DATA ASSOCIATION, LOCALISATION AND TRACKING OF MULTIPLE TARGETS IN MULTI STATIC RADAR SYSTEMS

A Project Report

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THESIS CERTIFICATE

This is to certify that the thesis titled **DATA ASSOCIATION, LOCALISATION AND TRACKING OF MULTIPLE TARGETS IN MULTI STATIC RADAR SYSTEMS**, submitted by **C DEEPTI (EE16B134)**, to the Indian Institute of Technology, Madras, for the award of the degree of **Bachelor of Technology and Master of Technology**, is a bona fide record of the research work done by her under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

In the recent times, target detection has become a problem of high importance because of its increasingly vital applications in remote area surveillance, military reconnaissance, identification of UAVs (Unmanned aerial vehicles), highway traffic monitoring etc. There are a wide range of algorithms and approaches which address this problem. But most of them involve the use of highly complex methods and complicated concepts and hence, not suitable for practical implementations. In this thesis, a much simpler and novel algorithm to perform data association and localisation of multiple targets has been proposed which utilises the time of arrival values of the targets and leverages the underlying geometry of the multi static radar system. This thesis also introduces a non-parametric novel tracking method that manages to track and localise slowly maneuvering targets.

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ABBREVIATIONS

txr	Transmitter
rxr	Receiver
ToA	Time of Arrival
SVD	Singular Value Decomposition
ASOE	Absolute Sum of Errors
ISD	Inter site distance

CHAPTER 1

INTRODUCTION

1.1 MOTIVATION

Detection and tracking of multiple targets is a task which is of great importance because of its applications in military and surveillance fields. But the process of detecting, associating the data, localising and tracking the targets involves a lot of challenges. Some of these include, huge number of possible data associations for a target, misdetections, false alarms, lack of knowledge of the motion model of the targets, high complexity of the techniques which handle maneuvering targets etc. Over the past few years, many techniques and approaches have been proposed to solve this problem ranging from image processing, pattern recognition and machine learning algorithms, deep convolutional neural networks to fibre-optic gyroscopes etc.

The aim of this thesis report is to propose a much simpler and faster approach to handle this problem of data association by exploiting the underlying geometry of the system and come up with a non-parametric, less complex tracking method that could handle (slowly) maneuvering targets.

1.2 Scenario considered

We have considered the scenario where targets are present in a 7 cell region with hexagon-hexagon distance of 3km. Each hexagonal cell consists of four towers which will be transmitting and receiving at different instants of time. At any given instant of time, one tower from a cell will be transmitter and the other 3 towers will be receivers. Hence, we will have 7 transmitters and 21 receivers in the 7 cell region at every instant.

For each set of 7 cells there is a fusion centre which works independently and records all the Time of arrival (ToA) values and bistatic Doppler frequencies corresponding to the all the transmitter-receiver pairs of the 7 cells.

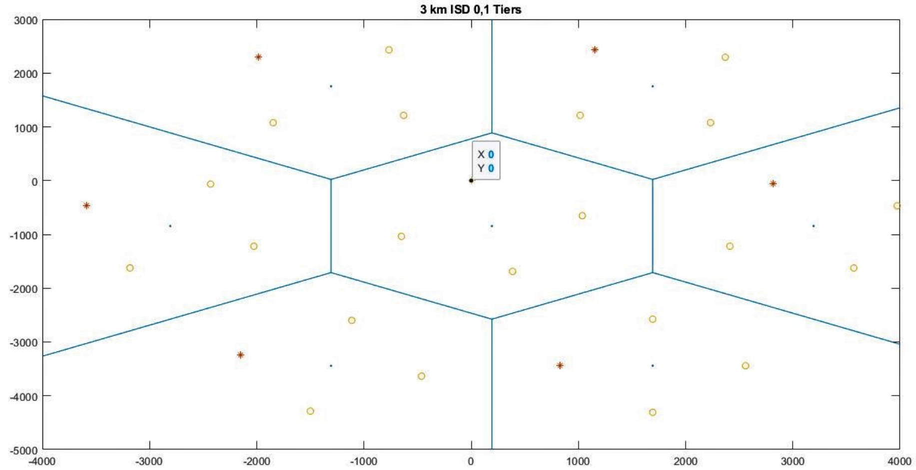

Figure 1.1: Position of transmitters and receivers in the 3km ISD 7cell region. The * represent transmitters and o represent receivers at a given instant.

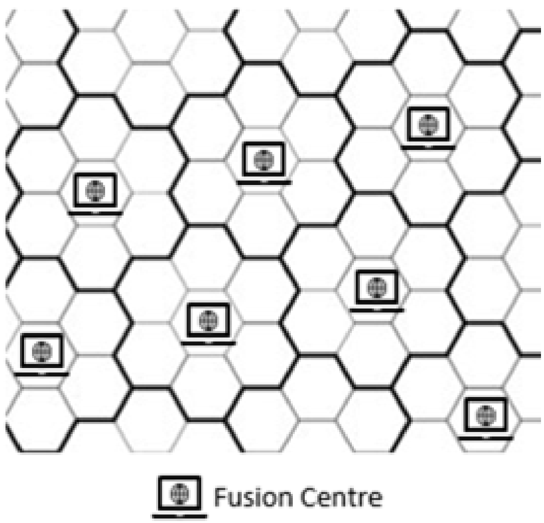

Figure 1.2: Fusion centres for different 7 cell regions

For every 6msec, the fusion centre would record a new set of ToA values (also called lags) and bistatic Doppler frequencies corresponding to a snapshot of the targets. We make use of these lag values and bistatic Doppler frequencies to identify the target's position and velocity. Using these position and velocity estimates we try to estimate the target's position in the next snapshot, thereby, tracking the target.

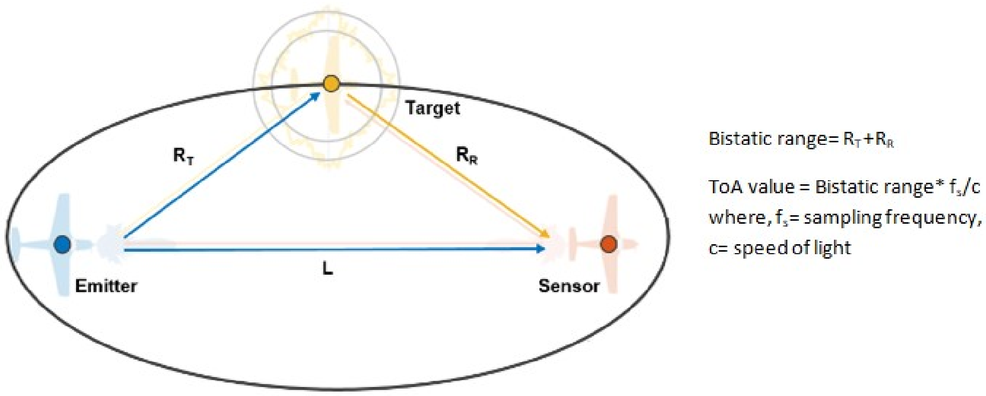

Figure 1.3: Bistatic range and Time of arrival relation

1.3 Outline of the thesis

The following concepts will be covered in the coming chapters in this thesis.

Chapter 2: The methodologies and approaches used to perform data association would be discussed.

Chapter 3: The procedure and steps followed to perform data association and localisation for two targets scenario would be discussed.

Chapter 4: The algorithm followed for three target localisation would be discussed.

Chapter 5: The implementation of a generalised data association and localisation algorithm for N targets would be discussed and the results of the algorithm for different values of N would be presented.

Chapter 6: This chapter would include the incorporation of tracking to the generalised data association algorithm and the results would be presented.

Chapter 7: This would be the final chapter which includes conclusion and future work.

CHAPTER 2

METHODOLOGY

In this thesis, we have only used time of arrival or lag values for the purpose of localisation. The bistatic Doppler frequenices along with the position estimates (calculated from localisation) have been used to find the velocity estimates (would be discussed later in chapter 6).

2.1 Separation by heights

For each bistatic pair (txr-rxr pair), a database of minimum peak samples for targets lying at different heights has been generated. Using the database, the maximum height within which the target lies corresponding to the measured lag can be found.

We can say that targets are **separable by height**s, if the range of maximum heights corresponding to the first set of lags does not overlap with the range of maximum heights corresponding to the second set of lags and so on. For example, consider a two target scenario with the following lag values and the maximum heights corresponding to them.

Table 2.1: Example table illustrating the lags and the corresponding maximum heights

Lags	Txr1	Txr2	Txr3
Rxr1	140, 300	143, 323	132, 312
Rxr2	120, 289	150, 278	130, 350

Max ht(Km)	Txr1	Txr2	Txr3
Rxr1	1.1, 3.5	1.8, 3.9	1.1, 4.2
Rxr2	1.6, 4.5	2.0, 3.6	2.1, 4.1

With reference to the example table: 2.1, the range of maximum heights corresponding to the first set of lags is (1.1km,2.1km) and corresponding to the second set of lags is (3.5km,4.5km). As we can see, these two ranges do not overlap and hence these two targets appear to be separated in heights for the given transmitter-receiver pairs.

If the targets are separable by heights in all the cells for all the hops, we can directly associate the corresponding lags and localize for both the targets individually.

If the criterion of non overlapping height ranges is satisfied only in one cell (for all the hops), then the targets appear to be separated for that one cell. In such situations, the lags corresponding to that separable cell can alone be used for localisation of targets. But the localisation accuracy would be low as only fewer equation (lesser number of lags) were used and as a result, these estimated target locations could be used as **intermediate locations** of the targets.

Based on this criterion of non-overlapping height ranges, the targets could be classified into the following categories:

1. Category 1: Height ranges corresponding to all the txr-rxr pairs of all the hops do not overlap.
2. Category 2: Height ranges corresponding to all the txr-rxr pairs of atleast 1 hop do not overlap.
3. Category 3: Height ranges corresponding to the txr-rxr pairs of atleast 1 cell for all the hops do not overlap.
4. Category 4: Height ranges corresponding to the txr-rxr pairs of atleast 1 cell do not overlap for atleast 1 hop.

2.2 Gating

Gating is a technique which is loosely based on the concept of multilateration.

Gating is mostly useful after finding an intermediate region along x,y plane over which the target might be present. The intermediate region is divided into smaller bins. For each (x,y) position in the bins, the possible z value for the target is calculated by making use of the recorded lags corresponding to a chosen bistatic pair according to the following expression:

$$bistatic\,range\,(r) = lag * c/f_s$$
$$r = \sqrt{(x-x_t)^2 + (y-y_t)^2 + (z-z_t)^2} + \sqrt{(x-x_r)^2 + (y-y_r)^2 + (z-z_r)^2}$$

where, (x, y) are bin positions in the intermediate region, (x_t, y_t, z_t) and (x_r, y_r, z_r) are Cartesian coordinates corresponding to the chosen transmitter and receiver pair, c is speed of light and f_s is sampling frequency

If the calculated z values are appropriate, then those (x,y,z) coordinates are regarded as possible positions for the target. Based on the calculated ranges of lags (found using the possible target positions) for all the bistatic pairs, the lags are grouped to the targets accordingly and the targets are localised with these associated lags.

The efficiency of this method depends on the resolution of bin sizes and the size of the intermediate region. Smaller the intermediate region, more compact the bins would be, leading to more lags getting grouped and, as a result, more accurate localisation of the targets.

2.3 Localisation

To perform localisation, the non-linear bistatic range expression must be converted into a linear relation in (x,y,z).

2.3.1 Approach 1

This approach of linearisation has been inspired by the method proposed in the paper[1]. Consider M transmitters located at positions $(x_{t_i}, y_{t_i}, z_{t_i})$ for $i = 1, 2, ...M$ and N receivers located at $(x_{r_j}, y_{r_j}, z_{r_j})$ for $j = 1, 2, ...N$. Let the target be present at (x, y, z) and r_{ij} be the bistatic range of the target with respect to transmitter i and receiver j. The bistatic range expession would be:

$$r_{ij} = \sqrt{(x - x_{t_i})^2 + (y - y_{t_i})^2 + (z - z_{t_i})^2} + \sqrt{(x - x_{r_j})^2 + (y - y_{r_j})^2 + (z - z_{r_j})^2}$$

Squaring on both sides and eliminating the terms would give,

$$r_{ij}\sqrt{(x - x_{t_i})^2 + (y - y_{t_i})^2 + (z - z_{t_i})^2} = x(x_{r_j} - x_{t_i}) + y(y_{r_j} - y_{t_i}) + z(z_{r_j} - z_{t_i}) + p_{ij}$$
$$where\ p_{ij} = \frac{1}{2}({r_{ij}}^2 - {x_{r_j}}^2 - {y_{r_j}}^2 - {z_{r_j}}^2 + {x_{t_i}}^2 + {y_{t_i}}^2 + {z_{t_i}}^2) \tag{2.1}$$

Taking the above equation (2.1) for $j = 1$ and $j = k$ $(1 < k \leq N)$ and cross multiplying by r_{ik} and r_{i1} respectively and subtracting the equations would yield:

[1] (Svecova *et al.*, 2008)

$$x[r_{ik}(x_{r_1} - x_{t_i}) - r_{i1}(x_{r_k} - x_{t_i})] + y[r_{ik}(y_{r_1} - y_{t_i}) - r_{i1}(y_{r_k} - y_{t_i})]$$
$$+ z[r_{ik}(z_{r_1} - z_{t_i}) - r_{i1}(z_{r_k} - z_{t_i})] = r_{i1}p_{ik} - r_{ik}p_{i1}$$

As x,y,z are unknowns the above expression can be written as: $a_{ik}x + b_{ik}y + c_{ik}z = g_{ik}$. Extending this relation to all the transmitters and receivers, the resulting system of equations could be written as :

$$\boldsymbol{As} = \boldsymbol{B}$$

$$where, \boldsymbol{A} = \begin{bmatrix} a_{12} & b_{12} & c_{12} \\ a_{13} & b_{13} & c_{14} \\ \vdots & \vdots & \vdots \\ a_{ik} & b_{ik} & c_{ik} \\ \vdots & \vdots & \vdots \end{bmatrix}, \boldsymbol{s} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} g_{12} \\ g_{13} \\ \vdots \\ g_{ik} \\ \vdots \end{bmatrix}$$

The least squares solution for the estimate is $\hat{\boldsymbol{s}} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{B}$.

Limitations:

- The initial number of non-linear equations were M*N. But after linearising by this approach we get only M*(N-1) linear equations.
- If the equation used for subtracting (here, corresponding to transmitter i and receiver 1) is erroneous, then that error would get propagated to all the other equations resulting in an erroneous final estimate.

2.3.2 Approach 2

This method of linearisation has been proposed in the paper[2]. The equation (2.1) could be re-written as:

$$(\vec{x}_{r_j} - \vec{x}_{t_i})^T\vec{x} = -p_{ij} + r_{ij}{\cdot}R_{t,tx_i}$$

where $\vec{x}_{t_i}$,$\vec{x}_{r_j}$ represent the position vectors of transmitter i and receiver j, $\vec{x}$ represents the target's position vector and R_{t,tx_i} denotes the distance between the transmitter i and target. Extending it to N receivers, we get:

[2] Noroozi and Sebt (2017)

$$\boldsymbol{S}_i\vec{x} = \vec{z}_i + \vec{r}_i R_{t,tx_i} \text{ where, } \boldsymbol{S}_i = \begin{bmatrix} \vec{x_{r_1}}^T - \vec{x_{t_i}}^T \\ \vdots \\ \vec{x_{r_N}}^T - \vec{x_{t_i}}^T \end{bmatrix}, \vec{z_i} = \begin{bmatrix} -p_{i1} \\ \vdots \\ -p_{iN} \end{bmatrix}, \vec{r_i} = \begin{bmatrix} r_{i1} \\ \vdots \\ r_{iN} \end{bmatrix}$$

Here, R_{t,tx_i} is a nuisance parameter which needs to be eliminated. Multiplication by a matrix $\boldsymbol{M}_i$ of dimensions $(N-1) \times N$, whose nullspace contains $\vec{r_i}$ (i.e.,$\boldsymbol{M}_i\vec{r_i} = 0$) would help eliminate this R_{t,tx_i} term. Such a matrix $\boldsymbol{M}_i$ is given by:

$$\boldsymbol{M}_i = \boldsymbol{V}^T\boldsymbol{D}_i$$

where, $\boldsymbol{D}_i = [diag(\vec{r_i})]^{-1}$ and

$\boldsymbol{V}$ is obtained from the SVD of matrix $(\boldsymbol{I}-\boldsymbol{Z})$ with $\boldsymbol{Z} = (N \times N)$ circular shift matrix.

Multiplication by $\boldsymbol{M}_i$ on both sides yields:

$$\boldsymbol{M}_i\boldsymbol{S}_i\vec{x} = \boldsymbol{M}_i\vec{z}_i + \underbrace{\boldsymbol{M}_i\vec{r}_i R_{t,Tx_i}}_{0} \Rightarrow \underbrace{\boldsymbol{M}_i\boldsymbol{S}_i}_{\boldsymbol{A}_i}\vec{x} = \underbrace{\boldsymbol{M}_i\vec{z}_i}_{\boldsymbol{b}_i} \Rightarrow \boldsymbol{A}_i\vec{x} = \boldsymbol{b}_i$$

The system of equations obtained after extending to all the M transmitters would be of the form: $\boldsymbol{A}\vec{x} = \boldsymbol{b}$, where $\boldsymbol{A} = [\boldsymbol{A}_1{}^T \ldots \boldsymbol{A}_M{}^T]^T$ and $\boldsymbol{b} = [\boldsymbol{b}_1{}^T \ldots \boldsymbol{b}_M{}^T]^T$.

The least squares solution for $\vec{x}$ is given by: $\hat{\vec{x}} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{b}$.

Improvements: From the method suggested above, the estimate $\hat{\vec{x}}$ obtained has lesser accuracy corresponding to the z coordinate. This is due to the heights between transmitter and receivers towers being comparable. To overcome this anomaly, a novel method to find the estimate of z coordinate has been adopted (but cannot be disclosed in this thesis).

Advantages:

- The number of linear equations obtained at the end are same as that of the initial number of non-linear bistatic equations (M*N).
- Since, it doesn't involve any cross operations among the equations the probability of an erroneous equation affecting other equations is very less. As a result, the issue of error propagation across equations is resolved.
- The accuracy of the estimate is as good (or even better than) the estimate obtained using approach 1.

2.4 Absolute sum of errors (ASOE)

It is defined as a metric to check for the correctness of the localisation estimate. It checks for consistency between the actually measured bistatic ranges and the bistatic ranges calculated from the position estimates. Let $r_{h,i,j}$ be the measured bistatic range corresponding to hop h, transmitter i and receiver j and let $\hat{r}_{h,i,j}$ be the bistatic range calculated from the position estimate of the target. Then, the ASOE is given by:

$$ASOE = \sum_{h=1}^{H} \sum_{i=1}^{M} \sum_{j=1}^{N} |r_{h,i,j} - \hat{r}_{h,i,j}| \tag{2.2}$$

where, H= number of hops, M= number of transmitters and N= number of receivers.

If ASOE value< Threshold[3], then the position estimate corresponds to a true target. Higher ASOE value implies greater error in localisation.

[3] (Hadi *et al.*, 2018)

CHAPTER 3

TWO TARGET DETECTION

The data association and localisation procedure for the scenario of two stationary targets present in our observation range has been discussed in this chapter. The steps and procedures followed are mainly built on the methods discussed in Chapter 2.

3.1 Algorithm

The data available to perform localisation of targets includes the positions of all transmitters and receivers present within the 7 cell region and the lag values and bistatic Doppler frequencies corresponding to the targets provided by the fusion center.

The database of minimum peak samples corresponding to all bistatic pairs for all hops for targets lying at different heights is generated. Based on this database, for each lag value the corresponding maximum height within which the target lies is recorded. With reference to this range of maximum heights, the targets are classified into different categories as mentioned in Section 2.1. Based on the category in which the targets lie, the data association and localisation would be performed as shown in figure 3.1.

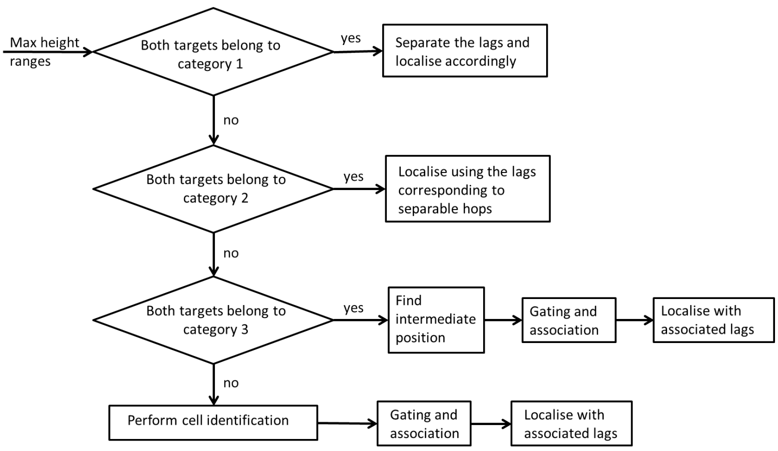

Figure 3.1: Flow of algorithm for data association and localisation of two targets

If the targets do not belong to either category 1 or 2 or 3, then we try to identify the cell over or around which the targets might be present (by identifying the txr-rxr pairs with least lag values). Now, the entire region of the cell (or cells) is taken as the intermediate region for the targets and gating is performed over this region.

3.2 Results

The algorithm proposed in the section 3.1 has been tested by considering targets lying at different heights and with different x,y separations within the 3km ISD 7 cell region. The simulations have been conducted assuming *isotropic antennas*, with sampling frequency f_s= 20MHz and stationary targets.

Table 3.1: Results for two target scenario

Ht diff between the targets	X-Y separation between the targets	Average RMSE
$\geq 3Km$	any separation	$\leq\ 2m \times 2m \times 2m$
$2Km$	any separation	$\leq\ 2m \times 2m \times 3m$
$1Km$	any separation	$\leq\ 3m \times 3m \times 6m$
$0.5Km$	well separated in either x or y	$\leq\ 3m \times 3m \times 4m$
$0.5Km$	less separated in x, y	high RMSE
$same\,height$	any separation	high RMSE

From table 3.1, it can be observed that targets with height difference > 1Km are localised with reasonably good accuracy. But the targets with height difference < 1Km and closely spaced in x,y and the targets lying at same heights are not being localised accurately.

3.3 Issues with the algorithm

- It has been observed that the algorithm fails for targets with height difference < 1Km. This could be attributed to the fact that most of these targets do not lie in either of 1,2 or 3 categories of section 2.1. To localise these targets, cell identification is performed. But since these targets are already closely spaced, gating fails to associate the lags properly owing to the large intermediate region provided by cell identification. Because of poor performance of gating, the localisation accuracy becomes low.
- Another issue would be the use of approach 2.3.1 to perform localisation. If the lag corresponding to the equation used for subtraction is wrongly grouped by gating, the error gets accumulated over all the other equations leading to a huge error in the final estimate.

CHAPTER 4

THREE TARGET DETECTION

The sequence of steps to be followed to perform data association and localisation of three stationary targets present in our observation region is discussed in this chapter.

4.1 Algorithm

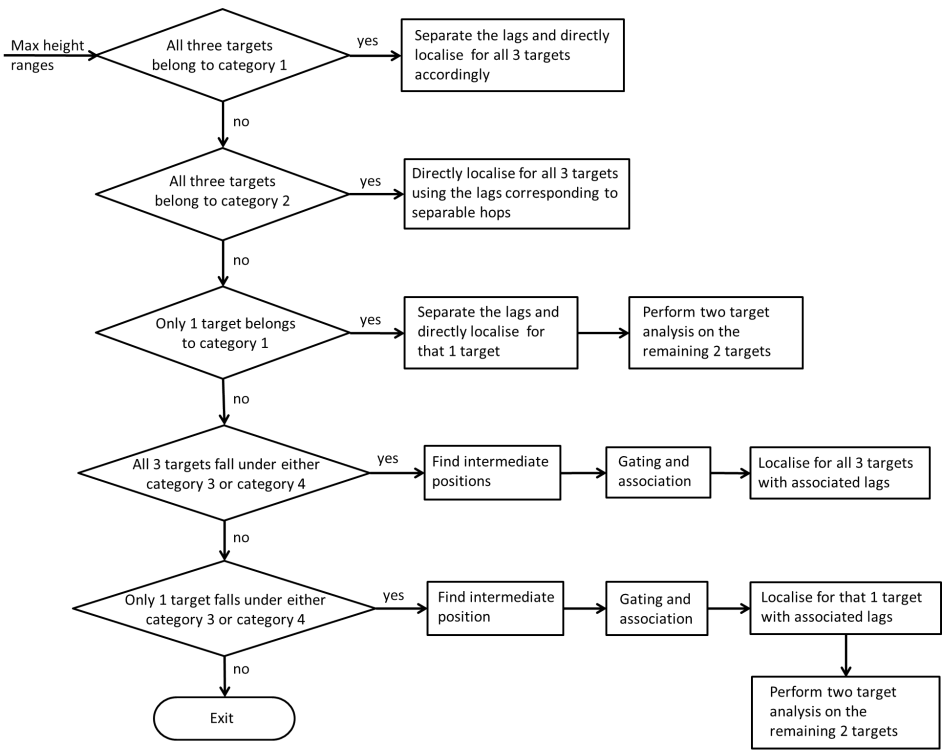

Figure 4.1: Flowchart explaining the algorithm for data association and localisation of three targets

The algorithm proposed in section 3.1 for the scenario of two targets has been refined and extended to the three target scenario. As we have observed previously, the cell identification step hasn't been of much help in localising targets with less separation. Therefore, while constructing the algorithm for three targets, the cell identification step has been dropped. Instead more emphasis has been given to accommodate various subcases now possible for the three target scenario, pertaining to targets being classified into different categories based on the maximum height ranges.

4.2 Results

The algorithm proposed in the section 4.1 has been tested for three stationary targets present in the 3km ISD 7 cell region over varying height differences between them. The simulations have been conducted assuming *isotropic antennas*, with sampling frequency f_s= 20MHz.

Table 4.1: Results for three target scenario

Ht diff between the targets	Target heights	Average RMSE
4Km	1Km, 5Km, 9Km	$\leq 1m \times 1m \times 1.5m$
3Km	1Km, 4Km, 7Km	$\leq 1m \times 1m \times 1.6m$
2Km	1Km, 3Km, 5Km	$\leq 1m \times 1m \times 1.6m$
1.5Km	1Km, 2.5Km, 4Km	$\leq 1m \times 1m \times 1.7m$
1Km	1Km, 2Km, 3Km	$\leq 2m \times 2m \times 2m$
same height	1Km, 1Km, 1Km	high RMSE or not identified

From table 4.1, it can be observed that targets with height difference $\geq$ 1Km are localised with an average RMSE within 2m. But the targets with height difference < 1Km are still a matter of concern.

4.3 Issues with the algorithm

- It is evident from the sections 3.1 and 4.1, that the sequence of steps followed to perform data association and localisation for two target and three target scenarios are very different. The number of stages in the three target algorithm are much greater than in the algorithm for two targets. If such a strategy is adopted to extend this algorithm to more number of targets (>3), then the number of stages would increase further and would even lead to increase in the computational load. So, there is a need to come up within a generalised algorithm for N targets (variable N) with limited number of stages.
- Even in the algorithm discussed in section 4.1, we were not able to identify and localise closely spaced targets. A new method has to be devised to address this problem.

CHAPTER 5

GENERALISED DATA ASSOCIATION AND LOCALISATION ALGORITHM FOR N TARGETS

This chapter discusses the proposal of a novel **Generalised algorithm to perform Data association and Localisation of N targets**. But firstly, as discussed in chapters 3 and 4, there is a need to introduce a method to identify and localise closely spaced targets. Therefore, before moving onto the main algorithm we will first address the issue of closely spaced targets.

5.1 Method for closely spaced targets

Closely spaced targets mostly refer to the targets which are present at same height or with < 500m height difference between them and an x,y separation < 500m.

This method involves trying to find at least 4 lags corresponding to targets which are close by, in order to estimate an intermediate position for them. For this purpose 1 transmitter and (minimum) 4 receivers are chosen and with the lag values corresponding to these bistatic pairs, different combinations of lags are generated. For each of these combinations, position estimates are found and only the ones with ASOE values< threshold are identified as true target position estimates. Since the accuracy of these position estimates would be low, they are considered as intermediate locations and processed further using gating and localisation.

5.1.1 Limitations

This method involves generating all possible lag combinations and finding position estimates corresponding to each of the combinations. If we have N targets then the maximum number of possible combinations would be N^4 (since 4 bistatic pairs are chosen). As the value of N increases, the complexity of this method increases exponentially.

This issue could be combated by performing the localisations corresponding to different combinations in parallel as it only involves computing 3x3 inverses.

5.1.2 Choosing the transmitter and receivers

The transmitter and receivers whose lag values would be used for generating these combinations could be chosen by the following two approaches.

- Approach A: Main approach
- Approach B: Backup approach

The details of these approaches cannot be disclosed in this thesis as patents for them are yet to be filed.

5.2 Generalised Algorithm for N targets

The **Generalised Data association and Localisation algorithm** is broken down into mainly the following 4 steps.

Step 1: involves trying to find the targets belonging to category 1 of section 2.1 and localising them.

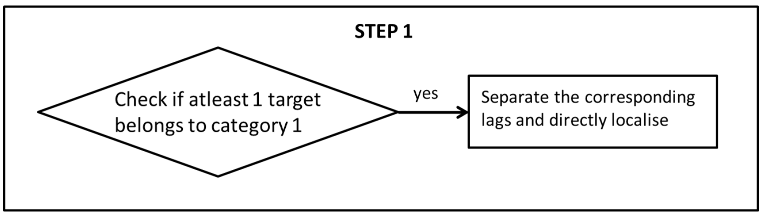

Figure 5.1: Step 1 of generalised algorithm

Step 2: involves finding the targets belonging to category 2 of section 2.1 and localising them.

Step 3: involves finding the targets belonging to either category 3 or 4 of section 2.1 and localising them.

Step 4: This step is exclusive for targets with less separation. This step utilises the method discussed in section 5.1 to identify and localise closely spaced targets. Step 4

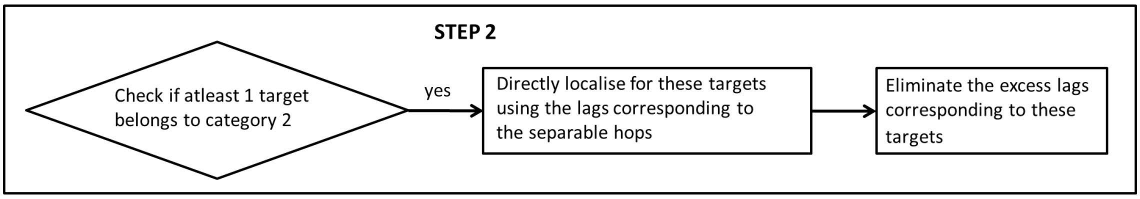

Figure 5.2: Step 2 of generalised algorithm

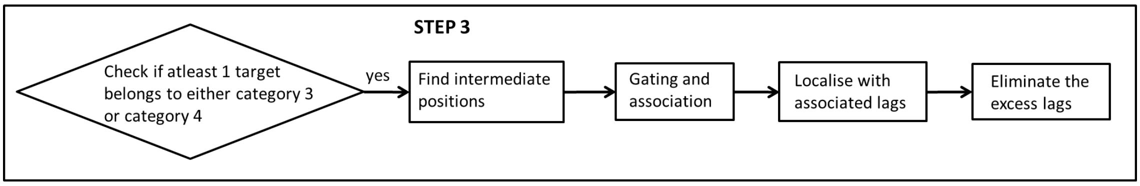

Figure 5.3: Step 3 of generalised algorithm

can be broken down to two parts: 4A and 4B. The first part (4A) involves choosing the transmitters and receivers according to approach A of section 5.1 and second part (4B) involves choosing the transmitters and receivers according to approach B of section 5.1.

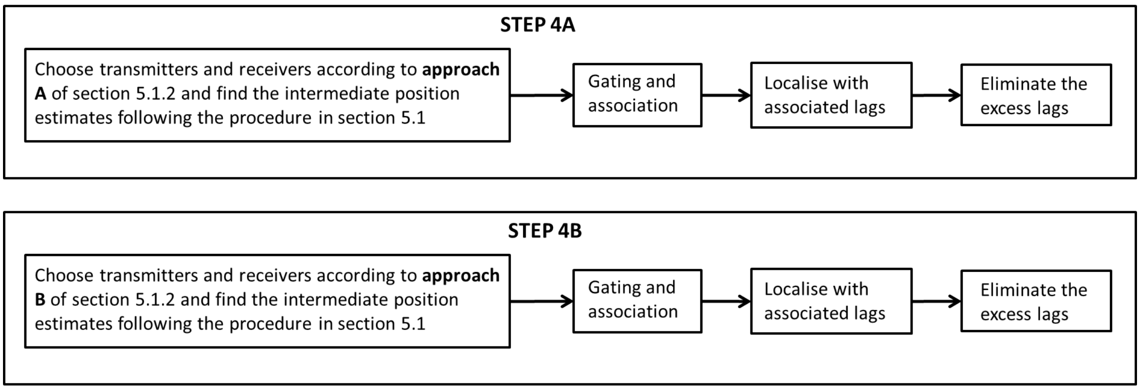

Figure 5.4: Step 4 of generalised algorithm

5.2.1 Algorithm

The entire flow of the steps followed in the **Generalised Data association and Localisation algorithm** is clearly explained in the figure 5.5.

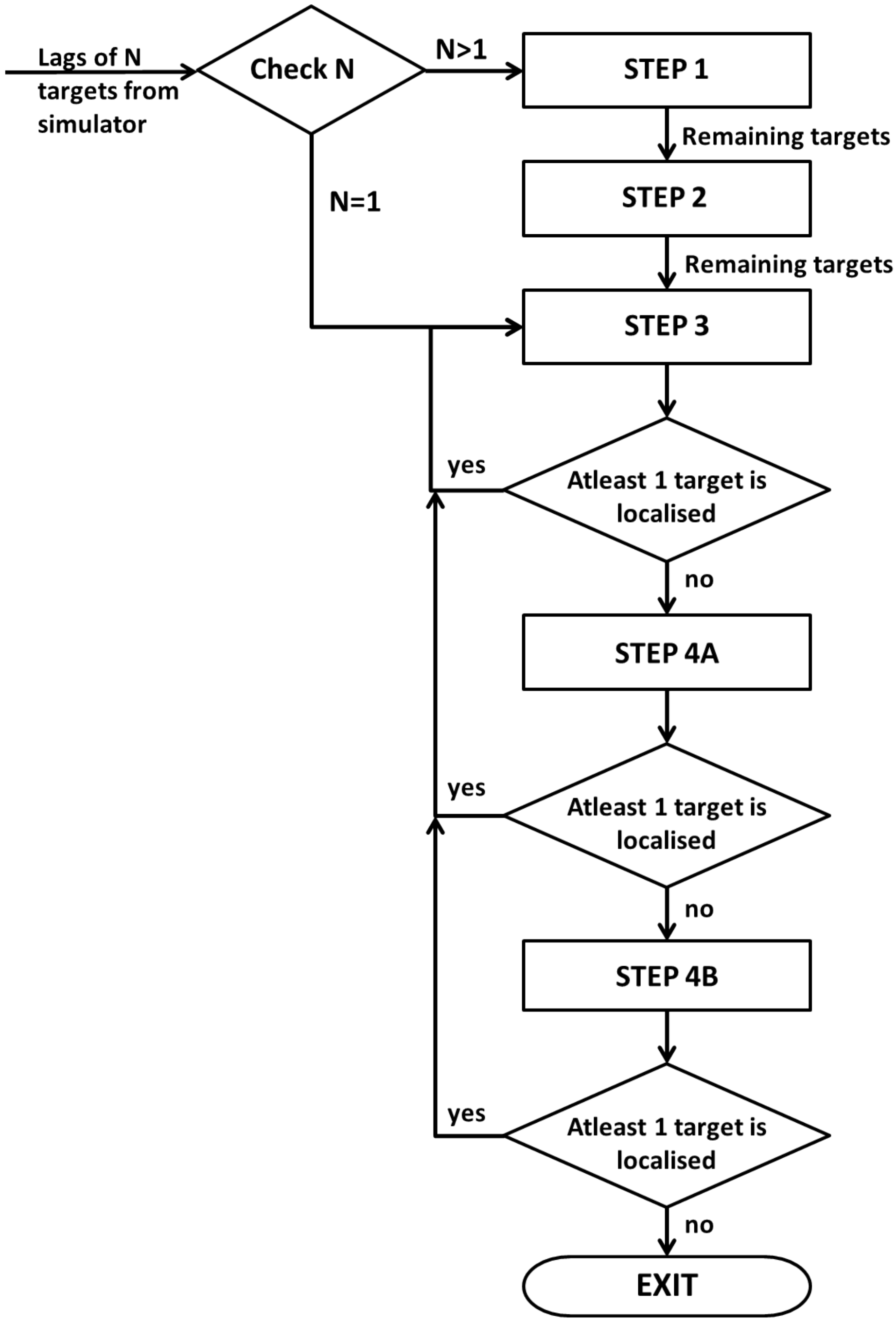

Figure 5.5: Flowchart of Generalised Data association and Localisation algorithm

5.3 Simulation results

The generalised algorithm described in section 5.2.1 has been tested for values of N= 2, 3, 4,i.e., by taking 2, 3, 4(stationary) targets at once within the 3km ISD 7 cell region. Scenarios of targets present with different height differences and varying x,y separations have been considered. The lag values used were generated by the *multistatic simulator*. As opposed to isotropic antennas, the transmitters and receivers of the simulator consist of an antenna pattern with certain gain along certain directions. As a result, not all targets would be seen with equal gain by all the antennas and this would result in missing lag values corresponding to some targets in some bistatic (txr-rxr) pairs. The generalised algorithm has been modelled to adapt to such situations and give reasonable outputs.

5.3.1 Two targets

First we would look at the results of a two target scenario for different height differences and x,y separations. For each x,y separation, 300 trials have been simulated with sampling frequency $f_s = 20MHz$. We say that the algorithm has failed if it fails to identify atleast 1 target.

Height difference between targets= 3Km

X-Y sep (m)	Max error (m)			Step 4A trials	Step 4B trials	X-Y sep (m)	Max error (m)			Step 4A trials	Step 4B trials
	x	y	z				x	y	z		
1000- 1000	3.65	2.84	4.90	0	0	**500 - 0**	4.53	3.91	4.01	0	0
500 - 500	4.25	3.60	9.77	0	0	**0 - 500**	5.29	2.62	5.77	0	0
200 - 200	4.45	3.89	5.36	0	0	**200 - 0**	3.65	3.24	7.46	0	0
100 - 100	3.67	2.90	4.67	0	0	**0 - 200**	3.29	4.34	6.78	0	0
1000 - 0	4.33	3.12	4.75	0	0	**100 - 0**	4.58	2.96	4.03	0	0
0 - 1000	3.83	4.00	6.16	0	0	**0 - 100**	4.21	3.64	5.49	0	0

Figure 5.6: Results for two targets with height difference= 3Km

- Targets are present at heights 1Km and 4Km.
- No. of failed trials= 0.

- No. of trials with error peaking= 0.
- No. of trials entering step 4A= 0.
- No. of trials entering step 4B= 0.
- Maximum error in position estimate is $\leq 6m \times 5m \times 10m$.

Height difference between targets= 1Km

X-Y sep (m)	Max error (m)			Step 4A trials	Step 4B trials	X-Y sep (m)	Max error (m)			Step 4A trials	Step 4B trials
	x	y	z				x	y	z		
1000- 1000	3.85	2.21	10.30	0	0	500 - 0	3.24	3.41	11.72	9	0
500 - 500	2.64	2.41	5.08	4	0	0 - 500	2.62	3.29	4.92	0	0
200 - 200	2.66	2.43	5.01	0	0	200 - 0	2.39	2.68	4.94	2	0
100 - 100	2.70	2.25	5.01	2	0	0 - 200	2.56	2.12	3.79	1	0
1000 - 0	2.59	3.55	5.66	2	0	100 - 0	2.77	3.16	8.05	2	0
0 - 1000	2.83	3.22	6.78	0	0	0 - 100	2.83	2.74	7.63	2	0

Figure 5.7: Results for two targets with height difference= 1Km

- Targets are present at heights 1Km and 2Km.
- No. of failed trials= 0.
- No. of trials with error peaking= 0.
- No. of trials entering step 4A= 24.
- No. of trials entering step 4B= 0.
- Maximum error in position estimate is $\leq 4m \times 4m \times 12m$.

Height difference between targets= 500m

- Targets are present at heights 1Km and 1.5Km.
- No. of failed trials= 0.
- No. of trials with error peaking= 0.
- No. of trials entering step 4A= 1897.
- No. of trials entering step 4B= 11.
- Maximum error in position estimate is $\leq 4m \times 4m \times 11m$.

X-Y sep (m)	Max error (m)			Step 4A trials	Step 4B trials	X-Y sep (m)	Max error (m)			Step 4A trials	Step 4B trials
	x	y	z				x	y	z		
1000- 1000	2.98	2.46	6.92	0	0	**500 - 0**	2.70	2.47	7.16	178	7
500 - 500	3.03	2.53	5.04	30	0	**0 - 500**	2.89	2.50	4.09	170	1
200 - 200	3.22	2.83	5.93	238	0	**200 - 0**	2.74	2.89	4.11	255	0
100 - 100	3.11	3.46	7.57	262	0	**0 - 200**	3.61	2.60	4.66	241	1
1000 - 0	2.35	2.36	5.92	17	1	**100 - 0**	2.77	2.68	7.97	251	1
0 - 1000	2.92	3.11	10.81	9	0	**0 - 100**	3.18	2.97	5.76	246	0

Figure 5.8: Results for two targets with height difference= 500m

X-Y sep (m)	Max error (m)			Step 4A trials	Step 4B trials	X-Y sep (m)	Max error (m)			Step 4A trials	Step 4B trials
	x	y	z				x	y	z		
1000- 1000	3.01	2.64	7.76	0	0	**500 - 0**	3.08	2.74	5.45	300	4
500 - 500	2.31	3.05	5.51	268	12	**0 - 500**	2.59	2.59	7.38	299	3
200 - 200	3.23	2.24	5.74	300	13	**200 - 0**	2.87	2.95	6.83	300	11
100 - 100	2.51	3.38	6.73	300	9	**0 - 200**	2.62	2.54	6.61	300	8
1000 - 0	2.69	2.36	6.19	14	0	**100 - 0**	5.50	2.94	9.89	300	3
0 - 1000	2.83	2.92	7.12	0	0	**0 - 100**	3.23	2.93	7.82	300	6

Figure 5.9: Results for two targets present at the same height

Targets at same height

- Both the targets are present at the same height of 1Km.
- No. of failed trials= 0.
- No. of trials with error peaking= 0.
- No. of trials entering step 4A= 2681.
- No. of trials entering step 4B= 69.
- Maximum error in position estimate is $\leq 6m \times 4m \times 10m$.

5.3.2 Three targets

Simulations for the three target scenario have been conducted by considering targets present at 3 different heights for varying height differences and x,y separations. 300 trials have been run with sampling frequency $f_s = 20MHz$ for each x,y separation. We say that the algorithm has failed if it fails to identify or localise atleast 1 target.

Height difference between targets= 2Km

X-Y sep (m)	Max error (m)			Step 4A trials	Step 4B trials	X-Y sep (m)	Max error (m)			Step 4A trials	Step 4B trials
	x	y	z				x	y	z		
1000- 1000	4.15	5.10	6.14	16	3	500 - 0	5.46	5.06	5.49	0	0
500 - 500	8.96	6.57	6.16	1	0	0 - 500	5.91	4.98	7.82	0	0
200 - 200	4.34	3.33	5.19	0	0	200 - 0	5.60	4.64	5.95	0	0
100 - 100	5.36	4.36	4.29	0	0	0 - 200	5.23	6.14	4.28	0	0
1000 - 0	5.55	3.66	6.06	1	0	100 - 0	6.20	4.61	7.75	0	0
0 - 1000	5.10	3.84	5.46	2	0	0 - 100	5.60	3.92	5.15	0	0

Figure 5.10: Results for three targets with height difference= 2Km

- Targets are present at heights 1Km, 3Km and 5Km.
- No. of failed trials= 0.
- No. of trials with error peaking= 0.
- No. of trials entering step 4A= 20.
- No. of trials entering step 4B= 3.
- Maximum error in position estimate is $\leq 9m \times 7m \times 8m$.

Height difference between targets= 1Km

X-Y sep (m)	Max error (m)			Step 4A trials	Step 4B trials	X-Y sep (m)	Max error (m)			Step 4A trials	Step 4B trials
	x	y	z				x	y	z		
1000- 1000	2.89	3.22	3.83	0	0	**500 - 0**	3.32	3.22	5.28	1	0
500 - 500	2.64	2.73	4.17	1	0	**0 - 500**	3.29	3.24	5.98	0	0
200 - 200	3.15	2.98	4.67	2	0	**200 - 0**	3.23	3.58	5.25	2	0
100 - 100	2.62	3.10	4.69	2	0	**0 - 200**	2.68	2.92	7.34	1	0
1000 - 0	2.79	2.65	6.08	1	0	**100 - 0**	3.26	3.03	5.14	1	0
0 - 1000	3.57	2.86	5.56	0	0	**0 - 100**	4.04	2.65	5.48	1	0

Figure 5.11: Results for three targets with height difference= 1Km

- Targets are present at heights 1Km, 2Km and 3Km.
- No. of failed trials= 0.
- No. of trials with error peaking= 0.
- No. of trials entering step 4A= 12.
- No. of trials entering step 4B= 0.
- Maximum error in position estimate is $\leq 4m \times 4m \times 8m$.

Height difference between targets= 500m

X-Y sep (m)	Max error (m)			Step 4A trials	Step 4B trials	X-Y sep (m)	Max error (m)			Step 4A trials	Step 4B trials
	x	y	z				x	y	z		
1000- 1000	3.13	2.97	5.88	0	0	**500 - 0**	3.84	3.36	7.32	148	5
500 - 500	3.04	3.82	7.02	20	1	**0 - 500**	3.90	2.83	6.92	153	2
200 - 200	3.41	2.93	8.03	245	4	**200 - 0**	3.73	2.40	5.93	247	0
100 - 100	3.55	3.43	5.85	263	1	**0 - 200**	2.77	3.36	11.34	254	2
1000 - 0	2.49	3.43	5.75	20	1	**100 - 0**	3.93	2.89	5.72	253	1
0 - 1000	2.73	2.63	7.60	14	0	**0 - 100**	3.21	3.06	5.86	245	0

Figure 5.12: Results for three targets with height difference= 500m

- Targets are present at heights 1Km, 1.5Km and 2Km.
- No. of failed trials= 0.

- No. of trials with error peaking= 0.
- No. of trials entering step 4A= 1862.
- No. of trials entering step 4B= 17.
- Maximum error in position estimate is $\leq 4m \times 4m \times 12m$.

Targets at same height

X-Y sep (m)	Max error (m)			Step 4A trials	Step 4B trials	X-Y sep (m)	Max error (m)			Step 4A trials	Step 4B trials
	x	y	z				x	y	z		
1000- 1000	3.74	2.76	9.24	6	0	500 - 0	3.27	2.63	7.14	300	14
500 - 500	2.91	2.33	6.30	273	6	0 - 500	2.82	3.35	7.58	300	7
200 - 200	2.90	2.78	6.25	300	10	200 - 0	3.55	2.56	7.52	300	12
100 - 100	3.30	18.72	43.36	300	11	0 - 200	4.60	3.90	10.84	300	10
1000 - 0	3.28	3.34	5.80	28	1	100 - 0	32.63	48.05	62.21	300	10
0 - 1000	2.62	2.80	7.03	8	0	0 - 100	7.66	14.85 **1 failed**	21.04	300	17

Figure 5.13: Results for three targets present at the same height

- All the three targets are present at the same height of 1Km.
- No. of failed trials= **1** (corresponds to 0m-100m x,y separation).
- No. of trials with error peaking= **12**.
  - 1 trial corresponds to 100-100 x,y separation case, 8 trials to 100-0 case and 3 trials to 0-100 case.
- No. of trials entering step 4A= 2715.
- No. of trials entering step 4B= 98.
- Maximum error in position estimate for separation between the targets $\geq 200m$ along either x or y direction is $\leq 5m \times 4m \times 11m$.

5.3.3 Four targets

Simulations for the four target scenario have been conducted for targets having different height differences and varying x,y separations. 300 trials have been run with sampling frequency $f_s = 20MHz$ for each x,y separation. We say that the algorithm has failed if it fails to identify or localise atleast 1 target.

Height difference between targets= 1Km

X-Y sep (m)	Max error (m)			Step 4A trials	Step 4B trials	X-Y sep (m)	Max error (m)			Step 4A trials	Step 4B trials
	x	y	z				x	y	z		
1000- 1000	4.52	3.21	4.97	32	2	500 - 0	4.24	4.24	5.30	3	0
500 - 500	3.95	3.86	5.06	23	0	0 - 500	3.19	4.34	5.14	6	0
200 - 200	3.71	3.77	6.01	3	0	200 - 0	3.92	3.56	6.81	3	0
100 - 100	3.74	4.54	5.39	2	0	0 - 200	2.97	3.02	7.20	1	0
1000 - 0	3.72	3.21	5.78	15	1	100 - 0	3.95	3.65	5.36	3	0
0 - 1000	4.11	3.89	5.85	9	1	0 - 100	3.32	3.19	5.87	1	0

Figure 5.14: Results for four targets with height difference= 1Km

- Targets are present at heights 1Km, 2Km, 3Km and 4Km.
- No. of failed trials= 0.
- No. of trials with error peaking= 0.
- No. of trials entering step 4A= 101.
- No. of trials entering step 4B= 4.
- Maximum error in position estimate is $\leq 5m \times 5m \times 8m$.

Height difference between targets= 500m

X-Y sep (m)	Max error (m)			Step 4A trials	Step 4B trials	X-Y sep (m)	Max error (m)			Step 4A trials	Step 4B trials
	x	y	z				x	y	z		
1000- 1000	2.76	3.59	4.16	7	0	500 - 0	7.02	4.43	10.40	122	0
500 - 500	3.35	3.16	4.16	25	0	0 - 500	4.71	4.09	6.54	122	1
200 - 200	4.71	3.53	4.77	207	4	200 - 0	3.16	3.11	5.68	240	3
100 - 100	3.59	3.49	6.71	247	0	0 - 200	3.70	4.13	6.45	244	2
1000 - 0	3.65	3.18	5.29	14	0	100 - 0	3.11	3.37	5.25	253	1
0 - 1000	3.03	2.54	5.24	10	0	0 - 100	3.61	6.15	6.19	267	1

Figure 5.15: Results for four targets with height difference= 500m

- Targets are present at heights 1Km, 1.5Km, 2Km and 2.5Km.
- No. of failed trials= 0.

- No. of trials with error peaking= 0.
- No. of trials entering step 4A= 1758.
- No. of trials entering step 4B= 12.
- Maximum error in position estimate is $\leq 7m \times 7m \times 11m$.

Targets at same height

X-Y sep (m)	Max error (m)			Step 4A trials	Step 4B trials	X-Y sep (m)	Max error (m)			Step 4A trials	Step 4B trials
	x	y	z				x	y	z		
1000- 1000	3.20	3.89	7.89	43	1	500 - 0	2.81	2.98	10.42	300	11
500 - 500	2.93	2.96	6.88	273	6	0 - 500	3.22	2.91	6.02	300	6
200 - 200	3.61	3.17	8.41	300	17	200 - 0	7.18	5.56	15.73	300	19
100 - 100	7.34	3.74	13.17	300	17	0 - 200	3.22	17.06	16.47	300	14
1000 - 0	3.03	2.70	8.46	138	1	100 - 0	15.13	26.85	43.93	300	13
0 - 1000	3.18	2.90	11.55	34	1	0 - 100	22.09	22.98	56.06	300	21

Figure 5.16: Results for four targets present at the same height

- All the four targets are present at the same height of 1Km.
- No. of failed trials= 0.
- No. of trials with error peaking= **24**.
  - 3 trials correspond to 100-100 x,y separation case, 10 trials to 100-0 case and 11 trials to 0-100 case.
- No. of trials entering step 4A= 2888.
- No. of trials entering step 4B= 127.
- Maximum error in position estimate for separation between the targets $\geq 500m$ along either x or y direction is $\leq 4m \times 4m \times 12m$.
- Maximum error in position estimate for a separation of $200m$ between the targets along either x or y direction is $\leq 8m \times 17m \times 17m$.

5.3.4 Summary of the results

The results for the simulations presented in section 5.3 can be summarized as follows:

- For 2 targets, present anywhere along the z direction and with a separation of atleast 100m along either x or y directions, we get a maximum error in position estimate to be $\leq 6m \times 5m \times 12m$.

- For 3 targets,
  - With a height difference of atleast 500m between them and separated along either x or y directions by atleast 100m, the maximum error in position estimates is found to be $\leq 9m \times 7m \times 12m$.
  - Present at the same height and separated along either x or y directions by atleast 200m, the maximum error in position estimate is $\leq 5m \times 4m \times 11m$.
  - Present at the same height with an x or y separation of 100m, the maximum error in position estimate spikes upto $33m \times 48m \times 63m$.
    - 12 out of 900 trials exhibit this peaking behaviour in position estimate's error. Out of these 12 peaking cases, 1 case corresponds to 100m-100m x,y separation case, 8 cases correspond to 100m-0m x,y separation and 3 cases correspond to 0m-100m x,y separation.
    - The algorithm fails for 1 trial (out of 300 trials) corresponding to 0m-100m x,y separation case.
- For 4 targets,
  - With a height difference of atleast 500m between them and separated along either x or y directions by atleast 100m, the maximum error in position estimates is found to be $\leq 7m \times 7m \times 11m$.
  - Present at the same height and separated along either x or y by atleast 500m, the maximum error in position estimate is $\leq 4m \times 4m \times 12m$.
  - Present at the same height with a separation of 200m along either x or y directions, the maximum error in position estimate slightly goes up to $8m \times 17m \times 17m$
  - Present at the same height with an x or y separation of 100m, the maximum error in position estimate spikes upto $22m \times 27m \times 56m$.
    - 24 out of 900 trials exhibit this peaking behaviour in position estimate's error. Out of these 24 peaking cases, 3 cases corresponds to 100m-100m x,y separation case, 10 cases correspond to 100m-0m x,y separation and 11 cases correspond to 0m-100m x,y separation.

As mentioned in section 5.1, the costliest step in the algorithm in terms of computational complexity is the step 4. We have divided step 4 into two parts: step 4A and step 4B, where, step 4A is considered as the main step and step 4B is a backup step incorporated to achieve better accuracy for the cases which fail to get localised by step 4A.

It has been observed that for all the considered values of N (2,3,4 targets), a substantial number of trials only corresponding to height difference of 500m or lesser and x-y separation $\leq 500m$, enter the step 4A. Out of these, only about $3-5\%$ of them actually get passed on to step 4B. Though initially the introduction of step 4B was considered as a trade-off for better accuracy of closely spaced targets, it is now observed from the simulations that it is not a very heavy penalty to pay in terms of computational efficiency.

5.4 Limitations

- The algorithm does not make use of the Doppler frequency information.
- We have observed from the simulations that for targets with height difference $\leq 500m$ and separation along x or y directions $\leq 500m$, the algorithm tends to enter step 4 for most number of trials. It is known that the complexity of step 4 increases exponentially with the number of targets (N^4). If in some scenario, we are intended to localise a large number of closely spaced targets in our observation region, the computational load of the algorithm increases immensely.
- Targets separated by a distance of less than 100m are still not identified.

CHAPTER 6

TRACKING

Tracking is a method where we try to estimate the target's position in the present instant by making use of the target's position and velocities corresponding to the previous time instants.

6.1 Target's Cartesian velocity estimation

The Cartesian coordinates of a target at a given instant can be estimated by following the algorithm described in section 5.2.1. In order to perform tracking, we also need to know the target's velocity at the given instant. The target's Cartesian velocity can be found by making use of the bistatic Doppler frequencies and the position estimate of the target by following the approach discussed in the paper[1].

Consider there are M transmitters and N receivers. Let $f_{D_{ij}}$ be the bistatic Doppler frequency recorded by the fusion center corresponding to transmitter i and receiver j. The range rate or bistatic velocity $\dot{r}_{ij}$ is given by the relation:

$$\dot{r}_{ij} = \frac{d}{dt}(R_{t,tx_i} + R_{t,rx_j}) = -f_{D_{ij}}\lambda$$

where, R_{t,tx_i} refers to the distance between transmitter i and the target, R_{t,rx_j} refers to the distance between receiver j and the target and λ corresponds to the wavelength of the signal. If we consider $\vec{v} = (\dot{x}, \dot{y}, \dot{z})^T$ to be the Cartesian velocity of the target, then the relation between range rate $\dot{r}_{ij}$ corresponding to transmitter i and receiver j and $\vec{v}$ can be expressed as:

$$\dot{r}_{ij} = \vec{v}^T\left(\frac{\vec{p} - \vec{p}_{r_j}}{\| \vec{p} - \vec{p}_{r_j} \|_2} + \frac{\vec{p} - \vec{p}_{t_i}}{\| \vec{p} - \vec{p}_{t_i} \|_2}\right)$$

Here, $\vec{p} = (x, y, z)^T$ is the position vector of the target, $\vec{p}_{r_j} = (x_{r_j}, y_{r_j}, z_{r_j})^T$ is the position vector of the receiver j and $\vec{p}_{t_i} = (x_{t_i}, y_{t_i}, z_{t_i})^T$ is the position vector corresponding

[1] Wei and Lun (2015)

to transmitter i. Extending the above relation to all the transmitter and receiver pairs, we arrive at a system of equations given by:

$$\begin{pmatrix} \frac{x-x_{r_1}}{\|\vec{p}-\vec{p}_{r_1}\|_2} + \frac{x-x_{t_1}}{\|\vec{p}-\vec{p}_{t_1}\|_2} & \frac{y-y_{r_1}}{\|\vec{p}-\vec{p}_{r_1}\|_2} + \frac{y-y_{t_1}}{\|\vec{p}-\vec{p}_{t_1}\|_2} & \frac{z-z_{r_1}}{\|\vec{p}-\vec{p}_{r_1}\|_2} + \frac{z-z_{t_1}}{\|\vec{p}-\vec{p}_{t_1}\|_2} \\ \vdots & \vdots & \vdots \\ \frac{x-x_{r_N}}{\|\vec{p}-\vec{p}_{r_N}\|_2} + \frac{x-x_{t_M}}{\|\vec{p}-\vec{p}_{t_M}\|_2} & \frac{y-y_{r_N}}{\|\vec{p}-\vec{p}_{r_N}\|_2} + \frac{y-y_{t_M}}{\|\vec{p}-\vec{p}_{t_M}\|_2} & \frac{z-z_{r_N}}{\|\vec{p}-\vec{p}_{r_N}\|_2} + \frac{z-z_{t_M}}{\|\vec{p}-\vec{p}_{t_M}\|_2} \end{pmatrix} \vec{v} = \begin{pmatrix} \dot{r}_{11} \\ \vdots \\ \dot{r}_{MN} \end{pmatrix}$$

$$\Rightarrow \boldsymbol{B}\vec{v} = \vec{z}_{\dot{r}}$$

From the above system of equations, the target's Cartesian velocity estimate is given by: $\hat{\vec{v}} = (\boldsymbol{B}^T\boldsymbol{B})^{-1}\boldsymbol{B}^T\vec{z}_{\dot{r}}$.

6.2 Tracking algorithm for Data association and Localisation of targets

The tracking algorithm utilizes the target's position and velocity estimates corresponding to the previous time instant, to associate the target's data corresponding to the present time instant and thereby, localise and obtain the position estimate of the target corresponding to the present instant.

Firstly, based on the velocity of the target corresponding to the previous instant, a range of lags corresponding to every bistatic pair is generated. The lags values of the present instant (taken from the fusion centre) are appropriately associated in accordance to these lag ranges. With these associated lags, an intermediate position for the target is calculated, upon which gating and localisation are performed to arrive at the final position estimate of the target corresponding to the present time instant.

If the tracking algorithm fails to localise any of the targets, then the residual lag values are sent into the generalised data association and localisation algorithm (5.2.1). After estimation of all the targets positions, the target velocities are estimated by utilising the bistatic Doppler frequencies (corresponding to the associated lags) according to the method given in section 6.1. The position and velocity estimates, thus calculated, are stored in order to be used for the next time instant.

The following flowchart explains the steps followed in the tracking algorithm and the integration of the tracking algorithm with the Generalised Data association and Localisation algorithm.

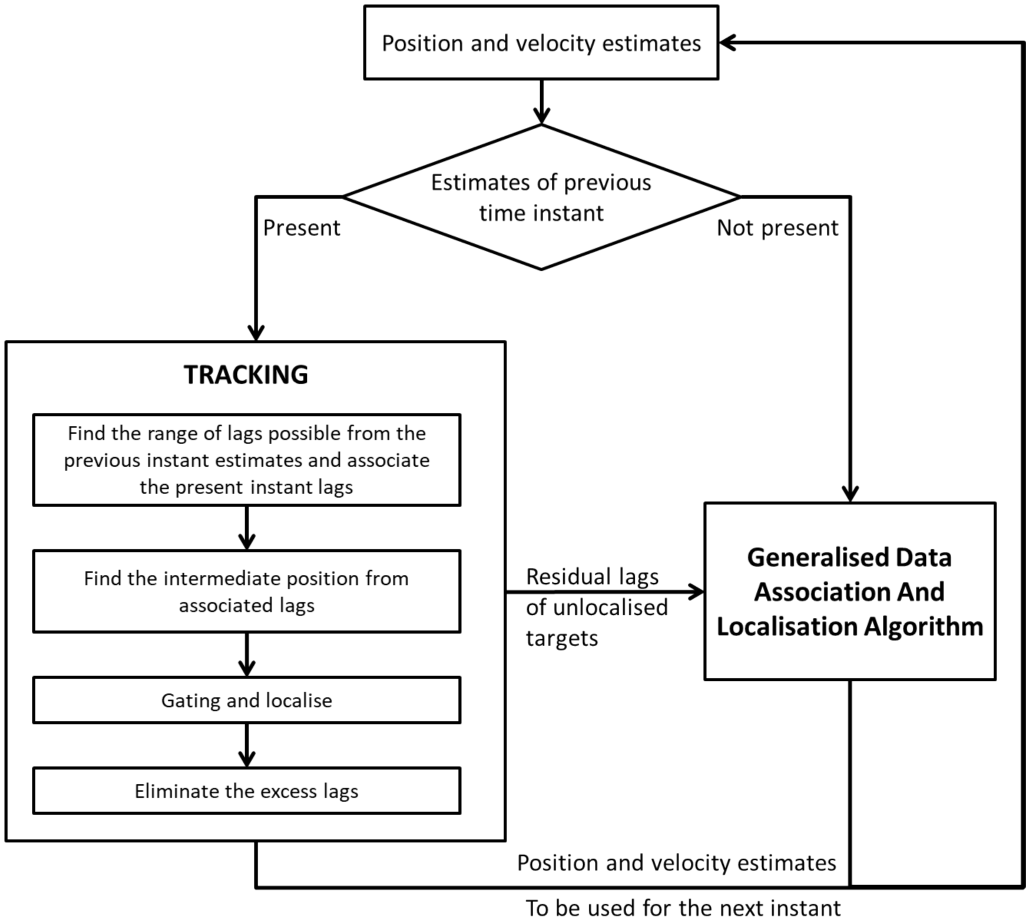

Figure 6.1: Flowchart of Tracking algorithm integrated with the generalised algorithm of section 5.2.1

6.3 Simulation results

6.3.1 Scenario 1

The first set of simulations have been conducted assuming the following scenario for 3 time instants.

At the 1^{st} instant of time, two targets fly into our observation region with an arbitrary velocity and separated by a height difference 'H' (varied in the simulations as 1km, 500m, 0m). During the 2^{nd} instant, a new third target flies in with some velocity and height difference 'H' into our observation region, in addition to the previously present two targets. The two targets which have entered in the 1^{st} instant have now moved to new positions. In the 3^{rd} instant, another new fourth target flies in (maintaining the height difference 'H') and the previous 3 targets have moved to newer locations within the observation region.

For each set of 3 time instants, targets present at different heights (for varying height differences 'H') and varying x,y separations within the 3km ISD 7 cell region have been considered. Corresponding to each height difference, 300 trials have been run for each x,y separation between the targets.
The lag values and bistatic Doppler frequencies have been generated by the *multistatic simulator* (with sampling frequency $f_s = 20MHz$) for every time instant. The duration between consecutive time instants (t_{max}) has been taken to be 100msec. The targets have been assumed to move only along x,y plane with constant velocities ranging from $50m/sec$ to $100\sqrt{2}m/sec$.

Height difference between the targets = 1Km

	X-Y sep (m)	Max error in position(m)			Max error in velocity(m/s)			Step 4A trials	Step 4B trials	X-Y sep (m)	Max error in position(m)			Max error in velocity(m/s)			Step 4A trials	Step 4B trials
		x	y	z	Vx	Vy	Vz				x	y	z	Vx	Vy	Vz		
Instant 1	1000 – 1000	3.03	2.21	6.37	4.66	5.47	8.37	0	0	500 – 0	3.29	2.43	5.61	4.56	4.60	11.13	2	0
Instant 2		3.72	3.34	4.13	4.98	5.94	7.87	0	0		2.52	3.70	5.79	4.77	6.13	12.11	0	0
Instant 3		4.43	5.17	3.82	5.73	5.82	9.62	1	0		4.16	3.34	4.97	6.62	6.48	12.64	0	0
Instant 1	500– 500	2.80	2.54	5.05	4.37	5.42	6.90	1	0	0 – 500	2.60	2.64	5.64	5.56	5.25	8.04	0	0
Instant 2		3.03	2.89	5.36	4.46	5.18	7.96	0	0		3.35	3.90	6.29	5.53	4.32	7.60	0	0
Instant 3		3.68	3.57	6.61	4.83	6.29	8.65	0	0		4.01	3.84	4.69	6.62	5.18	8.95	0	0
Instant 1	200– 200	4.36	3.29	6.05	4.43	4.96	10.61	2	0	200 – 0	2.60	2.40	4.10	4.50	4.56	8.81	0	0
Instant 2		2.69	2.68	5.92	4.55	6.27	8.23	0	0		3.15	3.04	5.80	4.92	4.94	8.37	0	0
Instant 3		5.01	3.84	5.16	5.62	6.21	7.77	0	0		3.43	3.28	7.53	5.68	4.87	8.99	0	0
Instant 1	100– 100	3.20	2.33	5.38	5.03	6.32	10.26	1	0	0 – 200	3.06	2.95	9.34	4.54	6.59	11.09	1	0
Instant 2		3.21	3.47	7.37	5.35	6.32	9.38	0	0		3.07	3.36	5.74	4.91	6.28	10.24	0	0
Instant 3		4.08	3.16	7.00	5.99	6.05	10.35	0	0		4.77	3.08	6.95	5.89	6.56	8.06	0	0
Instant 1	1000 – 0	2.47	2.80	8.87	5.16	4.64	7.78	0	0	100 – 0	3.65	3.63	8.36	4.94	5.55	11.71	1	0
Instant 2		3.65	2.86	4.29	6.22	6.76	8.39	0	0		3.13	2.80	4.53	6.25	5.44	11.53	0	0
Instant 3		3.13	3.58	5.61	6.03	6.80	8.03	0	0		3.97	3.40	6.24	6.22	5.57	12.37	0	0
Instant 1	0 – 1000	2.88	3.07	5.31	6.64	4.87	9.87	0	0	0 – 100	2.48	2.61	7.14	5.24	5.67	9.48	0	0
Instant 2		3.59	3.57	5.61	6.89	5.09	8.99	0	0		3.35	2.55	5.07	5.26	5.96	8.31	0	0
Instant 3		4.85	3.79	5.41	6.49	5.07	8.30	0	0		3.42	3.17	4.68	5.17	5.97	8.83	0	0

Figure 6.2: Scenario 1 tracking results for targets with height difference of 1Km

- Targets are present at heights 1Km, 2Km, 3Km and 4Km.

- No. of failed trials= 0.
- No. of trials with position error peaking= 0.
- No. of trials with velocity error peaking= 0.
- No. of trials entering step 4A= 9.
  - 8 trials correspond to 1^{st} instant and 1 trial of 3^{rd} instant.
- No. of trials entering step 4B= 0.
- Maximum error in position estimate is $\leq 5m \times 6m \times 10m$.
- Maximum error in velocity estimate is $\leq 7m/s \times 7m/s \times 13m/s$.

Height difference between the targets = 500m

	X-Y sep (m)	Max error in position(m)			Max error in velocity(m/s)			Step 4A trials	Step 4B trials	X-Y sep (m)	Max error in position(m)			Max error in velocity(m/s)			Step 4A trials	Step 4B trials
		x	y	z	Vx	Vy	Vz				x	y	z	Vx	Vy	Vz		
Instant 1	1000 – 1000	2.78	2.70	9.91	4.73	5.02	10.02	0	0	500 – 0	2.72	2.31	9.44	4.74	5.37	8.78	67	5
Instant 2		3.17	2.78	4.25	4.51	5.53	10.38	1	0		3.38	3.46	7.11	4.85	5.57	7.68	0	0
Instant 3		3.39	3.56	6.94	4.56	5.62	9.22	1	0		3.30	2.79	5.94	6.69	6.03	7.57	0	0
Instant 1	500– 500	2.49	2.29	3.91	4.33	5.39	9.29	7	0	0 – 500	2.97	2.52	5.18	5.11	6.68	9.23	89	0
Instant 2		3.38	2.31	6.29	4.95	5.53	8.28	0	0		2.88	2.53	5.20	4.81	6.68	10.09	1	0
Instant 3		2.86	3.30	6.79	4.54	5.11	7.02	0	0		4.12	2.56	6.58	7.69	6.87	11.17	1	0
Instant 1	200– 200	3.82	3.06	5.37	4.45	4.00	9.81	154	1	200 – 0	3.49	2.83	5.95	4.82	6.08	9.46	203	2
Instant 2		2.74	2.66	7.91	4.77	5.21	8.88	0	0		3.23	2.59	5.15	5.88	6.16	11.04	0	0
Instant 3		3.06	3.02	6.72	5.29	7.29	7.64	0	0		3.06	2.76	5.57	6.64	6.24	10.39	0	0
Instant 1	100– 100	2.67	2.76	5.69	6.39	5.61	12.79	247	0	0 – 200	3.45	3.00	6.05	5.85	5.67	9.98	207	1
Instant 2		3.48	2.55	5.73	6.52	5.69	10.55	1	0		3.85	6.32	10.43	6.11	5.57	11.34	0	0
Instant 3		4.07	3.29	6.62	6.66	6.51	10.27	0	0		3.11	4.19	7.42	6.58	5.95	12.65	0	0
Instant 1	1000 – 0	2.95	2.75	7.87	5.67	6.35	9.21	3	0	100 – 0	2.82	2.62	4.53	5.09	6.80	10.86	244	0
Instant 2		2.73	2.68	5.84	5.60	6.31	8.83	1	0		4.57	3.56	8.22	5.88	6.04	12.77	2	0
Instant 3		3.94	3.66	6.11	5.52	6.24	8.68	0	0		4.05	2.62	4.19	6.19	5.93	11.79	0	0
Instant 1	0 – 1000	3.24	2.54	6.43	4.87	6.70	18.99	1	0	0 – 100	2.94	3.11	5.14	6.58	4.58	6.63	243	0
Instant 2		3.24	2.59	7.71	5.73	6.52	18.10	0	0		4.27	3.45	8.45	6.53	5.13	7.48	2	0
Instant 3		2.76	4.00	5.69	5.61	6.67	18.71	0	0		2.92	3.54	7.05	6.49	6.04	7.34	1	0

Figure 6.3: Scenario 1 tracking results for targets with height difference of 500m

- Targets are present at heights 1Km, 1.5Km, 2Km and 2.5Km.
- No. of failed trials= 0.
- No. of trials with position error peaking= 0.
- No. of trials with velocity error peaking= 0.
- No. of trials entering step 4A= 1469.

- 1465 trials enter during 1^{st} instant, 8 trials during 2^{nd} instant and 3 trials during 3^{rd} instant (some trials enter in multiple time instants).

- No. of trials entering step 4B= 9 (all during 1^{st} instant).
- Maximum error in position estimate is $\leq 5m \times 7m \times 11m$.
- Maximum error in velocity estimate is $\leq 8m/s \times 8m/s \times 19m/s$.

Targets at same height

	X-Y sep (m)	Max error in position(m)			Max error in velocity(m/s)			Step 4A trials	Step 4B trials	X-Y sep (m)	Max error in position(m)			Max error in velocity(m/s)			Step 4A trials	Step 4B trials
		x	y	z	Vx	Vy	Vz				x	y	z	Vx	Vy	Vz		
Instant 1	1000 – 1000	2.30	2.90	8.73	4.42	4.39	7.81	0	0	500 – 0	2.51	2.95	8.23	7.22	4.82	19.36	133	6
Instant 2		3.26	3.46	6.79	4.47	4.60	8.69	0	0		2.64	3.30	6.39	7.02	4.85	18.49	1	1
Instant 3		3.94	2.69	8.15	5.37	4.32	9.67	1	0		3.80	2.68	6.83	6.34	5.15	15.70	1	0
Instant 1	500– 500	2.72	2.44	5.55	3.93	4.57	8.02	71	2	0 – 500	2.18	2.75	8.80	5.45	5.82	15.36	150	3
Instant 2		2.81	2.81	8.40	4.52	5.09	7.58	0	0		2.60	2.87	6.73	4.91	5.51	14.61	1	0
Instant 3		3.73	2.26	4.92	5.06	5.97	8.41	0	0		2.86	2.79	7.33	6.82	5.90	16.77	1	0
Instant 1	200– 200	2.77	2.47	4.30	5.55	5.58	12.46	289	10	200 – 0	2.90	3.14	6.26	5.19	6.74	9.35	300	9
Instant 2		2.55	3.09	7.79	5.35	5.63	10.16	1	0		3.65	5.73	8.39	5.02	6.34	10.52	2	0
Instant 3		4.06	2.91	8.46	5.06	5.69	10.43	2	0		4.34	3.32	10.72	4.74	6.20	9.46	2	0
Instant 1	100– 100 3 spikes	3.17	2.46	7.56	6.15	5.18	11.92	300	15	0 – 200	2.81	2.56	8.88	5.15	4.50	14.30	300	9
Instant 2		4.77	4.23	10.65	5.26	5.14	14.68	1	0		2.70	3.06	7.54	5.40	5.11	13.00	1	0
Instant 3		21.70	11.70	38.21	4.91	5.22	13.82	0	0		3.17	3.86	8.78	5.08	5.49	13.74	2	0
Instant 1	1000 – 0	3.03	2.19	6.62	7.17	5.59	14.06	0	0	100 – 0 20 spikes	3.67	3.25	6.14	6.26	5.56	13.42	300	7
Instant 2		3.41	2.70	6.43	7.03	5.99	13.31	0	0		14.64	12.62	60.25	6.13	6.07	14.18	1	0
Instant 3		3.53	2.84	7.54	6.88	6.12	12.33	1	0		17.33	76.92	209.42	8.72	5.81	21.25	2	0
Instant 1	0 – 1000	2.45	2.72	5.03	5.71	5.84	11.17	0	0	0 – 100 18 spikes	2.81	2.42	9.19	6.17	4.39	13.74	300	7
Instant 2		3.22	2.73	7.65	5.82	5.97	12.32	0	0		32.77	6.82	43.66	5.71	7.02	13.32	2	0
Instant 3		4.03	3.41	9.84	5.82	5.88	12.00	0	0		26.66	22.68	50.47	5.74	6.43	12.38	3	0

Figure 6.4: Scenario 1 tracking results for targets present at same heights

- All targets are present at the same height of 1Km.
- No. of failed trials= 0.
- No. of trials with position error peaking= **41**.
  - 3 trials correspond to 100-100 x,y separation case, 20 trials correspond to 100-0 case and 18 trials correspond to 0-100 case.
- No. of trials with velocity error peaking= **1** (corresponding to 100-0 case).
- No. of trials entering step 4A= 2149.
  - 2143 trials enter during the 1^{st} instant, 10 trials during the 2^{nd} instant and 15 trials during the 3^{rd} instant of time (some trials enter during multiple time instants).

- No. of trials entering step 4B= 69.
- Maximum error in position estimate for separation between the targets $\geq 200m$ along either x or y direction is $\leq 5m \times 6m \times 11m$.
- Maximum error in velocity estimate for separation between the targets $\geq 200m$ along either x or y is $\leq 8m/s \times 7m/s \times 20m/s$.
- For 100m separation along both x,y directions:
  - During 1^{st} instant, the maximum error in position estimate is $\leq 4m \times 3m \times 8m$ and during the 2^{nd} instant it is $\leq 5m \times 5m \times 11m$.
  - The position error spikes up during the third instant.
- For separation of 100m along only x or only y (100-0, 0-100 cases):
  - During the 1^{st} instant, the maximum error in position estimate is $\leq 4m \times 4m \times 10m$.
  - The position error spikes up during the second and third instants.

INFERENCE

The above results can be summarised as follows.

The targets separated by height differences of 1Km and 500m and with a separation of atleast 100m along either x or y directions are localised by the algorithm of section 6.2 with good accuracy.

In case of targets present at the same height, the ones with separation of atleast 200m along either x or y directions have low localisation error. The targets separated by 100m along both x and y directions are localised well during the first 2 instants, but the error in position estimate spikes up during the third instant. For targets with separation of 100m either along only x or only y directions, the error in position estimate is low for the first instant but spikes up during the second and third instants. This spiking up of localisation error can be explained by the decrease in separation between the targets caused due to their movements.

It can be observed that the entry of simulation to the steps 4A and 4B of algorithm 5.2.1 is restricted mostly only to the 1^{st} instant (even for close by targets), during which two targets need to be localised. During the 2^{nd} and 3^{rd} instants, the task of localisation is taken up by tracking, thereby, reducing the computational load.

6.3.2 Scenario 2

The second set of simulations have been conducted assuming the following scenario for 9 time instants.

At the 1^{st} instant of time, one target flies into our observation region with some arbitrary velocity. During the 2^{nd} instant, a new target flies in with some velocity and height difference of 'H' (varied as 500m and 0m) with respect to the first target into our observation region. The first target might have now moved to a new location. In the 3^{rd} instant, another new target flies in (maintaining the height difference 'H') and the previous 2 targets have moved to newer locations within the observation region. This goes on for 9 time instants and at the end of 9^{th} instant we would have 9 targets present in our observation region.

For each set of these 9 instants, targets present at different heights (for height differences of 500m and 0m) and varying x,y separations within the 3km ISD 7 cell region have been considered. Corresponding to each height difference, 50 trials have been run for each x,y separation between the targets.

The lag values and bistatic Doppler frequencies have been generated by the *multistatic simulator* (with sampling frequency $f_s = 20MHz$) for every time instant. The duration between consecutive time instants (t_{max}) has been taken to be 100msec for 500m height difference case and for same height targets t_{max} is varied as 20msec and 100msec. The targets have been assumed to move only along x,y plane with constant velocities ranging from $50m/sec$ to $100\sqrt{2}m/sec$.

Height difference between targets= 500m, t_{max}= 100msec

- Targets are present at heights 1Km, 1.5Km, 2Km, 2.5Km, 3Km, 3.5Km, 4Km, 4.5Km and 5Km.
- No. of failed trials= 0.
- No. of trials with position error peaking= 0.
- No. of trials with velocity error peaking= 0.
- No. of trials entering step 4A= 144.
- No. of trials entering step 4B= 7.
- Maximum error in position estimate is $\leq\ 8m \times 12m \times 15m$.
- Maximum error in velocity estimate is $\leq\ 10m/s \times 8m/s \times 14m/s$.

	X-Y sep (m)	Max error in position (m)			Max error in velocity (m/sec)			Step 4A trials	Step 4B trials
		x	y	z	Vx	Vy	Vz		
Instant 1		2.69	1.88	5.41	3.96	5.57	7.60	0	0
Instant 2		2.03	1.77	4.50	4.09	5.98	8.03	1	0
Instant 3		2.68	2.40	6.15	4.46	5.43	8.95	0	0
Instant 4		2.93	2.37	8.26	4.90	5.76	8.03	0	0
Instant 5	500-500	4.46	2.61	11.03	5.67	5.52	8.33	1	0
Instant 6		2.93	2.69	6.13	6.09	4.97	8.39	0	0
Instant 7		3.13	3.01	4.48	6.09	4.84	9.51	0	0
Instant 8		6.37	4.89	9.82	5.64	5.20	9.18	3	0
Instant 9		6.29	7.48	14.72	5.40	7.18	6.84	8	0
Instant 1		1.84	1.67	3.44	3.45	3.40	6.23	0	0
Instant 2		2.10	1.86	2.92	3.89	3.83	4.73	0	0
Instant 3		2.19	2.12	3.91	3.88	3.34	5.23	0	0
Instant 4		2.82	3.35	6.28	4.97	3.34	4.96	0	0
Instant 5	200-200	2.90	2.63	2.95	5.38	3.69	4.63	0	0
Instant 6		3.03	2.53	2.78	5.24	4.67	4.10	1	0
Instant 7		4.15	2.88	2.69	4.97	4.59	4.78	0	0
Instant 8		6.00	3.90	5.75	5.02	5.28	4.53	3	1
Instant 9		4.93	11.14	4.22	4.96	6.25	5.13	13	0
Instant 1		1.52	1.33	2.90	2.07	4.92	4.21	1	0
Instant 2		5.23	5.79	12.89	2.55	5.13	4.74	0	0
Instant 3		2.16	5.44	7.82	3.34	4.59	4.93	0	0
Instant 4		4.62	2.67	7.96	3.99	4.89	4.97	1	0
Instant 5	100-100	3.57	1.92	4.95	4.04	5.56	3.71	0	0
Instant 6		2.72	2.59	3.61	4.61	5.47	4.29	0	0
Instant 7		3.73	3.33	4.90	4.76	6.42	3.56	1	0
Instant 8		5.44	5.08	3.43	9.07	5.66	3.96	4	0
Instant 9		4.30	5.54	2.64	7.08	6.36	4.74	18	2

	X-Y sep (m)	Max error in position (m)			Max error in velocity (m/sec)			Step 4A trials	Step 4B trials
		x	y	z	Vx	Vy	Vz		
Instant 1		2.09	1.83	4.29	3.69	4.51	11.96	1	0
Instant 2		2.21	2.47	5.88	3.77	4.51	10.10	1	0
Instant 3		2.17	2.03	8.53	3.88	4.65	10.42	0	0
Instant 4		3.16	3.41	4.36	3.91	3.80	9.21	1	0
Instant 5	500-0	2.66	2.31	4.70	4.16	4.23	9.55	0	0
Instant 6		3.18	3.11	4.82	4.58	5.13	11.26	0	0
Instant 7		4.26	3.34	5.61	5.00	5.00	11.85	0	0
Instant 8		4.40	4.96	4.24	4.55	5.86	13.06	3	0
Instant 9		6.68	5.62	6.30	7.45	6.18	14.01	8	0
Instant 1		1.73	1.73	4.59	6.12	4.77	5.23	1	0
Instant 2		1.77	1.74	3.17	5.57	4.72	5.59	0	0
Instant 3		2.88	2.16	5.15	5.77	4.65	8.21	1	0
Instant 4		3.99	2.62	5.40	5.46	4.57	7.51	0	0
Instant 5	200-0	3.02	2.14	3.77	4.75	4.78	6.43	1	0
Instant 6		2.87	3.06	3.84	4.59	4.52	7.59	0	0
Instant 7		4.97	3.35	10.94	4.76	4.75	6.95	2	0
Instant 8		3.68	4.00	3.83	5.46	5.67	7.25	7	0
Instant 9		4.38	5.53	5.74	6.36	6.13	6.55	9	1
Instant 1		1.28	2.45	6.88	2.50	3.63	5.39	0	0
Instant 2		1.87	2.20	5.24	2.76	3.43	4.20	0	0
Instant 3		2.54	2.38	4.07	3.39	3.61	4.50	1	0
Instant 4		2.41	2.23	3.48	3.87	4.33	5.21	0	0
Instant 5	100-0	3.55	2.71	6.30	4.04	5.19	5.31	0	0
Instant 6		2.96	2.52	4.90	4.35	5.26	7.72	0	0
Instant 7		3.30	2.74	6.45	5.10	5.24	5.79	1	0
Instant 8		3.65	4.98	6.13	5.69	5.53	6.35	4	0
Instant 9		4.10	4.42	6.26	6.59	6.79	6.23	17	1

	X-Y sep (m)	Max error in position (m)			Max error in velocity (m/sec)			Step 4A trials	Step 4B trials
		x	y	z	Vx	Vy	Vz		
Instant 1		2.26	2.28	6.20	4.16	3.85	9.17	0	0
Instant 2		2.25	2.31	3.86	4.66	4.73	8.51	1	0
Instant 3		2.45	1.61	4.73	4.98	4.55	7.77	0	0
Instant 4		2.93	2.46	5.67	4.32	4.46	6.21	0	0
Instant 5	0-500	3.13	2.22	3.71	4.67	4.23	6.72	0	0
Instant 6		2.97	2.85	5.19	7.60	5.42	8.14	0	0
Instant 7		2.73	3.86	8.88	6.68	5.35	9.21	1	0
Instant 8		6.05	6.74	6.31	6.35	5.15	7.87	7	0
Instant 9		7.48	8.03	5.54	5.89	6.39	8.10	14	0
Instant 1		2.31	3.34	6.29	3.01	3.47	9.08	0	0
Instant 2		2.19	2.17	5.67	6.16	5.85	5.26	0	0
Instant 3		3.38	5.78	8.33	5.60	5.34	6.37	0	0
Instant 4		2.42	2.48	4.14	5.34	5.34	5.96	1	0
Instant 5	0-200	3.29	3.67	6.77	4.59	5.39	4.92	0	0
Instant 6		3.30	3.65	5.50	4.69	4.04	4.66	0	0
Instant 7		3.93	4.09	5.51	4.86	4.70	5.50	0	0
Instant 8		3.60	5.09	4.19	7.98	4.54	4.24	4	0
Instant 9		5.37	5.14	5.22	8.11	6.08	4.94	15	2
Instant 1		2.23	2.37	4.18	2.85	3.24	4.18	0	0
Instant 2		1.66	2.69	4.35	4.56	3.81	7.74	0	0
Instant 3		2.77	2.55	3.24	4.98	4.04	6.54	0	0
Instant 4		3.23	2.25	5.00	5.31	3.80	6.74	0	0
Instant 5	0-100	2.56	3.46	4.44	5.43	4.72	6.23	0	0
Instant 6		6.26	10.95	11.44	5.92	4.75	5.74	0	0
Instant 7		3.47	3.48	5.60	6.37	4.50	6.75	0	0
Instant 8		4.34	3.90	5.72	6.51	5.06	7.64	4	0
Instant 9		5.07	4.85	3.12	6.66	7.54	8.35	13	0

Figure 6.5: Scenario 2 results for targets with height difference of 500m, t_{max}=100ms

Targets present at same height, t_{max}=20msec

	X-Y sep (m)	Max error in position (m) x	y	z	Max error in velocity (m/sec) Vx	Vy	Vz	Step 4A trials	Step 4B trials
Instant 1		2.00	2.15	5.28	5.00	4.03	10.33	0	0
Instant 2		2.03	2.04	6.48	5.08	6.28	10.08	0	0
Instant 3		2.72	2.25	5.23	4.97	6.18	9.99	0	0
Instant 4		3.04	2.74	4.07	4.64	6.22	9.67	0	0
Instant 5	500-500	2.47	2.86	3.91	4.68	6.05	9.63	0	0
Instant 6		2.65	2.56	4.71	4.90	5.89	10.63	0	0
Instant 7		2.78	3.62	11.02	4.89	5.54	10.12	1	0
Instant 8		3.33	3.25	7.13	4.94	5.30	11.60	0	0
Instant 9		5.55	4.22	12.68	4.97	5.35	12.52	0	0
Instant 1		1.62	1.81	2.46	4.51	3.29	6.32	0	0
Instant 2		2.64	1.63	4.28	5.07	2.97	6.16	1	0
Instant 3		2.45	2.15	5.01	4.57	3.89	6.38	0	0
Instant 4		2.66	3.75	5.72	4.63	3.60	6.26	0	0
Instant 5	200-200	2.21	2.31	4.59	4.72	3.60	5.96	1	0
Instant 6		2.54	2.67	6.30	4.73	3.94	6.59	0	0
Instant 7		3.23	2.57	7.40	4.45	4.42	6.16	1	0
Instant 8		3.73	4.34	6.37	4.45	6.38	6.53	1	0
Instant 9		3.72	3.61	5.83	4.41	6.39	7.59	2	0
Instant 1		1.25	1.63	2.88	3.69	3.61	7.70	0	0
Instant 2		2.64	2.13	4.82	3.86	3.77	7.94	1	0
Instant 3		2.59	2.68	7.85	3.94	4.29	8.43	2	1
Instant 4		3.05	3.61	8.25	4.19	4.06	8.55	1	0
Instant 5	100-100	3.31	3.14	12.67	4.16	4.07	9.29	2	0
Instant 6		4.86	2.99	9.38	4.15	4.08	8.52	1	0
Instant 7		5.41	5.34	14.55	3.89	4.35	8.04	2	0
Instant 8		14.20	7.01	30.73	3.92	4.37	7.37	0	0
Instant 9		9.64	13.40	45.25	4.27	4.14	8.04	2	0

	X-Y sep (m)	Max error in position (m) x	y	z	Max error in velocity (m/sec) Vx	Vy	Vz	Step 4A trials	Step 4B trials
Instant 1		1.94	2.22	8.36	5.22	3.37	6.52	0	0
Instant 2		2.33	2.28	6.30	5.14	4.43	6.49	0	0
Instant 3		2.14	2.18	4.54	5.11	4.40	6.95	0	0
Instant 4		2.55	2.37	5.30	5.63	4.37	7.61	0	0
Instant 5	500-0	2.59	3.45	6.18	5.60	4.52	7.52	0	0
Instant 6		2.82	3.95	10.39	5.48	4.51	6.75	0	0
Instant 7		3.36	2.68	6.15	5.46	4.78	6.75	0	0
Instant 8		4.21	3.40	7.66	5.22	4.91	7.57	0	0
Instant 9		5.09	3.44	7.61	5.26	4.88	8.24	0	0
Instant 1		1.56	2.08	4.44	2.86	3.92	5.40	1	0
Instant 2		2.80	2.43	6.52	5.01	5.21	9.05	0	0
Instant 3		2.30	2.35	5.04	5.18	5.21	9.20	1	0
Instant 4		2.55	2.55	8.69	5.13	4.98	8.81	1	0
Instant 5	200-0	3.75	2.29	9.16	5.18	4.98	9.32	1	0
Instant 6		3.74	2.94	6.57	5.09	5.00	9.14	0	0
Instant 7		3.28	3.14	5.92	5.05	5.01	10.03	2	0
Instant 8		4.22	3.43	7.67	5.02	5.01	9.74	1	0
Instant 9		4.77	4.97	12.53	4.98	5.03	9.67	0	0
Instant 1		2.10	1.81	5.04	2.68	4.00	6.64	0	0
Instant 2		2.52	1.79	4.99	4.54	4.08	6.60	0	0
Instant 3		4.58	4.54	12.03	4.51	4.03	11.23	0	0
Instant 4		7.29	3.11	14.95	4.49	4.04	10.77	0	0
Instant 5	100-0	6.05	4.33	17.76	4.50	4.15	11.20	0	0
Instant 6		7.46	4.43	12.18	4.43	4.09	11.03	2	0
Instant 7		7.84	4.94	16.77	4.70	3.89	10.74	0	0
Instant 8		9.36	4.44	13.68	4.96	3.82	10.60	1	0
Instant 9		7.98	6.46	30.95	4.91	3.83	10.61	0	0

	X-Y sep (m)	Max error in position (m) x	y	z	Max error in velocity (m/sec) Vx	Vy	Vz	Step 4A trials	Step 4B trials
Instant 1		1.68	2.82	5.57	4.01	2.80	6.64	0	0
Instant 2		1.68	2.65	6.33	4.43	2.95	5.68	0	0
Instant 3		2.72	3.46	7.66	4.39	2.93	5.97	2	0
Instant 4		2.35	2.47	5.60	4.07	3.00	6.11	0	0
Instant 5	0-500	3.40	4.31	5.89	4.02	4.29	5.39	1	0
Instant 6		3.42	3.49	5.76	3.96	4.56	6.07	2	0
Instant 7		3.41	2.66	8.47	4.21	4.61	5.73	0	0
Instant 8		3.82	3.00	10.23	4.04	4.76	7.54	0	0
Instant 9		3.62	4.36	8.62	4.50	4.88	7.77	0	0
Instant 1		1.68	2.04	5.30	4.80	3.48	6.42	1	0
Instant 2		2.29	2.43	5.06	4.76	3.51	7.10	0	0
Instant 3		2.86	3.73	7.69	4.59	3.46	9.61	1	0
Instant 4		3.28	2.51	6.33	4.50	4.46	9.41	1	0
Instant 5	0-200	4.43	3.52	7.44	4.47	4.47	9.77	1	0
Instant 6		4.99	4.87	9.06	4.70	4.51	9.59	1	0
Instant 7		3.07	3.96	9.17	5.61	4.52	13.20	1	0
Instant 8		6.14	4.84	6.68	5.59	4.28	12.50	1	0
Instant 9		7.60	4.23	10.08	5.72	4.16	13.85	1	0
Instant 1		2.21	1.96	4.30	3.71	3.71	10.37	0	0
Instant 2		1.99	2.44	5.60	4.89	4.19	10.33	1	0
Instant 3		4.23	3.22	12.44	4.81	4.72	10.02	2	0
Instant 4		3.44	7.11	20.94	4.90	4.77	9.83	0	0
Instant 5	0-100	7.71	5.43	16.54	4.84	4.96	8.83	0	0
Instant 6		7.57	5.96	19.40	5.23	4.47	9.60	2	0
Instant 7		5.50	8.02	27.32	5.17	6.42	9.93	1	0
Instant 8		13.32	6.73	35.31	5.11	6.68	10.48	0	0
Instant 9		19.89	11.15	79.81	5.28	6.66	9.51	1	1

Figure 6.6: Scenario 2 results for targets present at the same height, t_{max}=20ms

- All targets are present at the same height of 1Km.
- No. of failed trials= 0.
- No. of trials with position error peaking= **13**.
  - Out of these 13, 3 trials correspond to 100-100 x,y separation case, 2 trials correspond to 100-0 case and 8 trials correspond to 0-100 case.
- No. of trials with velocity error peaking= 0.
- No. of trials entering step 4A= 46 .
- No. of trials entering step 4B= 2.
- Maximum error in position estimate for separation between the targets $\geq 200m$ along either x or y directions is $\leq\ 8m \times 5m \times 13m$.

- Maximum error in velocity estimate is $\leq 6m/s \times 7m/s \times 14m/s$.
- For 100m separation along both x,y directions:
  - For the first 7 instants, the maximum error in position estimate is $\leq 6m \times 6m \times 15m$.
  - The position error spikes up during the eighth and ninth instants.
- For separation of 100m along only x or only y directions (100-0, 0-100 cases):
  - During the first 3 instants, the maximum error in position estimate is $\leq 5m \times 5m \times 13m$.
  - From the 4^{th} instant onwards, the position error spikes (especially the error in z coordinate).
- From the earlier simulations discussed in this thesis, it has been observed that the performance of this algorithm becomes marginal for separation between the targets $\leq 100m$. The peaks in position errors observed in the 100m x,y separation cases can be attributed to the point that the initial separation between the targets (which was 100m) gradually decreases during the course of the simulation because of the target movements. As a result, the algorithm fails to identify and properly localise these targets leading to high errors.

Targets present at same height, t_{max}=100msec

	X-Y sep (m)	Max error in position (m)			Max error in velocity (m/sec)			Step 4A trials	Step 4B trials	X-Y sep (m)	Max error in position (m)			Max error in velocity (m/sec)			Step 4A trials	Step 4B trials	X-Y sep (m)	Max error in position (m)			Max error in velocity (m/sec)			Step 4A trials	Step 4B trials
		x	y	z	Vx	Vy	Vz				x	y	z	Vx	Vy	Vz				x	y	z	Vx	Vy	Vz		
Instant 1		1.78	1.59	6.05	5.34	4.97	9.67	1	0		1.98	2.09	4.19	4.33	3.03	10.41	0	0		2.02	2.06	4.68	6.95	2.58	8.69	0	0
Instant 2		2.16	2.14	4.74	5.53	4.80	9.15	1	0		2.09	1.85	6.22	4.33	3.57	11.17	0	0		3.20	1.74	6.58	6.84	2.62	8.09	0	0
Instant 3		3.04	2.81	4.67	5.39	4.90	9.81	0	0		2.76	2.35	4.38	4.25	3.98	11.55	2	1		3.66	2.33	6.39	6.30	3.91	7.04	0	0
Instant 4		3.72	2.41	4.82	5.28	4.70	9.66	0	0		2.90	2.56	4.71	4.68	3.49	11.92	1	0		2.62	2.70	5.76	5.97	4.32	6.88	0	0
Instant 5	500-500	2.73	2.56	5.63	5.28	4.80	10.55	0	0	500-0	3.30	3.28	9.64	3.98	3.23	10.44	1	0	0-500	3.79	2.61	5.25	6.13	5.23	8.68	0	0
Instant 6		2.73	2.23	3.84	5.06	4.34	8.32	0	0		2.91	2.57	5.82	3.75	3.26	6.02	1	0		2.84	2.75	6.13	6.05	5.32	8.21	2	0
Instant 7		4.99	2.69	6.41	5.68	4.12	9.07	2	0		2.84	2.73	5.64	3.61	3.48	6.59	1	0		4.39	3.85	7.68	6.23	4.97	9.44	1	1
Instant 8		3.92	4.61	7.06	5.68	4.12	9.20	1	0		3.20	3.55	5.74	4.13	3.32	6.95	0	0		3.53	3.64	5.88	6.39	4.56	9.76	1	0
Instant 9		5.09	6.10	12.80	5.48	6.16	13.63	2	0		5.02	3.88	9.90	3.89	3.53	7.64	0	0		3.62	3.26	5.69	5.72	4.62	10.41	0	0
Instant 1		2.64	2.18	11.29	3.49	3.47	7.77	1	0		1.45	1.66	5.15	3.67	3.89	8.51	0	0		2.21	1.78	5.07	6.95	3.36	11.38	0	0
Instant 2		1.94	3.05	3.75	3.68	4.91	7.26	1	0		1.87	2.45	4.88	4.21	4.10	9.24	1	0		2.17	1.82	6.17	7.12	3.97	11.64	1	0
Instant 3		2.47	2.30	4.65	3.97	4.98	6.69	2	0		3.46	1.87	4.85	4.97	4.30	8.79	1	1		3.84	2.52	5.18	6.68	4.03	11.37	1	0
Instant 4		2.82	2.58	5.89	4.06	5.04	8.05	0	0		2.46	2.66	5.79	4.80	4.28	7.56	3	0		3.43	2.78	8.34	5.90	4.33	8.79	0	0
Instant 5	200-200	2.93	4.16	6.61	4.93	5.10	7.13	1	0	200-0	3.28	3.00	7.18	4.88	4.23	8.08	1	0	0-200	4.35	3.26	7.30	6.09	4.61	10.15	3	0
Instant 6		2.72	2.62	5.04	5.08	5.15	7.03	1	0		4.10	3.16	10.92	4.61	4.76	8.18	2	0		2.69	4.50	5.31	5.97	4.79	9.53	2	0
Instant 7		4.80	2.74	7.51	4.83	5.01	8.74	2	0		2.92	3.32	8.17	4.70	4.82	7.90	0	0		5.97	4.12	15.34	5.55	4.69	8.62	1	0
Instant 8		5.42	3.63	16.99	5.70	4.65	8.67	1	0		5.49	3.47	11.81	5.06	4.76	7.52	0	0		3.01	5.21	8.72	5.41	4.30	6.74	1	0
Instant 9		4.17	3.29	5.85	5.66	4.33	7.87	3	0		9.69	6.58	36.94	5.17	4.60	9.23	0	0		34.09	59.97	32.07	5.51	4.31	7.62	1	1

Figure 6.7: Scenario 2 results for targets present at the same height, t_{max}=100ms

- All targets are present at the same height of 1Km.

- No. of failed trials= **1** (corresponds to 0-200 x,y separation).
- No. of trials with position error peaking= **2** (1 trial each corresponding to 200-0 and 0-200 x,y separation cases).
- No. of trials with velocity error peaking= 0.
- No. of trials entering step 4A= 40.
- No. of trials entering step 4B= 4.
- Maximum error in position estimate for a separation of 500m between the targets along either x or y directions is $\leq\ 6m \times 7m \times 13m$.
- Maximum error in position estimate for a separation of 200m between the targets along both x and y directions is $\leq\ 6m \times 5m \times 17m$ (error along z direction slightly increases).
- For targets, separated by 200m along only x or only y directions,
  - Till the 8^{th} instant, the maximum error in position estimate is $\leq\ 6m \times 6m \times 16m$.
  - During the 9^{th} instant, the position error peaks. The peak in position error can be attributed to the reduced x,y separation between the targets caused as a result of movements during the simulation period.
- Maximum error in velocity estimate is $\leq\ 8m/s \times 7m/s \times 14m/s$.
- The cases corresponding to 100m x,y separation are not reported, as the targets tend to come **very close** to each other (might even **cross over**) during the subsequent instants and our algorithm fails to identify such very close targets.

INFERENCE

The results obtained above can be summarised as follows.

For the height difference of 500m between the targets and $t_{max} = 100$ milliseconds, we are able to localise the targets with an x,y separation of atleast 100m between them for all the 9 instants with reasonably good accuracy.

When all the targets are present at the same height and t_{max} is 20 milliseconds, the targets separated by atleast 200m along either x or y directions are being localised well. But for targets with a separation of 100m along either x or y directions, we are able to localise them with a decent accuracy only till few instants (7 instants for 100m-100m separation case and 3 instants for 100m-0m, 0m-100m cases). This is because the initial separation between the targets was taken to be 100m and as time progresses the targets might move closer to each other thereby, reducing the separation between them. Since, the performance of our algorithm is not that good for separations <100m, it fails to identify these targets, resulting in errors peaking.

When targets are present at same height and t_{max} is taken as 100 milliseconds, the targets with separation of 500m along either x or y directions and the ones separated by 200m along both x,y directions are localised with reasonably good accuracy (error along z coordinate is slightly higher for 200m separation case). But the targets separated by 200m along only x or only y planes are localised with decent accuracy only till the first 8 instants. During the final (9^{th}) instant, the localisation error spikes up. This spiking in the error of position estimate can be attributed to the reduced x,y separation between the targets caused over the course of the simulation due to the motion of targets. The results for 100m x,y separation cases are not reported as for $t_{max} = 100ms$, the separation between the targets might become very low (even close to zero) and the targets might even cross over each other. Since our algorithm fails to identify such closely spaced targets, the results have been omitted.

It has also been observed that with the usage of tracking for localisation, the number of trials entering step 4A and step 4B of algorithm 5.2.1 has decreased.

6.4 Advantages of tracking

With the integration of tracking to the Generalised Data association and Localisation algorithm of section 5.2.1, the computational complexity has decreased as tracking makes use of previous instant's position and velocity estimates to perform associations rather than going through and processing the entire data from scratch.

But care must be taken to ensure the accuracy of the estimates used for tracking as an erroneous estimate may lead to the estimates of the coming instants also to be erroneous, thereby, leading to a chain of erroneous results.

CHAPTER 7

CONCLUSION

In this thesis, a generalised algorithm to perform data association and localisation of N targets using the ToA (or lag) values of the targets corresponding to different bistatic pairs has been proposed. The algorithm has been tested by taking a maximum of 4 stationary targets at once and the maximum error in the position estimate for targets present at any height difference with an x,y separation of atleast 200m, was found to be $\leq 10m \times 10m \times 12m$ most of the times (slightly increases for some corner cases). The performance holds for most 100m separation cases as well (fails for <4% of the cases).

In addition, an algorithm which performs tracking of the targets by making use of the position and velocity estimates of the previous instants has also been designed and integrated with the generalised algorithm. With this combined algorithm, we were able to localise maximum 9 moving targets coming into our observation region one-by-one with arbitrary velocities and an x,y separation of atleast 200m between them with a maximum error in position estimate to be $\leq 10m \times 10m \times 16m$ for most of the trials.

FUTURE WORK

- So far, Doppler values have only been used for tracking. The future work in this project would include using the bistatic Doppler frequency values for one level of association.
- When targets are present far from each other with x,y separations > 4km, the receivers see only a subset of the targets. Few parts in the algorithm have to be modified to accommodate the scenarios of more than two targets present with such far separation.
- The performance of the algorithm for 1Km ISD (where towers are brought closer) has to be investigated.
- Tracking of cross-over targets without ambiguity.
- Improving the ambiguity region with the current range resolution of the waveform and optimization of the algorithm (to run even faster).
- Exploring the concept of floating fusion centre, which might be helpful in localisation of targets present around the corners of outer cells.

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