# OPTIMAL PLACEMENT AND EXPANSION OF THE WIND FARM GENERATORS USING THE ALTERNATING PROJECTION METHOD

***A THESIS***

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*For the award of the degree of*

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**ELECTRICAL ENGINEERING**
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Special thanks to my family for all of the sacrifices that you have made on my behalf and supporting me throughout my study. Very special thanks to my little daughter NANDINI for cheering me up.

Rajender Singh

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## THESIS CERTIFICATE

This is to certify that the project report entitled “**OPTIMAL PLACEMENT AND EXPANSION OF THE WIND FARM GENERATORS USING THE ALTERNATING PROJECTION METHOD** “submitted by **RAJENDER SINGH (EE13M052)** to the Indian Institute of Technology Madras for the award of the degree of **MASTER OF TECHNOLOGY** in **POWER SYSTEM AND POWER ELECTRONICS** in Electrical Engineering is a bonafide record of research work carried out by him under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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## ABBREVIATIONS

1. LPP…………………………Linear programming problem
2. NLPP………………………Non linear programming problem
3. RESS……………………...Renewable energy storage system
4. REPTC…………………….Renewable electricity production tax credit
5. KKT………………………… Karush –Kuhn-Tucker conditions
6. CWET…………………….Center of wind energy technology
7. INWEA……………………Indian wind energy association
8. EUAC………………………Equivalent uniform annual cost
9. NPV……………………….Net present value
10. APM………………………Alternating projection method

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## ABSTRACT

Power system optimization is the one of the most important problem in large scale operation and planning of the Electric power networks. Proper location and sizing (capacity) of the generators is extremely important for obtaining the maximum cost benefits. Many methods had been developed to solve the power system optimization problems. This project presents the analytical method to determine the optimal placement and sizing of the additional wind farm generators assuming that the grid is no longer to supply the additional increase in the load demand.

We solve a non-convex optimization problem to find out the placements and power capacities and power flows of new generators that will minimize the sum of the transmission and generation costs in electric transmission systems. We analyze and compare the performance of two different algorithms – Interior point (IP) and Alternating projections (AP) - to handle the non-convexity of the problem. We will develop the algorithms in the case of nine bus transmission systems. Results indicate that even as demands at load buses increase, the transmission cost can be decreased by optimizing the placement of new generators to satisfy the increase in load demand.



**Key Words**:

Convex optimization, Wind farm generators, Transmission cost, Linear Programming problem (LPP), Simplex Algorithm, Newton Method, KKT Conditions.

# CHAPTER 1

# INTRODUCTION

Power companies throughout the world face significant problems, mainly due to increase in the load demands that require the generation capacities beyond the current generation's limits. For example, due to population increase in the US residential electricity demand is expected to rise by 20% in the next 20 years [1]. In India the rise in the electricity demand is about 5-6% every year. Fig.1.1 shows the rise in the electricity demand in India for the last 10 years.

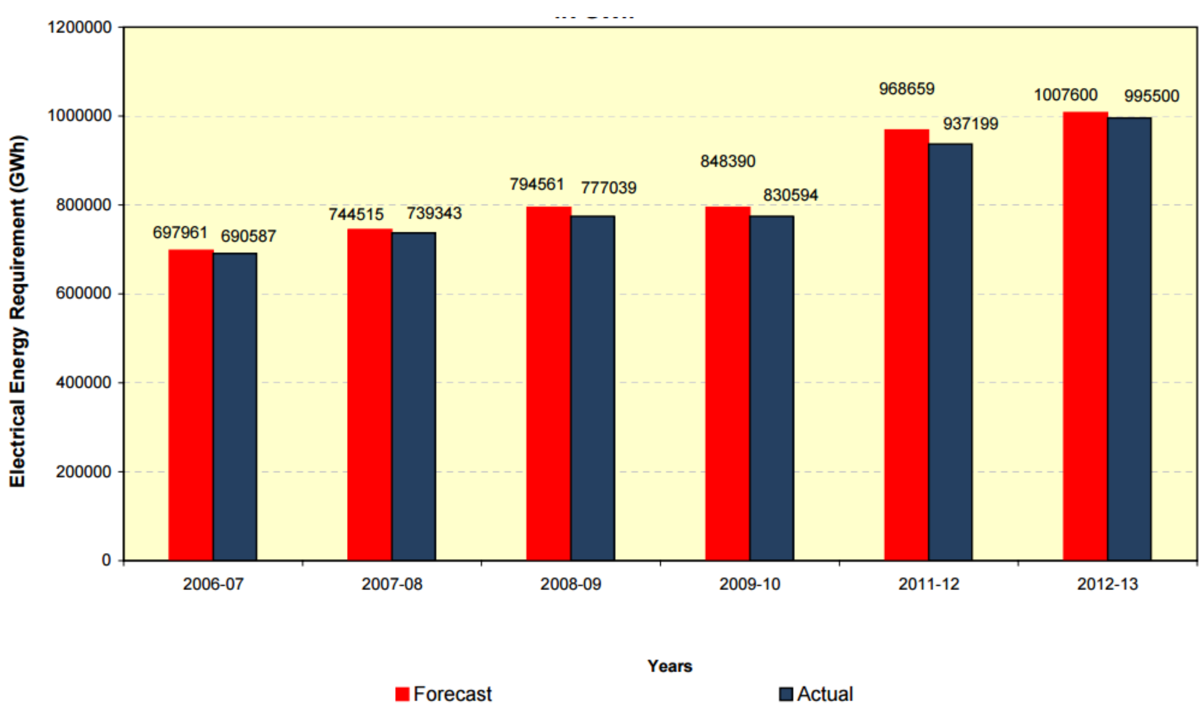


Fig.1.1: Rise in load demand in India

Source: Growth of Electricity in India 1947-2013 http://www.cea.nic.in/reports/planning/dmlf/growth.pdf

Other than upgrading the capacities of the current generators the only feasible solution is to supplement transmission grids with the addition of the new generators.

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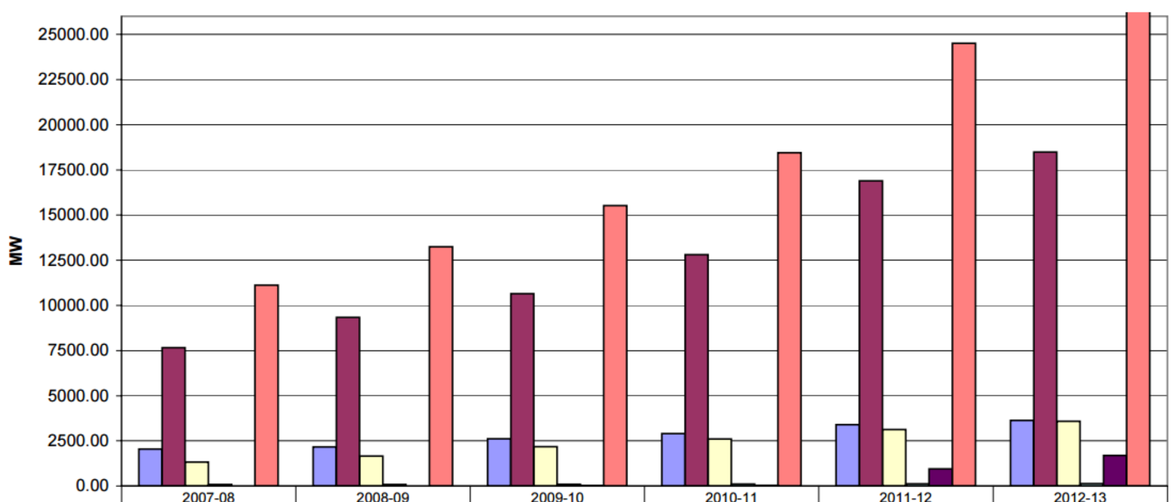
## 1.1 MOTIVATION:

Fig.1.2 shows the increase in the number of the renewable sources of energy in India from last 10 years. According to the report 'GROWTH OF ELECTRICITY SECTOR IN INDIA FROM 1947-2013'issued by the Government of India ,Central Electricity Authority ,Ministry of Power ,New Delhi the renewable sources of energy are 12% of the current installed capacity in India.




| | 2007-08 | 2008-09 | 2009-10 | 2010-11 | 2011-12 | 2012-13 |
|---|---|---|---|---|---|---|
| SHP | 2045.07 | 2160.48 | 2604.92 | 2913 | 3410.52 | 3643.17 |
| WIND | 7666.84 | 9344.13 | 10647.45 | 12806.54 | 16896.60 | 18484.99 |
| BIOMASS | 1325.63 | 1650.43 | 2167.73 | 2600.13 | 3135.33 | 3601.03 |
| U & I | 85.75 | 85.25 | 95.01 | 102.46 | 119.68 | 126.08 |
| SOLAR | 2.12 | 2.12 | 6.00 | 32.39 | 941.31 | 1686.44 |
| TOTAL | 11125.41 | 13242.41 | 15521.11 | 18454.52 | 24503.44 | 27541.71 |

Fig.1.2: Growth of generating capacity of the Renewable energy sources.

Source: Growth of Electricity in India 1947-2013 http://www.cea.nic.in/reports/planning/dmlf/growth.pdf

In particular, wind farms, which consist of many turbines, are shown to be the most cost effective form of renewable energy, and can function at the capacity levels of coal generators in high-voltage transmission systems [3]. In order to justify the concept of integrating wind (or solar) energy into the grid, renewable energy storage systems (RESS) are required to reduce the impact of the stochastic nature of these sources. The construction and maintenance of these renewable energy generators is very expensive, as even wind energy, averaging roughly $2,000 per kilowatt peak power, costs on average almost 40% more than conventional forms of generation such as coal [2].Many governments mandate that all states must generate a certain portion of their energy from

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renewable sources. Additionally, they fund a certain portion of the renewable energy cost. In the US, the standard for most companies is to get their credits from the Renewable Electricity Production Tax Credit (REPTC) mandated by the federal government, which offers 2.2 cents per KWh for wind energy production over a 10 year period [1]. This corresponds to $192,720 per MW each year. Interestingly, for a 270 MW wind farm generator, credit will cover roughly 35% of the yearly cost, which will cancel the added cost of wind versus conventional forms of generation for 10 years.

The Indian government is also supporting in the production of energy from the renewable sources of energy. Indian government is playing an active role in promoting the adoption of renewable energy resources by offering various incentives, such as generation- based incentives (GBIs), capital and interest subsidies, viability gap funding, concessional finance, fiscal incentives etc. Wind energy equipment prices have fallen tremendously due to technological innovation, increasing manufacturing scale. Prices for solar modules have also declined by almost 80% since 2008 and wind turbine prices have declined by more than 25% during the same period. The government has created a liberal environment for foreign investment in renewable energy projects. The establishment of a dedicated financial institution – the Indian Renewable Energy Development Agency (IREDA), is for development and extension of financial assistance for renewable energy and energy efficiency/conservation projects.

So in this thesis our focus is towards the establishment of the renewable sources of the energy (wind farm generators) if the current generators are not capable of supplying the increase in the load demand instead of increasing the capacity of the current generators.

## 1.2 OBJECTIVE AND SCOPE OF WORK:

### 1.2.1 OBJECTIVE :

The main objectives indentified in this work are:

1. Development and analysis of an optimization problem to find the placement and sizing of a fixed number of large scale wind farms at the transmission level to

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minimize the transmission and generation cost, given that the grid can no longer satisfy the increased load demands with its existing generation capacity.

2. Development of the technique to handle the non convexity of the optimization problem and compare it with traditionally adopted methods.

### 1.2.2 SCOPE :

The project deals with the development and formulation of the optimization problem in the existing grid where the current generators are not capable of supplying the increase in the demand. Then the new wind farm generators are introduced and location as well as sizing of these wind farm generators becomes the optimization problem variables. The objective function is the combination of the transmission (developed from the MW –Mile method) as well as generation cost of the wind farm generators. That is, the optimization problem seeks to find at which grid coordinates the wind farms should be placed, and what peak power level each should be capable of providing.

## 1.3 ORGANIZATION OF THESIS:

- Chapter 2 deals with the general concept of the optimization, different types of the optimization and different methods available to solve them. Also it explains the different algorithm used in the work like Simplex algorithm, Newton method and alternating projection algorithm. It also explains the concept of the convex and concave optimization.
- Chapter 3 explains the concept of the transmission pricing, different methods of the transmission pricing, EUAC (Equivalent uniform annual cost), Implementation of EUAC in the wind farm generators .This chapter basically forms the basis of the formulation of the optimization problem.
- Chapter 4 deals with the concept of the placement of the wind farm generators. With this concept we will formulate the optimization problem with the combination of the transmission pricing concept and EUAC of the wind farm generators explained in the chapter 3. Development of the power flow constraints and placement constraint for the wind farm generators with figures is also discussed.

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- Chapter 5 explains the above concept implemented to the practical nine bus system. This chapter basically explains the development of the solution of the optimization problem. We explain the different algorithm like, Newton method along with the KKT conditions, Alternating projection algorithm in this particular case to solve the optimization problem in the different scenarios. Also the most important aspect of the wind power plant i.e. geographical constraints (depends upon the availability of wind) and its effect on the optimal placement is also discussed.
- Chapter 5 explains the results and comparison between the different methods and concludes the best method. Appendix 1 explains the line parameters as well as the bus parameters of the nine bus system considered in the work.

The interconnection between the various chapters is shown in the Fig .1.3.Chapter 2 and 3 are interconnected and together they form the basis for the formulation of the optimization problem and the constraints. With this formulation the case study of nine bus system is taken into the consideration in chapter 5 and results are explained in the chapter 6.

## 1.4 OVERVIEW OF WORK :

The fig 1.4 also explains the overview of the work done in the thesis. The chapter 2 forms the phase 1 and contributes the 20% of the work. The phase 2 deals with the optimization problem and constraint formulation. Phase 3 deals with the different scenarios in which we have solve the optimization problem and major portion of the work (60%) is done in this section only.

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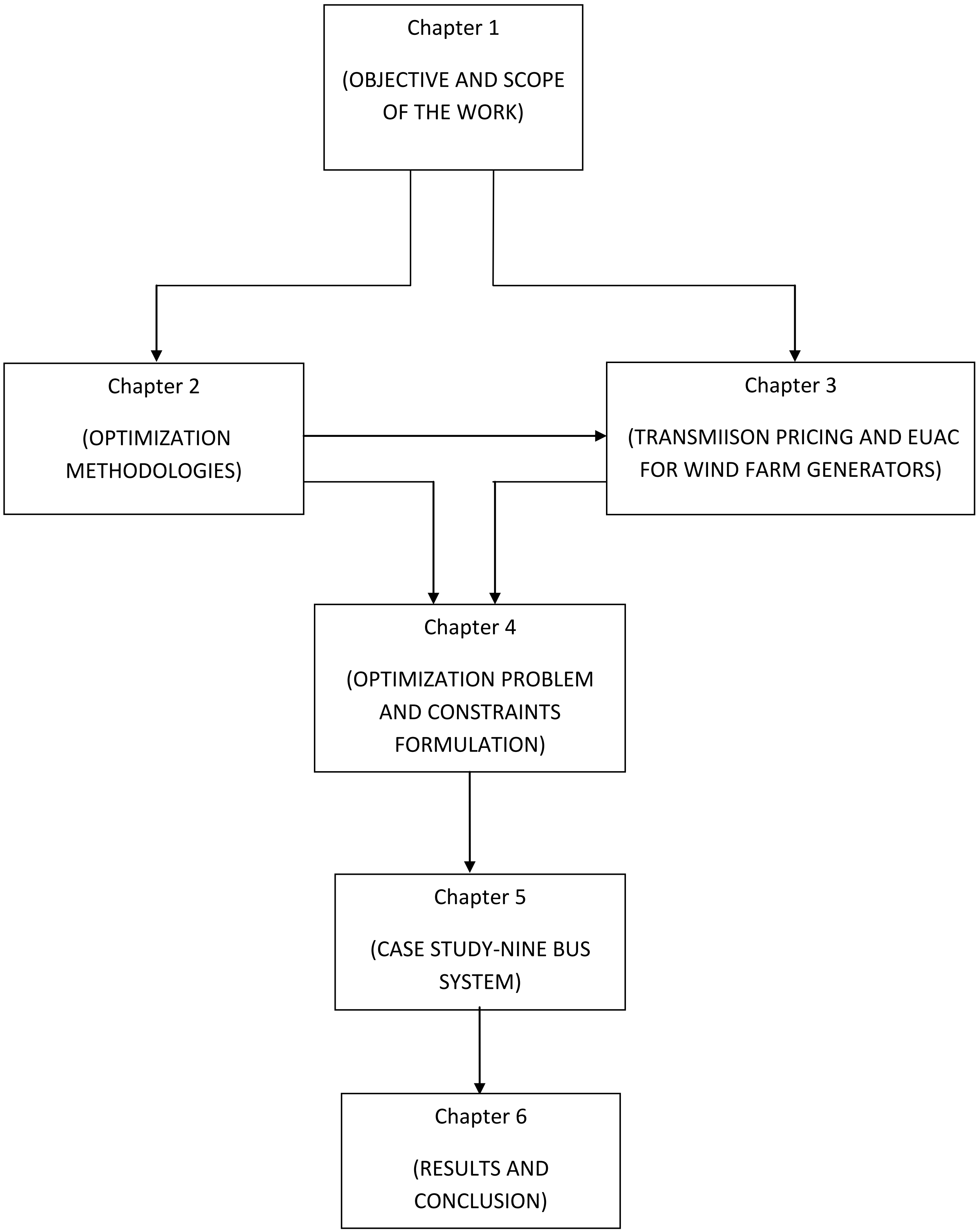


Fig.1.3: General organization of thesis

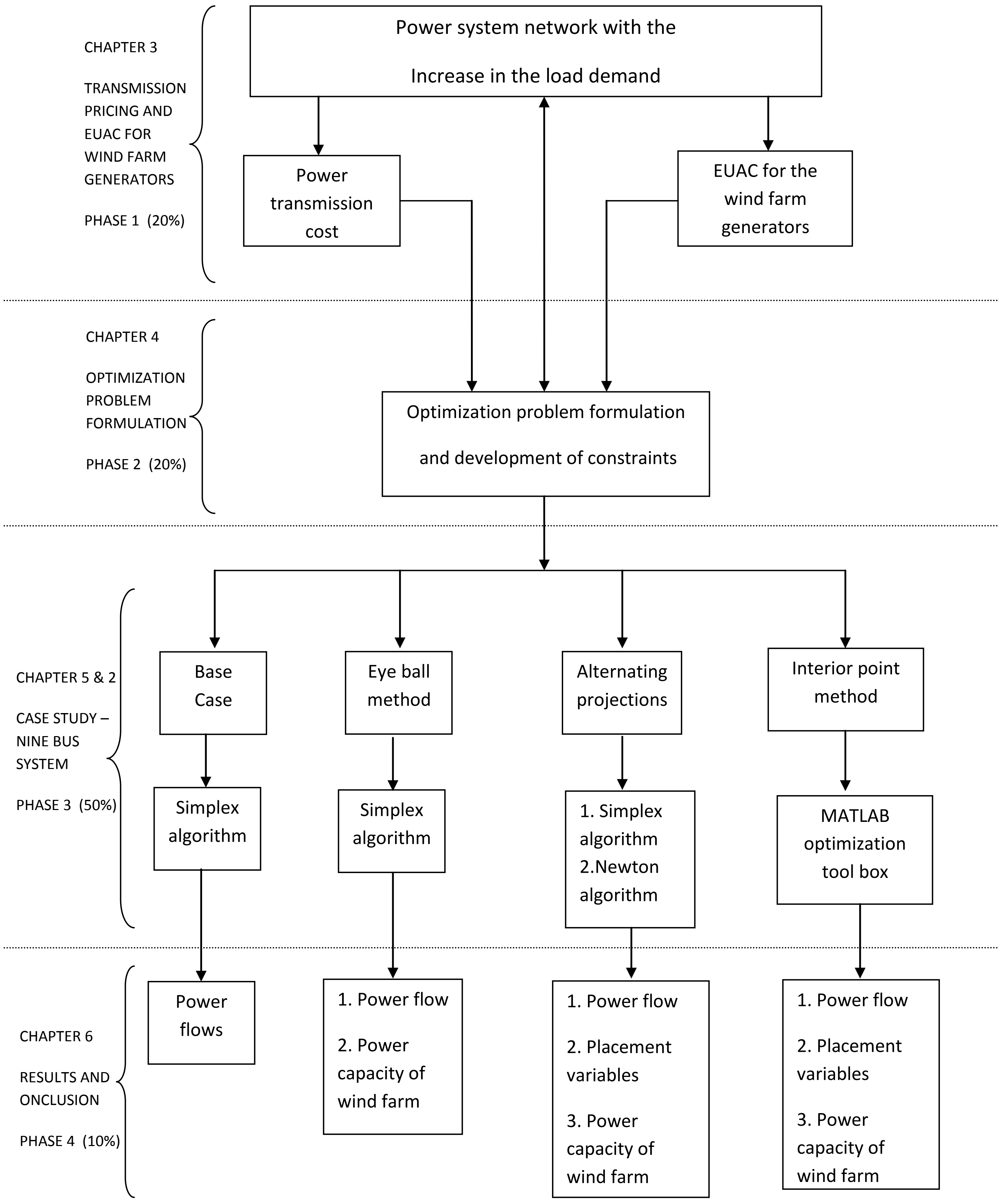


Fig.1.4 –Overview of the project work

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# CHAPTER 2

# OPTIMIZATION METHODOLOGIES

Optimization is a finding the alternative with the most cost effective or highly achievable performance under the given constraints, by maximizing the desired factors and minimizing the undesired factors. Optimization finds the variety of applications in the fields of the Power system e.g. Unit commitment, Economic load dispatch, Optimal power flow etc.

## 2.1 GENERAL OPTIMIZATION PROBLEM:

The standard form of the optimization problem is as

$$\text{Minimize} \quad f(x)$$

Subject to

$$g_i(x) \leq 0 \text{ , } i = 1, 2 \text{ …………m,}$$

$$h_i(x) = 0, i = 1,2 \dots \dots \dots \dots . n$$

where

$f(x)$ is the objective function to be minimzed over x

$g_i(x) \leq 0$ are called inequaity constraints , and

$h_i(x) = 0$ are called the equaity constraints .

## 2.2 TYPES OF THE OPTIMIZATION PROBLEM :

Optimization problems can be classified based on the various parameters like type of the constraints, number of solutions, nature of the equations, nature of the variables, values of the design variables and number of the objective functions. The different types of the optimization problems and theirs basis are shown in the Fig 2.1.However in our work is limited to the use of only linear, non- linear and convex and concave optimization and their methods to solve.

### 2.2.1 LINEAR OPTIMIZATION PROBLEM:

If the objective function and all the constraints are linear functions of the decision variables, the mathematical programming problem is called a linear programming (LPP) problem.

Standard LPP is defined as

Minimize

$$C^T x$$

Subject to

$$Ax = b$$

And $x \geq 0$

Where $x$ represents the vectors of the decision variables to be determined, C and b are vectors of the known coefficients, A is the matrix of the coefficients of the size $m \times n$.

n = Number of the decision variables.

m= Number of the constraints.

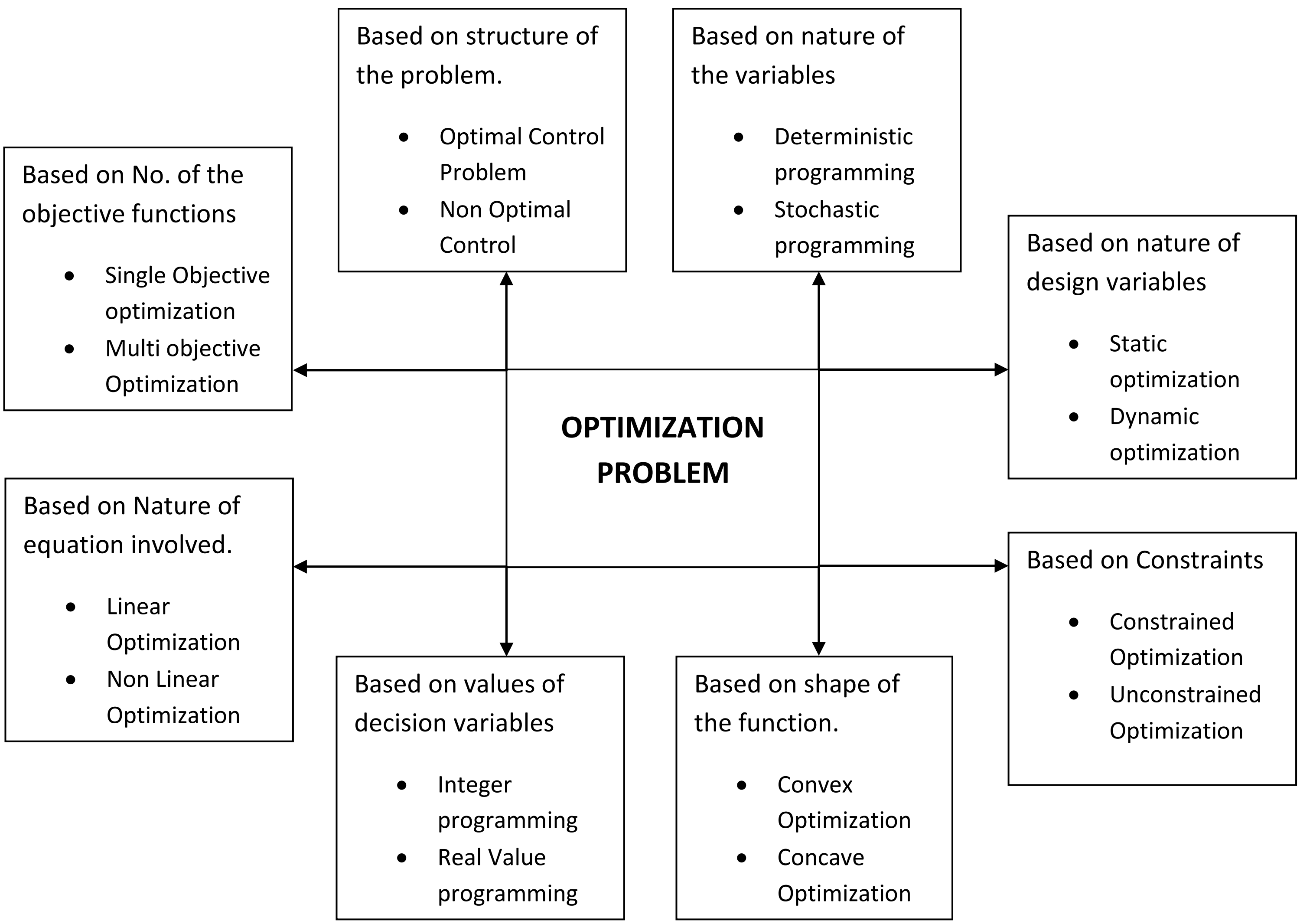


Fig.2.1: Types of the Optimization problem

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### 2.2.2 NON LINEAR OPTIMIZATION PROBLEM:

If any of the functions among the objectives and constraint functions is nonlinear, the problem is called a nonlinear programming/optimization (NLP) problem.

### 2.2.3 CONVEX OPTIMIZATION PROBLEM:

A function f (x) is called convex if, for every y and z and every $0 \le \lambda \le 1$,[10]

$$f[\lambda y + (1 - \lambda)z] \le \lambda f(y) + (1 - \lambda) f(z). \quad (2.1)$$

It is called strictly convex if, for every two distinct points y and z and every $0 < \lambda < 1$, [10]

$$f[\lambda y + (1 - \lambda)z] < \lambda f(y) + (1 - \lambda) f(z). ). \quad (2.2)$$

The left hand side in this definition is the function evaluation on the line joining x and y; the convex function definition is shown in the Fig.2.2.

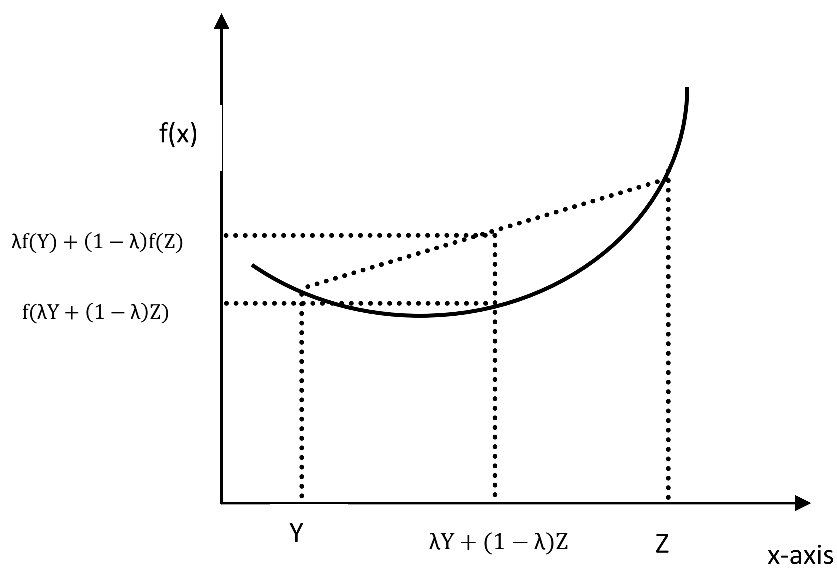


Fig.2.2: Convex Functions

There are some of the properties of the convex functions[10]:

`

- Every local minimum is a global minimum in the convex optimization.
- The set of the local/global minima of the convex optimization is the convex set.
- If all Eigen values of the Hessian are positive the function is strictly convex

A convex minimization problem is thus written as[10]

$$\text{Minimize } f(x)$$

Subject to

$$g_i(x) \leq 0, \; i = 1, 2 \ldots\ldots\ldots m,$$

$$h_i(x) = 0, \; i = 1,2 \ldots \ldots \ldots \quad n$$

where

$f(x)$ is the objective function to be minimzed over x and should be convex.

$g_i(x) \leq 0$ are called inequaity constraints and it should be convex .

$h_i(x) = 0$ are called the equaity constraints and should be affine.

### 2.2.4 CONCAVE OPTIMIZATION PROBLEM:

A function f (x) is called concave if, for every y and z and every $0 \leq \lambda \leq 1$,

$$f[\lambda y + (1 - \lambda)z] \geq \lambda f(y) + (1 - \lambda) f(z).\ ). \tag{2.3}$$

It is called strictly concave if, for every y and z and every $0 < \lambda < 1$,[10]

$$f[\lambda y + (1 - \lambda)z] > \lambda f(y) + (1 - \lambda) f(z).\ ). \tag{2.4}$$

Concave functions are simply the negative of convex functions.

Also the linear programming problem is convex and concave optimization problem

## 2.3 METHODS TO SOLVE OPTIMIZATION PROBLEM:

There are many methods available to solve this type of the optimization problem.

Table 2.1 lists the different techniques used in solving the different types of the optimization problems.

`

Table 2.1: Methods to solve the optimization problem

| Problem type | Unconstrained optimization | Constrained optimization | |
|---|---|---|---|
| | | Linear optimization problem | Non linear optimization problem |
| Methods available | Dichotomous search | Simplex method | Penalty factor method |
| | Fibonacci search | | Branch and bound method |
| | Golden section method | | Newton method (KKT conditions to be satisfied) |
| | Bisection method | Interior point method | Trust region algorithm |
| | Newton method | | Active set algorithm |
| | Steepest descent method | | Interior point method |

Apart from these methods there are many evolutionary methods which are used in the field of the optimization. These are listed below

- PSO (Particle swarm optimization)
- GA ( Genetic algorithm)
- Ant colony optimization
- Bat optimization
- Bee optimization
- Alternating projections

We will discuss some of the algorithms which are being used in the project work.

### 2.3.1 SIMPLEX ALGORITHM:

The Simplex algorithm for the linear programming problems is explained as,

For the linear programming problem,

$$\text{Minimize} \quad f(x)$$

Subject to

$$AX \leq b , i = 1, 2 \ldots\ldots\ldots m, x \geq 0 ;$$

The Simplex algorithm is as follows:

`

- For the Matrix A by adding the Slack variables in the equations so that the equation will be in the form of AX=b. The additional slack / surplus variables are called Basic variables. Make all other non basic variables zero and the values of the basic variables are called the initial basic feasible solution.
- Check the value of $C_j - Z_j \geq 0$ for the minimization problem then it is the current basic feasible solution and we have to stop here. The vice versa is applicable for the maximization problem.
- Select the number q such that $C_q - Z_q < 0$ . For Maximization problem we have to select with most positive value.
- If all the elements in the A matrix corresponding to the q i.e. $y_{iq} \leq 0 \; \forall \; i$ then the problem is unbounded then stop.
- If $y_{iq} \geq 0 \; \forall \; i$ then we have to do the ratio test and select the p = arg min$\left\{\frac{y_{i0}}{y_{iq}}\right\}$. So q will be the incoming variable and p will be the outgoing variable.
- Perform the row operations so that with the revised array so that the elements corresponding to basic variables will form the identity matrix.
- Repeat the above step until all $C_j - Z_j \geq 0$ .

Also in this algorithm the condition is that the entire element on the right hand side of the constraints i.e. b vector should be positive and also the decision variable should be positive.

### 2.3.2 ALTERNATING PROJECTION METHOD:

Let C and D are the closed convex sets in $R^n$ and let $P_c$ and $P_D$ denote the projection on the C and D respectively .The Algorithm starts with any $x_0 \in C$ and then alternatively projects onto the C and D[3]:

$$y_k = P_D(x_k) \,, \quad x_{k+1} = P_c(y_k) \quad , k=1, 2,3\ldots\ldots$$

This generates the sequence of the points $x_k \in C$ and $y_k \in D$ .

If $C \cap D \neq \emptyset$ then the sequences $x_k$ and $y_k$ both converge to a point $x^* \in C \cap D$. Alternating projections finds a point in the intersection of the sets, provided they intersect.

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Alternating projections is also useful when the sets do not intersect. In this case we can prove the following. Assume the distance between C and D is achieved (i.e., there exist points in C and D whose distance is dist(C, D)). Then $x_k \rightarrow x^* \in$ C, and $y_k \rightarrow y^* \in$ D, where $\| x - y \|_2$ = dist(C, D). In other words the alternating projection gives the pair of the point in C and D that has minimum distance. A simple example of the both cases is shown in the Fig.2.3 and 2.4.[3]

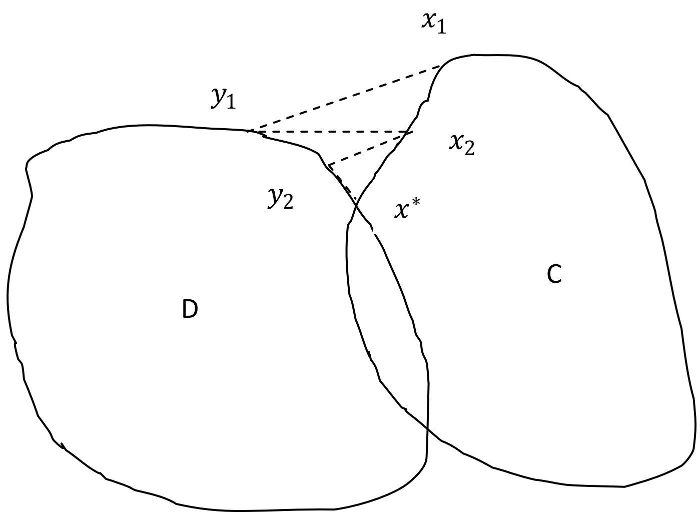


Fig.2.3: First few iteration of the alternating projections. Both sequences are converging to the point $x^* \in C \cap D$

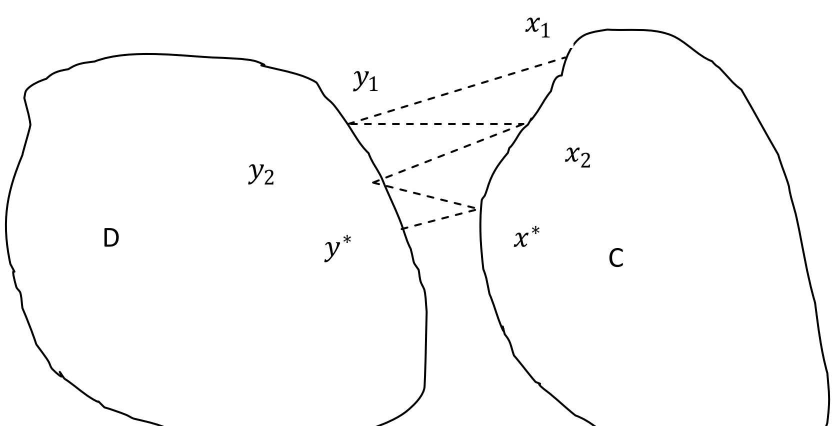


Fig.2.4: first few iteration of the alternating projections for the case $C \cap D = \emptyset$ . The sequence is converging to $x^* \in$ C and $y^* \in$ D.

`

### 2.3.3 NEWTON METHOD:

Let f(x) is the function to be minimized (here x represents the vector of variables) with the $g_i(x) \leq 0$ be the inequality constraints where i = 1, 2, 3…..m. If the inequality is not in the less than equal to sign then perform the necessary operations on the constraint equation to bring it into the required form.

**Step 1:** Define the Lagrange's function as

$$L = f(x) + \sum_{i=1}^{m} \lambda_i \, g_i(x) \qquad (2.5)$$

Where $\lambda_i$ represnt the decent direction for inequality function as well as for the function f(x) itself.

**STEP 2:** Make the Set of the equations by taking first order derivatives of $L$ with vector x equating them to zero and take the complementary slackness condition form the KKT Conditions.

$$\lambda_i g_i(x) = 0 \qquad (2.6)$$

These set of the equations are called Jacobians $J_k$.

**STEP 3:** Make the Hessian matrix $H_k$ by differentiating all the set of the equations with all the variables including all $\lambda_i$'s.

**STEP 4:** Calculate the direction d by taking the step size as 1.

$$d_k = -H_k^{-1} \times J_k \qquad (2.7)$$

Now the algorithm for the Newton Method is explained as,

- Initialize the set of the variables $x_k$ for all variables in the vector x and all $\lambda_i$'s.
- Calculate all $d_k$ as mentioned in the above equation.
- New Set of the variables are $x_{k+1} = x_k + d_k$.
- Check the values of all $\lambda_i$'s . if $\lambda_i < 0$ then stop ( As KKT Condition violated ).
- If $x_{k+1} - x_k \leq \in$ then Stop, where ∈ is the already defined error.

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# CHAPTER 3
# METHODOLOGY FOR TRANSMISSION PRICING AND WIND POWER PLANTS

## 3.1 TRANSMISSION PRICING CONCEPT:

In the past few years many studies have been done to compare the different transmission pricing schemes [6][5] and developed the various cost models and they have evaluated the potential of integrating the renewable energy generators. In terms of the transmission cost different countries have the different cost allocation method. The brief classification of the different transmission pricing method is explained in the next section.

### 3.1.1 CLASSIFICATION OF TRANSMISSION PRICING METHOD:

All the existing as well as proposed transmission pricing models are the cost based models. Based on this, transmission pricing paradigms can be defined which convert the transmission costs into transmission charges [5].The basic paradigms is the Embedded transmission pricing. This pricing is also known as Rolled in transmission pricing .All the costs including the infrastructure and future investment, network operating costs are summed up and then allocate to the various customers through the cost allocation philosophy[5].

The Basic structure of this type of the transmission pricing is as shown in the Fig.3.1.

The Commonly used cost allocation methods are discussed below:

- **Postage Stamp Method :**

  It is the simplest and easiest method of the transmission pricing. It allocates the fixed cost per megawatt of the power demand to each distribution substation. This method does not include the distance involved during the transmission of the power. The transaction or the charge /amount can be calculated based upon the transmission usage by both generators as well as loads or only through the loads.

  The transacted power for a particular transmission transaction is given as [5],

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$$R = TTP \times \frac{P_t}{P_{peak}} \qquad (3.1)$$

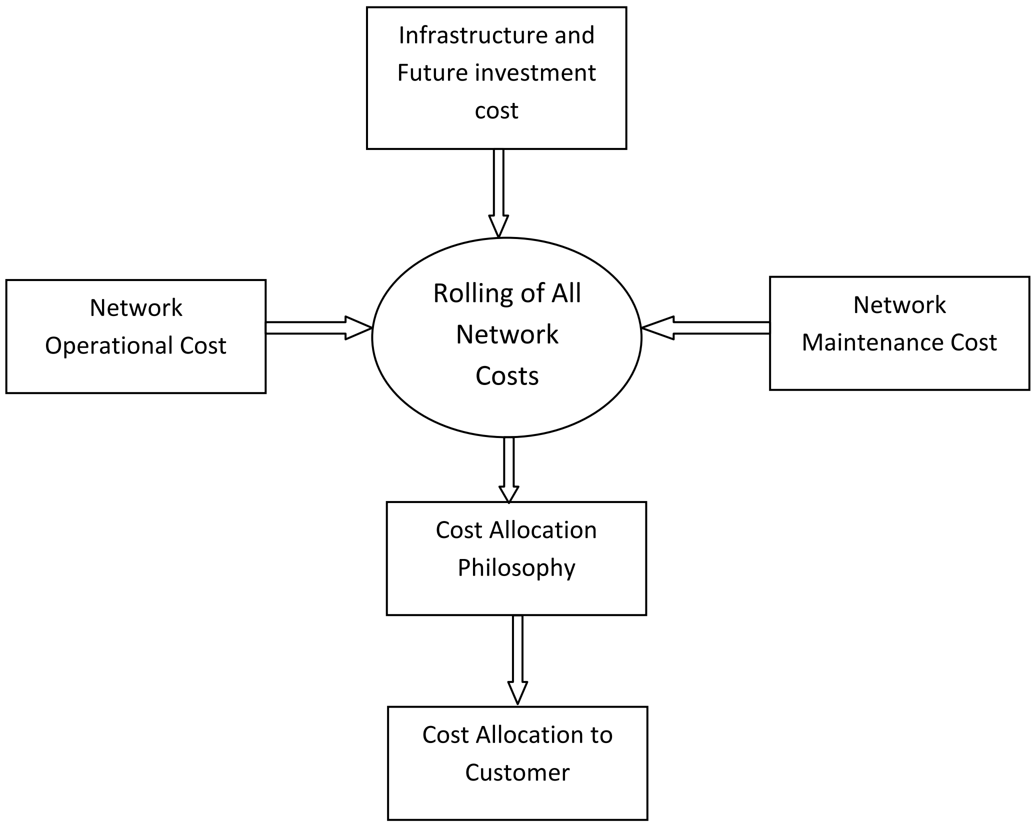


Fig 3.1: Rolled in /Embedded Transmission pricing

Where R = Transmission price for the transaction t

TTP = Total transmission Charges,

$P_t$ = Load during the transaction t,

$P_{peak}$ = Peak load during the transaction.

The main purpose of using this methodology is the entire system is assumed to be centrally operated integrated system. Since this method does not include the distance involved in transacting the power, so it does not give the correct economical signal to the customers.

- **MW – Mile Method:**

This method overcomes the problem we face in the postage stamp method. This method analyzes the each transmission line used by each customer and takes into the account power flow, cost, length and capacity of the each line [5]. The MW-mile method

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is considered as one of the most economically pricing schemes because it takes the power flows of each user, the capacities, and the distances of the network into account. The MW-mile transmission cost allocation is determined as follows:

**Step 1:** For each load k, the transaction power flows on all transmission lines $(i, j) \in Z$ are calculated, where Z is the set of all line connections.

**Step 2:** The cost allocated to each load is calculated.

**Step 3:** The total transmission cost is determined as:

$$\text{Transmission cost} = \sum_{k} \sum_{(i,j)}^{Z} C_{ij} \, L_{ij} \, \frac{|P_{ij}|}{P_{ij(m)}} \tag{3.2}$$

Where

| | |
|---|---|
| $C_{ij}$ | Cost per unit length of the line (i,j) |
| $P_{ij(k)}$ | Power flow over the line (i,j) as a result of the bus k |
| $P_{ij}$ | Power flow over the line (i,j). |
| $L_{ij}$ | Length of the line (i,j) |
| $P_{ij(m)}$ | Power capacity of the line (i,j). |

There are many variations in this method. Many books/references are suggesting that the power flows should come in the denominator of equation (3.2) rather than the line capacity. However it is considered unfair if we take the case in which the line is having the very low percent of utilization then the customer will be charged very heavily.

**3.2 WIND GENERATION PROGRESS IN INDIA:**

The Wind farm was developed in India in 1990s and has significantly increased in the last years. India has the fifth largest installed wind power capacity in the world. As of 31 March 2015 the installed capacity of wind power in India was 22,645 MW mainly across the states like Tamil Nadu, Gujarat, Maharashtra, Karnataka and Rajasthan.

Fig 3.2 is defining the progress in India’s installed wind power generation capacity since 2006. India still is having the huge potential for establishment of the wind farm plants. The total wind

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energy potential in India has been estimated up to 60000 MW. Currently more than 80% of the installed capacity from the renewable sources of energy is through wind power only

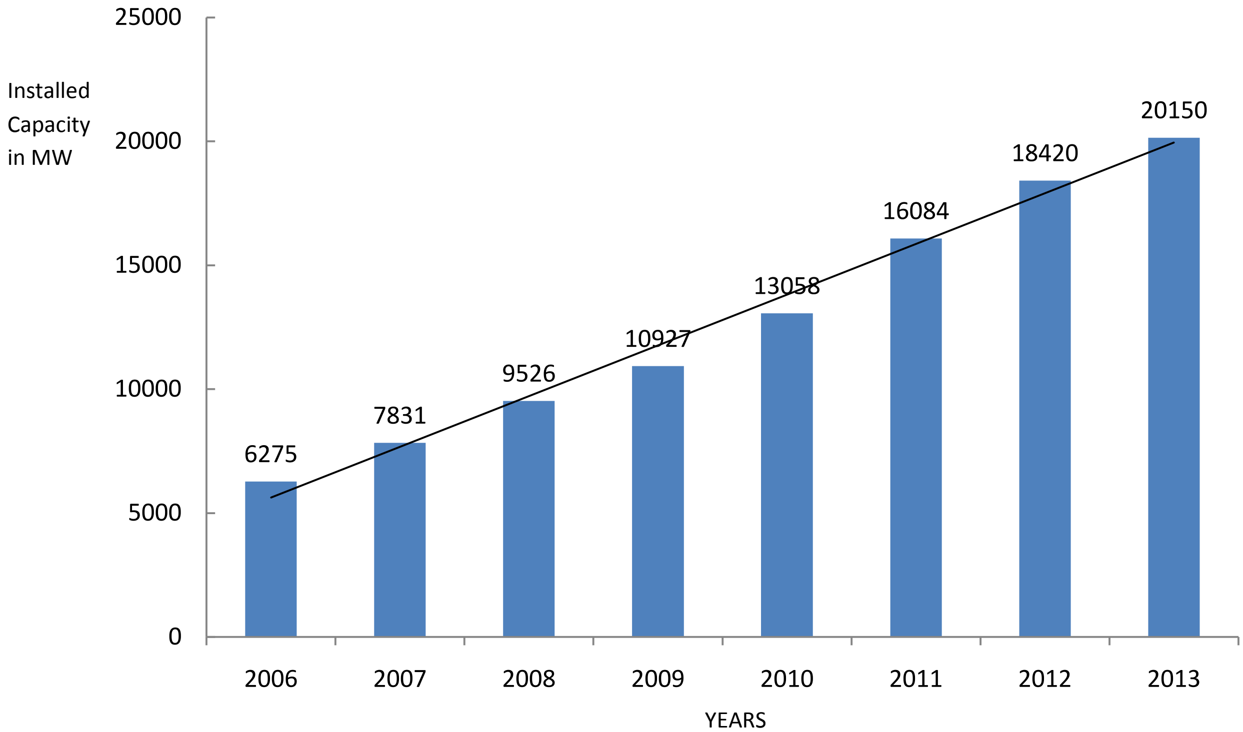


Fig.3.2: Progress in the Wind farm Generation capacity over the years

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There are many wind power institutions as well as associations like Center of Wind energy technology (CWET), Indian wind energy association (INWEA) Indian Wind turbine Manufacture Association (IWTMA) for promoting the growth of the wind power in India. Also from time to time there are different types of subsidies are being given for the wind power development by the Government of India.

The rapid growth in wind power can be attributed to the advancement in the technology and the manufacturing improvements in the field of the wind power. Wind power is emerging as an environment friendly alternative to meet the ever increasing world demand for electricity, at an affordable price. So this is the reason we are assuming, in our work the wind farm generators to be placed as compared to any other renewable source of energy.

`

In our work we will now assume that the additional increase in the demand will be meet by the wind farm generators as other generators connected in the grid are not capable of supplying the increase in the demand. So we will now place the wind farm generators in the existing grid to meet the additional rise in the demand. So in the next section we discuss the cost model of the wind farm generator which is included in the optimization problem formulation.

### 3.2.1 WIND FARM GENERATORS COSTS:

The basic block diagram of the wind farm generation is shown in the Fig 3.3. The first block consists of the many turbines which generate the AC output. Then it requires the AC output to be converted to the DC for the storage purpose. For this AC to DC converter is being used. This converted DC is store in the batteries. Following these batteries is a DC-AC inverter to convert this storage back to AC. Finally, a grid-interfacing transformer is needed to step up the medium voltage (MV) output of the wind farm and the inverter to the high voltage (HV) of the transmission system.

Wind farms are very expensive due to the high replacement costs of the components, especially because of the turbines and the batteries. . The other two components converter and inverter can be represented by the linear function of the power capacity of the farm.

The price rating characteristics of the transformer are explains as [11],

$$C_m = C_n \times [\frac{P_m}{P_n}]^s \tag{3.3}$$

Where $C_m$ , $C_n$ , $P_m$ and $P_n$ are the costs and power levels of the base case (n) and desired transformers (m) .Here s is constant and vary between 0.4 - 0.6. Here s = 0.4.

In the chapter 5, table 5.9 we have summarized the life expectancies and prices of the various components obtained from the available data [7], [2]. We have to include the each component of the wind in calculating the total wind farm costs. Since all the components have the unequal life span so best method to determine the cost effectiveness of the wind farm plant is to go with the Equivalent Uniform annual cost (EUAC) which is explained in next section.

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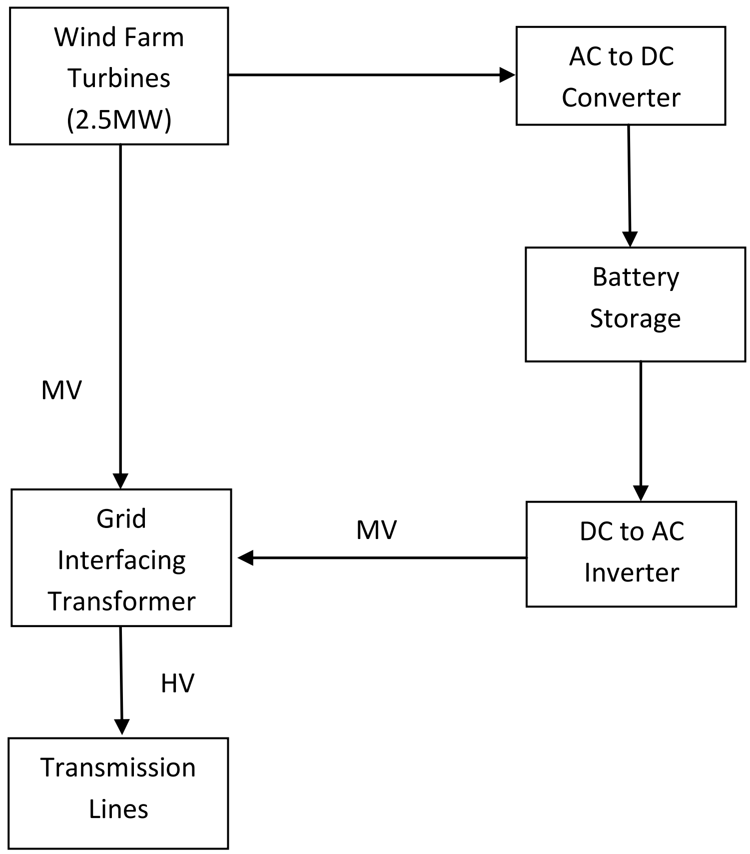


Fig.3.3: Wind farm Block diagram (include the major Components)

### 3.2.2 EUAC (EQUIVALENT UNIFORM ANNUAL COST) :

Equivalent uniform annual cost (EUAC) is the cost of the owning and maintain an asset over the entire life time .It basically converges all the cash flows into the annual payment in terms of the today's money. So it is easy to make the comparison between the two assets who have different life spans.

The EUAC is defined as,

$$\mathrm{EUAC} = \mathrm{A} \times \frac{\mathrm{t}(1+\mathrm{t})^{\mathrm{n}}}{(1+\mathrm{t})^{\mathrm{n}}-1} \qquad (3.4)$$

`

Where n is the useful life span of the component and t is the interest rate. A is the asset price/NPV. The equation (3.4) allows us to compare the cost effectiveness of various assets. It is best used where the investment projects do not have the equal lifespan. A is also called the net present value (NPV) [8].EUAC can be used in the following scenarios.

1. Assessing the alternative projects of the unequal life spans.
2. Assessing whether giving an asset on rent is more economical or purchasing.
3. Assessing whether high maintenance costs will change the useful life of an asset.
4. Comparing to estimate annual cost savings, in order to determine whether it makes economical to invest.
5. Estimating the cost savings in the purchase of new equipment/machinery.

In the chapter 5, table 5.9 we have the components of wind farm generators which have the different costs as well as the different life spans as explained in the chapter 5. So EUAC is the best method to calculate the wind farm generators costs. So the implementation of the EUAC in the wind farm generators to calculate the cost is explained in the next section.

**3.2.3 EUAC IN WIND FARMS:**

Table 5.9 in chapter 5 gives the details of the life span and the cost of the different components shown in the Fig.3.3. Since the cost of each component is in per MW, so the NPV of the each component are the multiplication of the cost ($C_c$) and the capacity of the wind farm. So the generation cost of integrating the w number of the wind farms in the existing grid is given as,

$$\text{Generation cost} = \sum_{w}\sum_{c}^{n} C_c \frac{t(1+t)^{n_c}}{(1+t)^{n_c}-1} P_{gw\,(m)} \qquad (3.5)$$

Where $n_c$ is useful life component of the component c and $C_c$ is the cost per MW of the component c and t is the interest rate = 7%.$P_{gw\,(m)}$ is the maximum capacity of the wind farm generator. n is the number of the components in the wind power plant.

`

# CHAPTER 4

# OPTIMIZATION PROBLEM & CONSTRAINT FORMULATION

Since as of now we have developed the cost model for the transmission cost of the network (Equation (3.2)) and also the cost model (with EUAC) (Equation (3.5)) for integrating the wind farm generators in the network.

We will define the optimization problem to find out placement and capacity of the fixed number of the wind farm generators at the transmission level so that our transmission cost as well as generation cost will get minimized because the existing generators in the grid are no longer to supply the increase in the demand. So we will find out the grid coordinates at which the wind farm should be placed and what is the peak power levels each should be capable.

## 4.1 CONCEPT OF INTEGRATION A WIND FARM:

Fig.4.1 explains the simple case in which the optimal placement of the wind farm within the existing grid can be seen as the network optimization problem [9].We will consider each bus as the node. The existing grid is shown in the black and the dotted lines are showing the potential connection of the wind farm to the bus. At initial stage the connection between the buses and the newly added buses is assumed to be with each and every bus shown by the red dotted lines in the Fig 4.1 .However these connections has to be included in the final distribution system depends upon the optimal power flow between these lines. This means if we are adding the new bus to the grid then it need not to be that all the busses present in the grid should be connected to the newly added bus .It entirely depends upon the optimal power flow between the lines i.e. optimal power flow has to be carried out for that particular system, then based upon the power flows between the lines will decide which connection between the existing buses and newly added buses has to be included in the final distribution system.

In other way the red line to be included in the final distribution system will depend upon the optimal power flow between the lines.

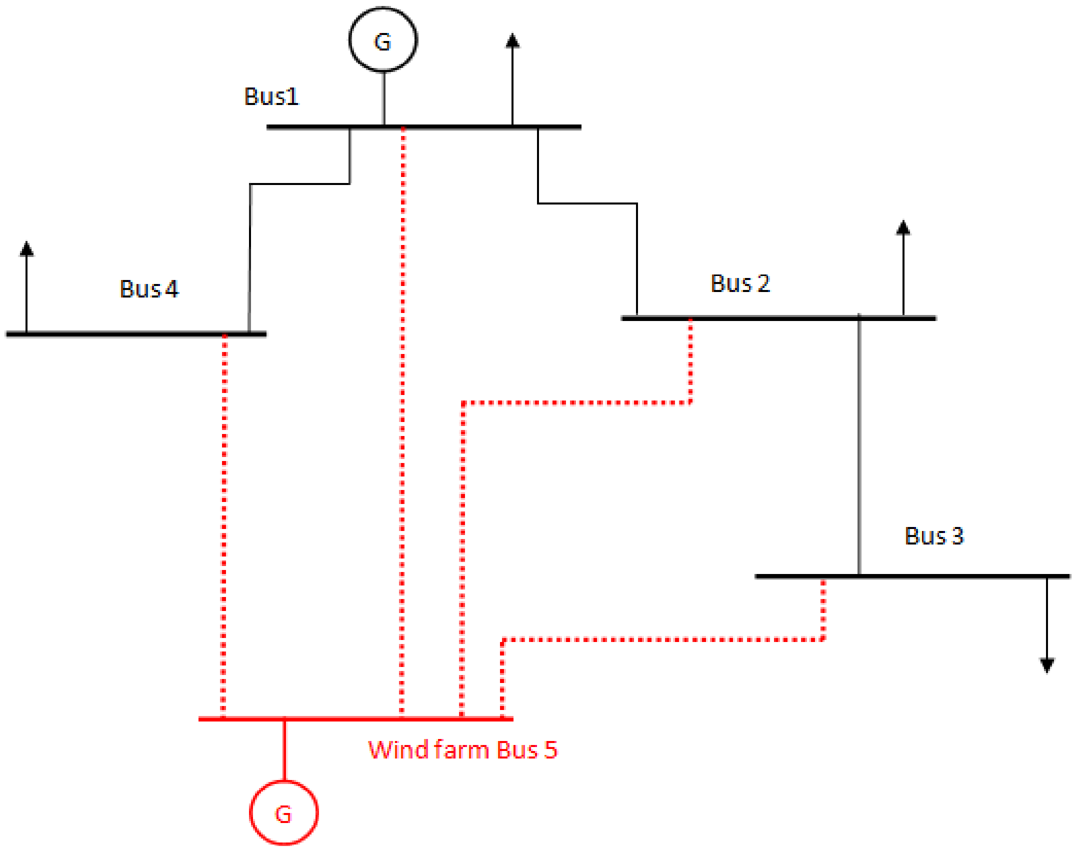


Fig.4.1: Illustration of the problem of integrating wind-farms into the current transmission system at the optimal location and power levels to minimize total cost.

So we have to find out the grid coordinates as well as the capacity of the wind farm and then integrate the wind farm in that location .Initially the connections has to be made with each and every bus and after conducting the optimal power flow, we will come to know the actual wind farm connections with the other buses which has to be connected in the final distribution system.

Before defining the problem and constraints, let us define the terms that will be used in define the objective function as well as the constraints.

Table 4.1 defines these parameters used in objective function as well in the constraint.

`

Table 4.1: Various parameters used in the objective function and constraints.

| Parameter | Definition |
|---|---|
| $(i, j) \in Z$ | Line (i,j) connecting buses i and j in the set of lines Z. |
| $w \in B$ | Wind farm bus w in the set of the all the buses B. |
| $i \in B$ | Bus i in the set of the all the buses B. |
| $C_{ij}$ | Cost per unit length of the line (i,j) |
| $P_{ij}$ | Power flow over the line (i,j). |
| $x_i$ | Position vector of the bus i in $R^2$ |
| $d_{min}$ | Minimum allowable distance between the buses. |
| $r \in R$ | A restricted area r in the set of all restricted areas. |
| $X_r$ | The rectangular set of the points in $R^2$ defining restricted area r. |
| $r_r$ | It defines the $X_r$. Its Components are 1.Horizontal coordinate of the center 2.Vertical coordinate of the center.3.Vertical dimension of the center .4.The horizontal dimension. |
| $L_{ij}$ | Length of the line (i,j) |
| $K_{wind}$ | Equivalent annual cost of wind farm per unit power capacity. |
| $P_{ij(m)}$ | Maximum power flow capacity of the line (i,j) |
| $P_{j(m)}$ | Maximum power flow capacity of the generator bus j. |
| $P_{L(j)}$ | Load at bus j |

## 4.2 OPTIMIZATION PROBLEM FORMULATION:

So our optimization problem states that for the given load increase at each bus, what should be the optimal placement and capacity of the wind farm of the wind farm generators to minimize the total cost while satisfying the load demand?

`

So our objective function is to minimize the sum of the equation (3.2) and equation (3.5).

Minimize

$$\sum_{k}\sum_{(i,j)}^{Z} C_{ij}\, L_{ij}\, \frac{|P_{ij}|}{P_{ij(m)}} + \sum_{w}\sum_{c}^{n} C_c \frac{t(1+t)^{n_c}}{(1+t)^{n_c}-1} P_{gw(m)} \qquad (4.1)$$

In this optimization problem we will use the $L_2$ norm of the difference $x_i - x_j$ to represent the line length .The $L_2$ norm is best suited for the transmission network which is large in the size, where the Euclidian distance is the best measure of the line length of the two buses. So the length of the line $L_{ij}$ can be defined the Euclidian distance between the buses i and j. Also we are assuming the transmission system to be very large so all the buses are assumed to be the node. So the length of the line is

$$L_{ij} = \left\||x_i - x_j|\right\|_2 \qquad (4.2)$$

Since for the wind farm generator the equivalent uniform annual cost will be the constant so we can take the EUAC as the constant as defined as,

$$K_{wind} = \sum_{c}^{n} C_c \frac{t(1+t)^{n_c}}{(1+t)^{n_c}-1} \qquad (4.3)$$

Also let's remove the outer summation in equation (4.1) as we are considering the aggregate cost over the each line. So after substituting the values of (4.2) and (4.3) in (4.1) we get,

Minimize

$$\sum_{(i,j)\in Z} C_{ij} \frac{|P_{ij}|}{P_{ij(m)}} \left\||x_i - x_j|\right\| + \sum_{w\in B} K_{wind}\, P_{gw(m)} \qquad (4.4)$$

The first term in the equation (4.4) is the transmission cost throughout the network .The second term in the equation (4.4) is the added generation cost, where the constant $K_{wind}$ is determined from the equation (3.5). The reason for including the second term is that the network should install only the amount of the capacity it needs to satisfy the increase in the demand.

`

### 4.3 DEVELOPMENT OF THE CONSTRAINTS

This optimization problem requires the following constraints.

- **Power flow Constraints :**

  In any transmission system, the sum of the total power entering in the bus and the total power generated at the bus must be equal to the total load demand. In other words, the difference between the load demand and the power flows must not exceed the bus capacity of the bus. This is shown in the Fig. 4.2.

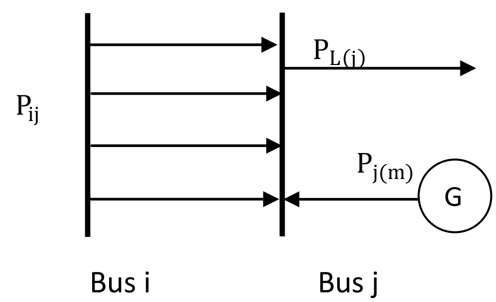


Fig.4.2: Power flow constraints.

This can be mathematically represented as,

$$P_{L(j)} - \sum_{i \in B} P_{ij} \le P_{j(m)} \qquad \forall\, j \in B \qquad (4.5)$$

Also the power generated at each bus is varied from 0 to $P_{j(m)}$ .At the load bus the difference between the load demand and the power flow must be equal to 0 (assuming the loss less line). It can be mathematically represented as,

$$P_{L(j)} - \sum_{i \in B} P_{ij} = 0 \text{ (at Load Bus)} \qquad \forall\, j \in B \qquad (4.6)$$

The power flow between any two lines should not exceed the line capacity .This can be mathematically represented as,

$$|P_{ij}| \le P_{ij(m)} \qquad \forall\, (i,j) \in Z \qquad (4.7)$$

- **Placement Constraints:**

  Since there should be always some distance between the buses to avoid the short circuit

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and safety purpose. The placement constraint equation states that the added generators cannot be within the certain radial distance $d_{min.}$ This is explained in the Fig.4.3.

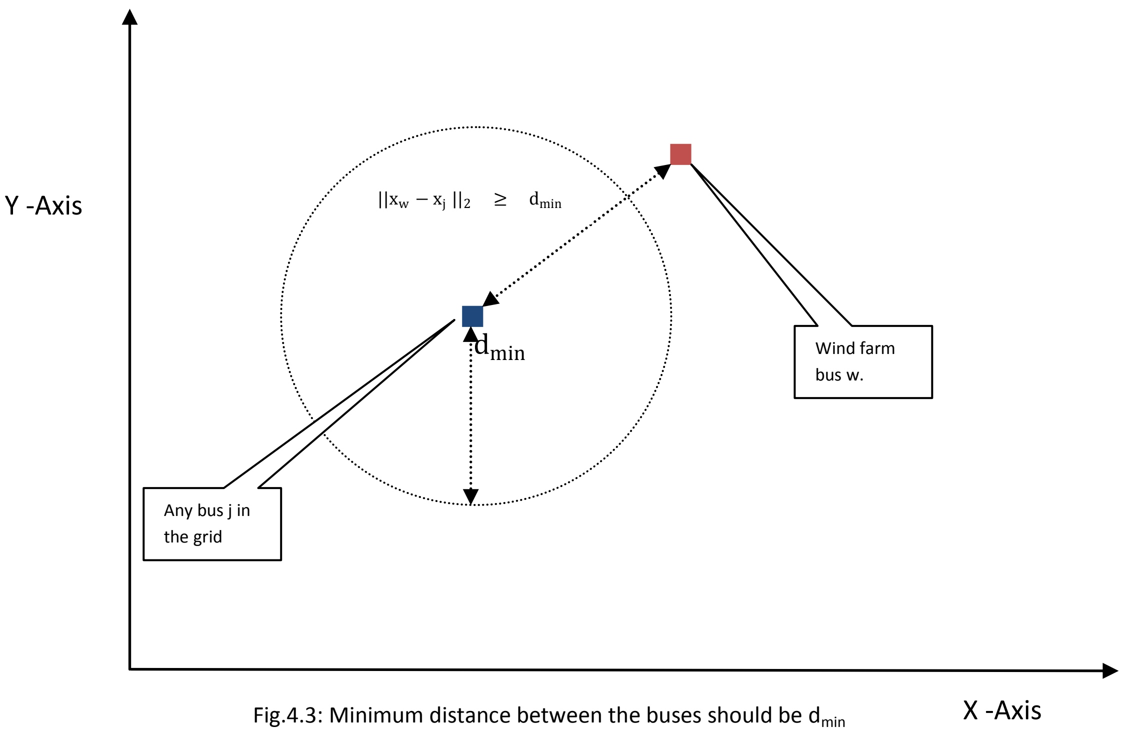


Fig.4.3: Minimum distance between the buses should be $d_{min}$

This can be represented in the mathematical form as,

$$||x_w - x_j||_2 \quad \geq \quad d_{min} \qquad \forall\ Z, j \in B; w \neq j \qquad (4.8)$$

Also we cannot place the generators where there is no wind. So for the wind farm generator the major constraint is the placement constraint. So the newly added generators cannot be within any restricted areas, each of which is defined by the rectangular set of the grid coordinates $X_{r.}$

The restricted areas can be pictorially defined as shown in the Fig .4.4. The dimension of the restricted areas includes the horizontal and vertical coordinates of the center and horizontal and vertical distance of the areas.

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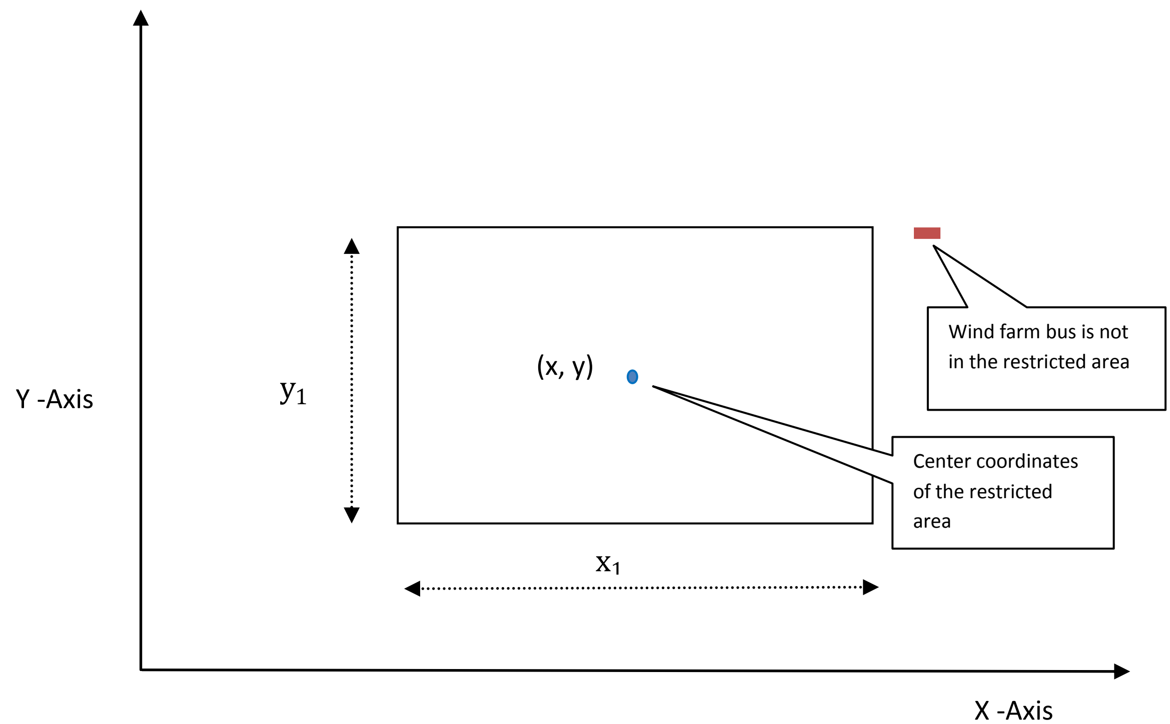



Fig.4.4: Restricted areas where wind farm generators cannot be placed

It can be mathematically represented as,

$$x_w \notin X_r \tag{4.9}$$

The variables in this optimization problem are,

$$x_w \;, P_{gw\,(m)},\; P_{ij}\;; \quad \text{for } w \in B \text{ and } (i,j) \in Z$$

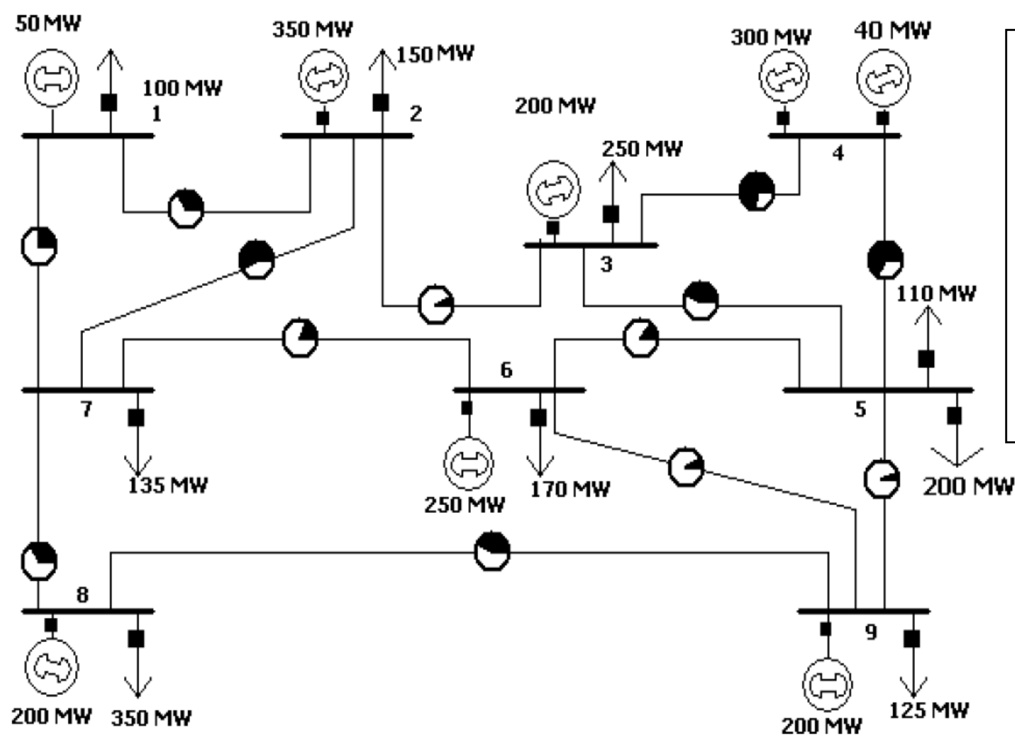
As we can see that the equation (4.4) is non convex as well as non linear and very challenging to solve. In addition to this the constraints (4.8) and (4.9) are also non linear. This inherent non linearity as well as non convexity will make the problem very difficult to solve. Also this problem solving with the other conventional methods like Interior point method is quite difficult.

So we proposed the new technique called Alternating projection method to handle this non convexity of the problem which is explained in the next chapter.

# CHAPTER 5

## CASE STUDY - NINE BUS TRANSMISSION SYSTEM

The nine bus test system analyzed shown in the Fig.5.1 [6]. Summary of the system is shown in the table 5.1 and full details and characteristics of each bus and line are provided in the Appendix 1. In this system we are using the $L_2$ norm as it is widespread sparse transmission system. The total load connected is 1590 MW is at the threshold which can be supplied by the generators at the initial state.




Number of generators = 7

Number of buses, B = 9.

Number of line connections, Z= 13

Total connected load = 1590 MW

Fig.5.1: Nine bus system used in the simulations.

Assuming the system to be lossless system, we need to define the location of each bus. For defining the location of the each bus we need to define the reference bus. Here the general method is to take the leftmost bus as the reference bus so that the whole network can be seen as the x - y plane. So we will take the bus 8 as the reference bus.

`

Now by taking the bus 8 as the reference bus the locations of the other buses are automatically defined. Appendix I give the details of location of the all buses.

The Fig.5.1 can be transformed to Fig.5.2 by considering the data given in the Appendix 1. The transformed network is shown in the Fig .5.2.

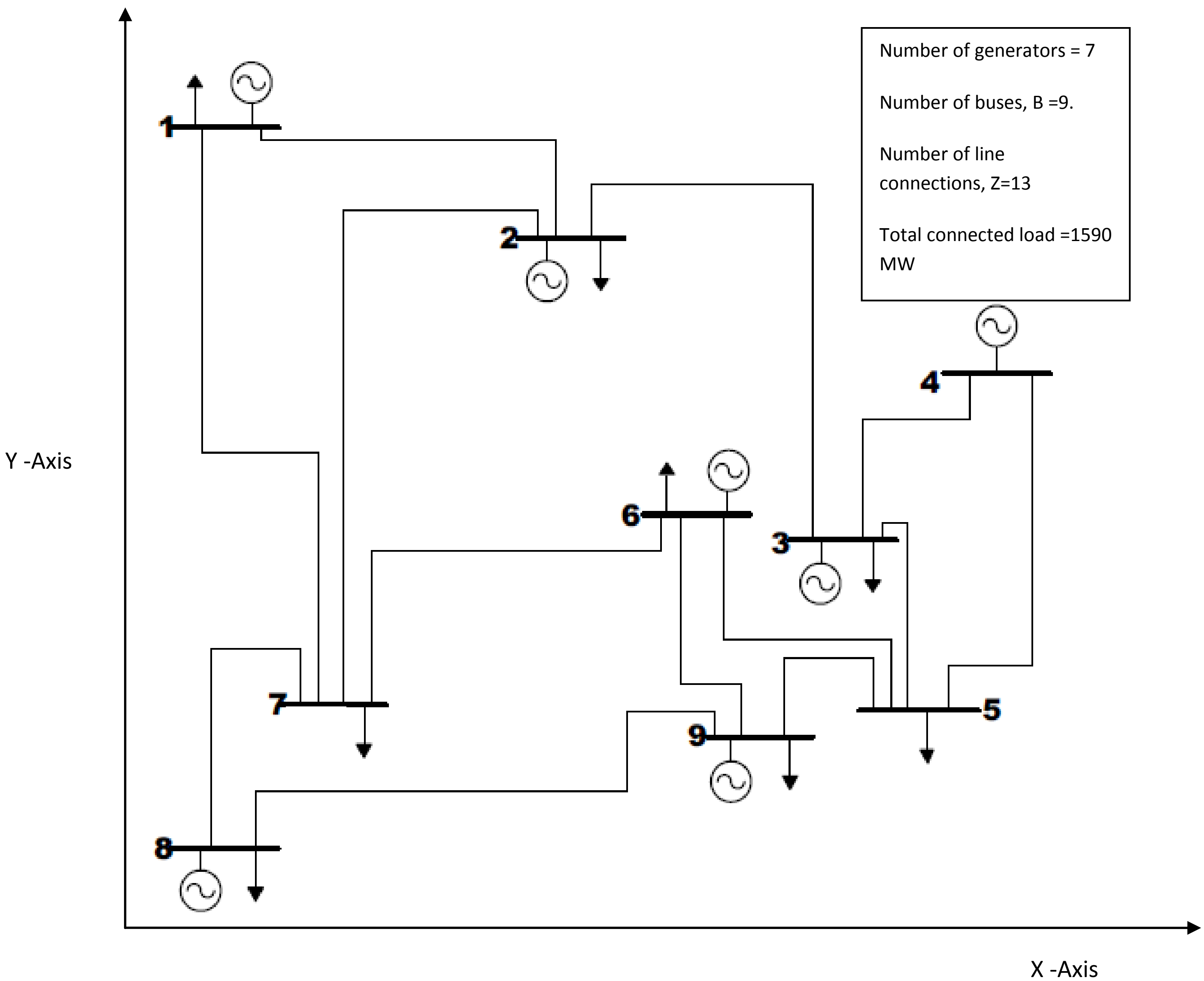



Fig.5.2: Nine bus transmission system taking bus8 as reference (0,0)

In this transformed Fig.5.2 the bus 8 is placed at the origin (0,0) and the other buses are defined by taking the bus 8 as the reference bus. The following table 5.1 defines the parameters of the lines and the buses in the transmission system as shown in the Fig.5.1.

`

Table 5.1: An overview of the parameters of the lines and buses of the transmission system

| Definition | Parameter | Range |
|---|---|---|
| Maximum value of the grid coordinates | $x_{max}$ | (300 KM, 1000 KM) |
| Load connected at the bus j. | $P_{L(j)}$ | 100 -350 MW |
| Generation capacity of the generator | $P_{g(m)}$ | 50-350 MW |
| Line flow capacity of the bus | $P_{ij\,(m)}$ | 200 -300 MW |
| Cost involved in transacting the power between the buses i and j per unit length | $C_{ij}$ | \$16 - 21$\times$ $10^3$ / KM |

The study of this nine bus transmission system is being divided in to the 3 – cases as discussed below.

- **Base Case :**

  In this case the optimal power flows are being determined before incrementing the load at the buses .So we are not integrating any wind farm bus into the grid, so the second term in the objective function becomes zero. So the objective function will become linear as well constraints are also linear. The power flows are calculated with the help of the Simplex algorithm.
- **Eye Ball Technique :**

  This method is based upon the intuition. By the past experience and examining the grid structure we will estimate the location of the wind farms and the corresponding power flows are being calculated. Since in this method the location of the wind farm is known, so the objective function becomes the linear and constraints also becomes linear .Again the same Simplex algorithm is being used to calculate the power flows in this case also.

- **Alternating Projection Algorithm:**

`

Alternating projection is a simple algorithm for computing the point of intersection of some convex sets, using the sequence of the projections on to the set as explained in chapter 2.

### 5.1 DEVELOPMENT OF SOLUTION:

The data for the nine bus system is given in the Appendix 1. The load demand on the system is 1590MW and is met by the generators. Let's suppose the load on each bus is increased by the 50 MW. Before incrementing the load on the each bus lets calculate the minimum transmission cost for the base case .So the details of the base case are as follows:

#### 5.1.1 BASE CASE:

Since we are going to calculate the minimum transmission cost when there is no load increment at the bus .So we don't need the wind farm generators to be placed in the network. As a result of this, our second term in the equation (4.4) becomes zero and the objective function will become as,

Minimize

$$\sum_{(i,j)\in Z} C_{ij} \frac{|P_{ij}|}{P_{ij(m)}} \left\| x_i - x_j \right\| \tag{5.1}$$

So variable in this equation will be $P_{ij}$ . Also since there is no wind farm bus so the placement constraints will also becomes zero. So the equations (4.8) and (4.9) become zero. So the constraints to this problem are power flow constraints only.

$$P_{L(j)} - \sum_{i\in B} P_{ij} \leq P_{j(m)} \qquad \forall\, j \in B \tag{5.2}$$

$$P_{L(j)} - \sum_{i\in B} P_{ij} = 0 \ \ (\text{at Load Bus}) \qquad \forall\, j \in B \tag{5.3}$$

$$|P_{ij}| \leq P_{ij(m)} \qquad \forall\, (i,j) \in Z \tag{5.4}$$

`

Using the data available in appendix I we will define our objective function as well as the constraints in the base case.

Since in this equation there are 13 line connections so the number of the variable will be 13. The objective function after substituting all the values of cost is given in 5.5,

Minimize

$$\left.\begin{aligned}&(13.97\,P_{12} + 81.21\,P_{17} + 42.37\,P_{23} + 66.34\,P_{27} + 26.52\,P_{34} + 10.95\,P_{35} + \\ &28.29\,P_{45} + 26.51\,P_{56} + 10.59\,P_{59} + 19.95\,P_{67} + 39.76\,P_{69} + 10.59\,P_{78} + \\ &26.45\,P_{89}) \times 10^3\end{aligned}\right\} \quad (5.5)$$

The corresponding power flow constraints will also become,

$$\left.\begin{aligned}
P_{12} + P_{17} &\le -50 \\
-P_{12} + P_{27} + P_{23} &\le 200 \\
-P_{23} + P_{34} + P_{35} &\le -50 \\
-P_{34} + P_{45} &\le 340 \\
-P_{45} - P_{35} + P_{56} + P_{59} &= -310 \\
P_{67} + P_{69} - P_{56} &\le 80 \\
P_{78} - P_{17} - P_{27} - P_{67} &= -135 \\
-P_{78} + P_{89} &\le -150 \\
-P_{89} - P_{69} - P_{59} &\le 75 \\
-300 \le P_{12} &\le 300 \\
-200 \le P_{17} &\le 200 \\
-200 \le P_{27} &\le 200 \\
-200 \le P_{23} &\le 200 \\
-200 \le P_{34} &\le 200 \\
-200 \le P_{35} &\le 200 \\
-300 \le P_{45} &\le 300 \\
-200 \le P_{56} &\le 200 \\
-200 \le P_{59} &\le 200 \\
-200 \le P_{67} &\le 200 \\
-200 \le P_{69} &\le 200 \\
-200 \le P_{78} &\le 200
\end{aligned}\right\} \quad (5.6)$$

`

$$-200 \leq P_{89} \leq 200$$

Since the above set of the equations (5.5) and (5.6) are said to be linear equation .So we have the linear programming problem as we have the linear objective function as well as the linear constraints.

These linear set of equation can be solved with the help of the **Simplex algorithm.**

If we compare our actual problem with the standard linear programming problem, discussed in chapter 2, used for the Simplex algorithm we find that our problem is not in the desired form. It has certain shortcomings which are as below,

- **b vector is not positive:**

  If we see right hand side of our constraints in equation (5.6) the some elements of the vector b are negative. So we need to multiply by -1 on both sides and as a result of this the inequality sign will get reverse. For greater than equality, we need to add the surplus variable to make the equation. Since negative sign on the basic variable will lead to the initial basic infeasible solution. So we need to do something to overcome this problem.

- **Power flows are unrestricted:**

  If we see standard linear programming problem the condition is that the decision variable should be strictly positive. But in our actual problem the decision variable is the power flow between the buses which can be either in any direction .So the power flow between the buses is unrestricted.

  **Equality constraints:**

  Since our actual problem consists of the equality constraints also. So it is not directly solvable by simplex method as it will not contain and basic variable to get the initial basic solutions.

With all above problems in the objective function and constraints we cannot input our optimization problem directly into the Simplex algorithm. So we need to modify our optimization problem to bring it into the linear programming problem. It can be done as,

`

- **b vector is not positive:**

  Multiply the constraint equation by -1 on both sides in which b elements are negative. As a result of this the less than equality sign will be change to the greater than equality sign. So then add the surplus variable (negative slack variable).Since these surplus variables will not give the initial basic solution we will add the additional variables called artificial variables to these constraints equations .These artificial variables should not affect the value of the objective function. So we multiply these artificial variables by a large quantity 'M' and add to the objective function (for minimization Problem) and subtract from the objective function for maximization problem. This method is also called as the Big M method. Another method to handle these artificial variables is by the two phase method .In this Method in the first phase the artificial variables will be send out by taking all the coefficient of the objective function variables as zero and in the second phase the problem is solved without taking the artificial variables into consideration.

- **Power flows are unrestricted:**

  For the unrestricted decision variable the unrestricted variable is being replaced by the subtraction of two new variables. Let $y_i$ be the unrestricted variables then it can be replace by two new variables such that,

$$y_i = y_i^{'} - y_i^{''} \qquad (5.7)$$

Where $y_i^{'} \geq 0$ ; $y_i^{''} \geq 0$ . Based upon these values, values of the $y_i$ is being calculated.

- **Equality constraints:**

  Since for the equality constraints we are not able to get the initial basic feasible solution as there are no basic variables. This problem also be solved taking the additional artificial variables in the equality constraint equation.

Now using all these methods to convert the optimization problem and using the Big M method and using the Simplex algorithm mentioned in chapter 2 the above optimization problem is being solved in the MATLAB. The result we got in this equation is the power flows between the

`

various lines in the base case .This power flows are substituted in the objective function and we got the minimum cost in the base case scenario.

The power flows found using this method are explained in the table 5.2 below:

Table 5.2: Power flow values in the Base case

| S. No. | From bus – To bus ($P_{ij}$) | Power flows (MW) | Maximum Line capacity (MW) |
|---|---|---|---|
| 1 | $P_{12}$ | 20 | 300 |
| 2 | $P_{17}$ | -70 | 200 |
| 3 | $P_{23}$ | 20 | 200 |
| 4 | $P_{27}$ | 200 | 200 |
| 5 | $P_{34}$ | -200 | 200 |
| 6 | $P_{35}$ | 170 | 200 |
| 7 | $P_{45}$ | 140 | 300 |
| 8 | $P_{56}$ | -200 | 200 |
| 9 | $P_{59}$ | 200 | 200 |
| 10 | $P_{67}$ | -45 | 200 |
| 11 | $P_{69}$ | -75 | 200 |
| 12 | $P_{78}$ | -50 | 200 |
| 13 | $P_{89}$ | -200 | 200 |

The total transmission cost for the base case is being calculated as $ $\mathbf{48.32 \times 10^{6}}$**.**As of now we don’t know whether the solution we got is the optimum or not (globally). For that we need to check that the given solution is satisfying the KKT conditions or not. For any optimization problem with equality ($h_j(x)$) as well as inequality ($g_i(x)$) constraints, the KKT conditions are to be satisfied.

For any optimization problem, there are three set of the equations which are known as KKT conditions to be satisfied for the point to be the optimal point.

Let f(x) be the function to be minimized and f(x) all the constraints to be differentiable then the Lagrange function is defined as,

`

$$L = f(x) + \sum_{i=1}^{m} \lambda_i \, g_i(x) + \sum_{j=1}^{n} \mu_j h_j \, (x) \qquad (5.8)$$

Then the KKT conditions are

1. Optimality condition

$$\frac{\partial f}{\partial x} + \sum_{i=1}^{m} \lambda_i \frac{\partial g_i}{\partial x} + \sum_{j=1}^{n} \mu_j \frac{\partial h_j}{\partial x} \ = 0 \qquad (5.9)$$

2. Feasibility condition.

$$g_i(x) \leq 0 \, , i = 1, 2 \, \ldots\ldots\ldots\ldots m, \qquad (5.10)$$

$$h_j(x) = 0, j = 1,2 \ldots \ldots \ldots \quad n \qquad (5.11)$$

3. Complementary slackness condition

$$\lambda_i g_i(x) = 0 \qquad \text{for all} \qquad i = \ 1, 2 \ \ldots \ldots \ldots \ldots . m, \qquad (5.12)$$

$$\lambda_i \ \geq 0 \ \text{ for all } i = \ 1, 2 \ \ldots \ldots \ldots \ldots . m, \qquad (5.13)$$

$$\mu_j \ \neq 0 \ \text{ for all } j = \ 1, 2 \ \ldots \ldots \ldots \ldots . n \qquad (5.14)$$

$\lambda$ Values are positive for the active constraints and zeros for the inactive constraints.KKT conditions are the necessary conditions. The KKT conditions will be the sufficient condition if f(x) is convex, $g_i(x)$ is convex and $h_j(x)$ is the affine.

So we got the power flows as the solution of the base case optimization problem and now it is required to check whether this solution is the KKT point or not.

From the values of the table 5.2 we can say that the 12 inequality constraints are the active constraints (violating the limits) and two are the equality constraints. The constraints which are violating the limit (active constraints) we have $\lambda_i > 0$ as per the complementary slackness condition and for the non active constraint $\lambda_i = 0$ Since we have total 12 active inequality constraint and 2 equality constraint so we need to calculate the $\lambda_{1,} \ \lambda_2 \ldots \ldots \lambda_{12}$ and $\mu_{1,} \ \ \mu_2$.So defining the Lagrange function (5.8) and taking only the active constraints ( $\lambda_{1,} \ \lambda_2 \ldots \ldots \lambda_{12}$ & $\mu_{1,} \ \ \mu_2$) into the account we differentiate the Lagrange function with respect to all the power flows .

`

This will gives us the 13 linear independent equations with 14 variables. These independent equations are being solved in the MATALAB and we got the following result for the $\lambda_{1,}\ \lambda_2 \ldots \ldots \lambda_{12}$ & $\mu_{1,}\ \ \mu_2$ in table 5.3.

Table 5.3: Lagrange multiplier values for the

Solution in table 5.2

| S.No. | Lagrange multiplier | Values |
|---|---|---|
| 1 | $\lambda_1$ | 0.02 |
| 2 | $\lambda_2$ | 13.96 |
| 3 | $\lambda_3$ | 56.34 |
| 4 | $\lambda_4$ | 39 |
| 5 | $\lambda_5$ | 61.26 |
| 6 | $\lambda_6$ | 91.8 |
| 7 | $\lambda_7$ | 101.2 |
| 8 | $\lambda_8$ | 0.9 |
| 9 | $\lambda_9$ | 23.14 |
| 10 | $\lambda_{10}$ | 43.86 |
| 11 | $\lambda_{11}$ | 32.54 |
| 12 | $\lambda_{12}$ | 17.23 |
| 13 | $\mu_1$ | 67.29 |
| 14 | $\mu_2$ | 81.21 |

As we can see from the values of the Lagrange's multiplier are positive this means the solution we got in the base case is the KKT point .In other words we can say that the optimization problem we solved is the convex optimization problem.

As of now we have assumed the base case only in which the load demand is being met by the power supplied by the generators in the grid .Now we will assume the increase in the load demand to be 50 MW at each bus .Since there are total nine buses so the total increase in the demand is 450 MW. We have already assumed that the current generators are not capable of supplying this increase in the demand. So we need to supply this increase in the demand with the help of the new generators. These new generators are nothing but the wind farm

`

generators. So we will assume there is only a one wind farm generator (as a simplified case) which has to be placed at the certain location in the grid so that our overall transmission cost and generation cost (wind farm) should get reduced. So our optimization problem state that what should be the location as well as capacity of this one wind farm generator so that it will meet the increase in the demand as well as our transmission cost and generation cost (wind farm generator) also get reduced.

We will proceed further in this section with our best of our guesses. We will guess the location of the wind farm generators with best of our ability. This method is known as Eye ball method which is explained in the next mentioned below section.

**5.1.2 EYE BALL TECHNIQUE:**

This method is totally based upon the intuition. By examine the grid structure we can guess the location of the wind farm buses. So if we place our wind farm generators near to these buses provided they satisfy the placement constraints then they can supply the increase in the demand. Please note that in this case we are assuming there is no geographical constraint i.e. that the wind farm generators can be placed at any part of the entire network i.e. $R = \emptyset$. So we need to see only that there should be minimum distance between the wind farm buses and the other buses. Appendix 1.3 will explain the properties of these wind farm generators and also the minimum distance between the buses.

So we will choose the location of the wind farm generators as (25,108) which is directly underneath the bus 7. This placement of the wind farm buses satisfy the placement constraint i.e. the minimum distance between the buses is 5KM. Let us assume the wind farm generator is being placed in that location .So now our B = 10 i.e. number of the buses become 10. Since we know the location of the wind farm buses already so $x_w$ is no longer the variable. The variables in the optimization problem will be only power flow between the lines $P_{ij}$ as well as the capacity of the wind farm generators $P_{gw(m)}$.

Next thing is, we need to determine value of the constant $K_{wind}$. It can be determined from the equation (3.5), which is given as

`

$$\text{Generation Cost} = \sum_{w}\sum_{c}^{n} C_c \frac{t(1+t)^{n_c}}{(1+t)^{n_c}-1} P_{gw\,(m)} \quad (5.15)$$

where

$$K_{wind} = \sum_{c}^{n} C_c \frac{t(1+t)^{n_c}}{(1+t)^{n_c}-1}$$

c is the number of the component in the wind farm generators. The details of these components are given in the following table 5.9. t =7% interest rate.

Table 5.9: Life expectancy and cost of each

Component in wind farm turbine

| Component | Life expectancy (years) | $C_c(\$ \times 10^6)$ |
|---|---|---|
| Turbines | 26 | 1.76 |
| Batteries | 15 | 3 |
| Inverter | 11 | 0.5 |
| Converter | 12 | 0.036 |
| Transformer | 20 | 0.018 |

After substituting the values of the life expectancies as well as cost of these components we calculate the value of the $K_{wind}$.

$$K_{wind} = 0.5430 \times 10^6 \text{ \$ /MW.} \quad (5.14)$$

Now we have to see the connection of these wind farm buses to the other buses in the grid. As we can see the Fig .5.3 the potential connection to the new bus (wind farm bus) is with the all the buses and after the optimal power flow only the connection to a particular bus is being decided.

Also we have assumed that the power flow from the wind farm bus is in the outward direction only. If the power flow is being taken in both directions then if the power flow is in the inward direction, the connection to that bus will unnecessarily add the cost to the total transmission cost.

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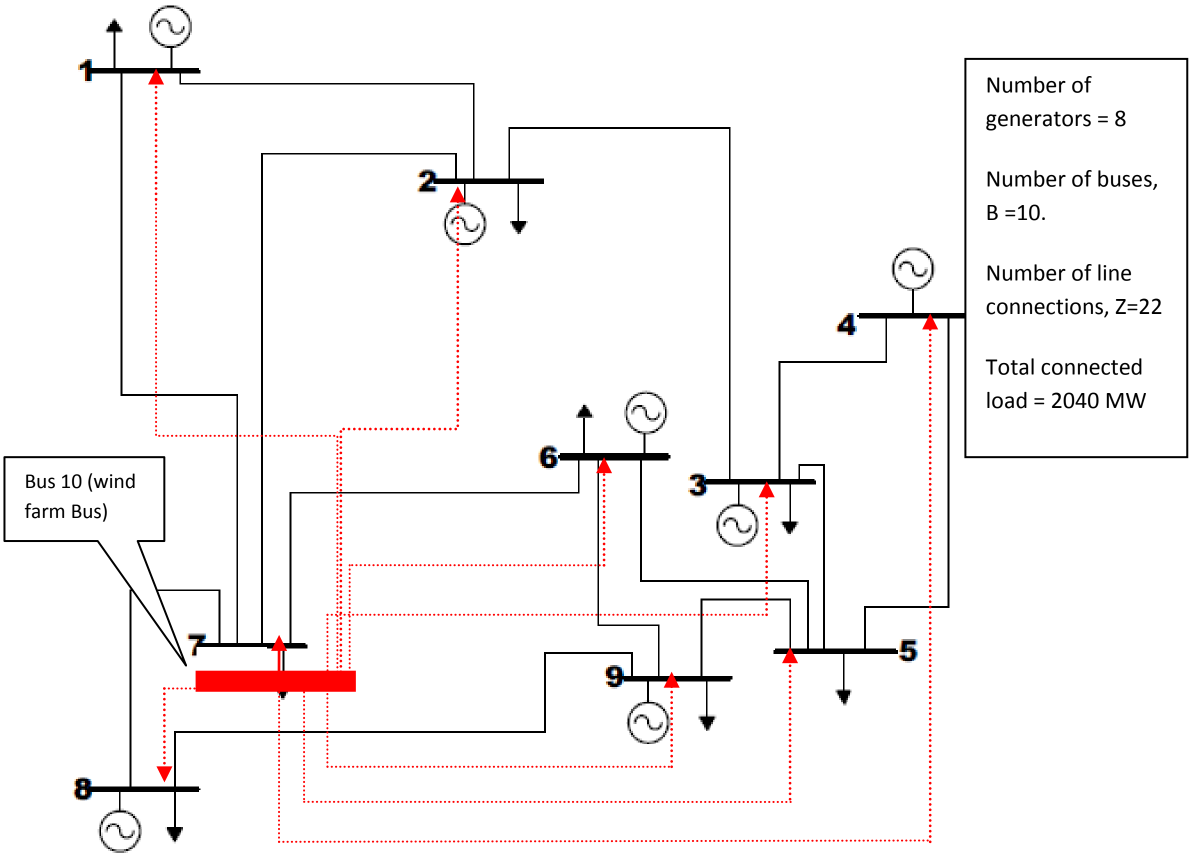


Fig.5.3: Placement of the wind farm bus and potential connections of the wind farm bus with the other Buses in Eye ball Method

Fig.5.3 shows the placement of the wind farm bus directly underneath the bus 7 and the potential connection of the wind farm bus with the other buses. The red mark bus is the wind farm bus and all the dotted red lines show the potential connection of the wind farm bus to the all other buses. Whether all the lines should be included in the final distribution system is being decided by the optimal power flow between the lines.

So in our optimization problem in this case have the 22 power flows and 1 wind farm generator capacity as the variable. So the total number of the variables in this case is 23. Since the placement of the wind farm i.e. $x_w$ is known so it is no longer variable in our optimization problem.

So again the whole optimization problem will become linear with the linear equality as well inequality constraints. So B= 10 and Z = 13+9 = 22.

`

So the optimization problem becomes,

Minimize

$$(13.97\,P_{12} + 81.21\,P_{17} + 42.37\,P_{23} + 66.34\,P_{27} + 26.52\,P_{34} + 10.95\,P_{35} + 28.29\,P_{45} + 26.51\,P_{56} + 10.59\,P_{59} + 19.95\,P_{67} + 39.76\,P_{69} + 10.59\,P_{78} + 26.45\,P_{89} + 81.67P_{10,1} + 66.79\,P_{10,2} + 31.95\,P_{10,3} + 56.20P_{10,4} + 26.68\,P_{10,5} + 45.73\,P_{10,6} + 0.457\,P_{10,7} + 10.14\,P_{10,8} + 20.87P_{10,9} + 543\,P_{gw\,(m)}) \times 10^3. \quad (5.16)$$

Subject to the following constraints,

$$\left.\begin{aligned}
&P_{12} + P_{17} - P_{10,1} \le -100\\
&-P_{12} + P_{27} + P_{23} - P_{10,2} \le 150\\
&-P_{23} + P_{34} + P_{35} - P_{10,3} \le -100\\
&-P_{34} + P_{45} - P_{10,4} \le 290\\
&-P_{45} - P_{35} + P_{56} + P_{59} - P_{10,5} = -360\\
&P_{67} + P_{69} - P_{56} - P_{10,6} \le 30\\
&P_{78} - P_{17} - P_{27} - P_{67} - P_{10,7} = -185\\
&-P_{78} + P_{89} - P_{10,8} \le -200\\
&-P_{89} - P_{69} - P_{59} - P_{10,9} \le 25\\
&P_{10,1} + P_{10,2} + P_{10,3} + P_{10,4} + P_{10,5} + P_{10,6} + P_{10,7} + P_{10,8} + P_{10,9} \le 450\\
&0 \le P_{gw\,(m)} \le 450\\
&-300 \le P_{12} \le 300\\
&-200 \le P_{17} \le 200\\
&-200 \le P_{27} \le 200\\
&-200 \le P_{23} \le 200\\
&-200 \le P_{34} \le 200\\
&-200 \le P_{35} \le 200\\
&-300 \le P_{45} \le 300\\
&-200 \le P_{56} \le 200\\
&-200 \le P_{59} \le 200\\
&-200 \le P_{67} \le 200\\
&-200 \le P_{69} \le 200\\
&-200 \le P_{78} \le 200
\end{aligned}\right\} \quad (5.17)$$

`

$$-200 \leq P_{89} \leq 200$$

$$0 \leq P_{10,1} \leq 200$$

$$0 \leq P_{10,2} \leq 200$$

$$0 \leq P_{10,3} \leq 200$$

$$0 \leq P_{10,4} \leq 200$$

$$0 \leq P_{10,5} \leq 200$$

$$0 \leq P_{10,6} \leq 200$$

$$0 \leq P_{10,7} \leq 200$$

$$0 \leq P_{10,8} \leq 200$$

$$0 \leq P_{10,9} \leq 200$$

Since this is also the linear programming problem with the linear equality as well as inequality constraints but this problem is not in the standard linear programming problem. To convert this optimization problem into the standard linear programming problem the methods mentioned in the section 5.1.1 are being used.

Using the Big M method/Two phase simplex method and using the above mentioned Simplex algorithm the above optimization problem is being solved in the MATLAB. The result we got in this equation is the power flows between the various lines in the Eye ball method .This power flows are substituted in the objective function and we got the minimum cost.

The power flows found using this method are explained in the table 5.4 below:

So by checking the optimal power flow equations we can conclude that there is no power flow between the buses 10 and 1 - 6. So the final distribution system will take the power flows between the buses 10 - 7, 10 - 8 and 10 - 9.The final distribution system is shown in the Fig.5.4. The connection of the wind farm bus is with only 8, 9 and 7 buses only.

The power flows through these lines are shown in the above table and they sum to the 450 MW which is increase in the load also.

`

Table 5.4: Power flow values in the

Eye ball method.

| S. No. | From bus to bus | Power flows (MW) | Maximum Capacity (MW) |
|---|---|---|---|
| 1 | $P_{12}$ | 100 | 200 |
| 2 | $P_{17}$ | -200 | 200 |
| 3 | $P_{27}$ | 100 | 200 |
| 4 | $P_{23}$ | 150 | 200 |
| 5 | $P_{34}$ | -200 | 200 |
| 6 | $P_{35}$ | 200 | 200 |
| 7 | $P_{45}$ | 90 | 300 |
| 8 | $P_{56}$ | -200 | 200 |
| 9 | $P_{59}$ | 130 | 200 |
| 10 | $P_{67}$ | -165 | 200 |
| 11 | $P_{69}$ | -5 | 200 |
| 12 | $P_{78}$ | -200 | 200 |
| 13 | $P_{89}$ | -200 | 200 |
| 14 | $P_{10,1}$ | 0 | 200 |
| 15 | $P_{10,2}$ | 0 | 200 |
| 16 | $P_{10,3}$ | 0 | 200 |
| 17 | $P_{10,4}$ | 0 | 200 |
| 18 | $P_{10,5}$ | 0 | 200 |
| 19 | $P_{10,6}$ | 0 | 200 |
| 20 | $P_{10,7}$ | 200 | 200 |
| 21 | $P_{10,8}$ | 200 | 200 |
| 22 | $P_{10,9}$ | 50 | 200 |
| 23 | $P_{gw\,(m)}$ | 450 | 450 |

Also the capacity of the wind farm generator is found to be 450 MW in this case. So the transmission cost of the grid is found to be \$ $\mathbf{68.42} \times \mathbf{10^6}$**.** The generation cost can be found as mentioned below:

Using equation (3.5),

`

$$\text{Generation cost } = \sum_{w}\sum_{c}^{n} C_c \frac{t(1+t)^{n_c}}{(1+t)^{n_c}-1} P_{gw\,(m)}$$

$$K_{wind} = \sum_{w}\sum_{c}^{n} C_c \frac{t(1+t)^{n_c}}{(1+t)^{n_c}-1}$$

$$K_{wind} = 0.5430 \times 10^6 \text{ \$ /MW}$$

$$\text{Generation cost } = K_{wind} \times P_{gw\,(m)}$$

$$\text{Generation cost } = \$\, \mathbf{244.9} \times \mathbf{10^6} \qquad (5.18)$$

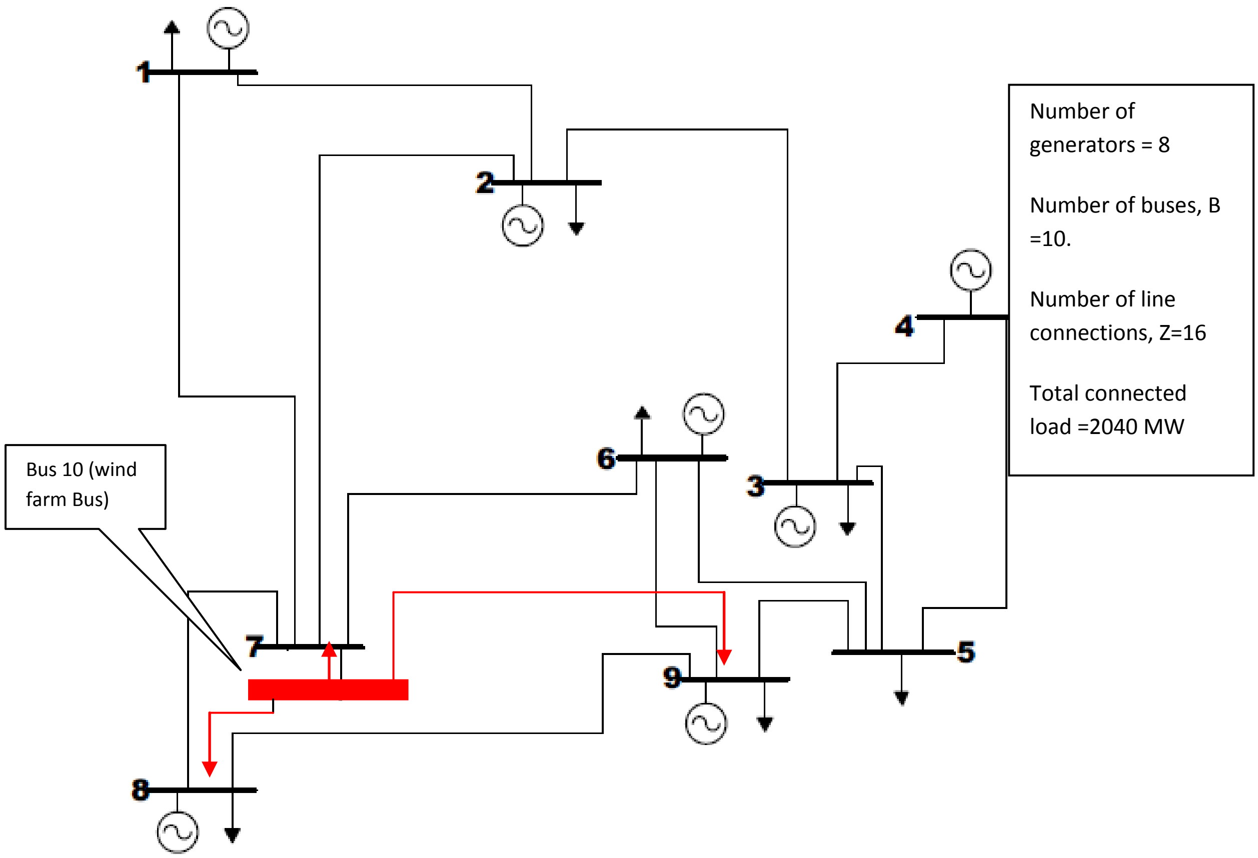


Fig.5.4: Placement of the wind farm bus and final connections of the wind farm bus with the other Buses in Eye ball Method

As we can see the transmission cost is being increased in the method as compared to the base case which is expected to be increase as the number of the lines (Z = 16 ) is increased. Since as of now our motive in not being solved as we have not found out the minimum cost .Now we will move to our proposed method to solve this optimization problem i.e. Alternating Projection method.

`

### 5.1.3 ALTERNATING PROJECTION METHOD:

As this method is being discussed in the previous sections the Alternating projection is the method to separate the portion of the optimization problems each of which is directly solvable by using the Linear programming as well as the Non linear programming Technique .This method basically repeats the process of holding the solution obtained for the variables of one of the problems as constant, and then projecting the onto the solution space of the other [3].

For the alternating projection we need to convert the original non-convex /non-linear optimization problem into the two sets of the convex optimization problem. This can be done as follows:

- **Convex Optimization 1:**

  Take the $x_w$ as constant then the optimization problem will become the linear problem with the linear set of the constraints .Since this problem is minimizing we can say the problem is the convex optimization problem. The variables will be power flow and capacity of the wind farm bus.

- **Convex Optimization 2:**

  The power flows as well as the capacity of the wind farm is taken as constant the resulting problem is having only the position of the wind farm bus $x_w$ as variable. This problem is also a convex optimization problem (Proof given in the later part of the section).

So now we have the set of the two convex optimization problems now we can use the method of the Alternating projection to find out the location of the wind farm bus. The alternating projection algorithm for this particular problem is explained below:

- **STEP 1:**

  Set k=1 and initialize the wind farm placements $x_w^k$ to the preset values.

- **STEP 2:**

  Check the placement constraint (4.8).

  $x_w^k$ Should not violate the above mentioned placement constraint .If it is violating the constraint then change the value of the $x_w^k$ as follows:

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Let j be the violated bus having the coordinates having the coordinates $(x_j(1),\ x_j(2))$ this is shown in red in the Fig.5.5. The blue colored bus is the reference bus for the violated bus.

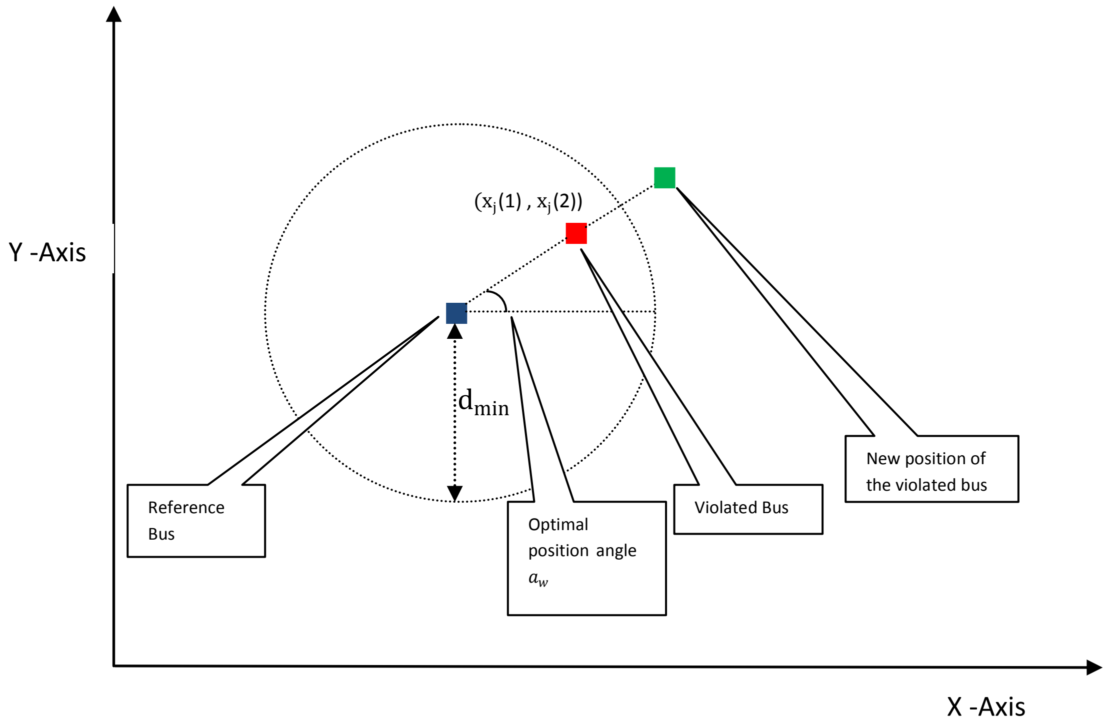


Fig.5.5: New position of the wind farm bus if the placement constraint (4.8) not satisfied.

Let $a_w$ be the optimal position angle for the violated bus. Since $d_{min}$ is the minimum distance between the buses, so the new position of the violated bus is calculated by projecting the $d_{min}$ on the x-axis as well as y-axis and adding on to the current position of the violated bus. So the position of the new bus is given by,

$$x_w = \left( x_j(1) + d_{min} \cos a_w , x_j(2) + d_{min} \sin a_w \right) \quad (5.19)$$

- **STEP 3:**

Check for the placement constraint (4.9).

where $X_m$ is the set of the rectangular sets of the restricted areas. Let us suppose the (x, y) be the center coordinates of the restricted areas.

Let us suppose $(x_1, y_1)$ be the width and height of the restricted area which is shown in the Fig.5.6. Let the wind farm generator is placed in the restricted area (shown with the red mark). Now we have to find out the new position of the wind farm bus so that it

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should not lie in the restricted area. The new position of the wind farm generator is calculated as given in the equation (5.20).

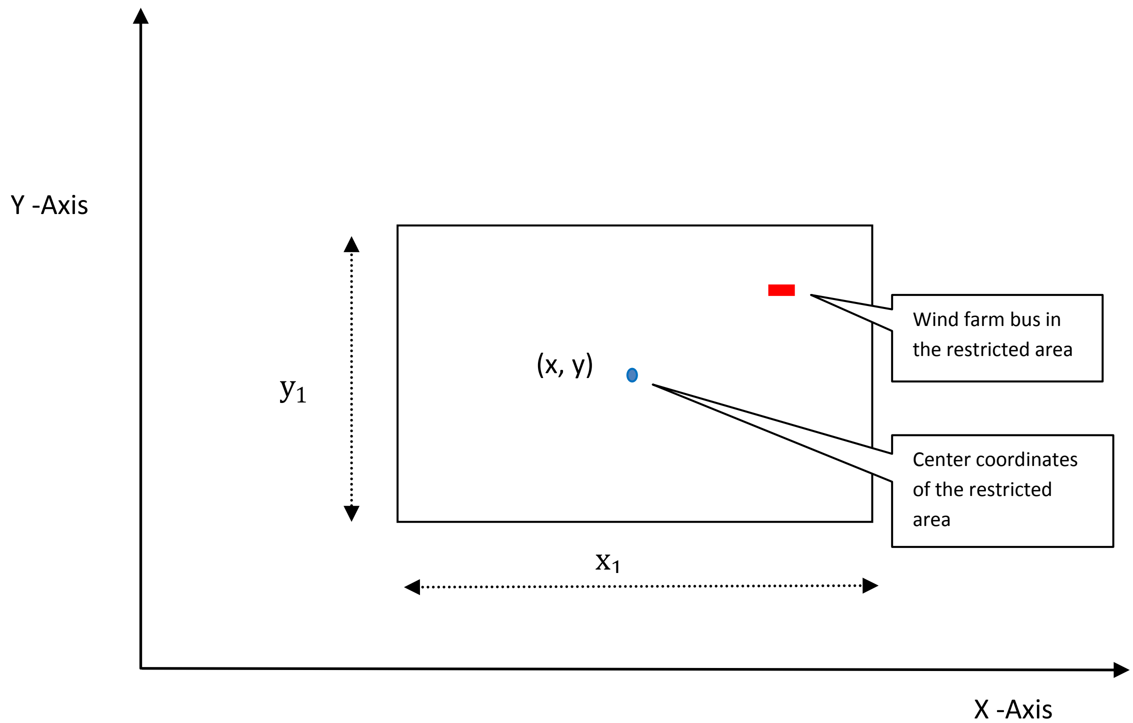



Fig.5.6: New position of the wind farm bus is the placement constraint (4.9) not satisfied.

$$x_w = \left( x \pm \left(\frac{x_1}{2}\right), y \pm \left(\frac{y_1}{2}\right)\right) \quad (5.20)$$

- **STEP 4:**

  If the $x_w$ satisfy the placement constraints and if not satisfied then use equation (5.19) and (5.20) to calculate the new value of the $x_w$.Taking these values as constant perform the convex optimization 1. As now this convex optimization problem is the linear programming problem with the linear set of the constraints and can be solved using the Simplex method/Big M method as explained in the previous section. Calculate the values of the power flows as well as the capacity of the wind farm generators in this section and hold the values.

- **STEP 5:**

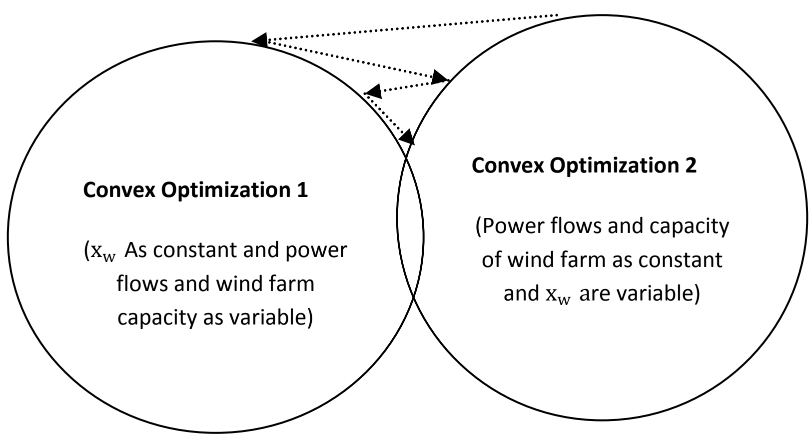
`

Take the values of the power flows and the wind farm capacity as the constant and take $x_w$ as the variable. Again this problem is the convex optimization problem (proof is in the later part of this section) and can be solved using the Newton method.

- **STEP 6:**

  Repeat all the steps for the certain number of the iterations.

The whole process completes the Alternating projection method. The Simple concept of the alternating projection in this case can be explained by the below mentioned Fig.5.7




. Fig.5.7: Alternating projection in the Nine Bus system

Basically we are trying to find out the point of intersection of the two optimization problems.

The flow chart diagram of the alternating projection is shown in the Fig.5.8.

The alternating projection is taking the less number of the iterations as compared to the conventional method available to solve this type of the non linear optimization problem like Interior Point method.

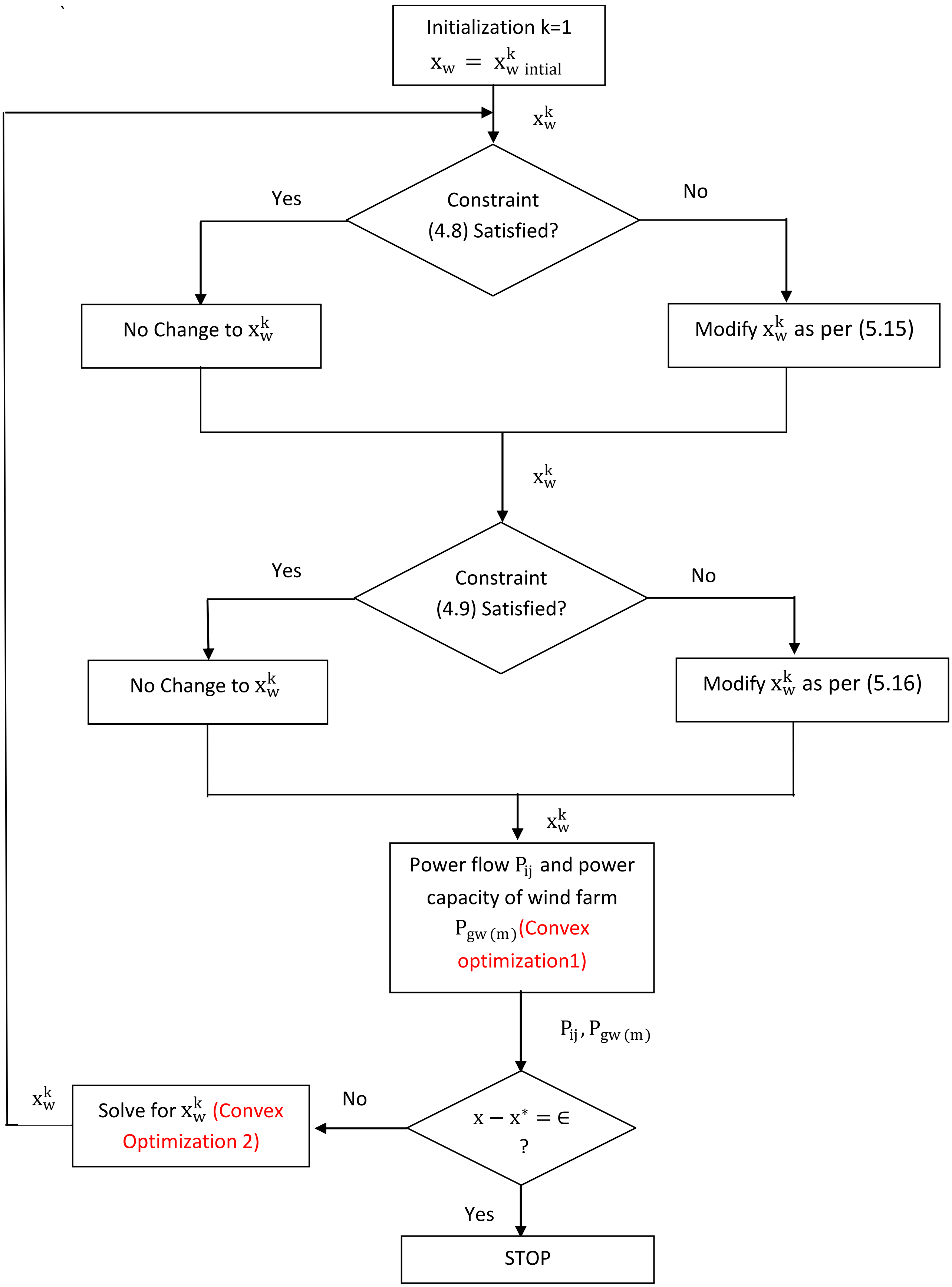



Fig.5.8: Alternating projection algorithm

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**5.1.4 ALTERNATING PROJECTION METHOD IN NINE BUS SYSTEM:**

Since we have defined the convex optimization 1 and convex optimization 2 for our original optimization problem defined in the earlier section. So we will implement the alternating projection method to our original optimization problem. We will implement each and every step as mentioned in the earlier section. In this current section we are assuming there is no geographical constraints .The problem with the geographical constraints is explained later in this chapter.

- **STEP 1:**

  Set k = 1 and let's take the $x_w = (200,250)$.

- **STEP 2:**

  Since $x_w$ is satisfying the placement constraint (4.8) with bus 8.So no need to apply the equation (5.19) and calculate the new value of the $x_w$.

- **STEP 3: Convex optimization 1**

  So now we have the similar case what we got in the Eyeball method. Our objective function is linear and the constraints are also linear. The objective function is given as,

  Minimize

$$\left.\begin{aligned}&(13.97\, P_{12} + 81.21\, P_{17} + 42.37\, P_{23} + 66.34\, P_{27} + 26.52\, P_{34} + 10.95\, P_{35} + \\ &28.29\, P_{45} + 26.51\, P_{56} + 10.59\, P_{59} + 19.95\, P_{67} + 39.76\, P_{69} + 10.59\, P_{78} + \\ &26.45\, P_{89} + 71.04 P_{10,1} + 53 P_{10,2} + 12.32\, P_{10,3} + 38.59\, P_{10,4} + 6.9032\, P_{10,5} + \\ &29.83\, P_{10,6} + 20.34 P_{10,7} + 29.30 P_{10,8} + 10.64 P_{10,9} + 543\, P_{gw\,(m)}) \times 10^3.\end{aligned}\right\} \quad (5.21)$$

Subject to the constraints

$$\left.\begin{aligned}P_{12} + P_{17} - P_{10,1} &\leq -100 \\ -P_{12} + P_{27} + P_{23} - P_{10,2} &\leq 150 \\ -P_{23} + P_{34} + P_{35} - P_{10,3} &\leq -100 \\ -P_{34} + P_{45} - P_{10,4} &\leq 290 \\ -P_{45} - P_{35} + P_{56} + P_{59} - P_{10,5} &= -360 \\ P_{67} + P_{69} - P_{56} - P_{10,6} &\leq 30 \\ P_{78} - P_{17} - P_{27} - P_{67} - P_{10,7} &= -185 \\ -P_{78} + P_{89} - P_{10,8} &\leq -200\end{aligned}\right\} \quad (5.22)$$

`

$$\left.\begin{array}{c}
-P_{89} - P_{69} - P_{59} - P_{10,9} \leq 25 \\
P_{10,1} + P_{10,2} + P_{10,3} + P_{10,4} + P_{10,5} + P_{10,6} + P_{10,7} + P_{10,8} + P_{10,9} \leq 450 \\
0 \leq P_{gw\,(m)} \leq 450 \\
-300 \leq P_{12} \leq 300 \\
-200 \leq P_{17} \leq 200 \\
-200 \leq P_{27} \leq 200 \\
-200 \leq P_{23} \leq 200 \\
-200 \leq P_{34} \leq 200 \\
-200 \leq P_{35} \leq 200 \\
-300 \leq P_{45} \leq 300 \\
-200 \leq P_{56} \leq 200 \\
-200 \leq P_{59} \leq 200 \\
-200 \leq P_{67} \leq 200 \\
-200 \leq P_{69} \leq 200 \\
-200 \leq P_{78} \leq 200 \\
-200 \leq P_{89} \leq 200 \\
0 \leq P_{10,1} \leq 200 \\
0 \leq P_{10,2} \leq 200 \\
0 \leq P_{10,3} \leq 200 \\
0 \leq P_{10,4} \leq 200 \\
0 \leq P_{10,5} \leq 200 \\
0 \leq P_{10,6} \leq 200 \\
0 \leq P_{10,7} \leq 200 \\
0 \leq P_{10,8} \leq 200 \\
0 \leq P_{10,9} \leq 200
\end{array}\right\} \quad (5.23)$$

Since this is also the linear programming problem with the linear equality as well as inequality constraints but this problem is not in the standard linear programming problem. To convert this optimization problem into the standard linear programming problem the methods mentioned in the section 5.1.1 are being used.

Now using all these methods mentioned in 5.1.1 are used to convert the optimization problem into the standard form we have total 36 variables and the size of the A matrix is $33 \times 36$. The power flows found using this method are explained in the table 5.5 below:

Table 5.5: Power flow values in the Alternating Projection Method – Convex optimization 1

| S. No. | From Bus to Bus | Power flows (MW) | Maximum Capacity of line (MW) |
|---|---|---|---|
| 1 | $P_{12}$ | 100 | 200 |
| 2 | $P_{17}$ | -200 | 200 |
| 3 | $P_{27}$ | 95 | 200 |
| 4 | $P_{23}$ | 155 | 200 |
| 5 | $P_{34}$ | -200 | 200 |
| 6 | $P_{35}$ | 195 | 200 |
| 7 | $P_{45}$ | 90 | 300 |
| 8 | $P_{56}$ | -200 | 200 |
| 9 | $P_{59}$ | 175 | 200 |
| 10 | $P_{67}$ | 30 | 200 |
| 11 | $P_{69}$ | -200 | 200 |
| 12 | $P_{78}$ | -200 | 200 |
| 13 | $P_{89}$ | -200 | 200 |
| 14 | $P_{10,1}$ | 0 | 200 |
| 15 | $P_{10,2}$ | 0 | 200 |
| 16 | $P_{10,3}$ | 0 | 200 |
| 17 | $P_{10,4}$ | 0 | 200 |
| 18 | $P_{10,5}$ | 50 | 200 |
| 19 | $P_{10,6}$ | 0 | 200 |
| 20 | $P_{10,7}$ | 0 | 200 |
| 21 | $P_{10,8}$ | 200 | 200 |
| 22 | $P_{10,9}$ | 200 | 200 |
| 23 | $P_{gw\,(m)}$ | 450 | 450 |

Using the Big M method and using the above mentioned Simplex algorithm the above optimization problem is being solved in the MATLAB.

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The total cost with this placement is found to be$ $\mathbf{73.42} \times \mathbf{10^6}$ .Now we have got the power flows $P_{ij}$ and the wind farm capacity $P_{gw(m)}$ . In this system also the value of the Lagrange multiplier is calculated and found to be positive. So we can say that the above problem is the convex optimization problem. Now we will move to the convex optimization 2 as per the flow chart.

- **STEP 4: Convex optimization 2**

We will take the values of the $P_{ij}$ as well as the wind farm capacity $P_{gw(m)}$ as constant and will take the $x_w$ as variables. In this optimization problem since the connections are with only 5, 8 and 9 bus, so substitute the values of the above power flows in the equation (4.4). So we will now form the convex optimization problem 2. The optimization problem is given as

Minimize

$$\left.\begin{aligned} &18.305 \times \sqrt{x_1^2 + x_2^2} + 18.305 \times \sqrt{(x_1 - 250)^2 + (x_2 - 145)^2} + 4.5762 \times \\ &\sqrt{(x_1 - 275)^2 + (x_2 - 258)^2} + 306758.1 \end{aligned}\right\} \tag{5.24}$$

Subject to

$$\left.\begin{aligned} x_1^2 + (x_2 - 1000)^2 &\geq 25 \\ (x_1 - 50)^2 + (x_2 - 827)^2 &\geq 25 \\ (x_1 - 250)^2 + (x_2 - 375)^2 &\geq 25 \\ (x_1 - 290)^2 + (x_2 - 662)^2 &\geq 25 \\ (x_1 - 275)^2 + (x_2 - 258)^2 &\geq 25 \\ (x_1 - 200)^2 + (x_2 - 576)^2 &\geq 25 \\ (x_1 - 25)^2 + (x_2 - 113)^2 &\geq 25 \\ (x_1)^2 + (x_2)^2 &\geq 25 \\ (x_1 - 250)^2 + (x_2 - 145)^2 &\geq 25 \\ x_1 &\geq 0 \\ x_2 &\geq 0 \\ x_1 &\leq 300 \\ x_2 &\leq 1000 \end{aligned}\right\} \tag{5.25}$$

`

Now we need to see whether the above mentioned problem is convex or not. So we plot the objective function in the MATLAB and check the convexity of the function. The function found to be of the shape shown in the Fig.5.9.

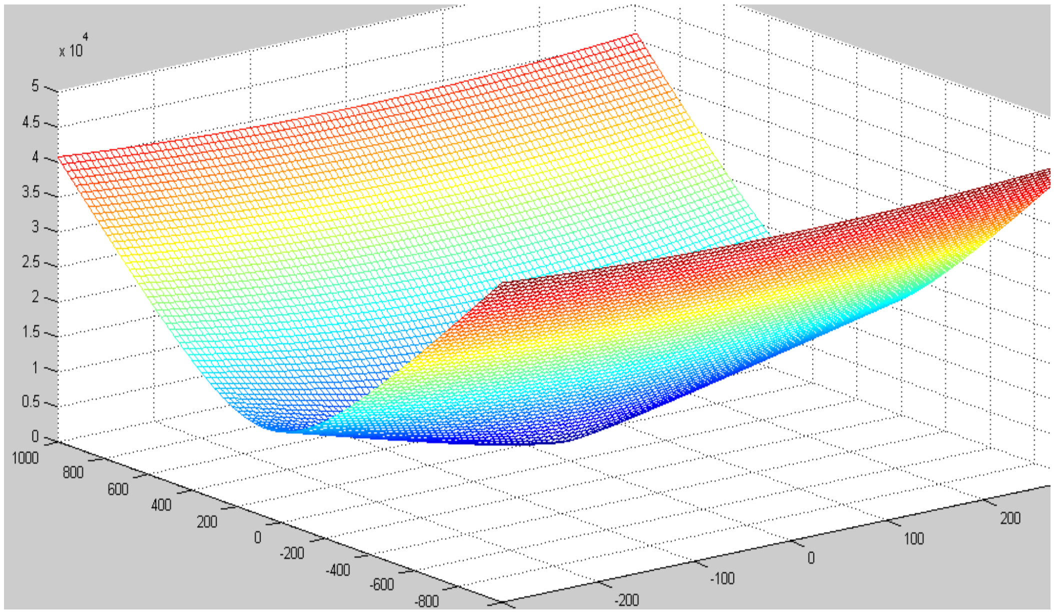


Fig.5.9: Shape of the objective function in Convex optimization 2..

As per the definition of the convex function explained in the previous chapter the given problem is the convex. We will prove the convexity of the optimization problem in the Newton method also while calculating the sufficient conditions for the optimization.

This is non linear programming problem and can be solved using the Newton method along with the KKT conditions to be satisfied.

Using the algorithm explained in the chapter 2 we find out the $x_w^1 = (250\,,290\,)$ . Now we have completed the one iteration. Now in the second iteration we will take $x_w^1 = (250\,,290\,)$ as the input and start processing as per the flow chart.

After the three iterations the $x_w^3$ = (250, 265).Now the values of the $\lambda_i$'s is found to be positive in this case which means this point is the KKT point .Since satisfying the KKT conditions are the necessary conditions only so we need to check the sufficient conditions also. The sufficient condition for multivariable optimization problem is that

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its Hessian should be positive definite. The Hessian matrix for the objective function at the (250,265) is given as,

$$H = \begin{bmatrix} 0.1712 & 0.0403 \\ 0.0403 & 0.1686 \end{bmatrix}$$

Since the trace of the matrix is positive, so it is a positive definite matrix. So we can say that the optimization problem is the convex optimization problem. The power flow values for the point $x_w^3$ = (250, 265) are calculated from the above mentioned method. The power flows are shown in the table.5.6.

The corresponding transmission cost is found to be $\$56.76 \times 10^6$.

Table 5.6: Power flow values in the Alternating projection method.

| S. No. | From Bus to Bus | Power flows (MW) | Maximum Capacity of line (MW) |
|---|---|---|---|
| 1 | $P_{12}$ | -5 | 200 |
| 2 | $P_{17}$ | -95 | 200 |
| 3 | $P_{27}$ | -55 | 200 |
| 4 | $P_{23}$ | 200 | 200 |
| 5 | $P_{34}$ | -200 | 200 |
| 6 | $P_{35}$ | 45 | 200 |
| 7 | $P_{45}$ | 90 | 300 |
| 8 | $P_{56}$ | -200 | 200 |
| 9 | $P_{59}$ | 175 | 200 |
| 10 | $P_{67}$ | 30 | 200 |
| 11 | $P_{69}$ | -200 | 200 |
| 12 | $P_{78}$ | -50 | 200 |
| 13 | $P_{89}$ | -200 | 200 |
| 14 | $P_{10,1}$ | 0 | 200 |
| 15 | $P_{10,2}$ | 0 | 200 |
| 16 | $P_{10,3}$ | 0 | 200 |
| 17 | $P_{10,4}$ | 0 | 200 |
| 18 | $P_{10,5}$ | 200 | 200 |
| 19 | $P_{10,6}$ | 0 | 200 |

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| | | | |
|---|---|---|---|
| 20 | $P_{10,7}$ | 0 | 200 |
| 21 | $P_{10,8}$ | 50 | 200 |
| 22 | $P_{10,9}$ | 200 | 200 |
| 23 | $P_{gw\,(m)}$ | 450 | 450 |

The Generation cost is found to be same as the Eye ball method i.e. $\$244.9 \times 10^6$. The optimal placement of the wind farm generator is shown in the fig and the connection to the bus is also shown in the same Fig.5.10.

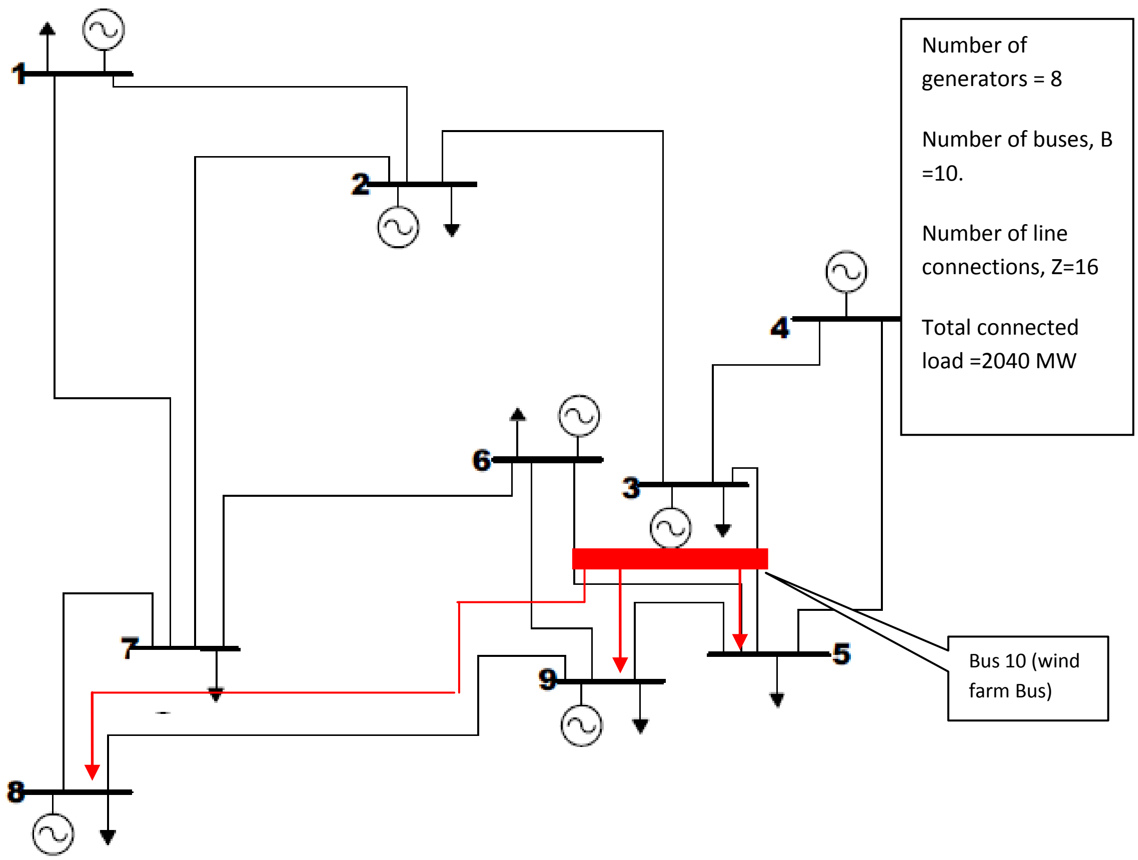


Fig.5.10: Optimal placement of the Wind farm bus as per the Alternating Projection method and its connection to the buses.

So far we have discussed the different scenarios to deal with the non convex and non linear optimization and find out the solution. In MATLAB one algorithm named as Interior point algorithm exists to deal with non convex /non linear optimization.

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For the comparison purposes we have directly implemented the objective function as well as the constraints in the MATLAB optimization tool box and determine the cost as well as the optimal placement of the wind farm generators.

The optimal placement found to be $x_w^k = (200,310\,)$ and the corresponding cost to be found to be $\$\,59.90 \times 10^6$.The Corresponding power flows for this are shown below in the table 5.7.

Table 5.7: Power flow values using the MATLAB in built function.

| S. No. | From Bus to Bus | Power flows (MW) | Maximum Capacity of line (MW) |
|---|---|---|---|
| 1 | $P_{12}$ | -5 | 200 |
| 2 | $P_{17}$ | -95 | 200 |
| 3 | $P_{27}$ | -55 | 200 |
| 4 | $P_{23}$ | 200 | 200 |
| 5 | $P_{34}$ | -200 | 200 |
| 6 | $P_{35}$ | 45 | 200 |
| 7 | $P_{45}$ | 90 | 300 |
| 8 | $P_{56}$ | -200 | 200 |
| 9 | $P_{59}$ | 175 | 200 |
| 10 | $P_{67}$ | 30 | 200 |
| 11 | $P_{69}$ | -200 | 200 |
| 12 | $P_{78}$ | -50 | 200 |
| 13 | $P_{89}$ | -200 | 200 |
| 14 | $P_{10,1}$ | 0 | 200 |
| 15 | $P_{10,2}$ | 0 | 200 |
| 16 | $P_{10,3}$ | 0 | 200 |
| 17 | $P_{10,4}$ | 0 | 200 |
| 18 | $P_{10,5}$ | 200 | 200 |
| 19 | $P_{10,6}$ | 0 | 200 |
| 20 | $P_{10,7}$ | 0 | 200 |
| 21 | $P_{10,8}$ | 50 | 200 |
| 22 | $P_{10,9}$ | 200 | 200 |
| 23 | $P_{gw\,(m)}$ | 450 | 450 |

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### 5.1.5 OPTIMAL PLACEMENT WITH GEOGRAPHICAL CONSTRAINTS:

Now we will consider the practically most important constraint on the placement feasibility we will take R = 1. The restricted areas with the center coordinates as well as dimensions are shown in the following Table 5.8.

Table 5.8: Center coordinates as well as dimensions of the restricted areas.

| Area | Center Coordinates | Width(KM) | Height(KM) |
|---|---|---|---|
| r1 | (250,265) | 100 | 100 |

The optimal placement we got in the previous section of this chapter is not feasible solution Using the equation (12) we find out the new value of the $x_w^1 = (200, 215)$.Now we have to repeat the whole Alternating projection algorithm along with the initial value of the placement variable. The optimum placement after the 3 iterations is found to be $x_w^3 = (100, 112\ )$ which results in the cost of the \$ $62.26 \times 10^6$ .

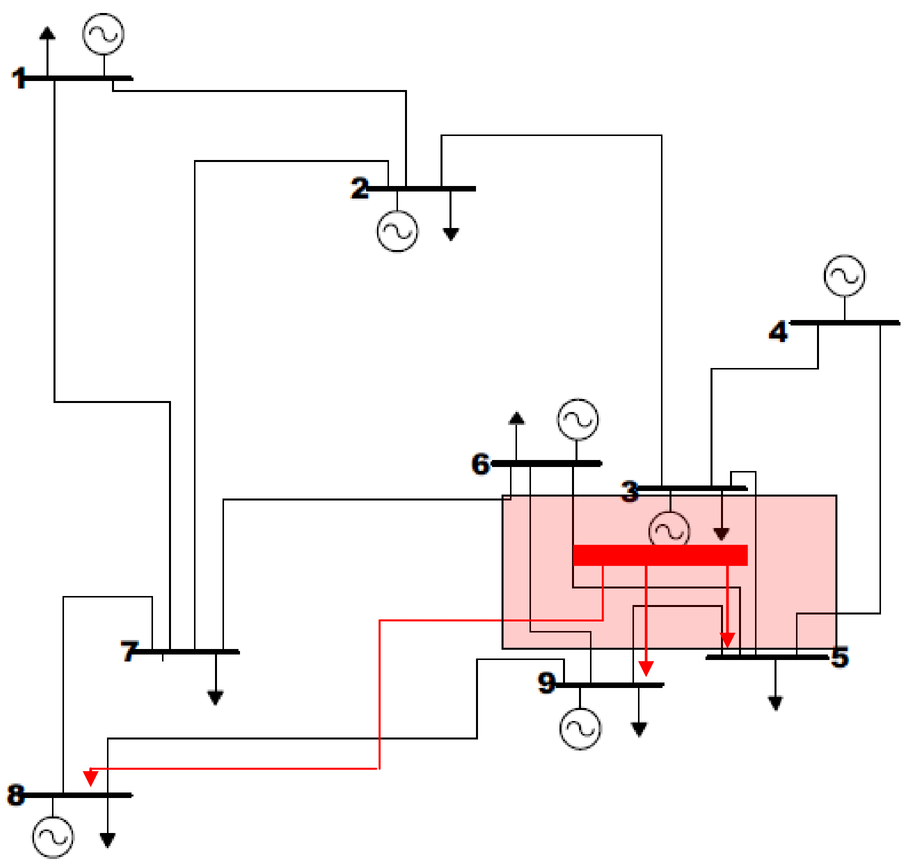


Fig.5.11: Restricted areas and wind farm bus placement in the nine bus system

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# CHAPTER 6

# SIMULATIONS RESULTS AND CONCLUSIONS

As of now we have seen the case of nine bus system in the in which we have use the Alternating projection, MATLAB optimization tool box and Eye ball technique method. The following Table 6.1 and the Fig 6.1 show the comparison of the result obtained from these methods in the nine bus system.

## 6.1 SIMULATION RESULT:

Table 6.1: Comparison of different algorithms for
the system considered.

| Algorithm | Alternating projection Method | MATLAB optimization tool box | Eye Ball technique |
|---|---|---|---|
| $x_w$ (Km, Km) | (250,265) | (200,310) | (25,108) |
| Connections to the Wind farm bus | 5,8,9 | 5,8,9 | 7,8,9 |
| Base case Cost ( $\$\times 10^6$) | 48.23 | 48.23 | 48.23 |
| Transmission Cost ( $\$\times 10^6$) | 56.76 | 59.90 | 68.42 |
| Generator cost ( $\$\times 10^6$) | 244.39 | 244.39 | 244.39 |

The IP algorithm also shows a decrease in transmission cost, but is clearly sub-optimal. Also, the EB placement results in a slight increase in the transmission cost, while the least-optimal placement at $x_{10}$ = (300, 1000) almost doubles it (data not shown). This shows how important optimization is in this problem, due to the sensitivity of placement of the wind farm generators.

Table 6.2 shows a comparison between the solutions found using each method for the cases when R = Ø and |R| = 1. The transmission cost increases by roughly $\$3.7\times10^6$ per year between R = Ø and |R| = 1.

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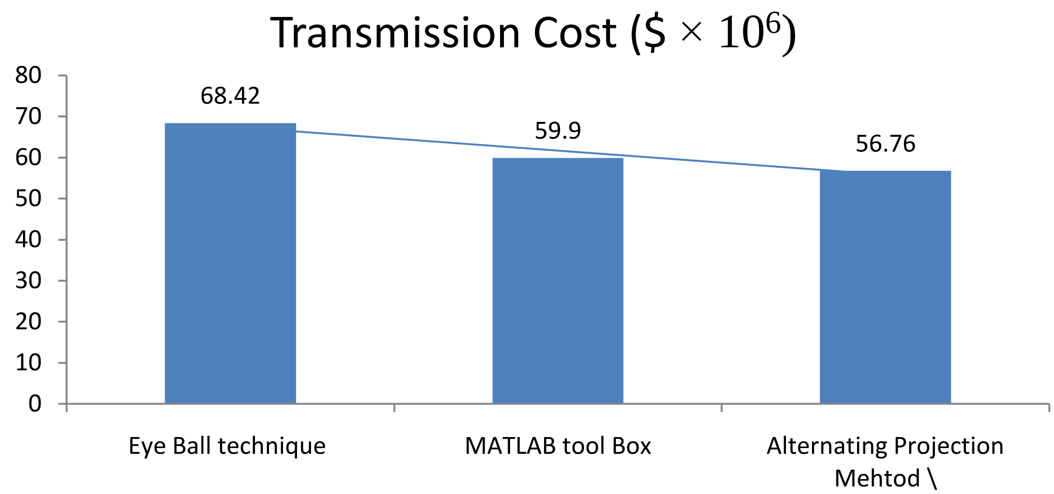


Fig 6.1: Comparison of transmission cost of different algorithms for the system considered

Table 6.2: Comparison of transmission cost with the restricted areas

| Test Case | R = Ø | \|R\| = 1 |
|---|---|---|
| Algorithm | AP | AP |
| $x_{10}$(Km,Km) | (200,265) | (100,112) |
| Connections to $x_{10}$ | 5,8,9 | 5,7,9 |
| Transmission Cost ($\$ \times 10^6$) | 56.26 | 62.26 |

### 6.2 CONCLUSION:

We have formulated and solved an optimization problem to find the optimal placement and capacity of wind farm generators to minimize the sum of the transmission and generation cost in transmission systems. Overall, our results show that while there is a lot of potential variation in the transmission cost based on generator placement and power flow, the optimal solution can actually reduce transmission cost .The addition of the geographical constraints give the direct impact on the transmission cost .The transmission cost gets increased due to addition of these important constraints. To this end, we have developed an AP algorithm which shows method of finding the optimal placement and power capacities in virtually any scenario.

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# APPENDIX 1

## 1.1 LINE PARAMETERS FOR THE NINE BUS SYSTEM:

| (i,j) | $C_{ij}$ ($\$ \times 10^6$) | $P_{ij(m)}$ |
|---|---|---|
| (1, 2) | 18.306 | 300 |
| (1, 7) | 18.306 | 200 |
| (2, 3) | 18.307 | 200 |
| (2, 7) | 18.306 | 200 |
| (3, 4) | 18.306 | 200 |
| (3, 5) | 18.306 | 200 |
| (4, 5) | 21.000 | 300 |
| (5, 6) | 16.228 | 200 |
| (5, 9) | 18.305 | 200 |
| (6, 7) | 8.065 | 200 |
| (6, 9) | 18.306 | 200 |
| (7, 8) | 18.305 | 200 |
| (8, 9) | 18.306 | 200 |

## 1.2 BUS PARAMETERS FOR THE NINE BUS SYSTEM:

| j | $x_j$ (KM, KM) | $P_{j(m)}$ (MW) | $P_{L(j)}$ (MW) |
|---|---|---|---|
| 1 | (0,1000) | 50 | 100 |
| 2 | (150,827) | 350 | 150 |
| 3 | (250,375) | 200 | 250 |
| 4 | (290,662) | 340 | 0.00 |
| 5 | (275,258) | 0.00 | 310 |
| 6 | (200,576) | 250 | 170 |
| 7 | (25,113) | 0.00 | 135 |
| 8 | (0,0) | 200 | 350 |
| 9 | (250,145) | 200 | 125 |

## 1.3 PROPERTIES OF THE WIND FARM GENERATORS:

| Parameter | Value |
|---|---|
| $C_{wj}$ ($\$\times 10^3$ /KM) | 18.305 |
| $P_{wj\,(m)}$ | 200 |
| $d_{min}$ | 5 |


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