Two-Dimensional Infrastructure-Based Wireless Networks: Coverage and Percolation Properties

A Project Report

submitted by

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THESIS CERTIFICATE

This is to certify that the thesis titled **Two-Dimensional Infrastructure- Based Wireless Networks: Coverage and Percolation Properties**, submitted by **Timmadasari Sumanth**, to the Indian Institute of Technology, Madras, for the award of the degree of **Master of Technology**, is a bonafide record of the project work done by him under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

KEYWORDS: Infrastructure Based Wireless Network; Independent disc model; Percolation

We consider an infrastructure-based wireless network comprising two types of nodes, namely *relays* and *sinks*; the sink nodes are connected to a wireline infrastructure, while the relay nodes are used to extend the network coverage by providing multi-hop paths to the sink nodes. We are interested in understanding the coverage and percolation properties of such infrastructure-based wireless networks; In this thesis, we present results from an extensive simulation study that provide valuable insights towards understanding these properties.

Coverage Properties: We first consider the SNR (Signal to Noise Ratio) model that yields circular coverage disks around each node. Thus, we say that a location is covered if it simply lies within the range of some node that has a multi-hop path to a sink node; otherwise the location is said to be uncovered or vacant. We compute the fraction of vacant region (average vacancy) created in the SNR model as a function of the sink and relay node densities. We also evaluate the average vacancy created in a traditional coverage processes model, referred to as the *independent-disk model*; we observe that the average vacancy created in the independent-disk model serves as a lower bound for that created in our model. Our other results for the SNR model includes a study on the hop-count constrained vacancy, and a network optimization problem (where we minimize the average deployment cost of the network subject to a constraint on the average vacancy). We next extend our results to a more general SINR (Signal to Interference plus Noise Ratio) model which takes the interference (caused by other nodes in the network) into account while evaluating the nodes' coverage regions.

Percolation Properties: For an all-infrastructure (or all-sink) network it is known (from continuum percolation) that there exists a finite-valued threshold on sink density such that above this threshold the network contains a giant connected component, while such a component does not exist for sink densities below the threshold. This non-

trivial threshold on sink density where the network undergoes a sudden transition from a disconnected to an almost-connected network is referred to as the percolation threshold. Following the procedure devised in Martens and Moore, we first compute the value of the percolation threshold for an all-infrastructure network. Next, we extend their procedure to a 1-hop constrained, infrastructure-based network; here, since both sink and relay nodes are involved, we obtain a percolation boundary (instead of a single percolation threshold) such that giant-components are formed for values of sink and relay density pairs beyond the boundary. Finally, we have also computed the percolation boundary for a restrictive version of our model (referred to as the Poisson AB model) where two sink nodes are connected only via. a relay node. We compare the above percolation threshold with the upper bound proposed by Iyer and Yogeshwaran; we find that the latter bound is weak for smaller values of sink densities, while being a good approximation at higher values.

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NOTATION

L	Side of square region
λ	Node density (Nodes/unit area)
β	Probability that a node is sink
r	Coverage radius of a node
SNR	Signal to Noise ratio
SINR	Signal to Interference plus Noise Ratio
P	Transmitting power
η	Path Loss Exponent
N_0	Noise Power
SNR_{th}	Threshold SNR
$SINR_{th}$	Threshold SINR
$v_{\lambda,\beta}$	Average vacancy in SNR case
h	Number of hops
$v_{\lambda,\beta,h}$	Hop constrained vacancy in SNR case
$w_{\lambda,\beta}$	Lower bound for vacancy in SNR case
C_S	Cost of sink node
C_R	Cost of relay node
$c_{\lambda,\beta}$	Optimal network installation cost per unit area
λ^*	Optimal node density
β^*	Optimal sink node probability
$\overline{v}$	Vacancy constraint
γ	Interference constant
$v^i_{\lambda,\beta,\gamma}$	Average vacancy in SINR case
$w^i_{\lambda,\beta,\gamma}$	Lower bound for vacancy in SINR case
λ_S	Sink node density
λ_R	Relay node density
$R_L^{ac}(\lambda_S, \lambda_R)$	'Any' percolation probability CDF
$R_L^{hc}(\lambda_S, \lambda_R)$	'Horizontal' percolation probability CDF
$R_L^{bc}(\lambda_S, \lambda_R)$	'Both' percolation probability CDF
$R_L^{ac}(\lambda_S)$	'Any' percolation probability CDF for $\beta = 1$
$R_L^{hc}(\lambda_S)$	'Horizontal' percolation probability CDF for $\beta = 1$
$R_L^{bc}(\lambda_S)$	'Both' percolation probability CDF for $\beta = 1$

CHAPTER 1

Introduction

Coverage is generally defined as the fraction of network area where the signal strength is greater than some threshold. Vacancy is uncovered part of the network area i.e., where the signal strength is less than the threshold. Vacancy is complement of coverage. Coverage is one of the most important metrics of a wireless network. Increase in the number of users and expansion of the urban areas demands the high coverage of the network. With the modern advancement of robotics and Internet of things, there is a need for good coverage for the reliable communication between the robots. So, coverage has become an important issue for the network engineers to make the communication more reliable. Let us know first the different types of wireless networks.

Wireless networks are generally classified into two categories (generally called topologies), all-Infrastructure and Infrastructure-based. In all-infrastructure wireless network, all the devices or users are connected to one of the access points among a set of access points (nodes) that are connected directly to the wireline infrastructure. Here, a device is said to be connected if it lies in the coverage region of one of the access points. This is a centralized network where there is no need of connectivity between the nodes or access points. Most of the networks used are all-infrastructure networks as these are easy to deploy.

On the other hand, Infrastructure-based wireless networks comprises of two types of nodes, namely sink and relay nodes. Sink nodes are similar to the nodes in all-infrastructure case and relay nodes are used to extend the network coverage by providing multi-hop paths to the sink nodes that are directly connected to the wireline infrastructure. A device is said to be covered if it is connected directly to a sink node or to a relay node that is connected to a sink node through multihops. Here, there is a need of connectivity between sink-relay and relay-relay nodes unlike in all-infrastructure case. So, it is not easy to deploy these networks without proper planning. With the demand of more and more coverage, the deployment of infrastructure-based wireless networks is drawing attention.

Study of coverage properties of one-dimensional infrastructure-based wireless networks is already presented in previous literature [K. P. Naveen and Kumar Anurag (2016)]. They can be used in real world applications like sensor nodes deployed along the border for intrusion detection, vehicular network along a highway with vehicles as relay nodes and base stations installed along the highway as sink nodes, etc. This study is applicable only for the one-dimensional applications. As most of the required practical applications are in two-dimensions, we are extending this to study the coverage properties of Two-dimensional infrastructure-based wireless networks. Our study is applicable to a femto-cellular setting where a large number of femto cell base stations (relays) to extend the coverage of existing cellular base stations (sinks), for example 5G and next generations. It can be applied to a vehicular network in a city or town where vehicles themselves act as relays to extend the coverage of the base stations installed. It can also be applicable to the reliable communication between the robots where robots act as relays and routers acts as sinks. Robots themselves are used to extend the coverage of the routers.

As we are using more and more number of nodes to increase the coverage, one of the important factor we need to consider seriously is network deployment cost. It is one of the important issues for the networking or telecom companies as they have to raise the call and internet charges increasing burden on the users. We have studied how the optimal cost varies with vacancy constraint for different relay and sink node costs.

In the next part we will see another property of network called Percolation. Percolation depends on the connectivity between the nodes. A network is said to be percolated if there exists a connection from one side to another side of the network i.e., there exists a cluster of nodes that spans both sides of the network. The minimum density of the nodes required for the system to percolate is called percolation threshold. If there is high coverage, there is a greater chance of getting percolation of the network. Percolation theory has been used to find connectivity properties of many disordered physical systems, for example lattice percolation. In [Mertens Stephan and Moore Cristopher (2012)], authors used union-find algorithm to compute the percolation thresholds for two dimensional percolation of arbitrary shaped objects like discs, randomly rotated sticks, aligned and rotated squares. We used this algorithm to find the percolation threshold of two-dimensional infrastructure-based wireless network i.e., at what densities of sink and relay nodes the percolation happens. This study gives us the idea of how dense the relays and sink nodes should be there in order to the message to travel from one end to the other.

1.1 Thesis Outline

The rest of the thesis has been organized as follows. In Chapter 2, we broadly review the existing literature related to the coverage and percolation properties of wireless networks. Chapter 3 describes the system model for vacancy, hop-constrained vacancy and cost optimization.

In Chapter 4, we simulate average vacancy and lower bound in SNR case. We study the hop-constrained vacancy in SNR case. We minimize the network deployment cost subject to a vacancy constraint. In chapter 5, we simulate average vacancy and lower bound for vacancy in SINR case. In Chapter 6, we study the percolation properties of the network. Percolation thresholds of ’any’, ’horizontal’ and ’both’ percolations are found for the following cases: (i) $\beta = 1$ (ii) $\beta \in [0, 1]$ and one-hop connectivity (iii) $\beta \in [0, 1]$ and strictly one-hop connectivity.

CHAPTER 2

Literature Survey

2.1 Coverage survey

Traditional coverage processes, where all nodes are of same type (sinks), have been extensively studied in the book [P. J. Diggle (1990)]. Application of coverage processes to wireless communication have been discussed in [Jeffrey G. Andrews, Francois Baccelli and R. K. Ganti (2010), H. S. Dhillon, R. K. Ganti and J. G. Andrews (2011), Baccelli Francois and Blaszczyszyn Bartlomiej (2000)]. In [Jeffrey G. Andrews, Francois Baccelli and R. K. Ganti (2010)], authors considered SINR model where the region covered by a node depends on both the signal power and the interference power received from all other nodes. In [H. S. Dhillon, R. K. Ganti and J. G. Andrews (2011)], nodes are heterogeneous in terms of their transmit power and their SINR threshold.

Desai and Manjunath in [M. Desai and D. Manjunath (2002)] considered a network with a finite number of nodes deployed on a line of finite length and obtained the exact formula for the probability that the entire network is connected. Miorandi and Altman in [Miorandi Daniele and Altman Eitan (2006)] considered a queuing theoretic approach to compute the coverage probability for one-dimensional networks. One of the early work considering an infrastructure-based architecture is that of Dousse [Olivier Dousse, Patrick Thiran and Martin Hasler (2002)] in which the relay nodes are Poisson distributed and the sink nodes are placed equi-distance from each other. In [S. I. Sou (2010)], Suo obtained the probability that all vehicles (relay nodes) within a road segment of finite length are connected to both road side units (sink nodes) located at either ends of the road segment. In [S. C. Ng, G. Mao and B. D. O. Anderson (2012)], more than two sink nodes are deployed at arbitrary locations within a segment of finite length and the relay nodes are Poisson distributed; the coverage probability is obtained.

In [K. P. Naveen and Kumar Anurag (2016)], authors considered a one-dimensional infrastructure based wireless network with sink and relay nodes, poisson distributed along the positive real line. The authors drawn an analogy between the connected components of the network and the busy periods of an M/D/∞ queue, and using renewal theoretic arguments, they obtain an explicit expression for the average vacancy in SNR case. They also computed an upper bound for vacancy by introducing the notion of left-coverage. They proved a lower bound by coupling the model with an independent-disk model, where the sinks' coverage regions are i.i.d. They studied the problem of minimizing network deployment cost subject to a constraint on the average vacancy. They also conduct simulations to understand the properties of hop-constrained coverage in SNR case.

2.2 Percolation survey

In [Mertens Stephan and Moore Cristopher (2012)], authors found precise values of the percolation transition for disks, squares, rotated squares, and rotated sticks in two dimensions, and confirmed that these transitions behave as conformal field theory predicts. They also measured the finite-size exponent, giving strong evidence that these continuum models are in the same universality class as lattice percolation. They found that the probability of a wrapping cluster at criticality is precisely that predicted by conformal field theory.

Two models, the Boolean model and the random connection model, are treated in detail in [R. Meester and R. Roy (1996)], and related continuum models are discussed. Important techniques are applied to obtain results on the existence of phase transitions, equality and continuity of critical densities, compressions, rarefaction, and other aspects of continuum models. In [Iyer, K. Srikanth and D. Yogeshwaran (2012)], authors studied the percolation properties of AB Poisson Boolean model which is a generalization of the AB percolation model on discrete lattices. They showed the existence of percolation for all dimensions greater than 2 and derive bounds for a critical intensity. For AB random geometric graph, they derived a weak law result for the largest nearest-neighbor distance and almost-sure asymptotic bounds for the connectivity threshold. In [Penrose and D. Mathew (2014)], continuum percolation of bipartite random geometric graph is discussed. Authors give a strong law of large numbers for the connectivity threshold of this graph.

CHAPTER 3

System Model

3.1 Vacancy

We consider a square area of size L. We place a large number of sink and relay nodes according to poisson distribution with mean parameter λ. Each node is independently a sink node with probability $\beta \in (0,1]$. Each node has a circular coverage region of radius r (r > 0). The area is shown in the figure below. Nodes which are transmitting are sinks and rest of them are relays. The blue (sink) and green (relay) regions constitutes the covered region. The remaining white and red (relay node connected to any sink) portions is the uncovered or vacant region (vacancy is complement of coverage). As the author in the one-dimensional Infrastructure Based Wireless Network considered vacancy instead of coverage, we are also considering vacancy. The vacancy is denoted by $\upsilon_{\lambda,\beta}$.

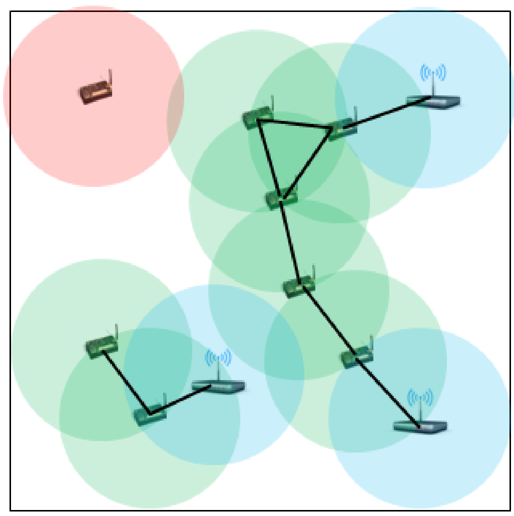


Figure 3.1: 2-D Infrastructure based wireless network: coverage

3.2 Hop-constrained vacancy

As the number of hops increases, the time for communication also increases. In order to reduce time for communication, we are introducing hop-constrained vacancy considering maximum number of hops required for the connection. Suppose maximum number of hops allowed is 'h', then a point is said to be covered if it is directly connected to a sink node or is connected to a relay node which is then connected to a sink node in less than or equal to h hops. Hop-constrained vacancy is denoted by $v_{\lambda,\beta,h}$. It can be seen in the figure below. Here also, only the blue and green regions constitutes the hop-constrained covered region.

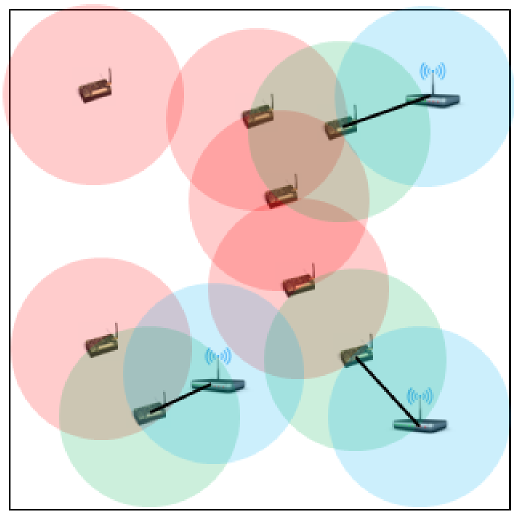


Figure 3.2: Coverage with one hop constraint

3.3 Cost-optimization

Let C_S be the cost of a sink node and C_R be the cost of a relay node. Deployment cost per unit area is given by $c_{\lambda,\beta} := \lambda\beta C_S + \lambda(1-\beta)C_R$. Our objective is to minimize this cost subject to the average vacancy constraint.

$$Minimize_{(\lambda,\beta)}\ c_{\lambda,\beta}$$

$$\text{subject to } v_{\lambda,\beta} \leq \overline{v}$$

CHAPTER 4

SNR case

We will first consider Signal-to-Noise ratio (SNR) to define the coverage of a node. The definition of SNR is as follows:

SNR at a point due to a node is defined as

$$SNR = \frac{P * d^{-\eta}}{N_0} \tag{4.1}$$

where

P = Transmitting power,

d = Distance between node and the point,

η = Path Loss exponent,

N_0 = Noise Power.

The point is said to be connected to the node if this SNR is greater than some threshold SNR (SNR_{th}) i.e., $SNR > SNR_{th}$. then the formula (4.1) will give

$$d < (\frac{P}{SNR_{th} * N_0})^{\frac{1}{\eta}} \tag{4.2}$$

We will take right hand side of equation (4.2) as 'r' where r is same as we get from

$$r = argmax_d(SNR > SNR_{th}) \tag{4.3}$$

So, a point is said to be connected to a node if it is at a distance less than 'r' from the node. The coverage of the node is the circle of radius 'r' around it. Two nodes are connected if the distance between them is less than 'r'. Finally, to complete the definition of coverage, a point is said to be covered if it is directly connected to a sink node or to a relay node that is connected to any sink node through multi-hops.

4.1 Algorithm to find average vacancy of network

Let λ nodes/unitarea be the node density and β is the fraction of sink nodes among all nodes. The average vacancy for a fixed λ and β is denoted by $v_{\lambda,\beta}$. The algorithm to find the average vacancy for a square area of side L units for different values of λ and β is as follows

1. Consider an area of side L units.

2. Fix one value of λ and β.

3. Generate N number of nodes according to a poisson random variable with mean parameter λL^2.

$$N \sim poi(\lambda L^2) \tag{4.4}$$

4. Select each as sink node independently according to a bernoulli random variable with probability of success β ($0 < \beta \leq 1$). It means probability of being a relay node is 1-β.

(It means, sink and relay node densities are $\lambda\beta$ and $\lambda(1-\beta)$ respectively. This is similar to generating N_S number of sink nodes according to poisson random variable with mean parameter $\lambda\beta L^2$ and N_R number of relay nodes according to poisson random variable with mean parameter $\lambda(1-\beta)L^2$)

5. Place the nodes in the square region of area L^2 with x and y coordinates of nodes distributed uniformly i.e., x $\sim$ unif(0,L) and y $\sim$ unif(0,L). It means nodes are distributed uniformly in the region. The region with L=10 will look like as shown in the figure 4.1.

6. We will find whether the mid-point of the region is vacant or not in order to find the average vacancy of the region. First, find the initial set of nodes that are connected to the mid-point.

7. If the set is empty then the midpoint is not connected. Therefore, we will take $v_{\lambda,\beta} = 1$.

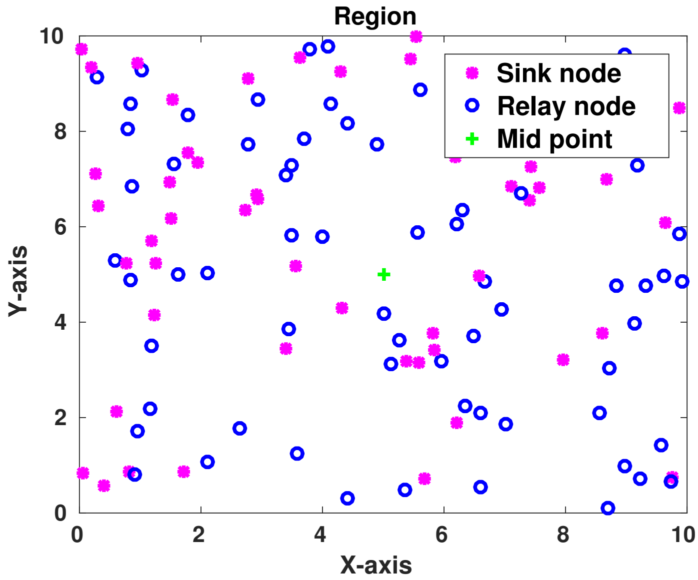


Figure 4.1: Square Region of side L=10 with λ=1 and β=0.4

8. if the set has one or more sink nodes, then the midpoint is connected. Therefore, we will take $v_{\lambda,\beta} = 0$.

9. If the initial set doesn't satisfies above 2 conditions, then find the new set of nodes that are connected to the initial set of nodes and check the above 2 conditions and take the value of $v_{\lambda,\beta}$ accordingly.

10. Repeat this finding of new set of nodes until the new set is empty or has atleast one sink node and take the value of $v_{\lambda,\beta}$ accordingly.

11. Repeat the steps from 3 to 10 for large number of iterations and find the average $v_{\lambda,\beta}$ for that number of iterations. This gives the probability that the mid-point is vacant (average vacancy) for a particular value of λ and β.

12. Find the $v_{\lambda,\beta}$ for different values of λ and β.

4.2 Algorithm to find lower bound of vacancy

Lower bound of vacancy (denoted by $w_{\lambda,\beta}$) is obtained by placing independent coverage discs (regions) around sink nodes. It is obtained by keeping each sink node at a time, place poisson randomly generated relay nodes around it and finding the coverage region for each sink node. The coverage region is the union of the coverage regions of all the sink nodes. This region is larger than the region obtained by placing all poisson randomly generated (as in average vacancy case) sinks and relays at the same time. The vacancy calculated in this model is a lower bound to the average vacancy found in previous section (This is proved in one-dimensional case [K. P. Naveen and Kumar Anurag (2016)] based on a coupling argument).

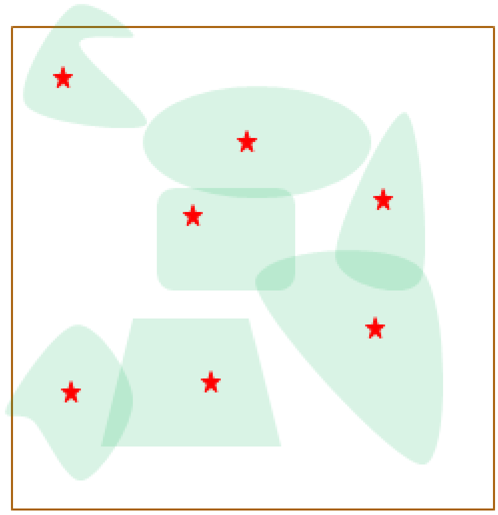


Figure 4.2: Independent discs around sink nodes for finding lower bound

In the average vacancy case, the coverage regions of sink nodes are dependant as the relays nodes are generated by a single poisson process. Hence, this model is called *Dependant-Disc model*. Whereas in finding the lower bound, the coverage regions of sink nodes are independent and identically distributed. Hence this model for finding lower bound is called *Independant-Disc model*.

The algorithm for finding the average vacancy for each iteration is ended by the step 10. The algorithm for finding the lower bound is to add the steps given below before step 11. The following is the algorithm to find lower bound of vacancy for each iteration for a fixed λ and β.

1. Keep one sink node at the same place as in the previous realization and remove all other sink and relay nodes.

2. Generate N_R number of relay nodes according to a poisson random variable with mean parameter $\lambda(1-\beta)L^2$ and place these relay nodes in the square region of side L uniformly as before. This will look like the figure 4.3.

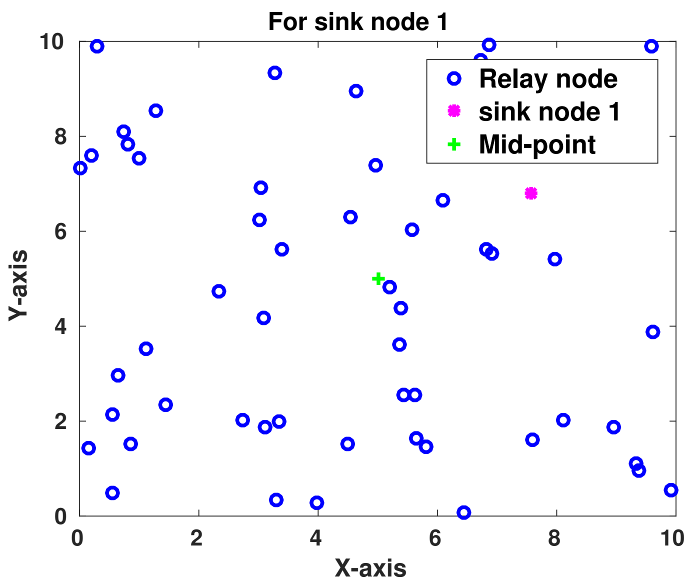


Figure 4.3: One sink node with relay nodes

3. Find whether the mid-point of the region is connected to that sink node.

4. Repeat this by placing the other sink nodes one by one with different independent realizations of relay nodes and find the connectivity of mid-point to each sink node.

5. If the mid-point is connected to atleast one of the sink nodes, then it is said to be connected and take $w_{\lambda,\beta} = 0$ else $w_{\lambda,\beta} = 1$.

4.2.1 Simulation Results

Figure 4.4 shows average vacancy vs node density λ for different values of β for L=10. The parameters considered are P = 2W, η = 4, N_0 = 5W, $SINR_{th}$ = 0.4. It gives r = 1unit. (These parameters are same for all simulations in SNR case)

Analysis:

* For a particular value of β, average vacancy decreases with increase in λ. It is because, as node density λ increases, number of sink and relay nodes increases which increases the coverage. So, vacancy decreases.

* The average vacancy goes to 0 after λ = 2 even for small β values. It means we need λ > 2 and λ > 3 for the coverage of atleast 80% and 95% respectively.

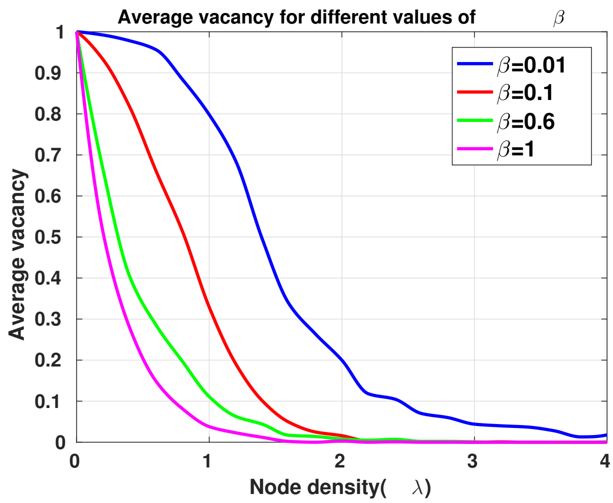


Figure 4.4: Average vacancy vs λ for different β values

* For a particular value of λ, as β value increases, the average vacancy decreases. This is because, increase in β increases the fraction of sink nodes thereby increasing coverage.

* For a high value of β, low value of λ is enough to get more coverage and for a low value of β we need high value of λ.

Figure 4.5 shows the average vacancy $v_{\lambda,\beta}$ and lower bound $w_{\lambda,\beta}$ for different values of λ and β for L=10.

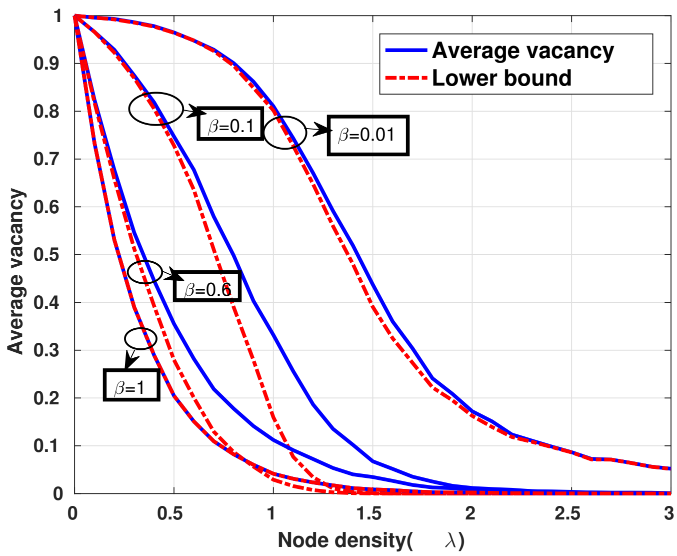


Figure 4.5: $v_{\lambda,\beta}$ and $w_{\lambda,\beta}$

Analysis:

* For a fixed value of λ, the difference between $v_{\lambda,\beta}$ and $w_{\lambda,\beta}$ increases as β increases except for β=1. Hence the lower bound is a good approximation to the average vacancy for smaller values of β. This is because, as β is small, the distance between sink nodes is more so that overlapping between adjacent coverage discs is less. So, it essentially appears as independent disc model where i.i.d coverage discs are placed around each sink node.

* For a fixed β, the difference between $v_{\lambda,\beta}$ and $w_{\lambda,\beta}$ increases as λ increases. This is because, as λ increases, number of nodes increases which increases area of the coverage discs. So, probability of overlapping of adjacent coverage discs increases thus increasing dependency in the model.

4.3 Hop-constrained vacancy

If the number of hops between the user and the sink node is high, the time required for communication is more. This delay in communication is not a good criteria as it causes breaks in the communication if the users are moving. So, we introduce hop-constrained vacancy where an user is said to be connected if it is directly connected to a sink node or is connected to a relay node which is further connected to any sink node that is less than fixed number of hops away. It is denoted by $v_{\lambda,\beta,h}$, where h denotes maximum number of hops allowed for connectivity.

4.3.1 Simulation results

Figure 4.6 shows the average vacancy $v_{\lambda,\beta}$ and hop-constrained vacancy $v_{\lambda,\beta,h}$ (from 0 to 9 hops) for $\beta = 0.01$ and $\beta = 0.1$ for L=10.

Analysis:

* As expected, as the h value increases, the coverage increases as we are allowing more number of hops.

* As the h value increases to ∞, $v_{\lambda,\beta,h}$ converges to $v_{\lambda,\beta}$.

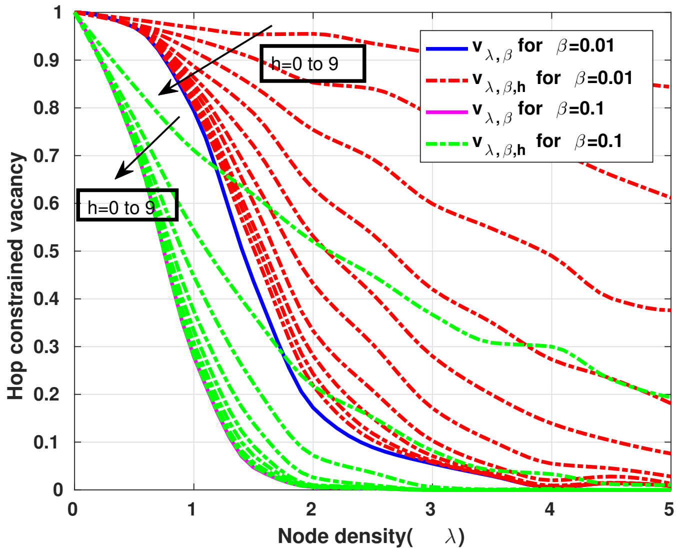


Figure 4.6: $v_{\lambda,\beta}$ and $v_{\lambda,\beta,h}$ for β=0.01 and β=0.1

* The value of h for the $v_{\lambda,\beta,h}$ to be best approximation to $v_{\lambda,\beta}$ is high for β=0.01 compared to β=0.1 (9 for β=0.01 and 5 for β=0.1) because as β increases, less number of hops are required to get the same coverage as unconstrained coverage. When β is 0.01, the sink nodes are sparse, so large number of hops are required to get the same coverage as unconstrained coverage.

4.4 Cost Optimization

As number of nodes in the network are more, one of the important factor we have to keep in mind is network deployment cost. As we have two types of nodes, sink and relay, the costs of sink and relay are different. Generally the cost of sink node is higher than relay node because sink node is connected to the wireline infrastructure and it has more hardware complexity, whereas relay node is just to transmit whatever is received.

Let C_S be the cost of a sink node and C_R be the cost of a relay node. Deployment cost per unit area is given by $c_{\lambda,\beta} := \lambda\beta C_S + \lambda(1-\beta)C_R$ where $\lambda\beta$ is the sink density and $\lambda(1-\beta)$ is the relay density. Our objective is to minimize this cost subject to the average vacancy constraint.

$$Minimize_{(\lambda,\beta)}\ c_{\lambda,\beta}$$

$$\text{subject to } v_{\lambda,\beta} \leq \overline{v}$$

It gives the optimal deployment cost for the average vacancy less than or equal to $\overline{v}$. It is equal to the optimal cost for the coverage more than or equal to $1 - \overline{v}$.

We will also find optimal node density λ^* and sink probability β^* for an average vacancy constraint.

4.4.1 Simulation results

Figure 4.7 gives the relation between optimal cost $c_{\lambda,\beta}$ and vacancy constraint $\overline{v}$ for different relay node costs. Here, we considered relay node cost as 1 unit ($C_R = 1$) without loss of generality. We took 6 sink node costs C_S as multiples of C_R and L=10.

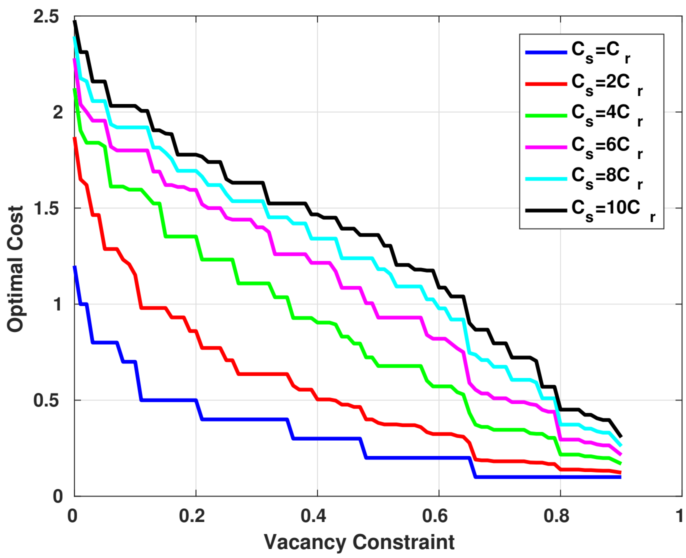


Figure 4.7: Optimal cost vs vacancy constraint ($\overline{v}$) for different relay node costs

Analysis:

* For a fixed value of C_S, optimal cost reduces with increase in average vacancy constraint. This is because, as constraint on vacancy increases, we are giving relaxation to vacancy which requires less number of nodes i.e., low node density.

* If the sink node cost is more, then further reduction in sink node cost do not significantly reduce network cost. If the sink node cost is less, then further reduction in sink node cost will have much impact on the optimal cost. From the figure, for $\overline{v} = 0.2$, network cost reduced by 2.8% (from 1.75 to 1.7) if sink node cost changes from $10C_R$

to $8C_R$ and reduced by 39% (from 1.4 to 0.85) if sink node cost changes from $4C_R$ to $2C_R$.

Figures 4.8 and 4.9 gives optimal node density λ^* and optimal sink node probability β^* for $C_S = 4C_R$ and $C_S = 10C_R$ respectively.

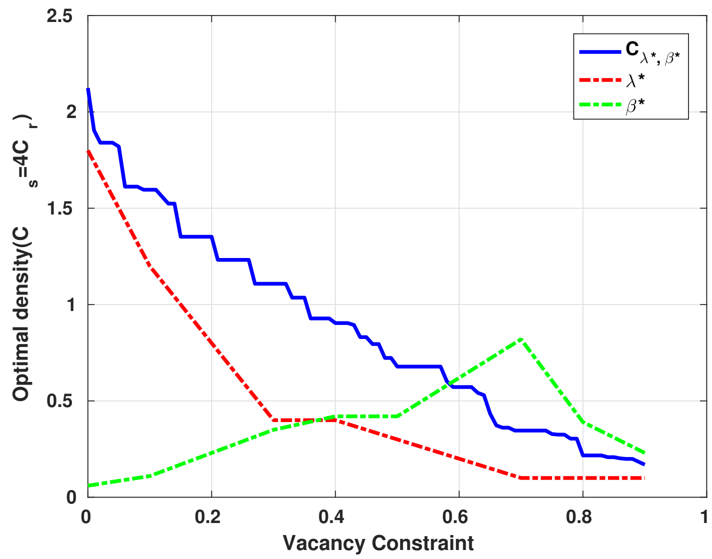


Figure 4.8: Optimal node density and sink probability for $C_S = 4C_R$

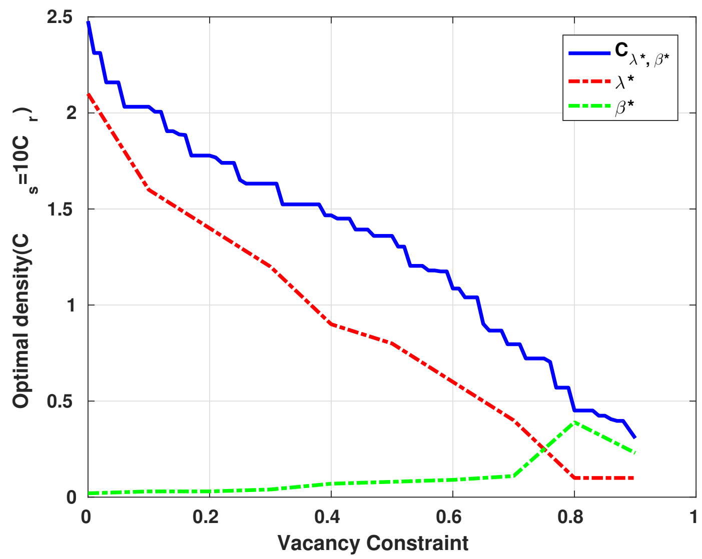


Figure 4.9: Optimal node density and sink probability for $C_S = 10C_R$

Analysis:

* In both figures, as the vacancy constraint increases, the optimal node density λ^* decreases because increase in vacancy constraint means relaxation on the vacancy which requires less number of nodes are required i.e, low node density.

* In both figures, β^* increases as vacancy constraint increases. It means, as vacancy constraint increases (i.e., for less coverage), we need higher fraction of sink nodes compared to that of relay nodes for achieving optimal cost. But for lower vacancy constraint (for high coverage), we need lower fraction of sink nodes and higher fraction of relay nodes for achieving optimal cost. (For $C_S = 4C_R$ and vacancy $\leq$ 0.1 (i.e., coverage $\geq$ 0.9), $\beta^* = 0.1$).

* β^* increases faster for $C_S = 4C_R$ compared to $C_S = 10C_R$ because, as the sink cost is already high for $C_S = 10C_R$, we don't need higher fraction of sinks for achieving optimal cost for a particular vacancy constraint.

* β^* increases upto $\overline{v} = 0.7$ for $C_S = 4C_R$ and upto $\overline{v} = 0.8$ for $C_S = 10C_R$ and decreases after that. This is a special criteria which we are considering it for future analysis.

CHAPTER 5

SINR case

In the previous chapter we considered Signal to Noise Ratio (SNR) for defining the coverage. But in practical applications, there will be interference from the other nodes. This interference will cause great impact on the coverage. We generally use frequency reuse to reduce the interference. This interference will reduce the coverage of a node. So, here in this chapter, we introduce Signal to Interference plus Noise Ratio (SINR) to define coverage.

In general, there will be interference only from some adjacent nodes. But, we are considering interference from all the other nodes. It means, we are considering worst-case interference. Though we are considering interference from all nodes, interference from farer nodes will be very low since the distance is very large. So, major fraction of interference comes from the nearer nodes.

SINR at a point 'a' due to a node 'i' is given as

$$SINR = \frac{P * {d_{ai}}^{-\eta}}{N_0 + \gamma * \sum_{k \neq i} P * {d_{ak}}^{-\eta}} \tag{5.1}$$

where

P = Transmitting power of a node,

d = Distance between point and node,

η = Path Loss Exponent,

N_0 = Noise power,

k = Interfering nodes,

γ = gamma depends on modulation used .$(0 \leq \gamma \leq 1)$

The point is connected to node if $SINR > SINR_{th}$ (threshold SINR). Two nodes i and j are connected if and only if these two conditions are satisfied:

$$SINR_{ij} > SINR_{th} \tag{5.2}$$

$$SINR_{ji} > SINR_{th} \tag{5.3}$$

where

$$SINR_{ij} = \frac{P * {d_{ij}}^{-\eta}}{N_0 + \gamma * \sum_{k \neq i} P * {d_{kj}}^{-\eta}} \tag{5.4}$$

i.e., SINR at node j due to node i, and k be the nodes other than i and j.

To conclude the definition of coverage, a point is said to be covered if it is connected to a sink node directly or to a relay node which is further connected to a sink node through multiple hops.

5.1 Average vacancy

The average vacancy of the square region of side L units with node density λ and sink probability β is denoted by $v^i_{\lambda,\beta,\gamma}$, where i refers to interference. The algorithm for finding average vacancy is same as in SNR case but here we use SINR in order to check the connectivity between nodes.

5.1.1 Simulation results

We considered L = 10, P = 2W, η = 4, N_0 = 5W, $SINR_{th}$ = 0.4. (All are same as in SNR case, $SINR_{th}$ = SNR_{th} without loss of generality)

Figure 5.1, 5.2 and 5.3 shows average vacancy $v^i_{\lambda,\beta,\gamma}$ vs node density λ for β = 1, β = 0.5 and β = 0.1 respectively.

Analysis:

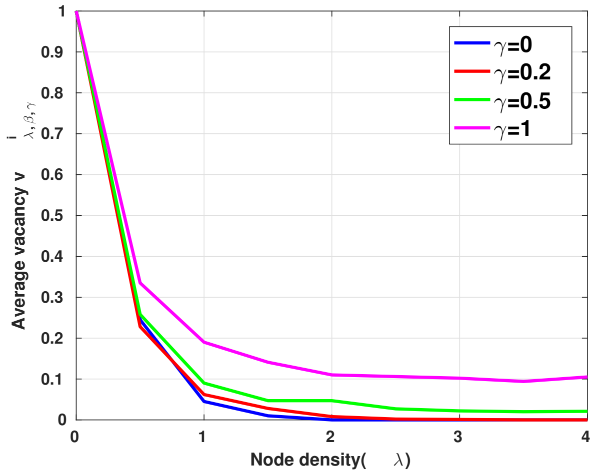


Figure 5.1: Average vacancy vs node density for $\beta = 1$ for different values of γ

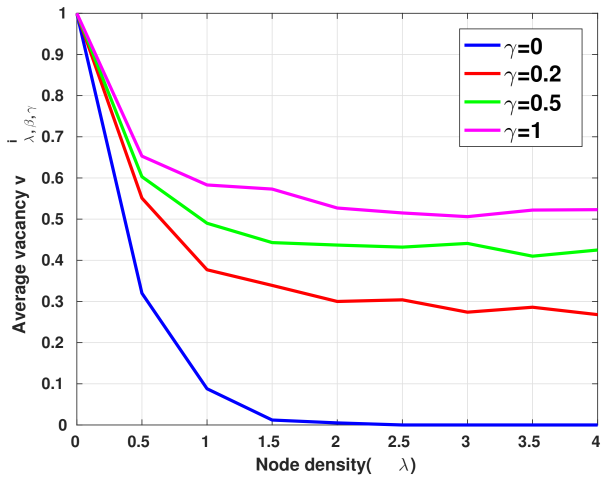


Figure 5.2: Average vacancy vs node density for $\beta = 0.5$ for different values of γ

* In all the three figures, blue curve refers to $\gamma = 0$. It is same as the average vacancy curve for SNR case.

* For a particular value of β, as γ increases, average vacancy increases due to the increase of interference from the other nodes.

* For a particular value of λ and γ, average vacancy decreases as β increases, because as β increases, the fraction of sink nodes increases maintaining constant number of total number of nodes. So, coverage increases and vacancy decreases.

* The average vacancy for $\gamma \neq 0$ is not going to 0 as λ increases, because as λ increases, increase in coverage is compensated by increase in interference. So, the average vacancy converges to a constant after $\lambda = 3$ instead of converging to 0.

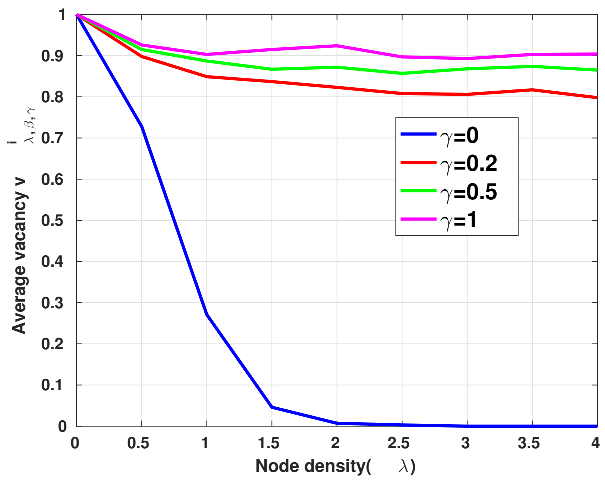


Figure 5.3: Average vacancy vs node density for $\beta = 0.1$ for different values of γ

5.2 Lower bound

Lower bound of vacancy in SINR case $w^i_{\lambda,\beta,\gamma}$ is obtained by the same Independent-Disc model used in SNR case. The algorithm followed is same as that of in SNR case except we have to consider SINR for connectivity. It is obtained by keeping each sink node at a time, place poisson randomly generated relay nodes around it and finding the coverage region for each sink node. The coverage region is the union of the coverage regions of all the sink nodes. This region is larger than the region obtained by placing all poisson randomly generated (as in average vacancy case) sinks and relays at the same time. The vacancy calculated in this model is a lower bound to the average vacancy found in previous section.

5.2.1 Simulation results

We simulated average vacancy and lower bound for three values of β. We considered L=10 and $\gamma = 1$.

Analysis:

* For a particular value of λ, the difference between $v^i_{\lambda,\beta,\gamma}$ and $w^i_{\lambda,\beta,\gamma}$ increases as β increases. So, the lower bound is a good approximation to average vacancy for lower values of β. This is because, if β is low, the distance between sink nodes is more.

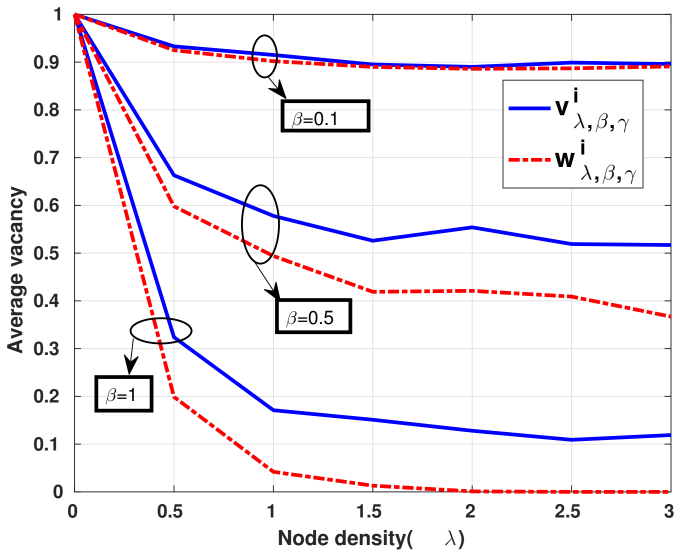


Figure 5.4: Average vacancy and lower bound in SINR case, $\gamma = 1$

So, the coverage discs around sink nodes are less likely to overlap. It appears as i.i.d coverage discs are placed around each node.

* Lower bounds also will converge to a constant.

* In SNR case, average vacancy and lower bound are same for $\beta = 1$ but in SNR case they are not equal. Because for $\beta = 1$ in SINR case, the coverage discs of individual sink nodes are circles of radius 'r' in independent disc model and the coverage of each sink node is less than the circle of radius 'r' in dependant disc model due to the interference from the other sink nodes. So, vacancy in Independent disc model is less than that in dependant disc model in SINR case.

CHAPTER 6

Percolation Properties of Network

Over a past five decades, percolation theory has been used to model disordered physical systems. Many methods have been used to find the percolation thresholds. A physical system is said to be percolated if there exists a direct link from end to other end of the system. The percolation threshold is the minimum density of elements in the system so that the system will percolate. In [Mertens Stephan and Moore Cristopher (2012)], the authors found the two-dimensional percolation thresholds of a disks, squares that are aligned or randomly rotated, and randomly rotated sticks. They used the most efficient union-find algorithm to find these thresholds.

We found the percolation probability and percolation thresholds of two dimensional infrastructure based wireless networks using the same union-find algorithm. Here, we can define, a network is said to be percolated if there exists a direct connection from one side to opposite side of the network. It can also be defined as if there exists a cluster of nodes whose coverage region spans the two opposite sides of the network, then the network is said to be percolated. It means, if we send a message from one side, then it will reach the opposite side of the network.

As this is two dimensional network, there will be horizontal and vertical percolations. We define three types of percolations namely 'any' percolation, 'horizontal' percolation and 'both' percolation. Any percolation means the first percolation occurred (either horizontal or vertical). Both percolation means both horizontal and vertical occurred.

Union-find algorithm is used to find these wrapping cluster of nodes. Our aim is to find the percolation (all three percolations) probability CDF $R_L(\lambda_S, \lambda_R) \in$ [0,1] of the network for λ_S and λ_R. It can also be defined as the probability that the the network is percolated for $\lambda_S < a$ and $\lambda_R < b$ where a and b are constants. It is the CDF of the percolation probability for a particular value of λ_S and λ_R. From this CDF, we can obtain the percolation threshold.

Next three sections deal with the percolation probability CDF for (i) $\beta = 1$, (ii) $\beta \in [0,1]$ and one-hop connectivity (iii) $\beta \in [0.1]$ and strictly one-hop connectivity.

6.1 Percolation probability CDF $R_L(\lambda_S)$ for $\beta = 1$

First we will see the percolation probability CDF for $\beta = 1$ i.e., $\lambda_R = 0$ and $\lambda_S = \lambda$. Here only sinks nodes exist. The percolation probability CDF is defined as the probability of percolation of the network for λ_S less than a given value. 'Any' percolation probability CDF is defined as

$$R_L^{ac}(\lambda_S) = \sum_{i=0}^{\infty}(\frac{\lambda_S^i}{i!}e^{-\lambda_S})\frac{N_{ac}(i)}{N} \tag{6.1}$$

$$N_{ac}(i) = \sum_{j=0}^{i} n_{ac}(j) \tag{6.2}$$

where n_{ac} is an one dimensional array with $n_{ac}(i) = 1$ if any percolation happens at 'i' number of sink nodes else $n_{ac}(i) = 0$, N = number of iterations. The same formula will be applicable to horizontal and both percolations.

Percolation threshold denoted by λ_c is the minimum value of λ_S (sink density) required for the percolation to occur if there are only sink nodes in the network i.e, $\beta = 1$. We will find the percolation thresholds λ_c^{ac}, λ_c^{hc} and λ_c^{bc} for any, horizontal and both percolations from $R_L^{ac}(\lambda_S)$, $R_L^{hc}(\lambda_S)$ and $R_L^{bc}(\lambda_S)$ respectively.

6.1.1 Algorithm to find $R_L(\lambda_S)$

1. Consider a square region of side L units and coverage range of each node r = 1 unit. Take $\beta = 1$.
2. Fix one value of λ_S.
3. Place a sink node randomly in the region with uniform distribution of x and y coordinates.
4. Find all the clusters of sink nodes present in the region by union-find algorithm.
5. Find if any cluster is vertically or horizontally percolated. And check one of the below.

a. If there are no either percolations, add one sink node and check for percolations.

b. If there is horizontal percolation and no vertical percolation then increment $n_{ac}(ns, 1)$ and $n_{hc}(ns, 1)$ by 1, where ns is the number of sink nodes at that time. And add more sink nodes to check vertical percolation and if it happens, increment $n_{bc}(ns, 1)$ by 1.

c. If there is vertical percolation and no horizontal percolation then increment $n_{ac}(ns, 1)$ by 1, where ns is the number of sink nodes at that time. And add more nodes to check horizontal percolation and if it happens increment $n_{hc}(ns, 1)$ and $n_{bc}(ns, 1)$ by 1.

6. Repeat steps 3, 4 and 5 for large number of iterations (N) and obtain matrices nac,nhc and nbc.

7. Now find $R_L^{ac}(\lambda_S)$ by the formula

$$R_L^{ac}(\lambda_S) = \sum_{i=0}^{\infty} (\frac{\lambda_S^i}{i!} e^{-\lambda_S}) \frac{N_{ac}(i)}{N} \tag{6.3}$$

where

$$N_{ac}(i) = \sum_{j=0}^{i} n_{ac}(j) \tag{6.4}$$

(In simulation, we take 1 to number of rows in matrix n_{ac} instead of 0 to ∞ and $n_{ac}(j) = n_{ac}(j, 1)$)

8. Similarly we can find $R_L^{hc}(\lambda_S)$ and $R_L^{bc}(\lambda_S)$.

9. Plot $R_L^{ac}(\lambda_S)$, $R_L^{hc}(\lambda_S)$ and $R_L^{bc}(\lambda_S)$ vs λ_S.

6.1.2 Union-find algorithm to find clusters

1. First node in the region is said to be the one cluster having only one node

2. Second node added is to be checked if it connected to the first cluster. if it connects, the second node is taken into first cluster else it is considered as a second cluster.

3. The next added nodes are to be checked if it is connected any of the existing clusters. Do one of the three things below.

a. If it is connected to any single cluster, the new node is taken into that cluster.

b. If it is connected to two or more clusters, those clusters are combined into a single cluster and this new node is taken into that cluster.

c. If it is not connected to any of the clusters, then the new node is taken as a new cluster.

6.1.3 Simulation results

Figures 6.1 shows ’any’, ’horizontal’ and ’both’ percolation probability CDF’s for $\beta = 1$.

Analysis:

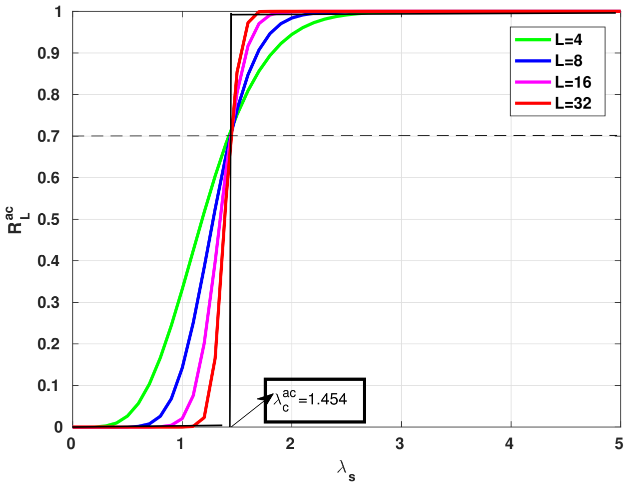
* For all ’any’,’horizontal’, and ’both’ percolations, the percolation probability CDF becomes steeper as L increases i.e., the CDF will go from 0 to 1 faster as L increases.

* So, at L = ∞, there will be abrupt change of CDF from 0 to 1 at a threshold value of λ_S i.e., before this threshold value of λ_S, the value of CDF is 0 and after that it is 1.

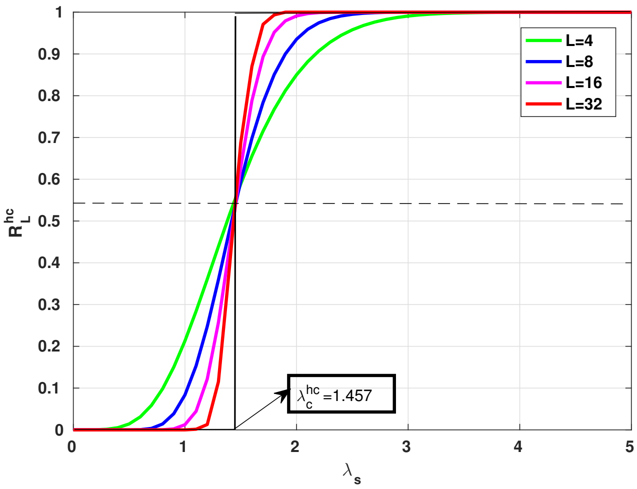
* The threshold sink densities denoted by λ_c are marked in the figures as $\lambda_c^{ac} = 1.454$ for any percolation, $\lambda_c^{hc} = 1.457$ for horizontal percolation and $\lambda_c^{bc} = 1.46$ for both percolation; there is no much difference in thresholds for different percolations.

* $\lambda_c^{ac} < \lambda_c^{hc} < \lambda_c^{bc}$. It means any percolation happens at lower values of λ_S and both percolations happens at higher values of λ_S. As we increase the number of nodes, any percolation occurs first then occurs horizontal percolation and finally both percolations occur.

* It shows that if $\lambda_S \geq 1.5$, all types of percolations occur.



(a)



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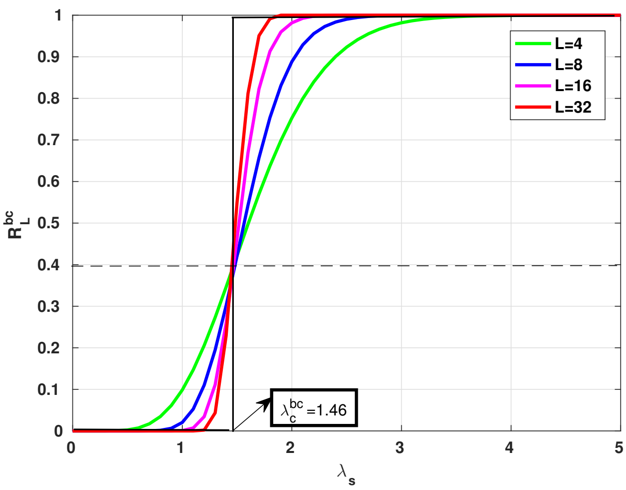


(c)

Figure 6.1: Percolation probability CDF vs λ_S for β=1 (a): Any percolation $R_L^{ac}(\lambda_S)$ (b): Horizontal percolation $R_L^{hc}(\lambda_S)$ (c): Both percolation $R_L^{bc}(\lambda_S)$

6.2 Percolation probability CDF for one-hop connectivity and $\beta \in [0, 1]$

Now we will see Percolation probability CDF for $\beta \in [0, 1]$, it means we also consider $\lambda_R = 0$ and $\lambda_S = 0$. We consider maximum number of hops allowed to be is 1 (Hops ≤ 1). It means horizontal (vertical) percolation happens when there exists a part of SRSRSS..., RSSRSSS... etc. from left to right (top to bottom). SRRSRRR... is not allowed. The percolation probability CDF $R_L(\lambda_S, \lambda_R)$ is defined as

$$R_L^{ac}(\lambda_S, \lambda_R) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}(\frac{\lambda_S^i}{i!}e^{-\lambda_S}\frac{\lambda_R^j}{j!}e^{-\lambda_R})\frac{N_{ac}(i,j)}{N} \tag{6.5}$$

where

$$N_{ac}(ns, nr) = \sum_{i=0}^{ns}\sum_{j=0}^{nr} n_{ac}(i,j) \tag{6.6}$$

where n_{ac} is an two dimensional array with $n_{ac}(ns, nr) = 1$ if any percolation happens at 'ns' number of sink nodes and 'nr' number of relay nodes else $n_{ac}(ns.nr) = 0$, N = number of iterations.

The Percolation boundary (λ_S and λ_R pair) is a boundary after which we obtain a giant connected component for the percolation to happen. We will obtain this percolation boundary for any, horizontal and both percolations from the plots $R_L^{ac}(\lambda_S, \lambda_R)$, $R_L^{hc}(\lambda_S, \lambda_R)$ and $R_L^{bc}(\lambda_S, \lambda_R)$ respectively (We consider boundary at CDF=0.9 for all types of percolations).

6.2.1 Algorithm to find $R_L(\lambda_S, \lambda_R)$

1. Consider a square region of side L units and coverage range of each node r = 1 unit. Consider maximum number of hops to allow to be 1.

2. Fix one value of λ_S and λ_R. Calculate β.

$$\beta = \frac{\lambda_S}{\lambda_S + \lambda_R} \tag{6.7}$$

3. Place a node randomly in the region with uniform distribution of x and y coordinates and consider it as sink node with probability β.

4. Find all the clusters of nodes present in the region by union-find algorithm given in the last section.

5. Find if any cluster is vertically or horizontally percolated. And check one of the below.

   a. If there are no either percolations add one node and check for percolations.

   b. If there is horizontal percolation and no vertical percolation then increment $n_{ac}(ns, nr)$ and $n_{hc}(ns, nr)$ by 1, where ns and nr are the number of sink and relay nodes at that time. And add more nodes to check vertical percolation and if it happens increment $n_{bc}(ns, nr)$ by 1.

   c. If there is vertical percolation and no horizontal percolation then increment $n_{ac}(ns, nr)$ by 1, where ns and nr are the number of sink and relay nodes at that time. And add more nodes to check horizontal percolation and if it happens increment $n_{hc}(ns, nr)$ and $n_{bc}(ns, nr)$ by 1.

6. Repeat steps 3, 4 and 5 for large number of iterations (N) and obtain matrices nac,nhc and nbc.

7. Now find $R_L^{ac}(\lambda_S, \lambda_R)$ by the formula

$$R_L^{ac}(\lambda_S, \lambda_R) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}(\frac{\lambda_S^i}{i!}e^{-\lambda_S}\frac{\lambda_R^j}{j!}e^{-\lambda_R})\frac{N_{ac}(i,j)}{N} \tag{6.8}$$

   where

$$N_{ac}(ns, nr) = \sum_{i=0}^{ns}\sum_{j=0}^{nr} n_{ac}(i,j) \tag{6.9}$$

   (In simulation, we take 1 to number of rows in matrix n_{ac} instead of 0 to ∞.)

8. Similarly we can find $R_L^{hc}(\lambda_S, \lambda_R)$ and $R_L^{bc}(\lambda_S, \lambda_R)$.

6.2.2 Simulation results

Figure 6.2 and 6.3 shows percolation probability CDF for any, horizontal and both percolations vs λ_S and λ_R for L=8 and L=16 respectively. These figures $R_L(\lambda_S, \lambda_R)$ vs λ_S and λ_R are 3-D plots shown as 2-D plots. The value of $R_L(\lambda_S, \lambda_R)$ is given by the colour bar. Blue colour represents 0 and red colour represents 1.

Analysis:

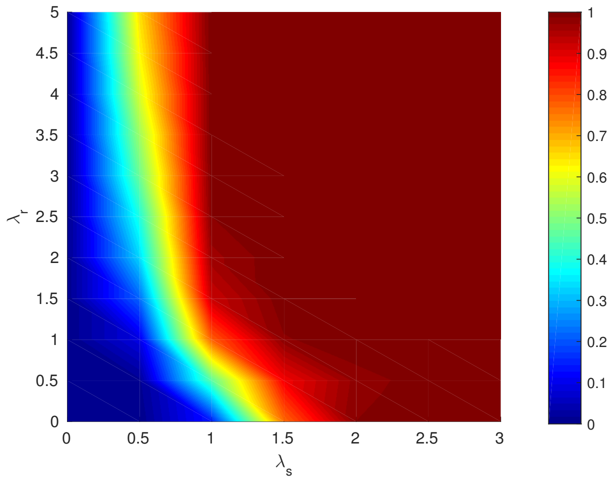
* For a particular percolation, as L increases (see 6.2(a) and 6.3(a)), the $R_L(\lambda_S, \lambda_R)$ rises faster (steeper) from 0 to 1. If L = ∞, there will be direct transition from 0 to 1 giving a transition boundary. This boundary is what is called percolation threshold.

As we can not simulate for L = ∞, we took percolation threshold from L = 16. These thresholds are shown in fig 6.3.

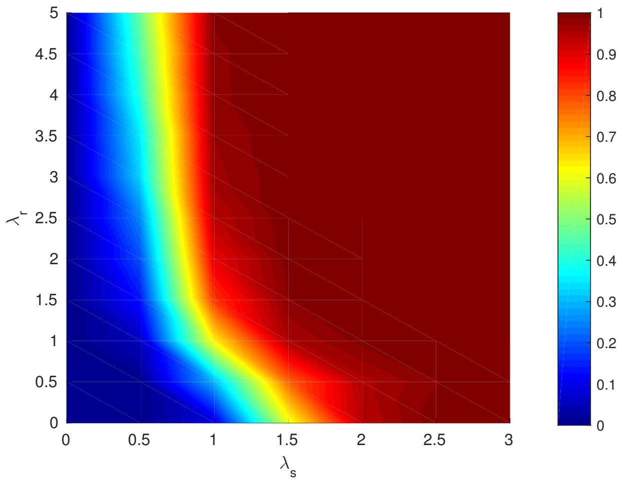
* The threshold increases (moves from left to right) from any, horizontal to both percolations.

* For $\lambda_r = 0$, all types of percolations occur (with probability > 0.9) for around $\lambda_S > 1.5$. This is same as when $\beta = 1$ in previous section, the percolation occurs after $\lambda_S = 1.5$. Even if there are no relay nodes, sink nodes with density 1.5 is enough for the network to percolate.

* For lower values of λ_S (<1), there is a very low probability (≤ 0.6) that the system can percolate even if relay density increases. It means, it requires minimum number of sink nodes for the system to percolate irrespective of number of relay nodes. Because, we are considering one-hop connectivity which requires less distance between sink nodes.



(a)



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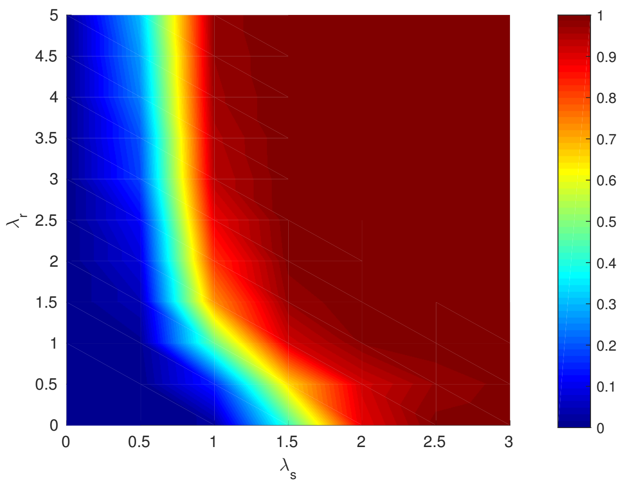
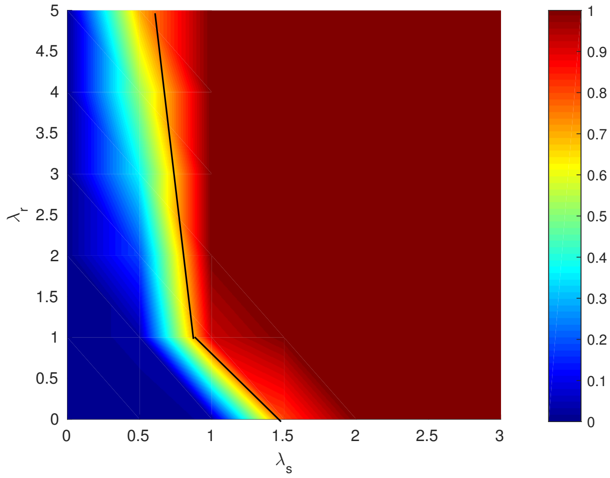
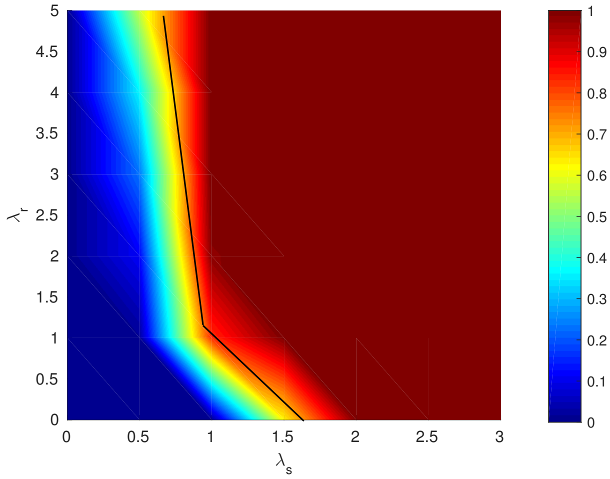


(c)

Figure 6.2: Percolation probability CDF vs λ_S and λ_R for L=8 (a): Any percolation $R_L^{ac}(\lambda_S, \lambda_R)$ (b): Horizontal percolation $R_L^{hc}(\lambda_S, \lambda_R)$ (c): Both percolation $R_L^{bc}(\lambda_S, \lambda_R)$



(a)



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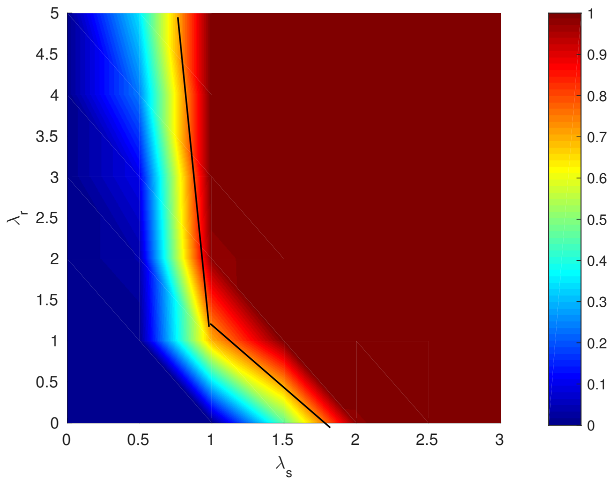


(c)

Figure 6.3: Percolation probability CDF vs λ_S and λ_R for L=16 (a): Any percolation $R_L^{ac}(\lambda_S, \lambda_R)$ (b): Horizontal percolation $R_L^{hc}(\lambda_S, \lambda_R)$ (c): Both percolation $R_L^{bc}(\lambda_S, \lambda_R)$

6.3 Percolation probability CDF for strictly one-hop connectivity and $\beta \in [0, 1]$

Here, we are considering number of hops to be strictly 1 (Hops=1). It is a restrictive version of previous case. So, a network is said to be horizontally (vertically) percolated if there exist a sequence of SRSRSR... or RSRSRS... nodes from left to right (top to bottom) of the network. It means two sink nodes are connected if there exists a relay node (connected to both sink nodes) between them. If percolation occurs with hops =1 for a particular λ_S and λ_R, then it will surely occur for hops ≤ 1 for that λ_S and λ_R.

The algorithm for finding $R_L(\lambda_S, \lambda_R)$ is same as before except finding the clusters with this definition of connectivity.

6.3.1 Simulation results

Figure 6.4 and 6.5 shows percolation probability CDF (strictly one hop) for L=8 and L=16 respectively. The value of $R_L(\lambda_S, \lambda_R)$ is given by the colour bar. Blue colour represents 0 and red colour represents 1.

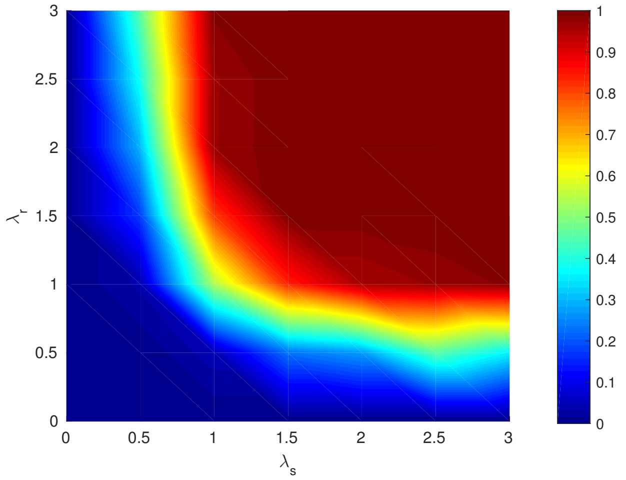
Analysis:

* Similar to previous plots, for a particular percolation, as L increases, the $R_L(\lambda_S, \lambda_R)$ rises faster (steeper) from 0 to 1. If L = ∞, there will be direct transition from 0 to 1 giving percolation threshold density. These boundaries are shown in fig 6.5. The threshold increases (moves from left to right) from any, horizontal to both percolations.

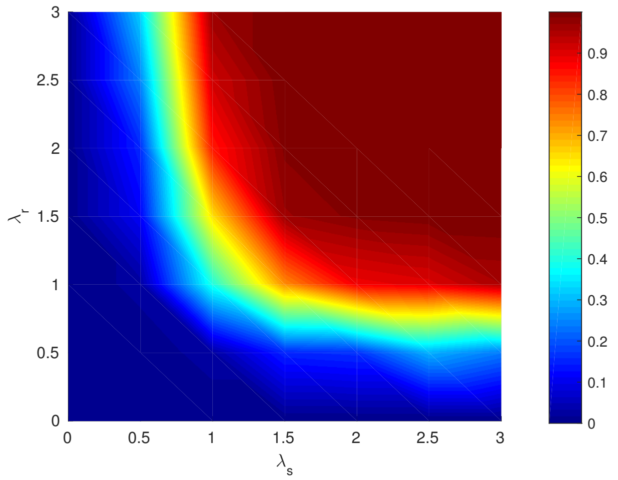
* All types of percolations occur (with probability > 0.9) only after around $\lambda_S > 1.5$ and $\lambda_R > 1.5$.

* For lower values of λ_S (<1) and/or λ_R(<1), there is a very low probability (≤ 0.6) that the system can percolate. It means, it requires minimum number of sink and relay nodes for the system to percolate. Because, we are considering strictly one-hop connectivity which requires sufficient number of sink and relay nodes.

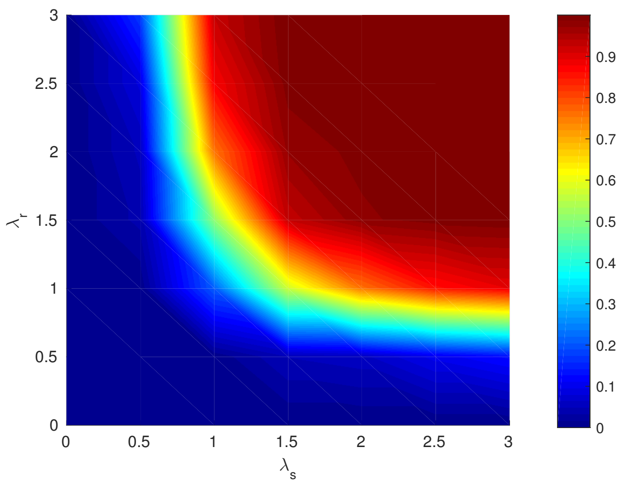
* Even large number of sink nodes are not enough for the system to percolate except if there are sufficient number of relay nodes (unlike as in previous section).



(a)

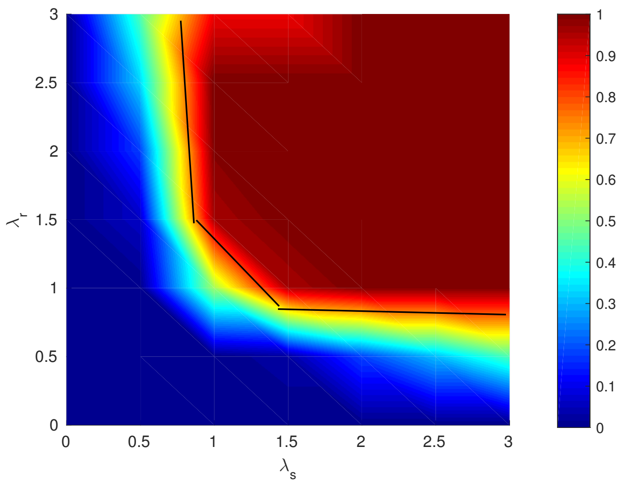


(b)

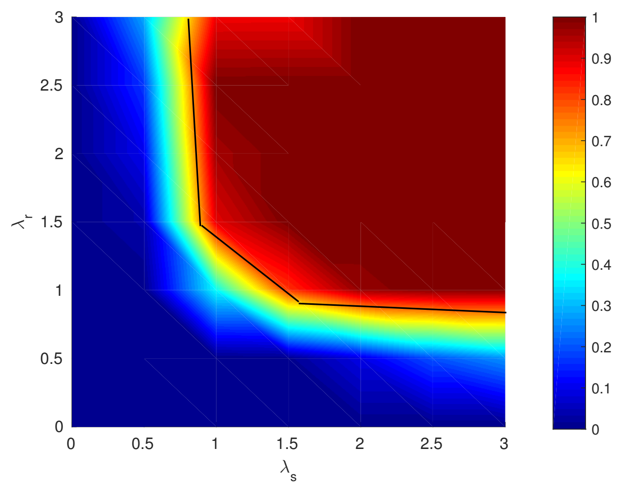


(c)

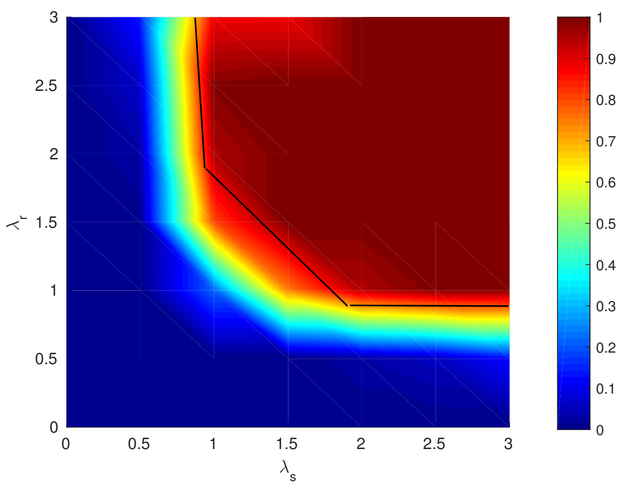
Figure 6.4: Percolation probability CDF (strictly one hop) vs λ_S and λ_R for L=8 (a): Any percolation $R_L^{ac}(\lambda_S, \lambda_R)$ (b): Horizontal percolation $R_L^{hc}(\lambda_S, \lambda_R)$ (c): Both percolation $R_L^{bc}(\lambda_S, \lambda_R)$



(a)



(b)



(c)

Figure 6.5: Percolation probability CDF (strictly one hop) vs λ_S and λ_R for L=16 (a): Any percolation $R_L^{ac}(\lambda_S, \lambda_R)$ (b): Horizontal percolation $R_L^{hc}(\lambda_S, \lambda_R)$ (c): Both percolation $R_L^{bc}(\lambda_S, \lambda_R)$

6.3.2 Iyer and Yogeshwaran upper bound

Iyer and Yogeswaran proposed an upper bound for percolation boundary (any percolation) for Poisson AB model (strictly one-hop connectivity) as

$$\lambda_R = -\frac{1}{a(d,2r)} log_e[1 - (\frac{p_c(d)}{1 - e^{-\lambda_S a(d,2r)}})] \tag{6.10}$$

for $\lambda_S \geq 0.843$.

d = number of dimensions of region = 2

r = coverage radius of node = 1

a(d,2r) = a(2,2) = 0.8227 as given by Iyer and Yogeshwaran

$p_c(d)$ = Critical percolation probability = 0.5 as calculated by Iyer and Yogeshwaran.

We will compare this upper bound with the 0.9 percolation boundary in Fig. 6.6. We observe that this bound is weak for smaller values of sink densities and is a good approximation at higher values.

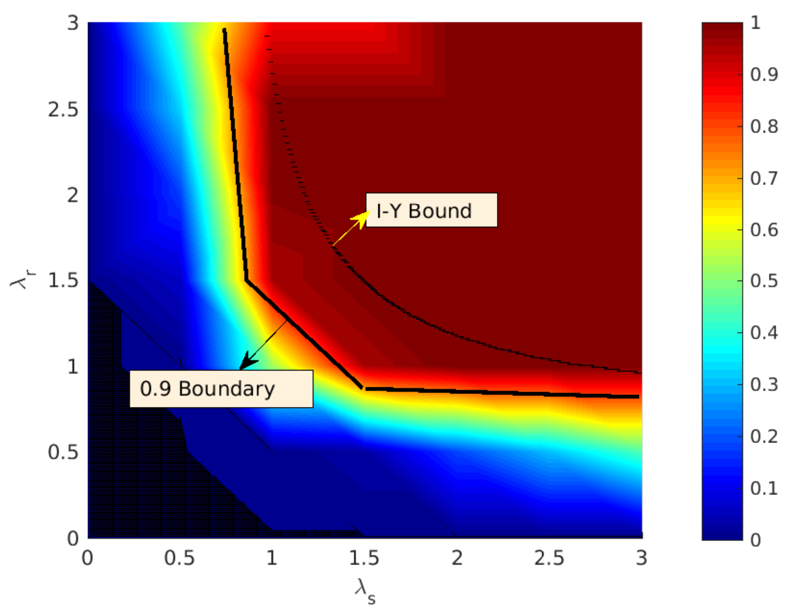


Figure 6.6: Iyer upper bound and 0.9 percolation boundary

CHAPTER 7

Conclusions and Future Scope

Conclusions: We simulated average vacancy and lower bound for vacancy in SNR case. We require atmost $\lambda = 2$ and $\lambda = 3$ for the coverage of atleast 80% and 95% respectively even for small values of β. We simulated average vacancy and lower bound for vacancy in SINR case. As γ increases, the average vacancy increases. The average vacancy will not goes to 0 as λ increases rather it converges to a constant. In both SNR and SINR cases, lower bound is a good approximation to average vacancy for lower values of β. We simulated hop constrained vacancy in SNR case. We studied the optimal cost of the network subject to a vacancy constraint. Decreasing sink node cost will not further highly reduce optimal cost unless the sink node cost is already low. Later, we found that the system will percolate only if $\lambda_S \geq 1.5$ for $\beta = 1$. If $\beta \in [0, 1]$ and for one-hop connectivity, percolation occurs if $\lambda_S \geq 1.5$ irrespective of relay node density. If $\beta \in [0, 1]$ and for strictly one-hop connectivity, percolation occurs only if $\lambda_S \geq 1.5$ and $\lambda_R \geq 1.5$.

Future scope: Our future study is to find the mathematical equations for average vacancy and lower bound in SNR and SINR cases. We need to analyse the optimal sink probability for optimal cost.We have to study the cost optimization problem in SINR case. We need to find the percolation thresholds analytically.

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