EE6190

Project Report
on
Overheads for Asynchronous Communication

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1 Introduction

The research in wireless techonolgies is driven by the growing number of mobile broadband users, demand of high data rate services, real time low latency applications, and the increasing realm of internet of things (**IoT**). According to **IEEE**, Internet of Things alone will consist of billions of devices by 2020. Many of these **IoT** devices, and sensors will run on batteries, which means that these devices must be able to operate in a low-power, low-latency state, which is often a conflicting requirement (see [McC17]). The devices and sensors developed for **IoT** will typically be low rate systems, meaning that the data packets will be transferred intermittently over a large transmission window. The detection of such small data packets is affected not only by noise, but also by the size of the transmission window. The impact of noise on such short packets is the focus of *short packet* communications (see [KHP17],[Dur+16]), whereas the impact of the transmission window size is the focus of *asynchronous* communications (see [CTW08; CM06]). Our work is focussed on the impact of transmission window size on these data packets i.e. *asynchronous* communications.

1.1 Asynchronous Communication

Asynchronous communication is the exchange of messages/information by reading and responding to schedules rather than according to some clock that is synchronized for both the Transmitter and Receiver. In asynchronous communication the data is transmitted in bursts over a large time horizon. Many emerging technologies such as machine to machine (**M2M**) communications, and Internet of Things (**IoT**) fall under the umbrella of asynchronous communication as the data rate involved is very low, and the transmissions are intermittent. Most of the communication between devices within computers and between computer and external devices is also asynchronous.

In asynchronous communication, it becomes necessary for the receiver to be able to distinguish between valid data transmission and noise otherwise it could result in a false alarm or missed detection. Also the cost of acquiring synchronization becomes significant for asynchronous communication because the number of bits transmitted per burst is relatively small, which is not the case with synchronous communication, where the cost of initially acquiring synchronization is amortized over the many symbols transmitted [CTT13]. In asynchronous communication,the receivers job is to locate the sync packet, embedded in noise, over a large time window. This problem, referred to as *frame synchronization* in the literature [Mas72], has received significant attention in recent years (see [CTW08; CM06]). An optimal strategy (for error minimization) at the receiver is to correlate the received data with a local copy of the sync packet over the entire length of transmission, in which the sync packet is known to occur almost surely, and choose the position of the sync packet to be the position with maximum correlation (see [Mas72]). The above mentioned strategy, though optimal in terms of locating the sync packet, is not optimal in situations where there is a delay constraint (such as sensor nodes transmitting some information which needs immediate attention).

In [CTW08], the authors consider the problem of locating a sync packet of length N, within some interval of size A, on the basis of sequential observations, which is also referred to as *one-shot frame synchronization problem*, and show that a sequential decoder can locate the sync pattern optimally(i.e. exactly, without delay and almost surely) with $N \to \infty$ as long as $A \sim O(e^{N\alpha})$, with α below the *synchronization threshold* (see [CTW08]). For a general DMC with channel transition probabilities Q, the authors in [CTW08] provide the scaling needed of the sync packet for error free frame synchronization. The necessary scaling of the sync packet for more general channels such as the finite state Markov channel is studied in [Sun+17].

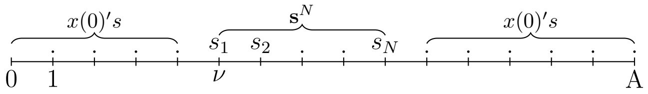

Figure 1: A sync frame $\mathbf{s}^N = (s_1, \cdots, s_N)$ of length N symbols is transmitted at a random time ν.

1.2 Motivation for our work

The works on asynchronous communication thus far consider the asynchronism only due to the random arrival time of data within a certain time window, also known as *source asynchronism*, assuming perfect clock synchronization between the transmitter and receiver. The other type of asynchronism (which is likely to be present) is due to the lack of a common clock between the transmitter and receiver, also known as *channel asynchronism*.

As discussed in the introductory paragraph, most of the sensors and devices for **IoT** are low-power devices. The low power requirement in these devices can be achieved with duty cycling, turning the radio on (”idle listening mode”) and off (”power save mode”) periodically to save energy. In idle listening mode, three main schemes are used for power saving, namely pure asynchronous, synchronous and pseudo synchronous, of which pure asynchronous communication is considered to be the most energy efficient mechanism (see [Pre+16]). Asynchronous communication in these devices is achieved by using an ultra-low power, highly sensitive second front-end known as a wake-up radio receiver (see [McC17]), which can be combined with the existing radio transceivers to reduce overall power consumption. This sleep and wake-up mode of operation in the devices is a source of asynchronism between the transmitter and receiver. Also, in order to enhance the battery life of these devices the synchronization algorithms employed will be of lower complexity resulting in poorer estimates of sync parameters such as timing, epoch, channel delays and clock offsets.

In this work, we incorporate the *channel asynchronism* or the clock misalignment as a random variable and study the scaling needed of the sync packet to achieve reliable communication.

2 Source Asynchronism

We first discuss the setup when there is only *source asynchronism* and show the tools and techniques used to derive the already known results ([CTW08], [SJR17]). This will ensure that we are on solid ground when we include *channel asynchronism* into the picture.

2.1 Setup

Consider a slotted communication model between the transmitter and receiver over a discrete memoryless channel (**DMC**). The **DMC** is characterized by finite input and output alphabet sets, $\mathcal{X}$ and $\mathcal{Y}$ respectively, and channel transition probability matrix $Q(y|x)\ \forall (x, y) \in (\mathcal{X}, \mathcal{Y})$.

Consider a *one shot synchronization* probelm where a sync packet $\mathbf{s}_N$ of length N symbols is transmitted at some random time ν, uniformly distributed in $\{1, 2, \cdots, A\}$, where the integer A characterizes the asynchronism level. The receiver knows A but not ν, otherwise there is no asynchronism. Assume that a symbol transmission occupies a single slot and the transmission of the sync packet $\mathbf{s}^N = (s_1, \cdots, s_N)$ occupies slots $\{\nu, \nu + 1, \cdots, \nu + N - 1\}$ (as illustrated in Figure 1). The channel input $\{x_n\}$ in slots $\{\nu, \nu + 1, \cdots, \nu + N - 1\}$ is given by $s_{n-\nu+1}$, and the channel input in all other slots is $x_n = x(0)$, which is the zero input.

The reciever seeks to identify the instant of the sync packet transmission, i.e. ν, and employs a sequential decoder in the form of a stopping time τ w.r.t to the received sequence $\{y_n\}$. If $\tau = t$, the receiver declares that the sync packet started being sent at time $t - N + 1$.

The associated error event is thus defined as, $\mathcal{E} = \{\tau \neq \nu + N - 1\}$.

The main result from the paper [CTW08] says that error free frame synchronization can be achieved (in the asymptotic sense as $N \to \infty$) iff $A < e^{\alpha(Q)N}$, where $\alpha(Q) = \max_{x \in \mathcal{X}} D\big(Q(.|x)||Q(.|x(0))\big)$ is defined as the *synchronization threshold* for the **DMC** and $D\big(Q(.|x)||Q(.|x(0))\big)$ denotes the KL divergence between $Q(.|x)$ and $Q(.|x(0)$. Thus, the *synchroization threshold* characterizes the scaling needed of the sync packet to achieve error free frame synchronization. Once, the maximally divergent symbol, $x(1)$, is known assume that the input alphabet $\mathcal{X} = \{x(1), x(0)\}$.

2.2 Proof of Achievability

We prove only the achievability of the result presented in [CTW08] using the setup discussed above, a detailed proof of the converse can be found in the paper itself.

Codeword:

The sync packet $\mathbf{s}_N = (s_1, \cdots, s_N)$ of length N satisfies:

- All s_n's are equal to $x(1)$, where $x(1) = arg \max_{x \in \mathcal{X}} D\big(Q(.|x)||Q(.|x(0))\big)$, except for a fraction at most equal to $1/K$, where K is large, that are equal to $x(0)$.
- The Hamming distance between the sync packets and any of its shifts is linear in N.

The sync packet s_N is constructed as follows. Pick some large K that satisfies $\lfloor N/K \rfloor = 2^m - 1$, for some postive integer m. Set $s_i = x(1)$ for all $\lfloor N/K \rfloor \leqslant i \leqslant N$. To specify the rest of the sync packet, pick an **MLSR** sequence $m_1, m_2, \cdots, m_{\lfloor N/K \rfloor}$ of length $\lfloor N/K \rfloor$ and set $s_i = x(1)$ if $m_i = 0$, and $s_i = x(0)$ if $m_i = 1$ for all $i \in [1, 2, \cdots, \lfloor N/K \rfloor$. A sequence constructed in this manner satisfies both the above mentioned properties.

Decoder:

Consider a sequential joint typicality decoder, which operates as follows. At time t, it computes the empirical distribution $\hat{\mathbb{P}}$ induced by the sync packet and the previous N output symbols y_{t-N+1}, y_{t-N+2},
$\cdots, y_t$, where the empirical distribution means the type of the distribution (see Chapter-11 in [CT06]). If this empirical distribution is close enough to $\mathbb{P}$, where $\mathbb{P}$ is the type of the sync packet, in the sense that $|\hat{\mathbb{P}}(x, y) - \mathbb{P}(x, y)| \leqslant \mu$ for all x, y and some small $\mu > 0$, the decoder stops and declares $t - N + 1$ as the time of trasmission of the sync packet. Otherwise, it moves one step ahead and repeats the same procedure.

Error Event:

Let τ_N denote the stopping time of the decoder and ν be the instant of sync packet transmission. The error event $\{\tau_N \neq \nu + N - 1\}$ can be broken down as a union of the following three events.

- False alarm event $\mathcal{E}_1$. N output symbols generated entirely by random data, i.e. $x(0)$'s, are typical with the sync packet. This imples that $\tau_N \in \mathcal{S}_1$, where $\mathcal{S}_1 = \{N, \cdots, \nu - 1\} \cup \{\nu + 2N - 1, \cdots, A + N - 1\}$.
- False alarm event $\mathcal{E}_2$. N output symbols generated partly by random data and the sync packet are typical with the sync packet. This imples that $\tau_N \in \mathcal{S}_2$, where $\mathcal{S}_2 = \{\nu, \cdots, \nu + N - 2, \nu + N, \cdots, \nu + 2N - 2\}$.

- Missed detection event $\mathcal{E}_3$. N output symbols generated entirely by the sync packet are not typical with the sync packet.

Performance Evaluation:
The error event $\mathcal{E} = \{\tau_N \neq \nu + N - 1\}$ is a union of the union of the above three events. From the union bound, we have:

$$Pr(\mathcal{E}) \leqslant Pr(\mathcal{E}_1) + Pr(\mathcal{E}_2) + Pr(\mathcal{E}_3) \tag{1}$$

Consider $Pr(\mathcal{E}_1)$. In this event, the output symbols are generated by the distribution $Pr\big(x(0)\big)Q\big(.|x(0)\big) = Q\big(.|x(0)\big)$. The type of the sync packet is

$$\Big(Pr\big(x(1)\big), Pr\big(x(0)\big)\Big) = \Big(\frac{N\big(1-1/K\big)}{N}, \frac{N/K}{N}\Big) = \Big(1 - \frac{1}{K}, \frac{1}{K}\Big)$$

From the union bound, $Pr(\tau_N \in \mathcal{S}_1) \leqslant |\mathcal{S}_1|\ Pr(\tau_N = t)$, where $|.|$ denotes cardinality of the set and $t \in \mathcal{S}_1$. Let $\mathbf{y}_t^N = (y_{t-N+1}, \cdots, y_t)$ be the received sequence correspoding to $\tau_N = t$.
Let $\mathcal{B}_\mu := \{\mathbf{y}^N \in \hat{\mathbb{P}}(y) : |\hat{\mathbb{P}}(x,y) - \mathbb{P}(x,y)| \leqslant \mu\}$, where $\hat{\mathbb{P}}(y) = \sum\limits_{x\in\mathcal{X}} \hat{\mathbb{P}}(x,y)$ and $\mathbb{P}(x,y) \triangleq \hat{\mathbb{P}}(x)Q(y|x)$, where $\hat{\mathbb{P}}(x)$ is the empirical distribution of the sync packet. Now, $Pr(\tau_N = t) = Pr(\mathbf{y}_t^N \in \mathcal{B}_\mu)$.

$$Pr(\mathbf{y}_t^N \in \mathcal{B}_\mu) = \prod_{i=0}^{N-1} y_{t-i} = |\mathcal{B}_\mu| \prod_{y_1\in\mathcal{Y}} \big[Q\big(y_1|x(0)\big)\big]^{N(y_1|\mathbf{y}_t^N)} \tag{2}$$

where $N(y_1|\mathbf{y}_t^N)$ denotes the number of times y_1 occurs in $\mathbf{y}_t^N$ (see Chapter-11 in [CT06]).
Now, for the sequence $\mathbf{y}_t^N$ to lie in $\mathcal{B}_\mu$, it must be true that

$$\begin{aligned}
\hat{\mathbb{P}}(x,y) &\leqslant \mathbb{P}(x,y) + \mu \\
\sum_{x\in\mathcal{X}} \hat{\mathbb{P}}(x,y) &\leqslant \hat{\mathbb{P}}\big(x(1)\big)Q\big(y|x(1)\big) + \hat{\mathbb{P}}\big(x(0)\big)Q\big(y|x(0)\big) + 2\mu \\
&\leqslant \big(1-1/K\big)Q\big(y|x(1)\big) + \big(1/K\big)Q\big(y|x(0)\big) + 2\mu
\end{aligned} \tag{3}$$

From the definition of type of a sequence, we have

$$\hat{\mathbb{P}}(y_1) = \frac{N(y_1|\mathbf{y}_t^N)}{N} = \sum_{x\in\mathcal{X}} \hat{\mathbb{P}}(x, y_1)$$

Now, from eqn. 3, we have

$$\begin{aligned}
N(y_1|\mathbf{y}_t^N) &\leqslant N\big[\big(1-1/K\big)Q\big(y_1|x(1)\big) + \big(1/K\big)Q\big(y_1|x(0)\big) + 2\mu\big] \\
Pr(\mathbf{y}_t^N \in \mathcal{B}_\mu) &\leqslant |\mathcal{B}_\mu| \prod_{y_1\in\mathcal{Y}} \big[Q(y_1|x(0))\big]^{N\big[(1-1/K)Q(y_1|x(1)) + (1/K)Q(y_1|x(0)) + 2\mu\big]} \\
&\leqslant |\mathcal{B}_\mu| \prod_{y_1\in\mathcal{Y}} e^{N\big[(1-1/K)Q(y_1|x(1)) + (1/K)Q(y_1|x(0)) + 2\mu\big] \log\big[Q(y_1|x(0))\big]} \\
&\leqslant |\mathcal{B}_\mu| e^{\sum\limits_{y_1\in\mathcal{Y}} N\big[(1-1/K)Q(y_1|x(1)) + (1/K)Q(y_1|x(0)) + 2\mu\big] \log\big[Q(y_1|x(0))\big]}
\end{aligned} \tag{4}$$

On simplification, eqn. 4 reduces to

$$Pr(\mathbf{y}_t^N \in \mathcal{B}_\mu) \leqslant |\mathcal{B}_\mu| e^{-N(1-1/K)(\alpha(Q)-\delta)+I}$$

where $\alpha(Q) = D\big(Q(y_1|x(1))||Q(y_1|x(0))\big)$, $\delta = \frac{2\mu K \sum\limits_{y_1 \in \mathcal{Y}} \log\big(Q(y_1|x(0))\big)}{\big(K-1\big)}$, and $I = -NH(\mathbf{y}^N|\mathbf{s}^N)$, $H(\mathbf{y}^N|\mathbf{s}^N)$ denotes the conditional entropy of all sequences $\mathbf{y}^N$ that have a type *close* to the sync packet $\mathbf{s}^N$. Using the fact that $|\mathcal{B}_\mu| = (polyN)e^{NH(\mathbf{y}^N|\mathbf{s}^N)}$ (see Chapter-11 in [CT06]), we get

$$Pr(\mathbf{y}_t^N \in \mathcal{B}_\mu) \leqslant (polyN)e^{-N(1-1/K)(\alpha(Q)-\delta)}$$

This implies,

$$\begin{aligned} Pr(\tau_N \in \mathcal{S}_1) &\leqslant |\mathcal{S}_1|\ Pr(\mathbf{y}_t^N \in \mathcal{B}_\mu) \\ &\leqslant (polyN)(A-2N+1)e^{-N(1-1/K)(\alpha(Q)-\delta)} \\ &\leqslant (polyN)(A)e^{-N(1-1/K)(\alpha(Q)-\delta)} \end{aligned} \tag{5}$$

From eqn. 5, $Pr(\mathcal{E}_1) = Pr(\tau_N \in \mathcal{S}_1) \to 0$ as $N \to \infty$ if $A < e^{N\alpha(Q)}$.

Using similar argumnets, it can be shown that

$$\begin{aligned} Pr(\mathcal{E}_2) &\leqslant (polyN)e^{-\Omega(N)(\alpha(Q)-\delta')} \\ Pr(\mathcal{E}_3) &\leqslant (polyN)e^{-\frac{N}{K}\big(D(Q(.|x(0))||Q(.|x(1)))-\delta''\big)} \end{aligned} \tag{6}$$

where $\Omega(N)$ denotes a term linear in N. Thus, we see that the probability of all the three error events go to zero as $N \to \infty$ if $A < e^{\alpha(Q)N}$, which implies (from eqn. 1) that $Pr(\tau_N \neq \nu + N - 1) \to 0$ as $N \to \infty$ if $A < e^{\alpha(Q)N}$.

The above condition is sufficient as well, i.e. if $A > e^{\alpha(Q)N}$, a **ML** decoder that is revealed the complete output sequence of size $A+N-1$ makes a decoding error with probability tending to one as $N \to \infty$.

The above result from the authors in [CTW08] has been extended, to the case of a general arrival distribution for ν, by the authors in [SJR17].The authors in [SJR17] shows that error free frame synchronization can be achieved (in the asymptotic sense as $H \to \infty$) for a general arrival distribution for ν if $\bar{N} > (1+\beta)\frac{H}{\alpha(Q)}$, where $\bar{N}$ is the average sync packet length, $\alpha(Q)$ is the *synchronization threshold* for the **DMC**, and $\beta > 0$ is a constant. The authors in [SJR17] also provide a variable length sync packet design that achieves the result. The result shown by the authors in [CTW08] is a subset of the result given in [SJR17] as $H = \log(A)$ when ν is distributed uniformly in $\{1, 2, \cdots, A\}$.

3 Source and Channel Asynchronism

As discussed in the introduction, it is very common to have a clock misalignment between the transmitter and receiver, which may result from various factors as discussed already. We now consider the case when there is a clock misalignment (or *channel asynchronism*) also present along with the random arrival of the sync packet at the transmitter (or *source asynchronism*).

3.1 Setup

Consider again a slotted communication model between a transmitter and a receiver in a discrete memoryless channel (**DMC**). The **DMC** is characterized by finite input and output alphabet sets $\mathcal{X}$ and $\mathcal{Y}$ respectively, and channel transition probabilities $Q(y|x)$ defined for all $(x, y) \in (\mathcal{X}, \mathcal{Y})$.

We consider a one shot frame synchronization problem where a sync frame $\mathbf{s}_\nu$ of length N_ν symbols is transmitted at some random time ν. We assume that a symbol transmission

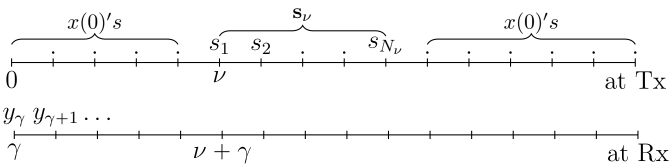

Figure 2: A sync frame $\mathbf{s}_\nu = (s_1, \cdots, s_{N_\nu})$ of length N_ν symbols is transmitted at a random time ν. We assume that the transmitter and receiver clocks are misaligned by a random value γ.

occupies a single slot and the transmission of the sync frame $\mathbf{s}_\nu = (s_1, \cdots, s_{N_\nu})$ occupies slots $\nu, \nu+1, \cdots, \nu+N_\nu-1$ (as illustrated in Figure 2). The channel input $\{x_n\}$ in slots $\nu, \nu+1, \cdots, \nu+N_\nu-1$ is $x_n = s_{n-\nu+1}$ and the channel input before and after the sync frame transmission is assumed to be $x_n = x(0)$. Assume that the random transmission time ν is distributed as $\{a_n : n \in \mathcal{A}\}$ and the distribution is known to both the transmitter and receiver. The entropy of the distribution $H_a = -\sum_n a_n \log(a_n)$ characterizes the uncertainty (in transmission time of the sync frame) at the source.

Assume a random misalignment, γ, between the transmitter clock and the receiver clock (see Figure 2). Let $\{y_n\}$ denote the channel output at the receiver (with respect to the clock at the receiver). Then, the distribution of the channel output (at the receiver) conditioned on the random transmission time ν (w.r.t the transmitter clock), the sync sequence $\mathbf{s}_\nu$ and clock misalignment γ is given by $Q(\cdot|s_{n-\nu+1-\gamma})$ for $n \in \nu+\gamma, \cdots, \nu+\gamma+N_\nu-1$ and $Q(\cdot|x(0))$ otherwise. We assume that γ is independent of ν (which is a fair assumption in many situations) and the misalignment is the same for all time slots (hence, the sync word is received contiguously at the receiver). Let the random misalignment γ be distributed as $\{b_n : n \in \mathcal{B}\}$ and let $H_b = -\sum_n b_n \log(b_n)$ denote the entropy of its distribution. Now, the uncertainty at the receiver regarding the sync frame time, $\nu+\gamma$, has a distribution $\{c_n\} = \{a_n\} * \{b_n\}$ with entropy $H_c \leqslant H_a + H_b$.

We study the problem of frame synchronization where the receiver seeks to identify the location of the sync packet, $\nu+\gamma$, from the channel output $\{y_n\}$. We consider a sequential receiver that employs knowledge of the distributions of ν and γ, the channel transition probabilities $Q(\cdot|\cdot)$ and of the codebook used. Let $\hat{v}$ denote the estimate of $\nu+\gamma$ at the receiver. Then, the probability of error in frame synchronization is denoted as $\mathsf{P}(\hat{v} \neq \nu+\gamma)$. We seek to characterize the scaling necessary of the average sync frame length for error-free frame synchronization as the uncertainty (measured in terms of entropy or cardinality of the distributions) at the source and due to misalignment tends to infinity.

We consider a general distribution for ν and γ with possibly infinite state space. In [V C13] (see Theorem 5 in [V C13]), it is shown that it is sufficient to consider a (minimal) finite subset of the sample space whose probability must tend to 1 as the uncertainty increases. Henceforth, we will assume that the distributions and the definitions (of entropy and cardinality) correspond to such (minimal) finite subsets only.

3.2 Results

In this section, we present some of the results that we have obtained.

3.2.1 Source Uncertainty Only

As discussed already, in [CTW08] it was shown that the probability of error in frame synchronization $\mathsf{P}(\hat{v} \neq \nu) \to 0$ as $A \to \infty$ if and only if $A < e^{N\alpha}$ for some $\alpha < \alpha(Q)$. Here,

$\alpha(Q)$, defined as the synchronization threshold of the DMC, was shown to be,

$$\alpha(Q) = \max_{x \in \mathcal{X}} D(Q(\cdot|x)||Q(\cdot|x(0))) \tag{7}$$

where $D(Q(\cdot|x)||Q(\cdot|x(0)))$ is the Kullback-Leibler distance between the distributions $Q(\cdot|x)$ and $Q(\cdot|x(0))$. In [SJR17], Sundaram et al, generalize the framework in [CTW08] and study optimal frame synchronization for a general arrival distribution $\{a_n\}$ (with entropy H_a). The authors propose a variable length sync frame for the general arrival distribution, with length N_n for slot n, and show that $\mathsf{P}(\hat{v} \neq \nu) \to 0$ iff $H_a < \bar{N}\alpha$ for some $\alpha < \alpha(Q)$, where $\bar{N}$ denotes the average sync frame length, $\bar{N} = \sum_n a_n N_n$.

3.2.2 Clock Misalignment Only

Here, we study optimal frame synchronization with zero uncertainty at the source ($\nu = 0$ with probability one) but a random misalignment γ between the transmitter clock and the receiver clock.

In [V C13], Chandar et al discuss the use of a constant length and common sync frame $\mathbf{s}_n = \mathbf{s}$ for all n for a general arrival distribution $\{a_n\}$. Let $A = |\mathcal{A}|$ denote the cardinality of the arrival distribution $\{a_n\}$ and let N denote the common sync frame length. It was shown that the probability of error in frame synchronization $\mathsf{P}(\hat{v} \neq \nu) \to 0$ as $A \to \infty$ iff $A < e^{N\alpha}$ for some $\alpha < \alpha(Q)$ (where $\alpha(Q)$ is the synchronization threshold of the **DMC**.

In our current setup, with $\nu = 0$, we are restricted to a single sync word at the transmitter, i.e., $\mathsf{s}_0 = \mathsf{s}$, even for a general distribution $\{b_n\}$ for γ. The setup at the receiver (with uncertainty distribution $\{b_n\}$ and a common sync frame s) is now similar to [V C13] and the following result holds.

Theorem 1 *Consider a DMC $(\mathcal{X}, \mathcal{Y}, Q(\cdot|\cdot))$ with synchronization threshold $\alpha(Q)$. A sync frame s of length N symbols is transmitted at time 0. Let γ denote the misalignment between the transmitter clock and the receiver clock, where γ is distributed as $\{b_n : n \in \mathcal{B}\}$ with cardinality $B = |\mathcal{B}|$. Then, the probability of error in frame synchronization $\mathsf{P}(\hat{v} \neq \nu) \to 0$ as $B \to \infty$ iff $B < e^{N\alpha}$ for some $\alpha < \alpha(Q)$.*

3.2.3 A General Framework for Asynchronous Communication

In this section, we will study optimal frame synchronization with uncertainty at the source and a random misalignment between the transmitter and receiver clocks.

The following lower and upper bound for the average sync frame length holds true for any general distribution $\{a_n\}, \{b_n\}$ and $\{c_n\} = \{a_n\} * \{b_n\}$.

Lemma 2 *Consider a DMC $(\mathcal{X}, \mathcal{Y}, Q(\cdot|\cdot))$ with synchronization threshold $\alpha(Q)$. A sync frame s_ν of length N_ν symbols is transmitted at a random time ν, where ν is distributed as $\{a_n\}$. Let γ denote the random misalignment between the transmitter clock and the receiver clock, where γ is distributed as $\{b_n\}$. Let $\{c_n : n \in \mathcal{C}\}$ denote the distribution at the receiver, where $\{c_n\} = \{a_n\} * \{b_n\}$ and let $C = |\mathcal{C}|$ denote the cardinality and H_c denote the entropy of the distribution $\{c_n\}$. Then,*

1. *$\mathsf{P}(\hat{v} \neq \nu) \to 0$ as $H_c \to \infty$ only if $H_c < \bar{N}\alpha$ for some $\alpha < \alpha(Q)$.*
2. *$\mathsf{P}(\hat{v} \neq \nu) \to 0$ as $C \to \infty$ if $C < e^{\bar{N}\alpha}$ for some $\alpha < \alpha(Q)$.*

Remark 3.1 *Note that, for the setup in Section 3.2.1, Lemma 2-1) is a sufficient condition as well. And, for the setup in Section 3.2.2, Lemma 2-2) is a necessary condition as well.*

We will consider the following framework to study optimal frame synchronization with a general distribution for ν and γ.

Setup:

A sync packet of length N_ν is transmitted at a random time ν over a DMC channel $\{\mathcal{X}, \mathcal{Y}, Q(\cdot|\cdot)\}$. We assume a random misalignment γ between the transmitter clock and the receiver clock.

Codeword:

We will construct a variable length sync word s_n of length N_n for time slot n as follows. The sync word begins with a maximal length shift register (MLSR) sequence of length M (independent of slot n and $M \leqslant N_n$ for all n). Here, 0 in the MLSR sequence is replaced by $x(0)$ and 1 in the MLSR sequence is replaced by $x(1)$, where

$$x(1) = \arg\max_{x \in \mathcal{X}} D(Q(\cdot|x)||Q(\cdot|x(0)))$$

is the symbol with the maximum divergence at the output (in comparison to default zero input). The rest of the sync word (of length $N_n - M$) is filled with the symbol $x(1)$.

The average sync frame length $\bar{N}$ is given by $\bar{N} = \sum_{n \in \mathcal{A}} a_n N_n$. In the following discussions, we will derive optimal sync frame lengths $\{N_n\}$ subject to the above constraint.

Decoder:

Consider any n (with respect to the clock at the receiver) such that $c_n > 0$ and let $\mathcal{A}_n$ denote the set of all epochs (with respect to the clock at the transmitter) whose transmission can appear at time n, i.e.,

$$\mathcal{A}_n = \{p : p + q = n, a_p > 0, b_q > 0\}$$

The cardinality of the set $\mathcal{A}_n$ can be greater than one and the sync frame length at these transmission epochs can be same or different. The receiver seeks only to detect the presence of the sync frame, i.e., identify the epoch $\nu + \gamma$ and not ν or γ separately. Hence, we recommend the following multi-stage sequential typicality decoder for the setup.

Consider any time n such that $c_n > 0$. Define $N_n^{\min}$ as

$$N_n^{\min} = \min\{N_p : p \in \mathcal{A}_n\}$$

the length of the smallest sync frame possible at time n. We note that the length of the smallest sync frame would correspond to a position with the largest a_p such that $p \in \mathcal{A}_n$. The number of receiver stages at time n is equal to $\lfloor \log_M(N_n^{\min}) \rfloor + 1$. In stage $s \in \{1, \cdots, \lfloor \log_M(N_n^{\min}) \rfloor + 1\}$, the receiver computes the empirical distribution $\hat{P}_{s,n}$ of the received word (i.e., fraction of time an output alphabet occurs) in $\{y_{n+(2^{s-1}-1)M}, \cdots, y_{n+(2^s-1)M}\}$ corresponding to those positions with input alphabet $x(1)$ for the smallest sync frame. If the empirical distribution is close to the expected distribution $Q(\cdot|x(1))$, i.e., if $|\hat{P}_{s,n} - Q(\cdot|x(1))| < \mu$ for some small $\mu > 0$, then the receiver proceeds to the next stage. If the received word is typical with $Q(\cdot|x(1))$ in all the stages, then the receiver declares the presence of a sync frame in slot n. Else, the receiver declares that the sync frame was not received in slot n and moves forward in time.

Error events:

The error event $\{\hat{v} \neq \nu\}$ can be partitioned into the following three categories.

- False Alarm Event $\mathcal{E}_1$: This corresponds to the event where the output generated entirely by the default (zero) input is typical with $Q(\cdot|x(1))$ in all the stages.
- Overlap Error Event $\mathcal{E}_2$: This corresponds to the event where the output generated partially by some sync frame and the default (zero) input is typical with $Q(\cdot|x(1))$ in all the stages.
- Missed Detection Event $\mathcal{E}_3$: This corresponds to the event where the output generated by a sync frame in slot n is not typical with $Q(\cdot|x(1))$ in at least one of the stages.

Performance Evaluation:
The probability of error in frame synchronization can be upper bounded as follows.

$$\mathsf{P}(\hat{v} \neq \nu) \leqslant \mathsf{P}(\mathcal{E}_1) + \mathsf{P}(\mathcal{E}_2) + \mathsf{P}(\mathcal{E}_3)$$

We will now upper bound the probabilities of the different error events.

Consider the false alarm event $\mathcal{E}_1$. The probability of error of type $\mathcal{E}_1$ at time slot n with the multi-stage receiver is upper bounded by the probability of error with a single stage receiver using the entire length. Then, using a union bound for all n, we have,

$$\mathsf{P}(\mathcal{E}_1) \leqslant \sum_{n \in \mathcal{C}} e^{-(N_n^{\min} - M)(\alpha(Q) - \delta)} \tag{8}$$

where $\delta > 0$ tends to zero as $\mu \to 0$.

Consider the overlap error event $\mathcal{E}_2$. Suppose that the sync frame is received in slot ν. Then, the number of positions n such that the sync frame s_n overlaps with the sync frame s_ν may be unbounded. However, we note that the number of positions n that overlap (for the first time) at stage s is upper bounded by $2^s M$. As the minimum Hamming distance between s_n and s_ν is at least $\Omega(2^s M)$ for all stages (including stage one due to the MLSR sequence), we have the following upper bound for $\mathsf{P}(\mathcal{E}_2)$ based on the arguments in [CTW08].

$$\mathsf{P}(\mathcal{E}_2) \leqslant \sum_{s=1}^{\infty} 2^s M e^{-\Omega(2^s M)(\alpha(Q) - \delta')} \tag{9}$$

where $\delta' > 0$ and tends to zero as $\mu \to 0$.

Consider the miss detection event $\mathcal{E}_3$. The probability of missed detection at any time n can be upper bounded using a union bound over all the stages as follows.

$$\mathsf{P}(\mathcal{E}_3) \leqslant \sum_{s} e^{-\Omega(2^s M)\delta''} \tag{10}$$

where $\delta'' > 0$. We note that an exception for the last stage can be made if we assume that the sync frame lengths are integer multiples of M.

Clearly, $\mathsf{P}(\mathcal{E}_2) \to 0$ and $\mathsf{P}(\mathcal{E}_3) \to 0$ as $M \to \infty$. With a suitable choice of M such as $M = \Omega(\epsilon \bar{N})$ (for some $\epsilon > 0$), we can ensure that the probability of the error events $\mathcal{E}_2$ and $\mathcal{E}_3$ tend to zero as long as $\bar{N} \to \infty$. Hence, in the rest of the discussion, we will focus only on an optimal choice of $\{N_n\}$ that minimizes (the upper bound of) $\mathsf{P}(\mathcal{E}_1)$ subject to a constraint on the average sync frame length.

3.2.4 An Optimization Framework

Consider the minimization of the upper bound of $\mathsf{P}(\mathcal{E}_1)$ in equation (8) subject to a constraint on the average sync frame length. Let us simplify the expression for the false alarm event to $\sum_{n \in \mathcal{C}} e^{-N_n^{\min}\alpha(Q)}$ as in [SJR17]. Then, the optimization framework for the general setup is given by

$$\begin{aligned} \text{minimize} \quad & \textstyle\sum_{n \in \mathcal{C}} e^{-N_n^{\min}\alpha(Q)} \\ \text{such that} \quad & \textstyle\sum_{n \in \mathcal{A}} a_n N_n \leqslant \bar{N} \end{aligned} \tag{11}$$

where $N_n^{min} = \min\{N_p : p \in \mathcal{A}_n\}$. We can now solve the optimization framework and also obtain the minimum $\bar{N}$ necessary for asymptotic optimal frame synchronization. The following sufficient scaling for the average sync frame length can be derived from the above formulation.

Lemma 3 *Consider a DMC $(\mathcal{X}, \mathcal{Y}, Q(\cdot|\cdot))$ with synchronization threshold $\alpha(Q)$. A sync frame s_ν of length N_ν symbols is transmitted at a random time ν, where ν is distributed as $\{a_n\}$. Let γ denote the random misalignment between the transmitter clock and the receiver clock, where γ is distributed as $\{b_n\}$. Then, $\mathsf{P}(\hat{v} \neq \nu) \to 0$ as $H_a + \log(B) \to \infty$ if $H_a + \log(B) < \bar{N}\alpha$ for some $\alpha < \alpha(Q)$.*

Proof:
Upper bound the expression for $\mathsf{P}(\mathcal{E}_1)$ in (11) using the inequality $e^{-N_n^{\min}\alpha(Q)} \leqslant \sum_{p\in\mathcal{A}_n} e^{-N_p\alpha(Q)}$. Then, we get,

$$\sum_{n\in\mathcal{C}} e^{-N_n^{\min}\alpha(Q)} \leqslant B \sum_{n\in\mathcal{A}} e^{-N_n\alpha(Q)}$$

Now, the result follows using arguments similar to those presented earlier.

3.2.5 Uniform Distribution for ν and γ

The following theorem generalizes the results in [CTW08] for uniform distribution when there is a random misalignment between the transmitter and receiver clocks.

Theorem 4 *Consider a DMC $(\mathcal{X}, \mathcal{Y}, Q(\cdot|\cdot))$ with synchronization threshold $\alpha(Q)$. A sync frame s_ν of length N_ν symbols is transmitted at a random time ν, where ν is distributed uniformly between 1 and A. Let the random misalignment γ between the transmitter clock and the receiver clock be distributed uniformly between 1 and B. Then, the probability of error in frame synchronization $\mathsf{P}(\hat{v} \neq \nu) \to 0$ as $\max(A, B) \to \infty$ iff $\max(A, B) < e^{\bar{N}\alpha}$ for some $\alpha < \alpha(Q)$.*

Proof:
Let $\{c_n\} = \{a_n\} * \{b_n\}$ and let C denote the cardinality of $\{c_n\}$. Without loss of generality, let us assume that $A \geqslant B$. Then, $A \leqslant C \leqslant 2A$ and $\log(A) \leqslant H_c \leqslant \log(2) + \log(A)$. Now, consider $N_n = N$ for all $n = 1, \cdots, A$. Then, $\bar{N} = N$.

From the necessary condition in Lemma 2-1), we know that $\mathsf{P}(\hat{v} \neq \nu) \to 0$ only if $\log(A) \leqslant H_c < N\alpha$ for some $\alpha < \alpha(Q)$.

Suppose that $\log(A) < N\alpha$ for some $\alpha < \alpha(Q)$. Then, for large N (as $A \to \infty$), there exists a $\delta > 0$ such that $\log(2) + \log(A) < N(\alpha + \delta)$ and $(\alpha + \delta) < \alpha(Q)$. Now, from the sufficient condition in Lemma 2-2), we have $\mathsf{P}(\hat{v} \neq \nu) \to 0$ as $C \leqslant 2A < e^{N(\alpha+\delta)}$ for some $(\alpha + \delta) < \alpha(Q)$.

4 Future Work

We are currently working on the application of asynchronism for *covert communication* and also extending the work on *channel asynchronism*.

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