B**

	μ_D	σ_D	μ_{LT}	σ_{LT}
B1	5.91	10.02	99.22	11.84
B2	2.76	5.96	20.28	3.59
B3	3.92	9.43	60.00	18.30
B4	2.68	6.48	18.40	2.63
B5	0.31	1.41	21.00	13.74

The following table contains the optimal pooled safety stock values for various pooling permutations

Table 4.29: **Optimal Pooling Site and Pooled Safety Stock for each pooling permutations**

Sites Pooled	Optimal Pooling Site	Pooled Safety Stock
B1	B1	273
B2	B2	71
B3	B3	236
B4	B4	71
B5	B5	20
B1 + B2	B2	149
B1 + B3	B1	413
B1 + B4	B4	133
B1 + B5	B5	221
B2 + B3	B2	140
B2 + B4	B4	101
B2 + B5	B2	74
B3 + B4	B4	132
B3 + B5	B5	184
B4 + B5	B4	72
B1 + B2 + B3	B2	201
B1 + B2 + B4	B4	161
B1 + B2 + B5	B2	152
B1 + B3 + B4	B4	178
B1 + B3 + B5	B5	344
B1 + B4 + B5	B4	134
B2 + B3 + B4	B4	153
B2 + B3 + B5	B2	148
B2 + B4 + B5	B4	102
B3 + B4 + B5	B4	138
B1 + B2 + B3 + B4	B4	202
B1 + B2 + B3 + B5	B2	207
B1 + B2 + B4 + B5	B4	162
B1 + B3 + B4 + B5	B4	183
B2 + B3 + B4 + B5	B4	159
B1 + B2 + B3 + B4 + B5	B4	207

The following table contains the optimal combined safety stock values for each pooling style

Table 4.30: **Optimal Pooling Combination and Combined Safety Stock for each pooling style. The pooling site is highlighted in bold**

Pooling Style	Optimal Pooling Combination	Combined Safety Stock
1+1+1+1+1	**B1** | **B2** | **B3** | **B4** | **B5**	671
1+1+1+2	**B2** | **B3** | **B5** | B1 + **B4**	460
1+1+3	**B2** | **B5** | B1 + B3 + **B4**	269
1+2+2	**B5** | B1 + **B4** | **B2** + B3	293
1+4	**B5** | B1 + B2 + B3 + **B4**	222
2+3	**B2** + B5 | B1 + B3 + **B4**	252
5	B1 + B2 + B3 + **B4** + B5	207

ISS Order: B1 (273) > B3 (236) > B4 (71) B2 (71) > B5 (20)

PSP Order: B4 (207) > B2 (232) > B5 (489) > B1 (583) > B3 (652)

4.2 Key Insights

In every result produced, the Optimal Pooling Combination is obtained in the following fashion

- Fix the first site from PSP Order as the pooling site. Once fixed, remove from the site from ISS Order and PSP Order
- Choose the largest number (say n) from pooling style. Once chosen, remove the number from pooling style
- The first $n-1$ sites from ISS Order will be pooled at the chosen pooling site. Add this pooled set of n sites to the the Optimal Pooling Combination. Remove them from ISS Order and PSP Order
- Repeat this process till no sites are left in the ISS Order and PSP Order

The algorithm for this **Optimal Pooling Strategy** is explained in detailed below

Inputs: Same as inputs of Algorithm 2 and P: Pooling Style (entered as an array)

Algorithm 3: $F(N, D, LT, P, p)$ Time Complexity: O(n^2)

Compute PSP Order and ISS order [3.3.3]
Arrange numbers in P from highest to lowest (non-increasing)
i=0; CSS_opt = 0; Comb_opt = $\emptyset$
while $N \neq \emptyset$ **do**
    $m = \emptyset$
    Set pooling site as first element in PSP
    Add the pooling site to m
    Remove the pooling site from N,ISS and PSP
    Choose first $P[i] - 1$ sites from ISS
    Add these $P[i] - 1$ sites to m
    Remove these $P[i] - 1$ sites from N,ISS and PSP
    Comb_opt $\cup = m$
    CSS_opt $+ =$ SS(m) [1]
    i++
Return CSS_opt , Comb_opt

Output: Same as outputs of Algorithm 2

Table 4.31: **Experiment v. Recommended Strategy**

	Method Used	Time Complexity
Experiment	Algorithm [2]	NP Complete
Recommended Strategy	Algorithm [3]	O(n^2)

CHAPTER 5

CONCLUSION

In this thesis, we developed an optimal pooling strategy based on safety stock values. We first implemented functions to calculate important quantities such as Demand distribution, Lead-time distribution, Lead-time Demand distribution, Reorder point, Safety stock and Pooling combinations. We then developed models to calculate pooled safety stock values and to compute Optimal Pooling Combination.

Managerial Insights

The following observations explain why the **Optimal Pooling Strategy** should be implemented immediately at a global scale

- The optimal pooling strategy implemented through Algorithm [3] that emphasizes on pooling the sites based on **PSP Order and ISS Order** will produce optimal combined safety stock level (globally)
- implementing this strategy is much more **time-efficient** [$O(n^2)$] than implementing the conventional method [NP Complete] 4.31

Future Scope

The following possibilities could be explored to broaden the solution's potential application

- Integrating other costs: Optimizing holding costs alone is effective for an MEIO within an echelon closer to the end customer. With the help of other costs, such as transportation, ordering, etc., MEIO can be implemented successfully within echelons closer to the supplier.
- MEIO across echelons: In this thesis, we predominantly discussed only about MEIO within an echelon. With the help of other costs (as mentioned above) and geographical location of all sites, we will be able to generate an appropriate cost function to run the MEIO across echelons.

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