# MMSE-FDE Single-Carrier Broadband Wireless System

*A Project Report*

*submitted by*

**ALOOB T**

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# THESIS CERTIFICATE

This is to certify that the thesis titled **MMSE-FDE Single-Carrier Broadband Wireless System**, submitted by **ALOOB T**, to the Indian Institute of Technology, Madras, for the award of the degree of **Master of Technology**, is a bona fide record of the research work done by him under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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# ABSTRACT

This report provide a detailed description of the MMSE-FDE Single-Carrier Broadband Wireless System. The Single-carrier frequency domain equalization (SCFDE) and orthogonal frequency division multiplexing (OFDM) communication systems are briefly explained. The frame level description and receiver algorithms are well explained with block diagrams. The LDPC code is used for error correction. This report contains extensive simulation results. The simulation results includes the BER performance, FER performance, mean square error of channel estimation, mean square error of timing offset and Peak to average power ratio performance.

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# ABBREVIATIONS

| | |
|---|---|
| **SCFDE** | Single-Carrier Frequency Domain Equalization |
| **OFDM** | Orthogonal Frequency Division Multiplexing |
| **ISI** | Inter-Symbol Interference |
| **QAM** | Quadrature Amplitude Modulation |
| **FFT** | Fast Fourier Transform |
| **IFFT** | Inverse Fast Fourier Transform |
| **LLR** | Log-Likelihood Ratio |
| **LDPC** | Low Density Parity Check (code) |
| **FDE** | Frequency Domain Equalization |
| **SIMO** | Single input multiple output |
| **CP** | Cyclic prefix |
| **FDE** | Frequency Domain Equalization |
| **MMSE** | Minimum Mean Square Error |
| **SC** | Single Carrier |

# CHAPTER 1

# INTRODUCTION

In recent years single carrier modulation (SCM) has again become an interesting and complementary alternative to multicarrier modulations such as orthogonal frequency division multiplexing (OFDM). SC radio modems with frequency domain equalization have similar performance, efficiency, and low signal processing complexity advantages as OFDM, and in addition are less sensitive than OFDM to RF impairments such as power amplifier nonlinearities. OFDM suffers from drawbacks such as a large peak to average power ratio (PAPR), intolerance to amplifier nonlinearities, and high sensitivity to carrier frequency offsets.

This report discusses an alternative approach based on more traditional single-carrier (SC) modulation methods. When combined with frequency domain equalization (FDE), this SC approach delivers performance similar to OFDM, with essentially the same overall complexity. In addition, SC modulation uses a single carrier, instead of the many typically used in OFDM, so the peak-to-average transmitted power ratio for SC-modulated signals is smaller. This in turn means that the power amplifier of an SC transmitter requires a smaller linear range to support a given average power (in other words,requires less peak power backoff). As such, this enables the use of a cheaper power amplifier than a comparable OFDM system; and this is a benefit of some importance, since the power amplifier can be one of the more costly components in a consumer broadband wireless transceiver.

SC and OFDM systems can potentially coexist for mutual benefit and cost reduction, because of the obvious similarities in their basic frequency domain signal processing functions. Single carrier techniques can easily be combined with multiple-input, multiple-output (MIMO) techniques, in which both transmitting and receiving ends use arrays of antenna elements. MIMO techniques can potentially achieve enormous spectral efficiencies (b/s/Hz), limited only by the number of diversity antenna elements that can be implemented practically . This, in turn, relieves the delay-spread issues, since the desired bit rate is achieved without increasing the symbol rate.

## 1.1 Organization of thesis

Chapter 2 explains about Single-Carrier Frequency Domain Equalization (SCFDE)and Orthogonal Frequency Domain Equalization . The drawbacks and advantages of SCFDE and OFDM are also discussed.

Chapter 3 provide a detailed description of the MMSE-FDE Single-Carrier Broadband Wireless System. The frame level description and receiver algorithms are well explained with block diagrams.

Chapter 4 deals with the simulations results of MMSE-FDE Single-Carrier Broadband Wireless System. The simulation results contains BER performance, mean square error of channel Estimation, mean square of timing offset and Peak to average power ratio performance.

Chapter 5 discusses the conclusions and future works.

# CHAPTER 2

# Single-Carrier Frequency Domain Equalization

Single-carrier frequency domain equalization (SCFDE) modulation [1] is a promising technique for highly dispersive channels in broadband wireless communications. A well known approach to mitigate ISI in highly dispersive channels is Orthogonal frequency division multiplexing(OFDM). OFDM suffers from drawbacks such as a large peak to average power ratio (PAPR), intolerance to amplifier nonlinearities, and high sensitivity to carrier frequency offsets. An alternative low-complexity approach to ISI mitigation is the use of single-carrier (SC) modulation combined with frequency domain equalization (FDE). Systems employing FD equalization are closely related to OFDM systems. In fact, in both cases digital transmission is carried out blockwise, and relies on FFT/inverse FFT (IFFT) operations. Therefore, SC systems employing FDEs enjoy a similar complexity advantage as OFDM systems without the stringent requirements of highly accurate frequency synchronization and linear power amplification as in OFDM. It is also worth noting that FDEs usually require a substantially lower computational complexity than their TD counterparts. In addition, recent results indicate that SC systems with FD equalization can exhibit similar or better performance than coded OFDM systems in some scenarios.

OFDM is a particular form of Multi-carrier transmission and is suited for frequency selective channels and high data rates. This technique transforms a frequency-selective wide-band channel into a group of non-selective narrowband channels, which makes it robust against large delay spreads by preserving orthogonality in the frequency domain. Moreover, the ingenious introduction of cyclic redundancy at the transmitter reduces the complexity to only FFT processing and one tap scalar equalization at the receiver. The major virtues of OFDM are 1) its resilience to multipath propagation providing a viable low-complexity and optimal (in the maximum likelihood sense) solution for intersymbol interference (ISI) mitigation, 2) the possibility of achieving channel capacity if the transmitted signal is adapted to the state of the communication channel (i.e., if energy and bit-loading procedures are adopted), and 3) the availability of strategies for frequency diversity scheduling in multiuser communication systems.

The block diagram of an SC wireless communication system employing FD equalization [2] is depicted in Figure 2.1. Serial-to-parallel (S/P) conversion produces data blocks, each consisting of $N$ symbols. Then, each block is cyclically extended, inserting at its beginning a repetition of its last $N_{cp}$ symbols, i.e., a cyclic prefix (CP), transmitted during the so-called guard interval. This introduces the elegant mathematical property of periodicity over a limited observation interval in the transmitted signal, at the price of a bandwidth/energy loss due to the presence of data redundancy. The sequence of cyclically extended blocks undergoes parallel-to-serial (P/S) conversion, so that one complex symbol is available every $T_s$ s, with $T_s$ being the so-called channel symbol interval for digital transmission. This requires the usual operations of digital-to-analog (D/A) conversion, frequency up-conversion, and filtering implemented in any SC modulator. The resulting radio frequency signal is transmitted over a wireless channel, characterized by a time dispersion not exceeding L channel symbol intervals (this includes the contributions of transmit and receive filtering also). The signal at the output of the wireless channel undergoes frequency down conversion, filtering, and analog-to-digital (A/D) conversion, producing a sequence of noisy samples that are grouped into equal-length blocks, each associated with a transmitted data block. For each noisy data block, the CP samples are discarded and the resulting block is sent to an FFT block converting it to the FD. This is followed by an FDE compensating for channel distortion and by an IFFT block bringing the noisy signal vector back to the TD. Finally, data decisions are made on a block-by-block basis and sent to the data link layer after S/P conversion.

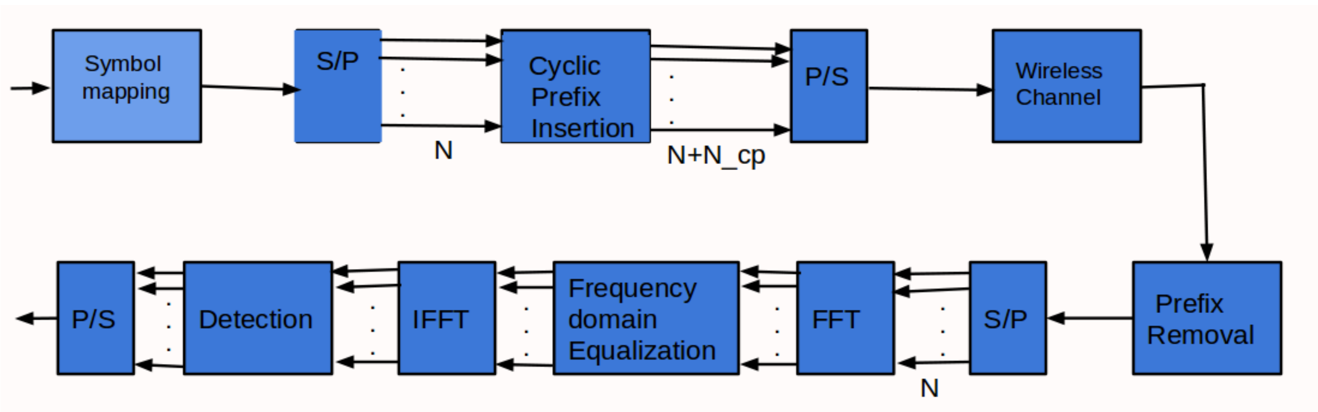


Figure 2.1: Block diagram of an SCFDE communication system

The block diagram of an OFDM system is illustrated in Figure 2.2. After symbol mapping and P/S conversion, blocks of N complex information symbols belonging to a M-ary complex constellation feed an Nth order inverse discrete Fourier transform (IDFT) block, implemented as an IFFT processor. Each block at the IFFT output, after P/S conversion, is cyclically extended,adding a prefix that consists of its last $N_{cp}$ symbols. The resulting sequence undergoes A/D conversion, frequency conversion, and filtering like in the SC system. The resulting radio frequency signal is transmitted over a wireless channel, characterized by a time dispersion not exceeding L channel symbol intervals (this includes the contributions of transmit and receive filtering also). The signal at the output of the wireless channel undergoes usual conversion and sampling operations already described for the SC system, demodulation can be accomplished via an FFT operation, separating the contributions associated with the different subcarriers. Then, after compensating for the phase rotations and the amplitude variations in the various subchannels, data decisions can be made, for a given data block, on a subcarrier-by-subcarrier basis.

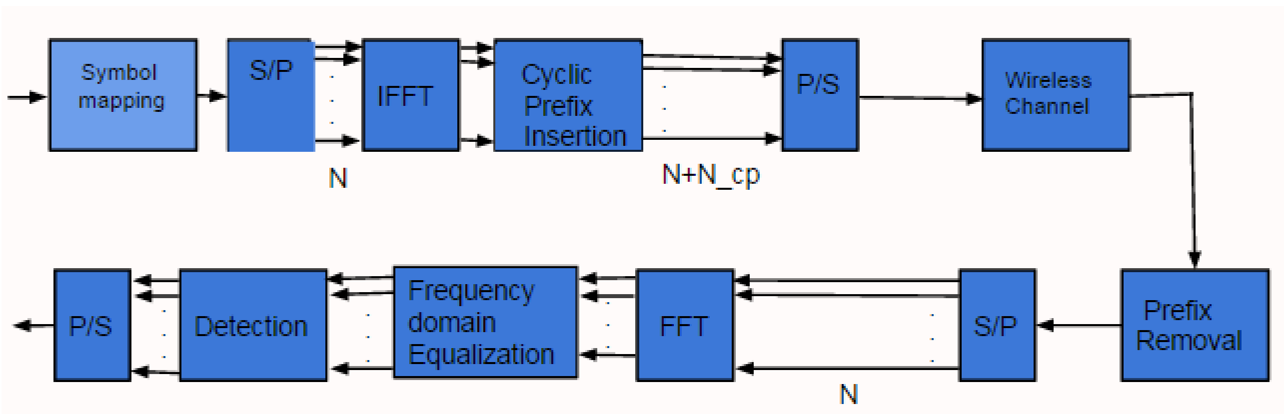


Figure 2.2: Block diagram of an OFDM communication system

# CHAPTER 3

# MMSE-FDE Single-Carrier Broadband Wireless System

In this Chapter, we provide a detailed description of the MMSE-FDE Single-Carrier Broadband Wireless System. Transmitter and Reciever blocks are explained in the subsequent sections. The frame structure description is included in the transmitter section.

## 3.1 TRANSMITTER

### 3.1.1 Frame Structure

Figure 3.1 provides the frame structure that is used in the transmitter. Each SCFDE symbol consists data symbols as well as the cyclic prefix (CP). The data symbols Block size is denoted by $N$ and the cyclic prefix length is denoted by $N_c$. A typical SCFDE frame consists of preamble, Pilot symbols and multiple SCFDE symbols. In this project an SCFDE frame consists of four SCFDE symbols in addition to the preamble and Pilot symbols. The pilot symbols block size is denoted by $N_p$ and the cyclic prefix length is denoted by $Np_{cp}$.


Figure 3.1: A typical SCFDE frame consisting of the preamble,Pilot symbols and multiple SCFDE symbols.

Preamble consists of two identical halves, each half is a pseudo-random number sequence. Channel is estimated with the use of pilot symbols inserted in the frame. The cyclic extension time length is denoted by $T_G$ and the sampling interval is denoted by $T_s$ .

## 3.2 Receiver Blocks

In this Section, a brief description of the blocks used in the receiver and the signal processing algorithms used in each block are provided.

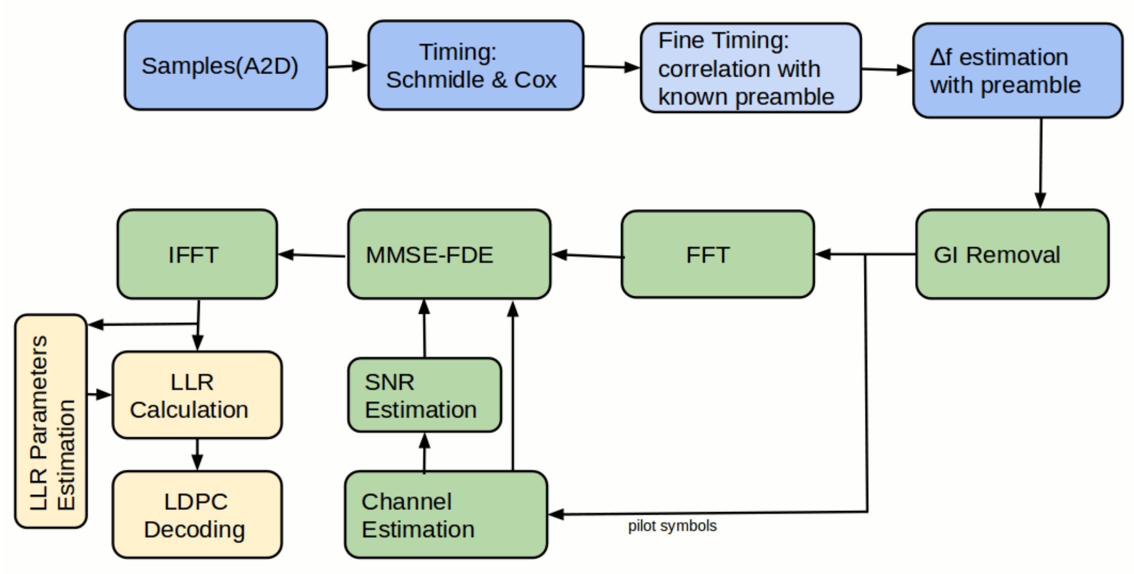


Figure 3.2: Basic blocks of the implemented receiver.

### 3.2.1 Timing and frequency Synchronization

Timing analysis is used to determine the point in time where the packet starts.This is also a method to recognize whether a packet has arrived or not. In the current scheme, the packet detection module uses the Schmidl and Cox method(Schmidl and Cox, 1997) [3] for timing analysis.For the receiver to be able to pick out a packet from ambient noise and interference, the transmitted packet must be designed for detection. All transmitted packets have a preamble, which consists of two identical halves. Each half is a pseudo-random-number sequence. The correlation of each half with with other (independent and hence uncorrelated) signals is expected to be low. However, its correlation with the other half will be high. Moreover, this property is well maintained even when the preamble is passed through an ISI channel with additive white gaussian noise.

The metric used to determine whether or not the correlation value is high enough is the normalized cross-correlation, defined as

$$\text{Metric} = \frac{|\text{Cross-correlation of the two windows}|}{\sqrt{\text{Product of the autocorrelations of the windows}}}$$

After sampling, the recieved complex samples are denoted as $r_m$. Let there be $L$ complex samples in one-half of the first training symbol (excluding the cyclic prefix), and let the sum of the pairs of products be

$$P(d)=\sum_{m=0}^{L-1}(r^*_{d+m}r^*_{d+m+L})$$

The autocorrelation for the second half symbol is defined by

$$R(d)=\sum_{m=0}^{L-1}|r_{d+m+L}|^2$$

Then the timing metric can be defined as $M(d) = \dfrac{|P(d)|}{|R(d)|}$.

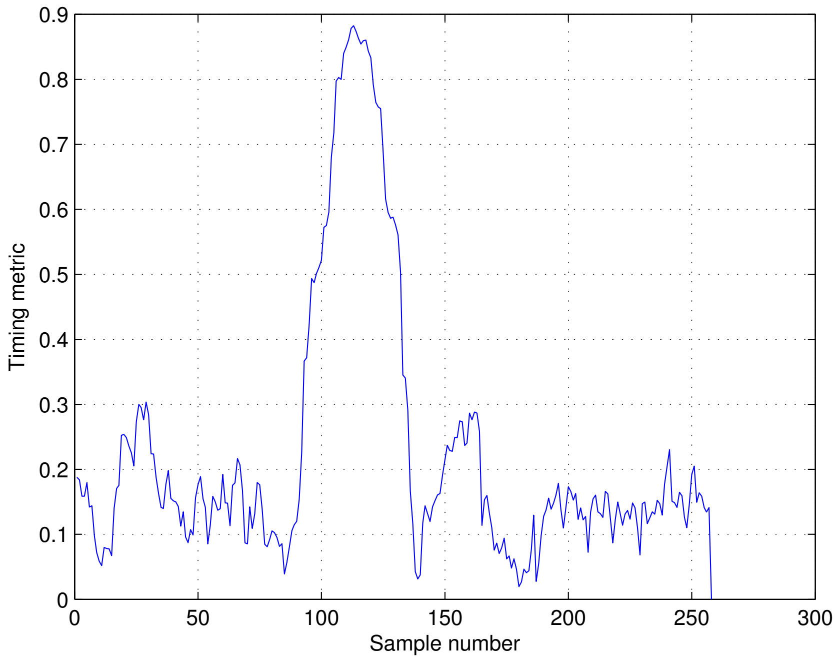


Figure 3.3: Timing metric for the 9-tap fading channel (SNR = 10 dB).

Consider the first training symbol where the first half is identical to the second half (in time order), except for a phase shift caused by the carrier frequency offset. The main difference between the two halves of the first training symbol will be a phase difference of, $\phi$=$\pi T \Delta f$

which can be estimated by $\hat{\phi}$=$angle(P(d))$ near the best timing point. If |$\hat{\phi}$| can be guaranteed to be less than $\pi$, then the frequency offset estmate is $\hat{\Delta f}$=$\hat{\phi}$/($\pi T$) and the even PN frequencies on the second training symbol would not be needed.

### 3.2.2 Channel Estimation

Two channel estimation methods are implemented in this project.

#### 1. Modified Least-square Channel Estimation

Estimates the channel with the use of OFDM modulated training symbols inserted in the SCFDE frame. Modified Least-square(LS) estimator [5] is used for channel estimation. The start of the frame is detected using the Preamble sequence. The OFDM system is modelled using the N-point discrete-time Fourier transform (DFT$_N)as$

$$y = DFT_N(IDFT_N(x) \otimes \frac{g}{\sqrt{N}} + n)$$

where $x_k$ are the transmitted symbols, $y_k$ are the recieved symbols, $n_t$ is the white complex gaussian channel noise , g(t) is the channel impulse response and $\otimes$ denotes circular convolution. The system described above can be written as a set of N independent Gaussian channels,

$$y_k = h_k x_k + n_k, \qquad\qquad k = 0....N-1$$

where $h_k$ is the complex channel attenuation given by $\mathbf{h} = [h_0 h_1 ... h_{N-1}]^T = DFT_N(g)$ and $\mathbf{n} = [n_0 n_1 ... n_{N-1}]^T = DFT_N(n)$ is an i.i.d. complex zero-mean Gaussian noise vector. In matrix notation y=XFg+n where X is a matrix wih the elements of x on its diagonal and F be the normalized FFT matrixof size $N$ x $N$ , whose $(k, n)$th element is given by (1/$\sqrt{N}$)exp(-j2$\pi(k-1)(n-1)$/$N$.

The Least square (LS) estimator for the cyclic impulse response g minimizes $(y - XFg)^H(y - XFg)$ and generates

$$\hat{h_{LS}}=FQ_{LS}F^H X^H y,$$

where $Q_{LS} = (F^H X^H XF)^{-1}$

so the LS estimate reduces to $\hat{h_{LS}} = X^{-1}y$

Most of energy in g is contained in, or near, the first $L$=$\lceil \frac{T_G}{T_s} \rceil$ taps, where $T_G$ is the cyclic extension time length and $T_s$ is the sampling interval. The LS estimator does not use the statistics of the channel. The energy of $g$ decreases rapidly outside the first $L$ taps, while the noise energy is assumed to be constant over the entire range. The matrix $T$ denotes the first $L$ columns of the DFT matrix $F$. Taking only the first $L$ taps of $g$ into account, thus implicitly using channel statistics, the modified LS estimator becomes

$$\hat{h_{LS}}=TQ_{LS}^{'}T^H X^H y$$

where

$$Q_{LS}^{'} = (\mathrm{T}^H X^H XT)^{-1}$$

Inorder to improve the estimate transform $\hat{h_{LS}}$ into the time domain with an IFFT and use an $L$-size window mask to remove the noise beyond the channel length, and then transform the time-domain channel coefficients back to the frequency domain with an FFT.

**2. Least squares (LS) based Frequency Domain Channel Estimation**

Estimates the channel with the use of QPSK modulated training symbols [7] inserted in the SCFDE frame. Let F be the normalized FFT matrixof size $N$ x $N$ , whose $(k, n)$th element is given by (1/$\sqrt{N}$)exp(-j2$\pi(k-1)(n-1)/N$.Taking the FFT of the transmitted and received signals and noticing that $F^hF$= $I_N$, we can obtain the frequency-domain representation as follows:

$$Y_p = Fy_p = FT_pF^hFx_p + Fv_p$$

ie, $Y_p = H_pX_p + Vp$ where $H_p = FT_pF^h$ is the frequency-domain channel matrix for the $p$th block.

if the block time duration $T_b$ is much smaller than the channel coherence time, i.e., the fading-channel coefficients remain approximately constant for the entire block, and then the time domain channel matrix $T_p$ is circulant and $H_p$ is diagonal.$Y_p(k)$, $X_p(k)$, and $V_p(k)$ are the normalized discrete Fourier transform (DFT) of the corresponding time domain transmitted,received and noise signals. For the training block (p = 1), both the transmitted training signal $X_1(k)$ and received signal $Y_1(k)$ are known.The frequency-domain channel transfer function $H_1(k)$ at the training block can be estimated by LS criterion as follows:

$$\widetilde{H}_1(k) = Y_1(k)/X_1(k) = H_1(k) + V_1(k)/X_1(k) \qquad k= 1, 2, \ldots, \text{N}.$$

The estimate $\widetilde{H}_1(k)$ can be improved by a frequency domain filter to reduce noise. Although various frequency domain filter to reduce noise.Although various frequency domain filters can be employed,a common technique is to transform $\widetilde{H}_1(k)$into the time domain with an IFFT and use an L-size window mask to remove the noise beyond the channel length, and then transform the time-domain channel coefficients back to the frequency domain with an FFT.

### 3.2.3 SNR Estimation

Estimates the SNR with the use of OFDM modulated training symbols inserted in the SCFDE frame. The average SNR estimate [8] is given by

$$\hat{\rho}_{avg}=\hat{S}/\hat{W},$$

$$\text{where } \hat{S}=\frac{1}{N}\sum_{k=1}^{N}| Y(k)\text{-}H(k)X(k)|^2$$

$$\text{and } \hat{W}=\frac{1}{N}\sum_{k=1}^{N}| H(k)X(k)|^2$$

$\hat{S}$ and $\hat{W}$ are the estimated signal power and noise power respectively. $Y(k)$ is the normalized discrete Fourier transform (DFT) of received pilot symbols and $X(k)$ is the OFDM modulated training symbols. $H(k)$ is the estimated frequency domain channel values.

### 3.2.4 FFT of SCFDE symbol

In this step the CP are removed and FFT is applied to obtain frequency domain samples of the SCFDE symbol.

### 3.2.5 Frequency Domain Equalization

Zero forcing equalization, Regularized zero forcing equalization and minimum mean square error (MMSE) equalization in Frequency domain are implemented. The estimated SNR values are used in the frequency domain MMSE equalization. The frequency-domain equalization can be achieved by one-tap linear equaliser. Let $H_l$ be the estimated frequency domain channel values . Then the frequency domain equalizer[2] in each case is defined as,

Zero-forcing:

$$W_l = \frac{H_l}{|H_l|^2} \qquad\qquad 0 \leq l \leq N-1$$

Minimum mean square error :

$$W_l = \frac{H_l}{|H_l|^2 + \frac{\sigma_v^2}{\sigma_s^2}} \qquad\qquad 0 \leq l \leq N-1$$

where $\sigma_s^2$ is the signal power and $\sigma_v^2$ is the noise variance .

Regularized zero forcing :

$$W_l = \frac{H_l}{|H_l|^2 + \alpha} \qquad\qquad 0 \leq l \leq N-1$$

where $\alpha$ is the regularization parameter .

### 3.2.6 IFFT of Equalized symbol

Inverse FFT returns the equalized signal to the time domain prior to the detection of data symbols. This time domain symbols are used for the log-likelihood ratio (LLR) calculation.

### 3.2.7 LLR CALCULATION

#### LLR CALCULATION WITH SINGLE ANTENNA AT THE RECIEVER

We use a non-virtual channel(NVC) based methodology [4] to calculate the LLR parameters. NVC means that the post effective channel gain and noisepower are directly calculated without inserting the UW. For simplicity, we first rewrite the system model in vector forms. After the CP/GI removal, the received k-th block length-N vector $r_k$ is expressed as (N denotes the length of the FFT block),

$$r_k = H_k x_k + n_k$$

Here, $H_k$ is a $N$x$N$ circulant matrix representing the channel at time $k$. More precisely,

$$H_k=\begin{bmatrix} h_0^k & 0 & \dots & h_{L-1}^k & \dots & h_1^k \\ \vdots & h_0^k & \ddots & 0 & \ddots & \vdots \\ h_{L-1}^k & \vdots & \ddots & \vdots & \ddots & h_{L-1}^k \\ 0 & h_{L-1}^k & \vdots & \ddots & \ddots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & h_{L-1}^k & \dots & h_0^k \end{bmatrix},$$

where $h_m^k$ is the m-th complex tap coefficient of the CIR at time $k$; $L$ is the channel length; $x_k$ is the length-N transmitted signal vector; $n_k$ is a length-N vector containing uncorrelated, zero-mean, complex white Gaussian noise samples, each with variance of $\sigma^2$. Performing the FFT to transform the time domain samplesto the frequency domain, we have

$$\hat{r_k} = FH_kx_k + Fn_k \ ,$$

where $F$ is the normalized FFT matrix of size $N$ x $N$ , whose $(k, n)$th element is given by (1/$\sqrt{N}$)exp(-j2$\pi(k-1)(n-1)/N$ . The above equation can be rewritten as,

$$\begin{aligned} \hat{r_k} &= FH_kF^HFx_k + Fn_k \\ &= \hat{H}_k\ \hat{x_k} + \hat{n_k} \end{aligned}$$

Here, $\hat{x_k}$=$Fx_k$ and $\hat{n_k}$=$Fn_k$ are frequency domain samples; $\hat{H}_k$=$FH_kF^H$ is a diagonal matrix with the CFR coefficients on its diagonal. In order to minimize the mean squared error between the received signal and the original signal, the MMSE-FDE is employed. The MMSE-FDE coefficients are calculated as,

$$C_k = \hat{H_k}^H(\hat{H_k}^H\hat{H}_k + I_N\tfrac{1}{\gamma})^{-1}$$

where $I_N$ is a $N$x$N$ identity matrix , $\gamma = \rho/\sigma^2$ is the received SNR with transmit signal power $E(x_kx_k^H) = \rho$. Then,the output signal of the MMSE-FDE is given as,

$$\bar{r_k} = C_k\hat{H}_k\ \hat{x_k} + C_k\hat{n_k}$$

After all, by performing the IFFT, the data samples are transformed from the frequency domain back to the time domain, i.e.,

$$\widetilde{r_k} = F^HC_k\hat{H}_k + F^H\hat{x_k} + F^HC_k\hat{n_k}$$

By substitution, we have

$$\begin{aligned} \widetilde{r_k} &= F^H\hat{H_k}^H(\hat{H_k}^H\hat{H}_k + I_N\tfrac{1}{\gamma})^{-1}\hat{H}_kFx_k + F^H\hat{H_k}^H(\hat{H_k}^H\hat{H}_k + I_N\tfrac{1}{\gamma})^{-1}Fn_k \\ &= x_k + [\mathrm{F}^H\hat{H_k}^H(\hat{H_k}^H\hat{H}_k + I_N\tfrac{1}{\gamma})^{-1}\hat{H}_kF - I_N]x_k + F^H\hat{H_k}^H(\hat{H_k}^H\hat{H}_k + I_N\tfrac{1}{\gamma})^{-1}Fn_k \end{aligned}$$

The first term of the RHS is the desired signal; the second term is the residual ISI; and the last term is the enhanced noise samples. Since the MMSE-FDE can well suppress the ISI, the second term of the RHS is negligible. so the last equation can be approximated as,

$$\tilde{r_k} \approx x_k + F^H \hat{H_k}^H (\hat{H_k}^H \hat{H}_k + I_N \tfrac{1}{\gamma})^{-1} F n_k$$
$$= x_k + \tilde{n_k},$$

where $\tilde{n_k} = F^H \hat{H_k}^H (\hat{H_k}^H \hat{H}_k + I_N \frac{1}{\gamma})^{-1} F n_k$ we can see that the post effective channel gain $\alpha$ actually corresponds to unity. The only remaining task is to obtain the noise power, $\tilde{\sigma}_k^2$. The covariance matrix of $\tilde{n_k}$ is calculated as,

$$\Sigma_k = E(\tilde{n_k}\tilde{n_k}^H) = \sigma^2 F^H C_k C_k^H F,$$

where $E(.)$ denotes the expectation operation . Then, we have

$$\tilde{\sigma}_k^2 = tr(\Sigma_k)/N$$
$$= tr(\sigma^2 F^H C_k C_k^H F)/N$$
$$= tr(\sigma^2 C_k C_k^H)/N$$
$$= tr(\sigma^2 \hat{H_k}^H (\hat{H_k}^H \hat{H}_k + I_N \tfrac{1}{\gamma})^{-1} \hat{H}_k)/N$$

where $tr(.)$ represents the matrix trace operation.

The log-likelihood ratio (LLR) of bit $r_i$, $i$=1,2 of the recieved symbol is defined as

$$LLR(r_i) = log\left(\frac{Pr(r_i=1|y)}{Pr(r_i=0|y)}\right)$$

Clearly, the optimum decision rule is to decide, $\hat{r}_i = 1$ if $LLR(r_i) \geq 0$, and 0 otherwise. Define two set partitions, $S_i^{(1)}$ and $S_i^{(0)}$ , such that $S_i^{(1)}$ comprises symbols with $r_i = 1$ and $S_i^{(0)}$ comprises symbols with $r_i = 0$ in the constellation. From the above equation, we have

$$LLR(r_i) = log\left(\frac{\sum_{\alpha \epsilon S_i^{(1)}} Pr(a=\alpha|y)}{\sum_{\beta \epsilon S_i^{(0)}} Pr(a=\beta|y)}\right)$$

Using Bayes' rule, we then have

$$LLR(r_i) = log\left(\frac{\sum_{\alpha \epsilon S_i^{(1)}} f_{y|a}(y|a=\alpha)}{\sum_{\beta \epsilon S_i^{(0)}} f_{y|a}(y|a=\beta)}\right)$$

since $f_{y|a}(y|a=\alpha) = \frac{1}{\sigma\sqrt{\pi}} exp\left(\frac{-1}{\sigma^2}||y-\alpha||^2\right)$ . So the LLR can be written as,

$$LLR(r_i) = \log\left(\frac{\sum_{\alpha \epsilon S_i^{(1)}} exp\left(\frac{-1}{\sigma^2}||y-\alpha||^2\right)}{\sum_{\beta \epsilon S_i^{(0)}} exp\left(\frac{-1}{\sigma^2}||y-\beta||^2\right)}\right)$$

Using the approximation[6] $log\left(\sum_j exp(-X_j)\right) \approx -min_j(X_j)$, we can approximate LLR as

$$LLR(r_i) = \frac{1}{\sigma^2}\left(\min_{\beta \epsilon S_i^{(0)}} ||y-\beta||^2 - \min_{\alpha \epsilon S_i^{(1)}} ||y-\alpha||^2\right)$$

**LLR CALCULATION WITH MULTIPLE ANTENNAS AT THE RECEIVER**

The LLR can be obtained from each recieve antenna seperately. The LLR from the $l$-th receive antenna is denoted as $LLR(r_l)$ . Let us assume that there are $L$ reciever antennas. After SCFDE demodulation, the recieved signal in each reciever antenna is approximated as,

$$r_l = x + \tilde{n}_l \qquad\qquad l = 0, 1, 2, ...L - 1$$

If the $\tilde{n}_l$ 's are independent , then we have

$$LLR(r_0, r_1, ....., r_{L-1}) = \sum_{l=0}^{L-1} LLR(r_l)$$

This shows that the joint LLR is simply a sum of individual LLRs.

# CHAPTER 4

# SIMULATION RESULTS

## 4.1 Simulation Parameters

- Number of QPSK symbols in one Symbol: 512
- Cyclic prefix length : 32
- Preamble length : 64
- Preamble Cyclic prefix length : 0
- No of pilots for channel estimation : 128
- Cyclic prefix for the channel estimation symbol : 32
- No of SCFDE symbols per frame : 4
- FFT/IFFT length : 512
- Timing metric threshold : 0.8
- Coding Parameters
  - LDPC channel coding is used
  - Rate of LDPC coding = 2/3
  - LDPC codeword length = 1920
- Bandwidth of the system : 10 MHz
- Channel model : Rayleigh
- Channel gains

$$[0.2772, 0.4130, 0.7077, 0.8518, 0.8184, 0.6713, 0.4813, 0.3055, 0.1730].$$

- Frequency domain equalization : MMSE, ZF
- Roll-off factor : 0.25
- SNR Range : 5 to 25 dB

## 4.2 Error Rate vs SNR Performance

Figure 4.1 shows the SCFDE Error Rate vs SNR performance in a 9-tap Fading channel wth zero forcing and MMSE equalization. Also, the Performance with LDPC channel coding is included.

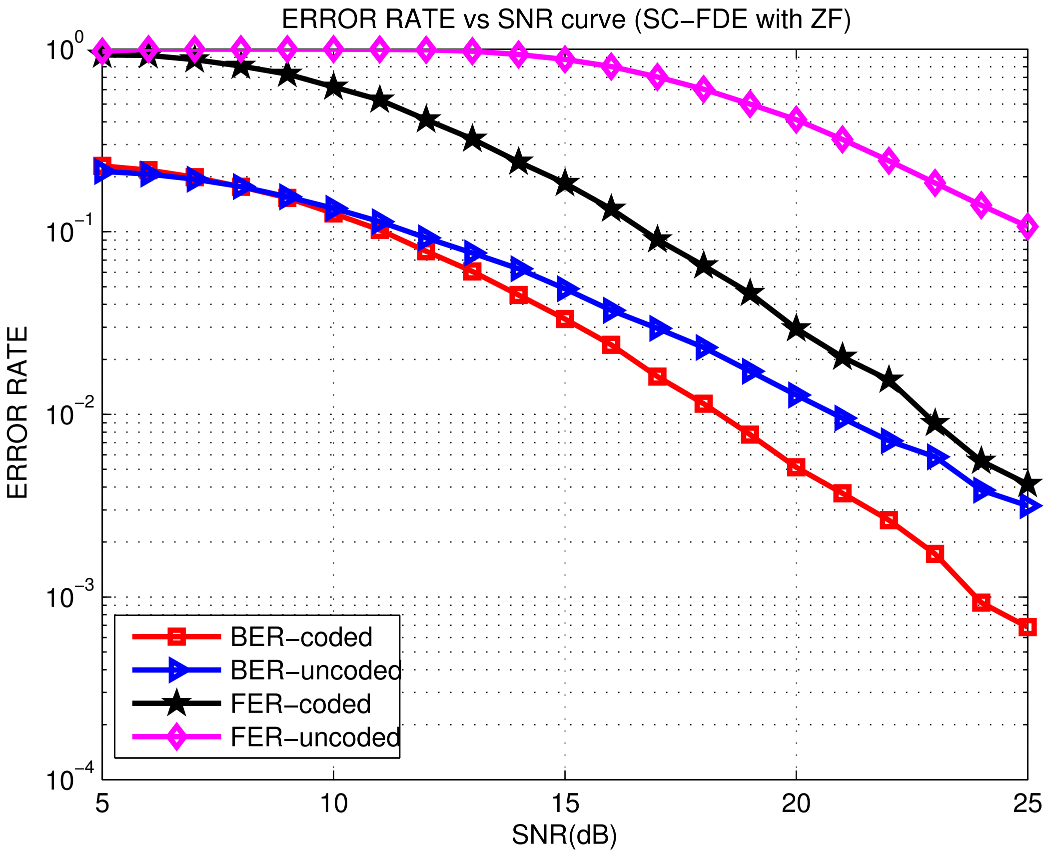


Figure 4.1: Bit error rate and Frame error rate in 9-tap Fading channel with zero forcing equalization.There is no frequency offset.

Figure 4.2 shows the Error Rate vs SNR performance of OFDM and SCFDE in a 9-tap Fading channel with MMSE equalization. The estimated SNR is used for MMSE frequency domain equalization. Also, the Performance with LDPC channel coding is included.

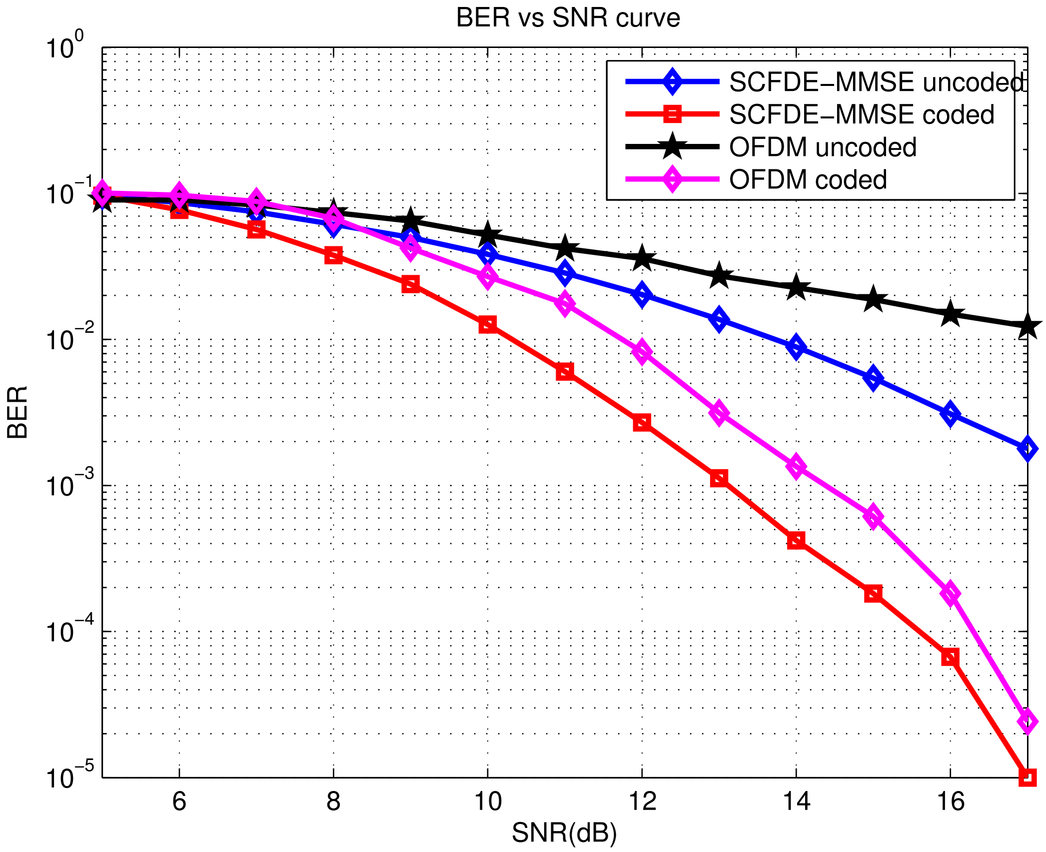


Figure 4.2: BER vs SNR performance of SCFDE and OFDM in 9-tap Fading channel. There is no frequency offset.

## 4.3 Error Rate vs SNR Performance in SIMO

Figure 4.3 shows the SCFDE Error Rate vs SNR performance with multiple antennas at the reciever. The 1xK representation corresponds to single antenna at the transmitter and K antennas at the reciever. The performance of 1x1,1x2 and 1x3 are shown in Figure 4.3. Also, the Performance with LDPC channel coding is included. The BER performance improves with the increase in number of antennas at the reciever.

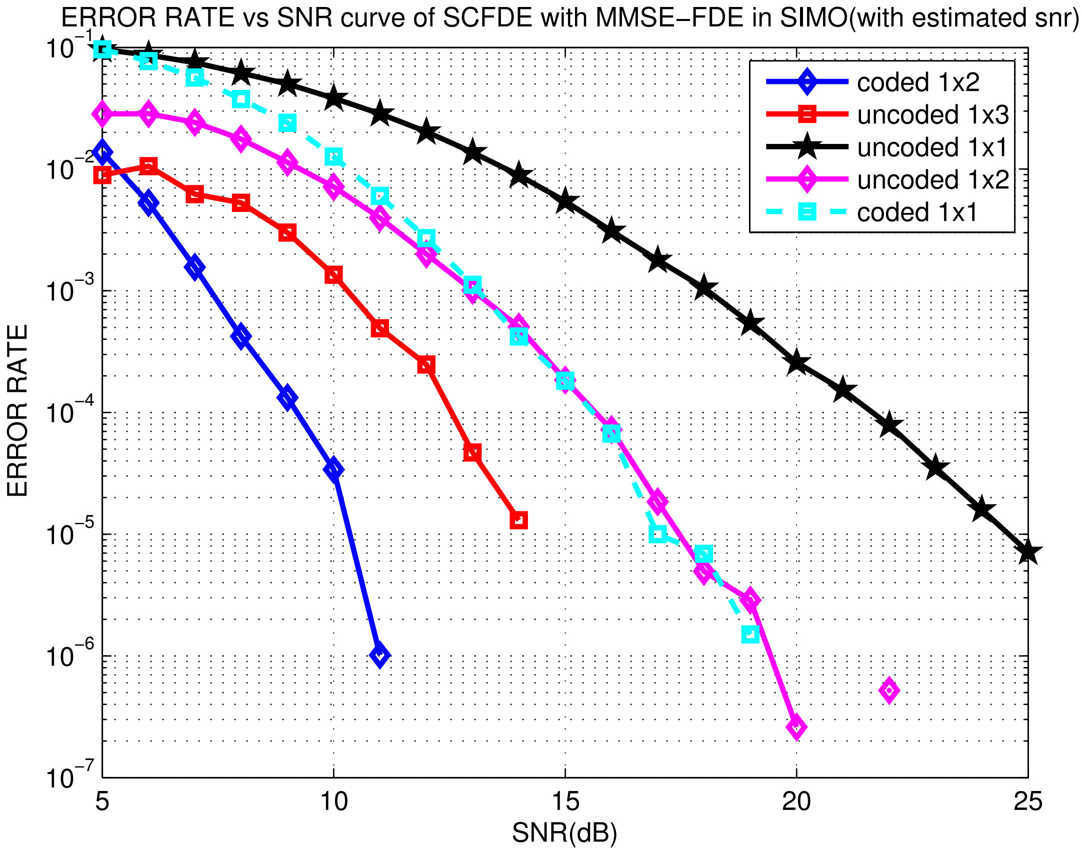


Figure 4.3: Bit error in 9-tap Fading channel with multiple antennas at the receiver and mmse(using estimated snr)frequency domain equalization. There is no frequency offset.

## 4.4 Mean Square Error of Channel estimation

Figure 4.4 shows the Mean Square error (MSE) of channel estimation in 9-tap Fading channel. The channel is estimated with the use of training symbols inserted in the SCFDE frame. Mean square error is calulated from the actual rayleigh channel used in the simulation and the estimated channel. Two channel estimation methods are implemented in this project, Modified Least-square channel estimation and Least squares based frequency domain channel estimation.

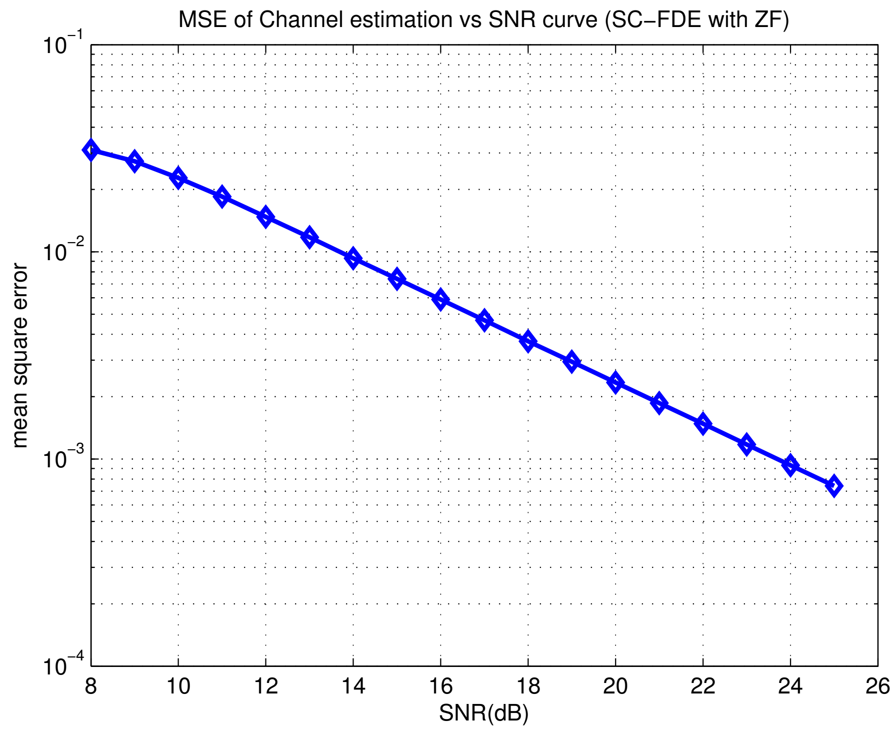


Figure 4.4: Mean Square error of channel estimation in 9-tap Fading channel

## 4.5 Mean Square Error of Timing offset

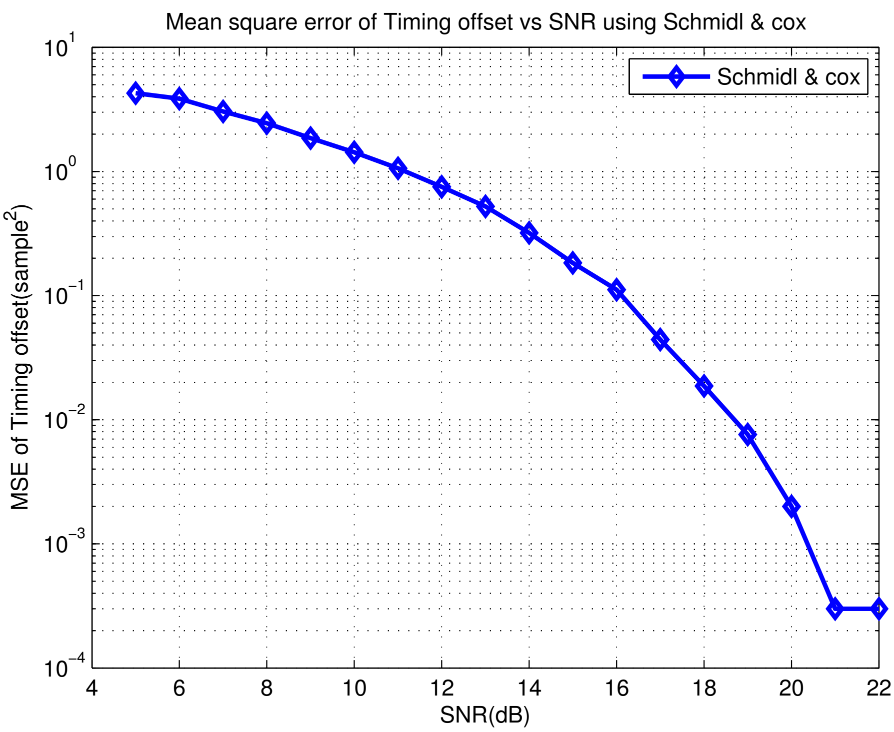


Figure 4.5: Mean square error of Timing offset in AWGN channel

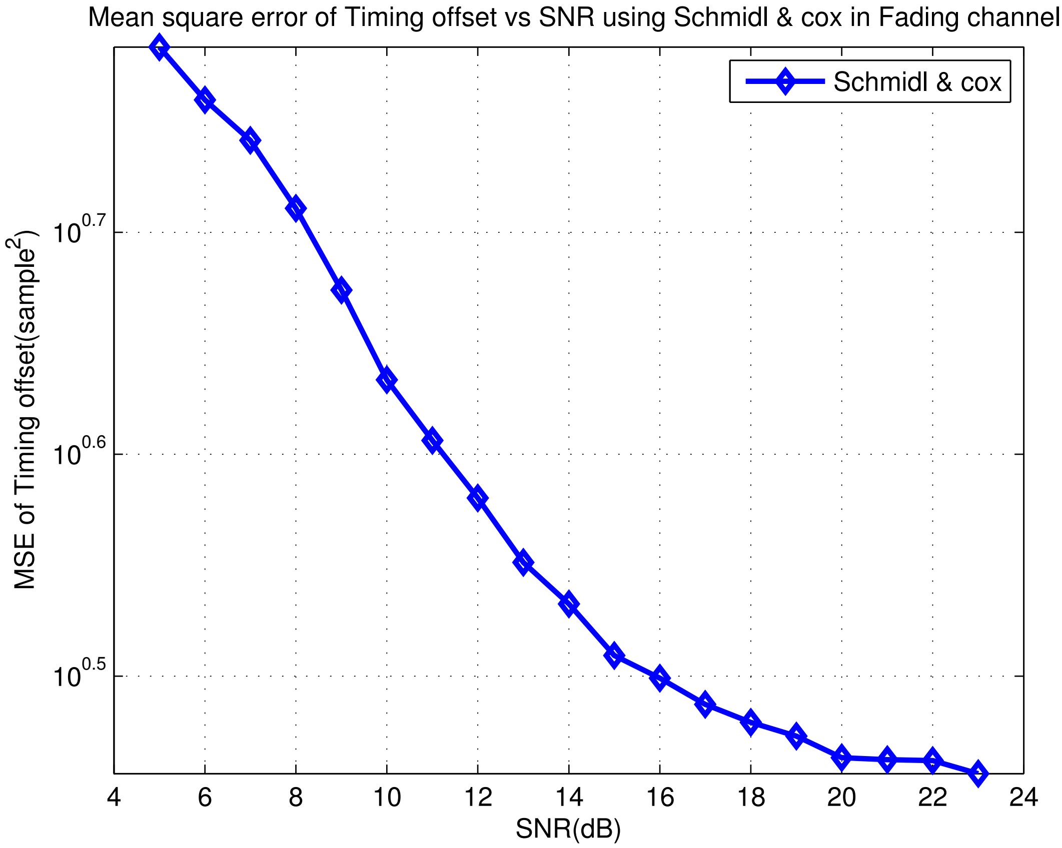


Figure 4.6: Mean square error of Timing offset in 9-tap Fading channel

## 4.6 Peak to average power Ratio

Figure 4.7 shows the Complementary Cumulative Distribution Function of Peak to average power Ratio for SCFDE and OFDM . The figure shows that the OFDM signal exhibits high PAPR compared to SCFDE. High PAPR values in OFDM stem from the superposition of a large number of usually statistically independent sub-channels that can constructively sum up to high signal peaks in the time domain. The SCFDE modulation use a Raised-cosine pulse shaping at the transmitter with a rolloff factor 0.25 . Figure 4.8 shows the Cumulative Distribution Function of Peak to average power Ratio for SCFDE and OFDM.

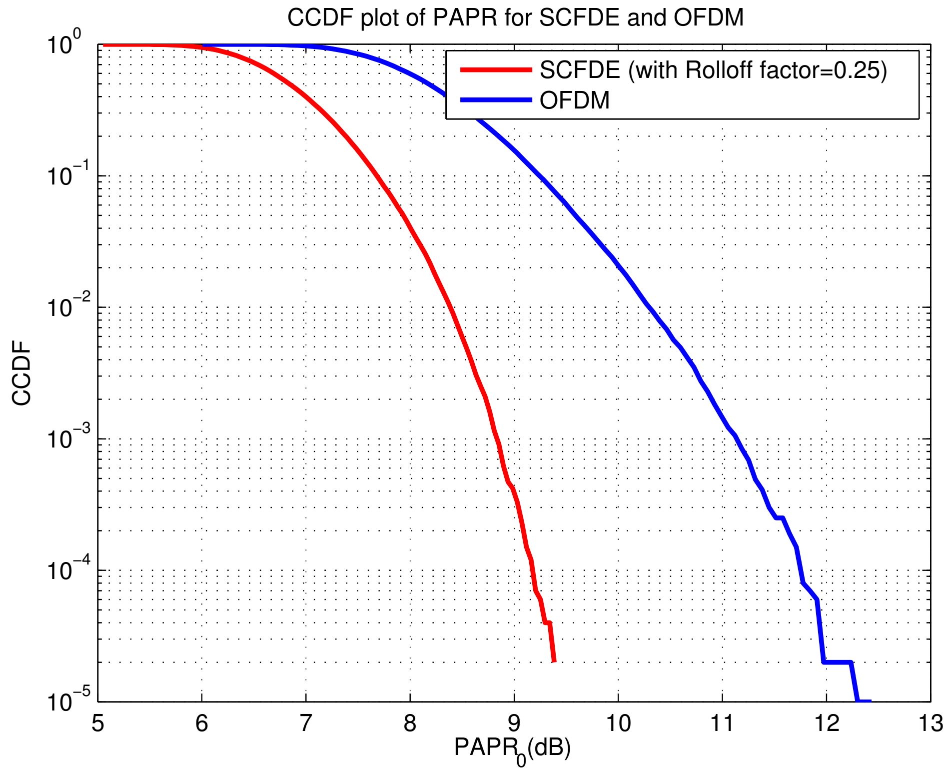


Figure 4.7: Complementary Cumulative Distribution Function of Peak to average power Ratio

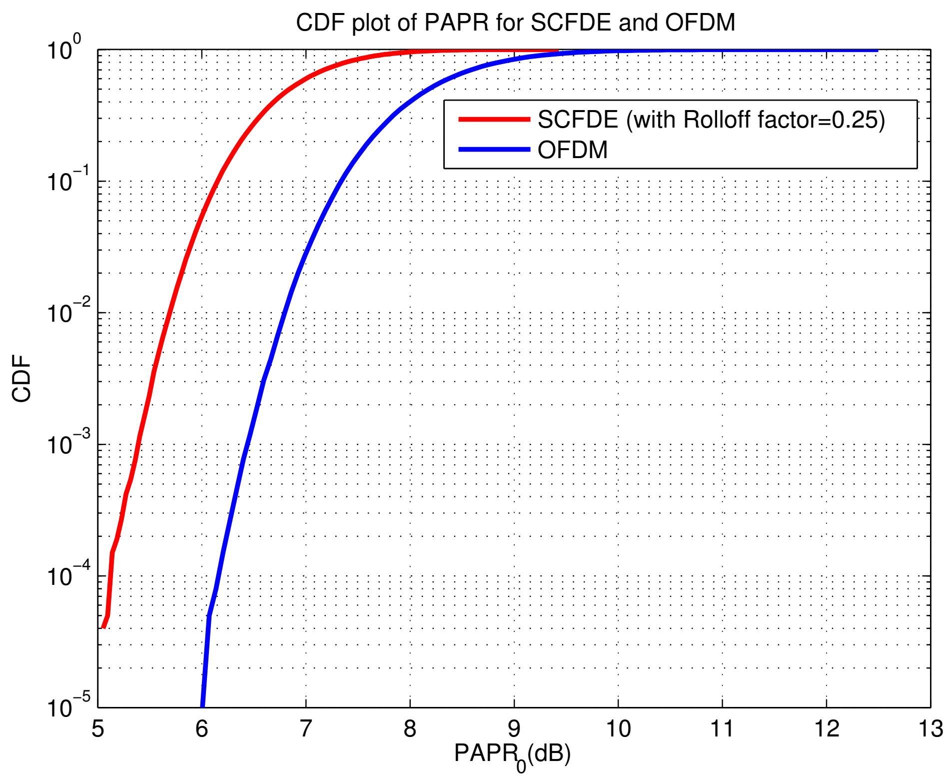


Figure 4.8: Cumulative Distribution Function of Peak to average power Ratio

# CHAPTER 5

# Conclusions and Future Work

In this thesis, a detailed description of the MMSE-FDE Single-Carrier Broadband Wireless System is provided. MMSE-FDE Single-carrier modulation avoids the inherent drawbacks of OFDM such as amplifier nonlinearities, carrier frequency offsets, and phase noise. The comparative performance analysis of MMSE-FDE Single-carrier modulation and OFDM schemes reveals that MMSE-FDE Single-carrier modulation achieves comparable (or even better in some scenarios) performance compared to its OFDM counterpart.

In this work, we didn't consider multiple antennas at the transmitter. We implemented single-input multiple-output (SIMO) and single-input single-output (SISO) scenarios. Multiple-input multiple-output (MIMO) systems can potentially achieve enormous spectral efficiencies (b/s/Hz), limited only by the number of diversity antenna elements that can be implemented practically.

The MMSE-FDE Single-carrier modulation technique have scope in future generation multi-band cognitive wireless systems. The interest in this topic is due to the fact that MMSE-FDE Single-carrier modulation techniques are less sensitive to RF impairment than OFDM; for this reason, they can applied to multiband transmissions exploiting cognitive radio principles.

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