Channel Estimation and 3-D Hybrid Precoding for Millimeter Wave Cellular Systems

A Project Report

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THESIS CERTIFICATE

This is to certify that the thesis titled **Channel Estimation and 3-D Hybrid Precoding for Millimeter Wave Cellular Systems**, submitted by **THOUTAM SAI CHARAN**, to the Indian Institute of Technology, Madras, for the award of the degree of **Master of Technology**, is a bonafide record of the project work done by him under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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I dedicate this thesis to my loving family who have always encouraged me and provided me with all the support for taking each step forward in this work. Last, but not the least, I thank Lord Almighty for being my strength in the successful completion of this work.

ABSTRACT

KEYWORDS: Millimeter-wave; Precoding; Beamforming; Right Singular Vector

Millimeter-wave systems support high date rates because of the large bandwidths available at mmWave frequencies. However, they experience more path loss. Therefore large antenna arrays are used in mmWave systems to employ beamforming and meet the required link margins. Right singular vector (RSV)-type beamforming structures are very sensitive to the channel estimate and not robust. Here, we consider directional beamforming. The channel estimation is done using a multi-resolution codebook for training beamforming vectors with different beamwidths. We design 3-D beamforming multi-resolution training codebook for channel estimation. Such a channel estimation method was proposed by Alkhateeb and others in 2014. In this project, we extend this scheme for a 3-D MIMO channel model. The results show that the estimated channel is close to the existing channel.

TABLE OF CONTENTS

LIST OF FIGURES

ABBREVIATIONS

BS	Base Station
MS	Mobile Station
MIMO	Multiple Input Multiple Output
SNR	Signal-to-Noise Ratio
CSI	Channel State Information
AoA	Angle Of Arrival
AoD	Angle Of Departure
ULA	Uniform Linear Array
MSE	Mean Square Error

NOTATION

$\mathcal{N}(\mu, \sigma^2)$	Gaussian distribution with Mean μ and Variance σ^2
$\mathcal{CN}(\mu, \sigma^2)$	Circularly symmetric complex Gaussian distribution with Mean μ and Variance σ^2
$(.)^H$	Hermitian Operator
$(.)^T$	Transpose Operator
$\|\|(.)\|\|_F$	Frobenius Norm
$(.)\backslash(.)$	Inverse Operator
$\|(.)\|$	Determinant
$(.)^{-1}$	Inverse Operator
$E(.)$	Expectation Operator
$\odot$	Dot Product
$\otimes$	Kronecker Product

CHAPTER 1

Introduction

With the increase in revolutionary technologies such as smart phones,IoT etc., the demand for high data rates is increasing.Millimeter Waves helps us in achieving high data rates using the carrier frequencies from 30-300 GHz range. Spectrum unavailability which is a problem at low frequencies is not at all a problem here. The only thing we need to worry is high frequency-dependent propagation and shadowing losses. This can be reduced by limiting our coverage to small areas and using the antenna arrays.

At mmWave frequencies the large antenna array can be packed into small form factors making us to realize large antenna arrays required for precoding gains. Directional precoding is used in mmWave communications to get sufficient SNR at the receiver. Design of precoding matrices is mainly based on channel state information which is difficult to achieve. This forces us to develop new channel estimation methods for mmWave systems.

Initially people proposed analog beamforming method where the main idea is to control the phase of the signal transmitted from each antenna by using analog phase shifters. Several beam training algorithms were proposed to design this beamforming coefficients without the channel state information (CSI) at the transmitter. However using analog beamforming we can communicate only single beam (because of single RF chain) where by loosing multiplexing gains by transmitting parallel data streams.

To achieve high precoding gains by transmitting parallel data streams people started to divide the precoding operations between analog and digital domain calling it as hybrid precoding. Here in this paper we generate a multi-resolution codebook for the purpose of training precoders. This codebook relies on joint analog/digital processing to generate training beamforming vectors.

We generate directional beamforming vectors using this codebook. Each precoding vector in the codebook is confined to certain angle and has certain beamwidth. We consider quantized angles here. But, as the values of AoA/AoD are actually continuous other algorithms such as least squares or newton refinement can be used to reduce the quantization error.

In this paper we talk about channel estimation and precoding algorithms for mmWave systems with antenna arrays both at the base station (BS) and the mobile station (MS).These algorithms are under the assumptions that

1. analog phase shifters have constant modulus and quantized phases,
2. The number of RF chains are less than the number of antennas of BS or MS.

Here we use an adaptive algorithm where the transmitter and receiver combinedly design or select their beamforming vectors reducing the beam training time. The main process is like the receiver calculates the received SNR using different combiners and sends the feedback to the transmitter about which precoder and combiner is giving the maximum SNR value at the receiver.

1.1 Thesis Outline

The rest of the thesis has been organized as follows. In Chapter 2, we broadly review the existing literature related to the methods used for precoding and channel estimation algorithms. Chapter 3 describes about the mmWave cellular system and the 3-D channel model considered in this paper. We also talk about the step by step transmission and receiving process happening in this system.

In Chapter 4, We design hybrid analog/ digital based multi-resolution codebook. Initially we develop a code book for single-path systems and then change the codebook slightly for a multi-path system. Here we deal with the design of beamforming vectors for codebook using matrix formulation.

Chapter 5 deals with mmWave channel estimation algorithms for the single-path and multi-path cases. This chapter uses the codebook designed in chapter 4.

Simulation results obtained and observations made from these results are discussed in Chapter 6. Finally, the summary and conclusion of the work done makes Chapter 7. Future research possibilities in this area are also mentioned as a final note.

CHAPTER 2

Literature Survey

mmWave communication is an important technology for the future outdoor cellular systems. Directional precoding with large antenna arrays are inevitable for mmWave systems to support the link margin. Initially, my work started with the study of matching pursuit algorithm for the purpose of frequency estimation over the continumm from the B. Mamandipoor, D. Ramasamy and U. Madhow (2016). Here in this paper it uses single refinement and cyclic refinement steps where it uses the Newtons update algorithm to reduce the frequency estimation errors.

Then for the further study of estimation in mmWave systems, I referred to Zhinus Marzi, Dinesh Ramasamy and Upamanyu Madhow (2016). This paper deals with the channel estimation for mmWave cellular systems with large antenna arrays. This paper also uses the same algorithm of B. Mamandipoor, D. Ramasamy and U. Madhow (2016) for the purpose of estimation.

Then, We have considered the problem of beamforming in mmWave systems. Analog beamforming solutions were proposed in many papers but they have problem of converging towards single communication beam. These techniques are not capable of achieving multiplexing gains. So, We considered hybrid beamforming to achieve larger precoding gains. For this purpose I have referred to Ahmed Alkhateeb, Omar El Ayach, Geert Leus and Robert W.Heath (2014) which is the backbone reference for this project. They have designed codebook for 2-D beamforming and proposed the channel estimation algorithms. I have extended the same codebook and channel estimation algorithms for 3-D beamforming case.

Here in this project we have considered hybrid precoding where in we divide the precoding operations between the analog and digital domains. But in consideration of precoding matrices for the codebook design we have considered the total precoding matrix. To split the total precoding matrix between the analog and digital domains we use the algorithm mentioned in Omar El Ayach, Sridhar Rajagopal, Shadi Abu-Surra, Zhouyue Pi, Robert W.Heath (2014) .

CHAPTER 3

System Model

Consider the mmWave cellular system as shown in below Fig.

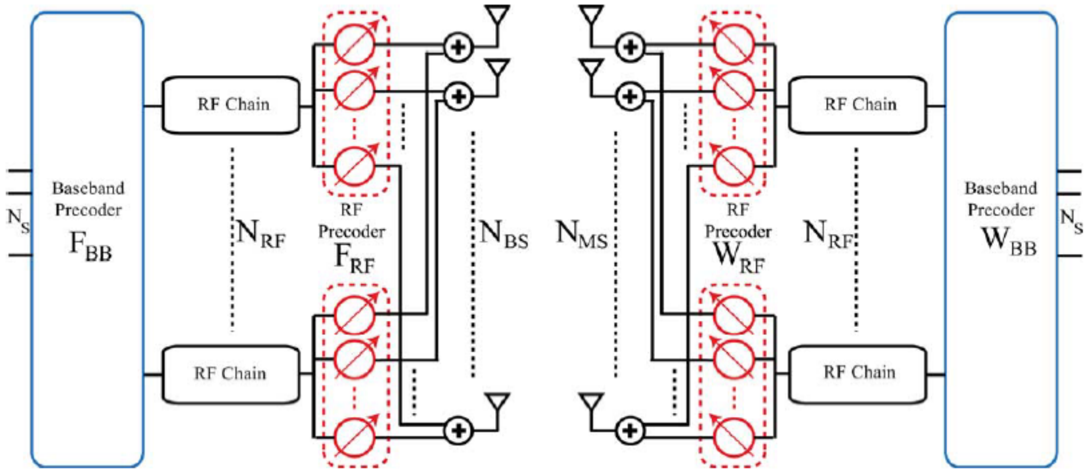

Figure 3.1: Block Diagram of BS-MS transceiver

A base station (BS) with N_{BS} antennas and N_{RF} RF chains is communicating to a mobile station (MS) with N_{MS} antennas and N_{RF} RF chains. The BS and MS communicate via N_S data streams, such that $N_S \leq N_{RF} \leq N_{BS}$ and $N_S \leq N_{RF} \leq N_{MS}$.

Base station applies an $N_{RF} \times N_S$ baseband precoder denoted as $\mathbf{F}_{BB}$ followed by an $N_{BS} \times N_{RF}$ RF precoder denoted by $\mathbf{F}_{RF}$. If the total precoding matrix is defined as $\mathbf{F}_T = \mathbf{F}_{RF} \times \mathbf{F}_{BB}$ then the transmitted signal is given by

$$\mathbf{x} = \mathbf{F}_T\mathbf{s}$$

where $\mathbf{s}$ is the $N_S \times 1$ vector of transmitted symbols satisfying the power constraints

we use a channel $\mathbf{H}$ through which the mobile station observes the received signal as

$$\mathbf{r} = \mathbf{H}\mathbf{F}_T\mathbf{s} + \mathbf{n}$$

The channel $\mathbf{H}$ between the mobile station and base station is denoted by a $N_{MS} \times N_{BS}$ matrix, and $\mathbf{n} \sim \mathcal{N}(0, \sigma^2 \mathbf{I})$ is the gaussian noise.

Similarly at the mobile station the receiver uses, $N_{MS} \times N_{RF}$ RF combiner denoted as $\mathbf{W}_{RF}$ followed by an $N_{RF} \times N_S$ baseband combiner denoted by $\mathbf{W}_{BB}$. If the total combiner matrix is defined as $\mathbf{W}_T = \mathbf{W}_{RF} \times \mathbf{W}_{BB}$ then the received signal is given by

$$\mathbf{Y} = \mathbf{W}_T^H \mathbf{H} \mathbf{F}_T \mathbf{s} + \mathbf{W}_T^H \mathbf{n}$$

In this project we have considered downlink model for the algorithms.This algortihms can be used to the uplink model with roles of preocder $(\mathbf{F}_{RF}, \mathbf{F}_{BB})$ and combiner $(\mathbf{W}_{RF}, \mathbf{W}_{BB})$ interchanged. Another assumption here is that we don't consider the interfering basestations.

3.1 Channel Model

We consider a geometric channel model with $\mathbf{L}$ scatterers, Where each scatterer contributes a single path between the base station and mobile station. The channel is expressed as

$$\mathbf{H} = \sqrt{\frac{N_{BS} N_{MS}}{\rho}} \sum_{l=1}^{L} \alpha_l \mathbf{a}_{MS}(\phi_{al}, \theta_{al}) \mathbf{a}_{BS}^H(\phi_{dl}, \theta_{dl})$$

Where,

ρ - average path loss between the base station and mobile station,

α_l - complex gain of the l^{th} path,

(ϕ_{al}, θ_{al}) - l^{th} path azimuth and elevation angle of arrival (AoA) at the mobile,

(ϕ_{dl}, θ_{dl}) - l^{th} path azimuth and elevation angle of arrival (AoD) at the base station, and

$\mathbf{a}_{MS}(\phi_{al}, \theta_{al}), \mathbf{a}_{BS}(\phi_{dl}, \theta_{dl})$ - antenna array response vectors at the MS and BS respectively

Here, the path gain α_l is assumed to be complex gaussian and the l^{th} path azimuth angles $\phi_{al}, \phi_{dl} \in [0, 2\pi]$, the elevation angles $\theta_{al}, \theta_{dl} \in [0, \pi]$. The azimuth and elevation angles are actually continuous but we are considering quantized angles only allowing the scope for quantization error. Including the azimuth and elevation angles says that the base station and mobile station should implement 3-D beamforming.

If the antenna array is assumed to be uniform linear array (ULA) then the array response vector is defined as

$$a(\phi, \theta) = [1, \ldots, e^{jkd(msin(\phi)sin(\theta)+ncos(\phi)}, \ldots, e^{jkd((W-1)sin(\phi)sin(\theta)+(H-1)cos(\phi)}]^T$$

Where,

k = $2\pi/\lambda$, λ is the wavelength,

d - Distance between the antenna elements and

WH = N (total no. of antennas at the BS or MS accordingly)

As said earlier, the estimated AoA's and AoD's are taken from a uniform grid of N points with N $\gg$ L i.e.,

$$\phi_{al}, \phi_{dl} \in [0, 2\pi/N, \ldots, 2\pi(N-1)/N]$$

$$\theta_{al}, \theta_{dl} \in [0, \pi/N, \ldots, \pi(N-1)/N]$$

We will neglect the quantiztion error here.

CHAPTER 4

Multi-Resolution Codebook

Here, In this chapter we design a hybrid analog/ digital based multi-resolution codebook.The approach for constructing the codebook is same for both ULA/ non-ULA structures. This variant beamwidth beamforming codebook is just an extension to the codebook proposed in Ahmed Alkhateeb, Omar El Ayach, Geert Leus and Robert W.Heath (2014), as said earlier.

This codebook is used for mmWave channel estimation, which is nothing but finding the AoA,AoD and gains of L paths. To do it correctly with minimum training time we need to design the codebook of precoders and combiners very carefully. The section 4.1 deals with the design of base station training precoding codebook $\mathcal{F}$. Similar approach can be followed to construct the mobile station training combiner codebook $\mathcal{W}$.

4.1 Codebook Structure

The proposed code book contains $\mathbf{S}$ levels, $\mathcal{F}_s$, $s = 1, 2, \ldots, S$. To acheive a resolution of $2\pi/N$ the no. of levels in the codebook should be $\mathrm{s} = log_{\sqrt{K}}N$. Each level contains beamforming vectors with different beamwidths, which are used in training stage of the algorithm. Fig. 4.1 shows the first two levels of the codebook with $\mathbf{N} = 4$, and $\mathbf{K} = 4$ beamforming vectors. Here, the no. of beamforming vectors is in the form of i^2, where the traditional codebook has it in the form of i.

In each codebook level s, the beamforming vectors are divided into $\mathbf{K}^{(s-1)}$ subsets, with $\mathbf{K}$ beamforming vectors in each of them. Each subset $\mathcal{K}$ of the codebook level s is associated with a unique range of angle of departure (AoD) i.e., $(2\pi u/N, \pi m/N)$ with u, m belonging to certain range. This AoD range in each subset is further divided into $\mathbf{K}$ sub-parts, and each of the $\mathbf{K}$ beamforming vectors in this subset should have an equal projection on the vector $\mathbf{a}_{BS}(\overline{\phi_u}, \overline{\theta_m})$ with u, m belonging to this sub-parts and zero projection on other vectors.

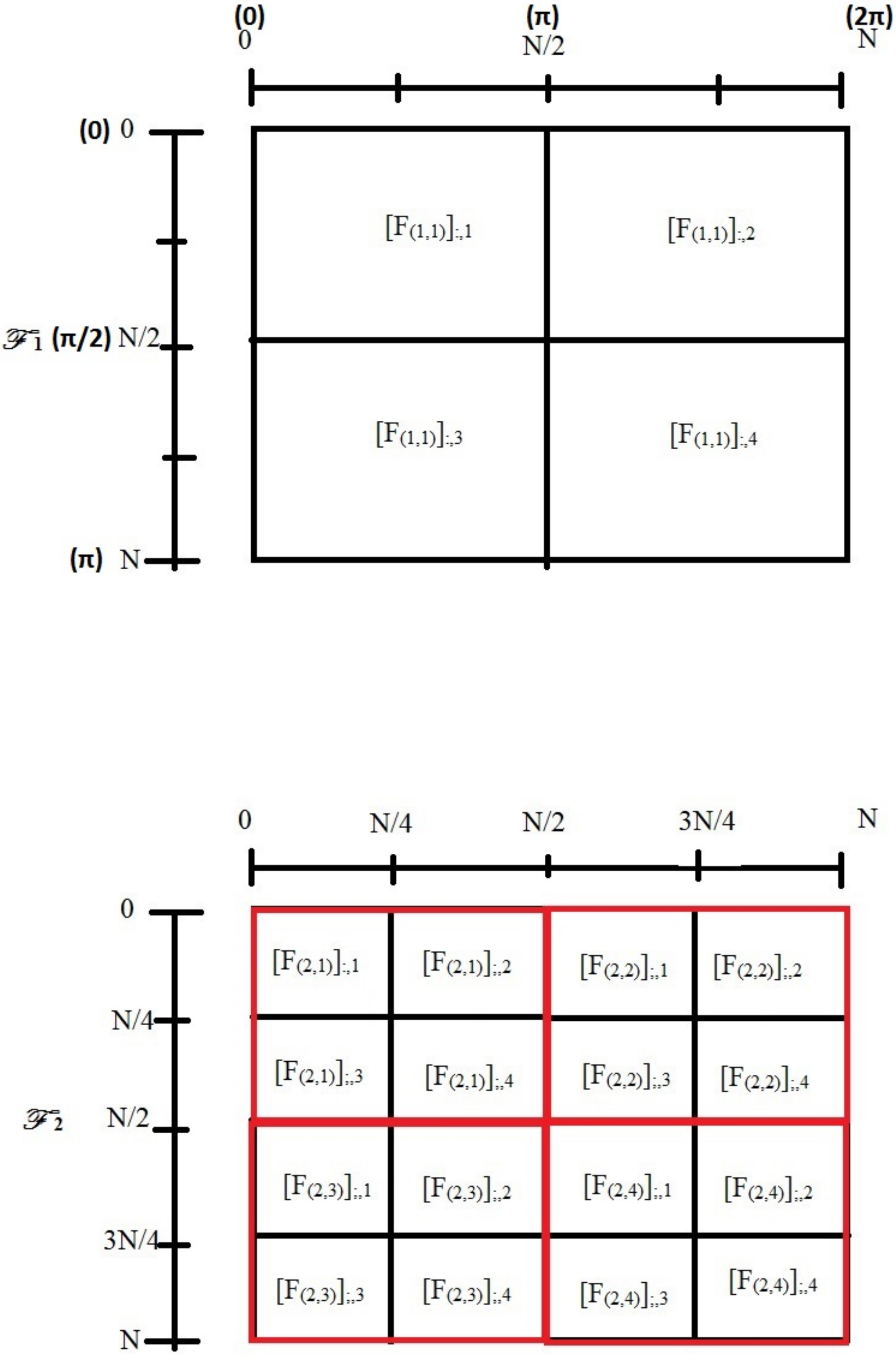

Figure 4.1: Codebook with N=4, and K=4 beamforming vectors

4.2 Design of the Beamforming Vectors for Codebook

The beamforming vectors $[F_{(s,k)}]_{:,m}$,m = 1,...,$\mathbf{K}$ in each codebook level s, and subset $\mathcal{K}$ is designed such that

$$[F_{(s,k)}]^H_{:,m}\ \mathbf{a}_{BS}(\overline{\phi_u}, \overline{\theta_m}) = \begin{cases} C_s, & \text{if } u, m \in \mathcal{I} \\ 0, & \text{otherwise} \end{cases}$$

Where, $\mathcal{I}$ denotes the sub-parts of the AoD's corresponding to the particular beamforming vectors, and C_S denotes the normalization constant such that $\|\mathbf{F}_{(s,k)}\|_F = \mathbf{K}$.

In other way we can write $\mathbf{F}_{(s,k)}$ as the solution of :

$$\mathbf{A}^H_{BS,D}\mathbf{F}_{(s,k)} = C_s\mathbf{G}_{\mathbf{BS}(s,k)}$$

Where, $\mathbf{G}_{(s,k)}$ is an $\mathbf{N^2} \times \mathbf{K}$ matrix where each coloumn contains 1's in the location where $(u, m) \in \mathcal{I}$ and zero's in the other location. As u and m have $\mathbf{N}$ possibilities the no. of rows of $\mathbf{G}_{(s,k)}$ matrix would be $\mathbf{N^2}$. $\mathbf{A}_{BS,D}$ is an over-complete dictionary of the base stations AoD matrix. In, the matlab you can just get the solution of above equation i.e., bemaforming vectors by doing $\mathbf{F}_{(s,k)} = \mathbf{A}^H_{BS,D}\backslash C_s\mathbf{G}_{\mathbf{BS}(s,k)}$.

As said in the chapter 3, $\mathbf{F}_{(s,k)} = \mathbf{F}_{RF,(s,k)}\mathbf{F}_{BS,(s,k)}$. So, we can design hybrid analog and digital precoders by using the algorithm stated in Omar El Ayach, Sridhar Rajagopal, Shadi Abu-Surra, Zhouyue Pi, Robert W.Heath (2014).

4.3 Modified Codebook for Multi-path Case

In multi-path cases we need to make little changes to the above proposed codebook. Here, there will be $\mathbf{K} = (\widetilde{\mathbf{K}}\mathbf{L}_d)^2$ at the basestation and mobilestation instead of $\mathbf{K}$ at each stage, where $\widetilde{\mathbf{K}}$ is the no. of beamforming vectors present in the traditional 2-D beamforming codebook and $\mathbf{L}_d$ is the no. of propagation paths. In each stage, $(\mathbf{L}_d)^2$ of the $\mathbf{K}$ partitions are selected for the refinement in next stage.

Here, we consider this $\mathbf{K}$ partitions in the form of 2-D matrix. In each stage we divide the matrix into four parts, by splitting the matrix along the half of coloumns and along the half of rows. Then the required $(\mathbf{L}_d)^2$ partitions are then selected from one of this four parts. The selected part is divided into $\mathbf{K}$ partitions again for the next stage.

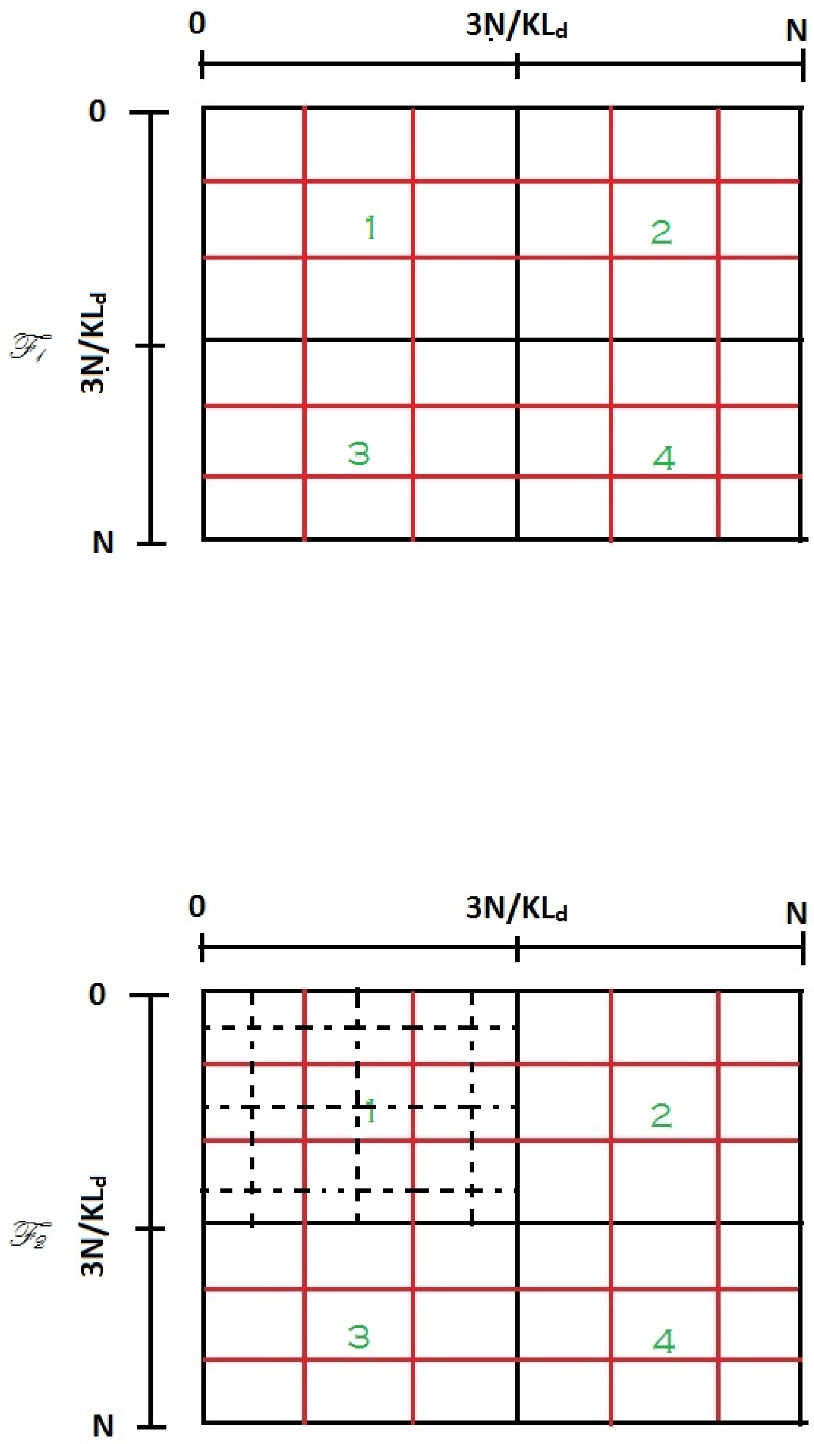

Figure 4.2: Codebook with $\widetilde{\mathbf{K}} = 2$ beamforming vectors and $\mathbf{L}_d = 3$ paths

CHAPTER 5

mmWave Channel Estimation algorithm

In this chapter, We talk about the mmWave channel estimation algorithms for the single-path and multi-path cases. This algorithms utilizes the codebook described in the previous chapter for both the cases accordingly. Initially, We talk about the single path algorithm and then go to the multipath case.

For the training phase which is done in the following estimation algorithms, We assume that all the transmitted symbols are equal namely, $\mathrm{s} = \mathbf{I}_{M_{BS}}$. For the below algorithms I have considered that the power of symbols is equal to 1. Under the above constraints the received vector can be written as:

$$\mathbf{Y} = \mathbf{W}^H\mathbf{H}\mathbf{F} + \mathbf{Q}$$

5.1 Channel Estimation Algorithm for Single-Path

In the initial stage, the base station uses the $\mathbf{K}$ training precoding vectors of the first level of codebook $\mathcal{F}$. The first level of the codebook has single subset only. For each of those precoding vectors, the mobile station uses the $\mathbf{K}$ combining vectors of the first level of codebook $\mathcal{W}$.

After the $\mathbf{K}^2$ measurment steps of this stage, the mobile station compares the power of these $\mathbf{K}^2$ measurments. Then the mobile station determines the precoding and combining vector which gives the maximum received SNR and feedback the index of precoding vector to the base station. This selection of precoding and combining vectors is nothing but selecting the range of quantized AoD and AoA.

The output of the maximum power problem is then used to determine the subsets of beamforming vectors of level $\mathrm{s} + 1$ of $\mathcal{F}$ and $\mathcal{W}$ to be used in the next stage. As the beamforming vectors of next levels have higher and higher resolution, the AoD/AoA ranges are further refined adaptively until the desired resolution of $2\pi/N$ is achieved with the no. of levels, $\mathbf{S} = log_{\sqrt{K}}N$.

Algorithm 1 Adaptive Estimation Algorithm for Single-Path mmWave Channels

$\mathbf{Input}$: BS and MS know N,K,L_d, and have $\mathcal{F}, \mathcal{W}$
$\mathbf{Initialization}$: $K_{BS} = [\emptyset], K_{MS} = [\emptyset]$, $\mathbf{S} = log_{\sqrt{K}}N$
for $s \leq$ **S do**
  for $m_{BS} \leq$ **K do**
    BS uses $[\mathbf{F_{(s,K_{BS})}}]_{:.m_{BS}}$
    for $m_{MS} \leq$ **K do**
      BS uses $[\mathbf{W_{(s,K_{MS})}}]_{:.m_{MS}}$
    After MS measurments:
    $y_{m_{BS}} = [\mathbf{W_{(s,K_{MS})}}]\mathbf{H}[\mathbf{F_{(s,K_{BS})}}]_{:.m_{BS}} + n_{m_{BS}}$
  $Y_{(s)} = [y_1, y_2, \ldots, y_K]$
  $(m_{BS}, m_{MS}) = \underset{m_{BS},m_{MS}}{\text{argmax}} [Y \odot Y^*]_{m_{MS},m_{BS}}$
$K_{BS} = find(G_{BS}(:, m_{BS}) == 1)$
$K_{MS} = find(G_{MS}(:, m_{MS}) == 1)$
$\mathbf{AoDestimation}$:
$temp = [matrix(G_{BS}(:, m_{BS}))]^T$
$[rows_{max}, cols_{max}] = find(temp == 1)$
$AoD_{Elevation} = \pi \times (cols_{max} - 1)/N$
$AoD_{Azimuth} = 2\pi \times (rows_{max} - 1)/N$
$\mathbf{AoAestimation}$:
Same as above with slight modifications

5.2 Channel Estimation Algorithm for Multi-path

In the multi-path case we need to estimate $\mathbf{L}_d$ dominant paths of the channel. So, we perform an algorithm similar to the single-path algorithm with $\mathbf{L}_d$ outer loop iterations. Every single iteration gives one dominant path of the channel.

If we see the traditional 2-D beamforming codebook we observe that first stage has $\widetilde{\mathbf{k}}$ partitions with $\mathbf{L}_d$ partitions in each of them. Then for the next stage we select $\mathbf{L}_d$ partitions i.e., nothing but one of the $\widetilde{\mathbf{k}}$ partitions that gives the maximum SNR at the receiver. The selected $\mathbf{L}_d$ partitions, each one are further divided into $\widetilde{\mathbf{k}}$ partitions for further refinement in next stage.

We extend the same method for our 3-D beamforming case. Here, we will have $\widetilde{\mathbf{k}}^2$ partitions with $\mathbf{L}_d^2$ partitions in each of them. Then for the next stage we select one of these $\mathbf{L}_d^2$ partitions and divide each of these partitions into $\widetilde{\mathbf{k}}^2$ partitions for further refinement.

The above process is continued for each of the dominant path until a resolution of $2\pi/N$ is achieved with no. of levels, $\mathbf{S} = log_{\widetilde{\mathbf{k}}} N/L_d$. With $\mathbf{L}_d$ outer loop iterations we can estimate the dominant paths of the channel, realizing the total mmWave channel.

Algorithm 2 Adaptive Estimation Algorithm for Multi-Path mmWave Channels

Input: BS and MS know N,K,L_d, and have $\mathcal{F}, \mathcal{W}$

Initialization: $K_{BS} = [\emptyset], K_{MS} = [\emptyset]$, $\mathbf{S} = log_{\widetilde{\mathbf{k}}} N/L_d, \mathbf{K} = (\widetilde{\mathbf{k}} L_d)^2$

for $l \leq L_d$ **do**

  for $s \leq$ **S do**

    for $m_{BS} \leq$ **K do**

      BS uses $[\mathbf{F}_{(\mathbf{s},\mathbf{K_{BS}})}]_{:.m_{BS}}$

      for $m_{MS} \leq$ **K do**

        BS uses $[\mathbf{W}_{(\mathbf{s},\mathbf{K_{MS}})}]_{:.m_{MS}}$

      After MS measurments:

      $y_{m_{BS}} = [\mathbf{W}_{(\mathbf{s},\mathbf{K_{MS}})}]\mathbf{H}[\mathbf{F}_{(\mathbf{s},\mathbf{K_{BS}})}]_{:.m_{BS}} + n_{m_{BS}}$

    $y_{(s)} = [y_1^T, y_2^T, \ldots, y_K^T]^T$

    for $p = 1 \leq length(K_{BS})$ **do**

      Project out previous path contributions

      $A1 = [\mathbf{F}_{(\mathbf{s},\mathbf{K_{BS}})}]^T [A_{BS,D}]^*_{:,K_{BS}(p)}$

      $A2 = [\mathbf{w}_{(\mathbf{s},\mathbf{K_{BS}})}]^H [A_{MS,D}]^*_{:,K_{MS}(p)}$

      $g = A1 \otimes A2$

      $y_{(s)} = y_{(s)} - {y_{(s)}}^H g(g^H g) g$

    Y=matrix($y_{(s)}$)

    $(m_{BS}, m_{MS}) = \underset{m_{BS},m_{MS}}{\operatorname{argmax}} [Y \odot Y^*]_{m_{MS},m_{BS}}$

  $K_{BS} = [K_{BS} \quad find(G_{BS}(:, m_{BS}) == 1]$

  $K_{MS} = [K_{MS} \quad find(G_{MS}(:, m_{MS}) == 1]$

AoDestimation:

  $temp = [matrix(G_{BS}(:, m_{BS}))]^T$

  $[rows_{max}, cols_{max}] = find(temp == 1)$

  $AoD_{Elevation} = \pi \times (cols_{max} - 1)/N$

  $AoD_{Azimuth} = 2\pi \times (rows_{max} - 1)/N$

AoAestimation:

  Same as above with slight modifications

CHAPTER 6

Simulation Results

In this chapter, we present the results to evaluate the performance of the proposed hybrid analog/ digital muti-resolution codebook and the channel estimation algorithm. As said earlier we consider BS-MS link i.e., downlink channel for the simulation of mmWave channel model.

6.1 Simulation Set-up

6.1.1 System Model

We adopt the architecture shown in Fig.3.1. We consider that base station has $\mathbf{N}_{BS} = 36$ antennas and the mobile station has $\mathbf{N}_{MS} = 4$ antennas. The antenna array is considered to be ULA, with spacing between antennas as $\lambda/2$ and the phase shifters are assumed to have quantized phases only.

6.1.2 Channel Model

We consider the channel model as described in section 3.1 of chapter 3 with power in each stage as $\mathbf{P} = 1$. We consider $\mathbf{L}_d = 2$ paths. The AoA/ AoD are assumed to take continuous values uniformly distributed in $[0, 2\pi]$ and $[0, \pi]$ for the azimuth and elevation angles respectively. The system operates ar 28GHz with a bandwidth of 100 MHz, and with a pathloss exponent of $\mathbf{n}_{pl} = 3$.

6.1.3 Simulation Scenario

The simulations show us that the mean square error between the estimated channel and the exact channel are as low as 0.4 and the probability that the error between the exact dominant angles and estimated angles will be less than 0.1 can reach as high upto the value of 0.85 at the SNR of 20 dB.

For the single path case, the channel estimation algorithm specified in the section 5.1 is used. We consider the resolution of $\mathbf{N} = 81$ with $\mathbf{k} = 9$ beamforming vectors. For the Multi path case, the channel estimation algorithm specified in the section 5.2 is used. We consider the resolution of $\mathbf{N} = 162$ with $\widetilde{\mathbf{k}} = 3$ beamforming vectors and $\mathbf{L}_d = 2$ paths.

To capture the MSE or probability of angle error, we perform over 100 iterations for each SNR value.

We also plot the spectral efficiency of the estimated channel and compare it with the original channel. We show that estimated channel spectral efficiency is closer to the original. What we do is initially we estimate the channel parameters using the algorithms specified above and then reconstruct the estimated channel from the estimated parameters.

As we know that the optimal precoder($\mathcal{F}$) and combiners($\mathcal{W}$) are nothing but the right and left singular vectors of the channel. So, We find the optimal precoders and combiners for both the original channel and estimate channel and then calculate the rate using

$$\mathbf{R} = \log_2 |(\mathbf{I}_{L_d} + \frac{P}{N_s}\mathbf{R}_n^{-1}GG^H)|$$

Where, $G = \mathbf{W}^H\mathbf{HF}$ and $\mathbf{R}_n$ is the noise covariance matrix and $\mathbf{I}_{L_d}$ is the identity matrix of the size of no. of paths to be estimated.

For the following figure the parameters considered are $\mathbf{N} = 162$ with $\mathbf{k} = 3$ beamforming vectors for $\mathbf{L}_d = 2$ paths. Here we calculate the MSE error of the channel which is done as $E(H - \hat{H})^2$ for SNR (dB) values from 0 to 20dB in steps of 5dB, Where H,$\hat{H}$ are the original and estimated channels respectively. Its done over a loop of 100 iterations approximately.

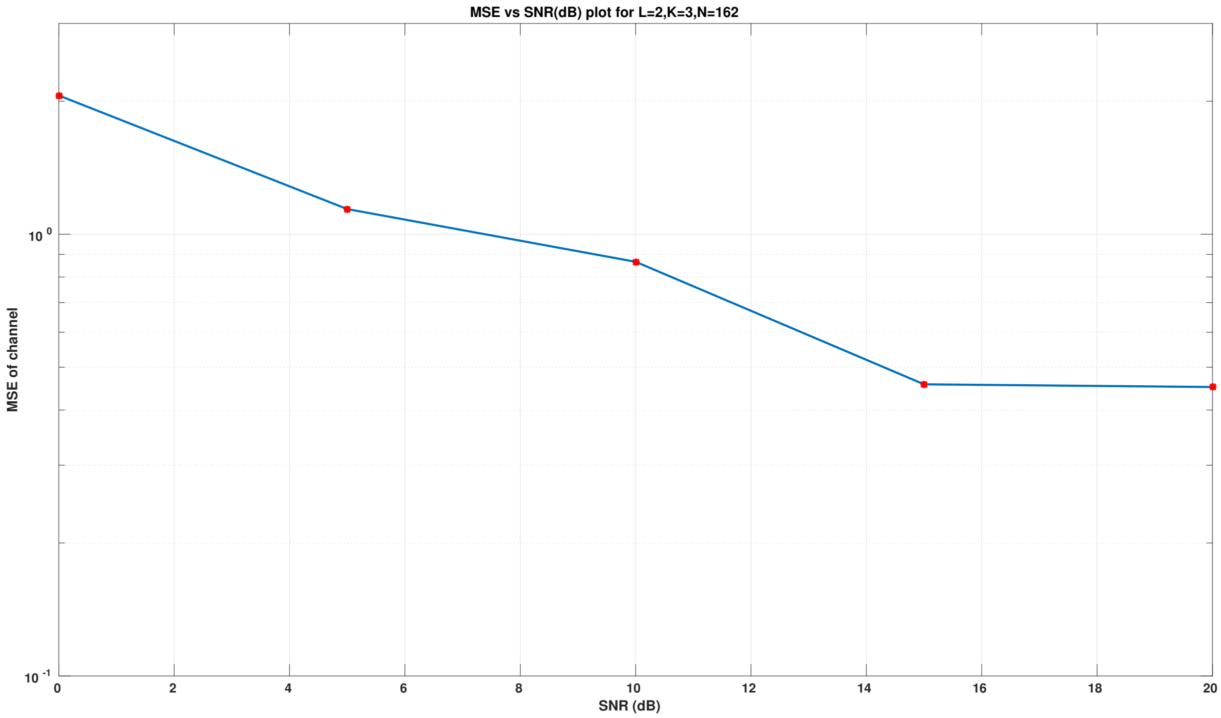

Figure 6.1: MSE of channel vs SNR (dB)

From the above figure, We observe that with increase in SNR (dB) values the mean square error (MSE) of the channel is decreasing. It makes complete sense as the increase in the SNR (dB) values indicate the decrease of the noise power added to the transmitted signal there by giving us the better signal for estimating. From the SNR = 15 dB, we observe that the curve is flattening from which we can infer that it is the least MSE we can achieve.

For the following figure the parameters considered are $\mathbf{N} = 162$ with $\mathbf{k} = 3$ beamforming vectors for $\mathbf{L}_d = 2$ paths. Its done over a loop of 200 iterations approximately. Here we calculate the prob(angle error)<0.1 with respect to the SNR (dB). Initially we calculate the angle error i.e., $(\alpha - \hat{\alpha})^2$, where

$$\alpha = [AoD_{azimuth} \quad AoD_{elevation}; AoA_{azimuth} \quad AoA_{elevation}]$$

$\hat{\alpha}$ is same matrix as above with the entries as estimated values

Then we calculate the probability(mean($(\alpha - \hat{\alpha})^2$) < 0.1) for each SNR value over 200 iterations.

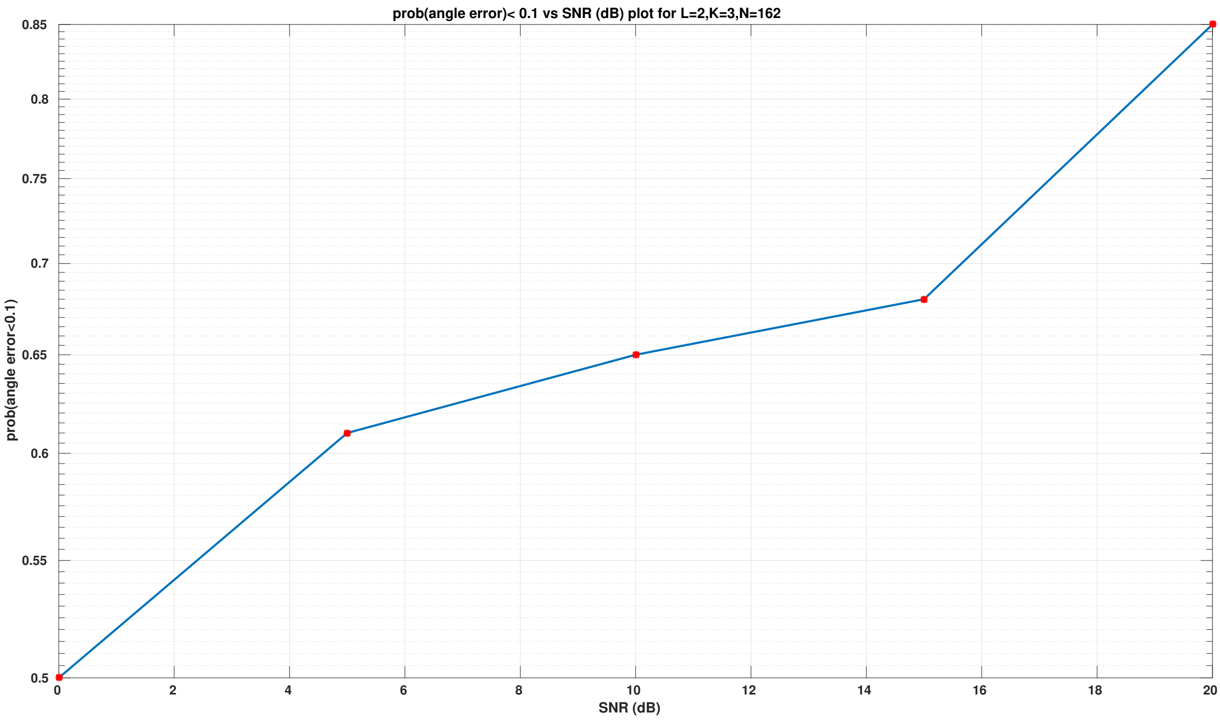

Figure 6.2: Probability(angle-error < δ) vs. SNR

From the above figure, We observe that value of the probability is increasing with the increase in SNR value. The reason is same as the reason for the above plot. With increase in SNR values we hope that our estimated angles get closer to the original angles, which is proven in the above plot.

For the following figures 6.3 and 6.4, the parameters considered are $\mathbf{N} = 162$,$\mathbf{k} = 3$, $\mathbf{L}_d = 2$ paths and $\mathbf{N} = 162$,$\mathbf{k} = 2$, $\mathbf{L}_d = 3$ paths respectively. Here, the spectral efficiency is calculated using the same formula stated in the simulation setup section i.e.,

$$\mathbf{R} = \log_2 |(\mathbf{I}_{L_d} + \frac{P}{N_s}\mathbf{R}_n^{-1}GG^H)|$$

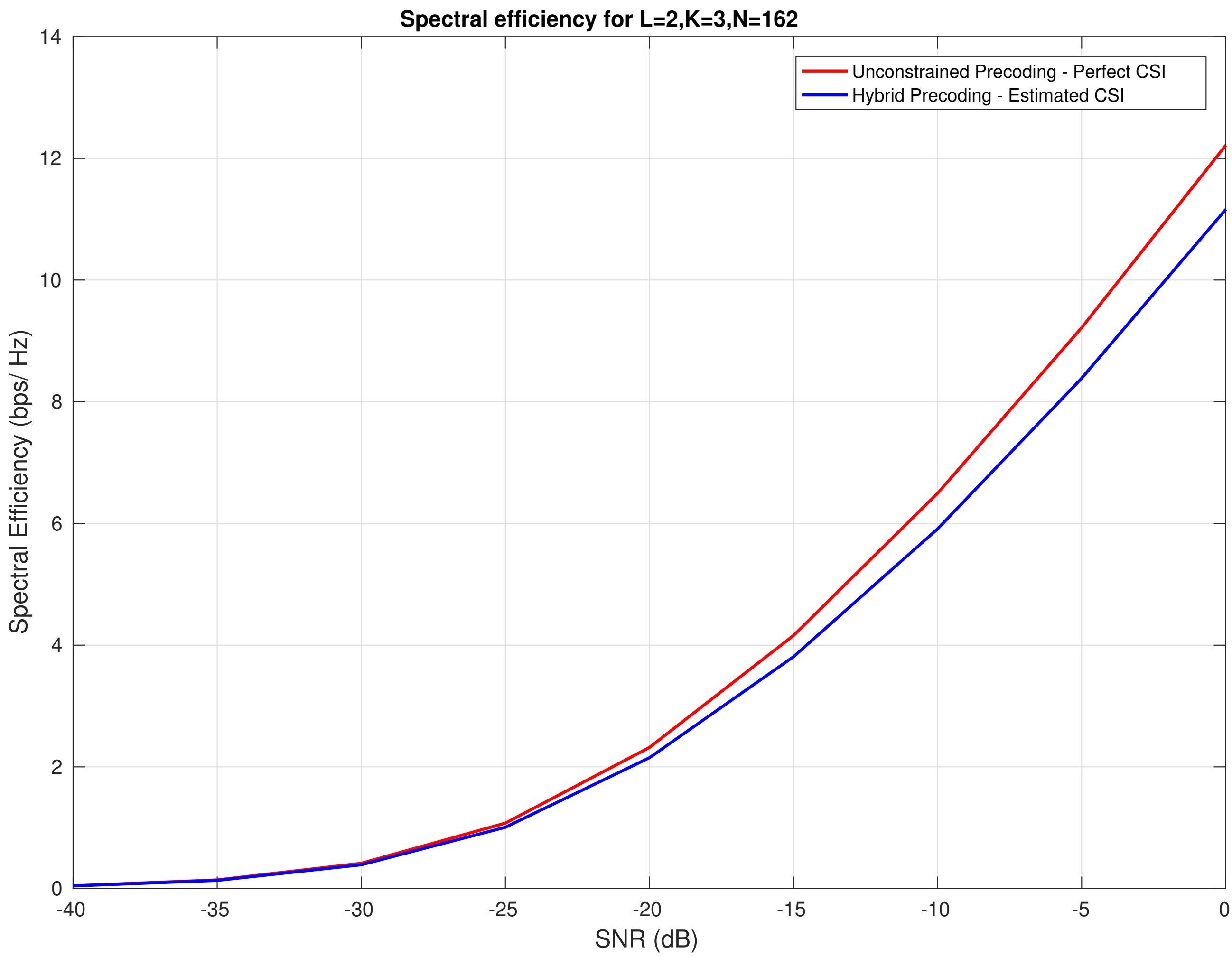

Figure 6.3: Spectral efficiency of the channel for $L_d = 2$,$\widetilde{K} = 3$

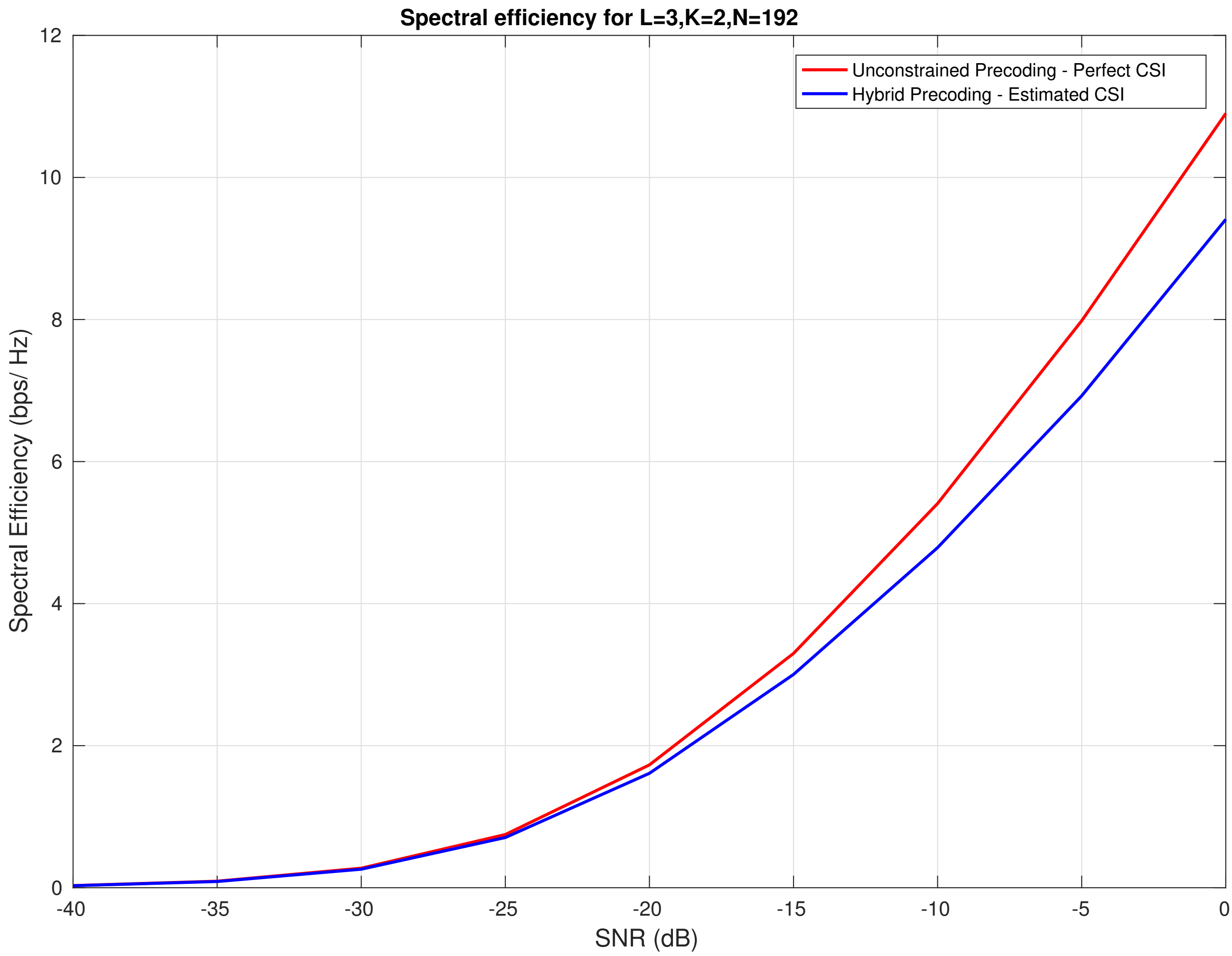

Figure 6.4: Spectral efficiency of the channel for $L_d = 3, \widetilde{K} = 2$

The above results indicate that comparable gains can be achieved using the proposed algorithms. From the above plots we can observe that the Hybrid Precoding Estimated channel spectral efficiency is approximately close to the spectral efficency of the original channel

6.1.4 Simulations done as a part of Literature Survey

I have simulated a part of B. Mamandipoor, D. Ramasamy and U. Madhow (2016) paper, Where we evaluated the performance of Newtonized orthogonal matching pursuit algorithm considering the squared frequency estimation error. My simulation setup included a mixture of $\mathbf{K} = 16$ sinusoids of length $\mathbf{N} = 256$. I performed 300 simulations runs for SNR = 25 dB. I have considered a oversampling factor, $\gamma = 4$, 1 single refinement stage and 3 cyclic refinement stages. I have considered minimum frequency separation as $\Delta\omega_{min}/\Delta_{dft} = 2.5$. Finally, I have plotted the CCDF of squared freuqncy estimation error.

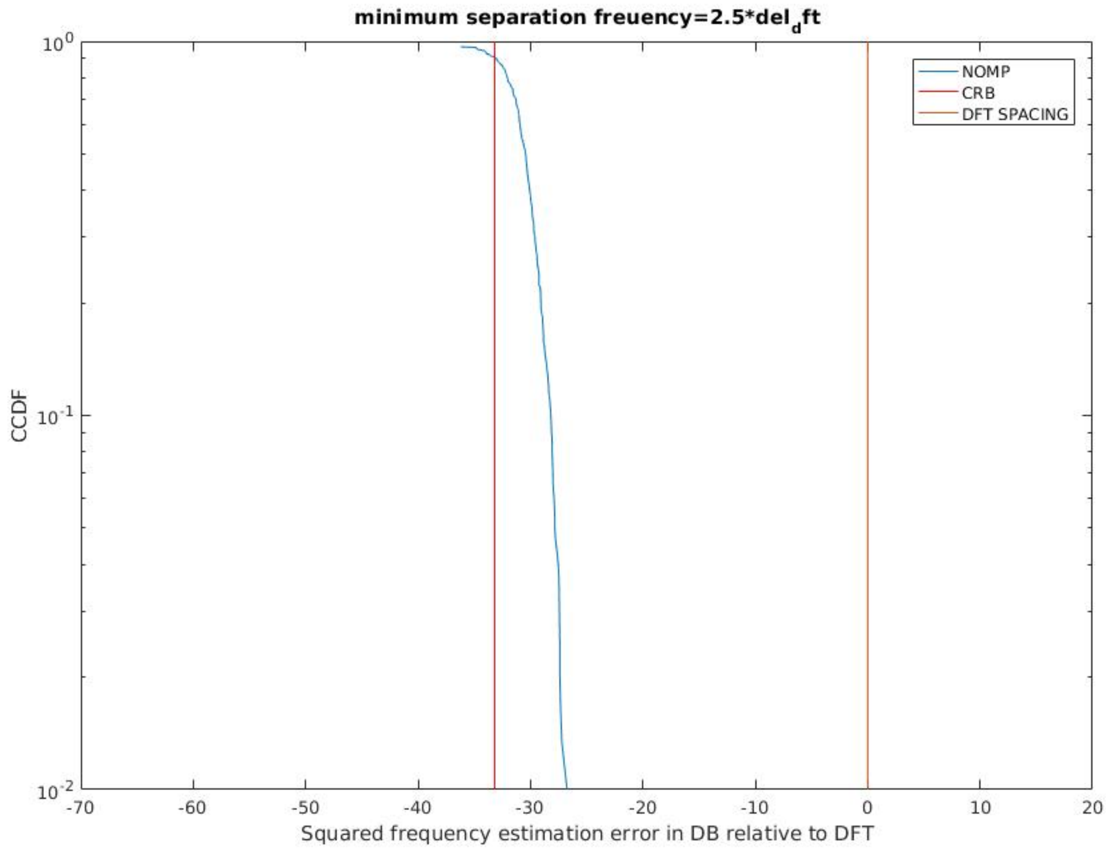

Figure 6.5: CCDF of frequency MSE

As we consider the case of well-separated frequencies, We hope to get the estimation accuracy of a single sinusoid. We therefore use the CRB corresponding to a single sinusoid i.e., $CRB(SNR) = 6/(SNR \times (N^2 - 1))$ as a measure of optimality. From the above figure we observe that our NOMP algorithm performance is close to the CRB.

For the next paper Zhinus Marzi, Dinesh Ramasamy and Upamanyu Madhow (2016), We consider 2-D sinusoids and perform compressive channel estimation algorithm for large arrays in mmWaves. For the simulation setup we have considered $\mathbf{N}_t = 8$ transmit antennas, $\mathbf{N}_r = 4$ receive antennas. The no. of transmit beacons, $\mathbf{M} = 24$ and the no. of virtual receive antennas,$\mathbf{L}$ are considered to be 6. Here also we consider the oversampling factor $\gamma = 4$ and look at the CCDF of frequency estimation error.

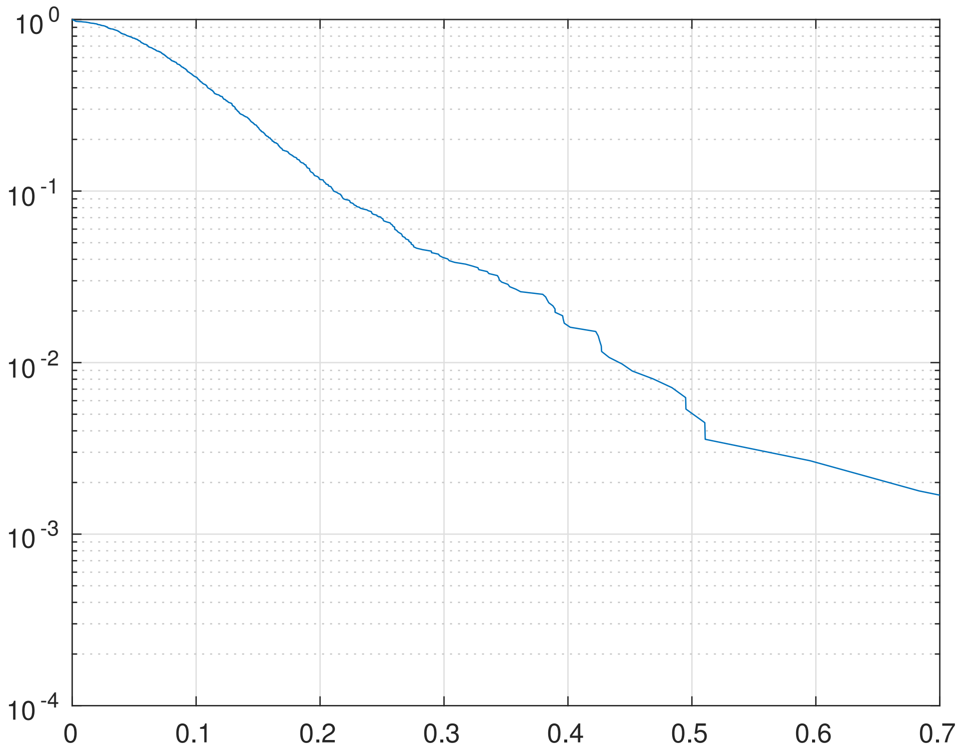

Figure 6.6: CCDF of frequency estimation error for 2-D sinusoids

The goal of this paper is to track the mm-Wave spatial channel as seen from the base station. So, Every mobile in the picocell needs to feedback a portion of the measured virtual channel. In this paper they consider two feedback strategies for the purpose of measured virtual channel i.e., feedback of dominant singular vectors and feedback of the entire matrix. Here for the simulations, I have considered the entire matrix feedback strategy but the performance when considering the dominant singular vectors is as efficient as feedbacking the entire matrix.

CHAPTER 7

Conclusions and Future Scope

7.1 Conclusions

Here in this paper we have considered a single user system. Initially we proposed a multi-resolution codebook for precoding vectors and then proposed a channel estimation algorithm for mmWave system. From the results obtained from simulations we can say that our estimated channel achieves the spectral efficiency close to the original channel. The mean square error between the original channel and estimated channel is found to be low and the probability of the angle error between the estimated angles and original angles less than a threshold, δ is found to be satisfactory.

7.2 Future Scope

For the future work of this project it would be interesting to consider the quantization error present due to the selection of quantized angles. We can incorporate off-grid based algorithms like Continuous basis pursuit, total least squares and Newtonized orthogonal matching pursuit (NOMP) to reduce this error.

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