EXPECTATION MAXIMIZATION BASED CHANNEL ESTIMATION USING QUANTIZED ADCS

A Project Report

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THESIS CERTIFICATE

This is to certify that the thesis titled **Expectation Maximization based channel estimation using quantized ADCs**, submitted by **Vaishnavi Adella**, to the Indian Institute of Technology, Madras, for the award of the degree of **Master of Technology**, is a bona fide record of the research work done by her under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

The increasing data rates push us to go for higher bandwidth systems. While the data needs of everyone can be taken care of, higher bandwidth systems need very high sampling rates for the ADCs being used. This becomes a major problem as ADCs consume high power as the sampling rate increases. So to avoid this situation, instead of using 8 or 12 bit ADCs, lower resolution ADCs using 1-4 bits can be used. But then the problem of channel estimation from the quantized data comes into picture. This motivates us to develop advanced estimation algorithms that take the quantizer effect on the system into consideration. This project is aimed to design one such method using Expectation Maximization algorithm to estimate the channel impulse response in case of lower resolution ADCs(1-4 bits).

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CHAPTER 1

INTRODUCTION

1.1 Motivation

The ever increasing need for higher data rates led to the invention of Millimeter wave systems which utilize the frequency bands between 30 and 300 GHz. Because of the small wavelength of operation, the dimensions of the transmitter and receiver can be reduced by packing antenna arrays in small area. But with increase in the bandwidth of operation, the sampling rate also increases dramatically. This in turn makes the Analog-to-Digital(ADC) converters more and more power hungry. ADCs become the bottleneck to the system.

The main challenge here is to be efficient with our signal processing while avoiding high power losses. To reduce the power consumption we have to resort to using low resolution ADCs. Channel State Information(CSI) is needed at the receiver to get back the transmitted signal. So the aim of this project is to improve the estimation algorithm to work efficiently even with low precision ADCs.

1.2 Literature Survey

There are two ways in which we can optimize the performance even while using low bit ADCs.

- To improve the signal processing before passing through the ADC so that the lower resolution might not affect the whole system.[1] In this paper, a dither signal is used for closed loop estimation based on linear feedback, with the dither signal being the MMSE estimator of the signal which is fed back to the quantizer. This reduces the dynamic range of the signal entering the quantizer. (In the thesis ADC and quantizer are used interchangeably; quantizer denotes the inner function of the Analog to Digital converter while ADC denotes the physical block used for the function.)

- To take the lower resolution as given and improve the estimation methods to better demodulate the data.[2] In this paper, Expectation Maximization(EM) algorithm is used to estimate the channel impulse response. This method is used in this project.

1.2.1 Why EM algorithm?

EM algorithm is an iterative algorithm to find Maximum Likelihood(ML) or Maximum A posteriori(MAP) estimates. It is known to give good results for parameter estimation from an observed variable; especially when the observed signal is incomplete or has missing data.That is specifically the case with the CSI estimation; the signal after passing through the ADC is incomplete. And also as we increase the resolution of the quantizer, we can get close to the performance of the full precision ADCs. EM method was first proposed in[3]. It is also used for channel estimation with multipath doppler channel model[4]. In this case too, the signal at the receiver is the sum of the delayed versions of the transmitted signal and is missing the complete information of the transmitted signal. Details about how EM algorithm is used are discussed in Chapter 3.

1.3 Thesis outline

The rest of the thesis has been organized as follows. Chapter 2 of the thesis describes the system model and the assumptions considered. In Chapter 3 the estimation methods and the procedure to obtain optimal channel impulse response is discussed briefly. Chapter 4 gives the simulation results and observations made from the comparison of EM based method with existing LS method. Also the future possibilites are discussed. Finally the project is concluded in Chapter 5.

CHAPTER 2

System Model

Figure 2.1 gives the block diagram of the system implemented. In this project, the EM method id compared with the already existing Least Squares(LS) Estimate method of channel estimation for OFDM system with low precision ADCs(1,2,4 and 8 bits). In LS method, the channel estimation is done after taking the FFT on the receiver side. But while using EM method the time domain impulse response of the channel is being estimated instead of the frequency response. So channel estimation is done just after passing the received signal through the quantizer, before Cyclic Prefix(CP) removal.

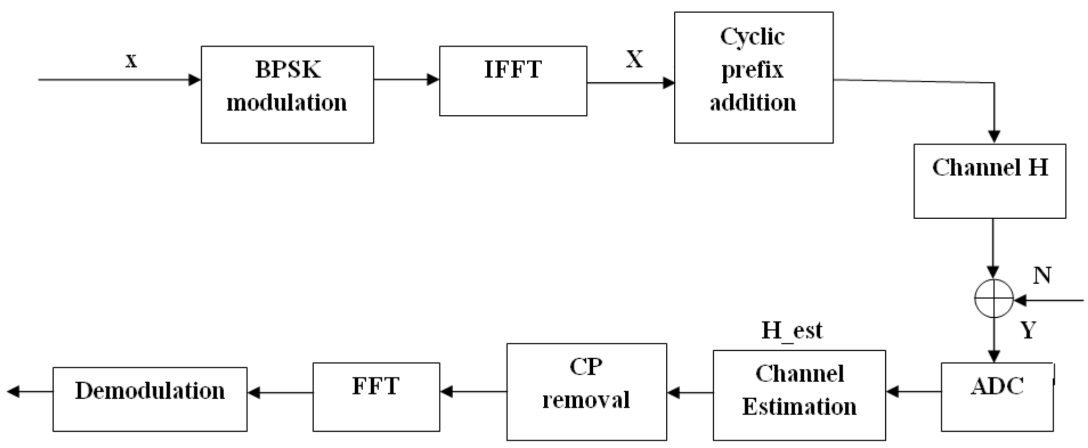

Figure 2.1: System Block Diagram

2.1 Assumptions

The noise added is assumed to be Additive White Gaussian(AWGN) in nature. The number of taps in the channel impulse response is assumed to be known at the receiver. Also, the channel response is assumed to be constant over an OFDM symbol. The estimation is pilot-aided. All the 1024 subcarriers of an OFDM symbol are used for pilot samples. And finally, the cyclic prefix length is assumed to be greater than the channel taps.

2.2 System equations

The following section gives the mathematical equations the describe the system shown in the figure above. $\mathbf{x}$ denotes the Nx1 input signal vector (here N=1024), $\mathbf{X}$ denotes the signal after taking the IFFT, $\mathbf{h}$ denotes the channel impulse response, $\mathbf{N}$ is the AWGN noise added, $\mathbf{Y}$ is the signal after passing through the channel, $\mathbf{h_{ls}}$ is the estimated channel response for LS estimation and $\mathbf{h_{em}}$ is the estimated channel response for EM method. L_h is the length of $\mathbf{h}$.

The cyclic prefix addition refers to the prefixing of a symbol with a repetition of the end. It helps in reducing the Inter symbol interference and also lets us model the linear convolution operation of the input signal and the channel with circular convolution. In our system, the circular convolution is modeled as a matrix operation where the input signal vector is modified into a NxL_h matrix. For example, if

$$\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

and if L_h=3, the modified matrix will be

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

The convolution of $\mathbf{x}$ and $\mathbf{h}$ will be equivalent to $\mathbf{x_{modified}h}$. So, the system equations are given by,

$$\mathbf{Y} = \mathbf{Xh} + \mathbf{N} \tag{2.1}$$

The signal is complex and can be written as the combination of its real and imaginary parts.

$$\mathbf{Y} = \mathbf{Y_r} + \mathbf{jY_{im}} \tag{2.2}$$

$$\mathbf{X} = \mathbf{X_r} + \mathbf{jX_{im}} \tag{2.3}$$

$$
\begin{aligned}
\mathbf{Y_r} + \mathbf{jY_{im}} &= \mathbf{Xh} + \mathbf{N} \\
&= (\mathbf{X_r} + \mathbf{jX_{im}})\,\mathbf{h} + (\mathbf{N_r} + \mathbf{jN_{im}}) \\
&= (\mathbf{X_r h} + \mathbf{N_r}) + \mathbf{j}\,(\mathbf{X_{im} h} + \mathbf{N_{im}})
\end{aligned} \tag{2.4}
$$

which implies that

$$\mathbf{Y_r} = (\mathbf{X_r h} + \mathbf{N_r}) \tag{2.5}$$

and

$$\mathbf{Y_{im}} = (\mathbf{X_{im} h} + \mathbf{N_{im}}) \tag{2.6}$$

This shows that we can use just the real or the imaginary part of the received signal alone to estimate **h** as it holds the whole information about the channel impulse response. At the receiver, the signal is passed through the quantizer. The quantizer function is denoted by Qu.

$$\mathbf{r_r} = Qu(\mathbf{Y_r}) \tag{2.7}$$

The quantizer function is described in the following section.

2.3 Quantizer function

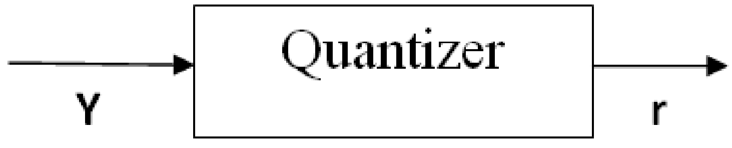

Figure 2.2: Quantizer function

The quantizer used here is uniform symmetric mid-riser type which takes values from the receive alphabet

$$r_i \in \left\{(-2^b/2 - 1/2 + k)\Delta; k = 1, 2, ...2^b\right\} \tag{2.8}$$

where $\Delta = [max(\mathbf{y}) - min(\mathbf{y})]/(2^b - 1)$ is the quantizer step size and b is the number of bits of the quantizer. In this project, b= 1, 2, 4 and 8 bits are considered and their performance is compared. As the number of bits increases, the power consumed and the complexity increases but the performance goes closer to the full precision quantizer as the quantization error decreases. Each quantization level has an upper and a lower quantization threshold which is defined as,

$$r_i^{lo} = \begin{cases} r_i - \Delta/2, r_i \geq -\frac{\Delta}{2}(2^b - 2) \\ -\infty, otherwise \end{cases} \tag{2.9}$$

$$r_i^{up} = \begin{cases} r_i + \Delta/2, r_i \leq \frac{\Delta}{2}(2^b - 2) \\ \infty, otherwise \end{cases} \tag{2.10}$$

For simulation purposes infinity is approximated as 10^5.

CHAPTER 3

Channel Estimation

3.1 Least-Squares Method

This is the basic channel estimation method used in OFDM systems. Frequency response of the channel is estimated here. It is done after the FFT step in the block diagram above. Let **y** be the N point FFT of the received signal **r** and **x** be the N point FFT of the BPSK modulated input signal. Then the Least Squares estimate is given by

$$\mathbf{H_{ls}} = \mathbf{y}/\mathbf{x} \tag{3.1}$$

An index wise division is done and this gives the frequency response of the channel which is a Nx1 vector. To get the time domain impulse response, we just take the N point IFFT of $\mathbf{H_{ls}}$ and take the first L_h samples from it. This gives us $\mathbf{h}_{ls}$.

The main advantage of this method is that it is simple to execute and it does not involve any time consuming matrix inversions. It just operates on FFT and IFFT which are linear matrix multiplication operations. The only problem with this; it is inefficient when low bit quantizers are used. Because, the signal **r** does not contain the whole information about the received signal **Y**. This method does not take into fact the quantizer or its effect on the system. This is why we go for another method of estimating the channel response.

3.2 Expectation Maximization Method

3.2.1 Algorithm Explained

EM algorithm presents a general approach to iterative computation of maximum-likelihood estimates when the observations can be viewed as incomplete data. Since each iteration of the algorithm consists of an expectation step followed by a maximization step we call it the EM algorithm. The EM process is remarkable in part because of the simplicity and generality of the associated theory, and in part because of the wide range of examples which fall under its umbrella. This sections explains the general idea of the EM algorithm and its applications.

The Maximum Likelihood estimation involves finding the Likelihood function and maximizing it to find the value of the parameter at which the maxima occurs. But this way of getting the parameter value is not always feasible. It is not always necessary that solving the differential equation gives a closed form expression in getting the parameter. And it might not always be simple to solve the differential equation itself. This is the reason to go to an iterative approach. The idea behind this algorithm is even though we do not have the exact values of the complete data, we try to get the value of the parameter from the conditional probability distribution of the complete data variable given the current parameter value and the value of the incomplete data. And this is done iteratively till the parameter converges to some value. This is explained with equations below. A block diagram in figure 3.1[1] shows the steps of the algorithm.

Let **x** be the complete data which is not observable. Let **y** be the incomplete data or data with missing information; This is the observed data. The parameter estimation is to be done using this data. let θ be the parameter to be estimated. The algorithm starts with randomly assuming an initial value for the parameter. Let this initial value be $\theta^{(0)}$ and $\theta^{(k)}$ denotes the value of the parameter at k^{th} iteration.

Let f(**x**|θ) be the pdf of the complete data and g(**y**|θ) be the pdf of the incomplete data. If the complete data is available for us, the problem would be to find θ that maximizes log(f(**x**|θ)); the log likelihood function. But since we do not have **x**, we in turn maximize the expected value of log(f(**x**|θ)) given **y** and the current value of θ.

Expectation Step (E-step): Compute

$$Q(\theta; \theta^{(k)}) = E[log(f(\mathbf{x}|\theta)|\mathbf{y}, \theta^{(k)}] \tag{3.2}$$

Here θ is the variable and $\theta^{(k)}$ is the parameter value in k^{th} iteration. $\theta^{(k)}$ is known at every iteration.

[1]This figure is taken from[5]

Maximization Step (M-step) : Update the value of $\theta^{(k+1)}$ as the value of θ that maximizes the function Q.

$$\theta^{(k+1)} = \underset{\theta}{\operatorname{argmax}}[Q(\theta; \theta^{(k)})] \tag{3.3}$$

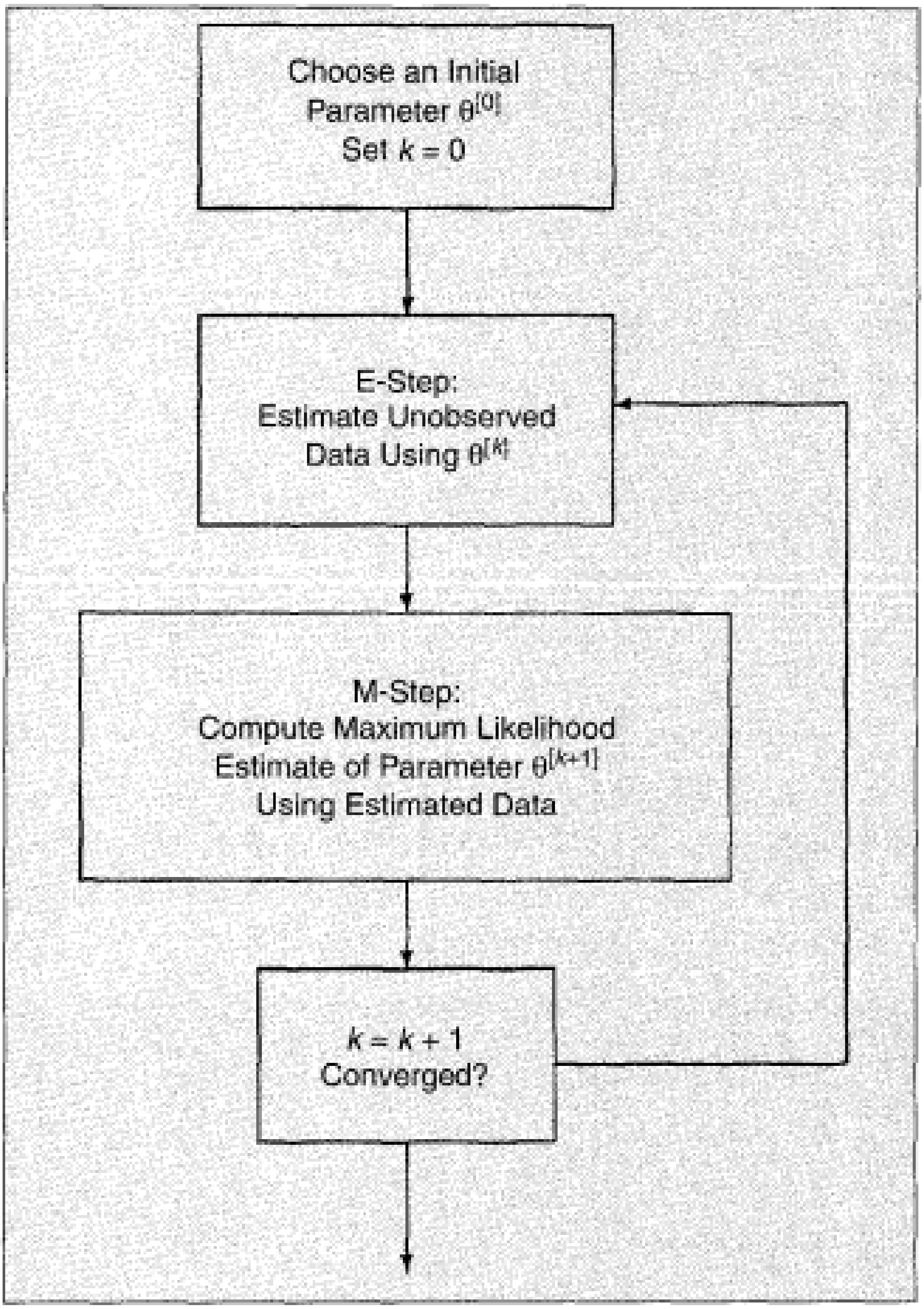

Figure 3.1: Block diagram for EM algorithm

These two steps are iteratively repeated till θ value converges. The proof that this convergence is possible is discussed in[3]. The main factors that determine the rate of convergence are the *initial value* and the *stopping criterion*. The algorithm might not always converge to a particular value. Even if it does, it might change for the next iteration and the system might start diverging from the optimal value. Or in some cases we might not need the value to be so accurate and we cannot spend our resources for too many iterations; We are fine if the value is close to the actual optimal value. So a condition is put to stop the iterations once it is satisfied. This is called a stopping criterion. The convergence also depends on the initial value of θ chosen. Because if the initial value is close to a local optima, it might converge to a local maxima instead of global maxima. If that is not the case, it might take longer time and more number of iterations for convergence if the initial value is not chosen carefully.

3.2.2 Applications

A typical application area of this algorithm is in genetics, where the observed data (the phenotype) is a function of the underlying unobserved gene pattern (the genotype). Another area is estimating parameters of the mixture distributions. In the area of signal processing, it is used in maximum likelihood tomographic image reconstruction, to train hidden markov models for speech recognition. Other applications include parameter estimation, pattern recognition, image reconstruction, neural network training, noise suppression, simultaneous detection and estimation among many others.

3.2.3 Channel estimation using EM

Coming to the problem at hand, **Y** is the complete data, **r** is the incomplete data and the parameter to be estimated is the channel impulse response **h**. As given by eqn(2.7), the observed variable is $\mathbf{r}_r$. The log likelihood equation is

$$L(\mathbf{h}) = log[p(\mathbf{r}_r|\mathbf{h})] \tag{3.4}$$

So the estimated value $\mathbf{h}_{ls}$ is given by

$$\mathbf{h}_{ls} = \underset{\mathbf{h}}{\text{argmax}}[L(\mathbf{h})] \tag{3.5}$$

We define a function

$$Q(\mathbf{h};\mathbf{h}^{(i)}) = \int g^{(i)}(\mathbf{Y}_r) log[p(\mathbf{Y}_r, \mathbf{r}_r|\mathbf{h})/g^{(i)}(\mathbf{Y}_r)]d\mathbf{Y}_r \tag{3.6}$$

where $g^{(i)}(\mathbf{Y}_r) = p(\mathbf{Y}_r|\mathbf{h}^{(i)}, \mathbf{r})$. By applying Jensen's Inequality to the above equation,

$$\begin{aligned} Q(\mathbf{h};\mathbf{h}^{(i)}) &\leqslant log[\int g^{(i)}(\mathbf{Y}_r)p(\mathbf{Y}_r, \mathbf{r}_r|\mathbf{h})/g^{(i)}(\mathbf{Y}_r)]d\mathbf{Y}_r \\ &= log[p(\mathbf{r}_r|\mathbf{h})] \end{aligned} \tag{3.7}$$

which is the log likelihood function in eqn(3.4). So maximizing Q($\mathbf{h}$;$\mathbf{h}^{(i)}$) maximizes L($\mathbf{h}$) and eqn(3.5) can be written as,

$$\mathbf{h}_{ls} = \underset{\mathbf{h}}{\text{argmax}}[Q(\mathbf{h};\mathbf{h}^{(i)})] \tag{3.8}$$

Since we have assumed the noise to be AWGN, the probability distribution

$$p(\mathbf{Y}_r, \mathbf{r}_r|\mathbf{h}) = \int_{r^{\text{lo}}}^{r^{\text{up}}} \frac{1}{(2\pi)^{\frac{N}{2}}\sigma^N} e^{-\frac{\|\mathbf{Y}_r - \mathbf{Xh}\|^2}{2\sigma^2}} d\mathbf{Y}_r \tag{3.9}$$

where σ is the noise variance and r^{lo} and r^{up} are the lower and upper quantization thresholds respectively as described in section 2.3.

Now Q($\mathbf{h}$;$\mathbf{h}^{(i)}$) can be decomposed as

$$Q(\mathbf{h};\mathbf{h}^{(i)}) = Q_1(\mathbf{h};\mathbf{h}^{(i)}) + H \tag{3.10}$$

where

$$Q_1(\mathbf{h};\mathbf{h}^{(i)}) = \int g^{(i)}(\mathbf{Y}_r) log[p(\mathbf{Y}_r, \mathbf{r}_r|\mathbf{h})]d\mathbf{Y}_r \tag{3.11}$$

and

$$H = -\int g^{(i)}(\mathbf{Y}_r) log[g^{(i)}(\mathbf{Y}_r)]d\mathbf{Y}_r \tag{3.12}$$

Since H does not depend on $\mathbf{h}$, we need not consider it while maximizing $Q(\mathbf{h};\mathbf{h}^{(i)})$. So eqn(3.8) can be rewritten as

$$\mathbf{h}_{ls} = \underset{\mathbf{h}}{\text{argmax}}[Q_1(\mathbf{h};\mathbf{h}^{(i)})] \tag{3.13}$$

Substituting the value of $p(\mathbf{Y}_r, \mathbf{r}_r|\mathbf{h})$ in the eqn(3.11) and evaluating eqn(3.13), we get

$$\mathbf{h}^{(i+1)} = (\mathbf{X}_r{}^T\mathbf{X}_r + \sigma^2\mathbf{R}_h{}^{-1})^{-1}\mathbf{X}_r{}^T(\mathbf{X}_r\mathbf{h}^{(i)} + \mathbf{b}^{(i)}) \tag{3.14}$$

where

$$\mathbf{b}^{(i)} = -\frac{\sigma}{\sqrt{2\pi}}.\frac{e^{-\frac{(\mathbf{r}_r{}^{up}-\mathbf{X}_r\mathbf{h}^{(i)})^2}{2\sigma^2}} - e^{-\frac{(\mathbf{r}_r{}^{lo}-\mathbf{X}_r\mathbf{h}^{(i)})^2}{2\sigma^2}}}{\Phi(\frac{\mathbf{r}_r{}^{up}-\mathbf{X}_r\mathbf{h}^{(i)}}{\sigma}) - \Phi(\frac{\mathbf{r}_r{}^{lo}-\mathbf{X}_r\mathbf{h}^{(i)}}{\sigma})} \tag{3.15}$$

$\mathbf{R}_h$ denotes the channel covariance matrix and $\Phi(x)$ represents the cummulative Gaussian distribution function given as

$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{(-t^2/2)}dt \tag{3.16}$$

3.2.4 Channel estimation in case of complex h

In the previous section the channel impulse response is assumed to be real. But when it comes to complex **h**, the estimation changes slightly. The real and the imaginary parts of **h** are to be estimated separately. This is illustrated from the system equations given below.

$$\mathbf{Y} = \mathbf{X}\mathbf{h} + \mathbf{N} \tag{3.17}$$

$$\mathbf{Y_r} + \mathbf{jY_{im}} = (\mathbf{X_r} + \mathbf{jX_{im}})\,(\mathbf{h}_r + \mathbf{h}_{im}) + (\mathbf{N_r} + \mathbf{jN_{im}}) \tag{3.18}$$

$$\mathbf{Y_r} = \mathbf{X_r}\mathbf{h}_r - \mathbf{X_{im}}\mathbf{h}_{im} \tag{3.19}$$

$$\mathbf{Y_{im}} = \mathbf{X_r}\mathbf{h}_{im} + \mathbf{X_{im}}\mathbf{h}_r \tag{3.20}$$

$$\mathbf{r_r} = Qu(\mathbf{Y_r}) \tag{3.21}$$

$$\mathbf{r_{im}} = Qu(\mathbf{Y_{im}}) \tag{3.22}$$

From the above equations it can be shown that both $\mathbf{h}_r$ and $\mathbf{h}_{im}$ cannot be obtained from either the real or the imaginary parts of the observed variable. So, the real and the imaginary parts are to be jointly estimated from both $\mathbf{r}_r$ and $\mathbf{r}_{im}$.

The initial values for the first iteration are taken randomly for both $\mathbf{h}_r$ and $\mathbf{h}_{im}$. The observed variables here would be $\mathbf{r}_r$ and $\mathbf{r}_{im}$ from both the ADCs. Following the same procedure as done for real **h** case, we will have two equations, one from maximizing the Q_r function with respect to $\mathbf{h}_r^i$ and one from maximizing the Q_{im} function with respect to $\mathbf{h}_{im}^i$ where

$$Q_r(\mathbf{h}_r; \mathbf{h}_r^{(i)}) = \int g_r^{(i)}(\mathbf{Y}_r) log[p(\mathbf{Y}_r, \mathbf{r}_r | \mathbf{h}_r)/g_r^{(i)}(\mathbf{Y}_r)] d\mathbf{Y}_r \tag{3.23}$$

$$Q_r(\mathbf{h}_{im}; \mathbf{h}_{im}^{(i)}) = \int g_{im}^{(i)}(\mathbf{Y}_{im}) log[p(\mathbf{Y}_{im}, \mathbf{r}_{im} | \mathbf{h}_{im})/g_{im}^{(i)}(\mathbf{Y}_{im})] d\mathbf{Y}_{im} \tag{3.24}$$

Solving these two equations we get,

$$\mathbf{h}_r{}^{(i+1)} = (\mathbf{X}_r{}^T\mathbf{X}_r + \mathbf{X}_{im}{}^T\mathbf{X}_{im} + \sigma^2\mathbf{R}_h{}^{-1})^{-1}(\mathbf{X}_r{}^T\mathbf{p} + \mathbf{X}_{im}{}^T\mathbf{q}) \tag{3.25}$$

$$\mathbf{h}_{im}{}^{(i+1)} = (\mathbf{X}_r{}^T\mathbf{X}_r + \mathbf{X}_{im}{}^T\mathbf{X}_{im} + \sigma^2\mathbf{R}_h{}^{-1})^{-1}(\mathbf{X}_r{}^T\mathbf{q} - \mathbf{X}_{im}{}^T\mathbf{p}) \tag{3.26}$$

where

$$\mathbf{p} = \mathbf{Y_r}^{\mathbf{(i)}} + \mathbf{b_r}^{\mathbf{(i)}} \tag{3.27}$$

$$\mathbf{q} = \mathbf{Y_{im}}^{\mathbf{(i)}} + \mathbf{b_{im}}^{\mathbf{(i)}} \tag{3.28}$$

CHAPTER 4

Results

The metric used to decide if the system is better is Mean Square Error(MSE) which is given by,

$$\mathbf{MSE}_{em} = \| \mathbf{h} - \mathbf{h}_{em} \|^2 \tag{4.1}$$

and

$$\mathbf{MSE}_{ls} = \| \mathbf{h} - \mathbf{h}_{ls} \|^2 \tag{4.2}$$

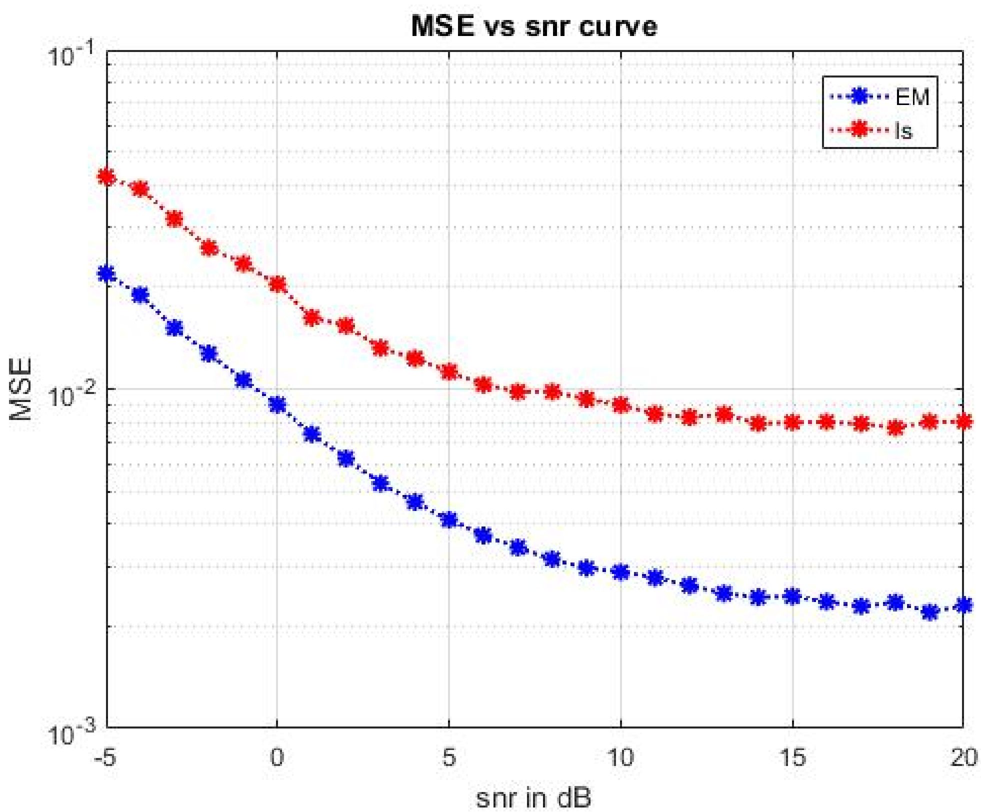

Figure 4.1: MSE vs SNR plots for EM and LS methods

The initial value is taken randomly and the stopping criterion is set as

$$\| h^{(i+1)} - h^{(i)} \|^2 < 10^{-6} \tag{4.3}$$

The above figure is plotted for a 3 tap channel when 2 bit quantizer is used. It can be seen that Expectation Maximization method gives better performance than Least Squares method as it gives lower MSE than LS.

The performance of the algorithm when different resolution ADCs are used is shown by plotting the MSE vs SNR curves for different bits. This is shown in the figure below.

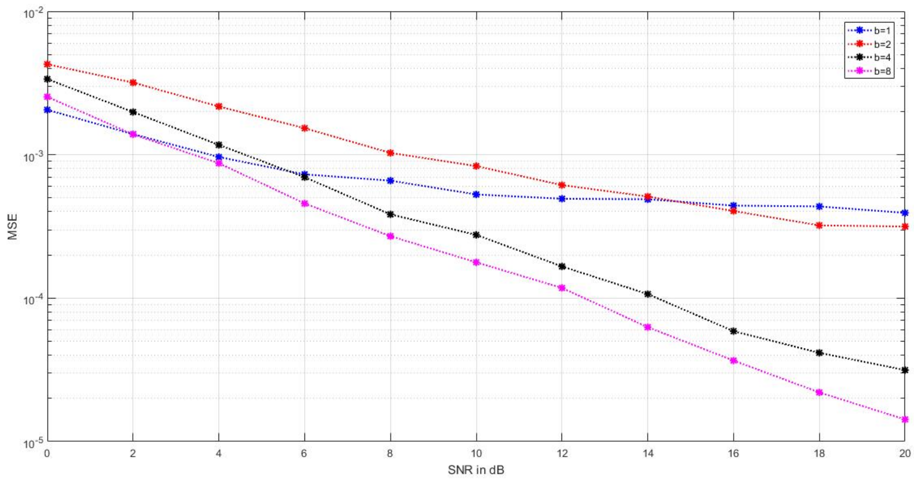

Figure 4.2: MSE vs SNR plots for different bit ADCs using EM method

As it can be seen, at very low snr, the one bit ADC is giving lower MSE compared to higher resolution ADCs. It is attributed to a phenomenon called *stochastic resonance* where even noise helps in getting a better estimate at low snrs.

The MSE curve for complex **h** is also plotted when 1 bit ADC is used.This is shown in the figure below. When complex case is considered, several factors come into play. The random initialization may affect the system performance since estimating both real and imaginary components has to be done.

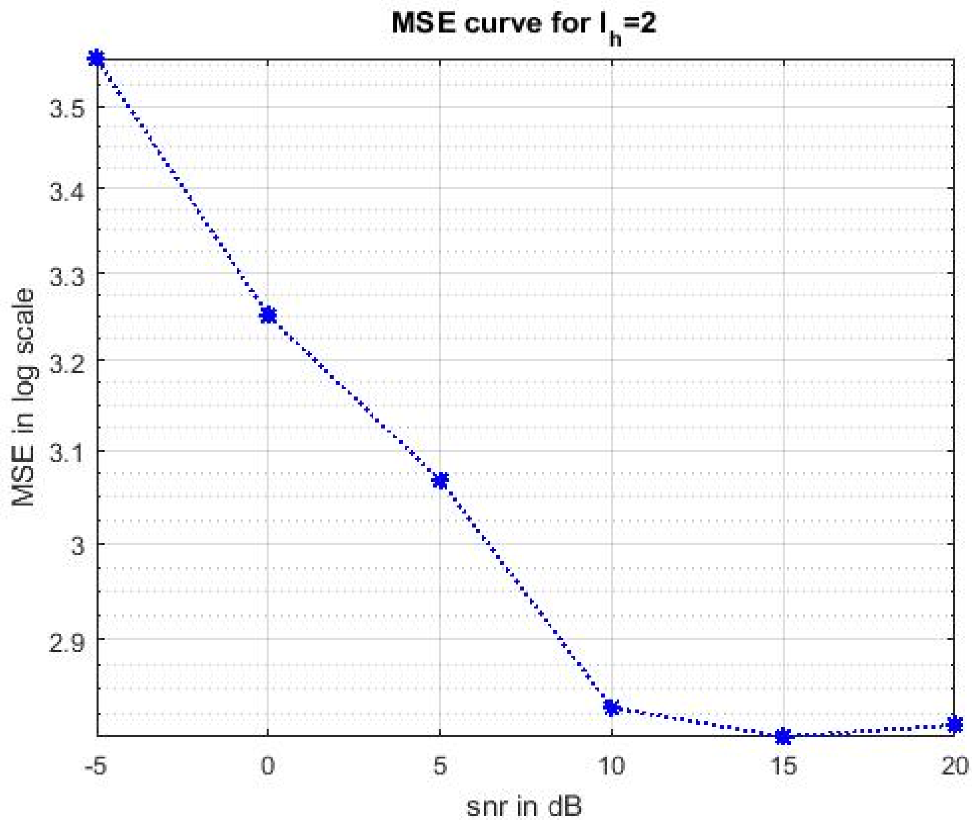

Figure 4.3: MSE vs SNR plot for complex **h** when 1 bit ADC is used

4.1 Further Improvements

The system can be further improved by taking the Least Squares estimate as the initial value for the iterations rather than randomly initializing. This might help getting better performance for the complex **h**. This might also reduce the number of iterations it takes to converge to a final desired value of the channel impulse response **h**. Also this can be extended to the MIMO OFDM case too.

CHAPTER 5

Conclusion

A new approach has been proposed to estimate the channel impulse response when quantized ADCs are used. This method involves iteratively obtaining the maximum likelihood estimate using an algorithm called Expectation Maximization. OFDM system is considered and the estimation method for the cases of both real and imaginary $\mathbf{h}$ has been discussed. A general quantizer function that works for different bit resolutions has been defined. Mean Square Error is taken as the performance metric and MSE curves for different ADC resolutions are compared. It is shown that EM method gives a better performance when compared to the Least Squares method. This is because EM method take into account the non linear quantizer function and maximizes the pdf function accordingly.

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