**Project**
Report

# Study of Super Resolution Algorithms For Remote Sensing

*Submitted in partial fulfillment of*
*the requirements for the award of the degree of*

**Master of Technology**
**in**
**Electrical Engineering**

Submitted by
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EE16M085

Under the guidance of
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# Department of Electrical Engineering

INDIAN INSTITUE OF TECHNOLOGY,MADRAS

## *Certificate*

This is to certify that this is a bonafide record of the project presented by Ayushi Jain (EE16M085) in partial fulfilment of the requirements of the degree of Master of Technology in Electrical Engineering.

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Date:

**Abstract**

Image resolution has been a topic of an attractive research for quite some time since it has many applications in different fields like medical imaging, remote sensing, video surveillance, astrophysics and many more. Image resolution represents the detailed description of an image.The higher the resolution, the more details we can get about the image.

Super resolution is class of techniques to get high resolution image from multiple or single low resolution images.In this thesis, we have learned about the techniques of getting high resolution image from multiple overlapped images since we will get overlapped data from Scatterometer. Three resolution techniques have been tried to learn in great detail.The first and second method i.e. Algebraic Reconstruction technique and Simultaneous Algebraic Reconstruction technique as name suggests, is reconstruction based method. The third one is Super Resolution via Sparse Representation which is learning based method.

# Acknowledgments

I would like to sincerely thank my guide Dr. Uday K Khankhoje for his valuable technical guidance as well as for the much needed encouragement and support. It has been a great opportunity working under him.

I am very thankful to KNGB Yashwanth for his valuable support during the course of my work. His knowledge in Image Processing helped me for the successful completion of the thesis. His patience to listen to my silly questions was really commendable which helped me in understanding my area of work. I am very grateful to him for teaching me various things during the course of this project.

I would like to thank all the members of the Microwave lab for their help and support during the project.

I express my special gratitude to Mukund Mishra with whom I have smiled even in the toughest phase and laughed for no reasons.

My sincere gratitude to Kamiya Jain, Aditi, Vivek, Ankit, Pranav, Meena, Sakees and all my friends for giving good motivation and suggestions and also helping me in various stages of my project work by keeping the mood light.

Last but not the least, I would also like to thank my parents for their continued support during the course of this work.

Dedicated to my Jeetumonies............

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# Chapter 1

# Introduction

## 1.1 Super Resolution

Super resolution means obtaining high resolution image from one or more low resolution images. High resolution images have high pixel density information i.e. more details are available for an image whereas low resolution images have low pixel density. High resolution images are needed in many areas like medical imaging, satellite and aerial imaging, ultrasound imaging, compressed images, facial images improvement and many more. High resolution can be achieved by collecting more information while capturing the image itself i.e. using extra sensors. But increasing the number of sensors to achieve high resolution is not the cost effective solution. We need super resolution algorithms which can reconstruct the degraded image. Super resolution problems mainly can be categorized in two main streams. One is multi-frame super resolution in which many low resolution images is being taken and we have to get high resolution image. Many low resolution images implying different looks of the same object but this doesn't ensure extra information. Low resolution images have to be different pixel shifted or overlapped to contain extra information needed to super resolve them into one. Second is single image super resolution where only one image is available as data and we have to super resolve it.

## 1.2 Remote Sensing and Scatterometer

Remote Sensing is a means of obtaining information about an object or medium without coming into physical contact with it. Two types of remote sensors:
Active: Sensors which use artificial sources that emit radiation, directing it

towards the object of interest and collecting the return signal.
Passive: Sensors which use the natural radiation emitted by the Sun, by stars, and by the Earth and its atmosphere.
Scatterometers are an active non imaging sensors which were typically used to measure radar backscatter from oceans surface to determine wind direction and its speed over the ocean. Scatterometer can cover large areas of earths surface within two/three days and is relatively immune to clouds and precipitation. There are different types of scatterometers with different spatial resolution capacity like Seasat-A-Scatteromter(SASS) has typically 50km resolution while QuickScat has 25km. This resolution is good enough to study wind vectors over ocean but not good for land or ice studies. However, by applying resolution enhancement algorithms i.e. image reconstruction methods to the data,images with sufficient detail for land and ice studies can be obtained.

## 1.3 Literature Review And Motivation

Super resolution problems come under Image Reconstruction class. The idea of super resolution is based on the linear imaging model which relates between low resolution images and high resolution image. The low resolution image is generally considered to be some decimated blur version of high resolution image with some noise added to it.

$$Y = BSX + n \tag{1.1}$$

where Y represents low resolution image. S is a decimation operator, B represents blur function, X is high resolution image, n is the noise in the system.

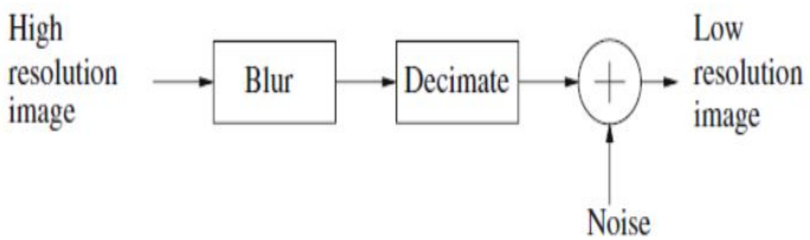



Figure 1.1: Imaging Model

So, super resolution algorithms are inverse problems where Y is the observed data and we need to find out the original image X i.e. reconstruct/restore it given the information of decimation and blur function. According to Jacques Hadamard, a mathematical model for any physical phenomena can be considered as well posed if it follows following condition:

- A solution exists.
- Solution is unique.
- Solution's behavior changes continuously with input data.

Our inverse problem doesn't belong to well posed category.It is an ill posed problem since a unique solution doesn't exist in our case. But in case of reconstruction, let say $SB = H$, the most feasible solution would have been

$$X = H^{-1}Y. \tag{1.2}$$

in case of noiseless system. But the inverse doesn't exist for our case. In inverse problems, eq 1.1 represents system of linear equations, we don't have enough information to determine the solution of given equations.So, we need some algorithms to solve eq 1.1 to get our reconstructed image from the observed one. Though there exists lot of approaches to solve this problem, the two categories are Reconstruction based methods and Learning based methods.

### 1.3.1 Reconstruction Based Methods

In this type of techniques, image reconstruction is being done using projections on the image as data. The Algebraic Reconstruction Technique(ART) and Simultaneous Algebraic Reconstruction Technique(SART) are the techniques which is being studied for reconstruction of image from projections. The basic of image reconstruction from projection was introduced by Herman, (1980), Natterer and Wubbeling (2001) and Kak and Malcolm (2001). The basic idea behind reconstruction is divide the whole space which one wants to reconstruct into finite number of pixels and each pixel represent a constant value.

### 1.3.2 Learning based Methods

Various regularization methods were proposed to stabilize the inversion of such an ill-posed problem, such as [1-3]. However, the performance of these reconstruction-based super-resolution algorithms degrades rapidly when the desired magniflcation factor is large or the available input images are limited. In these cases, the results may be overly smooth. In learning based techniques, an over sized dictionaries corresponding to high and low resolution images are being learned using test images. We try to find out the most sparseset solution for our overlapped images with respect to learned dictionaries. [4]

## 1.4 Research Objective

The main aim is to reconstruct high resolution image from the low resolution overlapped images. Since, this inverse problem is highly ill posed and system of equations form an underdetermined system, so, the study of different iterative algorithms which are capable to reconstruct efficiently and easy to implement is being carried out. Solve the eq:

$$y = Hx + n \tag{1.3}$$

by ART , SART and Sparse representation.

## 1.5 Thesis Organization

Chapter 1 is the introduction of super resolution , remote sensing , motivation and research objective of the project.

Chapter 2 is the detailed discussion of the techniques tested in the project. ART , SART and Sparse representation to the application of high resolution is being explained.

Chapter 3 is the results and discussions of the techniques mentioned in the chapter 2.

Chapter 4 is the conclusion.

# Chapter 2

# Background Theory

## 2.1 Algebraic Reconstruction Technique

It is an iteration based technique in which reconstruction problem is modelled as set of linear equations i.e. it is applied to solve large system of linear equations. In this method, reconstruction of image is being done by taking measurements from the projection on the object. A linear model is being proposed in between measurements and no of unknown variables. The object is being divided into discrete finite grid where every cell represent constant value, x. Every projection onto it gives the measurement, y. The linear relation is as follows:

$$\sum_{i=1}^{M} H_{ji} x_i = y_j \tag{2.1}$$

$j = 1$ to N

where $H_{ij}$ is the weight function which accounts for the contribution of every cell to particular projection. M is the no of cells in the grid and N is the no of projections taken.

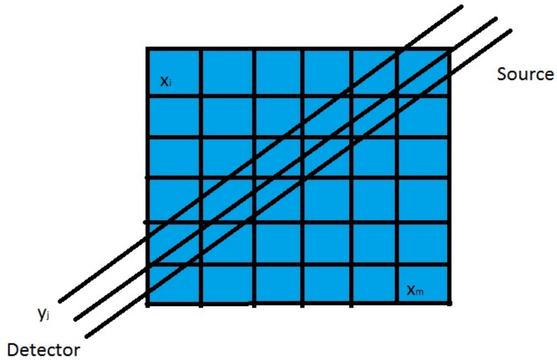



Figure 2.1: Projection as sum of intersected pixels

Since, its an iterative method, initial guess to x plays a role to the convergence of solution. If initial x is chosen to be zero, then ART converges to minimum norm solution.
$min||x||^2$ such that $Hx = y$

The geometrical interpretation of the technique is as follows:
From eq 2.1, we can say every equation represents a hyper plane. Projections onto hyper planes one after the other till it converges to the solution. It is also known as row action technique as one row of H is needed at one single iteration. The unique solution to the system will be existing if the equations are consistent. The uniqueness will depend on the rank of H. But in case, if the measurements are less or system is inconsistent/ under determined, we can get minimum norm solution.

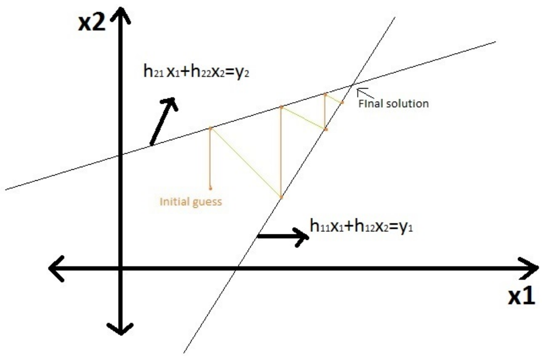



Figure 2.2: Geometrical interpretation when M=N=2

In ART, one projection is considered at a time i.e. in one iteration, one projection is taken and initial guess is modified according to that which makes the algorithm slower towards convergence as number of iterations is very large.

Initialisation: $x^0$=0
Iterative step :

$$x_j^{k+1} = x_j^k + \frac{y_i - \langle h_i, x^k \rangle}{||h_i||^2} h_{ij} \tag{2.2}$$

where $x_j^k$is the jth element of the vector x at the kth iteration, $h_i$ is the ith row of H, and $h_{ij}$ is the (i,j)th element of H.[2] From the equation, we can see that only immediate predecessor x is needed and one row of H. Since the correction from the projection term is being added to the initial guess and so on, this ART is also known as Additive ART.

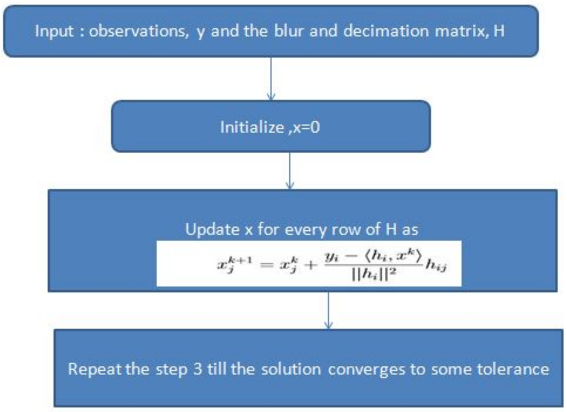



Figure 2.3: Flowchart for the ART Algorithm

## 2.2 Simultaneous Algebraic Reconstruction Technique

In this technique, average projection is added to the iterative term i.e. in one iteration itself, initial guess is projected onto every hyperplane and average of all those projections is being added to initial guess to update x. Initialisation: $x^0$=0
Iterative step :

$$x_j^{k+1} = x_j^k + \frac{\sum_{i=1}^{m} \frac{h_{ij} d_i^k}{\sum_{j=1}^{n} h_{ij}}}{\sum_{i=1}^{m} h_{ij}} \tag{2.3}$$

where $d_i^k = y_i - \langle h_i, x^k \rangle$. [3]

SART is different from ART because in this, single pixel update requires information from every measurement as it normalizes each update $x_j$ by the sum of the elements in the jth column of H. It is also known as column action technique as one column is needed to update x at one time.

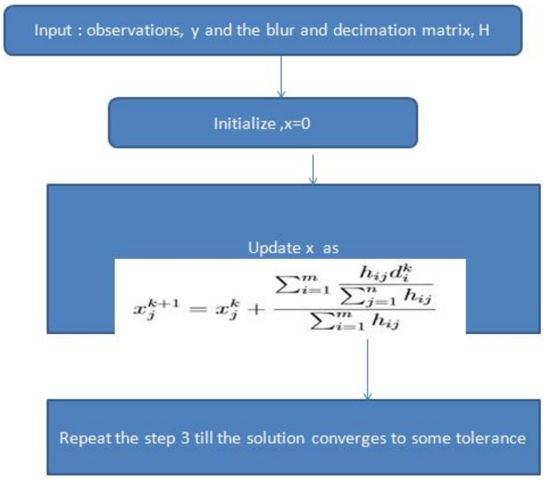



Figure 2.4: Flowchart for the SART Algorithm

## 2.3 Sparse Representation Technique

It is a learning based technique in which sparsity of natural images has been exploited for super resolution i.e. any natural images can be represented as sparse signal in the chosen over-complete dictionary. It is also based on the fact that there exists a linear relationship between high resolution images and its low dimensional projections. Let D be an over-complete dictionary of k bases and a signal x can be represented as

$$x = D\alpha \tag{2.4}$$

where $\alpha$ determines the nonzero coefficients and it tells the sparsity of signal with respect to D. The details regarding dictionary is explained in next section. In this technique, we refer to our overlapped measurements as patch

since one low resolution image from Scatterometer corresponds to one patch of the complete area under its scanning. In this method, we will work with the two coupled dictionaries $D_L$ and $D_H$ for low and high resolution patches respectively. This algorithm is based on patch wise reconstruction. We will find out sparse representation of low resolution patches with respect to $D_L$ and reconstruct the high resolution patch using $D_H$ and sparse representation found out from $D_L$.
Since, we know every low resolution patch, y, can be expressed as sparse representation with respect to $D_L$ locally. And globally, it follows:

$$y = Hx \tag{2.5}$$

considering the noiseless case. First we will try to find out the most sparseset representation of y. In general, L0 minimization has to be done to find out the most sparse solution but since L0 minimization is non-deterministic polynomial hard problem, we will try to find out the solution by L1 minimization which is as follows:

$$\min_{\alpha} ||FD_l\alpha - Fy||_2^2 + \lambda||\alpha||_1 \tag{2.6}$$

where $\lambda$ represents the sparsity of solution and F represents the feature extraction operator which is used to get texture from the image than the intensity. Every patch is being convoluted with first order and second order filters to get four different patches and all the patches are being concatenated into a matrix to form feature extraction operator for that patch.
The equation 2.6 doesn't explore the overlapped information in between different patches. To ensure that the reconstructed patches follow over overlapped region, we minimize the following eq:

$$\min_{\alpha} ||PD_H\alpha - w||_2^2 + \lambda||\alpha||_1 \tag{2.7}$$

where P extracts the overlapped region between current patch and previously reconstructed patch and w is the previously reconstructed high resolution image on the overlap.
We can find the most sparseset solution by optimizing the above two equations which puts local constraint on patches that every patch can be expressed as sparse representation of dictionaries.
The two equations can be combined as:

$$\min_{\alpha} ||\tilde{D}\alpha - \tilde{y}||_2^2 + \lambda||\alpha||_1 \tag{2.8}$$

$$\tilde{D} = \begin{bmatrix} FD_l \\ \beta PD_h \end{bmatrix} \tag{2.9}$$

$$\tilde{y} = \begin{bmatrix} Fy \\ \beta w \end{bmatrix} \tag{2.10}$$

After finding sparse representations for every patch, we can reconstruct the patch in high resolution by:

$$x = D_H \alpha + m \tag{2.11}$$

where x represents the high resolution reconstructed patch.
After reconstructing all the patches, we can reconstruct the image and to ensure that every patch follow global constraint i.e. y=Hx, we can use steepest gradient descent method and the update equation for that is given by:

$$X^* = arg \min_X ||BSX - Y||_2^2 + c||X - X_0||_2^2 \tag{2.12}$$

where $X_0$ represents the highly reconstructed image from patch wise reconstruction.
$X^*$ is the final high resolution reconstructed image.[4]

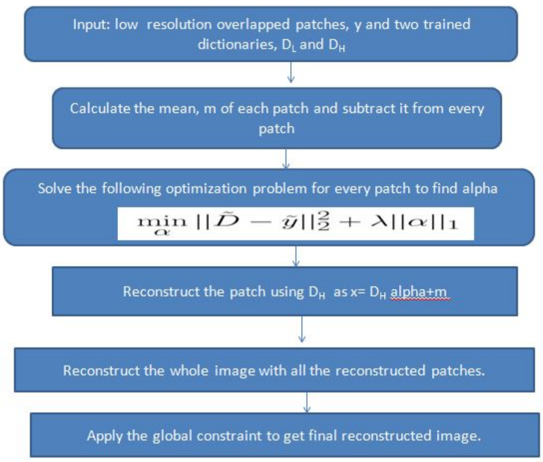



Figure 2.5: Flowchart for the Sparse Representation Algorithm

### 2.3.1 Dictionary Learning

Dictionary is a matrix in which signal can be represented as sparse representation of 'atoms'. Atoms are the bases vectors as columns vectors in the dictionary. To learn coupled dictionaries, we need two training sets,

$X^h = [x_1, x_2...]$and $Y^l = [y_1, y_2..]$ which corresponds to the set of sampled high resolution and low resolution patches correspondingly. We can learn dictionaries by optimizing the following eq:

$$arg \min_{D_h, Z} ||X^h - D_h Z||_2^2 + \lambda||Z||_1 \tag{2.13}$$

$$arg \min_{D_l, Z} ||Y^l - D_l Z||_2^2 + \lambda||Z||_1 \tag{2.14}$$

such that

$$||D_i||_2^2 \leq 1, i = 1, 2....k$$

But these equations are non convex but fixing D and then optimize for Z and fixing Z and optimizing for D till it converges.

# Chapter 3

# Results and Discussions

## 3.1 Simulation Results

All the simulations for this project is being carried out in MATLAB. For some part of the simulations, CVX toolbox and SPAMS toolbox is being used. SPAMS toolbox is used for dictionary learning. CVX toolbox is used for L1 norm minimization. Algorithms are being tested on the self generated overlapped coarse data, y i.e. undersampled low resolution data.

## 3.2 Generation of coarse data

Coarse data from image is being generated by taking patch of dimension 10 $\times$10 and summing it up to get one coarse measurement.Similarly many overlapped patches were taken to get the coarse data. This means resolution achieved is 10 times the coarse data given to the algorithm. Since, no of measurements is less than no of pixels needed to be reconstructed, the system of equations formed an underdetermined system. The performance of the algorithms is being tested by calculating the normalised error as:
Normalised error=(Reconstructed image - Original Image)/Original Image
Percentage error in the reconstruction can be calculated as:
Percentage error=Normalised error $\times$ 100.

## 3.3 Algebraic Reconstruction Technique's Results

Reconstruction of 2D image using ART technique.
Original image size 512$\times$512 = 262144 pixels.

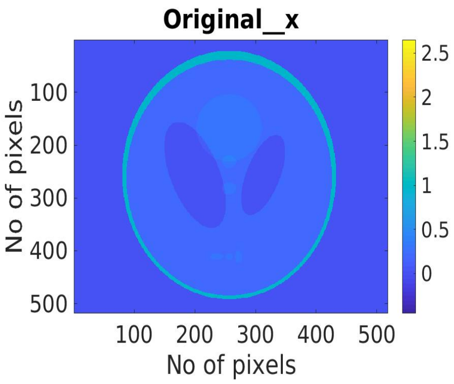
| Coarse measurements | |
|---|---|
| Overlap% | coarse pixels |
| 60 | 16129 |
| 80 | 63504 |
| 90 | 253009 |

Table 3.1: Table to show no of coarse observations generated

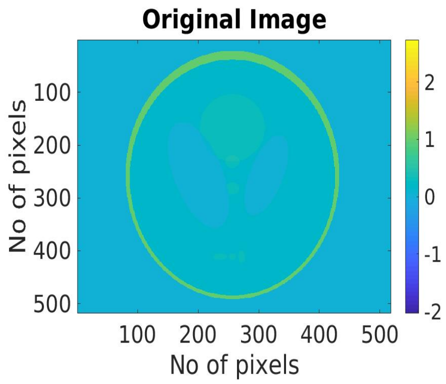
| Normalised error in Reconstructed Images | | |
|---|---|---|
| | Data(y) without noise | Data with SNR=10dB |
| Overlap% | k=30 | k=30 |
| 10 | 31.54 | 32.56 |
| 20 | 37.13 | 37.81 |
| 30 | 36.74 | 37.46 |
| 40 | 30.63 | 31.45 |
| 50 | 14.38 | 17.88 |
| 60 | 21.50 | 23.39 |
| 80 | 09.38 | 23.83 |
| 90 | 38.04 | 44.82 |

Table 3.2: Table to show normalised error. k=no of times algorithm is being run.

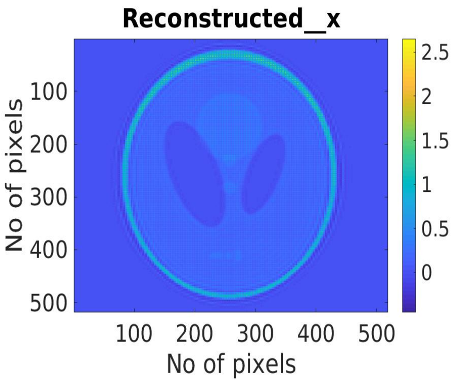
From the table, we can see as the overlap increases, the error is decreasing as more information is available to reconstruct and geometrically we can say as overlap increases, no of equations increases which increases the probability of converging to minimum norm solution. But since simulations are being tested for fixed no of iterations i.e. k=30, the error in 90 % overlap is high as in this case , the no of equations is increased and convergence will be not be achieved in 30 iterations.When k=50 was used,then error goes to 20.53. We can say , as no of equations increases, the convergence gets slow and more no of iterations is needed.But ART is susceptible to noise. As noise in the image is added, the error is increased and its effect is more visible in higher overlap. Mathematically we can say, the condition no of H is high i.e. small change in measurements bring large change in output.



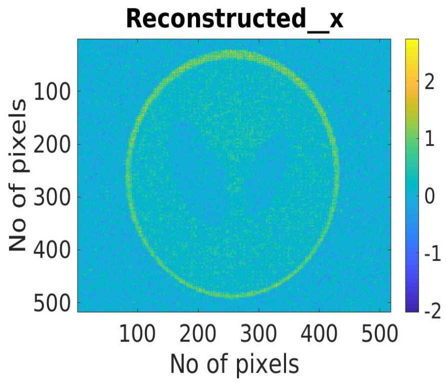

(a) Original image



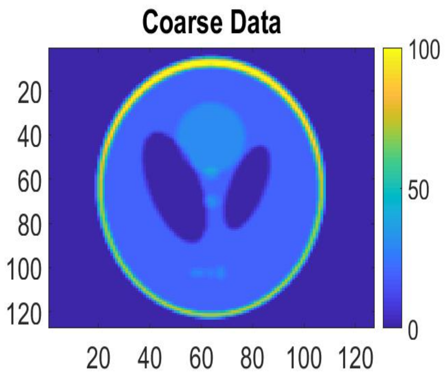

(b) Original image




(c) Reconstructed image when k=30 and without noise

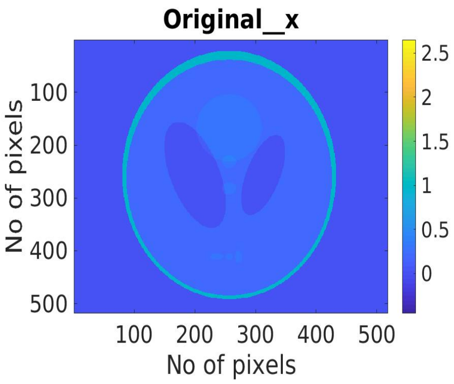



(d) Reconstructed image when k=30 and with noise

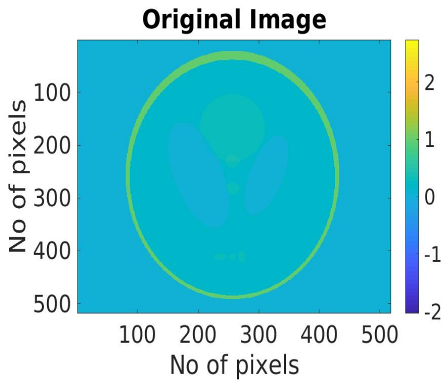



(e) Coarse data

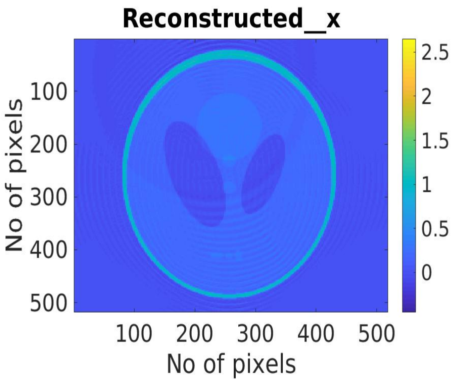
Figure 3.1: Images corresponding to overlap 60%



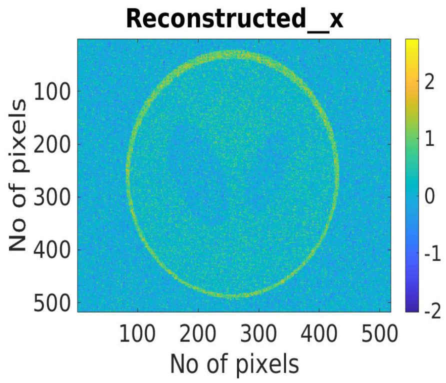

(a) Original image



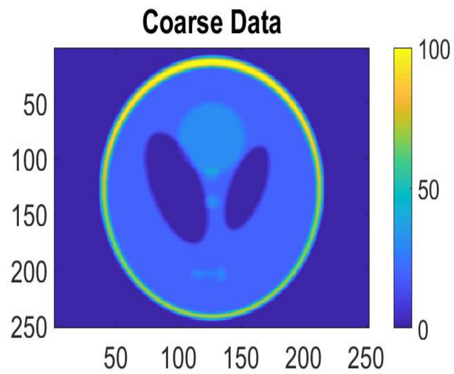

(b) Original image




(c) Reconstructed image when k=30 and without noise

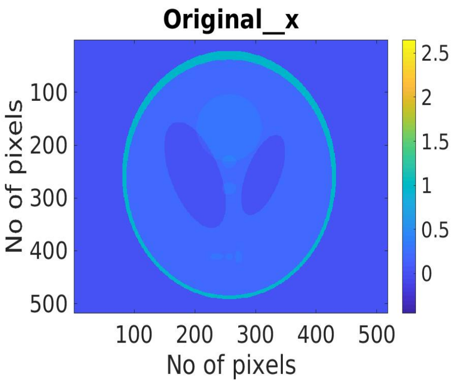



(d) Reconstructed image when k=30 and with noise

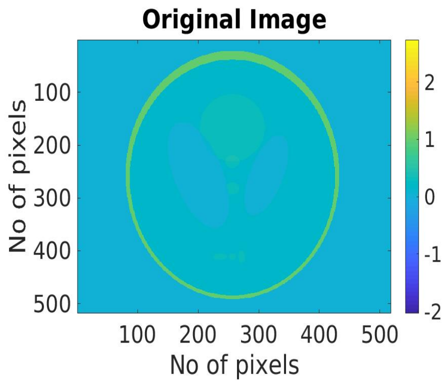



(e) Coarse data

Figure 3.2: Images corresponding to overlap 80%




(a) Original image

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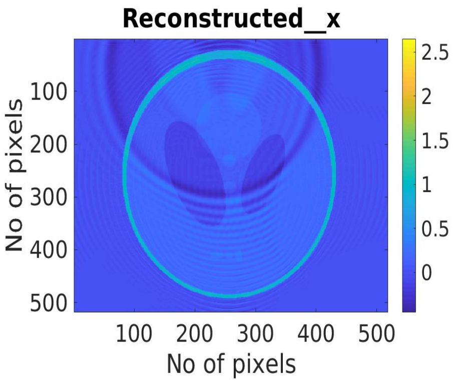
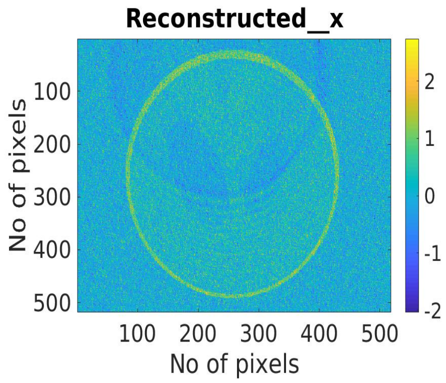
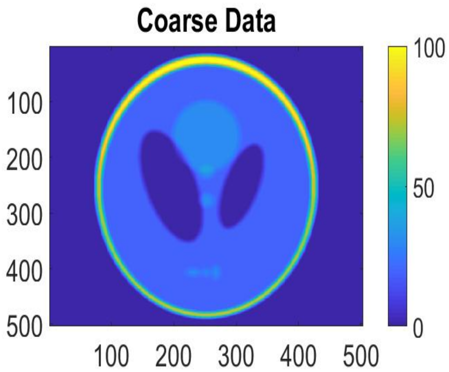



(c) Reconstructed image when k=30 and without noise

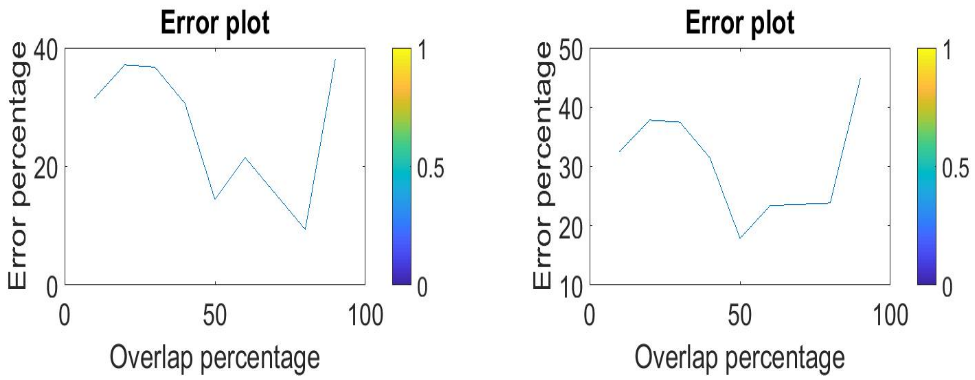



(d) Reconstructed image when k=30 and with noise




(e) Coarse data

Figure 3.3: Images corresponding to overlap 90%




(a) Error, when k=30 (b) Error when k=30

Figure 3.4: Reconstructed error when signal has a. no noise b. 10dB noise

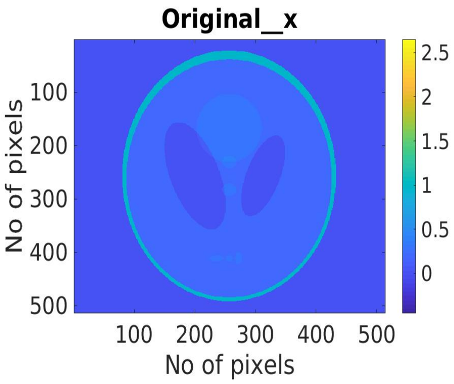
## 3.4 Simultaneous Algebraic Reconstruction Technique's Result

Reconstruction of 2D image using SART technique.
Original image size 512×512 = 262144 pixels.

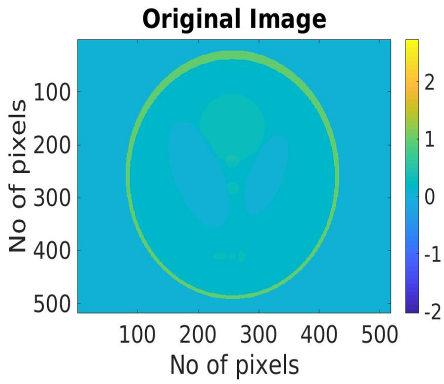
| Normalised error in Reconstructed Images | | |
|---|---|---|
| | Data(y) without noise | Data with SNR=10dB |
| Overlap% | k=30 | k=30 |
| 10 | 21.50 | 22.61 |
| 20 | 19.01 | 20.25 |
| 30 | 20.68 | 22.14 |
| 40 | 16.87 | 30.62 |
| 50 | 14.71 | 21.55 |
| 60 | 13.76 | 23.61 |
| 80 | 10.68 | 18.47 |
| 90 | 10.01 | 12.18 |

Table 3.3: Table to show normalised error. k=no of times algorithm is being run.

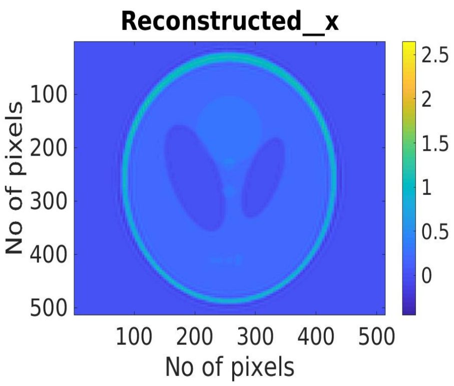
From the table, we can see the same trend as in ART technique but we can see the SART technique converges faster and SART is less prone to error as compare to ART. As in SART, we take all the projections at one time and take the average to update the the reconstructive step which makes this technique better than ART.



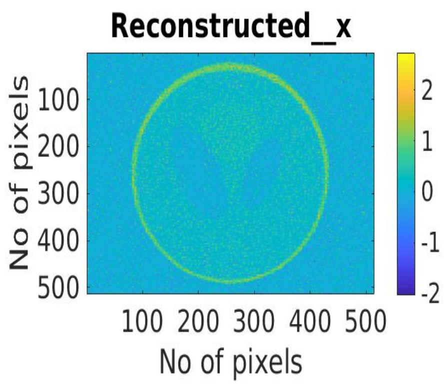

(a) Original image



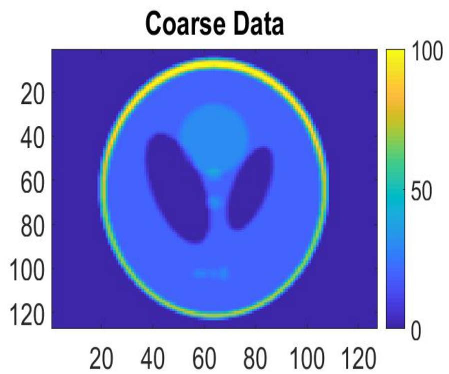

(b) Original image




(c) Reconstructed image when k=30 and without noise

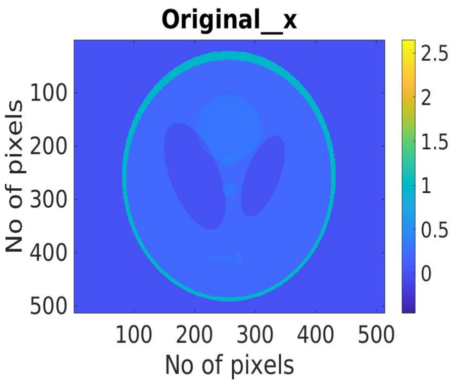



(d) Reconstructed image when k=30 and with noise

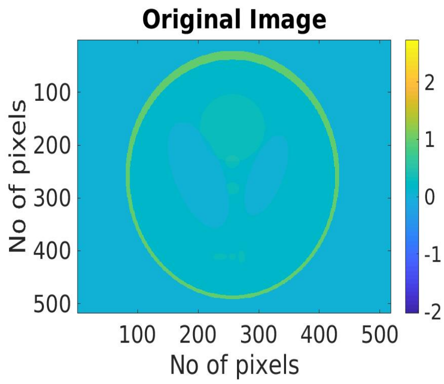



(e) Coarse data

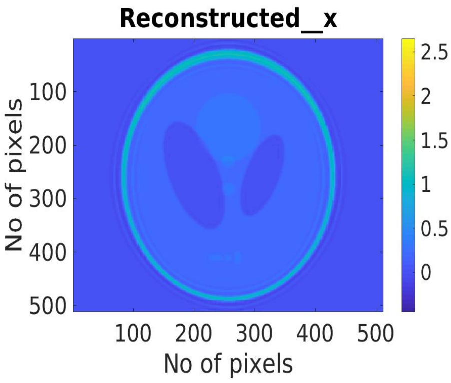
Figure 3.5: Images corresponding to overlap 60%



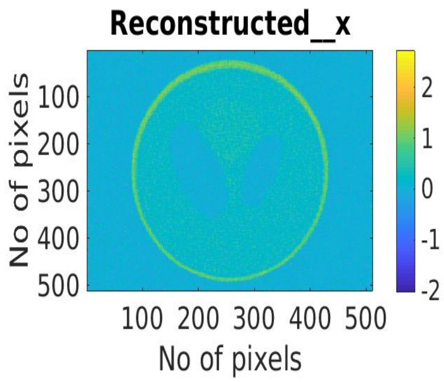

(a) Original image



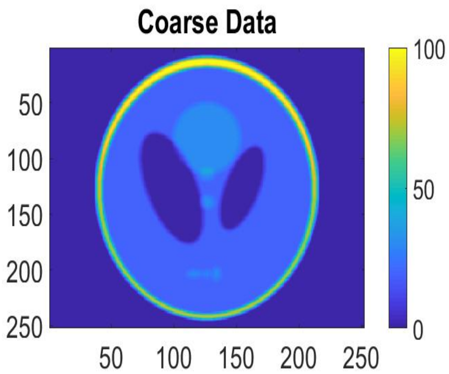

(b) Original image




(c) Reconstructed image when k=30 and without noise

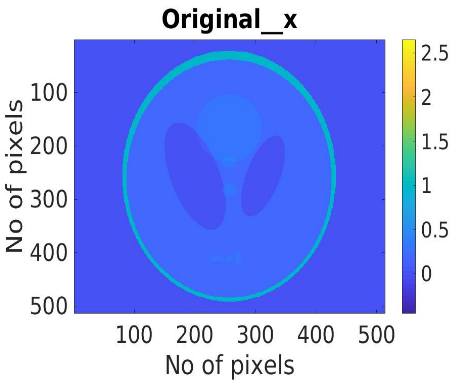



(d) Reconstructed image when k=30 and with noise

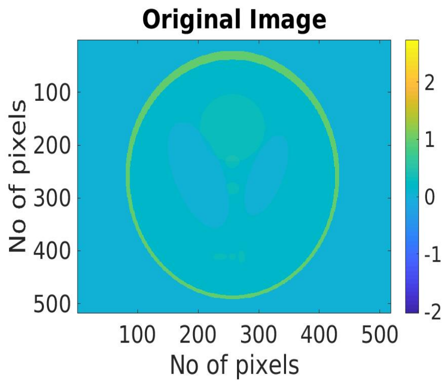



(e) Coarse data

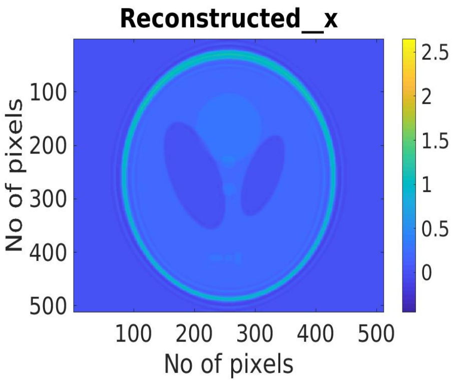
Figure 3.6: Images corresponding to overlap 80%



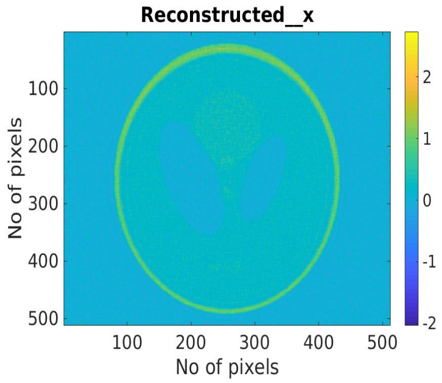

(a) Original image



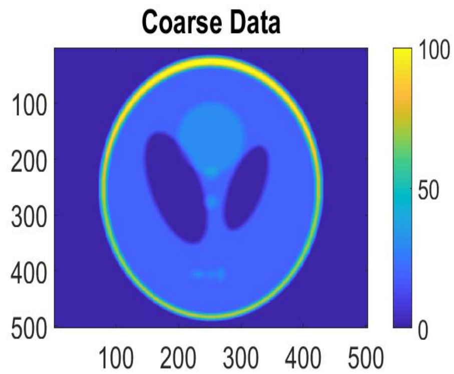

(b) Original image




(c) Reconstructed image when k=30 and without noise

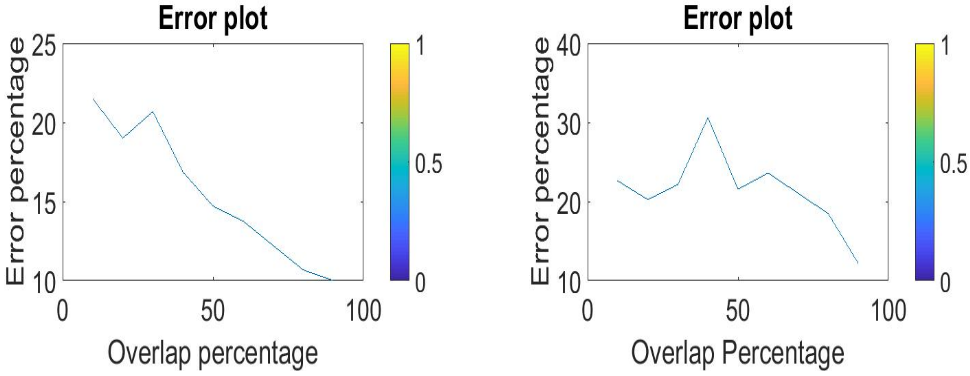



(d) Reconstructed image when k=30 and with noise




(e) Coarse data

Figure 3.7: Images corresponding to overlap 90%




(a) Error, when k=30 (b) Error when k=30

Figure 3.8: Reconstructed error when signal has a. no noise b. 10dB noise

# Chapter 4

# Conclusion

## 4.1 Conclusion

The super resolution techniques are being tested and found out that in case of noiseless measurements, SART converges to solution with a more monotonic behaviour than ART. In ART, the order in which rows of H has been picked also matters. Resolution by a factor of 100 is being achieved since for every 100 pixels in high resolution, we have one pixel information in the low resolution grid. As the overlap increases, the error will decrease as more information is available for reconstruction. But as overlap increases, the number of hyperplanes increases for the projections, so we need to take more projections in case of ART to get better results. In SART, as overlap increases, we have got expected results.
But in case of measurements having noise, both the algorithms don't show promising results.In case of noisy measurements also, SART performs better than ART.
SART converges faster and the error is less in reconstruction than ART.

## 4.2 Future work

We can apply these techniques to real world data by overcoming the challenges in it. After super resolving the data , we can use the information to model for various applications in remote sensing like wind vector retrieval over oceans, soil moisture etc.

# References


[1] Image Reconstruction from Projections: The Fundamentals of Computerized Tomography (Computer Science Applied Mathematics), aHerman, Gabor T,

*1980*

[2] A tutorial on ART (algebraic reconstruction techniques), Gordon, Richard, *journal=IEEE Transactions on Nuclear Science, volume=21, number=3, pages=78–93, year=1974, publisher=IEEE*

[3] Andersen, Anders H and Kak, Avinash C, Simultaneous algebraic reconstruction technique (SART): a superior implementation of the ART algorithm

*journal=Ultrasonic imaging, volume=6, number=1, pages=81–94, year=1984, publisher=SAGE Publications Sage CA: Los Angeles, CA*

[4] Image super-resolution via sparse representation, Yang, Jianchao and Wright, John and Huang, Thomas S and Ma, Yi, *journal=IEEE transactions on image processing, volume=19, number=11, pages=2861–2873, year=2010, publisher=IEEE*

[5] Long, David G and Early, David S.Image reconstruction and enhanced resolution imaging from irregular samples *year=2001, publisher=IEEE*

[6] Rafel C. Gonzalez and Richard E Woods.Digital Image Processing