A Novel GO-PO Based Optimization Method for mm-Wave Antenna Lens Design

A Project Report

submitted by

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in partial fulfilment of the requirements
for the award of the degree of

BACHELOR OF TECHNOLOGY

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY MADRAS.

May 2019

THESIS CERTIFICATE

This is to certify that the thesis titled **A Novel GO-PO Based Optimization Method for mm-Wave Antenna Lens Design**, submitted by **Sankalp Chapalgaonkar**, to the Indian Institute of Technology, Madras, for the award of the degree of **Bachelor of Technology**, is a bona fide record of the research work done by him under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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0.1 Proposed Contents of the Report

The outline of this report is as follows:

1. Abstract
2. Introduction
3. Design Technique
4. 3-D Lens Design using Geometric optics
    - power consumption principle
    - Vector's Snells law
    - Solution to problem
        - Differentiation
        - Linearization
        - Discretization
5. Algorithm
6. Conclusion
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0.2 Abstract

This report presents the extensive study and the optimization technique for generating 3-D Lens profile designed for transforming radiation pattern of the primary feed into the desired amplitude shaped pattern. The design of the lens is made keeping in mind the working frequency of operation to be 28 GHz. Hence using lens antenna we can obtain the gain upto 15 dB by simply using 5 dB microstrip antenna. In this project, we implement the optimization algorithm proposed by (1) for designing 3-D lens profile. The inverse scattering problem (to determine the characteristics of an object based on the data of how it scatters the particles or incoming radiation, in our case its radiation from 2x2 microstrip patch antenna) is approached using techniques of geometric optics (GO) and physical optics (PO). Firstly, we derive extensive geometry for the lens using GO and generate second-partial differential equation which is strongly non-linear and of Monge-AmpÃĺre (M.A) type. Then to solve this problem we use iterative algorithm and the error is minimize by upper bounding it with required threshold. Second step is of surface optimization and analysis based on PO to come up with a 3-D lens profile which comply with the prescribed required radiation pattern. By varying the inputs for radiation pattern of primary antenna and the required radiation pattern, we can obtain the precise lens profile using the algorithm proposed.
Keywords-inverse scattering problem, Geometric optics and Physical optics, Monge-AmpÃĺre equation, microstrip antenna, optimization

0.3 Introduction

As per the data rates promised by 5G wireless system and the proposed high bandwidth of usage, antenna plays an integral role in fulfilling this requirement. Hence the need to make advancements in the field of antenna and innovating the existing conventional designs is at uppermost priority. To increase data throughput in 5G cellular system includes improving spectral efficiency using techniques such as MIMO transmission, Beam forming, beam steering, defining small cells i.e. increasing the number of basestation in a region. Using conventional antenna for the transmission and receptions require far more specification and to meet the expectations, the design becomes too use case specific and very much complex. Plus as the frequency of operation increases the fading increases as well along the distance. Hence 5G system assumes multiple antenna for serving in small area. The problem of cost optimization is also of major concern. Building conventional antennas that support the required specification and using multiple such antennas within a close range will eventually increase the total cost of operation. Considering this factor, lens antenna solves major of the issues with conventional antennas. Lens antenna was first used by Oliver Lodge in 1888 for his experiments at 1mm wavelength (Lodge and Howard 1888). But it was not until World War II that the research on lens antenna was progressed. Since then lens antennas are used to transform the radiation pattern of primary antenna into some high gain radiation pattern depending upon requirement of fixed or scanning beam applications. Lens antennas have greater design tolerance hence the need of having precise design is not needed. Even having design tending to the ideal required one gives the expected results. Lens antennas can be of various types such as Di-electric lens, H-plane metal plate lens, Delay lens (to introduce delay in the path of travelling waves using lens material), E-plane metallic plate lens, Non-mettalic di-electric lens, etc. Feed and feed support of the lens antenna does not obstruct it's aperture. Moreover, dielectric lenses have wide-band capabilities and a very low dissipative loss.

Apart from using lens as a separate entity, they can be designed to have the feed directly in contact with the flat bottom surface and these are call as integrated lens antennas (ILA). The use of integrated lens antennas started by using hemispherical lenses added on top of integrated circuit antennas to increase radiation efficiency. Nowadays lens antenna can be excited with a small antenna element, typically a patch antenna or an open-ended waveguide. To provide equal amount of gain as compared to complex conventional antenna, we use the lens which can be mounted on simple 2x2 microstrip antenna and the original pattern of this microstrip antenna can be amplified and transformed into the required far field electric field pattern.

Beam steering is required especially in two dimensions (azimuthal and elevation) to ease the deployment and cope up with the small cell requirement in 5G cellular network. Beam steering is done by switching between multiple feed antennas at beneath the lens. Beam steering principle is shown along with the feeding array of antenna in Fig. 1. The collimating part which is either hemispherical or elliptical is used to focus the radiation from the lens and to the lens accordingly. The part of the lens between the flat part and collimating part is known as extension and is designed so as to minimize the internal

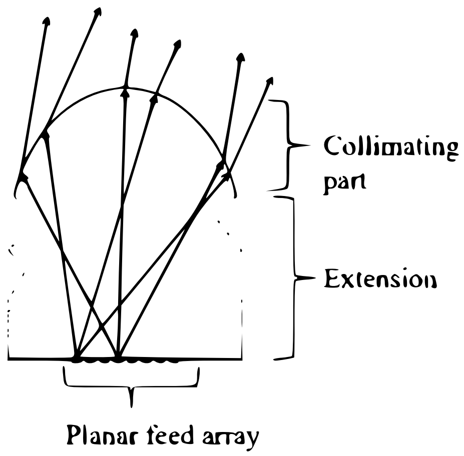

Figure 1: Beam-steering using planar array of antenna

reflection. Mostly this extension part is cylindrical in nature and it is very important in low permitivity lenses. The loss through this extension part is known as spillover loss and is responsible for the side-lobe power. Absorbers can be used around the extension part to minimize the side-lobe levels.

0.4 Design Technique

Various lens antennas are manufactured mostly using Teflon as main material. Figure 2 shows the final product of the design process. Designing the lens antenna without strong mathematical tool and precise optimization method seems to be quite difficult. Hence in this project we design the mathematical tool (/method) which can be used to design the precise 3-D lens antenna which can be manufactured using different material which are selected based upon their relative permittivity or dielectric constant which range from 1.2 to approximately 10. Second most important thing to consider while choosing the material is the dielectric loss tangent ($tan\ \delta$), which is the material loss. Hence to minimize the material losses, lower loss tangent material is preferred. Mechanical consideration such as mechanical hardness, melting temperature or fracture hardness plays vital role in deciding the material for lens antenna.

Figure 3 and Figure 4 shows the calculation of maximum scan angle and corresponding beam with main lobe and side lobe. In this report we present the method to generate the 3-D lens profile using following algorithm as shown in figure 5. The output

Figure 2: Example of finished lens antenna
This lens was manufactured at Sameer, Centre for Electromagnetics, Chennai not as part of this project.

of this design process is the lens profile r^k at every point (θ, ϕ), which finally can be piloted in 3-D coordinate system. Later based on the results, the profile can be improved by lowering the error upper-bound. This will take more iterations but will give more precise lens profile as per requirement.

0.5 3-D Lens Design using GO

In the Fig. 6, the design of arbitrary lens with homogeneous dielectric ($\varepsilon_{r,d}$) is shown. The 2x2 microstrip patched antenna is kept at bottom surface of the lens ($z = 0$). Then the unknown lens profile $r(\theta, \phi)$ is determined in 3 dimensional space and the central thickness is defined as $e = r(\theta = 0, \phi)$. Using GO, $r(\theta, \phi)$ is calculated so that the radiation intensity $g(\theta, \phi)$ of primary feed matched with the intensity field just outside the lens surface after refraction through the lens. We apply power conservation principle at the surface and equate the power on both the sides. The far field pattern, $h(\alpha, \beta)$ is useful in calculating the field just outside the lens surface using reverse calculation method. Here (θ, ϕ) and (α, β) are the directions of incident ray and refracted ray respectively.

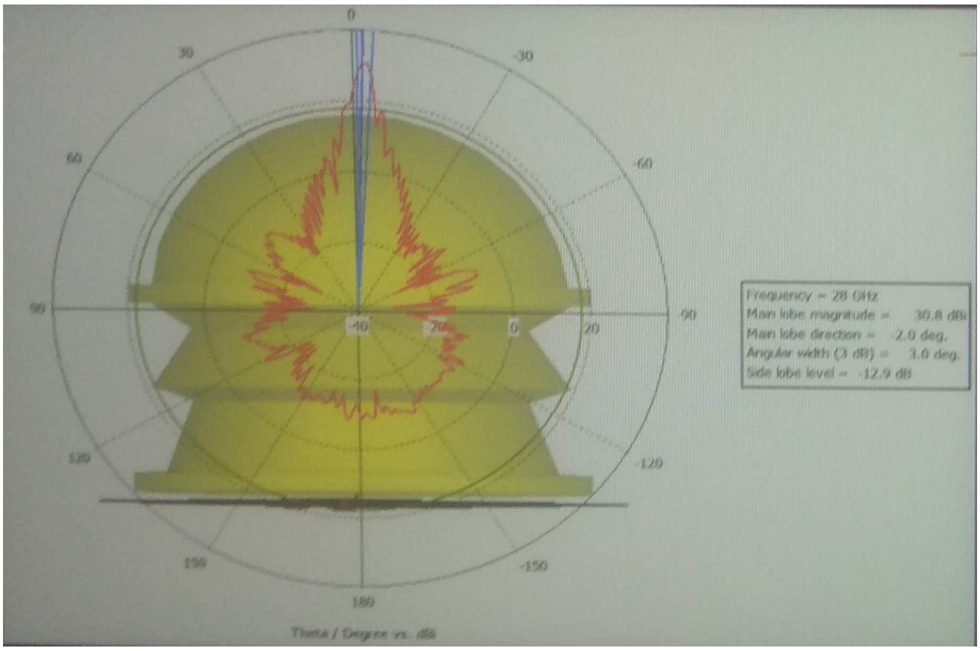

Figure 3: Hemispherical Lens with corresponding beam at central location

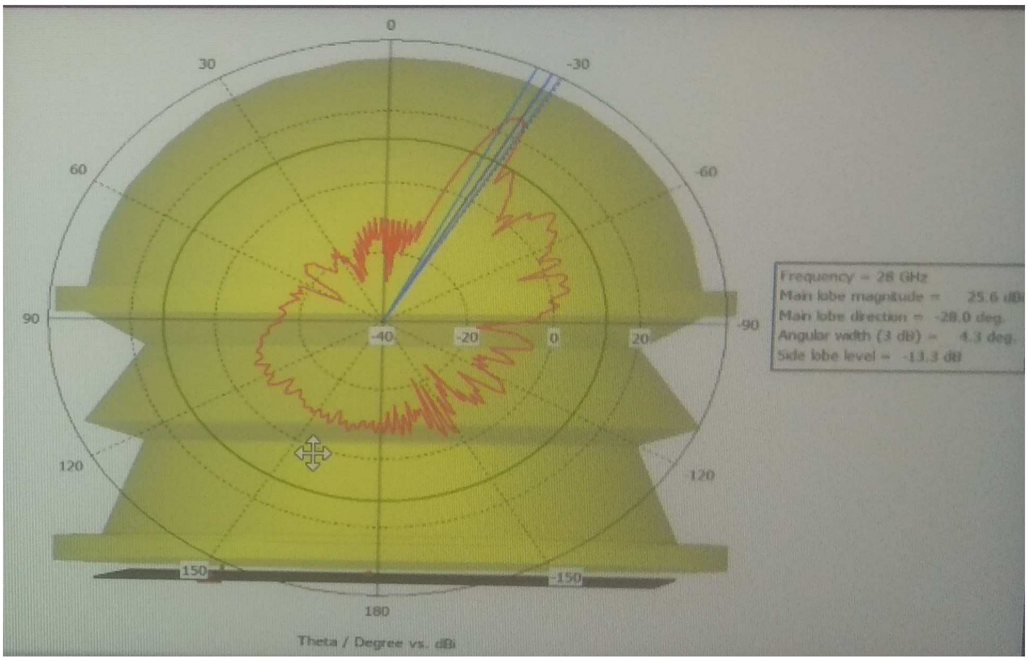

Figure 4: Maximum scan angle in grooved hemispherical lens antenna

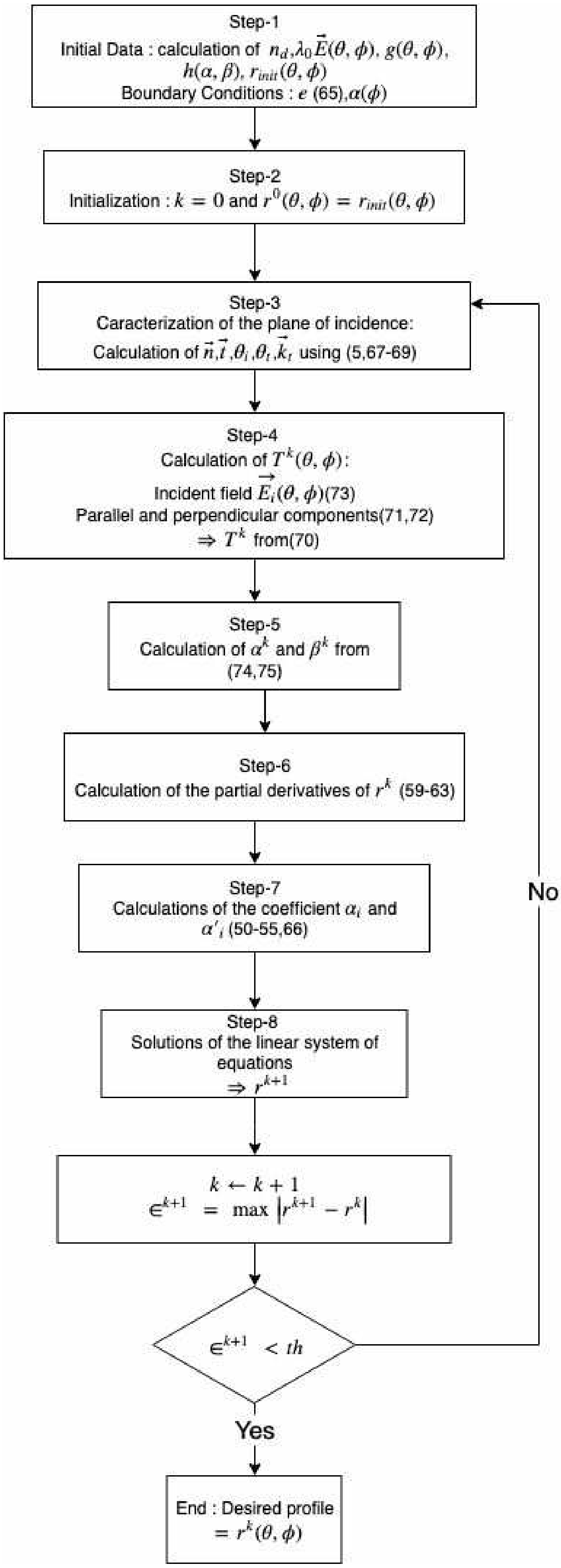

Figure 5: Algorithm used to calculate $r^k(\theta, \phi)$

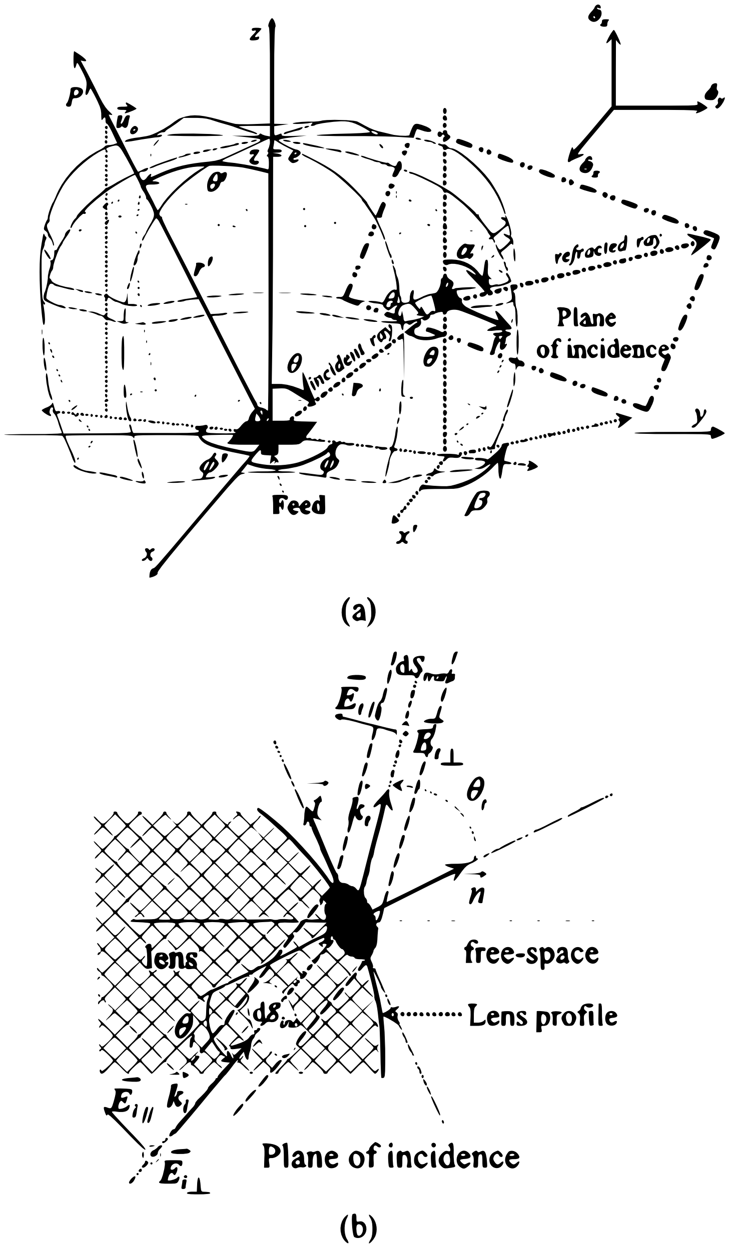

Figure 6: (a) Arbitrary lens geometry (b)vector and angular notation in plane of incident source: paper (1)

0.5.1 Power consumption principle

As stated above, we use power consumption principle at the lens surface as follows. Right hand side of equation (1) shows the power transmitted from primary source antenna to an elementary external surface of the lens in the direction (θ, ϕ).

$$h(\alpha, \beta)sin(\alpha)d\alpha d\beta = KT(\theta, \phi)g(\theta, \phi)sin(\theta)d(\theta)d(\phi) \tag{1}$$

$$K = \frac{\int_{-\pi}^{\pi}\int_{0}^{\frac{\pi}{2}} h(\alpha^{'}, \beta^{'})sin(\alpha^{'})d\alpha^{'}d\beta^{'}}{\int_{-\pi}^{\pi}\int_{0}^{\frac{\pi}{2}} T(\theta^{'}, \phi^{'})g(\theta^{'}, \phi^{'})sin(\theta^{'})d\theta^{'}d\phi^{'}} \tag{2}$$

T(θ, ϕ) is the ratio of transmitted power to the incident power at the lens surface. LHS of equation (1) denotes power radiated in the direction (α, β). K is normalization constant calculated by equating the total power radiated outside the lens and the total power radiated by the primary feeding source as given in equation (2). Further dielectric losses can be added using $exp[\frac{-2\pi n_d}{\lambda_0}\delta r(\theta, \phi)]$. where n_d is refractive index of the lens and is calculated as given in equation (3), δ is loss angle and λ_0 is wavelength.

$$n_d = \sqrt{\varepsilon_{r,d}} \tag{3}$$

But in this computation we will neglect the effect of dielectric losses.

0.5.2 Vector's Snells law

We apply snells law at the boundary of the lens as per equation (4).

$$n_d\vec{k_i}\Lambda\vec{n} = \vec{k_t}\Lambda\vec{n} \tag{4}$$

where $\vec{n}$ is outer vector orthogonal to the lens surface and $\vec{k_i}, \vec{k_t}$ are normalized incident and transmitted vectors. At the point of contact,

$$\vec{n}(\theta, \phi) = \frac{\frac{\partial\vec{r}}{\partial\theta}\Lambda\frac{\partial\vec{r}}{\partial\theta}}{\|\frac{\partial\vec{r}}{\partial\theta}\Lambda\frac{\partial\vec{r}}{\partial\theta}\|} \tag{5}$$

$$\frac{\partial \vec{r}}{\partial \theta} = \begin{pmatrix} \frac{\partial r}{\partial \theta} \\ r \\ 0 \end{pmatrix} (\hat{e_r}, \hat{e_\theta}, \hat{e_\phi}) \tag{6}$$

$$\frac{\partial \vec{r}}{\partial \phi} = \begin{pmatrix} \frac{\partial r}{\partial \phi} \\ 0 \\ r sin\theta \end{pmatrix} (\hat{e_r}, \hat{e_\theta}, \hat{e_\phi}) \tag{7}$$

Feed at point source ($\vec{k_i} = \vec{e_r}$),

$$\vec{k_t} = \begin{pmatrix} sin\theta cos\phi sin\alpha cos\beta + sin\theta sin\phi sin\alpha sin\beta + cos\theta cos\alpha \\ cos\theta cos\phi sin\alpha sin\beta + cos\theta sin\phi cos\alpha sin\beta - sin\theta cos\alpha \\ -sin\phi sin\alpha cos\beta + cos\phi sin\alpha cos\beta \end{pmatrix} (\hat{e_r}, \hat{e_\theta}, \hat{e_\phi}) \tag{8}$$

From equation (4) and (8) we get,

$$\frac{\partial r}{\partial \theta} = \frac{(sin\ \alpha\ cos\ \theta\ cos\ (\beta - \phi) - cos\ \alpha\ sin\ \theta)\ r}{n_d - (sin\ \alpha\ sin\ \theta\ cos\ (\beta - \phi) + cos\ \alpha\ cos\ \theta)} \tag{9}$$

and

$$\frac{\partial r}{\partial \phi} = \frac{sin\ \alpha\ sin\ (\beta - \phi)\ r\ sin\ \theta}{n_d - (sin\ \alpha\ sin\ \theta\ cos\ (\beta - \phi) + cos\ \alpha\ cos\ \theta)} \tag{10}$$

Using schwartz condition for the smooth surface we have,

$$\frac{\partial^2 r}{\partial \theta\, \partial \phi} = \frac{\partial^2 r}{\partial \phi\, \partial \theta} \tag{11}$$

By considering axis symmetry we have $\beta = \phi$. Hence equations (1), (2) and (9) transforms into (12), (13) and (14) respectively.

$$h(\alpha) sin(\alpha) d\alpha = K^{'} T(\theta) g(\theta) sin(\theta) d(\theta) \tag{12}$$

$$k^{'} = \frac{\int_0^{\frac{\pi}{2}}\ h(\alpha^{'})\ sin\alpha^{'}\ d\alpha^{'}}{\int_0^{\frac{\pi}{2}}\ T(\theta')\ g(\theta')\ sin(\theta')\ d\theta'} \tag{13}$$

$$\frac{\partial r}{r} = \frac{sin(\theta - \alpha)}{cos(\theta - \alpha) - n_d} d\theta \tag{14}$$

0.5.3 Solution to problem

We use three steps for solving this numerical problem :- differentiation, Linearization and Discreatization.

Differentiation

Here θ and ϕ are state variables and hence we consider α and β to be their functions. So, equation (1) takes following shape.

$$h(\alpha(\theta,\phi),\beta(\theta,\phi))sin(\alpha(\theta,\phi))\left[\frac{\partial\alpha(\theta,\phi)}{\partial\theta}\,\frac{\partial\beta(\theta,\phi)}{\partial\phi}-\frac{\partial\alpha(\theta,\phi)}{\partial\phi}\,\frac{\partial\beta(\theta,\phi)}{\partial\theta}\right]=KT(\theta,\phi)g(\theta,\phi)sin(\theta) \tag{15}$$

applying theorem of implicit functions $\{f(\theta,\phi,\alpha,\beta)=0\}$ on equations (9) and (10) we get partial differentiation of α and β with respect to θ and ϕ. Let's denote $F_x = \frac{\partial F}{\partial x}$ and $F_{xy} = \frac{\partial^2 F}{\partial x\,\partial y}$, we obtain

$$\alpha_\theta(\theta,\phi) = \frac{f_{1_\theta}\,f_{2_\beta} - f_{2_\theta}\,f_{1_\beta}}{f_{1_\alpha}\,f_{2_\beta} - f_{2_\alpha}\,f_{1_\beta}} \tag{16}$$

$$\alpha_\phi(\theta,\phi) = \frac{f_{1_\phi}\,f_{2_\beta} - f_{2_\phi}\,f_{1_\beta}}{f_{1_\alpha}\,f_{2_\beta} - f_{2_\alpha}\,f_{1_\beta}} \tag{17}$$

$$\beta_\theta(\theta,\phi) = \frac{f_{1_\theta}\,f_{2_\alpha} - f_{2_\theta}\,f_{1_\alpha}}{f_{1_\beta}\,f_{2_\alpha} - f_{2_\beta}\,f_{1_\alpha}} \tag{18}$$

$$\beta_\phi(\theta,\phi) = \frac{f_{1_\phi}\,f_{2_\alpha} - f_{2_\phi}\,f_{1_\alpha}}{f_{1_\beta}\,f_{2_\alpha} - f_{2_\beta}\,f_{1_\alpha}} \tag{19}$$

where as equation for $f_{1_\theta}, f_{1_\phi}, f_{1_\alpha}, f_{1_\beta}, f_{2_\theta}, f_{2_\phi}, f_{2_\alpha} and f_{2_\beta}$ are given in paper (1). So when we substitute equations (16), (17), (18) and (19) in (15) we obtain second order partial differential equation for r of the following form.

$$E : r_{\theta\theta}r_{\phi\phi} - {r_{\theta\phi}}^2 - ar_{\theta\theta} - br_{\theta\phi} - cr_{\phi\phi} - H = 0 \tag{20}$$

whereas, a,b,c and H are the functions of $r, r_\theta, r_\phi, n_d, \theta$ and ϕ. For calculating these equation we solve the system of equation from (16) to (19) and converts into (21) to (24)

$$\alpha_\theta(\theta,\phi) = \frac{f_{1_\theta}\ f_{2_\beta} - f_{2_\theta}\ f_{1_\beta}}{f_{1_\alpha}\ f_{2_\beta} - f_{2_\alpha}\ f_{1_\beta}} = \frac{A}{E-F} r_{\theta\theta} + \left(\frac{-C}{E-F}\right) r_{\theta\phi} + \frac{B-D}{E-F} \tag{21}$$

$$\alpha_\phi(\theta,\phi) = \frac{f_{1_\phi}\ f_{2_\beta} - f_{2_\phi}\ f_{1_\beta}}{f_{1_\alpha}\ f_{2_\beta} - f_{2_\alpha}\ f_{1_\beta}} = \frac{G}{E-F} r_{\theta\phi} + \left(\frac{-I}{E-F}\right) r_{\phi\phi} + \frac{H-J}{E-F} \tag{22}$$

$$\beta_\theta(\theta,\phi) = \frac{f_{1_\theta}\ f_{2_\alpha} - f_{2_\theta}\ f_{1_\alpha}}{f_{1_\beta}\ f_{2_\alpha} - f_{2_\beta}\ f_{1_\alpha}} = \frac{-K}{E-F} r_{\theta\theta} + \left(\frac{M}{E-F}\right) r_{\theta\phi} + \frac{N-L}{E-F} \tag{23}$$

$$\beta_\phi(\theta,\phi) = \frac{f_{1_\phi}\ f_{2_\alpha} - f_{2_\phi}\ f_{1_\alpha}}{f_{1_\beta}\ f_{2_\alpha} - f_{2_\beta}\ f_{1_\alpha}} = \frac{-P}{E-F} r_{\theta\phi} + \left(\frac{R}{E-F}\right) r_{\phi\phi} + \frac{S-Q}{E-F} \tag{24}$$

whereas A to S are the functions of $r, r_\theta, r_\phi, n_d, \theta$ and ϕ as follows.

$$\begin{aligned} A = (r_\phi - r)\Big[& n_d\ sin(\alpha)\ sin(\theta)\ sin(\beta-\phi) - \frac{sin^2(\alpha)sin^2(\theta)sin2(\beta-\phi)}{2} \\ & - \frac{sin(2\alpha)sin(2\theta)sin(\beta-\phi)}{4}\Big] \end{aligned} \tag{25}$$

$$\begin{aligned} B = {} & 2r_\theta(r_\phi - r)\left[\frac{sin(2\alpha)sin^2(\theta)sin(\beta-\phi)}{2} - \frac{sin^2(\alpha)sin(2\theta)sin2(\beta-\phi)}{4}\right] \\ & + r(r_\phi - r)\left[\frac{sin^2(\alpha)sin^2(\theta)sin2(\beta-\phi)}{2} - \frac{sin(2\alpha)sin(2\theta)sin(\beta-\phi)}{4}\right] \end{aligned} \tag{26}$$

$$
\begin{aligned}
C = r_\theta \bigg[& n_d \, sin(\alpha) \, sin(\theta) \, sin(\beta-\phi) - \frac{sin^2(\alpha) sin^2(\theta) sin2(\beta-\phi)}{2} \\
& - \frac{sin(2\alpha) sin(2\theta) sin(\beta-\phi)}{4} \bigg] + r \bigg[n_d \, sin(\alpha) \, cos(\theta) \, sin(\beta-\phi) \\
& - \frac{sin^2(\alpha) sin(2\theta) sin2(\beta-\phi)}{4} - \frac{sin(2\alpha) cos^2(\theta) sin(\beta-\phi)}{2} \bigg]
\end{aligned} \tag{27}
$$

$$
\begin{aligned}
D = & \, sin^2(\alpha) sin^2(\beta-\phi) \Big(r^2 cos^2(\theta) - r_\theta^2 sin^2(\theta) \Big) \\
& - r_\theta r_\phi \bigg[\frac{sin^2(\alpha) sin(2\theta) sin2(\beta-\phi)}{4} - \frac{sin(2\alpha) sin^2(\theta) sin(\beta-\phi)}{2} \bigg] \\
& + r \, r_\phi \bigg[\frac{sin^2(\alpha) cos^2(\theta) sin2(\beta-\phi)}{2} - \frac{sin(2\alpha) cos(2\theta) sin(\beta-\phi)}{4} \bigg]
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
E = & \, 2 r_\theta (r - r_\phi) \bigg[\frac{sin(2\alpha) sin^2(\theta) sin2(\beta-\phi)}{4} + \frac{sin^2(\alpha) sin(2\theta) sin(\beta-\phi)}{2} \bigg] \\
& + r(r - r_\phi) \bigg[\frac{sin(2\alpha) sin(2\theta) sin2(\beta+\phi)}{8} - sin^2(\alpha) sin^2(\theta) sin(\beta-\phi) \bigg]
\end{aligned} \tag{29}
$$

$$
\begin{aligned}
F = & - r_\theta r_\phi \bigg[\frac{sin(2\alpha) sin^2(\theta) sin2(\beta-\phi)}{4} + \frac{sin^2(\alpha) sin(2\theta) sin(\beta-\phi)}{2} \bigg] \\
& - r r_\theta \frac{sin(2\alpha) sin^2(\theta) sin^2(\beta-\phi)}{2} - r^2 \frac{sin(2\alpha) sin(2\theta) sin^2(\beta-\phi)}{4} \\
& - r r_\phi \bigg[\frac{sin(2\alpha) sin(2\theta) sin2(\beta-\phi)}{8} + sin^2(\alpha) cos^2(\theta) sin(\beta-\phi) \bigg]
\end{aligned} \tag{30}
$$

$$
\begin{aligned}
G = A = (r_\phi - r) \bigg[& n_d \, sin(\alpha) \, sin(\theta) \, sin(\beta-\phi) - \frac{sin^2(\alpha) sin^2(\theta) sin2(\beta-\phi)}{2} \\
& - \frac{sin(2\alpha) sin(2\theta) sin(\beta-\phi)}{4} \bigg]
\end{aligned} \tag{31}
$$

$$
\begin{aligned}
H =&(r^2 - r\, r_\phi)\frac{sin^2(\alpha)sin(2\theta)sin^2(\beta-\phi)}{2} + (r\, r_\theta - r_\theta\, r_\phi)sin^2(\alpha)sin^2(\theta)sin^2(\beta-\phi) \\
&+ (r\, r_\phi - r^2{}_\phi)\left[\frac{sin^2(\alpha)sin(2\theta)sin2(\beta-\phi)}{4} - \frac{sin(2\alpha)sin^2(\theta)sin(\beta-\phi)}{2}\right]
\end{aligned} \tag{32}
$$

$$
\begin{aligned}
I =&n_d\, r_\theta sin(\alpha)sin(\theta)sin(\beta-\phi) + n_d\, r\, sin(\alpha)cos(\theta)sin(\beta-\phi) \\
&- r_\theta\left[\frac{sin^2(\alpha)sin^2(\theta)sin2(\beta-\phi)}{2} + \frac{sin(2\alpha)sin(2\theta)sin(\beta-\phi)}{4}\right] \\
&- r\left[\frac{sin^2(\alpha)sin(2\theta)sin2(\beta-\phi)}{4} + \frac{sin(2\alpha)cos^2(\theta)sin(\beta-\phi)}{2}\right]
\end{aligned} \tag{33}
$$

$$
\begin{aligned}
J =&rr_\theta\frac{sin^2(\alpha)sin^2(\theta)sin2(\beta-\phi)}{2} + r^2\frac{sin^2(\alpha)sin(2\theta)sin2(\beta-\phi)}{4} \\
&- 2r_\theta\, r_\phi sin^2(\alpha)sin^2(\theta)sin^2(\beta-\phi) - r\, r_\phi sin^2(\alpha)sin(2\theta)sin^2(\beta-\phi)
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
K =& - r_\phi n_d\big(cos(\alpha)sin(\theta)cos(\beta-\phi) + sin(\alpha)cos(\theta)\big) - rn_d cos(\alpha)sin(\theta)sin(\beta-\phi) \\
&+ r_\phi\left[\frac{sin(2\alpha)sin^2(\theta)cos^2(\beta-\phi)}{4} + \frac{sin(2\theta)cos(\beta-\phi)}{2} + \frac{sin(2\alpha)cos^2(\theta)}{2}\right] \\
&+ r\left[\frac{sin(2\alpha)sin^2(\theta)cos2(\beta-\phi)}{4} + \frac{sin(2\theta)cos^2(\alpha)sin(\beta-\phi)}{2}\right]
\end{aligned} \tag{35}
$$

$$
\begin{aligned}
L =& - 2r_\theta r_\phi\left[\frac{cos(2\alpha) - cos(2\theta)}{2}cos(\beta-\phi) + \frac{sin(2\alpha)sin2(\theta)sin^2(\beta-\phi)}{4}\right] \\
&+ 2r_\theta r\left[\frac{sin(2\alpha)sin2(\theta)sin2(\beta-\phi)}{8} - sin^2(\theta)cos^2(\alpha)sin(\beta-\phi)\right] \\
&- r^2\left[\frac{sin(2\alpha)sin^2(\theta)sin2(\beta-\phi)}{4} + \frac{sin(2\theta)cos^2(\alpha)sin(\beta-\phi)}{2}\right] \\
&- r_\phi r\left[\frac{sin(2\alpha)sin^2(\theta)cos^2(\beta-\phi)}{2} + \frac{sin(2\theta)cos(\beta-\phi)}{2} + \frac{sin(2\alpha)cos^2(\theta)}{2}\right]
\end{aligned} \tag{36}
$$

$$
\begin{aligned}
M =& - n_d r_\theta \big(cos(\alpha)sin(\theta)cos(\beta-\phi) + sin(\alpha)cos(\theta)\big) \\
& - n_d r \big(cos(\alpha)cos(\theta)cos(\beta-\phi) + sin(\alpha)sin(\theta)\big) \\
& + r_\theta \Bigg[\frac{sin(2\alpha)sin^2(\theta)cos^2(\beta-\phi)}{2} + \frac{sin(2\alpha)cos^2(\theta)}{2} + \frac{sin(2\theta)cos(\beta-\phi)}{2}\Bigg] \\
& + r \Bigg[\frac{sin(2\alpha)sin(2\theta)}{4}\big(\frac{3+cos2(\beta-\phi)}{2}\big) + \frac{1+cos(2\alpha)cos(2\theta)}{2}cos(\beta-\phi)\Bigg]
\end{aligned} \tag{37}
$$

$$
\begin{aligned}
N =& r_\theta r \Bigg[\frac{sin(2\alpha)sin(2\theta)sin2(\beta-\phi)}{4} + sin^2(\alpha)sin(\beta-\phi)\Bigg] \\
& + r^2 \Bigg[\frac{sin(2\alpha)cos^2(\theta)sin2(\beta-\phi)}{4} + \frac{sin^2(\alpha)sin(2\theta)}{2}sin(\beta-\phi)\Bigg] \\
& + r_\theta^2 \Bigg[\frac{sin(2\alpha)cos^2(\theta)sin2(\beta-\phi)}{4} + \frac{sin^2(\alpha)sin(2\theta)}{2}sin(\beta-\phi)\Bigg] \\
& + r_\phi r_\theta \Bigg[\frac{sin(2\alpha)sin(2\theta)cos^2(\beta-\phi)}{4} - \frac{sin(2\alpha)sin(2\theta)}{4} + \big(cos^2(\theta) - cos^2(\alpha)\big)cos(\beta-\phi)\Bigg] \\
& + r_\phi r \Bigg[\frac{sin(2\alpha)cos^2(\theta)cos^2(\beta-\phi)}{2} - \frac{sin(2\alpha)sin^2(\theta)}{2} - \frac{cos(2\alpha)sin(2\theta)cos(\beta-\phi)}{2}\Bigg]
\end{aligned} \tag{38}
$$

$$
\begin{aligned}
P =& - n_d r_\phi \big(cos(\alpha)sin(\theta)cos(\beta-\phi) + sin(\alpha)cos(\theta)\big) - n_d r cos(\alpha)sin(\theta)sin(\beta-\phi) \\
& + r_\phi \big[\frac{sin(2\alpha)sin^2(\theta)cos^2(\beta-\phi)}{2} + \frac{sin(2\theta)cos(\beta-\phi)}{2} + \frac{sin(2\alpha)cos^2(\theta)}{2}\big) \\
& + r \big[\frac{sin(2\alpha)sin^2(\theta)sin2(\beta-\phi)}{4} + \frac{cos^2(\alpha)sin(2\theta)sin(\beta-\phi)}{2}\big)
\end{aligned} \tag{39}
$$

$$
\begin{aligned}
Q =& r_\phi r \Bigg[\frac{sin(2\alpha)sin(2\theta)sin2(\beta-\phi)}{4} + \big(cos^2(\theta) - cos^2(\alpha)\big)sin(\beta-\phi)\Bigg] \\
& + r^2 \frac{sin(2\alpha)sin(2\theta)sin^2(\beta-\phi)}{4} + r_\theta r \frac{sin(2\alpha)sin^2(\theta)sin^2(\beta-\phi)}{2} \\
& + r_\phi^2 \Bigg[\frac{sin(2\alpha)sin(2\theta)sin^2(\beta-\phi)}{4} + \big(cos^2(\theta) - cos^2(\alpha)\big)cos(\beta-\phi)\Bigg] \\
& + r_\theta r_\phi \Bigg[\frac{sin(2\alpha)sin^2(\theta)sin2(\beta-\phi)}{4} + \frac{sin^2(\alpha)sin(2\theta)sin(\beta-\phi)}{2}\Bigg]
\end{aligned} \tag{40}
$$

$$
\begin{aligned}
R =& - n_d r_\theta \big(cos(\alpha) sin(\theta) cos(\beta - \phi) + sin(\alpha) cos(\theta) \big) \\
& - n_d r \big(cos(\alpha) cos(\theta) cos(\beta - \phi) + sin(\alpha) sin(\theta) \big) \\
& + r_\theta \left[\frac{sin(2\alpha) sin^2(\theta) cos^2(\beta - \phi)}{2} + \frac{sin(2\theta) cos(\beta - \phi)}{2} + \frac{sin(2\alpha) cos^2(\theta)}{2} \right] \\
& + r \Bigg(\frac{3 + cos2(\beta - \phi)}{2} sin(2\alpha) sin(2\theta) + cos^2(\alpha) cos^2(\theta) cos(\beta - \phi) \\
& + sin^2(\alpha) sin^2(\theta) cos(\beta - \phi) \Bigg)
\end{aligned}
\tag{41}
$$

$$
\begin{aligned}
S =& 2 r_\phi r_\theta \left[\frac{sin(2\alpha) sin^2(\theta) sin2(\beta - \phi)}{4} + \frac{sin^2(\alpha) sin(2\theta) sin(\beta - \phi)}{2} \right] \\
& + 2 r_\phi r \left[\frac{sin(2\alpha) sin(2\theta) sin2(\beta - \phi)}{8} + sin^2(\alpha) sin^2(\theta) sin(\beta - \phi) \right] \\
& - r_\theta r \left[\frac{sin(2\alpha) sin^2(\theta) cos^2(\beta - \phi)}{2} + \frac{sin^2(\alpha) sin(2\theta) cos(\beta - \phi)}{2} \right] \\
& - r^2 \left[\frac{sin(2\alpha) sin(2\theta) cos^2(\beta - \phi)}{4} + sin^2(\alpha) sin(2\theta) cos(\beta - \phi) \right]
\end{aligned}
\tag{42}
$$

Hence, when we simplify the equation (15) using equations (21) to (24) we get following expression.

$$
\begin{aligned}
& \left[\frac{\partial \alpha(\theta, \phi)}{\partial \theta} \frac{\partial \beta(\theta, \phi)}{\partial \phi} - \frac{\partial \alpha(\theta, \phi)}{\partial \phi} \frac{\partial \beta(\theta, \phi)}{\partial \theta} \right] = \\
& \left(\frac{GK - AP}{(E - F)^2} \right) r_{\theta\theta} r_{\theta\phi} + \left(\frac{CP - GM}{(E - F)^2} \right) r_{\theta\theta}^2 + \left(\frac{AP - IK}{(E - F)^2} \right) r_{\phi\phi} r_{\theta\theta} \\
& + \left(\frac{MI - CR}{(E - F)^2} \right) r_{\phi\phi} r_{\theta\phi} + \left(\frac{I(N - L) - R(B - D)}{(E - F)^2} \right) r_{\phi\phi} + \left(\frac{A(S - Q) + K(H - J)}{(E - F)^2} \right) r_{\theta\theta} \\
& + \left(\frac{P(D - B) + C(Q - S) + M(J - H) + G(L - N)}{(E - F)^2} \right) r_{\theta\phi} \\
& + \frac{(B - D)(S - Q) - (H - J)(N - L)}{(E - F)^2}
\end{aligned}
\tag{43}
$$

In the above expression we get two extra terms which are not the part of the comparing equation which are $r_{\theta\theta} r_{\theta\phi}$ and $r_{\phi\phi} r_{\theta\phi}$. For now we will neglect these terms as lens design is allowed for little tolerance in design. So by replacing the partial derivatives equation in (15) by equation (43) we get following expressions for $a, b, candH$. We will divide by coefficient of $r_{\theta\theta}r_{\phi\phi}$ to make it equal to 1. While doing this we assume that coefficient of $r_{\theta\phi}^2$ approximates to 1.

$$a = -\frac{A(S-Q) + K(H-J)}{AR - IK} \tag{44}$$

$$b = -\frac{P(D-B) + C(Q-S) + M(J-H) + G(L-N)}{AR - IK} \tag{45}$$

$$c = -\frac{I(N-L) + R(B-D)}{AR - IK} \tag{46}$$

$$\begin{aligned} H = &- \frac{(B-D)(S-Q) - (H-J)(N-L)}{AR - IK} \\ &- \frac{KT(\theta,\phi)g(\theta,\phi)sin(\theta)}{h(\alpha,\beta)sin(\alpha)}\left[\frac{(E-F)^2}{AR - IK}\right] \end{aligned} \tag{47}$$

Linearization

Now that we have got the second order partial differential equation, we can use iterative method to linearize it and obtain the equation between r^{k+1} and r^k as follows.

$$\begin{aligned} E(r^k) &+ \frac{\partial E}{\partial r_{\theta\theta}}.(r_{\theta\theta}^{k+1} - r_{\theta\phi}^k) + \frac{\partial E}{\partial r_{\theta\phi}}.(r_{\theta\phi}^{k+1} - r_{\theta\phi}^k) + \frac{\partial E}{\partial r_{\phi\phi}}.(r_{\phi\phi}^{k+1} - r_{\phi\phi}^k) \\ &+ \frac{\partial E}{\partial r_\theta}.(r_\theta^{k+1} - r_\theta^k) + \frac{\partial E}{\partial r_\phi}.(r_\phi^{k+1} - r_\phi^k) + \frac{\partial E}{\partial r}.(r^{k+1} - r^k) = 0 \end{aligned} \tag{48}$$

combining both (48) and (15) we get following equation

$$\alpha_1 r_{\theta\theta}^{k+1} + \alpha_2 r_{\theta\phi}^{k+1} + \alpha_3 r_{\phi\phi}^{k+1} + \alpha_4 r_\theta^{k+1} + \alpha_5 r_\phi^{k+1} + \alpha_6 r^{k+1} + \alpha_7 = 0 \tag{49}$$

where α_1 to α_7 are the functions of $a, b, c, H, r_{\theta\theta}, r_{\theta\phi}, r_{\phi\phi}, r_\theta, r_\phi$ and r as given in equation (50) to (56). Here we compute the profile r^{k+1} from the lens profile in previous iteration i.e. r^k.

$$\alpha_1 = r_{\phi\phi}^k - a^k \tag{50}$$

$$\alpha_2 = -2r^k_{\theta\phi} - b^k \tag{51}$$

$$\alpha_3 = r^k_{\theta\theta} - c^k \tag{52}$$

$$\alpha_4 = -\left(\frac{\partial a^k}{\partial r_\theta}\right) r^k_{\theta\theta} - \left(\frac{\partial b^k}{\partial r_\theta}\right) r^k_{\theta\phi} - \left(\frac{\partial c^k}{\partial r_\theta}\right) r^k_{\phi\phi} - \left(\frac{\partial H^k}{\partial r_\theta}\right) \tag{53}$$

$$\alpha_5 = -\left(\frac{\partial a^k}{\partial r_\phi}\right) r^k_{\theta\theta} - \left(\frac{\partial b^k}{\partial r_\phi}\right) r^k_{\theta\phi} - \left(\frac{\partial c^k}{\partial r_\phi}\right) r^k_{\phi\phi} - \left(\frac{\partial H^k}{\partial r_\phi}\right) \tag{54}$$

$$\alpha_6 = -\left(\frac{\partial a^k}{\partial r}\right) r^k_{\theta\theta} - \left(\frac{\partial b^k}{\partial r}\right) r^k_{\theta\phi} - \left(\frac{\partial c^k}{\partial r}\right) r^k_{\phi\phi} - \left(\frac{\partial H^k}{\partial r}\right) \tag{55}$$

$$\alpha_7 = r^k_{\theta\theta} r^k_{\phi\phi} - (r^k_{\theta\phi})^2 - a^k r^k_{\theta\theta} - b^k r^k_{\theta\phi} - c^k r^k_{\phi\phi} - \alpha_1 r^k_{\theta\theta} - \alpha_2 r^k_{\theta\phi} - \alpha_3 r^k_{\phi\phi} - \alpha_4 r^k_\theta - \alpha_5 r^k_\phi - \alpha_6 r^k \tag{56}$$

Discretization

To get the new profile r^{k+1} from the lens profile in previous iteration i.e. r^k, we discretize the 3-D lens into (N+1)(2M+1) points and the angles θ and ϕ are also discretized as in equation (57) and (58). And then we take $i\Delta\theta$ and $j\Delta\phi$ where $i \in [0, 1, ..., N]$ and $j \in [-M, ..., -1, 0, 1, ..., M]$.

$$\Delta\theta = \frac{\pi}{2N} \tag{57}$$

$$\Delta\phi = \frac{\pi}{M} \tag{58}$$

and partial derivatives of r are also discretized as in equation (60) to (63)

$$r_\theta = \frac{r_{i+1,j} - r_{i-1,j}}{2\Delta\theta} \tag{59}$$

$$r_\phi = \frac{r_{i,j+1} - r_{i,j-1}}{2\Delta\phi} \tag{60}$$

$$r_{\theta\theta} = \frac{r_{i+1,j} - 2r_{i,j} + r_{i-1,j}}{\Delta\theta^2} \tag{61}$$

$$r_{\phi\phi} = \frac{r_{i,j+1} - 2r_{i,j} + r_{i,j-1}}{\Delta\phi^2} \tag{62}$$

$$r_{\theta\phi} = \frac{r_{i+1,j+1} - r_{i+1,j-1} - r_{i-1,j+1} + r_{i-1,j-1}}{4\Delta\theta\phi} \tag{63}$$

Boundary conditions must satisfy the continuity of the lens.

$$\begin{aligned} r_{i,M} &= r_{i,-M} \\ r_{i,M+1} &= r_{i,-M+1} \\ r_{i,M-1} &= r_{i,-M-1} \end{aligned} \tag{64}$$

As the central thickness of the lens is fixed to be e, we have one more condition.

$$r_{0,j} = e \tag{65}$$

Also maximum refraction angle is $\alpha(\theta = \frac{\pi}{2},) = \alpha_m(\phi)$. When this condition is imposed and is replaced in equation (9) and (10) we get more boundary condition as in equation (66)

$$\left(\frac{\alpha_4^{'}}{\Delta\theta} + \alpha_6^{'}\right) r_{N,j}^{k+1} + \frac{\alpha_5^{'}}{2\Delta\phi}\left(r_{N,j+1}^{k+1} - r_{N,j-1}^{k+1}\right) + \alpha_7^{'} = 0 \tag{66}$$

where as expressions for $\alpha_4^{'}$ to $\alpha_7^{'}$ are given in paper (1)

0.6 Algorithm

To solve M.A. type equation we use iterative algorithm proposed in paper (1). This algorithm calculates the lens profile at each iteration and keeps on counting until the error between current profile r^{k+1} and previous lens profile r^k is under the threshold value. The plane of incidence needs to be characterized for each (θ, ϕ), hence we use equation (5) to calculate unit transverse vector

$$\vec{t}(\theta, \phi) = \frac{-\vec{n}(\theta, \phi)\Lambda\big(\vec{n}(\theta, \phi)\Lambda\vec{k_i}(\theta, \phi)\big)}{\|\vec{n}(\theta, \phi)\Lambda\big(\vec{n}(\theta, \phi)\Lambda\vec{k_i}(\theta, \phi)\big)\|} \tag{67}$$

Hence $\vec{n}(\theta, \phi)$ and $\vec{t}(\theta, \phi)$ denote the orthonormal basis for particular (θ, ϕ). Further incident and transmitted angles can be found out as

$$\begin{aligned} \theta_i(\theta, \phi) &= \arctan\left(\frac{\vec{k_i}(\theta, \phi).\vec{t}(\theta, \phi)}{\vec{k_i}(\theta, \phi).\vec{n}(\theta, \phi)}\right) \\ \theta_t(\theta, \phi) &= \arcsin n_d sin\theta_i(\theta, \phi) \end{aligned} \tag{68}$$

The normalized transmitted wave vector,

$$\vec{k_t}(\theta, \phi) = cos\theta_t(\theta, \phi)\vec{n}(\theta, \phi) + sin\theta_t(\theta, \phi)\vec{t}(\theta, \phi) \tag{69}$$

Power transmission coefficient in k^{th} iteration is defined as follows

$$T^k(\theta, \phi) = 1 - \frac{r_{//}^2.|E_{i_{//}}|^2 + r_{\perp}^2.|E_{i_{\perp}}|^2}{|E_{i_{//}}|^2 + |E_{i_{\perp}}|^2} \tag{70}$$

$r_{//}$ and $r_{\perp}$ are fresnel reflection coefficients. The components of is computed as follows.

$$E_{i_{\perp}} = \vec{E_i}(\theta, \phi).\frac{\vec{n}\Lambda\vec{k_i}}{\|\vec{n}\Lambda\vec{k_i}\|} \tag{71}$$

$$E_{i_{//}} = \vec{E_i}(\theta, \phi) - E_{i_{\perp}}.\frac{\vec{n}\Lambda\vec{k_i}}{\|\vec{n}\Lambda\vec{k_i}\|} \tag{72}$$

whereas, $\vec{E_i}(\theta, \phi)$ can be computed from far field pattern, $\vec{E_f}(\theta, \phi)$ as follows

$$\vec{E_i}(\theta, \phi) = \vec{E_f}(\theta, \phi).\frac{e^{-jk_d r^k(\theta, \phi)}}{r^k(\theta, \phi)} \tag{73}$$

angles of refraction at k^{th} iteration is computed as follows

$$\alpha^k = \arccos\left(\vec{k_t}.\hat{e_z}\right) \tag{74}$$

$$\beta^k = \arctan\left(\frac{\vec{k_t}.\hat{e_y}}{\vec{k_t}.\hat{e_x}}\right) \tag{75}$$

Hence now that we know all the required terms we can proceed to the algorithm which is described as follows

Step 1: External Data

To feed the initial data of $n_d, \lambda_0, E_f(\theta,\phi), g(\theta,\phi), \vec{h}(\theta,\phi), r_init(\theta,\phi)$ and the boundary conditions, e and $\alpha_m(\phi)$

Step 2: Initialization of variables

initialize all required variables. Start the counter, $k = 0$ and initial lens profile $r^0(\theta,\phi) = r_{init}(\theta,\phi)$

Step 3: Plane of Incidence Characterization

Calculation of $\vec{n}, \vec{t}, \theta_i, \theta_t$ and $\vec{k_t}$. Now in this step we have entered the while loop with condition based on the error between lens profiles at successive iterations.

Step 4: calculation of transmission coefficient $T^k(\theta,\phi)$

Calculate incident field from far-field Electric field pattern $\vec{E_f}(\theta,\phi)$ first and then parallel perpendicular components of $E_i(\theta,\phi)$

Step 5: Calculation of α^k and β^k

Calculate the angle of refraction at each iteration

Step 6: Calculation of parallel derivatives of r^k

This involves computing $r_\theta, r_\phi, r_{\theta\theta}, r_{\phi\phi}$ and $r_{\theta\phi}$ in discretized form

Step 7: Calculation of α_i and $\alpha_i^{'}$

This involves computing the coefficients from A to S and then calculating the a, b, c and H.Finally we get α_i and $\alpha_i^{'}$

Step 8: Final Solution to the linear system of equations for r^{k+1}

replace k with $k+1$ and check if error defined as $\epsilon^k = max|r^{k+1} - r^k max|$ is less than desired threshold. If it is less then end the loop and we got the final lens profile i.e. Desired profile $= r^k(\theta, \phi)$. But if the error is more than threshold, loop goes back to step 3 and next iteration starts.

0.7 Conclusions

Rapid prototyping and the trend to move into mm-waves and sub-millimeter waves are bringing the interest on dielectric lens especially integrated lens antennas. Hence the requirement as well as the use of such lenses into our wireless systems are going to increase more in future. The algorithm used in this project is independent of external desired parameters. Hence for any required field, we can use this method and obtain the 3-D lens profile. Though the algorithm neglects the small error in the lens profile while calculating it, still meets our expectation. Manufacturing and implementing these lenses will extremely help 5G wireless system to achieve its capacity. The matlab code designed in such a way as to take the parameters such as far-field pattern $(\vec{E_f}(\theta, \phi))$, the incident radiation from primary source $(\vec{g}(\theta, \phi))$ and desired field pattern outside the lens surface $(\vec{h}(\alpha, \beta))$ as an external parameter. Hence the same code can be used to simulate the 3-D di-electric lens profile for different requirements.

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