STATISTICAL COMPACT MODEL EXTRACTION FOR SKEW-NORMAL DISTRIBUTIONS

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THESIS CERTIFICATE

This is to certify that the thesis titled **STATISTICAL COMPACT MODEL EX-TRACTION FOR SKEW-NORMAL DISTRIBUTIONS**, submitted by **REVANTH K**, **EE14B029**, to the Indian Institute of Technology, Madras, for the award of the degree of **Bachelor of Technology**, is a bonafide record of the research work done by him under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

KEYWORDS: skew-normal distribution, non-linear optimization, artificial neural network

A technique for extracting Statistical Compact Model(SCM) parameters for skewed Normal parameters is proposed. Existing techniques handle non-Gaussian variations through non-linearity in model equations. However, hardware data on certain technologies suggest that non-Gaussian variations are observed even on linear parameters like Idlin/Idsat. We propose to model such variations through skewed Normal random variables. Analytical expressions relating the statistics of the skewed Gaussian process and performance parameters are derived. Statistical measures of process parameters are extracted using back propagation algorithm.

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ABBREVIATIONS

- ANN Artificial Neural Network
- MC Monte Carlo
- **BPV** Back Propogation of Variance
- MIMO Multiple Input Multiple Output

Chapter 1

INTRODUCTION

As the semiconductor devices scale down, the process variations become more significant. Circuit designer needs to assess the impact of these variations on the performance of the circuit using existing statistical or analytical models. The classic approach of running the Monte-Carlo(MC) simulations to match the resulting parameter distributions with that of predicted or measured distributions is time-consuming trial and error method. As the technology further scales down these variations start deviating from the Gaussian distributions making the MC simulations even more cumbersome.

Building an analytical model relating the Process parameters and Performance parameters can help in speeding up the extraction process to a better extent. Analytical model helps in deriving the dependence of the statistical measures of Performcance parameters on Process parameters which can be used in extracting the required parameters.

Existing techniques of Statistical Compact Modelling(SCM) do handle the case when the performance parameters are non-Gaussian. However the reason is considered to be non-linear dependence between the process parameters and performance parameters while considering the process variations to be Gaussian. However this is not the case, since hardware data of some of the technologies show that even linear parameters like Idlin, Idsat have also skewed histograms. Attempts have be made to model the process parameters to be skew-Gaussian to model the skewness in the linear parameters. But considering skew-Gaussian variation in performance parameters have created difficulties in modelling non-linear parameters like Ioff. In this report we attempt to model the non-Gaussian variations in process parameters using skew-Normal variables with certain constraints which will also be able to model the non-linear parameters like Ioff.

1.1 Literature Review

Building analytical models to extract the variations in the process parameters is one of the major areas of research in semiconductor industry. The prime models as described in [1] and [3] implements the Back Propagation Algorithm (BPA) efficiently for nonlinear relationships that be accurately modelled using the quadratic and linear terms. These work well if the degree of non-linearity is not too high.

However as the device size keeps scaling down, the sub threshold leakage current becomes one of the considerable performance parameters as the leakage power contributes to nearly 50% of the total device power. It is also well known that the leakage current is exponentially dependent on process parameter variation[5]. The above described methods fail to model the leakage current efficiently.

The modelling in [6] shows that ANN can be used to efficiently model the performance parameters including the leakage current in a scenario of Multiple Input Multiple Output(MIMO) within an error of 1%. [7] uses the ANN based approach to extract the statistical parameters with an underlying assumption that the variation in process parameters are Gaussian. However as proposed by Kovac *et.al* with the scaling down of technology, the variations in process parameters deviates from Gaussian, which can be pretty much modelled using skew-normal distributions.

This report using the model described in [6] tries to build up on the footsteps of [7] to device an analytical model when the variations in process parameters is skew-normal.

1.2 Problem Formulation

Inline measurements determine the distributions of the performance parameters and some process parameters. Hence the inputs of ANN model generation are the statistical measures of performance parameters like Idlin(linear region ON current), Id-sat(saturation region ON current), Ioff(subthreshold leakage at Vgs = 0), Vtlin(linear region threshold voltage), Vtsat(saturation region threshold region). The goal is to obtain the statistical measures of process parameters like mobility, oxide thickness etc..., with that of the known parameters keeping constant at the measured values which result in the same variations as the measured results.

Let the process parameters which see the variations due to manufacturing process be denoted by $\mathbf{P} = [P_1 P_2 ... P_N]^T$. The Performance parameters which depend on these underlying process parameters through complicated functions be denoted by $\mathbf{Y} = [Y_1 Y_2 ... Y_M]^T$. The statistical measures of P be denoted by $s^P = [s_1^P s_2^P ... s_N^P]^T$ and that of Y be denoted by $s^Y = [s_1^Y s_2^Y ... s_M^Y]^T$.

The statistical measures of Y are measured inline and added with suitable guard bands to take into account for the varying process conditions with time and limited sample size are supplied as targets to the ANN model. Let these be denoted as s^{Target} = $[s_1^{Target}s_2^{Target}...s_M^{Target}]^T$. The problem that needs to be solved is to determine the statistical measures of the process parameters s^P that result in $s^Y = s^{Target}$. This can be expressed as inversion problem as follows

$$s^{req} = \{s^P : s^Y(s^P) = s^{Target}\}$$
(1.1)

The key to solve this problem is to evaluate s^Y as function of s^P which completely depends on f_i which relates Y_i and P as follows

$$Y_i = f_i(\mathbf{P}) \tag{1.2}$$

The physics based equations are quite complex. Hence [5] uses ANN to model these complex relations so that the equations relating the statistical measures can be analytically derived. In the next section, we look at ANN based modelling, statistical parameters involved, deriving the relation between s^{Y} and s^{P} which can be subsequently used in optimizing problem.

Chapter 2

ANN Modelling and Optimization

2.1 MIMO modelling using ANN

Neural network modelling involves two phases - training phase and testing phase.



Figure 2.1: Basic Structure of ANN used

The basic structure of a MIMO ANN with one hidden layer is shown in Fig. 1. As shown in the figure ANN consists of three layers: input, hidden and output. The hidden layer consists of hidden units where weighted sum of inputs is passed through a non-linear activation function. The activation function chosen is exponential-sigmoid as described in Eq. 2.1 in contrast to standard activation functions like tan-sigmoid and log-sigmoid to ease the analytical derivation of relationship between input and output.

$$\phi(x) \begin{cases} (1 - e^{-x}), & x \ge 0\\ -(1 - e^{x}), & x \le 0 \end{cases}$$
(2.1)

The weighted outputs of the hidden layers are again linearly combined to get the required outputs. The values (W^I, B^I) and (W^O, B^O) are unknown variables and are determined during training of ANN. The relation between output and input of an ANN is given by

$$\mathbf{Y} = W^O \times \phi(W^I \times \mathbf{P} + B^I) + B^O \tag{2.2}$$

The ANN is trained using the Levenberg and Marquardt algorithm as described in [2].

2.2 Statistical Parameters involved

As described in [4] skew-normal distributions can be best represented as linear combination of standard normal and half normal random variable

$$Z = \lambda |U| + V \tag{2.3}$$

$$P = \sigma Z \tag{2.4}$$

where U and V are N(0,1) and λ and σ are real numbers with $\sigma \ge 0$. The mean and variance of P are as follows

$$\mu = \lambda \sqrt{\frac{2}{\pi}} \tag{2.5}$$

$$\sigma^{2} = \lambda^{2} (1 - \sqrt{\frac{2}{\pi}}) + 1$$
 (2.6)

Hence a skew normal distribution can be best described with the help of λ and σ as parameters instead of mean and variance.

To ease the analytical derivation of propagation of skew random variables through ANN, all the process parameters distributions are considered to be represented by same half normal distribution.

$$\mathbf{P}_i = \sigma_i (\lambda_i |U| + V_i) \tag{2.7}$$

Since the propogation of variance of skew-normal distribution through ANN would be quite difficult, so instead of modelling the variance and higher moments of performance parameters, the squares and cubes of the performance parameters are modelled. Their expectations E[Y], $E[Y^2]$ and $E[Y^3]$ are used to achieve the targets in variance and higher order moments, instead of passing them through ANN.

2.3 Statistical Analysis

In the previous section, we built the ANN which models the required relations and also the stated the statistical measures of our interest. In this section our goal is to determine analytical relations between the statistical measures of Performance parameters and Process parameters.

The key for deriving the required relations is to evaluate the expectation at the output of the hidden layer for a given skew-normal input. The hidden layer can be broken down into two stages - Linear combination of weighted inputs and Passing through activation function.

The random variable description after stage 1 of hidden layer is as follows

$$S_{i} = \sum_{j=1}^{N} W_{ij}^{I} P_{j} + B_{j}^{I}$$

$$= \sum_{j=1}^{N} W_{ij}^{I} \sigma_{j} (\lambda_{j} |U| + V_{j}) + B_{j}^{I}$$

$$= \sum_{j=1}^{N} W_{ij}^{I} \sigma_{j} \lambda_{j} |U| + \sum_{j=1}^{N} W_{ij}^{I} \sigma_{j} V_{j}$$

$$= \lambda_{si} |U| + V_{si}$$
(2.8)

The form of random variable S describes it to be a skew-normal random variable whose mean and variance are given by

$$\mu_{si} = \lambda_{si} \sqrt{\frac{2}{\pi}} + \mu_{vsi} \tag{2.9}$$

$$\sigma_{si}^2 = \lambda_{si}^2 (1 - \sqrt{\frac{2}{\pi}}) + \sigma_{vsi}^2$$
(2.10)

The Probability distribution function of S_i is given by

$$f_{S_i}(z) = \alpha e^{-\frac{(z-\mu_{vsi})^2}{\sigma}} \operatorname{erfc}[(\mu_{vsi} - z)\gamma]$$
(2.11)

where,

$$\alpha = \frac{1}{\sqrt{2\pi(\sigma_{vsi}^2 + \lambda_{si}^2 \sigma_u^2)}}$$

$$\sigma = 2(\sigma_{vsi}^2 + \lambda_{si}^2 \sigma_u^2)$$

$$\gamma = \frac{\lambda_{si}\sigma_u}{\sigma_{vsi}\sqrt{2(\sigma_{vsi}^2 + \lambda_{si}^2 \sigma_u^2)}}$$
(2.12)

The random variable when passed through the exponential sigmoid activation function gives random variable K which has expectation given by

$$\mu_{ki} = \frac{\alpha\sqrt{\pi\sigma}}{2} \left[\operatorname{erfc}\left(-\frac{\mu_{vsi}}{\sigma}\right) - \operatorname{erfc}\left(\frac{\mu_{vsi}}{\sigma}\right) \right] + \alpha e^{\frac{\sigma}{4}} \left[f(\mu_{vsi}) - f(-\mu_{vsi}) \right] \\ + \alpha e^{\frac{\sigma}{4}} \left(\frac{\gamma\sigma^{\frac{3}{2}}}{2} + \frac{\gamma^{3}\sigma^{\frac{5}{2}}(6+\sigma)}{24} \right) \left[f(\mu_{vsi}) + f(-\mu_{vsi}) \right] \\ + \frac{\alpha\gamma e^{-\frac{\mu_{vsi}^{2}}{\sigma}}}{\sqrt{\pi}} \left(2\sigma \left(\frac{\mu_{vsi}^{4}\gamma^{4}}{10}\right) + 2\sigma^{2} \left(\frac{\mu_{vsi}^{2}\gamma^{4}}{5}\right) + 2\sigma^{3} \left(\frac{\gamma^{4}}{5} + \frac{\gamma^{2}}{12}\right) \right) \quad (2.13)$$

where,

$$f(x) = e^x \left(\operatorname{erfc} \left(\frac{x}{\sqrt{\sigma}} + \frac{\sqrt{\sigma}}{2} \right) \right)$$
(2.14)

The output of ANN as described is the linear combination of the weighted outputs of hidden layers whose description and expectation are given by

$$\mathbf{Y}_{i} = \sum_{j=0}^{R} K_{j} W_{ji}^{O} + B_{j}^{O}$$
(2.15)

$$\mu_{Yi} = \sum_{j=0}^{R} \mu_{Kj} W_{ji}^{O} + B_j^{O}$$
(2.16)

The Derivations of all the above equations are given in Appendix A

2.4 Non Linear Optimization Formulation

The objective function for non-linear optimization can be formulated as follows

$$\min_{\sigma,\lambda} \sum_{i=1}^{M} \left(\frac{s_i^{Target} - s_i^Y}{s_i^{Target}} \right)^2 + \sum_{i=1}^{N} \left(\lambda_i^3 \sigma_i^3 12 \sqrt{\frac{2}{\pi}} \right)^2$$

$$\text{Subject to } 0 \le \sigma_i^P \le s_i^{MAX} \text{ and } -\lambda_i^{MAX} \le \lambda_i^P \le \lambda_i^{MAX}$$

$$(2.17)$$

where σ_i^{MAX} and λ_i^{MAX} denotes the maximum deviation in process parameters with in which the targets are expected to be achieved.

The first term in the above formulation minimizes the relative error between the achieved value and target specified. The second term penalizes the deviation of the simulated distribution from the Gaussian distribution. The second term is necessary since the minimization of first term can solely be achieved by choosing large skew which is not generally observed in practice. So in order to keep check on the deviation from the Gaussian it is necessary to penalize this deviation.

Chapter 3

Results

In this section we validate the theory presented in the previous section starting with the training of ANN and then proceeding to the optimization results.

3.1 ANN training

The MIMO ANN is trained to model five performance parameters(Idlin, Idsat, Ioff, Vtlin, Vtsat) and their squares and cubes as function of nine process parameters.

Technology	20nm
No of Inputs(N)	9
No of hidden layers(R)	27
No of outputs(M)	14

Table 3.2: Summary of ANN Training Process

Parameter	Maximum Testing Error (%)	Mean Testing Error(%)
idlin	0.3801	0.01846
idsat	0.2198	0.01866
ioff	1.9712	0.0830
vtlin	0.0915	0.01349
vtsat	0.2517	0.02146
idlin ²	0.6208	0.02891
idsat ²	0.3307	0.0402
ioff ²	4.681	0.163
vtlin ²	0.1784	0.02582
vtsat ²	0.5120	0.04162
idlin ³	0.8285	0.0444
idsat ³	0.6565	0.0644
vtlin ³	0.5159	0.04146
vtsat ³	0.6108	0.06412

The training of ANN was performed using the Matlab neural networks toolbox. As described in Table 3.1 the ANN has 9 input layers, 27 hidden layers, and 14 output layers. From Table 3.2 one can observe that the maximum sample error on the testing set is less than 5% whereas the mean error is less than 0.2 %. This shows that the ANN can accurately model the required parameters. The training process takes less than 5 minutes on a typical laptop of 1.77GHz and 6GB RAM.

3.2 Optimization Results

We now look at the results of the optimization formulation, The optimization executed on two cases,

- 1. when the performance parameters are Guassian with skew less than 0.1.
- 2. When the Performance parameters deviate from Gaussian with skewness reaching 0.8 in idlin and idsat and other performance parameters remaining nearly Guassian.

The optimization results are presented for both the cases in Table. 3.3 and Table. 3.4 respectively.

Table 5.5. Results of Optimization for Gaussian distribution in performance parameter	Table 3.3: Result	s of Optimization	for Gaussian	distribution in	performance	parameters
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Process Parameters				Performance Parameters			
σ^P λ^P $\lambda^P\sigma^P$			μ_Y^{sim}	μ_Y^{target}	Error (%)		
P1	0.3333	0	0	idlin	0.999	0.9991	0.01
P2	0.2021	1.00E-04	2.03E-05	idsat	1.001	1.0013	0.02
P3	0.3333	0	0	ioff	1.187	1.187	0.04
P4	0.3575	-2.2E-02	-7.90E-03	vtlin	1.000	0.999	0.01
P5	0.3582	-1.06E-02	-3.78E-03	vtsat	0.997	0.997	0.00
P6	0.3333	0	0	idlin ²	1.001	1.001	0.01
P7	0.226	1.00E-01	2.28E-02	idsat ²	1.006	1.006	0.01
P8	0.3333	0	0	ioff ²	1.903	1.903	0.01
P9	0.228	-1.49E-01	-3.41E-02	vtlin ²	1.000	1.0005	0.00
				vtsat ²	0.997	0.997	0.02
				idlin ³	1.007	1.007	0.00
				idsat ³	1.013	1.014	0.02
				vtlin ³	1.003	1.003	0.01
				vtsat ³	1.000	1.000	0.04

Process Parameters				Performance Parameters			
	σ^P	λ^P	$\sigma^P \lambda^P$		μ_Y^{sim}	μ_Y^{target}	Error (%)
P1	0.3333	0	0	idlin	1.012	1.0148	0.32
P2	0.184	-2.7E-04	-5.14E-05	idsat	1.012	1.0145	0.29
P3	0.3333	0	0	ioff	1.211	1.2058	0.43
P4	0.3666	-1.61E-01	-5.93E-02	vtlin	0.995	0.9979	0.31
P5	0.3206	-1.73E-01	-5.55E-02	vtsat	0.994	0.995	0.12
P6	0.3333	0	0	idlin ²	1.027	1.0330	0.63
P7	0.3654	2.44E-01	8.94E-02	idsat ²	1.027	1.0326	0.57
P8	0.3333	0	0	ioff ²	1.963	1.964	0.11
P9	0.3657	2.87E-01	1.05E-01	vtlin ²	0.991	0.9971	0.59
				vtsat ²	0.991	0.9935	0.22
				idlin ³	1.046	1.0551	0.91
				idsat ³	1.046	1.0545	0.84
				vtlin ³	0.989	0.9975	0.86
				vtsat ³	0.991	0.9941	0.32

Table 3.4: Results of Optimization for skew-normal distribution in performance parameters

The process parameters (P1, P3, P6, P8) are measured inline and hence their statistical measures are fixed and used for forward propagation. Matlab's optimization toolbox is used to solve the non-linear optimization problem. The algorithm used for minimization is interior-point method.

The optimization routine when implemented on Gaussian distribution in Performance parameters produced skew-normal distributions with very small $\lambda\sigma$, such distribution can be considered as Gaussian distribution and the targets are modelled within an max error of 0.1%. The process parameters being Gaussian is highly expected since any linear combination of Gaussian distribution gives Gaussian distribution. This proves that the given equations are analytically correct for the propagation of mean through ANN.

Skew-normal implementation of optimization routine models the targets within an error of 1%. This error can be expected as a Taylor Series approximation of three terms for the pdf of skew-normal distribution is used in the derivation of results. The error can be further reduced by considering higher order terms of Taylor series thereby reducing the error in approximation.

Intuitively, one can observe that the process parameter P9 has the highest skew.

This is expected since only skew in Idlin, Idsat is considered for simulation and process parameter P9 is mobility, as the skew in mobility only effects Idlin and Idsat. The results can also be seen correct intuitively.

3.3 Conclusion

The ANN can be used to model multiple performance parameters of CMOS devices as a function of multiple process parameters. The numerical model can then be used to predict the statistical measures of performance parameters as a function of statistical measures of process parameters using accurate analytical expressions. It can also be observed that instead of passing higher order moments through ANN, higher powers of outputs can be modelled and their means can be considered for modelling higher order moments. The problem of statistical compact modelling can then be solved with a nonlinear optimization problem. Results showed that, with five performance parameters and nine process parameters the proposed algorithm matched with in an error of 0.5% in mean targets and with in an error of 1% for targets of higher moments.

Appendix A

A.1 Probability distribution function of S_i

$$f_{Si}(z) = \int_{-\infty}^{\infty} f_{\lambda_{si}|U|}(x) f_v(z-x) dx$$
$$= \frac{1}{\pi \lambda_{si} \sigma_{vsi}} \int_0^{\infty} e^{-\frac{x^2}{2\lambda_{si}^2}} e^{-\frac{(z-x-\mu_{vsi})^2}{2\sigma_{vsi}^2}}$$

Expanding the second exponent gives integral of form

$$=\beta \int_0^\infty e^{-(ax^2+bx)}dx$$

converting the exponent of e into perfect squares gives

$$=\beta e^{-k} \int_{h}^{\infty} e^{-ax^2} dx \tag{A.1}$$

Using the standard integral

$$\int_x^\infty e^{-u^2} du = \frac{\sqrt{\pi}}{2} \operatorname{erfc}(x)$$

gives

$$f_{Si}(z) = \alpha e^{-\frac{(z-\mu_{vsi})^2}{\sigma}} \operatorname{erfc}[(\mu_{vsi}-z)\gamma]$$

where α , σ and γ are given by (2.12)

A.2 Mean of K_i

$$\mu_{Ki} = E[\phi(s_i)]$$
$$= \int_{-\infty}^{\infty} \phi(x) f_{si}(x) dx$$

Substituting $\phi(x)$ and $f_{si}(x)$ as given by (2.1) and (2.11)

$$= -\int_{-\infty}^{0} (1-e^x)\alpha e^{-\frac{(z-\mu_{vsi})^2}{\sigma}} \operatorname{erfc}[(\mu_{vsi}-z)\gamma]dz$$
$$+ \int_{0}^{\infty} (1-e^{-x})\alpha e^{-\frac{(z-\mu_{vsi})^2}{\sigma}} \operatorname{erfc}[(\mu_{vsi}-z)\gamma]dz$$

Substituting $(\mu_{vsi} - z) = x$ in the equation gives

$$= \int_{-\infty}^{\mu_{vsi}} \alpha (1 - e^{x - \mu_{vsi}}) e^{-\frac{x^2}{\sigma}} \operatorname{erfc}(x\gamma) dx$$
$$- \int_{\mu_{vsi}}^{\infty} \alpha (1 - e^{\mu_{vsi} - z}) e^{-\frac{z^2}{\sigma}} \operatorname{erfc}(x\gamma) dx$$

Considering, Taylor series approximation for erfc(x) given by

$$\operatorname{erfc}(x) = 1 - \frac{2}{\sqrt{\pi}} \left(x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \ldots\right)$$
 (A.2)

Considering till power of 5 and Substituting gives

$$= \alpha \left(1 - \sqrt{\frac{2}{\pi}} \left(z - \frac{z^3}{5} + \frac{z^5}{10} \right) \right) \left[\int_{-\infty}^{\mu_{vsi}} e^{-\frac{z^2}{\sigma}} \operatorname{erfc}(z\gamma) dz - \int_{-\infty}^{\mu_{vsi}} e^{\left(-\frac{z^2}{\sigma} + z - \mu_{vsi}\right)} \operatorname{erfc}(z\gamma) dz - \int_{\mu_{vsi}}^{\infty} e^{-\frac{z^2}{\sigma}} \operatorname{erfc}(z\gamma) dz + \int_{\mu_{vsi}}^{\infty} e^{\left(-\frac{z^2}{\sigma} - z + \mu_{vsi}\right)} \operatorname{erfc}(z\gamma) dz \right]$$

Splitting the terms into individual integrals of the form

$$\int_{a}^{b} z^{m} e^{-kx} and \int_{a}^{b} z^{m} e^{-kx^{2}}$$

and then using Integration by Parts with appropriate substitutions

gives the Equation (2.13) as the mean of K_i

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