Adaptive resource allocation for two tier FFR-Aided cellular network with subscriber differentiation

A Project Report

submitted by

ANJANA N N

in partial fulfilment of the requirements
for the award of the degree of

MASTER OF TECHNOLOGY

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY MADRAS.

June 2017

THESIS CERTIFICATE

This is to certify that the thesis titled **Adaptive resource allocation for two tier FFR-Aided cellular network with subscriber differentiation**, submitted by **Anjana N N**, **EE15M013**, to the Indian Institute of Technology, Madras, for the award of the degree of **Master of Technology**, is a bona fide record of the research work done by her under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Dr. Sheetal Kalyani
Research Guide,
Associate Professor,
Dept. of Electrical Engineering,
IIT Madras, 600 036.

Place: Chennai

Date : June 9, 2017

ACKNOWLEDGEMENTS

My first gratitude is towards my parents for all their love and support throughout.

I was fortunate to have a wonderful mentor Dr. Sheetal Kalyani for my masters who was very supportive through the best and worst parts of my research. Her perseverance and motivation has driven me to give my best towards my masters research. Also, I express my sincere gratitude towards Dr.Suman Kumar (Asst. Professor, IIT Ropar) and Muralikrishnan S (PhD scholar, IIT Madras) for their consistent guidance and assistance. I acknowledge the support and care of my friends in this journey.

I am thankful to all those professors whose courses have been instrumental for my better understanding of the fundamentals.

ABSTRACT

KEYWORDS: Coverage probability,rate,frequency allocation scheme

Optimal spectrum allocation to users in cellular system is relevant in a network. This becomes more complicated when we introduce discrimination among users in the system. In this work, we consider two sets of users, as privileged users and unprivileged users in FFR aided cellular network and, we attempt to introduce new frequency allocation schemes such that the set of privileged users are allowed to enjoy better quality of service with least harm to the other set of users. Also, the scheme we implement depends on the percentage of privileged users in the network. Coverage probability and average rate are plotted for different resource allocation schemes and results are analysed. Also, we have come up with theoretical expressions for coverage and rate of a user in such a system.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABBREVIATIONS

BS	Base Station
FFR	Fractional Frequency Reuse
FRδ	Frequency Reuseδ
ICIC	Inter-cell interference coordination
SIR	Signal to Interference ratio
MIMO	Multi input multi output
pdf	Probability Density Function
CDF	Cumulative Distribution Function
i.i.d	Independent and Identically Distributed

NOTATION

ψ	Set of interferers in Frequency reuse 1
ϕ	Set of interferers in Frequency reuse 3
ϕ'	Set of interferers in Frequency reuse 4
α	Path loss factor
$\sim$	Distributed as, for example, $X \sim \mathcal{N}(0, 1)$ denotes X is a zero mean Gaussian random variable with variance 1
T_h	Desired SIR for privileged users

CHAPTER 1

Introduction

In recent years, mobile broadband systems have witnessed tremendous growth in both the number of subscribers and the traffic of each subscriber.The network providers are moving from single service (e.g.Internet access) to multiple service offering (e.g. multimedia telephony and mobile-TV) [2]. However, spectrum being an unreplenishable limited physical resource, it is difficult to have a single frequency sub-band for a user fulfilling the rising demand. Moreover, different users subscribing for the same service may receive different treatment from the network providers because of the subscriber differentiation provided by the service providers [3]. Subscriber differentiation may include high paid versus low paid user, corporate versus private user, post versus prepaid user etc. Users with higher priority typically expect better target coverage and rate levels compared to other users in the network.

3GPP-Long Term Evolution (LTE) is a standard for wireless communications to achieve high spectral efficiency, high peak data rates, as well as flexibility in frequency and bandwidth. High spectral efficiency is achieved using frequency reuse factor one. However, the Inter-Cell Interference (ICI) resulting from the frequency reuse factor one is the main limitation in these networks. Fractional Frequency Reuse (FFR) has been proposed as a possible solution and it is one of the Inter-Cell Interference Coordination (ICIC) technique in Orthogonal Frequency Division Multiple Access (OFDMA) based LTE networks. OFDMA inherits the immunity to inter symbol interference (ISI) wherein the users of the same cell are multiplexed in frequency with their data being transmitted over different orthogonal sub bands (orthogonal frequency division multiplexing (OFDM)). The inter-cell interference problem is a key issue in OFDMA based mobile cellular networks. The users at the outer ends of the cell suffer from ICI. In a OFDMA FDD system, the most commonly used interference coordination technique is a frequency reuse scheme wherein the use the same set of frequencies (sub-bands) is avoided in neighbouring cells. The ICI is therefore reduced at the expense of the available frequency resources for each cell. In order to balance a trade off between available frequency resources of each cell and interference reduction, the following solution, namely FFR, is proposed and currently being used in LTE [1]. The cell is divided into two areas namely cell-centre and cell-edge. The total system bandwidth is thus divided into two parts. The first part is universally reused (i.e. frequency reuse factor 1) by the cell-centre zone and the cell-edge zone experience a frequency reuse factor > 1 (shared among neighbouring cells).

In FFR, frequency reuse factor one is employed in areas close to the base station, and a higher reuse factor is employed in areas closer to the cell boundary. FFR is a combination of frequency reuse 1 (FR1) and frequency reuse δ (FRδ). FR1 allocates all the frequencies to each cell and hence results in a low-quality coverage due to the higher inter-cell interference. On the other hand, FRδ allocates a fraction ($\frac{1}{\delta}$) of the frequencies to each cell which reduces the area spectral efficiency, but improves the SIR. FFR exploits the advantages of both FR1 and FRδ by relying on FR1 for the cell-centre users i.e. for those users who are close to their serving base station (BS) (who experience less interference) and FRδ is invoked for the cell-edge users i.e. for those users who are far from their serving BS and hence, suffer co-channel interference adversely from the neighboring cells in FR1 . The optimal frequency reuse factor is FR3 for the cell-edge users [5].

Several ICIC techniques have been proposed to improve rate and coverage in a multi-cell network [1] [4]. There are different resource allocation schemes suggested in literature that take into account the subscriber differentiation in a cellular network [2] [3]. Mostly, there will be a single cell mobile system that consists of a single BS with M regular users and another N VIP users and they assign target rates to the VIP users and the corresponding resource allocation strategy is viewed as an optimisation problem.

In this report, we study subscriber differentiation in cellular networks when FFR is also used. The goal is to improve target rates of high priority users with minimal degradation to other users. Here, we have x percent of users in a cell as privileged users and we try to come up with a resource allocation scheme which is a function of x(mentioned above).

1.1 Purpose of the project

We analyse the resource allocation scheme to be designed when there are discrimination among users [2] as a set of privileged users versus a set of unprivileged users in a multi-cell environment as modelled in [1]. Here, we try to provide better service for so-called privileged user set in the system with least harm to the remaining set of users. We expect the set of privileged users to achieve a threshold SIR, T_h. Our key observations include:

1. When the number of privileged users in the system is very low, we use FFR with δ=3 and combine it with a starvation scheme(FR-1,3 plus starve scheme) wherein cell-centre users enjoy FR1, cell-edge users enjoy FR3. The cell-centre privileged users who didn't meet T_h demand are swapped with cell-edge unprivileged users. Again, among the cell-edge privileged users those who didn't meet T_h, we give them the provision to use one more sub-band by starving another cell-edge privileged user in that frame.

2. When the number of privileged users in the system is medium, say 2 to 4 on an average per cell, we go for both FR-1,3 and FR-1,3,4 schemes. To elaborate, we assume channel to remain constant over $n + 1$ frames.
   - The system follows FR1 for cell-centre, FR3 for cell-edge users and the cell-centre privileged users who didn't meet T_h demand are swapped with the cell-edge unprivileged users over n frames.
   - For $n+1$th frame, all privileged users who didn't meet the requirement (even after swapping) will enjoy FR4, other privileged users enjoy FR1 or FR3 depending on their position either at cell-centre or cell-edge respectively and remaining set of unprivileged users at cell-edge enjoy FR3 and others enjoy FR1.

The above said resource allocation schemes show very good performance and are explained in detail in chapter 4. Coverage probability expressions are theoretically derived for above schemes and we believe that one can use these expressions to decide which resource allocation strategy to be followed depending on the number of privileged users present in the system instead of going for time consuming simulations. Next chapter familiarises the system model and the parameters used.

Percentage of privileged users in a cell	Frequency Reuse scheme deployed	Explanation
Low (eg.5%)	FR-1,3 plus starve	All cell centre users uses FR-1 and cell edge users uses FR-3 and the privileged users who didn't meet T_h requirement uses one more FR3 sub-band
Medium (eg. 25 %)	FR-1,3 plus FR-1,3,4	All cell centre users uses FR-1,cell edge users uses FR-3 in every n frames and for every $n + 1$th frame, the privileged users in cell-edge who didn't meet T_h requirement uses FR-4 and all other cell-center users use FR-1 and cell edge users use FR-3.

Table 1.1: List of resource allocation schemes proposed for obtaining better performance for privileged users

CHAPTER 2

System Model

A two-tier 19 cell network with hexagonal tessellation is considered. Number of transmit antennas(N_t) and receive antennas(N_r) is 1. We consider three resource allocation patterns- FFR3(with FR1 and FR3), FFR4(with FR1 and FR4) and FFR34(with FR1,FR3,FR4). SIR of a user, in FR1, FR3 and FR4 schemes, at distance r metres from serving BS are given by,

$$\eta_1(r) = \frac{gr^{-\alpha}}{I_1}, \quad I_1 = \sum_{i\in\psi} h_i d_i^{-\alpha} \text{ and } \eta_3(r) = \frac{g'r^{-\alpha}}{I_3}, \quad I_3 = \sum_{i\in\phi} h_i' d_i^{-\alpha} \tag{2.1}$$

$$\eta_4(r) = \frac{\hat{g}r^{-\alpha}}{I_4}, \quad I_4 = \sum_{i\in\phi'} \hat{h}_i d_i^{-\alpha} \tag{2.2}$$

ψ , ϕ and ϕ'represent set of interferers in FR1, FR3 and FR4 schemes , α is the path loss constant, r and d_i are the distances from the user to the serving BS and to the i_{th} interfering BS, respectively, while g and h_i denote the corresponding channel fading power,which are independent and identically exponentially distributed (i.i.d.) with a unit mean, i.e., g $\sim$ exp(1) and hi $\sim$ exp(1) $\forall$i.

$$FFR - 1, 3$$

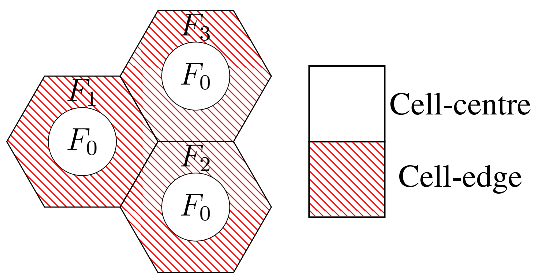

Figure 2.1: Frequency allocation in FFR for three neighbouring cells with $\delta = 3$. The cell-centre users of all the cells rely on a common frequency band F_0, while the cell-edge users of the three cells occupy different frequency bands, namely F_1, F_2 and F_3 [1].

The users in the network are classified according to their wide band SIR (WBSIR) as cell-edge or cell-centre users i.e. users with WBSIR $> S_{th}$ categorised as cell-centre

$FFR-1,4$

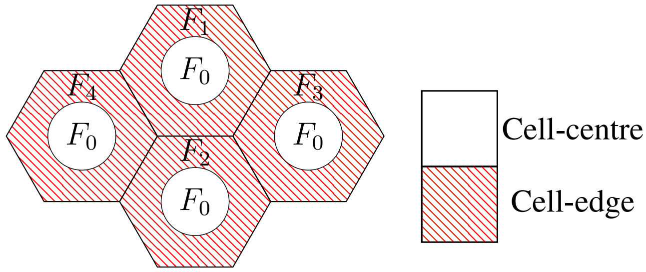

Figure 2.2: Frequency allocation in FFR for three neighbouring cells with $\delta = 4$. The cell-centre users of all the cells rely on a common frequency band F_0, while the cell-edge users of the three cells occupy different frequency bands, namely F_1, F_2 , F_3 and F_4.

user and others as cell-edge user similar to [1] . If the calculated SIR of a user is lower than S_{th}, the user is classified as a cell-edge user, else as a cell-centre user. Typically, FFR divides the whole frequency band into a total of (1+δ) parts, where F_0 is allocated to the cell-centre users of all the cells, as seen in figures 2.1 and 2.2. One of the F_1, ..,F_δ parts is assigned to the cell-edge users in each cell in a planned fashion. The users are assumed to be uniformly distributed in a cell. The transmit power is assumed to be fixed. A cell-centre user will experience the fading power, i.e., g and h_i from the user to the serving BS and to the i_{th} interfering BS, respectively. However, the cell-edge user is allocated another sub-band (from the set of sub-bands assigned to cell-edge users) and it experiences a new fading power, i.e., g' and h' in FR3 and $\hat{g}$ and $\hat{h}$ in FR4 schemes from the user to the serving BS and to the i_{th} interfering BS respectively.
Coverage probability($P_c(r)$) and rate($R(r)$) of a user, at distance r metres from the reference base station, with SIR $\eta(r)$ are given by,

$$P_c(r) = P(\eta(r) > T) \quad (2.3)$$

$$R(r) = E[\ln(1 + \eta(r))] \quad (2.4)$$

Here, T is the target SIR.

Figures 2.3 and 2.4 represent a two-tier 19 cell system with hexagonal shaped cells. The cell-centre users suffers interference from all 18 neighbouring cells whereas cell-

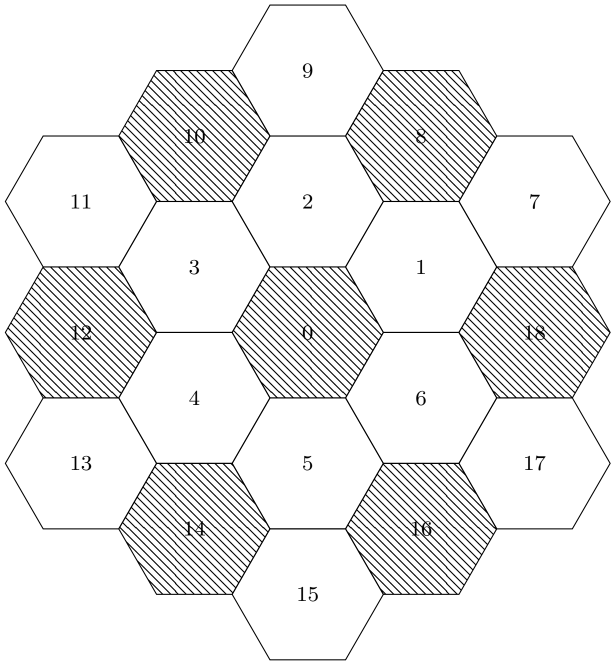

Figure 2.3: Hexagonal macrocell structure. The interference imposed by a FR1 system on cell 0 is contributed by all the 18 neighbouring cells, while only the shaded cells impose interference in a frequency reuse $\frac{1}{3}$ system [1].

edge users in FR3 suffer interference from 6 BSs in second tier of cells and cell-edge users in FR4 suffer interference from 4 BSs in second tier of cells. Since in FR3 and FR4, interference experienced is from second tier of cells, the SIR will be better for cell-edge users compared to cell-centre users in FR1 where some of the interferers are close to them(first tier).

2.1 Prior Art

The idea of classifying users as cell centre and cell edge users based on SINR threshold and the study on the optimal frequency reuse factor of the edge users as well as the bandwidth to be assigned to both centre and edge users are discussed [5] [6]. A user can be classified as cell-centre user or cell-edge user based on distance threshold or SIR threshold. System level simulations and theoretical analysis has been studied in detail [6] [7]. The expressions for the area spectral efficiency with both FFR and SFR systems under fully loaded and partially loaded scenerio for downlink and uplink is studied in [8]. An analytical statistical generic model of co-channel interference for uplink FFR-based OFDMA networks considering the effect of user scheduling algorithms is proposed in [9]. The authors of [10] showed coverage probability (CP)

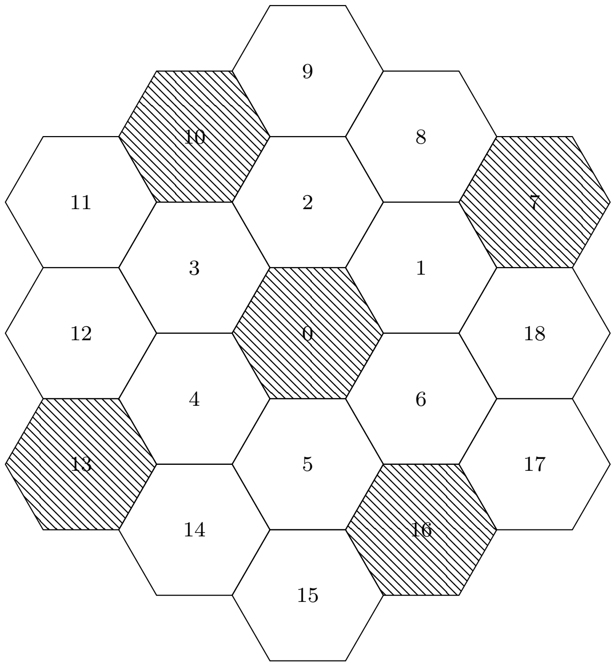

Figure 2.4: Hexagonal macrocell structure. The interference imposed by a FR1 system on cell 0 is contributed by all the 18 neighbouring cells, while only the shaded cells impose interference in a frequency reuse $\frac{1}{4}$ system .

and rate of the system is better in presence of correlated interferers than when the interferers are independent and derived the CP and the average rate expressions when FFR or soft frequency reuse (SFR) schemes are employed in single-inputâĂŞmultiple-output (SIMO) network. Expressions are derived for both the CP and average rate of multi user-multiple input multple output (MU-MIMO) and SIMO systems based on a planned FFR deployment, with and without frequency-domain correlation between the sub-bands allocated to the FR1 and FR3 regions [1]. Also, [1] showed the optimal choice of the SINR threshold, for which the CP of the FFR system is higher than that of its FR3 counterpart. Emerging system-level interference-reducing strategies based on cooperation is discussed in [11].

FFR and exploitation of the channel state information at the transmitter (CSIT) are effective approaches to enhance the spectrum efficiency of the outer coverage region. A new FFR pattern is proposed [12] for multi-cell OFDMA systems with frequency or time division duplexing (FDD/TDD) in time-varying channels wherein the frequency assignment is such that for the zones with high CSIT accuracy the amount of resources is larger compared to the reference schemes and the amount of resources allotted lessens with reduction in CSIT accuracy in later part of the frame whereas the total resource amount per frame remains the same.

So far, we were discussing about static resource allocation schemes whereas [13] discuss about a self-organized dynamic resource allocation scheme using enhanced FFR (SODRA-EFFR) where resources are allocated dynamically to cell inner and outer regions in LTE relay-based networks. The downlink power, frequency resources allocation for cell inner and outer regions , frequency resources allocation between eNBs and relay stations in each cell are dynamically allocated in [13] based on coordination between neighbouring eNBs and relay stations. A solution to enhance adaptive distributed FFR scheme based on the center of gravity(CoG), a point within the cell such that the sum of distance between this point and all user positions is minimum, of users in each cell is proposed in [14] employing cellular automata(CA) as a step towards achieving an emergent self-organized solution. Cell-centre users enjoy FR1 and cell-edge users enjoy FR3 with power amplification such that the amplification factor is calculated based on the state of each cell (dependent on CoG), the ratio of users located in cell edge to cell center, and the current state of neighbouring cells [14]. Attempts are made in [15] to find out the optimal bandwidth subdivision for cell centre users and cell edge users under a prescribed boundary function.

Influence of power control factor towards improvement of SFR system outage performance and resistivity of Strict FFR systems towards interior radius selection are studied [16]. We have seen cross tier interference as well as intra-cell interference. If the antennas itself experience spatially correlated interference, maximum ratio combining help improve performance [17]. When there are power disparities among power nodes, [18] provides an optimal fractional frequency reuse and power control scheme which coordinate the interference among high power and low power nodes efficiently. A 3- cell network MIMO with the rearranged tri-sector frequency partition strategy which outperforms the multi-cell network MIMO with omni-directional antennas is discussed in [19].

We have seen homogeneous cellular systems with hexagonal tessellation alone or BSs forming a spatial poisson point process alone. The outage probability and the spatially averaged throughput of heterogeneous cellular networks where the Distributed Antenna Systems(DAS)-aided macrocells and femtocells coexist and the performance of the system in DAS-aided unity frequency reuse (UFR) and fractional frequency reuse (FFR) transmission scenarios are discussed in detail in [20] and [21] respectively. An analytical evaluation of the performance of the mobile users according to their spatial locations is studied [22]. A hybrid model where the macrocells form a hexagonal grid whereas the femtocell base stations (FBSs) form a spatial Poisson point process (PPP) to model the two-tier network is adopted [22] . Resource allocation to femtocells in OFDMA systems employing fractional frequency reuse are studied [23], [18]. A two-tier OFDMA cellular network comprising co-channel deployed macrocells and femtocells is considered [24]. [24] aims at optimizing FFR-based parameters as a function of, for instance, the RB scheduling policy used in both tiers (e.g., roundrobin (RR) or maximum signal-to-interference-plus- noise ratio (MSINR)), the density of users per cell, the femto-BS (FBS) density per area unit or the power attenuation due to wall penetration losses.

We have seen the effect of FFR on reduction of ICI in multi-cellular network. The performance of coverage and average rate in FFR-aided cellular networks with uniformly weighted users have been discussed in literature. But the resource allocation strategy when there is subscriber differentiation in FFR-aided networks is still uncertain. In next chapter, we try to analyse the performance in such a network and derive expression for coverage probability. This attempt helps to understand the type of resource allocation scheme to be designed in a network where there are high priority users without going for high level simulations.

CHAPTER 3

Adaptive resource allocation for two tier FFR-Aided cellular networks with non-uniform weighted users

The users are classified as cell-centre and cell-edge users depending on their WBSIR. If WBSIR is greater than or equal to SIR threshold (WBSIR$\geq S_{th}$), the user is cell-centre user else a cell-edge user.

3.1 FR-1,3 plus starve

The SIR of a cell-centre and cell-edge user is given by 2.1 . We have x per cent of the available users as privileged users in each cell. If a privileged user is cell-centre user and didn't meet T_h requirement , first swap the cell-centre privileged user with cell-edge unprivileged user ('swap' implies a user in one frequency reuse scheme exchange sub-band with another user in different frequency reuse scheme) because $FR3$ scheme suffers less interference. If there are privileged users enjoying $FR3$ sub-band who still didn't meet T_h requirement, they will use one more sub-band and SIR becomes 3.8 and SIR of the corresponding starved unprivileged user becomes zero at that instant. The standard FFR coverage probability expression is obtained using [1]. Coverage probability of privileged users in the system, P_{F_p} is given by

$$P_{F_p} = P_{F_c,p} + P_{F_e,p} + P_{F_{sw,c},p} + P_{F_{st,c},p} + P_{F_{st,e},p} \quad (3.1)$$

- $P_{F_c,p}$ is coverage probability of cell-center privileged user $\times$ probability of user being a cell-centre user
- $P_{F_e,p}$ is coverage probability of cell-edge privileged user $\times$ probability of user being a cell-edge user

- $P_{F_{sw,c},p}$ is coverage probability of cell-center privileged user using FR-3 sub-band(swap) $\times$ probability of user being a cell-centre user
- $P_{F_{st,c},p}$ is coverage probability of cell-center privileged user using FR-3 sub-band who uses one-more sub-band in FR-3 (swap cum starve) $\times$ probability of user being a cell-centre user
- $P_{F_{st,e},p}$ is coverage probability of cell-edge privileged user who uses one-more sub-band in FR-3(starve) $\times$ probability of user being a cell-edge user

T is target SIR and S_{th} is the SIR threshold to classify a user as cell-center or cell-edge user. T_h represent the desired SIR for privileged users. Here, $P_{F_c,p}, P_{F_e,p}, P_{F_{st},p}$ represent coverage probability of cell-center privileged user, cell-edge privileged user and remaining privileged users who didn't meet T_h requirement even after using FR-3 sub-band .

3.1.1 Calculation of coverage probability of privileged users

Here, $\eta_1(r)$ is the SIR of a user, at distance r from the serving BS, in FR1 scheme 2.1, T_h is desired SIR threshold for a privileged user and S_{th} is SIR threshold to classify a user as cell-centre or cell-edge user. $P(\eta_1(r) > T, \eta_1(r) > T_h|\eta_1(r) > S_{th})$ gives the coverage probability of a privileged user given he/she is a cell-centre user. $P(\eta_1(r) > Sth)$ gives the probability that a user is a cell-centre user. The coverage probability of cell-center privileged user $\times$ probability of user being a cell-centre user, $P_{F_c,p}$, is given by

$$\begin{aligned}
P_{F_c,p} &= P(\eta_1(r) > T, \eta_1(r) > T_h|\eta_1(r) > S_{th})P(\eta_1(r) > Sth) \\
&= P(\eta_1(r) > T_h, \eta_1(r) > S_{th}, \eta_1(r) > T) \\
&= \prod_{i \epsilon \psi} \frac{1}{1 + max(T, T_h, S_{th}) r^{\alpha} d_i^{-\alpha}}
\end{aligned}$$

The other equations derived below follow similar logic.

$$\begin{aligned}
P_{F_{e},p} &= P(\eta_3(r) > T, \eta_3(r) > T_h | \eta_1(r) < S_{th}) P(\eta_1(r) < Sth) \\
&= P(\eta_3(r) > T_h, \eta_1(r) < S_{th}, \eta_3(r) > T) \\
&= P(\eta_3(r) > T_h, \eta_3(r) > T) P(\eta_1(r) < S_{th}) \\
&= \prod_{i \epsilon \phi} \frac{1}{1 + max(T, T_h) r^{\alpha} d_i^{-\alpha}} (1 - \prod_{i \epsilon \psi} \frac{1}{1 + S_{th} r^{\alpha} d_i^{-\alpha}})
\end{aligned}$$

$$\begin{aligned}
P_{F_{sw,c},p} &= P(\eta_3(r) > T, \eta_3(r) > T_h, \eta_1(r) < T_h | \eta_1(r) > S_{th}) \\
& \quad P(\eta_1(r) > S_{th}) \\
&= P(\eta_3(r) > max(T, T_h), \eta_1(r) < T_h, \eta_1(r) > S_{th}) \\
&= P(\eta_3(r) > max(T, T_h)) P(S_{th} < \eta_1(r) < T_h) \\
&= \prod_{i \epsilon \phi} \frac{1}{1 + max(T, T_h) r^{\alpha} d_i^{-\alpha}} (\prod_{i \epsilon \psi} \frac{1}{1 + S_{th} r^{\alpha} d_i^{-\alpha}}) \\
& \quad (1 - \prod_{i \epsilon \psi} \frac{1}{1 + T_h r^{\alpha} d_i^{-\alpha}})
\end{aligned}$$

$\eta_{st}(r)$ refers to the SIR of a user, at a distance r from serving BS, who uses additional sub-band by starving another user (3.8). A privileged user uses one more sub-band only if his/her SIR in FR3 didn't meet T_h demand. So, $P(\eta_{st}(r) > T, \eta_3(r) < T_h, \eta_1(r) < T_h | \eta_1(r) > S_{th})$ refers to the coverage probability of a privileged user who uses one more sub-band given he/she is a cell-centre user. Similar logic is followed in 3.5.

$$\begin{aligned}
P_{F_{st,c},p} &= P(\eta_{st}(r) > T, \eta_3(r) < T_h, \eta_1(r) < T_h | \eta_1(r) > S_{th}) P(\eta_1(r) > S_{th}) && (3.2) \\
&= P(\eta_{st}(r) > T, \eta_3(r) < T_h, \eta_1(r) < T_h, \eta_1(r) > S_{th}) && (3.3) \\
&= P(\eta_{st}(r) > T, \eta_3(r) < T_h) P(S_{th} < \eta_1(r) < T_h) && (3.4)
\end{aligned}$$

$$\begin{aligned}
P_{F_{st,e},p} &= P(\eta_{st}(r) > T, \eta_3(r) < T_h | \eta_1(r) < S_{th}) P(\eta_1(r) < S_{th}) && (3.5) \\
&= P(\eta_{st}(r) > T, \eta_3(r) < T_h, \eta_1(r) < S_{th}) && (3.6) \\
&= P(\eta_{st}(r) > T, \eta_3(r) < T_h) P(\eta_1(r) < S_{th}) && (3.7)
\end{aligned}$$

$$\eta_{st}(r) = \frac{gr^{-\alpha}}{I_r} + \frac{g'r^{-\alpha}}{I'_r} \tag{3.8}$$

$$I_r = \sum_{i \in \phi} h_i d_i^{-\alpha} \tag{3.9}$$

$$I'_r = \sum_{i \in \phi} h'_i d_i^{-\alpha} \tag{3.10}$$

Here, g and g' are fading power of two sub-bands used by the privileged user. ϕ is set of interferers in FR-3 scheme, N=$|\phi|$. Since there are two sub-bands used by the user, the interference experienced will be different in different sub-bands referred as I_r and I'_r. Here, h and h' are the corresponding channel fading power. d_i is distance between user and i_{th} interfering BS. Similar approach as in (3.8) is seen in [17]-Appendix A.

$$P(\eta_{st}(r) > T) \quad = \quad P(\frac{gr^{-\alpha}}{I_r} + \frac{g'r^{-\alpha}}{I'_r} > T) \tag{3.11}$$

Let X= $\frac{gr^{-\alpha}}{I_r}$ and Y= $\frac{g'r^{-\alpha}}{I'_r}$,hence we rewrite (3.2) and (3.5) as shown below.

$$\begin{aligned} P_{F_{st,c,p}} &= P(\eta_{st}(r) > T, \eta_3(r) < T_h)P(S_{th} < \eta_1(r) < T_h) \\ &= P(X + Y > T, X < T_h)P(S_{th} < \eta_1(r) < T_h) \\ &= P(X + Y > T, X < T_h)(\prod_{i \epsilon \psi} \frac{1}{1 + S_{th} r^{\alpha} d_i^{-\alpha}}) \\ &\quad (1 - \prod_{i \epsilon \psi} \frac{1}{1 + T_h r^{\alpha} d_i^{-\alpha}}) \end{aligned}$$

$$\begin{aligned} P_{F_{st,e,p}} &= P(\eta_{st}(r) > T, \eta_3(r) < T_h)P(\eta_1(r) < S_{th}) \\ &= P(X + Y > T, X < T_h)P(\eta_1(r) < S_{th}) \\ &= P(X + Y > T, X < T_h)(1 - \prod_{i \epsilon \psi} \frac{1}{1 + S_{th} r^{\alpha} d_i^{-\alpha}}) \end{aligned}$$

where we have c.d.f of X and Y as,

$$
\begin{aligned}
F_X(x) &= 1 - \prod_{i\epsilon\phi} \frac{1}{1 + xr^\alpha d_i^{-\alpha}} \\
F_Y(y) &= 1 - \prod_{i\epsilon\phi} \frac{1}{1 + yr^\alpha d_i^{-\alpha}} \\
f_X(x) &= \frac{dF_X(x)}{dx} \\
&= \prod_{i\epsilon\phi} \frac{1}{1 + x(\frac{r}{d_i})^\alpha} \sum_{i\epsilon\phi} \frac{(\frac{r}{d_i})^\alpha}{1 + x(\frac{r}{d_i})^\alpha} \\
f_Y(y) &= \frac{dF_Y(y)}{dy} \\
&= \prod_{i\epsilon\phi} \frac{1}{1 + y(\frac{r}{d_i})^\alpha} \sum_{i\epsilon\phi} \frac{(\frac{r}{d_i})^\alpha}{1 + y(\frac{r}{d_i})^\alpha}
\end{aligned}
$$

In FR-3 reuse, a user in reference cell experience interference from six interferers as shown in 2.3. Let $k_8 = (\frac{r}{d_8})^\alpha$, $k_{10} = (\frac{r}{d_{10}})^\alpha$, $k_{12} = (\frac{r}{d_{12}})^\alpha$, $k_{14} = (\frac{r}{d_{14}})^\alpha$ $k_{16} = (\frac{r}{d_{16}})^\alpha$ $k_{18} = (\frac{r}{d_{18}})^\alpha$where r is distance between user in reference cell-0 and d_i is the distance between user and set of interferers in FR-4 scheme (i,j ϵ ϕ={8,10,12,14,16,18}).

$$
f_X(x) = \prod_{i\in\phi} \frac{1}{(1 + k_i x)} \sum_{j\in\phi} \frac{k_j}{(1 + k_j x)}
$$

$$
f_Y(y) = \prod_{i\in\phi} \frac{1}{(1 + k_i y)} \sum_{j\in\phi} \frac{k_j}{(1 + k_j y)}
$$

and since X and Y being independent, we get $f_{X,Y}(x,y)$= $f_X(x)$ $f_Y(y)$

$$
P(X+Y > T, X < T_h) = \int_{y=0}^{T} \int_{x=T-y}^{T_h} f_X(x) f_Y(y) dy dx + \int_{y=T}^{\infty} \int_{x=0}^{T_h} f_X(x) f_Y(y) dx dy \tag{3.12}
$$

$$
\begin{aligned}
&= 1 - \left[\int_{y=0}^{T} \int_{x=0}^{T-y} f_X(x) f_Y(y) dx dy \right. \\
&\quad \left. + \int_{y=0}^{\infty} \int_{x=T_h}^{\infty} f_X(x) f_Y(y) dx dy \right] \\
&= 1 - \left[\int_{y=0}^{T} \int_{x=0}^{T-y} f_X(x) f_Y(y) dx dy \right. \\
&\quad \left. + \int_{x=T_h}^{\infty} f_X(x) dx \right]
\end{aligned}
$$

We solve $\left[\int_{y=0}^{T} \int_{x=0}^{T-y} f_X(x) f_Y(y) dx dy \right]$ using numerical integration and

$$
\int_{x=T_h}^{\infty} f_X(x) dx = \prod_{i \in \phi} \frac{1}{(1 + k_i T_h)} \tag{3.13}
$$

Results are obtained for different values of r and d_i. Hence, using all these results above, we get the coverage probability expression as mentioned in 3.1.

Before we proceed to calculation of coverage probability of unprivileged users, we need to know the probability with which a privileged user has been swapped or the probability with which a privileged user has starved some other user etc.
P_{swst} is the probability with which a cell-centre privileged user meet T_h demand by swapping a cell-edge unprivileged user and later starving a cell-edge unprivileged user, P_{sw} is the probability with which a cell-centre privileged user swap a cell-edge unprivileged user to meet T_h demand , P_{st} is the probability with which a cell-edge privileged user starve a cell-edge unprivileged user to meet T_h.

$$
\begin{aligned}
P_{st} &= P(\eta_3(r) < T_h, \eta_1(r) < S_{th}) \\
&= \left[(1 - \prod_{i \epsilon \phi} \frac{1}{1 + T_h r^{\alpha} d_i^{-\alpha}})(1 - \prod_{i \epsilon \psi} \frac{1}{1 + S_{th} r^{\alpha} d_i^{-\alpha}}) \right]
\end{aligned}
$$

$$
\begin{aligned}
P_{sw} &= P(\eta_3(r) > T_h, \eta_1(r) > S_{th}, \eta_1(r) < T_h) \\
&= \left[(\prod_{i\epsilon\psi} \frac{1}{1+S_{th} r^\alpha d_i^{-\alpha}})(1 - \prod_{i\epsilon\psi} \frac{1}{1+T_h r^\alpha d_i^{-\alpha}}) \right] \\
&\quad \times \prod_{i\epsilon\phi} \frac{1}{1+T_h r^\alpha d_i^{-\alpha}}
\end{aligned}
$$

$$
\begin{aligned}
P_{swst} &= P(\eta_3(r) < T_h, \eta_1(r) > S_{th}, \eta_1(r) < T_h) \\
&= \left[(\prod_{i\epsilon\psi} \frac{1}{1+S_{th} r^\alpha d_i^{-\alpha}})(1 - \prod_{i\epsilon\psi} \frac{1}{1+T_h r^\alpha d_i^{-\alpha}}) \right] \\
&\quad \times \left[1 - \prod_{i\epsilon\phi} \frac{1}{1+T_h r^\alpha d_i^{-\alpha}} \right]
\end{aligned}
$$

$$prob = (P_{sw} + P_{swst} + P_{st})$$

1-$prob$ gives the probability that the privileged users were neither swapped nor did they starve any other user. In other words, 1-$prob$ gives the probability that the unprivileged cell-edge users were not starved or swapped. If we have more than one privileged user, then P_{sw}, P_{swst} and P_{st} will be arrays (of size equal to number of privileged users), then

$$prob = mean(P_{sw} + P_{swst} + P_{st})$$

Using P_{sw}, P_{swst} and P_{st}, we can calculate coverage probability of unprivileged users as shown below 3.14.

3.1.2 Calculation of coverage probability of unprivileged users

- $P_{F_c,up}$ is the coverage probability of cell center unprivileged user $\times$ probability of user being a cell-centre user
- $P_{F_c,up}$ is the coverage probability of cell center unprivileged user $\times$ probability of user being a cell-centre user
- $P_{F_c,up}$ is the coverage probability of cell center unprivileged user $\times$ probability of user being a cell-centre user
- $P_{F_e,up}$ is the coverage probability of cell-edge unprivileged user $\times$ probability of user being a cell-edge user

P_{st} is the probability that a cell-edge unprivileged user will be starved, P_{sw} is the probability that a cell-edge unprivileged user will be swapped, P_{swst} is the probability that a cell-edge unprivileged user will be swapped and another unprivileged user will be starved, then total coverage probability of unprivileged user is given by

$$P_{F_{up}} = P_{F_c,up} + (P_{F_e,up} - prob \times \kappa) + P_{sw} \times \kappa \times P_{F_e,up} \tag{3.14}$$
$$P_{F_c,up} = P(\eta_1(r) > T | \eta_1(r) > S_{th}) P(\eta_1(r) > S_{th}) \tag{3.15}$$
$$P_{F_e,up} = P(\eta_3(r) > T | \eta_1(r) < S_{th}) P(\eta_1(r) < S_{th}) \tag{3.16}$$
$$\tag{3.17}$$

where $\kappa = \frac{no.of privileged users}{no.of unprivilged users}$

$$\begin{aligned} P_{F_c,up} &= P(\eta_1(r) > T | \eta_1(r) > S_{th}) P(\eta_1(r) > S_{th}) \\ &= P(\eta_1(r) > T, \eta_1(r) > S_{th}) \\ &= (\prod_{i \epsilon \psi} \frac{1}{1 + max(T, S_{th}) r^{\alpha} d_i^{-\alpha}}) \end{aligned}$$

$$\begin{aligned} P_{F_e,up} &= P(\eta_3(r) > T | \eta_1(r) < S_{th}) P(\eta_1(r) < S_{th}) \\ &= P(\eta_3(r) > T, \eta_1(r) < S_{th}) \\ &= (\prod_{i \epsilon \phi} \frac{1}{1 + T r^{\alpha} d_i^{-\alpha}})(1 - \prod_{i \epsilon \psi} \frac{1}{1 + S_{th} r^{\alpha} d_i^{-\alpha}}) \end{aligned}$$

Hence, using above results we get coverage expression for unprivileged users as mentioned in 3.14.

3.2 FR-1,4 plus starve

The users are classified as cell-centre and cell-edge users depending on their WBSIR. If WBSIR is greater than or equal to SIR threshold (WBSIR$\geq S_{th}$), the user is cell-centre user else a cell-edge user. The SIR of a cell-centre and cell-edge user is given by 3.18 whereas if a user uses one more sub-band in the same region by starving another user, his/her SIR is given by 3.19 and starved user has zero SIR at that instant.

$$\eta_1(r) = \frac{gr^{-\alpha}}{I_1}, \quad I_1 = \sum_{i\in\psi} h_i d_i^{-\alpha} \text{ and } \eta_4(r) = \frac{\hat{g}r^{-\alpha}}{I_4}, \quad I_4 = \sum_{i\in\phi'} \hat{h}_i d_i^{-\alpha} \tag{3.18}$$

$$\eta_{st_4}(r) = \frac{\hat{g}r^{-\alpha}}{I_4} + \frac{g_4 r^{-\alpha}}{I_{4_1}}, \quad I_{4_1} = \sum_{i\in\phi'} \hat{h}_{i_1} d_i^{-\alpha} \tag{3.19}$$

Here, g_4 and $\hat{h}_{i_1}$ refers to the channel fading power of the additional sub-band of the user in FR4 and corresponding interferers.We have x per cent of the available users as privileged users in each cell. If a privileged user is cell-centre user and didn't meet T_h requirement, first swap the cell-centre privileged user with cell-edge unprivileged user as mentioned earlier. Also, $FR4$ scheme suffers $\frac{2}{9}$ of the $FR1$ interference. The only difference in FR-1,4 combination compared to FR-1,3 combination is the number of interferers in cell-edge which reduces from 6 to 4.

In FR-4 reuse, a user in reference cell experience interference from four interferers as shown in 2.4. Similar logic as followed above in FR-13 plus starve is followed here. The only difference is the set of interfering BSs. Let $k_7 = (\frac{r}{d_7})^\alpha$, $k_{10} = (\frac{r}{d_{10}})^\alpha$, $k_{13} = (\frac{r}{d_{13}})^\alpha$, $k_{16} = (\frac{r}{d_{16}})^\alpha$ where r is distance between user in reference cell-0 and d_i is the distance between user and set of interferers in FR-4 scheme(i,j ϵ ϕ'={7,10,13,16}).

$$f_X(x) = \prod_{i\in\phi'} \frac{1}{(1+k_i x)} \sum_{j\in\phi'} \frac{k_j}{(1+k_j x)}$$

$$f_Y(y) = \prod_{i\in\phi'} \frac{1}{(1+k_i y)} \sum_{j\in\phi'} \frac{k_j}{(1+k_j y)}$$

$$\int_{x=T_h}^{\infty} f_X(x)dx = \prod_{i\in\phi'} \frac{1}{(1+k_i T_h)}$$

The privileged users enjoying $FR4$ sub-band who still didn't meet T_h requirement will use one more sub-band and SIR becomes 3.19 and the SIR of the corresponding starved unprivileged user zero at that instant. Expressions for coverage probability is similar to section 3.1. Since deploying FR4 in the system affect rate adversely, we implement this scheme in $n + 1$th frame and FR-1,3 scheme (wherein cell-centre user enjoy FR1 and cell-edge user enjoy FR3) in rest n frames.

3.3 FR-1,3,4 scheme

The SIR of a cell-centre and cell-edge user is given by 2.1 . We have x per cent of the available users as privileged users in each cell. If a privileged user is cell-centre user and didn't meet T_h requirement , first swap the cell-centre privileged user with cell-edge unprivileged user. The privileged users using $FR3$ sub-band who still didn't meet T_h requirement will switch to $FR4$ scheme with their SIR being 2.2 . Since deployment of FR4 in the system affect rate adversely, we implement this scheme once in $n + 1$ frames.
We proceed to simulation results in next chapter.

CHAPTER 4

Simulation results

4.1 Simulation Set-up

1. 19 cell system with 24 sub-bands and 24 users per cell is considered. The cell radius is R=500m.
2. SIR Threshold, S_{th}= 0 dB (used for CP plots)
3. SIR Threshold requirement for privileged user, T_h=15 dB
4. Path loss factor, α = 3.2
5. Channel fading power is independent and identically exponentially distributed with unit mean.
6. Standard FFR system refers to 19 cell FFR system (FR1 in cell centre and FR3 in cell edge) with uniformly weighted users.

We consider a two-tier 19 cell system with 24 users and 24 sub-bands per cell. All users are uniformly distributed in a cell and so are the sub-bands which are uniformly shared among the users. We generate channel fading power for each user in accordance with the serving BS as well as corresponding interfering BSs. We calculate SIR per user per sub-band. If the user's SIR $\geq S_{th}$, the user is a cell-centre user else cell-edge user. Once this classification is done, then we look into privileged users in the set of users and their corresponding SIR. If the privileged user is a cell-centre user who didn't meet T_h requirement, we will swap that user with an unprivileged cell-edge user('swap' is merely exchange of sub-bands in different frequency reuse schemes). Deployment of various resource allocation schemes mentioned in Chapter 3 depends on the number of privileged users in the system using FR3 who still didn't meet the T_h requirement. This is done over 50 iterations and in each iteration we find out the number of weak privileged users* in each of the 19 cells in the system. We go for section 3.1 if the percentage of privileged users per cell is very low (the average number of weak privileged users per cell be 1) else switch to FR-1,3 cum FR-1,3,4 scheme 3.3 as mentioned in the table 1.1. Also, we assume channel fading be constant for $n+1$

number of frames.

*A weak privileged user implies a privileged user in FRδ scheme who didn't meet T_h requirement.

4.2 FR-1,3 plus starve scheme

When the number of privileged users in the system is very small, we can employ FR-1,3 cum starve scheme. Each iteration comprises three frames over which channel fading is constant. Coverage probability($P_c(r)$) and rate($R(r)$) of a user, at distance r metres from the reference base station, with SIR $\eta(r)$ are given by,

$$P_c(r) = P(\eta(r) > T) \tag{4.1}$$

$$R(r) = E[\ln(1 + \eta(r))] \tag{4.2}$$

Here, T is the target SIR. Coverage probability and average rate, using this scheme, are plotted for different number of privileged users.

1. 5% privileged users in the system- Coverage plot is shown in 4.1 and rate plot is shown in 4.2.
2. 15% privileged users in the system- Coverage plot is shown in 4.3 and rate plot is shown in 4.4.

4.3 FR-1,3 plus FR-1,3.4 scheme

When there are more privileged users in the system, we go for FR-1,3 scheme for n frames and FR-1,3,4 in $n+1$th frame as mentioned in the table 1.1. Here, n=4. Here, SIR of a user is calculated using 2.1 and SIR of the weak privileged user in FR3 with SIR being 2.1 who enjoys FR4 in $n+1$th frame is calculated using 2.2. Coverage probability and rate is calculated using 4.1 and 4.2 respectively. Coverage probability and average rate, using this scheme, are plotted for different number of privileged users.

1. 15% privileged users in the system- Coverage plot is shown in 4.5 and rate plot is shown in 4.6.

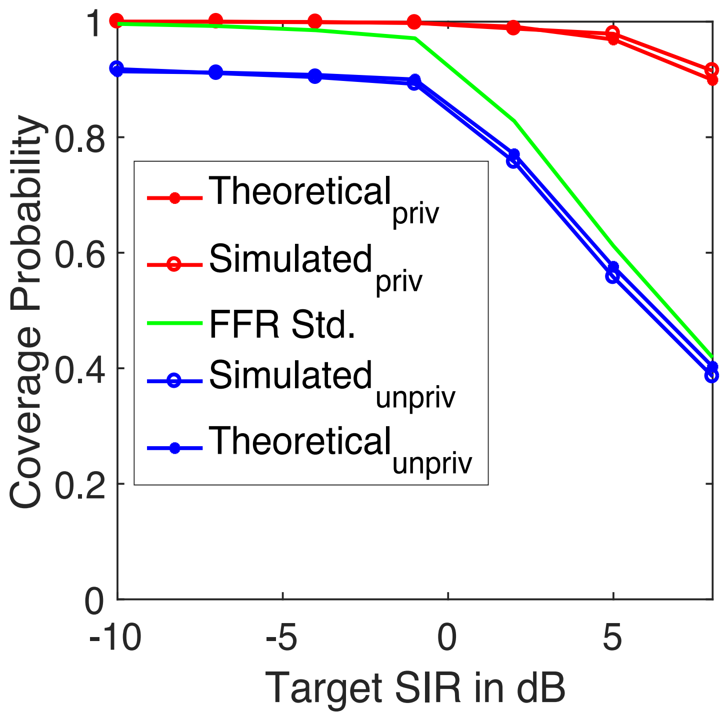

Figure 4.1: Coverage probability of unprivileged and privileged users in 19 cell FFR system is compared with Standard FFR system over varying Target SIR,T. For simulated plot, SIR of a user is calculated using 2.1 and SIR of the weak privileged user using an extra sub-band is calculated using 3.8 and the coverage probability is calculated using 4.1. Theoretical plot was obtained using the expressions 3.1 and 3.14 whereas FFR Std. plot uses the FFR coverage expression in [1]. Theoretical and simulated plots are matching.

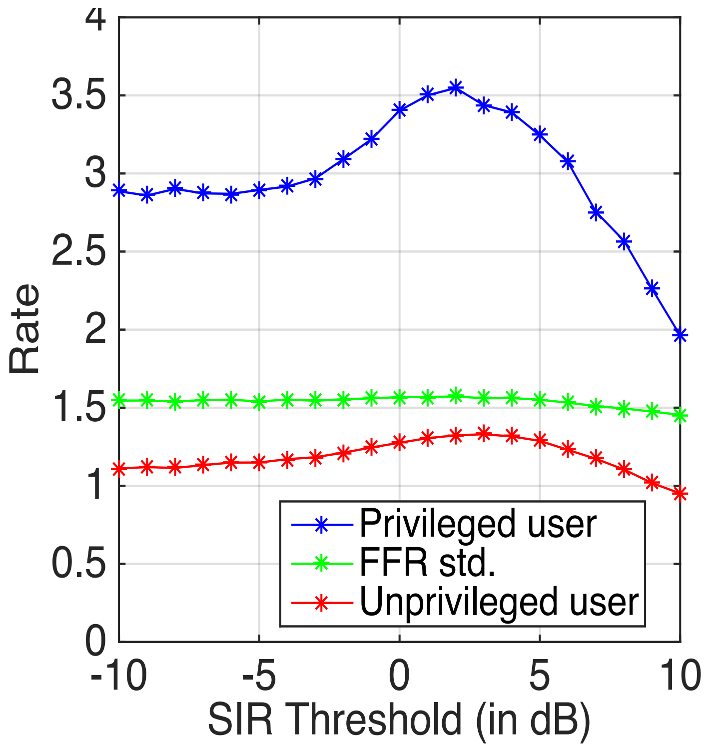

Figure 4.2: Rate of unprivileged and privileged users in 19 cell FFR system is compared with Standard FFR system over varying SIR threshold,S_{th}. SIR of a user is calculated using 2.1 and SIR of the weak privileged user using an extra sub-band is calculated using 3.8. Rate is calculated using 4.2. Unprivileged users didn't suffer worst in this frequency allocation scheme.

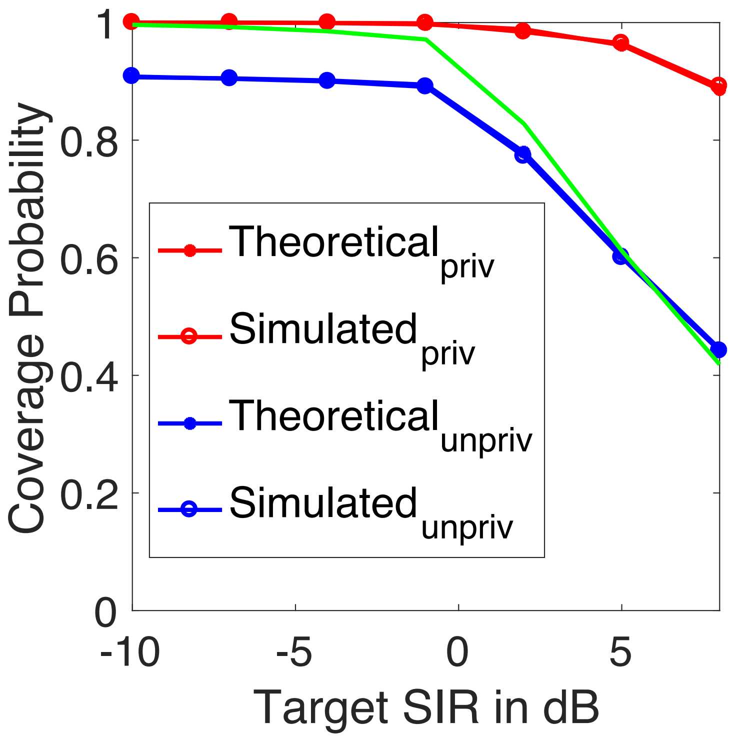

Figure 4.3: Coverage probability of unprivileged and privileged users in 19 cell FFR system is compared with Standard FFR system over varying Target SIR,T. For simulated plot, SIR of a user is calculated using 2.1 and SIR of the weak privileged user using an extra sub-band is calculated using 3.8 and the coverage probability is calculated using 4.1. Theoretical plot was obtained using the expressions 3.1 and 3.14 whereas FFR Std. plot uses the FFR coverage expression in [1]. Theoretical and simulated plots are matching.

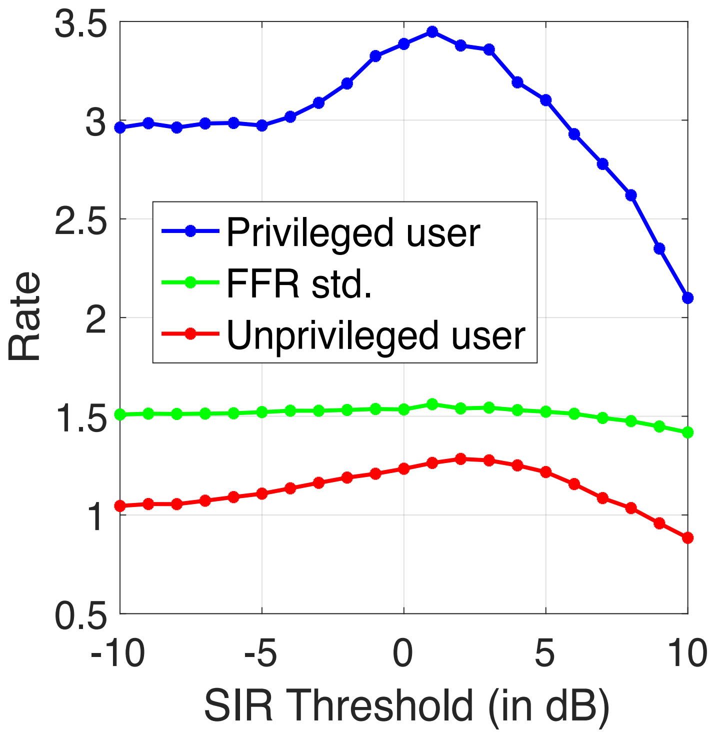

Figure 4.4: Rate of unprivileged and privileged users in 19 cell FFR system is compared with Standard FFR system over varying SIR threshold,S_{th}. SIR of a user is calculated using 2.1 and SIR of the weak privileged user using an extra sub-band is calculated using 3.8. Rate is calculated using 4.2. Unprivileged users didn't suffer worst in this frequency allocation scheme.

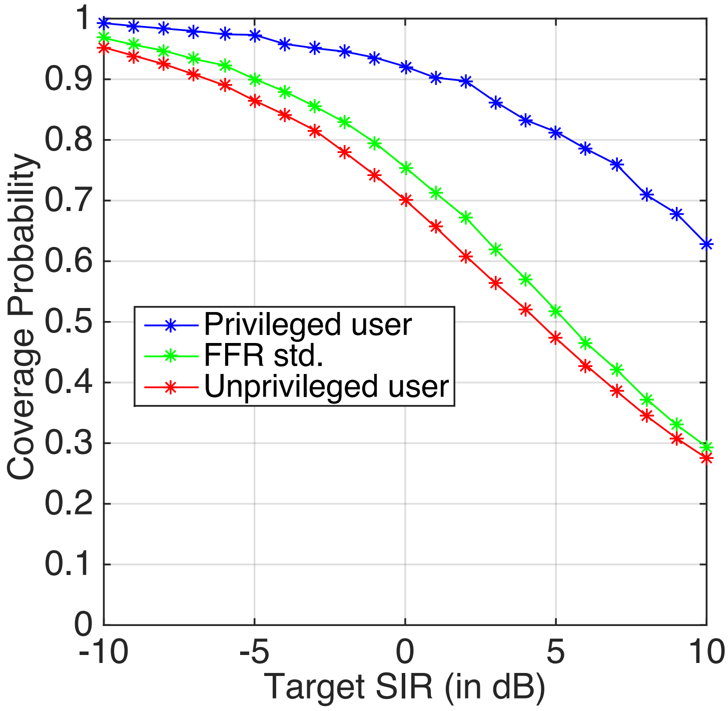

Figure 4.5: Coverage probability of unprivileged and privileged users in 19 cell FFR system is compared with Standard FFR system over varying Target SIR,T.

2. 25% privileged users in the system- Coverage plot is shown in 4.7 and rate plot is shown in 4.8.

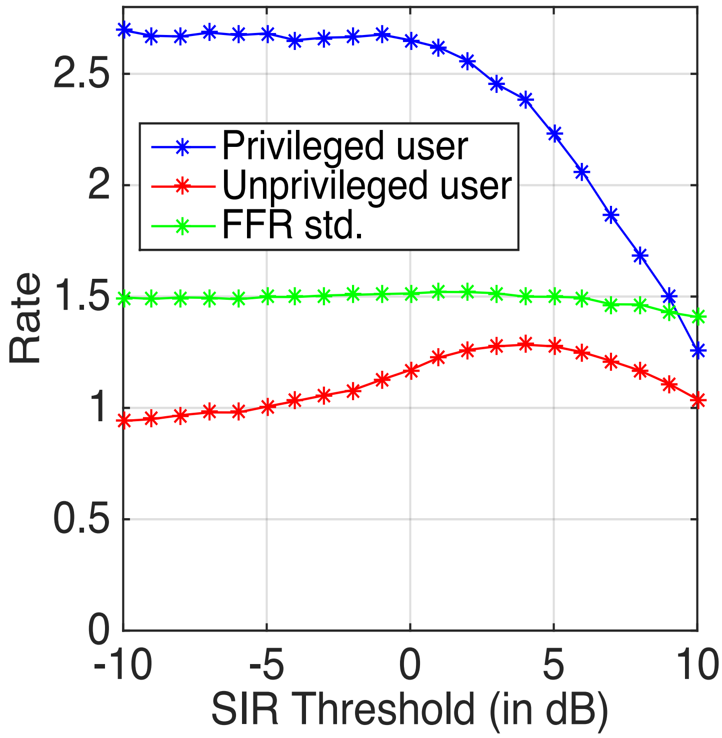

Figure 4.6: Rate of unprivileged and privileged users in 19 cell FFR system is compared with Standard FFR system over varying SIR threshold,S_{th}.Unprivileged users didn't suffer worst in this frequency allocation scheme.

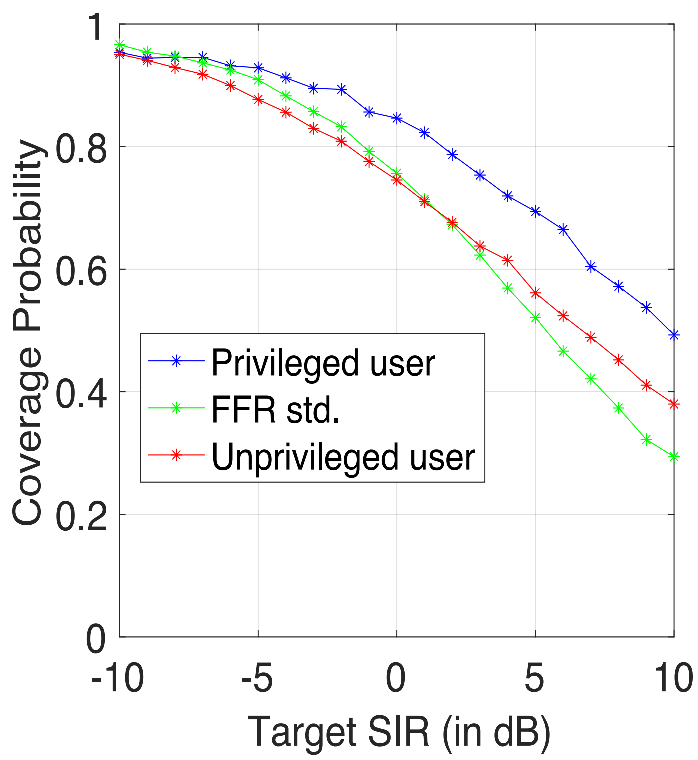

Figure 4.7: Coverage probability of unprivileged and privileged users in 19 cell FFR system is compared with Standard FFR system over varying Target SIR,T.

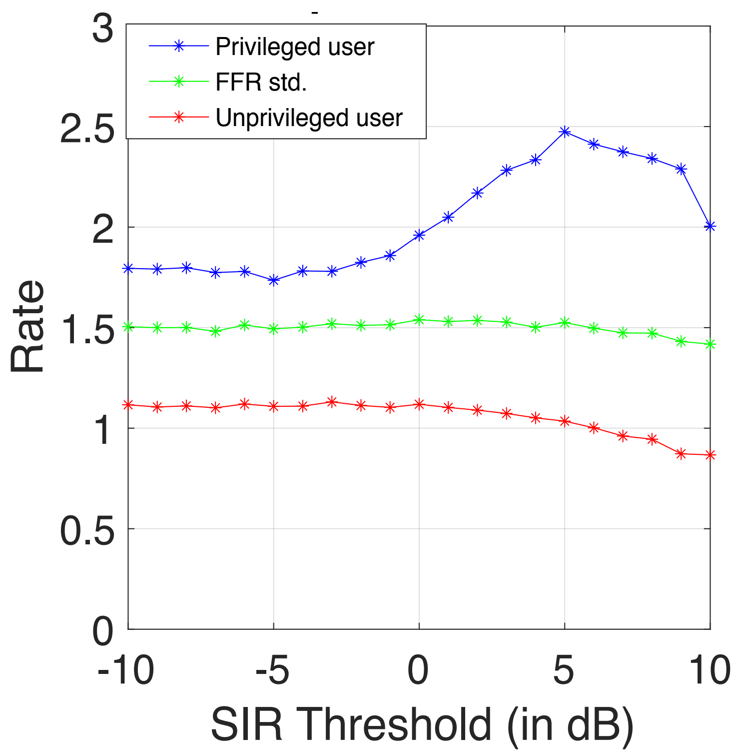

Figure 4.8: Rate of unprivileged and privileged users in 19 cell FFR system is compared with Standard FFR system over varying SIR threshold,S_{th}.Unprivileged users didn't suffer worst in this frequency allocation scheme.

From the above results, we infer that the FR-1,3 plus starve scheme holds good when there are less number of privileged users per cell and FR-1,3 plus FR-1,3,4 scheme performs better when there are more number of privileged users per cell. Also, we see that average rate suffers in FR-1,3 plus starve scheme with increasing per-cent of privileged users per cell.

CHAPTER 5

Conclusions and Future work

We have derived coverage probability expressions for $FR-1,3$ plus starve and $FR-1,4$ plus starve schemes. From simulation results, it is evident that for low percentage of privileged users in the system $FR-1,3$ plus starve shows good performance, for medium case we better opt $FR-1,3$ for n frames and $FR-1,3,4$ for $n+1$ th frame. Based on extensive simulations, we observed that if the percentage of privileged users in the system becomes very high, using $FR-1,3$ for n frames and $FR-1,4$ plus starve for $n+1$ th frame gives good results. These types of resource allocation schemes help improve performance of privileged users and save unprivileged users from loss of coverage and rate to some extent. We have derived expressions for coverage probability which shows the effect of different parameters in the performance of the system. Using our results, one can easily determine which resource allocation strategy to be employed in the FFR-aided system with subscriber differentiation and hence, avoid performing time consuming simulations. Future work may include deriving expression for optimal count of frames n wherein switching happens between resource allocation schemes.

REFERENCES

[1] S. Kumar, S. Kalyani, L. Hanzo, and K. Giridhar, "Coverage probability and achievable rate analysis of FFR-aided multi-user OFDM-based MIMO and SIMO systems," *IEEE Transactions on Communications*, vol. 63, no. 10, pp. 3869–3881, 2015.

[2] H. Shajaiah, A. Abdelhadi, and C. Clancy, "Multi-application resource allocation with users discrimination in cellular networks," *IEEE International Symposium on Personal, Indoor and Mobile Radio Communications, PIMRC*, vol. 2015-June, pp. 1163–1168, 2015.

[3] H. Ekstr??m, "QoS control in the 3GPP evolved packet system," *IEEE Communications Magazine*, vol. 47, no. 2, pp. 76–83, 2009.

[4] L. Liu, T. Peng, P. Zhu, Z. Qi, and W. Wang, "Analytical evaluation of throughput and coverage for FFR in OFDMA cellular network," *IEEE Vehicular Technology Conference*, vol. 2016-July, 2016.

[5] M. Assaad, "Optimal fractional frequency reuse (FFR) in multicellular OFDMA system," *IEEE Vehicular Technology Conference*, pp. 1–5, 2008.

[6] R. Y. Chang, Z. Tao, J. Zhang, and C.-C. Kuo, "A Graph Approach to Dynamic Fractional Frequency Reuse (FFR) in Multi-Cell OFDMA Networks," *2009 IEEE International Conference on Communications*, pp. 1–6, 2009.

[7] H. Zhuang and T. Ohtsuki, "A model based on poisson point process for analyzing MIMO heterogeneous networks utilizing fractional frequency reuse," *IEEE Transactions on Wireless Communications*, vol. 13, no. 12, pp. 6839–6850, 2014.

[8] A. Mahmud and K. A. Hamdi, "A Unified Framework for the Analysis of Fractional Frequency Reuse Techniques," *Ieee Toc*, vol. 62, no. 10, pp. 3692–3705, 2014.

[9] H. Tabassum, Z. Dawy, M. S. Alouini, and F. Yilmaz, "A generic interference model for uplink OFDMA networks with fractional frequency reuse," *IEEE Transactions on Vehicular Technology*, vol. 63, no. 3, pp. 1491–1497, 2014.

[10] S. Kumar, S. Kalyani, and K. Giridhar, "Impact of Sub-Band Correlation on SFR and Comparison of FFR and SFR," *IEEE Transactions on Wireless Communications*, vol. 15, no. 8, pp. 5156–5166, 2016.

[11] J. G. Andrews, W. Choi, and R. W. Heath, "Overcoming interference in spatial multiplexing mimo cellular networks," *IEEE Wireless Communications*, vol. 14, no. 6, pp. 95–104, 2007.

[12] P. Yen, Q. Zhan, and H. Minn, "New Fractional Frequency Reuse Patterns for Multi-Cell Systems in Time-Varying Channels," vol. 4, no. 3, pp. 253–256, 2015.

[13] A. S. Mohamed, M. Abd-Elnaby, and S. A. El-Dolil, “Self-organised dynamic resource allocation scheme using enhanced fractional frequency reuse in long term evolution-advanced relay-based networks,” *IET Communications*, vol. 10, no. 10, pp. 1163–1174, 2016.

[14] O. Aliu, M. Mehta, M. Imran, A. Karandikar, and B. Evans, “A New Cellular-Automata-Based Fractional Frequency Reuse Scheme,” *IEEE Transactions on Vehicular Technology*, vol. 64, no. 4, pp. 1535–1547, 2015.

[15] H. B. Chang and I. Rubin, “Optimal downlink and uplink fractional frequency reuse in cellular wireless networks,” *IEEE Transactions on Vehicular Technology*, vol. 65, no. 4, pp. 2295–2308, 2016.

[16] T. Novlan, J. G. Andrews, I. Sohn, R. K. Ganti, and A. Ghosh, “Comparison of fractional frequency reuse approaches in the OFDMA cellular downlink,” *GLOBECOM - IEEE Global Telecommunications Conference*, 2010.

[17] R. Tanbourgi, H. S. Dhillon, J. G. Andrews, and F. K. Jondral, “Effect of spatial interference correlation on the performance of maximum ratio combining,” *IEEE Transactions on Wireless Communications*, vol. 13, no. 6, pp. 3307–3316, 2014.

[18] Q. C. Li, R. Q. Hu, Y. Xu, and Y. Qian, “Optimal fractional frequency reuse and power control in the heterogeneous wireless networks,” *IEEE Transactions on Wireless Communications*, vol. 12, no. 6, pp. 2658–2668, 2013.

[19] L. C. Wang and C. J. Yeh, “3-cell network MIMO architectures with sectorization and fractional frequency reuse,” *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 6, pp. 1185–1199, 2011.

[20] J. Zhang, F. Jin, R. Zhang, G. Li, and L. Hanzo, “Analysis and Design of Distributed Antenna-Aided Twin-Layer Femto-and Macrocell Networks Relying on Fractional Frequency Reuse,” *IEEE Transactions on Vehicular Technology*, vol. 63, no. 2, pp. 763–774, 2014.

[21] J. Zhang, R. Zhang, G. Li, and L. Hanzo, “Distributed antenna systems in fractional-frequency-reuse-aided cellular networks,” *IEEE Transactions on Vehicular Technology*, vol. 62, no. 3, pp. 1340–1349, 2013.

[22] F. Wang and W. Wang, “Analytical evaluation of femtocell deployment in cellular networks using fractional frequency reuse,” *Communications, IET*, vol. 8, no. 9, pp. 1599–1608, 2014.

[23] J. Y. Lee, S. J. Bae, Y. M. Kwon, and M. Y. Chung, “Interference analysis for femtocell deployment in OFDMA systems based on fractional frequency reuse,” *IEEE Communications Letters*, vol. 15, no. 4, pp. 425–427, 2011.

[24] J. Garc??a-Morales, G. Femenias, and F. Riera-Palou, “Analysis and Optimization of FFR-Aided OFDMA-Based Heterogeneous Cellular Networks,” *IEEE Access*, vol. 4, pp. 5111–5127, 2016.