A STUDY OF WOLA-OFDM : AN OVERVIEW AND IMPLEMENTATION ASPECTS

A Project Report

submitted by

M GOPI KRISHNA SINGH

EE14B035

in partial fulfilment of requirements
for the award of the degree of

BACHELOR OF TECHNOLOGY

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY MADRAS

JUNE 2018

THESIS CERTIFICATE

This is to certify that the thesis titled **A Study of WOLA-OFDM : An Overview and Implementation Aspects**, submitted by **M Gopi Krishna Singh**, to the Indian Institute of Technology, Madras, for the award of the degree of **Bachelor of Technology**, is a bonafide record of the research work done by him under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

David Koilpillai
Project Guide
Professor
Dept. of Electrical Engineering
IIT-Madras, 600 036

Place: Chennai

Date: 6th June 2018

ACKNOWLEDGEMENTS

First and foremost, I would like to thank my guide, Prof.David Koilpillai, for giving me an opportunity to work under him. I would like to thank him for constantly guiding me thoughtfully and efficiently throughout this project. He allowed me to work at my own pace, devoted considerable portion of his time providing me with very useful insights and directions whenever necessary.

I am also grateful to all the professors at IITM who offered me invaluable knowledge and advice during my excellent study journey in IITM.

In addition, I would like to take this opportunity to thank my parents and all my friends who have been a great source of motivation and encouragement throughout.

ABSTRACT

KEYWORDS: OFDM ; WOLA-OFDM ; Spectrum ; Channel ; Equalization ; BER ; CFO ; ICI ; PAPR

Conventional OFDM has many advantages for LTE system but it suffers from serious limitations of high out-of-band emission, high peak-to-average-power ratio and strict synchronization requirements. In this paper, we study and implement a new waveform, Weighted Overlap and Add based OFDM (WOLA-OFDM) and investigate the spectrum, PAPR, BER and effects of CFO on the system. Indeed, its performance is studied and compared to OFDM in all these fronts. With WOLA-OFDM, the out-of-band emission is suppressed effectively and the tolerance to frequency offset is significantly improved, making it a promising 5G waveform candidate.

TABLE OF CONTENTS

ABBREVIATIONS

OFDM	Orthogonal Frequency Division Multiplexing
WOLA	Weighted Overlap and Add
OOB	Out Of Band
BER	Bit Error Rate
CFO	Carrier Frequency Offset
ICI	Inter Carrier Interference
ISI	Inter Symbol Interference
PSD	Power Spectral Density
RRC	Root Raised Cosine
AWGN	Additive White Gaussian Noise
ZF	Zero Forcing
MMSE	Minimum Mean Square Error
SNR	Signal to Noise Ratio
PAPR	Peak to Average Power Ratio
BPSK	Binary Phase Shift Keying
QPSK	Quadrature Phase Shift Keying
QAM	Quadrature Amplitude Modulation
(I)DFT	(Inverse) Discrete Fourier Transform
(I)FFT	(Inverse) Fast Fourier Transform
LTE	Long Term Evolution
CP	Cyclic Prefix
S/P	Serial to Parallel
P/S	Parallel to Serial
5G	5th Generation Wireless Systems
Tx	Transmitter
Rx	Receiver

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

1.1 Multi-Carrier Transmission

During the last two decades, the demand for wireless communication services has grown rapidly and the trend doesn't seem to cease in the near future. The multicarrier transmission techniques made it possible to handle the demand for higher data rates and spectrum efficient transmission. The first use of the multicarrier communication system dates back to late 1950s and early 1960s in the high frequency radio links‘of military. But these systems required analog oscillators and filters of wider bandwidth and sharp cut-offs which made it complex for commercial usage. Orthogonal Frequency Division Multiplexing (OFDM), a special form of spectral efficient multicarrier transmission scheme, with densely spaced subcarriers was patented in 1970. However, the consideration and popularity of OFDM increased only after the use of Inverse Discrete Fourier Transform (IDFT) to generate the orthogonal subcarriers at the transmitter. The use of IDFT/DFT totally eliminated the need of oscillators at the transmitter and the receiver. Further studies and developments were carried out and the use of fast DFT alogorithm (FFT) significantly reduced the implementation complexity of OFDM. Many further advancements were made over the years and OFDM now stands as the leading candidate for many wireless communication systems including the fourth generation (4G) mobile communication system called LTE (long term evolution).

1.2 Next Generation

5th Generation Wireless Systems (5G) is now extensively discussed in the wireless industry. At present, there is a dramatic change taking place in the mobile communication landscape with the applications requiring further increase of bandwidth. Also for the next generation communication system, machine-to-machine type communication with huge number of inter-connected devices are expected to be deployed. The 'Internet of

Things' promises to be the next big thing in the technological advancements and its applications include a wide range of fields like vehicle communications, security monitoring, remote medical services etc. However, machine type communications and Internet of Things evoke new requirements for the spectral characteristics, system delay, power consumption, and synchronization. So, the three trends Enhanced Mobile Broadband, Massive Machine Type Communication, Ultra-reliable and low latency communication have been identified as the requirements to be supported by the 5th Generation of mobile Communication [Rohde (2013)]. Though conventional OFDM has many advantages for LTE system, it suffers from serious limitations of high out-of-band emission and strict synchronization requirements. To satisfy the diverse requirements, a lot of research and work is being done including the revisit to waveforms that are the basis for current LTE and LTE-Advanced networks.

In this paper, WOLA-OFDM waveform, a promising 5G waveform candidate, is investigated and compared to the conventional OFDM in various fronts. The remaining chapters of the paper are organised as follows. We start with OFDM basics and discuss its advantages and disadvantages in Chapter-2. Next, Chapter-3 introduces the background of WOLA-OFDM and its uses. After that several implementation aspects and experimental results are discussed in Chapter-4.

CHAPTER 2

OFDM

2.1 Introduction

OFDM (Orthogonal Frequency Division Multiplexing), which is currently used in the LTE/LTE-Advanced is a multicarrier transmission scheme of data. The basic idea is to divide the available spectrum into several parallel sub channels which are referred as subcarriers. These narrowband subcarriers now experience almost flat fading which makes equalization easier at the receiver side. The spacing between the subcarriers is selected in a way that makes an efficient use of the available spectrum and maintaining orthogonality between them at the same time. A cyclic prefix (CP) is further introduced to increase the robustness of the signal against Inter Symbol Interference (ISI) even in a time dispersive channel. The cyclic prefix is a copy of the tail part of an OFDM symbol which is appended to its beginning, see figure 2.1.

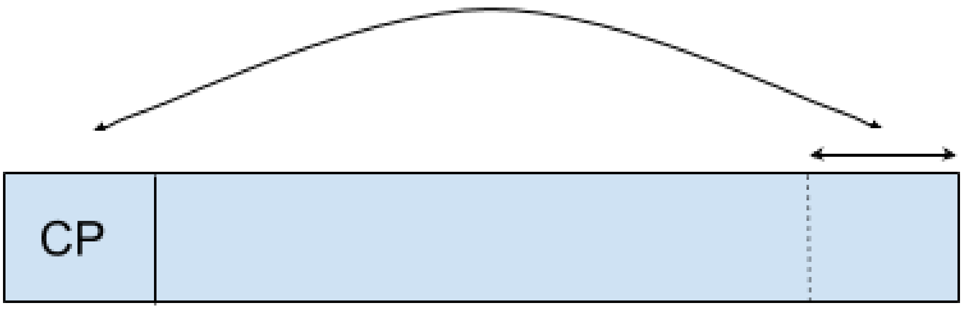

Figure 2.1: Forming a CP in OFDM symbol

2.2 Block Diagram

The figure 2.2 shows the basic Tx and Rx block diagrams of OFDM system. Firstly in the transmitter, the data bits are modulated into complex symbols such as QPSK,

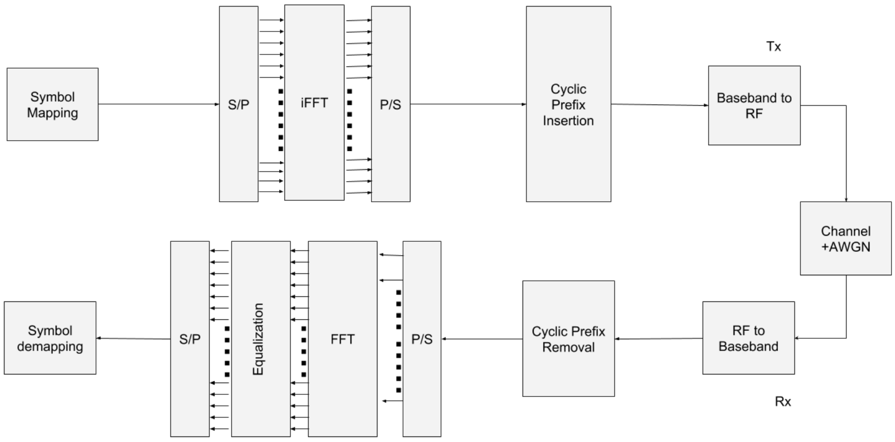

Figure 2.2: Block Diagram of OFDM system

16QAM, 64QAM etc. depending on the requirement. Next is the serial to parallel conversion of the data stream which is done by the S/P block. These symbols are now mapped to each subcarrier by the IFFT block which converts the signal from frequency domain to time domain. After the IFFT operation, the P/S block converts the parallel stream into series stream which is followed by the CP addition.

The receiver structure of the OFDM is basically back pedalling of the transmitter. It starts with CP removal followed by S/P block which feeds the data to the FFT block. Then the receiver equalizes the signal and the parallel to serial conversion of data is done in the P/S block. Finally the complex symbols are demodulated to get back the original data bits sent.

2.3 Advantages and Disadvantages of OFDM

2.3.1 Advantages

- Makes efficient use of the spectrum by allowing overlap between the subcarriers.
- The division of the channel into multiple narrowband flat fading subchannels makes OFDM more resistant to frequency selective fading.
- Eliminates ISI through the use of a cyclic prefix.
- Channel equalization becomes easier which is typically a single tap equalizer at the receiver side.

- OFDM is computationally efficient by using FFT techniques to implement the modulation and demodulation functions.

2.3.2 Disadvantages

- OFDM transmission spectrum experiences significant Out-of-Band (OOB) emission due to the poor localized frequency response of the rectangular transmit filter.
- Very sensitive to carrier frequency offsets and timing offsets.
- Cyclic prefix overhead due to double transmission of same content in a symbol.
- High PAPR due to the superposition of all individual subcarriers.

2.4 Variants of OFDM

In this section we will look at some of the variants of OFDM from the literature which have basic format of OFDM with some added attributes.

2.4.1 Coded OFDM

The uses and implementation of Coded OFDM (COFDM) is studied in Zou and Wu (1995). The basic idea of COFDM is to improve the performance of OFDM systems in a frequency selective channel by incorporating error control coding into the signal. An important step in COOFDM is to interleave and code the bits prior to IFFT. This would help in not getting a contiguous stream of error bits which helps the decoder to correct the errors. Trellis coded modulation (TCM) combined with frequency and time interleaving is considered the most effective means of channel coding for a frequency selective fading channel.

2.4.2 Wideband OFDM

From the study of Wideband OFDM (WOFDM) in EION (2007), the spacing between the channels in WOFDM system is made large enough so that any frequency errors between the transmitter and receiver have no effect on the performance of the system. It is particularly applicable to Wi-Fi systems. WOFDM allows several independent channels to operate within the same band. This creates an overlay of low-power, multipoint radio networks and point-to-point backbone systems.

2.4.3 Single Carrier - FDMA

The high PAPR in OFDM made a strong point to combine the advantage of a multicarrier transmission principle such as OFDM with the advantage of a single carrier transmission technique which provides much lower PAPR. The solution is to apply an N-point DFT, prior to the subcarrier carrier mapping. An N-point DFT is applied to the modulated symbols which are thus transformed into the frequency domain. Then an M-point IDFT is performed as in OFDM, where N<M, followed by P/S conversion and insertion of CP. This method is understood as 'DFT-spread-OFDM' as the DFT in the beginning spreads the modulated symbols over the subcarrier and thus every subcarrier carries a portion of each modulated symbol (Rohde (2013)).

There are many more variants of OFDM like Flash OFDM (Nikhil Ranjan (2012)), Vector OFDM (Ayanoglu *et al.* (2001)) etc. and upcoming 5G waveforms based on OFDM like Filter-Bank Multi-Carrier (FBMC), Universal Filter Multi-Carrier (UFMC) and Generalized Frequency Division Multiplex (GFDM) (Rohde (2013)). But in this paper, we will study, implement and analyze a potential 5G waveform called Weighted-Overlap and Add-OFDM (WOLA-OFDM).

CHAPTER 3

WOLA - OFDM

3.1 Literature Survey

In [7], Qualcomm (2016), discusses several waveform candidate proposals from literature to be further evaluated for various service/deployment scenarios for the new Radio Access Technologies. They analysed the spectrum and PAPR of various single carrier waveforms like constant envelope waveform, SC-QAM, SC-FDE, SC-DFT spread OFDM, Zero-tail DFT-OFDM. They have further studied the spectrum and PAPR details of multi-carrier waveforms like CP-OFDM, WOLA-OFDM, UFMC, F-OFDM, FBMC. They addressed the need to reduce OOB emission of OFDM by proposing transmit filters like RRC pulse in order to achieve that.

In [11], Zayani *et al.* (2016), investigate the performance, in relaxed synchronization scenario, of WOLA-OFDM. A comparison is done based on the aspects of OOB radiation, robustness to asynchronism and channel delay spread with CP-OFDM and UFMC. The results prove WOLA-OFDM to be a potential multi-carrier waveform for future wireless standards due to the simple implementation using time domain windowing, efficient OOB suppressions, better robustness to asynchronism between users and compatibility with MIMO.

In [5], Medjahdi *et al.* (2017), investigate the effectiveness of WOLA processing on windowed multicarrier waveforms. They have highlighted the advantages of block-transform schemes and have pointed out the need of windowing to reduce the important OOB radiation induced by inter-block discontinuities. They have introduced the application of WOLA processing to CP-OFDM and CP-COQAM schemes and proved the increased robustness to timing asynchronism of both schemes than before.

In [4], Gerzaguet *et al.* (2017), compares the performances of promising 5G waveforms namely WOLA-OFDM and BF-OFDM in terms of spectrum confinement and robustness in a typical multi-user asynchronous scenario. Also a comparison of transmitter and receiver complexity in terms of real multiplications per unit of time have been performed, showing the potential benefits of WOLA-OFDM and BF-OFDM.

3.2 WOLA-OFDM

WOLA (Weighted Overlap and Add) is a variant of OFDM and a potential 5G waveform which was initially introduced in [7] by Qualcomm Incorporated and has been studied in [4] and [11] with OFDM in asynchronous 5G scenario. In WOLA, the basic aim is to achieve a better containment of frequency response compared to OFDM. Hence, the idea is to replace the rectangular prototype filter with a pulse containing softer edges at both sides which helps in suppressing the OOB emission and achieving a well localized frequency response. These soft edges at the beginning and ending can be added to the originally transmitted signal with the help of a time domain windowing operation. Of the many possible time domain windowing functions, in this paper we chose to define the edge of the time domain window as Meyer RRC pulse as in Gaspar *et al.* (2015) combining the RRC time domain pulse with the Meyer auxiliary function as given below,

$$F_w[k] = \begin{cases} 1, & |k| \leq (1-\alpha_s)L/2 \\ \sqrt{0.5[1+\cos \pi v(\frac{|k|-(1-\alpha_s)\frac{L}{2}}{\alpha_s L})}, & (1-\alpha_s)L/2 < |k| \leq (1+\alpha_s)L/2 \\ 0, & otherwise \end{cases}$$

where $v(k)$ is the Meyer auxiliary function given by,

$$v(k) = k^4(35 - 84k + 70k^2 - 20k^3)$$

and α_s is the roll-off factor that controls the windowing function and is defined as ratio of window length to symbol length L.

Let us understand the WOLA processing at the transmitter end and the receiver end along with the block diagram of WOLA systems in the coming sections.

3.3 Block Diagram

The figure 3.1 shows the block diagram of WOLA system. It is actually inspired from the conventional OFDM system with a few variations. The CP block is replaced by CP+CS block which is followed by time domain windowing as discussed above.

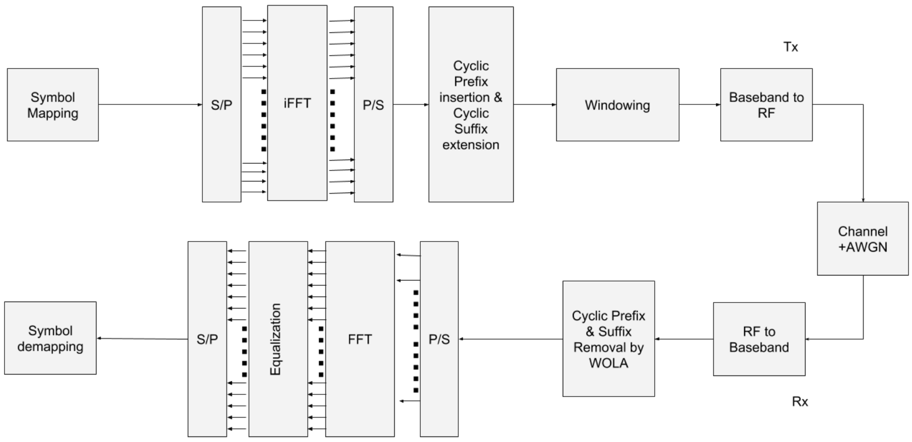

Figure 3.1: Block diagram of a WOLA-OFDM system

3.4 WOLA-OFDM Transmitter

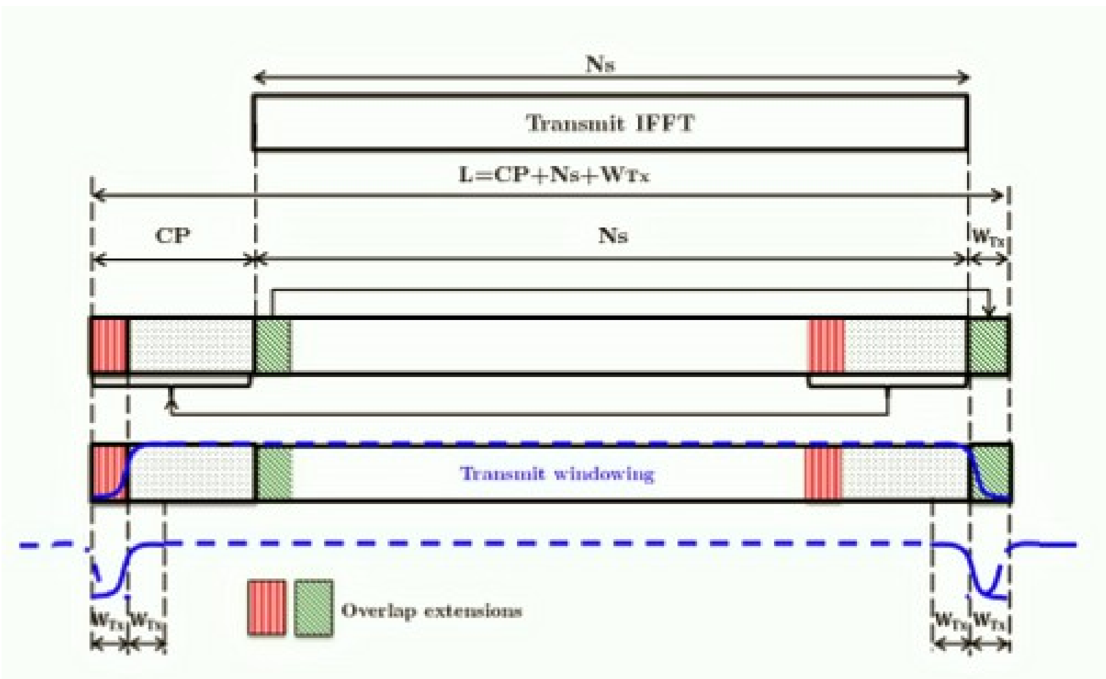

Figure 3.2: WOLA processing : Transmitter side

The figure 3.2 describes the transmitter operations of WOLA after performing IFFT. As is the case in OFDM, CP samples are appended at the beginning of the symbol whose length is Ns. In addition to this, some initial part of the original symbol is appended to its ending. It may be referred to as 'cyclic suffix' and let its length be W_{Tx}. So the total length of the time domain symbol is now,

$$L = Ns + CP + W_{Tx}$$

Now we soften the edges of this block with the help of a time domain windowing function of same length L. As discussed earlier, we use Meyer's RRC pulse as our windowing function and ensure a smooth transition between last sample of current symbol and the first sample of next symbol with point-to-point multiplication of the windowing function and the extended symbol. Also we notice that the adjacent symbols overlap with each other in the edge transition region. So the overhead of WOLA remains same as that of OFDM due to the overlap of the adjacent symbols as shown in the figure 3.2.

3.5 WOLA-OFDM Receiver

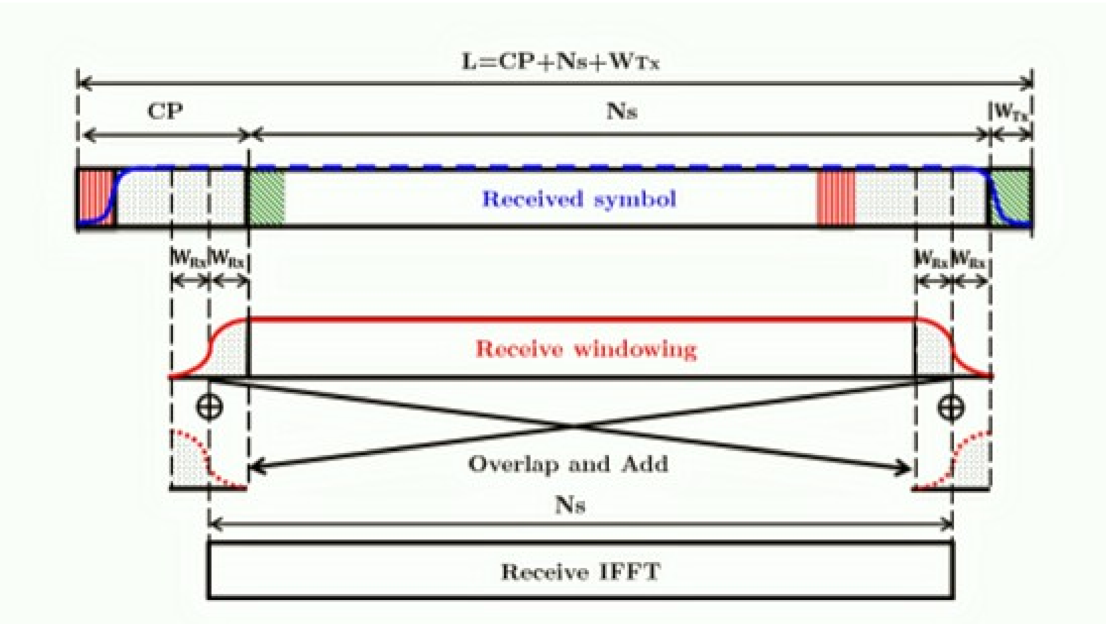

Figure 3.3: WOLA processing : Receiver side

The receiver side of the WOLA system also does a pre-processing of the symbol as shown in the above figure. Here a windowing function of length $Ns + 2 * W_{Tx}$ is chosen and applied on the received symbol. To reduce the effects of windowing on the symbol, we overlap and add the edge transition samples from the right to the left and vice-versa. Finally we remove the extension and form a symbol of length Ns which later undergoes the FFT operation.

3.6 Advantages w.r.t OFDM

- WOLA transmission spectrum has significant suppression of the Out-of-Band (OOB) emission which is a main problem in OFDM.
- Better robustness to asynchronous frequency environment.
- Better robustness to channel delay spread.
- Possibility of higher data rate transmission with trade-off in spectrum.

CHAPTER 4

Implementation Aspects

4.1 Spectrum

In an OFDM signal, the complex values modulating the subcarriers in each symbol period are statistically independent of each other. They are also independent of the values modulating any subcarrier in any previous or subsequent symbol period. Each carrier is modulated using BPSK or QPSK or QAM. Instead of separating each of the 52 carriers with a guard band, OFDM overlaps them. As a result, the power spectrum of the overall signal can be found by summing the power spectra of all individual subcarriers for any symbol period (van Waterschoot *et al.* (2010)). Power Spectral Density function (PSD) shows the strength of the variations (energy) as a function of frequency. It shows at what frequencies variations are strong and at what frequencies variations are weak. The unit of power spectral density is energy per frequency and energy can be obtained within a specific frequency range by integrating Power spectral density within that frequency range. The power spectral density is averaged across 20 MHz bandwidth as specified by WLAN. Power spectral density also depends on levels of modulation i.e, it increases with increase in levels of modulation.

In this paper, we will implement and analyze the performance of OFDM and WOLA systems loosely based on 802.11a specifications as shown in the table 4.1.

The figures 4.1 and 4.2 shows the spectrum of an OFDM and WOLA-OFDM transmission based on specifications given in table 4.1.

The PSD of the OFDM subcarrier is also affected by the symbol shaping function and the symbol rate. Hence, WOLA-OFDM replaces the rectangular prototype filter with Meyer RRC pulse, containing softer edges at both sides which helps in suppressing the OOB emission. From the two figures 4.1 and 4.2, we can clearly see that OOB suppression in WOLA-OFDM is substantially better than OFDM.

Table 4.1: Specifications for the implementation.

Parameter	Value
Number of subcarriers, N	64
Number of used subcarriers, N_d	52
Sampling frequency	20 MHz
Subcarrier spacing	312.5 KHz
Used subcarrier index	[-26 to -1, +1 to +26]
Cylclic prefix duration, T_{cp}	0.8 us
Cylclic suffix duration, T_{cs}	0.2 us
Data symbol duration, T_d	3.2 us

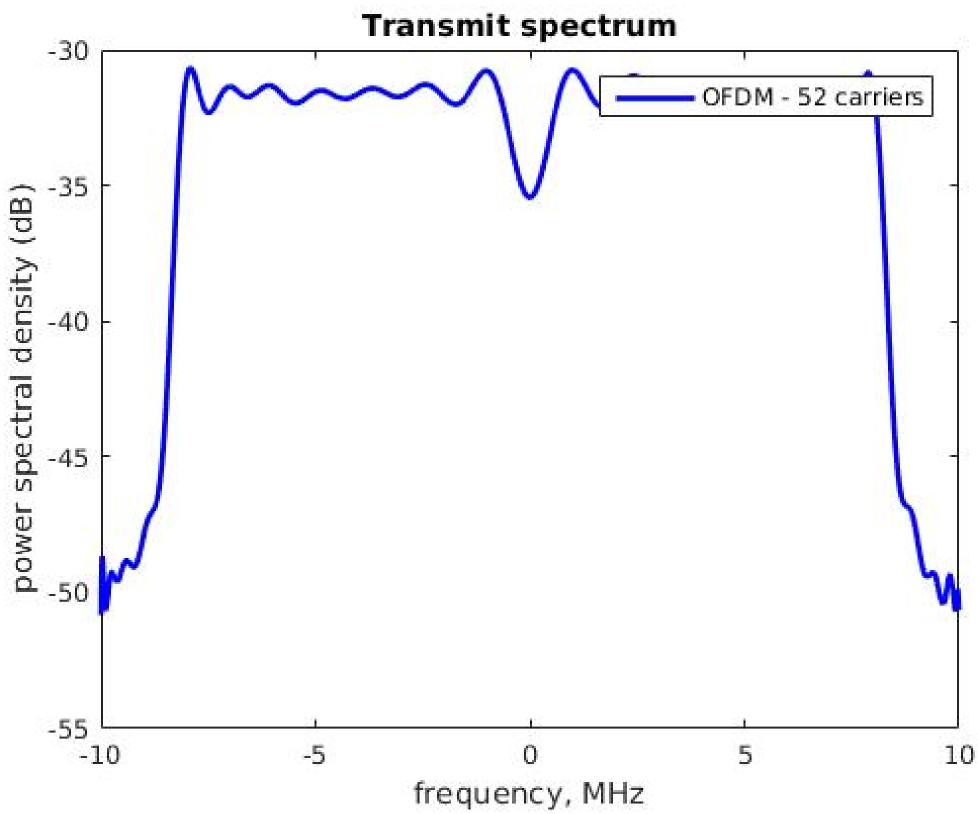

Figure 4.1: Transmit spectrum of OFDM

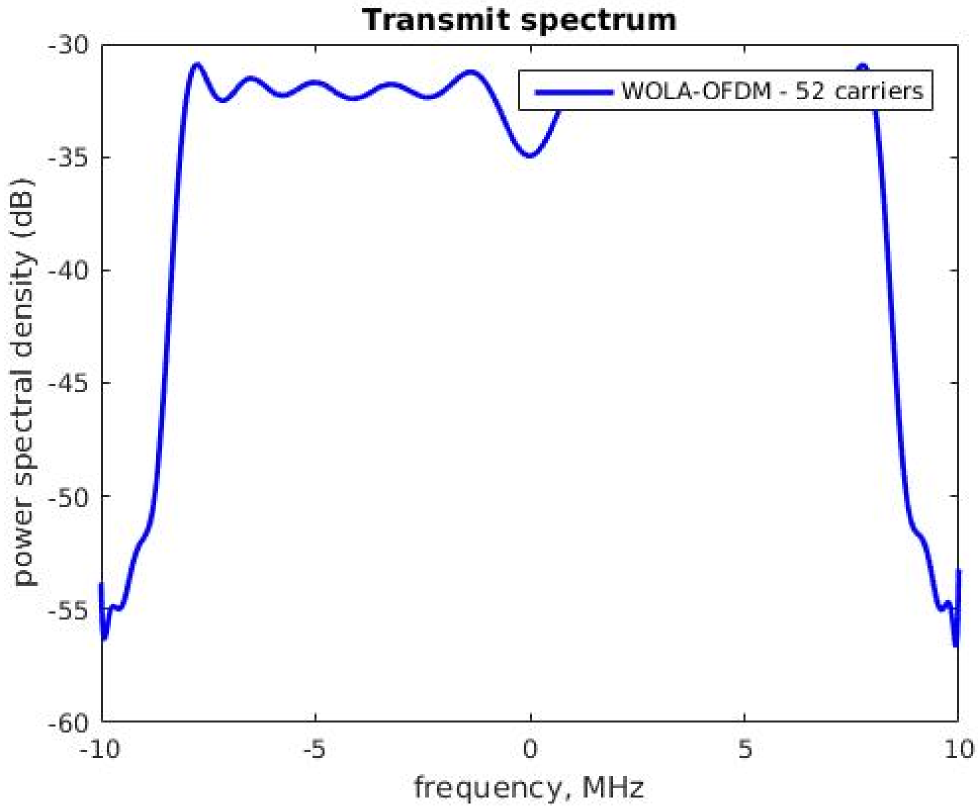

Figure 4.2: Transmit spectrum of WOLA-OFDM

4.2 PAPR

OFDM signal can be seen as a sum of many individual subcarrier tones with independent amplitude and phase factors.The high PAPR compared to a single-carrier transmission technique occurs due to the summation of the many individual subcarriers. Undeniably, the high PAPR characteristic is one of the most unfavorable aspects in OFDM system, which makes the OFDM system suffer from decrease in signal to quantization noise ratio (SQNR) at both analog-digital converter (ADC) and digital-analog converter (DAC).

4.2.1 Calculation of PAPR

The Peak to Average Power Ratio (PAPR) is maximum squared magnitude value of a baseband signal divided by average value of this signal (also known as mean-squared value), i.e., PAPR of a signal $x(t)$ can be defined as,

$$papr = \frac{max[x(t)x^*(t)]}{E[x(t)x^*(t)]}$$

where $x^*(t)$ represents the complex conjugate of the signal $x(t)$. The above equation when represented in decibels,

$$papr(dB) = 10\log_{10}(papr)$$

Let us now consider a complex sinusoidal periodic signal $x(t)$ with period T.

$$x(t) = e^{2\pi fT}$$

The peak power of the signal,

$$max[x(t)x^*(t)] = +1$$

The average power of the signal is,

$$\begin{aligned} E[x(t)x^*(t)] &= \frac{1}{T}\int_0^T e^{4\pi fT} \\ &= 1 \end{aligned}$$

So the $paprdb$ of a complex sinusoidal tone is 0 dB.

We are already pretty clear that an OFDM symbol is the sum of multiple subcarriers separated by $\delta f = 1/T$, where each subcarrier is modulated by an independent complex data symbol ak. Therefore, the transmitted baseband signal for an OFDM symbol can be mathematically represented as,

$$x(t) = \sum_{0}^{K-1} X(k)e^{j2\pi kt/T}$$

where, K is number of data subcarriers in our OFDM symbol and $X(k)$ is the modulated symbol. Let us now calculate the peak power,

$$\begin{aligned}\max[x(t)x^*(t)] &= \max_{0<k\leq K}[\sum_k X(k)e^{j2\pi kt/T}\sum_k X^*(k)e^{-j2\pi kt/T}] \\ &= \max_{0<k\leq K}[X(k)X^*(k)\sum_k e^{j2\pi kt/T}\sum_k e^{-j2\pi kt/T}] \\ &= K^2 \max_{0<k\leq K}[|X(k)|^2]\end{aligned}$$

and the average power,

$$\begin{aligned}E[x(t)x^*(t)] &= E_{0<k\leq K}[\sum_k X(k)e^{j2\pi kt/T}\sum_k X^*(k)e^{-j2\pi kt/T}] \\ &= E_{0<k\leq K}[X(k)X^*(k)\sum_k e^{j2\pi kt/T}\sum_k e^{-j2\pi kt/T}] \\ &= KE_{0<k\leq K}[|X(k)|^2]\end{aligned}$$

So, the PAPR of an OFDM system with K data subcarriers can be given by,

$$papr = K\frac{\max_{0<k\leq K}[|X(k)|^2]}{E_{0<k\leq K}[|X(k)|^2]}$$

And its maximum value is K, which occurs when all the subcarriers are equally modulated. But it is highly unlikely to happen as the modulated data on each subcarrier is random and uncorrelated to each other. Hence, we can study the PAPR of OFDM and WOLA systems as random variables which have distinct probability distribution functions. Figure 4.3 is the plot of cumulative distribution function (CDF) of PAPR for OFDM and WOLA with the specifications given in the table 4.1.

From the figure 4.3, we can observe that PAPR of WOLA is slightly greater than OFDM (about 0.5 dB).

4.3 Channel Estimation

The channel through which the OFDM signal passes from transmitter to receiver might be of flat fading type or frequency selective fading type. Regardless, at the receiver, the presence of additive white Gaussian noise (AWGN) is inherent. In this section, we assume perfect synchronization between the transmitter and receiver.

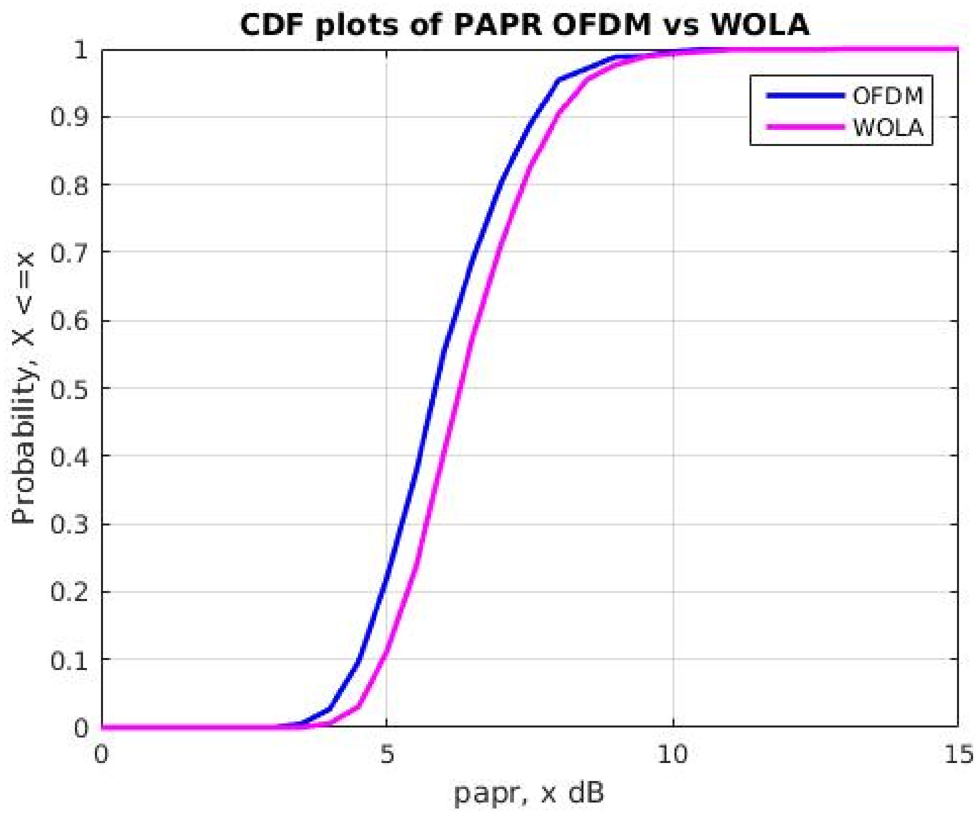

Figure 4.3: PAPR of OFDM vs WOLA-OFDM

The transfer function of the channel is a random process due to the existence of frequency selective fading, Doppler shift, etc. In order to address this problem and compensate the filtering effect, we first need to estimate the channel effect on each subcarrier. The channel estimation is commonly based on pilots, which are specific transmitted data symbols known by the receiver.

In pilot based channel estimation, we insert N_p pilot symbols uniformly into the modulated data symbols $X(k)$ as follows,

$$X(k) = X(mL + l) = \begin{cases} X_p(m), & l = 0 \\ data, & others \end{cases}$$

where $L =$ number of carriers$/N_p$ and $X_P(m)$ denotes the m^{th} pilot carrier value. The Least Square estimator gives the estimate of the frequency response of the channel at pilot subcarriers given by,

$$H_p(k) = Y_p/X_p \quad \text{for} \quad k = 0, 1, ...N_p - 1$$

where Y_p and X_p are the input and output at the k^{th} pilot subcarrier respectively. Further we use linear interpolation technique to estimate the channel frequency response at the remaining data subcarriers. At the data carrier k where $mL < k < (m+1)L$, the linear interpolation gives the channel estimate $H_e(k)$ as,

$$\begin{aligned} H_e(k) &= H_e(mL + l) \quad \text{for} \quad 0 \leq l \leq L \\ &= (H_p(m+1) - H_p(m))\frac{l}{L} + H_p(m) \end{aligned}$$

4.4 Equalization

The basic idea to equalize a received signal is to undo the distortions caused to the transmitted signal by a channel. One of the practical problems in wireless communications is Inter Symbol Interference (ISI) which is imposed on the transmitted signal due to the band limiting effect of the practical channel and also due to the multipath effects of the channel. Zero-Forcing (ZF) Equalizer and Minimum Mean Square Error (MMSE) Equalizer are the two conventionally used equalization techniques to nullify the effects of channel on the signal. After the equalization, we get back the complex symbols sent which can be demodulated to extract the original data sent.

4.4.1 Zero Forcing Equalizer

Zero Forcing Equalizer refers to a form of linear equalization algorithm used in communication systems which applies the inverse of the frequency response of the channel. The Zero-Forcing Equalizer applies the inverse of the channel frequency response to the received signal, to restore the signal after the channel. The name Zero Forcing corresponds to bringing down the ISI to zero in a noise free case. This will be useful when ISI is significant compared to noise. For a channel with frequency response $H(k)$, the

ZF equalizer $H_{zf}(k)$ is given by,

$$H_{zf}(k) = 1/H(k)$$

However, the Zero Forcing equalizer has its own disadvantages. Firstly, even if the channel impulse response has finite length, the impulse response of the equalizer needs to be infinitely long due to the inverse function. And secondly, the ZF equalizer amplifies the weak signal which in turn amplifies any noise added to the signal resulting in poor SNR.

4.4.2 MMSE Equalizer

To minimize the ISI and additive effects of noise, the equalizer coefficients can be optimized using the Minimum Mean Squared Error (MMSE) criterion. Suppose $x[k]$ is passed through the channel $h[k]$ with the inherent AWGN $n[k]$ of zero mean, then the received signal $y[k]$ can be represented as,

$$y[k] = x[k] \circledast h[k] + n[k]$$

In MMSE method, for each sample time k we would want to find a set of coefficients $c[k]$ which minimizes the mean square error between the desired signal $x[k]$ and the equalized signal $c[k] \circledast y[k]$. Let $e[k]$ be the error at sample time k, then

$$e[k] = x[k] - c[k] \circledast y[k])$$

And the mean square error will be

$$E(e[k])^2 = E(x[k] - c[k] \circledast y[k])^2$$

For solving the MMSE criterion, we need to find a set of coefficients c which minimizes above equation. So differentiating above equation w.r.t c and equating to zero, we get,

$$\begin{aligned} c &= R_{yy}^{-1}.R_{sy} \\ &= [R_{hh} + \frac{var(n)}{var(x)}]^{-1}.h \end{aligned}$$

where R_{ys} is the cross correlation between received sequence and input sequence, R_{yy} is the auto-correlation of the received sequence, R_{hh} is the auto-correlation of the channel, $var(n)$ and $var(x)$ denote the variance of noise and input signal respectively.

Let us now analyze the BER performance of both the equalizers WOLA based OFDM system based on 802.11a specifications in a 3-tap multipath channel with AWGN. The MMSE equalizer used has 7-taps.

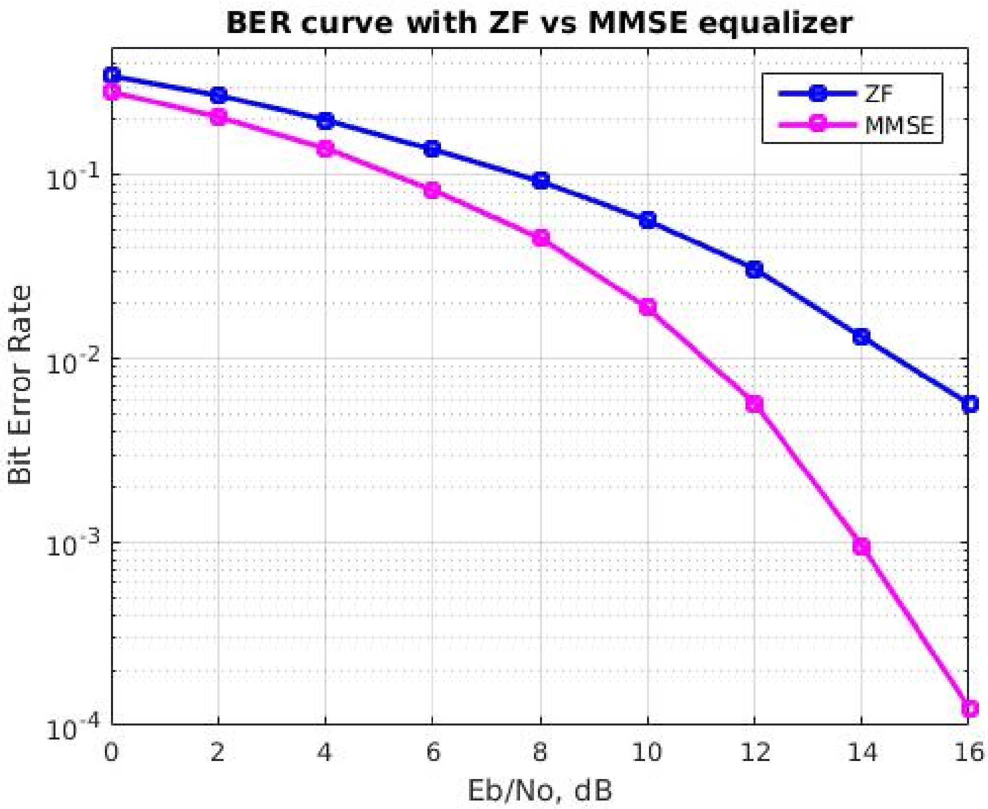

Figure 4.4: BER performance of ZF equalizer vs MMSE equalizer

As you can see in the figure 4.4, the MMSE equalizer outperforms the ZF equalizer in terms of requiring lesser SNR for a given BER. This may be attributed to the fact that MMSE equalizer takes into account both the noise and signal variance, makes to not amplify the noise as Zero Forcing does which in turn accounts to a more robust extraction of the original signal.

Hence, we will use MMSE equalization for analyzing the BER performance of OFDM and WOLA systems in the next sections.

4.5 Bit Error Rate

After equalization, the output that we have now is the receiver's estimate of the original OFDM/WOLA symbol. These complex symbols are now demodulated to get back the original data bits sent. In digital transmission, the number of bit errors is the number of received bits of a data stream over a communication channel that have been altered due to noise, interference, distortion or bit synchronization errors. BER can be mathematically defined as the number of bits that are erroneously decoded divided by the total number of bits transmitted. It is unitless and often expressed as a percentage.

BER in a wireless communication system is often expressed in terms of E_b/N_o which is the energy per bit to noise power spectral density ratio. BER curves which are generally the plots of BER vs E_b/N_o(dB) are used to describe the performance of a communication system. At the transmission end, signals are transmitted in the form of symbols. The connection between E_s (energy per symbol) and E_b (energy per bit) can be written as,

$$E_s = G.E_b$$

where G is the number of bits transmitted per symbol. But as discussed earlier, BER curve is based on E_b/N_o and hence, we should find the relation between E_b/N_o and SNR. We already know from the table 4.1 that only N_d out of N carriers are active subcarriers. So, the SNR for OFDM system can be given by,

$$\begin{aligned} SNR &= (\frac{N_d}{N}.\frac{E_s}{T_d}.\frac{T_d}{T_d + T_{cp}})/(\frac{N_o}{T_d}) \\ &= G.\frac{N_d}{N}.\frac{E_b}{N_o}.\frac{T_d}{T_d + T_{cp}} \end{aligned}$$

Similarly for WOLA, SNR in terms of E_b/N_o is given by

$$SNR = G.\frac{N_d}{N}.\frac{E_b}{N_o}.\frac{T_d}{T_d + T_{cp} + T_{cs}}$$

Let us now analyze the BER performance of OFDM and WOLA with specifications in table 4.1 under various modulation schemes namely BPSK, 4-QAM, 16-QAM and 64-QAM.

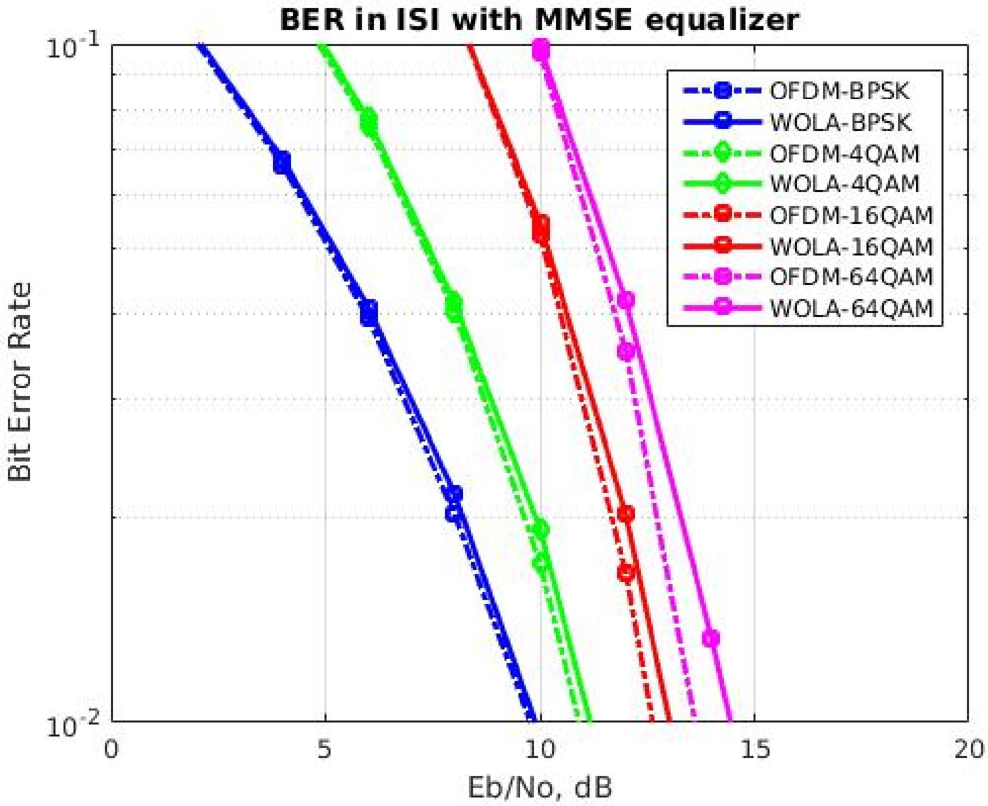

Figure 4.5: BER performance of OFDM vs WOLA-OFDM

We know that with the increase in the order of modulation, the distance between the points of the constellations becomes shorter and shorter. In the presence of noise, the probability of erroneous decoding of data increases as the distance between the constellation points decreases. Hence, for a given SNR 64-QAM has a higher BER compared to 16-QAM or BPSK. We can see that the figure 4.5 also has the same trend. Also, we observe that the performance of WOLA seems to be slowly diverging from that of OFDM with the increase in the order of modulation which may be accounted to the slender loss of orthogonality between subcarriers in WOLA-OFDM.

4.6 Effects of CFO

4.6.1 ICI comparison in the presence of CFO

In this section, let us evaluate the impact of carrier frequency offset (CFO) which results in ICI while receiving an OFDM/WOLA modulated symbol. In a typical wireless communication system, the signal to be transmitted is up-converted to a carrier frequency prior to transmission. The receiver is expected to tune to the same carrier frequency for down-converting the transmitted signal to baseband, prior to demodulation. CFO is caused due to the misalignment in carrier frequencies of the receiver and the transmitter. In this section, we used AWGN channel model.

Suppose $S(m)$ is the transmitted signal and $Z(m)$ be the received signal on subcarrier m for a N carrier OFDM system with AWGN channel in presence of CFO. If T is the symbol duration, then from Yang *et al.* (2004), the recovered signal on subcarrier m can be given by,

$$Z(m) = I_0 S(m) + \sum_{k=0,k\neq m}^{N-1} I_{k-m} S(k)$$

where

$$I_{k-m} = \frac{\sin \pi(\Delta f T + k - m)}{N \sin \frac{\pi}{N}(\Delta f T + k - m)} . e^{(j\pi(N-1)(\Delta f T + k - m)/N)}$$

In the above equation, I_0 represents attenuation and phase rotation of the desired signal which is independent of m and I_{k-m} represents the ICI coefficient from other subcarriers which depends on m.

It can be seen that the ICI power on subcarrier m increases as $|\Delta f T|$ increases. Accordingly, variance of $|z(m)|$ increases as $|\Delta f T|$ increases (Yang *et al.* (2004)). Hence, subcarrier variance can be used as a measure of ICI strength in OFDM. Similarly, for a WOLA system, if we can prove that the received signal on subcarrier m can be represented as a sum of transmitted signal and ICI, we can use the subcarrier variance to measure the strength of ICI in WOLA-OFDM.

Let $S(k)$ be the transmitted signal on subcarrier k, $s(n)$ be the transmit IFFT, $x(n)$ be the symbol after adding CP and CS, $w_t(n)$ be windowing function on transmitter side and $x_t(n)$ be windowed transmit IFFT after CP and CS addition as in figure 3.2.

And let $x_r(n)$ be the received symbol after applying receiver windowing function $w(n)$, and $y_r(n)$ be the symbol after WOLA processing as in the figure 3.3. T is the symbol duration and N, cp, cs are assumed to be the lengths of symbol, CP and CS respectively. Then,

$$s(n) = \sum_{k=0}^{N-1} S(k) e^{j2\pi kn/N}$$

Adding CP and CS,

$$\begin{aligned} x(n) &= s(n+N), \quad && -cp \leq n < 0 \\ &= s(n), && 0 \leq n < N \\ &= s(n-N), && N \leq n < cs \end{aligned}$$

After applying CFO,

$$x_t(n) = x(n)w_t(n)e^{j2\pi n\Delta fT/N}, \quad -cp \leq n < cs$$

Now at the receiver, after windowing,

$$x_r(n) = x_t(n)w(n)e^{j2\pi n\Delta fT/N}, \quad -2cs \leq n < N$$

and wola processing results in,

$$\begin{aligned} y_r(n) &= x_r(n) + x_r(n+N), \quad && -2cs \leq n < 0 \\ &= x_r(n), && 0 \leq n < N-2cs \\ &= x_r(n) + x_r(n-N), && N-2cs \leq n < N \end{aligned}$$

Removing CS and taking DFT,

$$Z(m) = \sum_{n=0}^{N-1} s(n)e^{j2\pi n(\Delta fT-m)/N} + (c-1)\sum_{n=N-2cs}^{N-1} s(n)e^{j2\pi n(\Delta fT-m)/N}w(n-N)$$

where $c = e^{-j2\pi\Delta fT}$.

In the above equation, the first term is same as calculating DFT for OFDM which has the transmitted signal and ICI. And for small CFO, the second term turns out to be cancelling a small part of the ICI present in the first term, hence reducing ICI for WOLA-OFDM. Let us plot the subcarrier variance as a function of normalized CFO for OFDM and WOLA systems in a AWGN channel.

We can see from figure 4.6 that for both OFDM and WOLA, the ICI power is nearly zero in the absence of CFO. With the increase in CFO, the ICI gradually increases for both the systems but WOLA-OFDM proves to have less ICI than OFDM.

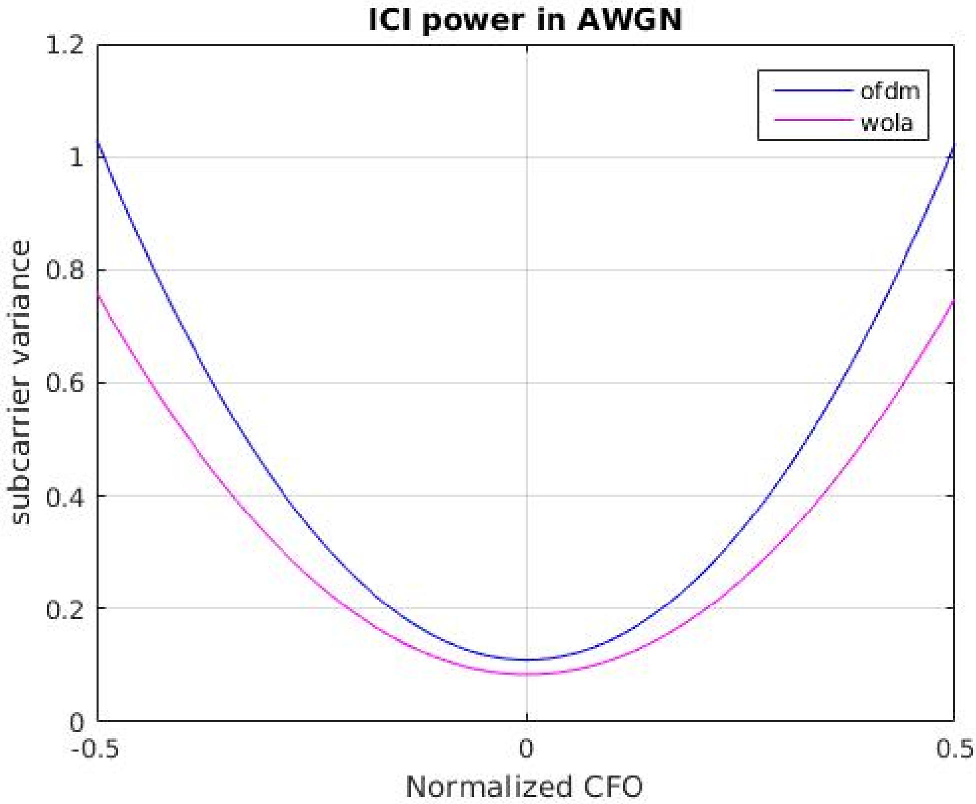

Figure 4.6: ICI power in presence of CFO, OFDM vs WOLA-OFDM

4.6.2 BER performance in the presence of CFO

In this section, we evaluate the BER performance of OFDM and WOLA-OFDM in a AWGN channel in the presence of CFO. The figure 4.7 plots the BER curves for OFDM and WOLA-OFDM for varying normalized CFO.

From the figure 4.7, we can see that when there is no CFO, both OFDM and WOLA have similar BER curves. But as CFO increases, WOLA-OFDM shows better BER performance compared to OFDM making it more robust to interference in asynchronous environments.

4.7 Increasing Data Rate

One more advantage of WOLA-OFDM is the possibility of increasing data rate compared to OFDM. In the section 4.1, we have already seen the transmission spectra of both OFDM and WOLA-OFDM and proved the effective suppression of OOB in

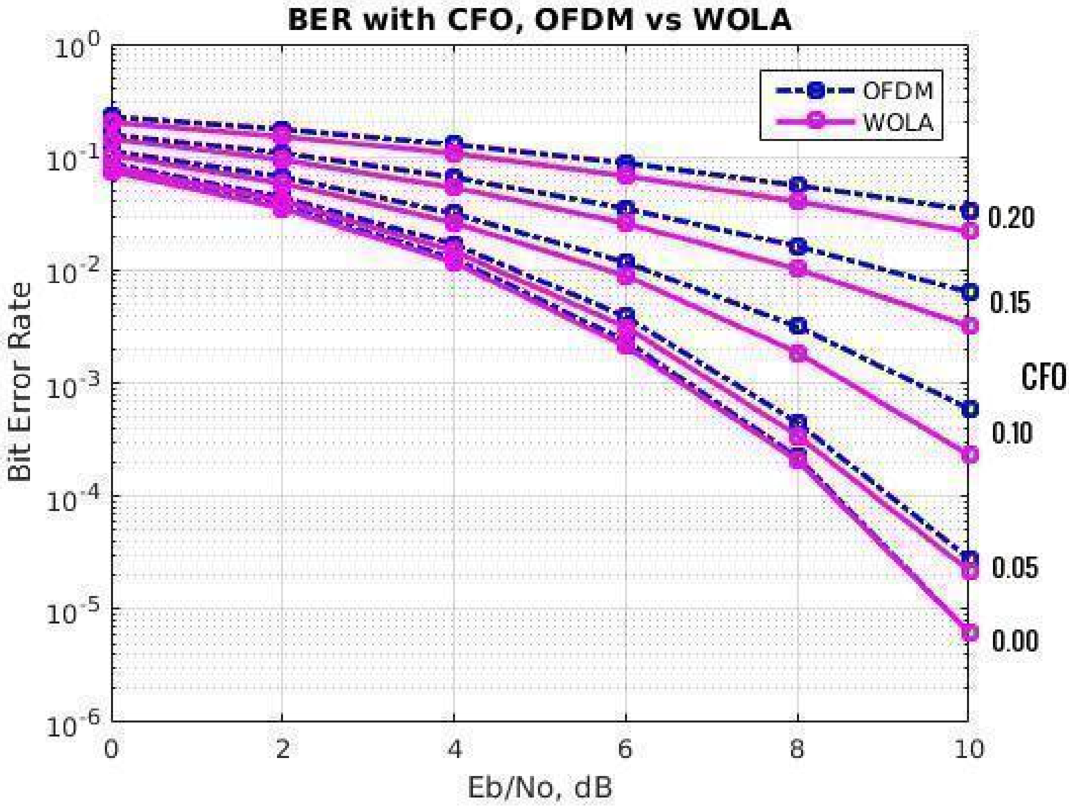

Figure 4.7: BER in presence of CFO, OFDM vs WOLA-OFDM

WOLA-OFDM. From table 4.1, we know that only 52 subcarriers out of the availabale 64 subcarriers are used to send the data. The used subcarrier have indices from [-26 to -1, +1 to +26]. Let us have a look at the PSD of OFDM and WOLA-OFDM signals at the transmission end in the figure 4.8.

We can the increase the number of data subcarriers but then we are effectively increasing the energy of the symbol. Since WOLA-OFDM has an advantage over the OFDM in terms of spectrum, WOLA-OFDM can afford to spend more data subcarriers than OFDM. Suppose we send the data in subcarriers having indices from [-28 to -1, +1 to +28] for WOLA-OFDM, the spectrum comparison of conventional OFDM and WOLA-OFDM is as shown in the figure 4.9. So, the trade-off between spectrum and data rate can be used to increase the data rate in a WOLA-OFDM system.

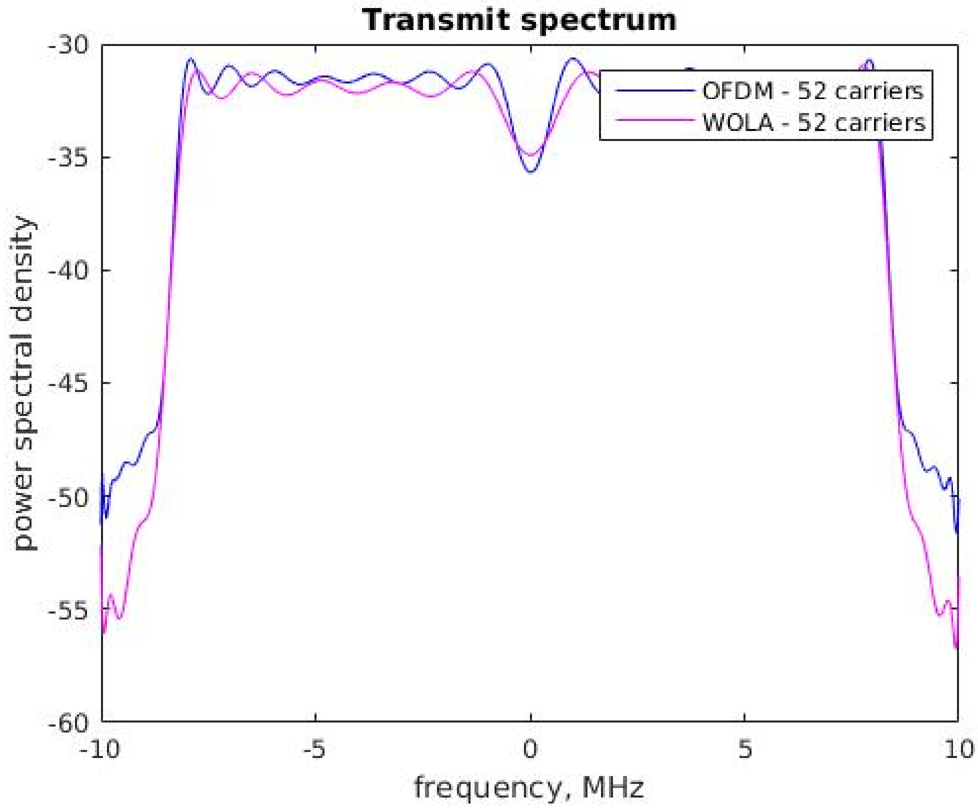

Figure 4.8: PSD at the transmitter

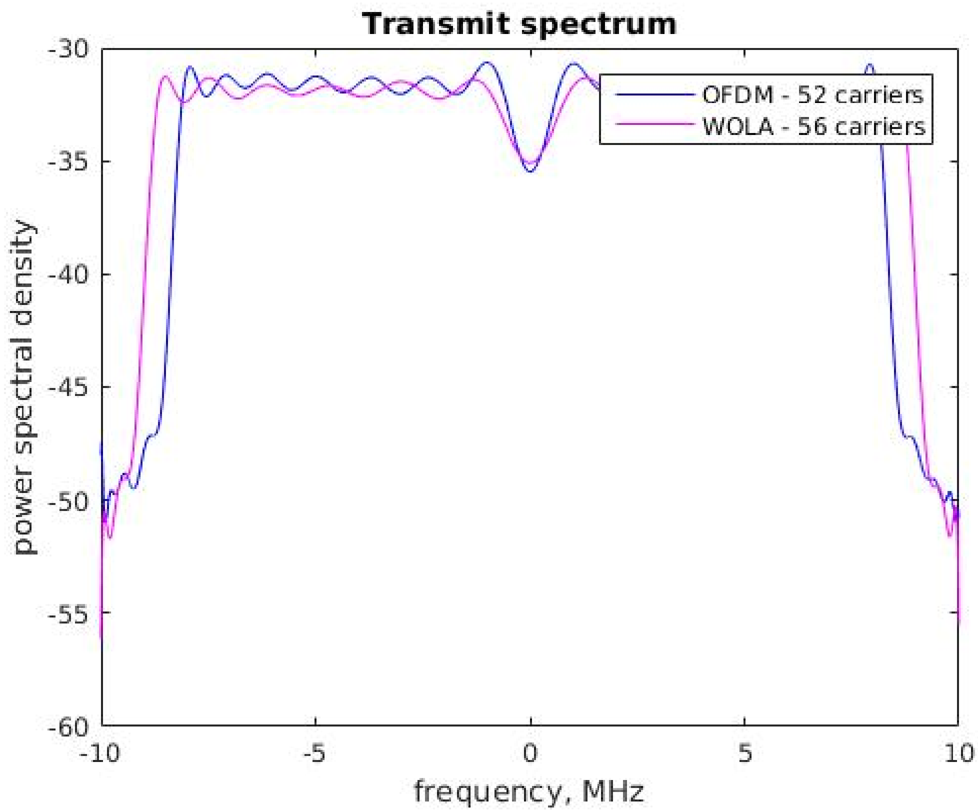

Figure 4.9: PSD at the transmitter

Conclusion

In this paper, we highlighted once again the limitations of conventional OFDM and emphasized the need for a waveform that can overcome these limitations to meet the requirements of future wireless systems. We studied the structure and working of WOLA-OFDM, which is considered to be a promising 5G waveform. We implemented both OFDM and WOLA-OFDM as per 802.11a specifications and investigated their performance in various aspects. The aspects considered in this study are : OOB emission, PAPR, BER performance in a multipath channel for different modulation schemes, effect of CFO on ICI, effect of CFO on BER, and data rates. Based on the obtained results, WOLA-OFDM seems to be a very attractive multicarrier waveform for future wireless standards due to the simple implementation using time domain windowing, efficient OOB suppression, better robustness to asynchronous frequency environment and possibility of higher data rate transmission compared to OFDM.

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