Recursive Beam and Channel Tracking

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ANJALI KASHYAP

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THESIS CERTIFICATE

This is to certify that the thesis titled **Recursive Beam and Channel Tracking**, submitted by **Anjali Kashyap**, to the Indian Institute of Technology, Madras, for the award of the degree of **Bachelor and Master of Technology**, is a bona fide record of the research work done by her under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Dr. Srikrishna Bhashyam
Research Guide
Professor
Dept. of Electrical Engineering
IIT-Madras, 600036
Place: Chennai
Date: 31^{st} May 2020

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ABSTRACT

KEYWORDS: mmWaves; Joint beam and channel tracking; Cramer Rao Lower Bound

The severe path loss in mmWaves makes highly directional communication imperative. The narrow beams produced require accurate beam alignment which can be realized with high pilot overhead. However, doing this becomes a challenge for fast-moving mobiles. In this report, we study a very efficient algorithm called Recursive Beam and Channel Tracking, proposed in [1], to jointly track channel coefficient and beam direction at the receiver side in an mmWave system. We consider two scenarios: Static and Dynamic. The algorithm can not only track the channel gain and beam direction in fading environments but also needs a low pilot overhead. Apart from this, it provides high tracking accuracy: in static scenarios, the Mean Square Error converges to minimum Cramer Rao Lower Bound, while in dynamic scenarios, it yields low tracking error even at high angular velocities.We also study a training scheme, Directional Training, proposed in [2], which is very effective for FDD massive MIMO.

TABLE OF CONTENTS

LIST OF FIGURES

ABBREVIATIONS

AWGN	Analog White Gaussian Noise
i.i.d.	Independent and Identically Distributed
MSE	Mean Square Error
SNR	Signal-to-noise ratio
CRLB	Cramer Rao Lower Bound
AoA	Angle of Arrival
AoD	Angle of Departure
RBCT	Recursive Beam and Channel Tracking
pdf	Probability density function
MIMO	Multiple Input Multiple Output
CSI	Channel State Information
CSIT	Channel State Information at Transmitter
CSIR	Channel State Information at Receiver
TDD	Time-Division Duplexing
FDD	Frequency-Division Duplexing
BS	Base station

NOTATION

s	Pilot Symbol
M	Number of antenna
λ	Wavelength
d	Distance between adjacent antenna
$\mathbf{a}(x)$	Steering vector
β	Channel Coefficient
θ	Angle of Arrival
x	sine of AoA
$\mathbf{w}_n$	Beamforming Vector
$\mathbf{W}_n$	Beamforming Matrix
$y_{n,i}$	Received signal
$z_{n,i}$	Noise
σ_0	Standard deviation
$E[.]$	Expected value operator
$\mathbf{I}(.)$	Fisher Information matrix
$\mathbf{A}^{-1}$	Inverse of a matrix A
$\mathbf{A}^{\mathrm{T}}$	Transpose of a matrix A
$\mathbf{A}^{\dagger}$	Pseudo-inverse of a matrix A
$\|\mathbf{x}\|$	Euclidean norm of complex vector$\mathbf{x}$
L_{max}	Number of dominant AoDs
K	Number of users
$\mathbf{h_{DL}}$	Downlink channel
$\mathbf{e}$	Error
$\mathbf{b}_k$	Channel Vector

CHAPTER 1

INTRODUCTION

1.1 Prerequisite

The rising demand of high data rates cannot be met with the currently used spectrum of frequencies below 6Ghz. This is because these frequency bands are heavily crowded. Thus, 5G systems will rely on millimeter waves. The mmWaves belong to the frequency of 3-300Ghz. However, the millimeter spectrum has its own set of problems. It is a known fact that higher the frequency of a wave, lower is its range. The mmWave channel faces severe path loss apart from other environmental obstacles. To counter these issues, 5G uses technologies like small cells, massive MIMO and beamforming.

Beamforming: Beamforming is, essentially, a technique using which an antenna sends/ receives a signal in a particular direction only. Beamforming not only increases the efficiency of 5G connections but also prevents interference. When a base-station receives a signal, it keeps track of the direction of arrival. Multiple antennas are kept close together and their weights are adjusted in such a way that there is constructive interference in the desired areas and destructive interference in the rest. Electromagnetic waves combine by coherence and thus form a “beam” towards where the previous signal came from. If the beam goes to unwanted locations, the phases collide and hence there is destructive interference. Thus, during transmission, the signal is directed towards a specific direction and while receiving a signal, sensors are calibrated such that the antennae receive from a specific direction.

Cramer Rao Lower Bound: Cramer Rao Lower Bound is a lower bound on the variance of any unbiased estimator of parameters which are deterministic in nature. For a scalar parameter θ, CRLB is given by the reciprocal of Fisher Information I(θ), where:

$$\mathrm{I}(\theta) = \mathrm{E}\left[\left(\frac{\partial \ln(p(\mathrm{x},\theta)}{\partial\theta}\right)^2\right] \tag{1.1}$$

So, the variance of any unbiased estimator $\hat{\theta}$ follows the below inequality:

$$var(\hat{\theta}) \geq \frac{1}{\mathrm{I}(\theta)}. \tag{1.2}$$

What is of interest to us is the extension of the above result to the case where a vector parameter is estimated. Let the vector parameter be $\boldsymbol{\theta} = [\theta_1\theta_2...\theta_p]^T$. Here, CRLB of i$^{\text{th}}$ element of the parameter is taken as [i,i] element of inverse of the Fisher information matrix $\mathbf{I}(\theta)$, where:

$$[\mathbf{I}(\boldsymbol{\theta})]_{ij} = \mathrm{E}\left[\left(\frac{\partial \ln(p(\mathrm{x}, \boldsymbol{\theta})}{\partial \theta_i}\right)\left(\frac{\partial \ln(p(\mathrm{x}, \boldsymbol{\theta})}{\partial \theta_j}\right)\right] \tag{1.3}$$

CSIT in FDD massive MIMO: Massive MIMO is a crucial part of 5G systems. And channel state information at the transmitter is needed to carry out transmit beamforming needed by massive MIMO. Getting CSIR is easy because the receiver already knows the pilot symbol sent by the transmitter. However, obtaining CSI at the transmitter is difficult because transmitter does not know the received symbol. In uplink communication, BS is the receiver. Therefore, it knows the uplink channel. But it does not know the downlink channel as it acts as the receiver. This is not an issue when we talk about time-division duplexing mode. In TDD mode, BS can get the downlink channel information by utilising uplink channel reciprocity. It just needs to conduct uplink transmission. Whereas, frequency-division duplexing does not follow channel reciprocity. Hence, when the terminals get to know the downlink channel, they send the information back to BS through uplink feedback.

1.2 Objective and Significance

In this report, we study two problems. The first one is in chapter 2 and the next one is in chapters 3 and after.

Because of high number of antennas, pilot overhead needed for uplink feedback in a massive MIMO system is quite high. Thus, obtaining CSI at the transmitter for massive MIMO in the FDD mode is considered to be a daunting task. Also, because FDD is a widely used communication mode, this challenge becomes all the more significant. There have been several works done for this challenge in the past, namely [3, 4, 5].

But these works assume that the we know the long-term channel statistics while in the real world, we may not know it. The work in [6] assumes small channel separation between the uplink and downlink channels which again, may not be the case in the real world. Therefore, in chapter 2, we study a method, Directional Training, proposed in [2], to find downlink channel information for FDD massive MIMO. This method uses instantenous CSI and does not need long-term statistics of the channel. It also works with high frequency separation.

To deal with severe path loss, mmWave systems heavily rely on directional communication. However, the beams formed are narrow. Even a slight beam misalignment can lead to considerable signal drop. The problem gets worse when the user is moving, i.e. in a dynamic scenario. One also has to limit the number of pilots for tracking, which gets tougher when the user is moving. Therefore, fast and efficient beam tracking strategy is imperative for an mmWave network. The key challenge is to track a large number of high-speed users and achieve high throughput along with low pilot overhead. A good amount of work has been done in this area. The algorithms based on compressed sensing used in [7, 8, 9], could work with low pilot overhead and achieved fast tracking speed. But these algorithms could only function in static scenarios and thus would not be suitable for high-mobility scenarios. The beam tracking algorithms used in [10, 11] take the prior information to track the beams. Unlike compressed sensing based algorithms, they can function in a dynamically changing environment. But they do not optimize beamforming vectors. This is why the tracking accuracy is not up to the mark in this case. The algorithm used in [12] optimizes the training beamforming vectors. However, it presumes that the channel coefficient is known. But channel coefficient can also be unknown in a real mmWave system. Therefore, in chapters 3 and after, we study an algorithm proposed in [1] that can track both channel coefficient and beam direction simultaneously in a high-mobility scenario and uses optimal beamforming vectors to achieve high tracking accuracy and low pilot overhead.

CHAPTER 2

DIRECTIONAL TRAINING

2.1 Angle-Based Channel Model

In this chapter, we characterize a received signal of wavelength λ incident on the BS at an angle of (θ, ψ) as:

$$\mathbf{a}(\theta,\psi) = \begin{bmatrix} 1 \\ e^{j\frac{2\pi}{\lambda}dsin\theta cos\psi} \\ . \\ . \\ . \\ e^{j\frac{2\pi}{\lambda}d((M_r-1)\cos\theta+(M_c-1)\sin\theta\cos\psi)} \end{bmatrix} \tag{2.1}$$

where M_r is the number of rows and M_c is the number of columns in the plane array used and d is the space between adjacent antennas.

A downlink channel can be represented both in angular domain and in antenna space. We will look at how it can be represented in the angular domain. If an l^{th} signal leaves the array at the angle of departure (θ_{dl}, ψ_{dl}) then the downlink channel, a total of L_d signals travel in, can be modeled as:

$$\mathbf{h}_{\mathrm{DL}} = \sum_{l=1}^{L_d} \beta_{dl}\mathbf{a}_{\lambda_d}(\theta_{dl}, \psi_{dl}) \tag{2.2}$$

where β_{dl} represents the complex channel coefficient of the l^{th} signal.

2.2 Dominant AoD set and Angle Correlation

In this section, we discuss two major results of a massive MIMO system in FDD mode. These two results will be utilized in directional training.

2.2.1 Number of dominant AoDs

It is found that one needs only a limited number of AoDs to represent the downlink channel, such angles can be called dominant AoDs. It is also found that the number of dominant AoDs is far less than the number of antennas which is quite large. This number also does not depend on the number of antennas. An intuitive explanation for this is that the channel energy is concentrated in a limited number of AoDs.

2.2.2 Angle Correlation

The FDD mode does not follow channel reciprocity because uplink and downlink connections are made at different frequencies and the difference between the center frequencies are quite larger than the coherence bandwidth. However, it is found that at the BS, AoAs are same in number and value as the AoDs. This is because the uplink and downlink signals face almost the same scattering. Thus, if one estimates the uplink angles one can use the same set of angles as the downlink angles.

2.3 Directional Training

This section outlines the main method used to estimate CSIT for downlink beamforming in FDD massive MIMO. This technique is applied using the two results highlighted in section 2.2. The following subsections discuss the step by step procedure for directional training.

2.3.1 Uplink Training and Estimation of AoD set

To implement directional training, we first need the dominant AoDs. Note that this number is very small, according to result 2.2.1. To get the AoDs we utilize the result 2.2.2 and calculate the uplink angles first which can be done by uplink training. Uplink training involves the users sending pilot symbols to the BS. Thus, BS knows the angles at which these signals arrive. The BS calculates these angles by applying a maximum likelihood estimator to the uplink CSI it has. This estimated dominant AoA set is taken as the dominant AoD set. The estimation procedure of AoA set is outlined below. The uplink channel of i-th subcarrier is represented by:

$$\hat{\mathbf{h}}_i = \sum_{l=1}^{L} \beta_{il}\mathbf{a}_i(\theta_l, \psi_l) + \hat{\mathbf{e}}_i \tag{2.3}$$

This equation can be rewritten as:

$$\hat{\mathbf{h}}_i = \mathbf{A}_i\mathbf{b}_i + \mathbf{e}_i \tag{2.4}$$

where $\mathbf{A}_i = [\mathbf{a}_i(\theta_1, \psi_1), \mathbf{a}_i(\theta_2, \psi_2)...\mathbf{a}_i(\theta_L, \psi_L)]$ and $\mathbf{b}_i = [\beta_{i1}\beta_{i2}...\beta_{iL}]^{\mathrm{T}}$. This can be written as:

$$\hat{\mathbf{h}} = \mathbf{R}\mathbf{s} + \mathbf{e} \tag{2.5}$$

with $\hat{\mathbf{h}} = \begin{bmatrix} \hat{\mathbf{h}}_1 \\ \hat{\mathbf{h}}_2 \\ . \\ . \\ \hat{\mathbf{h}}_I \end{bmatrix}$, $\mathbf{R} = \begin{bmatrix} \mathbf{A}_1 & . & . & . & 0 \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ 0 & . & . & . & \mathbf{A}_I \end{bmatrix}$ and $\mathbf{s} = \begin{bmatrix} \mathbf{b}_1 \\ . \\ . \\ . \\ \mathbf{b}_I \end{bmatrix}$ I is the total number of subcarriers used. The optimization problem being solved is:

$$\underset{(\theta_l, \psi_l, \beta_{il})}{\text{minimize}} \; ||\hat{\mathbf{h}} - \hat{\mathbf{R}}\hat{\mathbf{s}}||_2 \tag{2.6}$$

For a known $\hat{\mathbf{R}}$, equation 2.6 can be rewritten as:

$$\underset{(\theta_l, \psi_l, \beta_{il})}{\text{minimize}} \; ||\hat{\mathbf{h}} - \hat{\mathbf{R}}\hat{\mathbf{R}}^{\dagger}\hat{\mathbf{h}}||_2 \tag{2.7}$$

Since problem (2.7) is a non-convex optimization problem, to find the global minimum one needs a good initial point. 2D Unitary ESPRIT method is used to derive the initial angles. To find the final AoA set, one uses the interior point method. The tool used for the same is MATLAB fmincon.

2.3.2 Downlink Training and Feedback

Now that the BS has the dominant AoD set, pilot symbols are sent only in the direction of these downlink angles. If there are K users, K beams are sent at the same time.

If L_{max} is the number of dominant AoDs, then the base station sends L_{max} symbols. The users estimate the downlink CSI and send the information back in the form of feedback, estimated channel vector $\hat{\mathbf{h}}_{tk}$. We need to obtain the channel coefficients from this feedback.

2.3.3 Estimation of AoD Channel Coefficients

We cannot directly take $\hat{\mathbf{h}}_{tk}$ as the desired channel vector, $\mathbf{b}_k$ because of inter-path and inter-beam interference. Note that $\hat{\mathbf{b}}_k = [\hat{\beta}_{k1}, .., \hat{\beta}_{kL_k}]^{\mathrm{T}}$. BS gets the AoD channel vector, $\mathbf{b}_k$ through:

$$\hat{\hat{\mathbf{b}}}_k = (\mathbf{W}^{\mathrm{T}}\hat{\mathbf{A}}_k)^{-1}\hat{\mathbf{h}}_{\mathrm{t}k} \tag{2.8}$$

with

$$\hat{\mathbf{A}}_k = [\mathbf{a}(\theta_{k1}, \psi_{k1}), .., \mathbf{a}(\theta_{kL_k}, \psi_{kL_k})] \tag{2.9}$$

$$\mathbf{W} = \left[\sum_{k=1}^{K}\mathbf{a}^*(\theta_{k1}, \psi_{k1}), .., \sum_{k=1}^{K}\mathbf{a}^*(\theta_{kL_{max}}, \psi_{kL_{max}})\right] \tag{2.10}$$

One needs the AoD set and the channel vector to get the downlink CSI. So, once BS gets the channel vector, it can obtain the downlink CSI through:

$$\hat{\mathbf{h}}_{\mathrm{DL}k} = \hat{\mathbf{A}}_k\hat{\mathbf{b}}_k \tag{2.11}$$

2.4 Summary

In this chapter, we looked at the directional training scheme to acquire CSIT for transmit beamforming in FDD massive MIMO. This method is inspired by two main results: One is that the number of dominant downlink angles is quite small and is independent of the antenna-size. And the second is that the uplink angles and downlink angles are highly correlated. Since it uses very less number of training symbols, directional training outperforms full training.

CHAPTER 3

RECURSIVE BEAM AND CHANNEL TRACKING

3.1 Type of Beamforming used

Because of its short wavelength, mmWave network can employ a large number of antenna elements in an array with a small factor. This provides enough link margin. However, the cost of AD/DA devices is high and mmWave RF chains consume significant amount of energy [13]. Therefore, the number of RF chains should be very much lesser than the number of antenna elements, which is why we use analog beamforming with phased arrays in this work. In this beamforming technique, only one RF chain is available. Amplitude of the weights are constant and phases are different. So, when the received signal is multiplied with the vector consisting of the weights (or the beamforming vector) to achieve beamforming, phase shifters serve the purpose.

3.2 System Model and Beamforming Implementation

The Fig. 3.1 shows the phased array antenna system used [1]. All the antenna elements, M in total, are connected to one RF chain via phase shifters. Distance between each adjacent pair of antennas is d. In a particular time slot n, if a signal with wavelength λ arrives at an AoA θ_n, then $\mathbf{a}(x_n)$=$[1, e^{\frac{2\pi d}{\lambda}x_n}, ..., e^{\frac{2\pi d}{\lambda}(M-1)x_n}]^{\mathrm{H}}$ is called the steering vector of the received signal. x_n,the sine of the AoA, is also called as the beam direction. As we intend to track both channel coefficient(β_n) and beam direction at the same time, we send two pilot symbols per time slot. The steering vector multiplied with pilot symbol affected by fading acts as the received signal vector. Each element of the received signal vector gets delayed/advanced by the corresponding phase shifter. Combining the output of the phase shifters gives the final received signal below:

$$y_{n,i} = \beta_n \mathbf{w}_{n,i}^{\mathrm{H}} \mathbf{a}(x_n) s + z_{n,i} \tag{3.1}$$

$\mathbf{w}_{n,i} = \frac{\mathbf{a}(x_n+\delta_{n,i})}{\sqrt{M}}$ denotes the beamforming vector, s the pilot symbol sent and $z_{n,i}$ the noise such that $z_{n,i}$ is an i.i.d. circularly symmetric complex Gaussian random variable with variance σ_0^2.

Let $\mathbf{y}_n = [y_{n,1}, y_{n,2}]^{\mathrm{T}},\ \mathbf{W}_n = [\mathbf{w}_{n,1}, \mathbf{w}_{n,2}]$ and $\boldsymbol{\psi}_n = [\beta_n^{\mathrm{re}}, \beta_n^{\mathrm{im}}, x_n]^{\mathrm{T}}$. Then the conditional pdf of $\mathbf{y}_n$ is given as:

$$p(\mathbf{y}_n|\boldsymbol{\psi}_n, \mathbf{W}_n) = \frac{1}{\pi^2\sigma_0^4} e^{\frac{-\|\mathbf{y}_n - s\beta_n \mathbf{W}_n^{\mathrm{H}}\mathbf{a}(x_n)\|_2^2}{\sigma_0^2}} \tag{3.2}$$

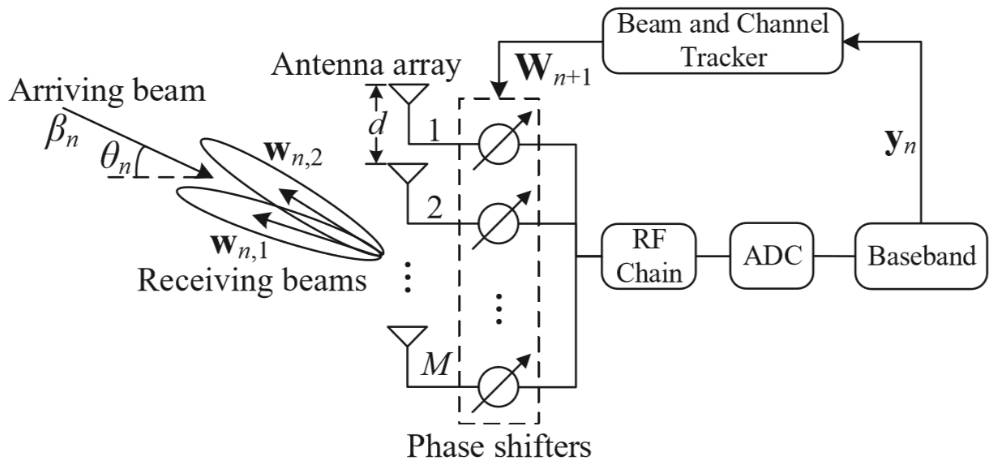

Figure 3.1: The System Model

3.3 The tracking method and Problem Formulation

In the first stage, the initial beam direction, x_n is estimated such that it is within the main lobe. Next, the tracking begins from this $\hat{x}_0$ to get more accurate beam directions. In each time slot n, the receiver chooses a beamforming matrix $\mathbf{W}_n$ on the basis of previously used beamforming matrices $(\mathbf{W}_1, ..., \mathbf{W}_{n-1})$ and observation vectors $(\mathbf{y}_1, ..., \mathbf{y}_{n-1})$. We obtain $\mathbf{y}_n$ by applying $\mathbf{W}_n$. We finally get the estimate of $\boldsymbol{\psi}_n$ by making use of all the available observation vectors and beamforming matrices [13].

We intend to minimise the error in beam direction tracking and thus we minimise the MSE for the same. Below is the joint beam and channel tracking problem:

$$\begin{aligned} \min \quad & \mathrm{E}\big[(\hat{x}_n - x_n)^2\big] \\ \text{subject to} \quad & \mathrm{E}\big[\hat{\beta}_n\big] = \beta_n,\ \mathrm{E}\big[\hat{x}_n\big] = x_n \end{aligned} \tag{3.3}$$

This problem is not easy to solve optimally. To begin with, $\mathbf{y}_n$ only gives us partial information about the system. Furthermore, we need to optimize both ψ_n and $\mathbf{W}_n$. And, both the optimization problems are non-convex.

3.4 CRLB of Beam Tracking

In this section, we find the minimum CRLB of MSE in static scenarios, wherein channel coefficient and beam direction both remain unchanged. This is how we write the CRLB inequality for MSE:

$$\mathrm{E}\big[(\hat{x}_n - x_n)^2\big] \geq \left[\left(\sum_{i=1}^{n} \mathbf{I}(\boldsymbol{\psi},\ \mathbf{W}_i)\right)^{-1}\right]_{3,3} \tag{3.4}$$

Here, $\mathbf{I}(\boldsymbol{\psi},\ \mathbf{W}_i)$ is the Fisher Information Matrix and is given by:

$$\mathbf{I}(\boldsymbol{\psi}, \mathbf{W}_i) = \mathrm{E}\left[\left(\frac{\partial \log(p(\mathbf{y}_i|\boldsymbol{\psi}, \boldsymbol{W}_i)}{\partial \boldsymbol{\psi}}\right)\left(\frac{\partial \log(p(\mathbf{y}_i|\boldsymbol{\psi}, \boldsymbol{W}_i)}{\partial \boldsymbol{\psi}^T}\right)\right] = \frac{2|s|^2}{\sigma_0^2}\begin{pmatrix} \|\mathbf{g}_i\|_2^2 & 0 & \mathrm{Re}\{\mathbf{g}_i^H \mathbf{e}_i\} \\ 0 & \|\mathbf{g}_i\|_2^2 & \mathrm{Im}\{\mathbf{g}_i^H \mathbf{e}_i\} \\ \mathrm{Re}\{\mathbf{g}_i^H \mathbf{e}_i\} & \mathrm{Im}\{\mathbf{g}_i^H \mathbf{e}_i\} & \|\mathbf{e}_i\|_2^2 \end{pmatrix} \tag{3.5}$$

in which $\mathbf{g}_i = \mathbf{W}_i^{\mathrm{H}} \mathbf{a}(x)$ and $\mathbf{e}_i = \beta \mathbf{W}_i^{\mathrm{H}} \frac{\partial \mathbf{a}(x)}{\partial x}$. To minimize the CRLB, we need optimal beamforming matrices,and by utilizing the linear additive property of Fisher Information Matrix, we can conclude that all the optimal beamforming matrices are the same.

$$\left[\left(\sum_{i=1}^{n} \mathbf{I}(\boldsymbol{\psi}, \mathbf{W}_i)\right)^{-1}\right]_{3,3} \geq \min_{\mathbf{W}_1,..,\mathbf{W}_n} \left[\left(\sum_{i=1}^{n} \mathbf{I}(\boldsymbol{\psi}, \mathbf{W}_i)\right)^{-1}\right]_{3,3} = \min_{\mathbf{W}_i} \frac{1}{n}\big[\mathbf{I}(\boldsymbol{\psi}, \mathbf{W}_i)^{-1}\big]_{3,3} \tag{3.6}$$

For any beamforming matrix $\mathbf{W}_i$, the $\mathbf{I}(\boldsymbol{\psi},\ \mathbf{W}_i)$ is as follows:

$$\big[\mathbf{I}(\boldsymbol{\psi}, \mathbf{W}_i)^{-1}\big]_{3,3} = \frac{\sigma_0^2}{2|s\beta|^2} \frac{\|\mathbf{g}_i\|_2^2}{\|\mathbf{g}_i\|_2^2 \|\mathbf{e}_i\|_2^2 - |\mathbf{g}_i^H \mathbf{e}_i|^2} \tag{3.7}$$

To obtain the optimal $\mathbf{W}$ for which the CRLB is minimized, we need to use the numerical method since the problem (2.6) is non-convex and makes it difficult to get an analytical solution.This is what we get:

$$\mathbf{W}^* = \frac{1}{\sqrt{M}}\big[\mathbf{a}(x-\delta^*), \mathbf{a}(x+\delta^*)\big] \tag{3.8}$$

in which δ^* is very close to $\frac{2\lambda}{3Md}$ as $M \to \infty$, and even at $M \geq 8$.

3.5 Coarse Beam Sweeping

The coarse beam sweeping is carried out to get the initial estimates of beam direction and channel coefficient. The method followed is orthogonal matching pursuit method which ensures that initial beam direction estimate is within the main lobe range:

$$\boldsymbol{B}(x_0) = \Big(x_0 - \frac{\lambda}{Md}, x_0 + \frac{\lambda}{Md}\Big) \tag{3.9}$$

We take all possible directions using $X = \{\frac{1-M_0}{M_0}, \frac{3-M_0}{M_0}, ..\frac{M_0-1}{M_0}\}$ and from this set we choose the initial estimates x_0 which is closest to the real x using equation 2.10. β_0 is calculated using x_0.

$$\hat{x}_0 = \arg\max_{x \epsilon X} |\mathbf{a}(\hat{x})^{\mathrm{H}}\tilde{\mathbf{W}}\tilde{\mathbf{y}}|, \hat{\beta}_0 = \big[\tilde{\mathbf{W}}^{\mathrm{H}}\mathbf{a}(\hat{x}_0)\big]^{\dagger}\tilde{\mathbf{y}} \tag{3.10}$$

3.6 Beam and Channel Tracking

In the previous sections, we got the minimum CRLB for a known x. But in a real scenario, x is unknown and thus, we adjust beamforming matrices dynamically [13] The first task is to choose beamforming vectors for the two symbols. To ensure that the beamforming vector does not go out of the main lobe range, we keep it close to the previously estimated beam direction, where δ^* acts as an offset. Below is the maximization likelihood problem that we follow:

$$\max_{\hat{\boldsymbol{\psi}}_n}\left\{\max_{\mathbf{W}_n}\sum_{i=1}^{n}\mathrm{E}\left[\log p(\mathbf{y}_i|\hat{\boldsymbol{\psi}}_n, \mathbf{W}_i)\middle|\hat{\boldsymbol{\psi}}_n, \mathbf{W}_1, .., \mathbf{W}_i, \mathbf{y}_1, .., \mathbf{y}_{i-1}\right]\right\} \tag{3.11}$$

The inner layer is equivalent to minimizing the CRLB to get $\mathbf{W}_n$. This gives:

$$\mathbf{w}_{n,1} = \frac{\mathbf{a}(\hat{x}_{n-1} - \delta^*)}{\sqrt{M}}, \mathbf{w}_{n,2} = \frac{\mathbf{a}(\hat{x}_{n-1} + \delta^*)}{\sqrt{M}} \tag{3.12}$$

Then, we solve the outer layer using the stochastic Newton's method and arrive at the following equation:

$$\hat{\boldsymbol{\psi}}_n = \hat{\boldsymbol{\psi}}_{n-1} + a_n \mathbf{I}(\hat{\boldsymbol{\psi}}_{n-1}, \mathbf{W}_n)^{-1} \frac{\partial \log(p(\mathbf{y}_n | \hat{\boldsymbol{\psi}}_{n-1}, \mathbf{W}_n)}{\partial \hat{\boldsymbol{\psi}}_{n-1}} \tag{3.13}$$

in which a_n is the step-size, and:

$$\frac{\partial \log(p(\mathbf{y}_n | \hat{\boldsymbol{\psi}}_{n-1}, \mathbf{W}_n)}{\partial \hat{\boldsymbol{\psi}}_{n-1}} = -\frac{2}{\sigma_0^2} \begin{bmatrix} \mathrm{Re}\{s^{\mathrm{H}} \hat{\mathbf{g}}_n^{\mathrm{H}} (\mathbf{y}_n - s\hat{\beta}_{n-1}\hat{\mathbf{g}}_n)\} \\ \mathrm{Im}\{s^{\mathrm{H}} \hat{\mathbf{g}}_n^{\mathrm{H}} (\mathbf{y}_n - s\hat{\beta}_{n-1}\hat{\mathbf{g}}_n)\} \\ \mathrm{Re}\{s^{\mathrm{H}} \hat{\mathbf{e}}_n^{\mathrm{H}} (\mathbf{y}_n - s\hat{\beta}_{n-1}\hat{\mathbf{g}}_n)\} \end{bmatrix} \tag{3.14}$$

where $\hat{\mathbf{g}}_n$=$\mathbf{W}_n^{\mathrm{H}}\mathbf{a}(\hat{x}_{n-1})$ and $\hat{\mathbf{e}}_n$=$\hat{\beta}_{n-1}\mathbf{W}_n^{\mathrm{H}} \frac{\partial \mathbf{a}(x)}{\partial x}\Big|_{x=x_{n-1}}$
Also, let l_n=$\|\hat{\mathbf{g}}_n\|_2 \|\hat{\mathbf{e}}_n\|_2$ and c_n=$\hat{\mathbf{g}}_n^{\mathrm{H}}\hat{\mathbf{e}}_n$ Then, we arrive at our final tracking equation:

$$\hat{\boldsymbol{\psi}}_n = \hat{\boldsymbol{\psi}}_{n-1} - k \begin{bmatrix} l_n^2 - \mathrm{Im}\{c_n\}^2 & \mathrm{Re}\{c_n\}\mathrm{Im}\{c_n\} & -\|\hat{\mathbf{g}}_n\|_2^2 \,\mathrm{Re}\{c_n\} \\ \mathrm{Re}\{c_n\}\mathrm{Im}\{c_n\} & l_n^2 - \mathrm{Re}\{c_n\}^2 & -\|\hat{\mathbf{g}}_n\|_2^2 \,\mathrm{Im}\{c_n\} \\ -\|\hat{\mathbf{g}}_n\|_2^2 \,\mathrm{Re}\{c_n\} & -\|\hat{\mathbf{g}}_n\|_2^2 \,\mathrm{Im}\{c_n\} & \|\hat{\mathbf{g}}_n\|_2^4 \end{bmatrix} \begin{bmatrix} \mathrm{Re}\{s^{\mathrm{H}} \hat{\mathbf{g}}_n^{\mathrm{H}} (\mathbf{y}_n - s\hat{\beta}_{n-1}\hat{\mathbf{g}}_n)\} \\ \mathrm{Im}\{s^{\mathrm{H}} \hat{\mathbf{g}}_n^{\mathrm{H}} (\mathbf{y}_n - s\hat{\beta}_{n-1}\hat{\mathbf{g}}_n)\} \\ \mathrm{Re}\{s^{\mathrm{H}} \hat{\mathbf{e}}_n^{\mathrm{H}} (\mathbf{y}_n - s\hat{\beta}_{n-1}\hat{\mathbf{g}}_n)\} \end{bmatrix} \tag{3.15}$$

where:

$$k = \frac{a_n}{\|s\hat{\mathbf{g}}_n\|_2^2 (l_n^2 - |c_n|^2)} \tag{3.16}$$

CHAPTER 4

SIMULATION

In this chapter, we give the framework for simulation and show the results of simulations. The theory for the simulations has been explored in the preceding chapter. The simulations are carried out for both static and dynamic scenarios. In both the cases,we take M=32 and d=0.5λ. We set the pilot symbol as s=0.5+0.5j and transmit SNR as 5dB. Also, we take β as a Rayleigh fading coefficient.

4.1 Static Scenario

The step-size used in this case was 1/n, where n is the time slot. We took AoA as 0.We plot MSE over time by taking 1000 realizations.The MSE curve obtained from the RBCT algorithm is shown in Fig. 4.1(a). As we can see, the MSE converges to the minimum CRLB.

4.2 Dynamic Scenario

The step-size used in this case was 1. We take real AoA at a time slot n, $\theta_n = \theta_{n-1} + \delta_{n-1}\omega$, with $\theta_0 = 0$, $\delta\epsilon\{-1, 1\}$ and $\omega\epsilon[0, 0.04]$. δ_n is chosen in such a way that θ_n lies between $[-\frac{\pi}{3}, \frac{\pi}{3}]$. The MSE over omega is calculated by averaging it over 1000 time-slots.The curve is shown in Fig. 4.2(a). As we can see, the RBCT algorithm supports high angular velocities. We also plot achievable rate over angular velocity. As per Fig. 4.2(b) , the RBCT algorithm achieves 83% of channel capacity at an angular velocity of 0.02 rad/time-slot. Thus,it can be said that the algorithm also supports high data rates.

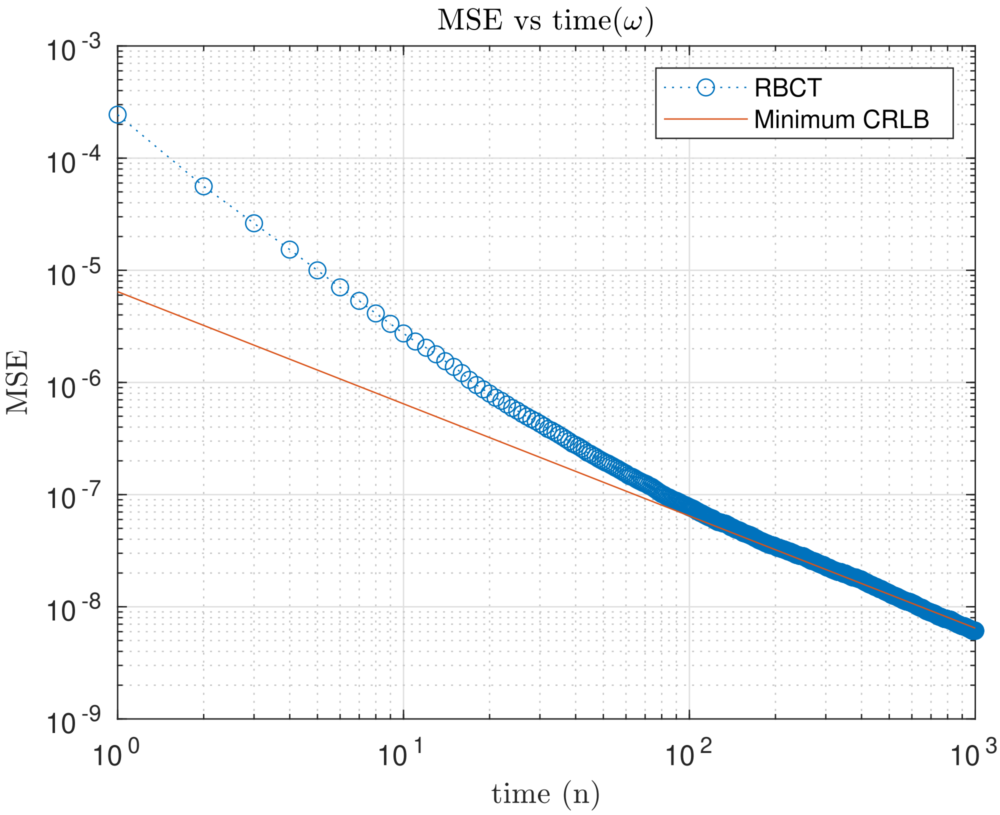

Figure 4.1: (a) Simulation result for Static Scenario: MSE vs. time-slot number

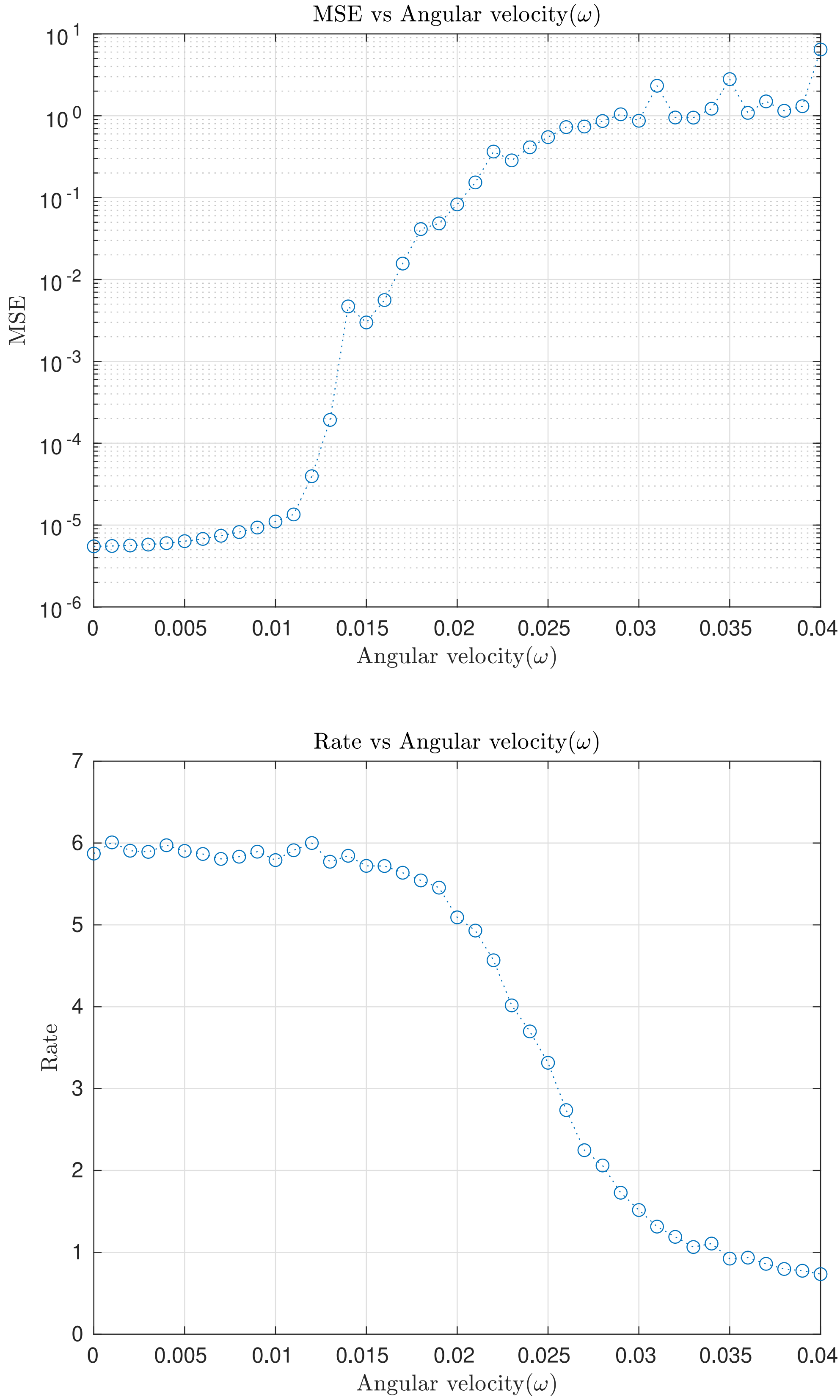

Figure 4.2: Simulation results for dynamic scenarios:(a) MSE vs. angular velocity (b) Data rate vs. angular velocity

CHAPTER 5

KEY RESULTS AND CONCLUSION

The mmWave network needs directional beamforming but is highly susceptible to signal drop because of narrower beams which gets more complicated as the value of ω increases. We simulated the MSE performance of RBCT algorithm for static scenarios. It was seen that the MSE curve achieves minimum CRLB as n increases. After this, we calculated MSE and achieveable data rates for different angular velocities. In those cases, we showed that the MSE is quite low for high angular velocities. Besides, the achievable rate is also decent at high angular velocities. Therefore, we can say that RBCT algorithm achieves low beam tracking error and high data rates along with low pilot overhead.

In full training, symbols are sent in all directions. The overhead simply depends on the number of antennas at the BS which is quite high for massive MIMO.However, in directional training, symbols are sent in specific directions only and the overhead depends on the number of AoDs used which is quite low. It costs $2L_{max}$ symbols while full training costs 2M symbols.Thus, directional training offers much lower overhead than full training.

CHAPTER 6

FUTURE WORK

Beam and Channel Tracking has a lot of avenues to be tried and examined. When we implemented the RBCT algorithm, we assumed that the channel coefficient and beam direction vary independently. However this may not be the case in real mmWave channels. The variation of channel coefficient and beam direction might be interrelated. This could be taken into account in the work ahead. We also assumed transmitter and receiver reciprocity and thus performed tracking only at the receiver. However, one can try to track both at the transmitter and receiver. Also, we focused on tracking one path. But jointly tracking multiple paths can also be considered in further research.

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