Low Complexity Millimeter-Wave Beamformed Full-dimensional MIMO Channel Estimation Based on Atomic Norm Minimization

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THESIS CERTIFICATE

This is to certify that the thesis titled **Low Complexity Millimeter-Wave Beamformed Full-dimensional MIMO Channel Estimation Based on Atomic Norm Minimization**, submitted by **RATHNA K K**, to the Indian Institute of Technology, Madras, for the award of the degree of **Master of Technology**, is a bonafide record of the research work done by her under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

KEYWORDS: Millimeter-wave; Full-dimensional (FD) MIMO; Channel estimation; Atomic norm; Uniform planar array (UPA); Alternating direction method of multipliers (ADMM).

Millimeter-wave systems are being employed in the latest 5th generation (5G) mobile networks because of its high data rates in the range of multi-gigabit. The consequence arises here compared to microwave band is the lesser wavelength and increased path loss. To overcome this problem 5G adds additional features of beamforming and massive MIMO. Both the user equipment and the base station in the cellular communication system may use uniform or non uniform planar arrays. In this project we are only interested in uniform planar array case and studying the techniques for channel estimation of millimeter-wave full dimensional MIMO. As we are interested in FD MIMO, the angle of arrivals (AoA) and angle of departures (AoD) are to be estimated not only in azimuth plane, but in elevation plane also.

This work is about studying the paper published by Yingming Tsai and Wang (2017). There the original large scale 4D atomic norm minimization is reformulated as semi-definite program (SDP) consisting of two decoupled two-level Toeplitz matrices. SDP is then solved using alternating direction method of multipliers (ADMM) with each iteration step having closed-form computations.

TABLE OF CONTENTS

LIST OF FIGURES

ABBREVIATIONS

ADMM	Alternating Direction Method of Multipliers
ANM	Atomic Norm Minimization
AoA	Angle of Arrival
AoD	Angle of Departure
AST	Atomic norm Soft Thresholding
AWGN	Additive White Gaussian Noise
CS	Compressed Sensing
CSI	Channel State Information
dB	Decibel
DFT	Discrete Fourier Transform
DoA	Direction of Arrival
FD	Full-dimensional
MIMO	Multiple Input Multiple Output
mmWave	Millimeter Wave
MSE	Mean Squared Error
MUSIC	Multiple Signal Classification
NMSE	Normalized Mean Squared Error
Rx	Receiver
SDP	Semi-definite Programming
SNR	Signal to Noise Ratio
Tx	Transmitter
UPA	Uniform Planar Array

NOTATION

$\mathcal{N}(\mu, \sigma^2)$	Gaussian distribution with mean μ and variance σ^2
$\mathcal{CN}(\mu, \sigma^2)$	Circularly symmetric complex Gaussian distribution with mean μ and variance σ^2
R	Set of Real Numbers
C	Set of Complex Numbers
$\mathcal{A}$	Vectorized Atomic Set
$\mathcal{A}_M$	Matrix Atomic Set
$\mathcal{O}$	Big O notation
$E(.)$	Expectation Operator
$\text{vec}(.)$	Vectorization of Matrix
$\text{vec}^{-1}(.)$	Inverse Vectorization of Matrix
$\text{conv}(.)$	Convex hull
$T_d(.)$	d-level Block Toeplitz Matrix
$(.)^T$	Transpose Operator
$(.)^H$	Hermitian Operator
$(.)^*$	Conjugate Transpose (Hermitian Operator)
$(.)^{-1}$	Inverse Operator
$\|(.)\|_F$	Frobenius Norm
$\|(.)\|_2$	L2 Norm
$\|(.)\|_{\mathcal{A}}$	Vectorized Atomic Norm
$\|(.)\|_{\mathcal{A}_M}$	Matrix Atomic Norm
$s.t.$	Subject to
$\in$	Element of
$\otimes$	Kronecker Product

CHAPTER 1

Introduction

In the 5th generation (5G) mobile networks, services in the range of multi-gigabit are attained with the usage of millimeter-wave in the physical layer. The important features of mmWave spectrum are the massive bandwidth and smaller wavelengths compared to sub-6GHz bands. For higher data rates and spectrum efficiency, Massive MIMO and mmWave technologies should be considered jointly. The mmWave full-dimensional MIMO places uniform or non-uniform planar arrays at both the mobile station and user equipment, and in the elevation angle domain we expect extra degree of freedom.

For mmWave the key challenge is to overcome the higher path loss and to reduce shadowing losses. To resolve this issue, introduced beamformed FD MIMO, where receiver obtains only beamformed channel state information (CSI) not the full CSI. For mitigating this issue one approach is to estimate mmWave channel by exploiting the sparse scattering nature of mmWave channels. The earlier techniques for solving this problem is CS-based methods and subspace methods. In CS-based methods the angular domain is divided into finite number of grids. Main drawback of this method is grid resolution limit. And the subspace-based mmWave MIMO channel estimation method that uses MUSIC algorithm is also proposed. Even though its capable of identifying multiple paths with high resolution, it is sensitive to antenna position, gain, and phase errors.

Recently, instead of on-the grid CS-based methods, atomic norm minimization based off-the grid CS methods are extensively used for spectral estimation, AoA estimation and for too many other applications. In light of this mmWave FD MIMO channel estimation can be formulated as atomic norm minimization problem. Under specific conditions exact frequency localization is achieved via atomic norm minimization thereby avoids the basis mismatch problem that existed in grid-based CS techniques. Uniform planar array is considered for our mmWave beamformed FD-MIMO channel, and both AoA and AoD estimation is involved. Here the problem becomes four-dimensional (4D) atomic norm minimization, i.e., both elevation and azimuth angle estimation at both transmitter and receiver. The original 4D atomic norm minimization involves block Toeplitz matrices and computational complexity is higher. Our objective is to reduce the computational complexity by reducing the dimensionality of block Toeplitz matrices. The problem is reformulated as a semi-definite program (SDP) and further an efficient algorithm computes the results based on ADMM algorithm.

1.1 Thesis Outline

This thesis is organized as follows:

Chapter 2 explains in detail the prior arts in the area of thesis work. Major developments till date based on the atomic norm minimization approach is discussed in detail.

Chapter 3 specifically explains application of atomic norm approach in line spectral estimation which is actually a 1-dimensional problem of my project work.

Chapter 4 tries to introduce the system model taken for the experiments of millimeter-wave channel estimation. All the necessary notations and explanations are given in this chapter.

Chapter 5 explains the underlying theories behind original 4-dimensional atomic norm minimization formulation for channel estimation.

In *Chapter 6*, mmWave beamformed full-dimensional MIMO channel estimation via approximate 4-dimensional atomic norm minimization is explained in detail. In this work we are going to compare original 4D and approximate 4D atomic norm minimization methods for channel estimation. The ADMM algorithm for approximate method provided at the end of this this chapter involves closed form computations and it reduces the complexity.

Chapter 7 details the simulation setting and the obtained simulation results. We get some great advantages using the proposed method of channel estimation.

Chapter 8 summarizes the work done and provides some concluding remarks and avenues for future work.

CHAPTER 2

Literature Survey

In next generation of wireless communication (5G) the major underlying technologies are millimeter-waves, massive MIMO and beamforming. My project work is a combination of these main foundations of 5G, that is millimeter-wave beamformed full-dimensional MIMO channel estimation based on atomic norm minimization (ANM).

For a better understanding of ANM approach, I referred Gongguo Tang and Recht (2015), which implements this method in the area of line spectral estimation. Research work on frequency recovery or estimation from discrete samples of superimposed sinusoidal signals is advanced by atomic norm techniques which exploit signal sparsity, work directly on continuous frequencies, and completely resolve the grid mismatch problem of previous compressed sensing off grid methods. This paper proposed an optimal algorithm, Atomic Norm Soft Thresholding (AST) for denoising a mixture of sinusoids from noisy equispaced samples. AST implementation needs efficient computation of atomic norm and it can be reformulated as a semi-definite program (SDP). And the constraint for this SDP involves projecting a matrix onto positive definite cone. This matrix itself contains one level Toeplitz matrix. Which further solved using Alternating Direction Method of Multipliers (ADMM). In this paper the Atomic norm minimization in applied for a 1-dimensional problem of finding component frequencies of a mixture of sinusoids. Explicit updating rules of ADMM algorithm is given in S. Boyd and Eckstein (2011). And additional details are given in the paper, atomic norm denoising with applications to line spectral estimation by B. N. Bhaskar and Recht (2013).

Later, as an extension into 4-dimensional case I referred Yingming Tsai and Wang (2017), which is the main paper I relied upon for my thesis work. The paper proposes 4D atomic norm minimization for millimeter-wave beamformed full-dimensional MIMO channel estimation. The original 4D problem consists of 4 level block Toeplitz matrices in the positive semi-definite matrix constraint. As this adds too much complexity for channel estimation, the decoupling of 2D direction of arrival (DoA) into 1D level proposed in the paper Z. Tian and Wang (2017), is extended to reduce the complexity involved with 4D problem by decoupling it into 2D level.

CHAPTER 3

Prior Art Based on Atomic Norm Minimization

At first, we will try to understand the 1D atomic norm minimization (ANM) in the area of line spectral estimation. The ANM approach is then reformulated using semi-definite programming and for efficient implementation, ADMM algorithm is provided for this 1D problem. The details are given below for this model. Our goal is to estimate the unknown complex amplitudes $\{c_l\}_{l=1}^{k}$ and corresponding k frequencies $\{f_l\}_{l=1}^{k}$ in the torus $T = [0, 1]$ of the signal $x(t), t \in R$, given as a mixture of k complex sinusoids.

$$x(t) = \sum_{l=1}^{k} c_l exp(i2\pi f_l t) \ \ ; t \in R \tag{3.1}$$

The intention of line spectral estimation is to estimate the frequencies and amplitudes of the signal $x(t)$ from the finite, noisy samples $y \in C^n$ given by

$$y_j = x_j + w_j \tag{3.2}$$

for $-m \leq j \leq m$, where $w_j \sim \mathcal{CN}(0, \sigma^2)$ is i.i.d. circularly symmetric complex Gaussian noise.

We take $n = 2m + 1$ line spectral observations, $x = [x_{-m}, ..., x_m]^T \in C^n$. It can be expressed as a sparse combination of "atoms" $a(f)$ which correspond to observations due to single frequencies. Defining the vector $a(f) \in C^n$ for any $f \in T = [0, 1]$

$$a(f) = \begin{bmatrix} e^{i2\pi(-m)f} \\ : \\ 1 \\ : \\ e^{i2\pi(m)f} \end{bmatrix} \in C^n \tag{3.3}$$

Then signal can be rewrite as,

$$x = \sum_{l=1}^{k} c_l a(f_l) = \sum_{l=1}^{k} |c_l| \, a(f_l) e^{i\phi_l} \tag{3.4}$$

Where ϕ_l is the phase of l-th frequency component. Thus the signal can be thought of as a sparse non-negative combination of elements from the atomic set $\mathcal{A}$ given by

$$\mathcal{A} = \left\{a(f)e^{i\phi}, f \in [0,1], \phi \in [0, 2\pi]\right\} \tag{3.5}$$

For a general atomic set $\mathcal{A}$, the atomic norm of a vector is defined as the gauge function associated with the convex hull conv($\mathcal{A}$) of atoms:

$$\begin{aligned} \|x\|_{\mathcal{A}} &= \inf\{t > 0 : x \in t\,\text{conv}(\mathcal{A})\} \\ &= \inf\left\{\sum_a c_a : x = \sum_a c_a a,\ a \in \mathcal{A},\ c_a > 0\right\} \end{aligned} \tag{3.6}$$

Gongguo Tang and Recht (2015) in their paper analyzed the performance of the atomic norm soft thresholding (AST) based on this model:

$$\hat{x} = \underset{x}{\text{minimize}}\ \frac{1}{2}\|x - y\|_2^2 + \tau\|x\|_{\mathcal{A}} \tag{3.7}$$

Here the regularization parameter is, $\tau = \eta\sigma\sqrt{n\,log(n)}$ for some $\eta \in (1, \infty)$. Efficient implementation of numerical procedures to solve the AST require efficient computation of the atomic norm. The atomic norm admits an equivalent semi-definite reformulation:

$$\|x\|_{\mathcal{A}} = \inf\left\{\begin{array}{c} \frac{1}{2}(u_1 + t) \\ \\ s.t. \begin{bmatrix} \text{Toep}(u) & x \\ x^* & t \end{bmatrix} \succeq 0 \end{array}\right\} \tag{3.8}$$

where $\text{Toep}(u)$ denotes a Hermitian Toeplitz matrix with u as its first row and u_1 is the first component of u. The AST problem is then rewritten as,

$$\hat{x} = \underset{t,u,x}{\text{minimize}} \left\{\begin{array}{c} \frac{1}{2}\|x - y\|_2^2 + \frac{\tau}{2}(t + u_1) \\ \\ s.t. \begin{bmatrix} T(u) & x \\ x^* & t \end{bmatrix} \succeq 0 \end{array}\right\} \tag{3.9}$$

Algorithm 1 Update steps of ADMM algorithm for the implementation of AST

1: $t^{l+1} = Z^l_{n+1,n+1} + (\Lambda^l_{n+1,n+1} - \frac{\tau}{2})\,/\rho$

2: $x^{l+1} = \frac{1}{2\rho+1}(y + 2\rho z^l_1 + 2\lambda^l_1)$

3: $u^{l+1} = W\left(T^*(Z^l_0 + \Lambda^l_0/\rho) - \frac{\tau}{2\rho}e_1\right)$

▷ ρ = 1, which is the penalty parameter in augmented Lagrangian

▷ W is the diagonal matrix with entries, $W_{ii} = \begin{cases} \frac{1}{n} & i = 1 \\ \frac{1}{2(n-i+1)} & i > 1 \end{cases}$

▷ Introducing the partitions, $Z^l = \begin{bmatrix} Z^l_0 & z^l_1 \\ z^{l^*}_1 & Z^l_{n+1,n+1} \end{bmatrix}$ and $\Lambda^l = \begin{bmatrix} \Lambda^l_0 & \lambda^l_1 \\ \lambda^{l^*}_1 & \Lambda^l_{n+1,n+1} \end{bmatrix}$

4: $Z^{l+1} := \underset{Z \succeq 0}{argmin} \left\| Z - \begin{bmatrix} T(u^{l+1}) & x^{l+1} \\ x^{l+1^*} & t^{l+1} \end{bmatrix} + \Lambda^l/\rho \right\|^2_F$

5: $\Lambda^{l+1} := \Lambda^l + \rho\left(Z^{l+1} - \begin{bmatrix} T(u^{l+1}) & x^{l+1} \\ x^{l+1^*} & t^{l+1} \end{bmatrix}\right)$

6: **return** x^{l+1}

CHAPTER 4

System Model

For the system model we are considering mmWave FD-MIMO system with *M* receive antennas and *N* transmit antennas that supports elevation and azimuth beamforming simultaneously. The resulting channel matrix can be expressed in terms of transmit and receive array response as:

$$\mathrm{H} = \mathrm{B}\Sigma\mathrm{A}^{\mathrm{H}} = \sum_{l=1}^{L} \sigma_l b(f_l) a(g_l)^H \tag{4.1}$$

The matrix $\Sigma = \mathrm{diag}(\sigma) = \mathrm{diag}([\sigma_1 \sigma_2 ... \sigma_L]^T)$ is a diagonal matrix with each $\sigma_l \in C$ denoting the *l*-th multipath gain; *L* denotes the number of paths; the matrices $\mathrm{B} = [b(f_1)...b(f_L)]$ and $\mathrm{A} = [a(g_1)...a(g_L)]$ denote the receive and transmit array steering responses. For a linear array with adjacent antenna element separation of half wavelength, the array response takes the form of a uniformly sampled complex sinusoid with frequency $x \in [-\frac{1}{2}, \frac{1}{2})$:

$$c_n(x) = \frac{1}{\sqrt{n}} \left[1 \;\; e^{j2\pi x} .... e^{j2\pi(n-1))x}\right]^T \in C^{n\times 1} \tag{4.2}$$

The transmitter (Tx) and receiver (Rx) array responses, with the assumption of both equipped with uniformly spaced planar antenna arrays (UPA), each with half-wavelength antenna element separations along the elevation and azimuth axis, can be expressed as:

$$a(g_l) = c_{N_1}(g_{l,1}) \otimes c_{N_2}(g_{l,2}) \tag{4.3}$$

$$b(f_l) = c_{M_1}(f_{l,1}) \otimes c_{M_2}(f_{l,2}) \tag{4.4}$$

with

$$g_l = \left\{ g_{l,1} = \frac{1}{2}\sin(\vartheta_l)\cos(\varphi_l), \;\; g_{l,2} = \frac{1}{2}\cos(\vartheta_l) \right\} \tag{4.5}$$

$$f_l = \left\{ f_{l,1} = \frac{1}{2}\sin(\theta_l)\cos(\phi_l), \;\; f_{l,2} = \frac{1}{2}\cos(\theta_l) \right\} \tag{4.6}$$

where ϑ_l, φ_l denote elevation and azimuth angles of the angle of departure (AoD) of the *l*-th path, respectively; and θ_l, ϕ_l denote elevation and azimuth angles of the angle of arrival (AoA), respectively. Here, N_1, N_2 denote the numbers of azimuth and elevation transmit antennas, respectively, and the total number of transmit antennas is $N = N_1 N_2$. Similarly, M_1, M_2 denote the numbers of azimuth and elevation receive antennas, respectively, and the total number of receive antennas is $M = M_1 M_2$. For the UPA configuration, it can resolve the AoA and AoD in $360°$ range, thereby $\vartheta_l, \varphi_l, \theta_l, \phi_l \in [-\pi, \pi]$ and $g_{l,1} = \frac{1}{2}\sin(\vartheta_l)\cos(\varphi_l) \in [-\frac{1}{2}, \frac{1}{2}),\ g_{l,2} = \frac{1}{2}\cos(\vartheta_l) \in [-\frac{1}{2}, \frac{1}{2}),\ f_{l,1} = \frac{1}{2}\sin(\theta_l)\cos(\phi_l) \in [-\frac{1}{2}, \frac{1}{2}),\ f_{l,2} = \frac{1}{2}\cos(\theta_l) \in [-\frac{1}{2}, \frac{1}{2})$.

To estimate the channel matrix, the transmitter transmits *P* distinct beams during *P* successive time slots. i.e., in the *p*-th time slot, the beamforming vector $\mathbf{p}_p \in C^{N\times 1}$ is selected from a set of unitary vectors in the form of Kronecker-product-based codebook, e.g., $\mathbf{p}_p = \mathbf{p}_{p,1} \otimes \mathbf{p}_{p,1}$ where $\mathbf{p}_{p,1} \in C^{N_1}$ and $\mathbf{p}_{p,2} \in C^{N_2}$ are selected from two DFT codebooks of dimensions N_1 and N_2, respectively as in D. Yang and Hanzo (2010). The *p*-th received signal vector can be expressed as

$$\mathrm{y}_p = \mathrm{H}\mathbf{p}_p s_p + \mathrm{w}_p \tag{4.7}$$

where $\mathrm{w}_p \sim \mathcal{CN}(\mathbf{0}, \sigma_w^2 \mathbf{I}_M)$ is the additive white Gaussian noise (AWGN) with $\mathbf{I}_M$ denoting the $M \times M$ identity matrix, and s_p denotes the pilot symbol in the *p*-th time slot. The receiver collects $\mathrm{y}_p \in C^{M\times 1}$ for *p* = 1...*P* and concatenates them to obtain the signal matrix

$$\mathrm{Y} = [\mathrm{y}_1\ \ \mathrm{y}_2 ... \mathrm{y}_P] = \mathrm{HPS} + \mathrm{W} = \mathrm{B}\Sigma\mathrm{A}^H\mathrm{PS} + \mathrm{W} \tag{4.8}$$

where $\mathrm{P} = [\mathbf{p}_1\ \mathbf{p}_2 .. \mathbf{p}_P] \in C^{N\times P}, \mathrm{W} = [\mathrm{w}_1\ \mathrm{w}_2 .. \mathrm{w}_P] \in C^{M\times P}$ and $\mathrm{S} = \mathrm{diag}([s_1\ s_2 .. s_P]) \in C^{P\times P}$. For simplicity, we assume that $\mathrm{S} = \sqrt{P_t}\mathrm{I}_P$, where P_t is the power of the pilot symbol. Then we have

$$\mathrm{Y} = \sqrt{P_t}\mathrm{HP} + \mathrm{W} = \sqrt{P_t}\mathrm{B}\Sigma\mathrm{A}^H\mathrm{P} + \mathrm{W} \tag{4.9}$$

Our goal is to estimate the channel matrix $\mathrm{H} \in C^{M\times N}$ from the measurements $\mathrm{Y} \in C^{M\times P}$. Note that the number of pilots is usually smaller than the number of transmit antennas, i.e., $P < N$. Hence, we need to exploit the sparsity of H for its estimation, which will be discussed in the next section.

CHAPTER 5

Channel Estimation via Atomic Norm Minimization

The performance of the mmWave channel estimators based on on-grid methods such as Compressed Sensing (CS) can be degraded due to grid mismatch. In this section, a new mmWave channel estimator based on an off-grid CS method is proposed, i.e., the atomic norm minimization method.

5.1 Background on Multi-dimensional Atomic Norm

First we briefly introduce the concept of multi-dimensional atomic norm in Z. Yang and Stoica (2016). A *d*-dimensional (*d*-dim) atom is defined as

$$q_d(x_1, ..., x_d) = c_{n_1}(x_1) \otimes .... \otimes c_{n_d}(x_d) \tag{5.1}$$

where n_i is the length of the normalized vector $c_{n_i}(x_i)$ defined in (4.2) and $x_i \in [-\frac{1}{2}, \frac{1}{2})$ for *i* = 1,2,..., *d*. The *d*-dim atomic set is then given by

$$\mathcal{A} = \left\{ \; q_d(x_1, ..., x_d) : x_i \in [-\frac{1}{2}, \frac{1}{2}), \;\; i = 1, ..., d \right\} \tag{5.2}$$

For any vector t_d of the form $t_d = \sum_l \alpha_l \; q_d(x_{l,1}, x_{l,2}, ..., x_{l,d})$, its *d*-dim atomic norm with respect to $\mathcal{A}$ is defined as

$$\begin{aligned} \|t_d\|_{\mathcal{A}} &= \inf \{t : t_d \in t \; \mathrm{conv}(\mathcal{A})\} \\ &= \inf_{\substack{x_{l,1}, x_{l,2}, ..., x_{l,d} \in [-\frac{1}{2}, \frac{1}{2}), \\ \alpha_l \in C}} \left\{ \sum_l |\alpha_l| : t_d = \sum_l \alpha_l \; q_d(x_{l,1}, x_{l,2}, ..., x_{l,d}) \right\} \end{aligned} \tag{5.3}$$

where conv($\mathcal{A}$) is the convex hull of $\mathcal{A}$. The d-dim atomic norm of t_d has following equivalent form:

$$\|t_d\|_{\mathcal{A}} = \inf_{\substack{U_d \in C^{(2n_d-1)\times(2n_{d-1}-1)\times\ldots\times(2n_1-1)} \\ t\in R}} \left\{ \begin{array}{l} \frac{1}{2n_1 n_2 \ldots n_d}\mathrm{Tr}(T_d(U_d)) + \frac{1}{2}t \\ \\ s.t. \begin{bmatrix} T_d(U_d) & t_d \\ t_d^H & t \end{bmatrix} \succeq 0 \end{array} \right\} \tag{5.4}$$

where $\mathrm{Tr}(.)$ is the trace of the input matrix, $U_d \in C^{(2n_d-1)\times(2n_{d-1}-1)\times\ldots\times(2n_1-1)}$ is d-way tensor and $T_d(U_d)$ is a d-level block Toeplitz, which is defined recursively as follows. Denote $\mathbf{n}_d = (n_d, n_{d-1}, ..., n_1)$ and $U_{d-1}(i) = U_d(i, :, :, .., :)$ for $i = -n_d + 1, -n_d + 2, ..., n_d - 1$. For $d = 1, \mathbf{n}_1 = (n_1)$ and $T_1(u_1) = \mathrm{Toep}(u_1)$ with $u_1 \in C^{(2n_1-1)}$ i.e.,

$$T_1(u_1) = \mathrm{Toep}(u_1) = \begin{pmatrix} u_1(0) & u_1(1) & \cdots & u_1(n_1-1) \\ u_1(-1)) & u_1(0) & \cdots & u_1(n_1-2) \\ \vdots & \vdots & \ddots & \vdots \\ u_1(1-n_1) & u_1(2-n_1) & \cdots & u_1(0) \end{pmatrix} \tag{5.5}$$

For $d \geq 2$, we have

$$T_d(U_d) = \begin{pmatrix} T_{d-1}(U_{d-1}(0)) & T_{d-1}(U_{d-1}(1)) & \cdots & T_{d-1}(U_{d-1}(n_d-1)) \\ T_{d-1}(U_{d-1}(-1)) & T_{d-1}(U_{d-1}(0)) & \cdots & T_{d-1}(U_{d-1}(n_d-2)) \\ \vdots & \vdots & \ddots & \vdots \\ T_{d-1}(U_{d-1}(1-n_d)) & T_{d-1}(U_{d-1}(2-n_d)) & \cdots & T_{d-1}(U_{d-1}(0)) \end{pmatrix} \tag{5.6}$$

5.2 Atomic Norm Minimization Formulation

In this subsection, we formulate the atomic norm minimization problem for channel estimation. First, we vectorize the mmWave FD-MIMO channel matrix H in (4.1) as,

$$\begin{aligned} \mathrm{h} &= \mathrm{vec}(\mathrm{H}) \\ &= \sum_{l=1}^{L} \sigma_l c_{N_1}^*(g_{l,1}) \otimes c_{N_2}^*(g_{l,2}) \otimes c_{M_1}(f_{l,1}) \otimes c_{M_2}(f_{l,2}) \end{aligned} \tag{5.7}$$

Comparing (5.3) and (5.7), for the mmWave FD-MIMO channel with uniform planar array (UPA) configuration, the atom has the form of,

$$q_4(g,f) = c^*_{N_1}(g_1) \otimes c^*_{N_2}(g_2) \otimes c_{M_1}(f_1) \otimes c_{M_2}(f_2) \tag{5.8}$$

and the set of atoms is defined as the collection of all normalized 4D complex sinusoids:

$$\mathcal{A} = \left\{ q_4(g,f) : f \in [-\frac{1}{2}, \frac{1}{2}) \times [-\frac{1}{2}, \frac{1}{2}), g \in [-\frac{1}{2}, \frac{1}{2}) \times [-\frac{1}{2}, \frac{1}{2}) \right\} \tag{5.9}$$

The 4D atomic norm for any h defined in (5.7) can be written as:

$$\|\mathrm{h}\|_{\mathcal{A}} = \inf_{\substack{f_l \in [-\frac{1}{2},\frac{1}{2}) \times [-\frac{1}{2},\frac{1}{2}), \\ g_l \in [-\frac{1}{2},\frac{1}{2}) \times [-\frac{1}{2},\frac{1}{2}), \\ \sigma_l \in C}} \left\{ \sum_l |\sigma_l| : \mathrm{h} = \sum_l \sigma_l q_4(g_l, f_l) \right\} \tag{5.10}$$

The atomic norm can enforce sparsity in the atom set $\mathcal{A}$. On this basis, an optimization problem will be formulated for the estimation of the path frequencies $\{f_l, g_l\}$. For the convenience of calculation, we will use the equivalent form of the atomic norm given by (5.4), i.e.,

$$\|\mathrm{h}\|_{\mathcal{A}} = \inf_{\substack{U_4 \in C^{(2N_1-1)\times(2N_2-1)\times(2M_1-1)\times(2M_2-1)} \\ t \in R}} \left\{ \begin{array}{l} \frac{1}{2MN} \mathrm{Tr}(T_4(U_4)) + \frac{1}{2} t \\ \\ s.t. \begin{bmatrix} T_4(U_4) & \mathrm{h} \\ \mathrm{h}^H & t \end{bmatrix} \succeq 0 \end{array} \right\} \tag{5.11}$$

where $T_4(U_4)$ is a 4-level Toeplitz matrix defined in (5.6). Define the minimum frequency separations as

$$\Delta_{min,f_i} = \min_{l \neq l'} \ \min \left\{ |f_{l,i} - f_{l',i}|, 1 - |f_{l,i} - f_{l',i}| \right\} \tag{5.12}$$

$$\Delta_{min,g_i} = \min_{l \neq l'} \ \min \left\{ |g_{l,i} - g_{l',i}|, 1 - |g_{l,i} - g_{l',i}| \right\} \tag{5.13}$$

for $i = 1, 2$. To show the connection between the atomic norm and the channel matrix, we states the Theorem 1 in Yingming Tsai and Wang (2017) via extending Theorem 1.2 in Candes and Fernandez-Granda (2014) for 1D atomic norm to 4D atomic norm.

If the path component frequencies are sufficiently separated, i.e.,

$$\Delta_{min,f_i} \geq \frac{1}{\lfloor (M_i - 1)/4 \rfloor} \tag{5.14}$$

$$\Delta_{min,g_i} \geq \frac{1}{\lfloor (N_i - 1)/4 \rfloor} \tag{5.15}$$

for $i = 1, 2$*, then we have* $\|\mathrm{h}\|_{\mathcal{A}} = \sum_l |\sigma_l|$*, so the component atoms of* h *can be uniquely located via computing its atomic norm.*

To estimate the mmWave FD-MIMO channel H in (4.1) based on the signal Y in (4.9) , we then formulate the following optimization problem:

$$\hat{\mathrm{h}} = \min_{\mathrm{h} \in C^{MN}} \ \mu \|\mathrm{h}\|_A + \frac{1}{2} \left\| \mathrm{y} - \sqrt{P_t}(\mathrm{P}^T \otimes \mathrm{I}_M)\mathrm{h} \right\|_2^2 \tag{5.16}$$

where y = vec(Y) and $\mu \propto \sigma_w \sqrt{MNlog(MN)}$ is a weight factor. Using (5.11), (5.16) can be equivalently formulated as a semi-definite program (SDP):

$$\hat{\mathrm{h}} = \min_{\substack{U_4 \in C^{(2N_1-1)\times(2N_2-1)\times(2M_1-1)\times(2M_2-1)}, \\ \mathrm{h} \in C^{MN}, t \in R}} \left\{ \begin{array}{c} \frac{\mu}{2MN} \mathrm{Tr}(T_4(U_4)) + \frac{\mu}{2} t \\ \\ + \frac{1}{2} \left\| \mathrm{y} - \sqrt{P_t}(\mathrm{P}^T \otimes \mathrm{I}_M)\mathrm{h} \right\|_2^2 \\ \\ s.t. \begin{bmatrix} T_4(U_4) & \mathrm{h} \\ \mathrm{h}^H & t \end{bmatrix} \succeq 0 \end{array} \right\} \tag{5.17}$$

The above problem is convex, and can be solved by using a standard convex solver. Suppose the solution to (5.17) is $\hat{\mathrm{h}}$. Then the estimated channel matrix is given by $\hat{\mathrm{H}} = \mathrm{vec}^{-1}(\hat{\mathrm{h}})$, where $\mathrm{vec}^{-1}(.)$ is the inverse operation of vec(.). In addition the positive semi-definite matrix in the constraint is of size $(MN+1) \times (MN+1)$ which is of high computational complexity and needs larger memory requirements. In the next section we will see an approximation to this method, which resolves this issue.

CHAPTER 6

Efficient Algorithm for Channel Estimation Under UPA

6.1 An Approximation to 4D Atomic Norm Minimization

Next proposed an approximation to the 4D atomic norm to reduce the computational complexity. In the paper by Z. Tian and Wang (2017), the authors explore the approximation of 2D atomic norm to improve the efficiency. Here, tries to extend the results from 2D atomic norm to 4D atomic norm case. The proposed approximation is calculated with input H. The channel matrix, H is the sum of $\sigma_l b(f_l)a(g_l)^H$, in which both $a(g_l)$ and $b(f_l)$ are Fourier bases. Different from the vectorized atomic norm, we introduce the matrix atom

$$\mathrm{Q}(f,g) = b(f)a(g)^H \tag{6.1}$$

and the matrix atom set,

$$\mathcal{A}_M = \left\{ \mathrm{Q}(f,g) : f \in [-\frac{1}{2},\frac{1}{2}) \times [-\frac{1}{2},\frac{1}{2}), g \in [-\frac{1}{2},\frac{1}{2}) \times [-\frac{1}{2},\frac{1}{2}) \right\} \tag{6.2}$$

The matrix atomic norm is then given by,

$$\|\mathrm{H}\|_{\mathcal{A}_M} = \inf_{\substack{f_l \in [-\frac{1}{2},\frac{1}{2})\times[-\frac{1}{2},\frac{1}{2}), \\ g_l \in [-\frac{1}{2},\frac{1}{2})\times[-\frac{1}{2},\frac{1}{2}), \\ \sigma_l \in C}} \left\{ \sum_l |\sigma_l| : \mathrm{H} = \sum_l \sigma_l \mathrm{Q}(f_l, g_l) \right\} \tag{6.3}$$

The matrix atom set is composed of rank-one matrices, and hence it amounts to atomic norm of low rank matrices. Since the operator vec(.) is a one-to-one mapping and the mapping $\mathcal{A}_M \to \mathcal{A}$ is also one-to-one, it is straightforward to conclude that $\|\mathrm{H}\|_{\mathcal{A}_M} = \|\mathrm{h}\|_{\mathcal{A}}$. Hence, if the component frequencies satisfy the sufficient separation condition given by (5.14) and (5.15), we have $\|\mathrm{H}\|_{\mathcal{A}_M} = \sum_l |\sigma_l|$ by Theorem 1.

Finding the harmonic components via atomic norm is an infinite programming problem over all feasible f and g, which is difficult. For better efficiency, we use SDP(H) in the Lemma 1 of Yingming Tsai and Wang (2017) to approximate $\|\mathrm{H}\|_{\mathcal{A}_M}$, which is stated as

For H *given by (4.1), we have* $\|\mathrm{H}\|_{\mathcal{A}_M} \geq \mathrm{SDP}(\mathrm{H})$ *, where*

$$\mathrm{SDP}(\mathrm{H}) = \inf_{\substack{U_2 \in C^{(2M_1-1)\times(2M_2-1)},\\ V_2 \in C^{(2N_1-1)\times(2N_2-1)}}} \left\{ \begin{array}{c} \frac{1}{2M}\mathrm{Tr}(T_2(U_2)) + \frac{1}{2N}\mathrm{Tr}(T_2(V_2)) \\ \\ s.t. \begin{bmatrix} T_2(U_2) & \mathrm{H} \\ \mathrm{H}^H & T_2(V_2) \end{bmatrix} \succeq 0 \end{array} \right\} \tag{6.4}$$

with $T_2(U_2)$ *and* $T_2(V_2)$ *being 2-level Toeplitz matrices defined in (5.6).*

The above lemma shows that SDP(H) is a lower bound of the matrix atomic norm. Moreover, the following Lemma 2 of Yingming Tsai and Wang (2017) states that if the component frequencies are sufficiently separated, then SDP(H) is equivalent to $\|\mathrm{H}\|_{\mathcal{A}_M}$.

If (5.14)-(5.15) hold, then $\|\mathrm{H}\|_{\mathcal{A}_M} = \mathrm{SDP}(\mathrm{H})$.

Therefore, instead of solving the original 4D atomic norm minimization in (5.17), we can solve the following SDP

$$\hat{\mathrm{H}} = \min_{\substack{\mathrm{H} \in C^{M\times N},\\ U_2 \in C^{(2M_1-1)\times(2M_2-1)},\\ V_2 \in C^{(2N_1-1)\times(2N_2-1)}}} \left\{ \begin{array}{c} \frac{\mu}{2M}\mathrm{Tr}(T_2(U_2)) + \frac{\mu}{2N}\mathrm{Tr}(T_2(V_2)) \\ \\ +\frac{1}{2}\left\|\sqrt{P_t}\mathrm{HP} - \mathrm{Y}\right\|_F^2 \\ \\ s.t. \begin{bmatrix} T_2(U_2) & \mathrm{H} \\ \mathrm{H}^H & T_2(V_2) \end{bmatrix} \succeq 0 \end{array} \right\} \tag{6.5}$$

The size of the positive semi-definite matrix in the constraint is $(M+N)\times(M+N)$, resulting in considerably lower computational complexity and memory requirement than (5.17).

Algorithm 2 Update steps of ADMM for Approximate 4D Atomic Norm Minimization

$\mathrm{H}^{l+1} = (\mathrm{Y}\mathrm{P}^H + 2\rho \mathrm{M}_2^l + 2\Upsilon_2^l)(\mathrm{P}\mathrm{P}^H + 2\rho \mathrm{I}_N)^{-1}$

$\mathrm{U}_2^{l+1} = \mathrm{T}_2^*(\mathrm{M}_0^l + \Upsilon_0^l/\rho) - \frac{\gamma}{2M\rho}\mathrm{e}_1$

$\mathrm{V}_2^{l+1} = \mathrm{T}_2^*(\mathrm{M}_1^l + \Upsilon_1^l/\rho) - \frac{\gamma}{2N\rho}\mathrm{e}_1$

▷ Introduced partitions are, $\mathrm{M}^l = \begin{bmatrix} \mathrm{M}_0^l & \mathrm{M}_2^l \\ (\mathrm{M}_2^l)^H & \mathrm{M}_1^l \end{bmatrix}$ and $\Upsilon^l = \begin{bmatrix} \Upsilon_0^l & \Upsilon_2^l \\ (\Upsilon_2^l)^H & \Upsilon_1^l \end{bmatrix}$

$\mathrm{M}^{l+1} = \underset{\mathrm{M} \in C^{(M+N)\times(M+N)} \succeq 0}{\operatorname{argmin}} \left\| \mathrm{M} - \tilde{\mathrm{M}}^{l+1} \right\|_F^2,$

▷ where $\tilde{\mathrm{M}}^{l+1} = \begin{bmatrix} \mathrm{T}_2(\mathrm{U}_2^{l+1}) & \mathrm{H}^{l+1} \\ (\mathrm{H}^{l+1})^H & \mathrm{T}_2(\mathrm{V}_2^{l+1}) \end{bmatrix} - \Upsilon^{l+1}/\rho$

$\Upsilon^{l+1} = \Upsilon^l + \rho \left(\mathrm{M}^{l+1} - \begin{bmatrix} \mathrm{T}_2(\mathrm{U}_2^{l+1}) & \mathrm{H}^{l+1} \\ (\mathrm{H}^{l+1})^H & \mathrm{T}_2(\mathrm{V}_2^{l+1}) \end{bmatrix} \right)$

return H^{l+1}

CHAPTER 7

Simulation Results

7.1 Simulation Setup

1D Atomic Norm Minimization for Line Spectral Estimation

Simulation results in Gongguo Tang and Recht (2015) is tried to plot and the simulation setup for AST and MUSIC algorithms are included below.

For each experiment, we generated k normalized frequencies $f_1, ..., f_k$ uniformly randomly chosen from [0, 1] such that every pair of frequencies are separated by at least $1/2n$. The signal $x \in C^n$ is generated according to (I.1) in reference paper with k random amplitudes independently chosen from $\chi^2(1)$ distribution (squared Gaussian). All of the sinusoids were then assigned a random phase (equivalent to multiplying the magnitude by random unit length complex number). The observation y is produced by adding complex white Gaussian noise w such that the input signal to noise ratio (SNR) is $-10, 5, 0, 5, 10, 15$ or 20 dB. We compared the average values of different metrics of the various algorithms in 300 random trials for the number of observations $(n = 256)$, and number of frequencies $(k = n/4, n/8, n/16)$.

AST needs an estimate of the noise variance σ^2 to set the regularization parameter according to (III.12) in paper. In the experiments, we do not provide AST with the true noise variance. Instead, we construct an estimate for σ with the following heuristic. We formed the empirical autocorrelation matrix using the MATLAB routine $corrmtx$ using a prediction order $n/3$ and averaging the lower 25% of the eigenvalues. We then use this estimate in equation (III.12) to determine the regularization parameter. We solved the SDP formulation of AST (III.15) using the Alternating Direction Method of Multipliers (ADMM). The stopping criteria described in S. Boyd and Eckstein (2011) was adopted and ρ = 2 for all experiments. We used the dual solution $\hat{z} = y - \hat{x}$ to determine the support of the optimal solution $\hat{x}$. Once the frequencies $\hat{f}_l$ are extracted, we ran the least squares problem $\underset{\alpha}{\text{minimize}} \|U\alpha - y\|^2$ where $U_{jl} = exp(i2\pi j \hat{f}_l)$ to obtain debiased estimates of the amplitudes.

Implemented the MUSIC method using the MATLAB routine *rootmusic*. MUSIC algorithm needs an estimate of the number of sinusoids. Rather than implementing a heuristic to estimate k, we fed the true k to our solvers. This provides a significant advantage to this algorithm. AST is not provided with the true value of k, and the noise variance σ^2 in regularization parameter is estimated from y.

Let $\hat{x}, \{\hat{c}_l\}, \left\{\hat{f}_l\right\}$ denote the signal, the amplitudes and the frequencies estimated by any of the algorithms - AST or MUSIC. As given in the paper the following error metrics are used to measure the denoising and frequency localization performance of various algorithms:

1. The mean squared-error,

$$\text{MSE} = \|\hat{x} - x\|_2^2 \tag{7.1}$$

2. Sum of absolute value of amplitudes in the far region F,

$$m_1 = \sum_{l:\hat{f}_l \in F} |\hat{c}_l| \tag{7.2}$$

3. The weighted frequency localization error,

$$m_2 = \sum_j \sum_{l:\hat{f}_l \in N_j} |\hat{c}_l| \left\{ \min_{f_i \in T} d\left(f_i, \hat{f}_l\right) \right\}^2 \tag{7.3}$$

4. Error in approximation of amplitudes in the near region,

$$m_3 = \max_j \left| c_j - \sum_{l:\hat{f}_l \in N_j} \hat{c}_l \right| \tag{7.4}$$

5. Let $\mathcal{P}$ be the set of experiments and let $e_s(p)$ be the value of an error metric e of experiment $p \in \mathcal{P}$ using the algorithm s. Performance profile $P_s(\beta)$ specifies the fraction of experiments where the ratio of the performance of the algorithm s to the minimum error e across all algorithms for the given experiment is less than β,

$$P_s(\beta) = \frac{\#\left\{p \in \mathcal{P} : \; e_s(p) \le \beta \; \min_s \; e_s(p)\right\}}{\#(\mathcal{P})} \tag{7.5}$$

4D Atomic Norm Minimization for Millimeter- Wave Beamformed FD-MIMO Channel Estimation

In this simulation setting, we evaluate the performance of the proposed method in the paper, i.e., approximate 4D channel estimator for mmWave FD MIMO links with UPA. We compare the channel estimation performance of the proposed algorithm with original 4D problem. The simulation parameters are set as follows. The numbers of transmit and receive antenna are N = 16 and M = 16, respectively. For UPA, we set N_1 = 4, N_2 = 4, M_1 = 4 and M_2 = 4. For the UPA case, the DFT codebooks at the transmitter for elevation and azimuth are given by,

$$\mathbf{P}_1 = [c_{N_1}(\psi_{1,0})\ \ c_{N_1}(\psi_{1,1})...c_{N_1}(\psi_{1,P_1-1})] \in C^{N_1 \times P_1} \tag{7.6}$$

$$\mathbf{P}_2 = [c_{N_2}(\psi_{2,0})\ \ c_{N_2}(\psi_{2,1})...c_{N_2}(\psi_{2,P_2-1})] \in C^{N_2 \times P_2} \tag{7.7}$$

where P_1 and P_2 are the sizes of azimuth and elevation codebooks, respectively. The DFT angles are $\psi_{1,i} = \frac{i}{P_1}$ for $i = 0, ..., P_1 - 1$ and $\psi_{2,i} = \frac{i}{P_2}$ for $i = 0, ..., P_2 - 1$. We take the Kronecker product of $\mathbf{P}_1$ and $\mathbf{P}_2$ to form the product codebook $\mathbf{P} = \mathbf{P}_1 \otimes \mathbf{P}_2$ with size $P = P_1P_2$. Each beamforming vector has a unit norm, i.e., $\|\mathbf{P}_p\| = 1$ for $p = 1, ..., P$ and rank $(\mathbf{P}) = P$.

The weight factor in (5.15) and (6.2) is set as $\mu \propto \sigma_w \sqrt{MNlog(MN)}$. The weight for the augmented Lagrangian is set as ρ = 0.05 as in the paper. g_l and f_l for each path are assumed to uniformly take values in $[-\frac{1}{2}, \frac{1}{2}) \times [-\frac{1}{2}, \frac{1}{2})$. The number of paths $L = 3$. The signal power is controlled by the signal-to-noise ratio (SNR) which is defined as SNR = $\frac{P_t}{\sigma_w^2}$ with σ_w^2= 1. In the simulation, we use the CVX package explained in Grant and Boyd (2014) to compute the original 4D atomic norm-based estimator.

The following metrics are considered for the performance analysis:

1. The normalized mean squared error,

$$\text{NMSE} = E\left\{\frac{\left\|\hat{\text{H}} - \text{H}\right\|_F^2}{\|\text{H}\|_F^2}\right\} \tag{7.8}$$

2. Running time

7.2 Simulation Results

1D Atomic Norm Minimization for Line Spectral Estimation

I have plotted all the frequency localization error metrics and performance profile plots as in the simulation setting part of 1D atomic norm minimization method for line spectral estimation. In Figure 7.1 we get a visual indication of superior performance of AST over subspace method MUSIC, which is obviously due to the off-grid algorithm of AST. From performance profile plots for all the metrics, which are MSE, m_1, m_2 and m_3, AST is proved to give better performance especially substantial margin for metrics m_1 and m_2. For the simulation we have fixed number of observations $n = 256$, number of frequencies $k = 16$ and signal to noise ratio (SNR) as -10 dB. The experiments are run over 400 realizations.

For $n = 256$ samples Figure 7.2-7.4 shows the average values for error metrics m_1, m_2 and m_3 respectively with increasing SNR. The top, middle and bottom rows of the plots corresponds to subset of experiments with number of frequencies $k = 16, 32$ and 64. For obtaining smoother curves the number of simulations are taken as 300. It can be inferred from these plots that AST always gives low frequency localization errors with large margin compared to MUSIC algorithm even for low SNRs.

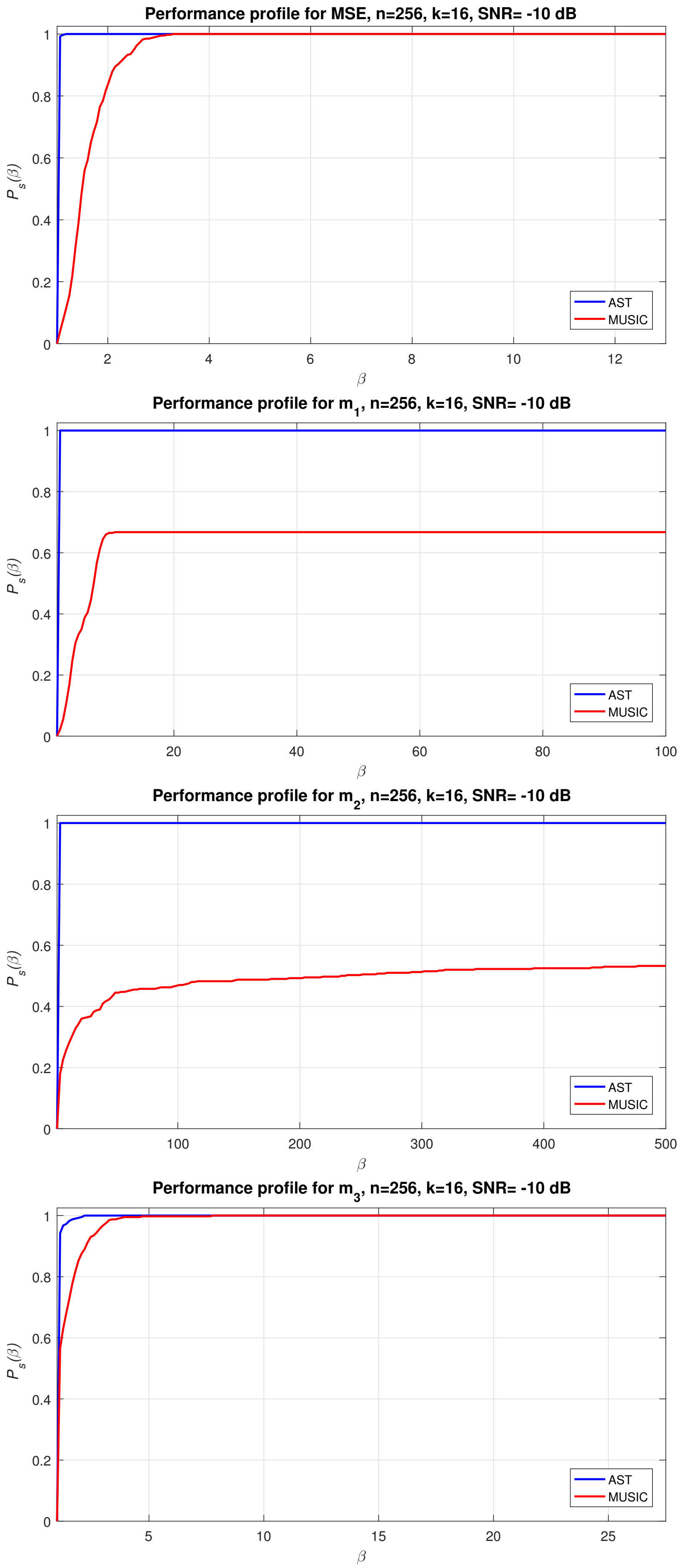

Figure 7.1: Performance profile plots for Mean squared error, m_1, m_2 and m_3.

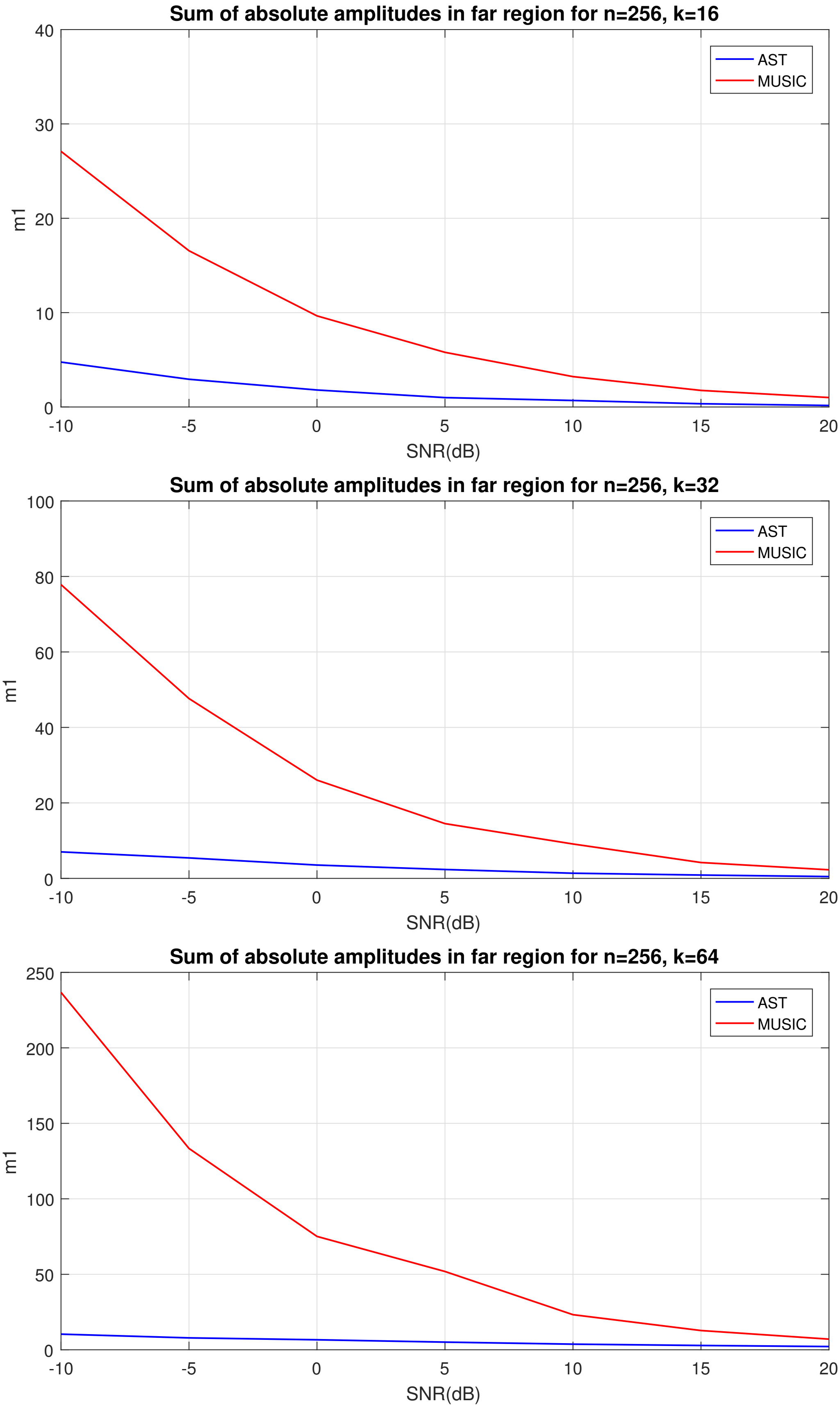

Figure 7.2: Error metric m_1 averaged over 300 random experiments.

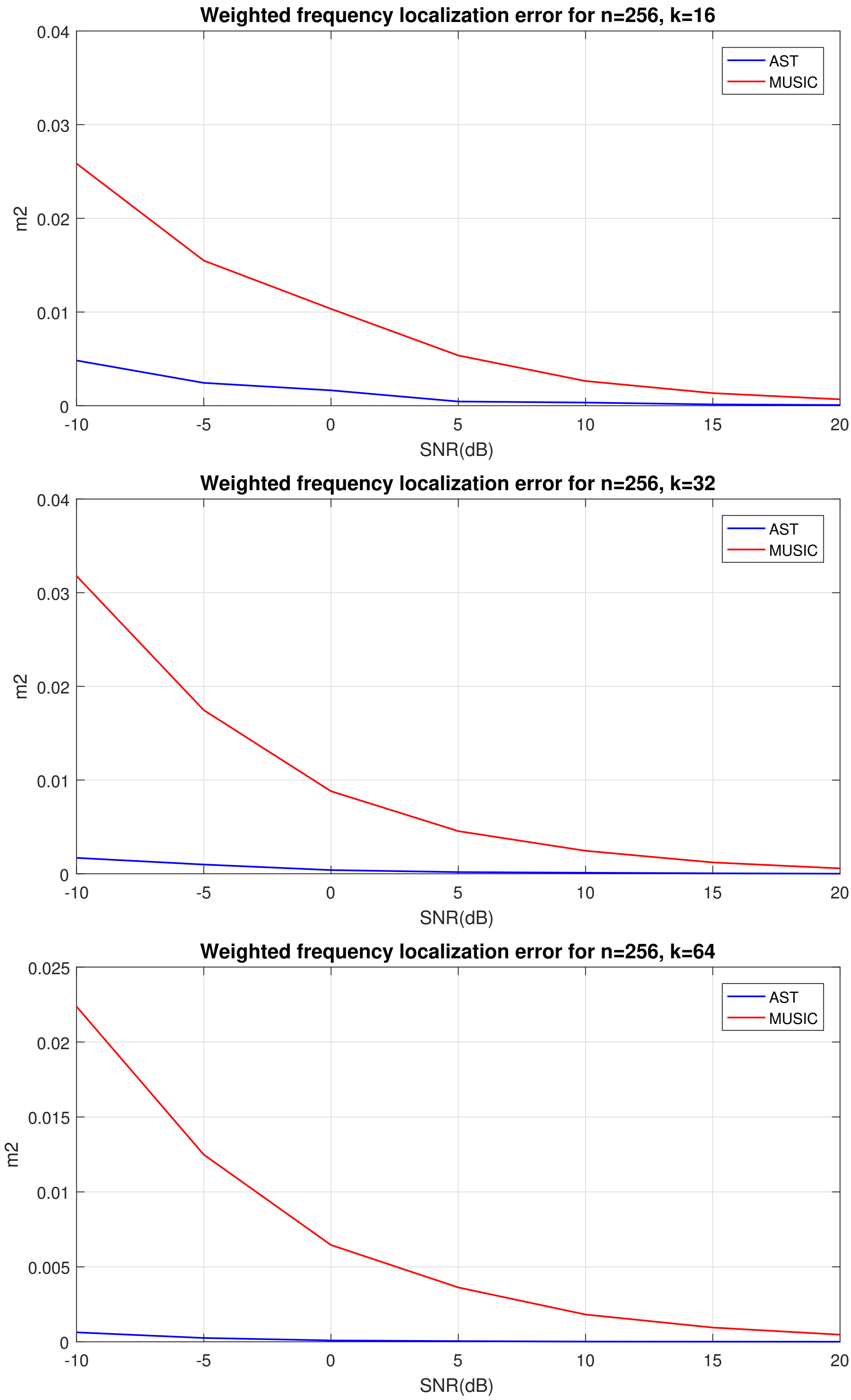

Figure 7.3: Error metric m_2 averaged over 300 random experiments.

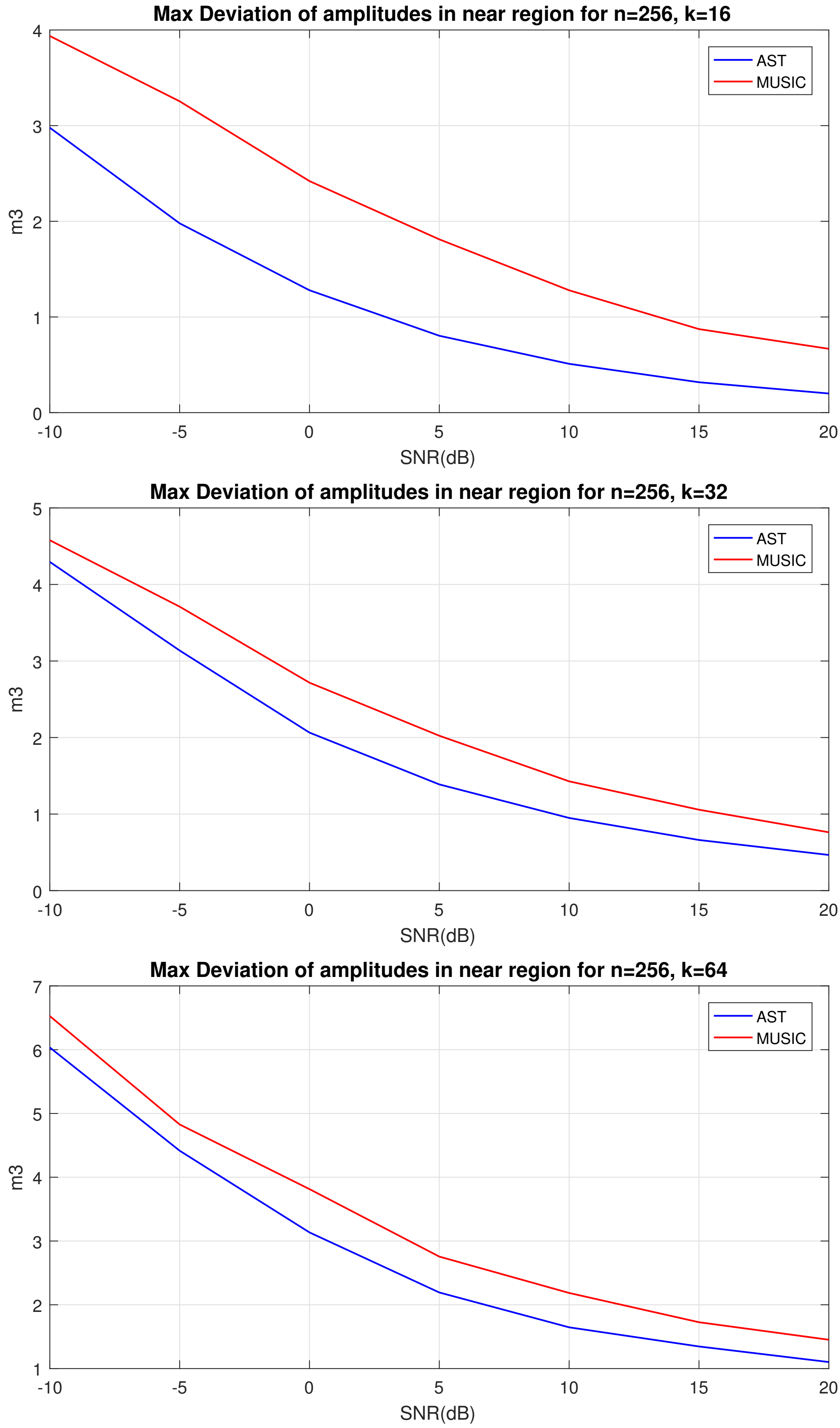

Figure 7.4: Error metric m_3 averaged over 300 random experiments.

Figure 7.5 and 7.6 shows the comparison of channel estimation and running time against grid size varied from 30 to 180. Here SNR is fixed with 10 dB. As both the original 4D atomic norm and approximate 4D atomic norm are off grid methods it is very straight forward that the curves are flat for these performance plots. Each curve is obtained by averaging over 100 realizations.

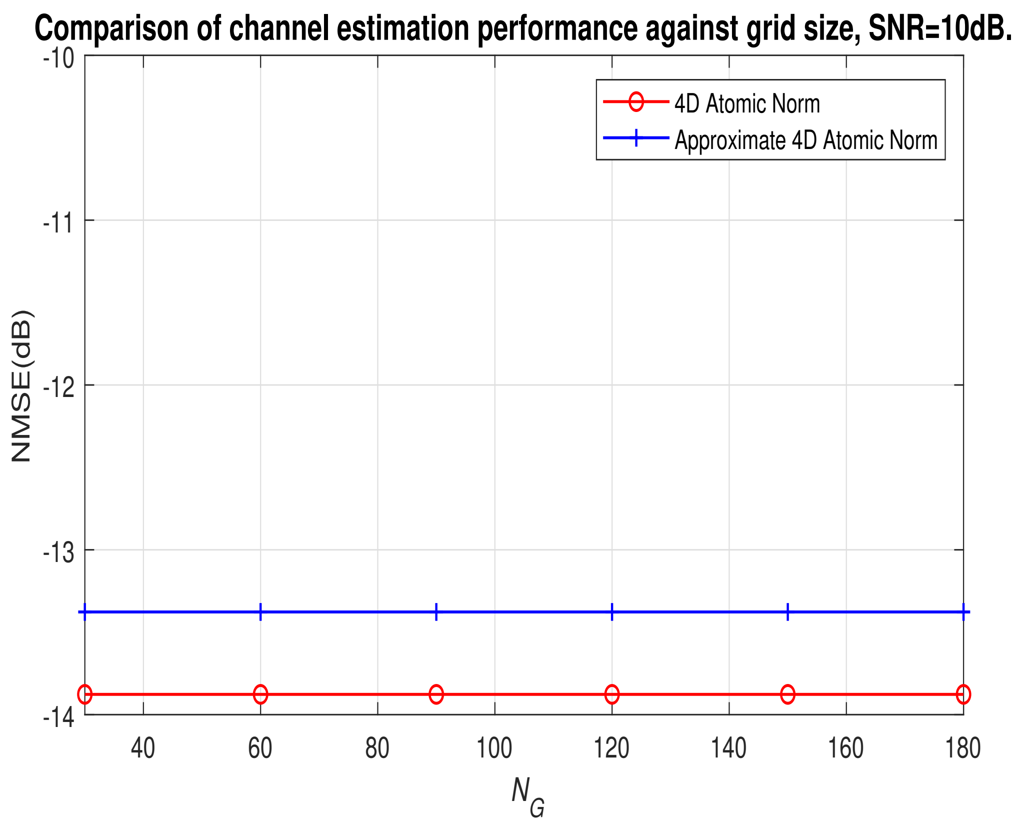

Figure 7.5: Comparison of channel estimation performance against grid size, SNR=10dB.

In Figure 7.5 normalized mean squared error (NMSE) curve for approximate 4D atomic norm proves that the error caused by the decoupling of Toeplitz matrices is not substantially large and it catches up with the original 4D problem which is solved using CVX package of MATLAB.

From Figure 7.6 we can conclude that the complexity and thereby the running time for proposed 4D atomic norm method is very low than the original 4D approach. The computational complexity of the proposed approximate 4D atomic-norm-based channel

Figure 7.6: Comparison of running time against grid size, SNR=10dB.

estimator is $\mathcal{O}((M+N)^3)$ per-iteration. In line with the Figure 7.5 this ensures comparable NMSE performance with lesser running time over original 4D method. And in this case also as the methods are off grid, we get flat curves as expected.

Figure 7.7 plots the NMSE (dB) performance with SNR increased from 2 to 10 dB. The simulations are averaged over 100 trials. We get approximately linearly shifted curve for approximate 4D atomic norm based method. The 4D atomic norm based channel estimator achieves better performance than the approximate 4D atomic norm based channel estimator by about 0.5 - 0.8 dB. This figure also illustrates that with the approximate method we have advantage in case of running time together the estimator error (NMSE) is not large.

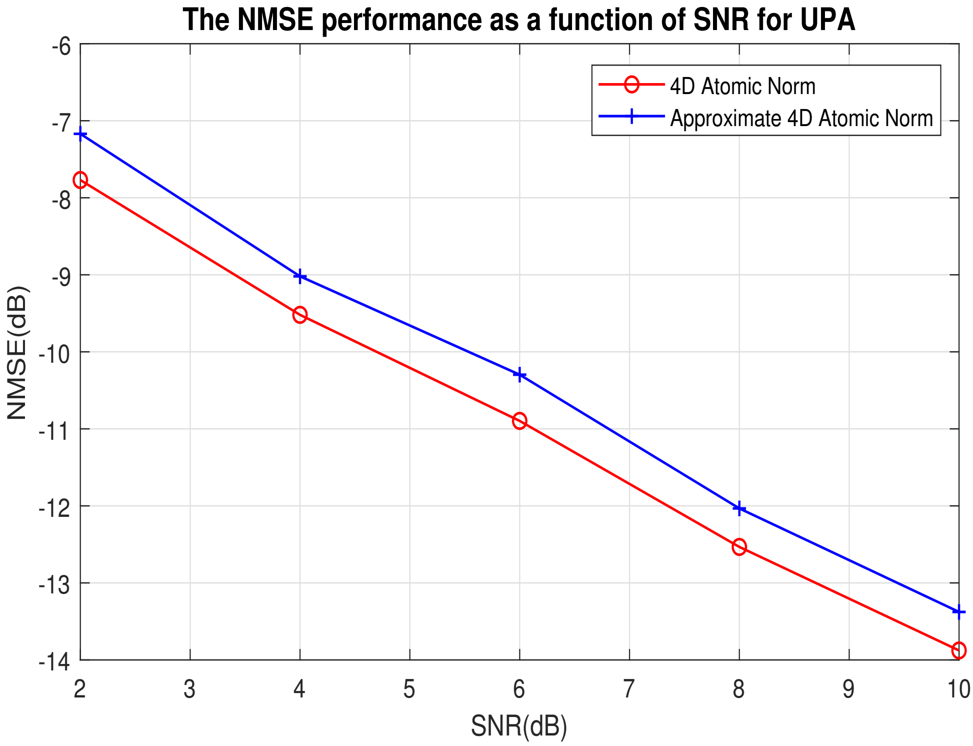

Figure 7.7: The NMSE performance as a function of SNR for UPA.

CHAPTER 8

Conclusions and Future Scope

8.1 Conclusions

Through this thesis work, we obtained the results that atomic norm based line spectral estimation is giving better performance than conventional subspace methods like MUSIC algorithm. This 1D problem is then extended to 4D case in which we are interested in millimeter wave beamformed full dimensional MIMO channel estimation under UPA case. The original 4D atomic norm minimization problem is solved using CVX package in MATLAB. Further as in 1D case, we obtained semi definite programming approximation, which later solved using closed form computations of ADMM algorithm. The approximation gives lesser NMSE difference compared to original 4D problem.

8.2 Future scope

This work can be extended to even lower complexity formulations, that decouples the original 4-level block Toeplitz matrix in the positive semi-definite constraint of SDP reformulation into four 1-level Toeplitz matrix constraint. This would be taking lesser running time and performance comparable to both the 4D algorithms discussed in this work. The decoupling should be dealt with great care since we are putting relaxations on transmitter and receiver array responses.

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