Predicting stochastic volatility in the financial markets using jump diffusion processes

A Project Report

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CERTIFICATE

This is to certify that the thesis entitled **Predicting stochastic volatility in the financial markets using jump diffusion processes**, submitted by **K.K.Prathyusha (EE15B098)**, to the Indian Institute of Technology Madras, for the award of the degree of Bachelors of Technology in Electrical engineering and Master of Technology in Electrical engineering, is a bonafide record of the research work carried out by her under my supervision. The contents of this report, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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LIST OF CONTENTS

ABSTRACT

- This study is about the impact of stock market cycles on the volatility of asian markets.
- It specifically addresses the combined effect of jumps and diffusion while predicting the market volatility.
- In this project we discuss Estimation of a stochastic volatility of asian market data using jump diffusion model.
- Building a jump diffusion model in matlab and comparing the results with jump symmetric model.
- The model used in this project is merton's jump diffusion model.

INTRODUCTION

What is the jump diffusion process ?

- Stochastic Volatility are those in which the variance of a stochastic process is itself randomly distributed.
- Jump diffusion is a stochastic process that involves jumps and diffusion.
- Jump diffusion processes have been used in modern finance to capture discontinuous behavior in asset pricing.
- In the stock market the diffusion index is usually measured from day to day and it refers to whether more stocks.
- Small day to day diffusion moments together with larger randomly occur jumps.
- As described in Merton (1976), the validity of Black-Scholes formula depends on whether the stock price dynamics can be described by a continuous-time diffusion process.

  (The merton's model is explained in the next section.

  If lambda = 0 in that explanation, it reduces to black - scholes model.)

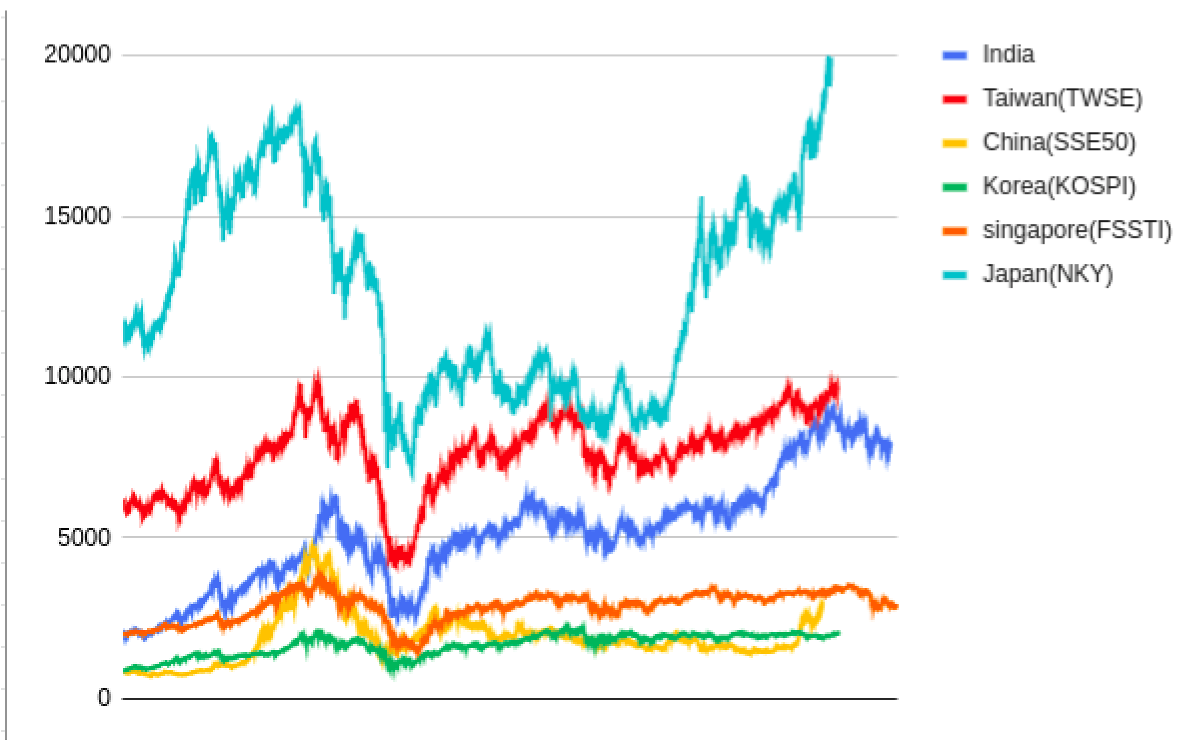
- Thus, if the stock price dynamics cannot be represented by a stochastic process with a continuous sample path, the Black-Scholes solution is not valid.
- In other words, as the price processes feature big jumps, i.e. not continuous, continuous-time models cannot explain why the jumps occur, and hence not adequate.
- In addition, Ahn and Thompson (1986) also examined the effect of regulatory risks on the valuation of public utilities and found that those ”jump risks” were priced even though they were uncorrelated with market factors.
- It shows that jump risks cannot be ignored in the pricing of assets. Thus, a ”jump” stochastic process defined in continuous time, and also called a ”jump diffusion model” was rapidly developed.

DATA

- The sample data consists of stock market indices of six asian countries.The indices considered in the study are NIFTY(India),KOPST(SouthKorea),TWSE(Taiwan),FSSTI(Singapore),Nikkei(Japan) and China.

- In this study daily data of stock indices over a period of ten years from 3rd jan 2005 to 31st dec 2014 have been considered.

Asian markets data



METHODOLOGY

Merton’s jump diffusion Model

- This model Super imposes a jump component on a diffusion component.
- It combines both jumps and diffusion into the standard stochastic volatility equation(SVJD Model).
- The Merton model is an analysis model used to assess the credit risk of a company's debt. Analysts and investors utilize the Merton model to understand how capable a company is at meeting financial obligations, servicing its debt, and weighing the general possibility that it will go into credit default.
- In 1974, Robert Merton proposed a model for assessing the credit risk of a company by modeling the company's equity as a call option on its assets.
- The Merton model provides a structural relationship between the default risk and the assets of a company.

- **The formula for merton model is**

  The risk-neutral jump-diffusion process for the stock price follows :

  S(t) - current price i.e stock price at time t

  Sigma - volatility of diffusion component

  r - risk free factor

  Lambda - poisson rate

$$\frac{dS_t}{S_t} = (r - \lambda \bar{k})\, dt + \sigma\, dW_t + k\, dq_t.$$

$$\ln(1+k) \sim N\left(\gamma, \delta^2\right)$$

with mean $\bar{k} \equiv E(k) = e^{\gamma + \delta^2/2} - 1.$

$$S_t = S_0 e^{(r - \lambda \bar{k} - \sigma^2/2)\, t + \sigma W_t} U(n(t)),$$

where

$$U(n(t)) = \prod_{i=0}^{n(t)} (1 + k_i)\,.$$

- k_i is the magnitude of the ith jump with $\ln(1+k_i) \sim N(\gamma, \delta^2)$.
- $k_0 = 0.$

n(t) - poisson process with intensity lambda

RESULTS & CONCLUSION

Inputs

cp - [1,-1] Call,Put

S - Current Price

K - Strike Vector

T - Time-to-Maturity Vector

sigma - Volatility of Diffusion

r - Risk-free-Rate

q - Div Yield - lambda*K

lambda - Poisson Rate

a - Jump Mean

b - gamma - Jump Std Deviation

n - Event Count

Code

```
function P = calcMJDOptionPrice(cp,S,K,T,sigma,r,q,lambda,a,b,n)
[K,T] = meshgrid(K,T);
[u,v] = size(K);
K = K(:,:,ones(1,1,n));
T = T(:,:,ones(1,1,n));
n = one,1,n);
n(:)=0:size(n,3)-1;
factn = factorial(n);
n = n(ones(u,1),ones(1,v),:);
factn = factn(ones(u,1),ones(1,v),:);
m = a+0.5*b.^2;
lambda_prime = lambda.*exp(m);
```

```
r_n = r - lambda*(exp(m)-1) + n.*(m)./T;
sigma_n = sqrt(sigma.^2 + (n*b^2)./T);
dfcn = @(z,sigma,r)((1./(sigma.*(T.^(0.5)))).*(log(S./K) + (r - q + z.*0.5*(sigma.^2)).*T));
callfcn =
@(sigma,r)(((1./factn).*exp(-lambda_prime.*T).*(lambda_prime.*T).^n).*(exp(-q.*T).*S.*Nfcn(
dfcn(1,sigma,r)) - K.*exp(-r.*T).*Nfcn(dfcn(-1,sigma,r))));   % Call
P = sum(callfcn(sigma_n,r_n),3);
if cp==-1
   P = P - S.*exp(-q.*T(:,:,1)) + K(:,:,1).*exp(-r.*T(:,:,1));  % Convert Call to Put by Parity Relation
end
end
%% NORM INVERSE
function p = Nfcn(x)
p=0.5*(1.+erf(x./sqrt(2)));
end
```

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