Resource Allocation and Dynamic Cell Formation in Cellular Networks

A Project Report

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SHASHANK YADAV

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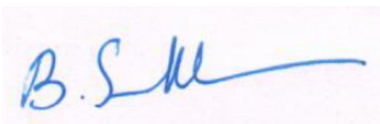
THESIS CERTIFICATE

This is to certify that the thesis titled **Resource Allocation and Dynamic Cell Formation in Cellular Networks**, submitted by **Shashank Yadav**, to the Indian Institute of Technology, Madras, for the award of the degree of **Dual Degree**, is a bona fide record of the research work done by him under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.



Dr. Srikrishna Bhashyam
Research Guide,
Professor,
Dept. of Electrical Engineering,
IIT-Madras, 600 036

Place: Chennai

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ABSTRACT

KEYWORDS: Resource Allocation, Performance Region, User Performance, Dynamic Cell, Single Objective Optimization, Multi Objective Optimization, Directional Communications

Nowadays, the adoption of multiple antennas at base stations has become a very crucial factor in the architecture of the cellular communication systems in order to meet the large capacity demands of the user equipments. The enhancement in the gain achieved by employing this much needed scheme of multiple transmit antennas at base stations is quite well-identified. The performance of the multi-cell systems depends largely on resource allocation; that is basically defining the way in which the frequency, power and spatial resources are divided among the many users being served in the network. This problem of resource allocation can be posed as a constrained multi objective optimization problem and can be solved through various optimization algorithms. Also, the usage of large number of antenna elements gives rise high directivity gains and fully directional communication; and thus gives birth to the concept of dynamic cell which provides a fairly new user-centric cell formation design.

This thesis work analyses the work done by Björnson and Jorswieck (2013) on the problem of optimal resource allocation in multi-cell communication system in depth, by using the system utility metric to achieve practical feasibility. We solve the presented optimization problem using an optimization algorithm known as Polyblock Outer Approximation Algorithm. Also, in the later part, we dig deeper into the concept of dynamic cell presented in Shokri-Ghadikolaei *et al.* (2015) to analyze the performance gain of the dynamic cell network due to the availability of new degrees of freedom. Finally, we present the simulation results in order to support the improvement of the performance of the dynamic cell system over the traditional cell system.

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ABBREVIATIONS

BS	Base Station
UE	User Equipment
SINR	Signal-to-Interference-and-Noise ratio
PR	Performance Region
RSRP	Reference Signal Received Power
RSSI	Received Signal Strength Indicator
PA	Polyblock Outer Approximation Algorithm
MS	Mobile Station

NOTATION

$\mathbf{x}$	Bold face letters denote column vectors or matrices
$\mathbf{x}^H$	Hermitian of $\mathbf{x}$
$\mathbf{x}_n^{(k)}$	n^{th} element of k^{th} vector
$\|x\|$	absolute value of x
x	absolute value of x
$diag(\mathbf{x})$	$N \times N$ diagonal matrix with diagonal elements given by $\mathbf{x}$
$\mathbf{I}_N$	$N \times N$ identity matrix
MS_k	k^{th} Mobile Station
$\mathcal{U}$	set of User Equipments
$\mathcal{B}$	set of Base Stations
$\boldsymbol{\theta_i^b}$	operating bandwidth of the BS i
$\boldsymbol{p}$	transmission power of a BS
$\mathcal{B}$	set of Base Stations
$\boldsymbol{\theta_i^b}$	operating bandwidth of the BS i
$\boldsymbol{\theta_j^u}$	operating bandwidth of UE j
$\boldsymbol{\zeta_{ij}^b}$	angle between the positive x-axis and the direction in which BS i sees UE j
$\boldsymbol{\zeta_{ij}^u}$	angle between the positive x-axis and the direction in which UE j sees BS i
$\boldsymbol{\phi_i^b}$	boresight angle of BS i relative to the x-axis
$\boldsymbol{\phi_j^u}$	boresight angle of UE j relative to the x-axis
$\boldsymbol{g_{ij}^c}$	channel gain between BS i and UE j capturing both the path loss and and shadowing
$\boldsymbol{g_{ij}^u}$	directivity gain UE j adds to the link between BS i and UE j (reception gain)

CHAPTER 1

INTRODUCTION

We all are well familiar with the basic purpose of *communication*, that is to transfer the data between devices through a physical medium known as the *channel*. Now, this transfer of data can take place through numerous mediums. However, here we would just like to focus on the wireless transfer of the data as the electromagnetic waves propagating through the environment between the devices, which is well known by the term wireless communications. The frequency spectrum, as we all know, being limited, is a universal resource. This spectrum is quite crowded as it is used for a large number of applications such as computer networks, cellular communications, radio/television, satellite communications, military applications and many more. And hence, the licenses are very expensive, especially for the frequency bands suited for the long range applications. Thus, the architecture design of the wireless communication systems should be such that the frequency resources are used as efficiently as possible. And this is the part where the concept of efficient resource allocation becomes a very important factor. The process of optimal resource allocation consists of various crucial steps such as accurate modelling of different types of multi-cell communication systems and measuring their performance, not just the system utility but also the individual user performance because its not just the whole system utility we are interested in, but also the individual user satisfaction is also equally important in this concept. And hence there arises a tradeoff between the use of these two performance measures to efficiently allocate the available resources among the users.

Here, in this thesis, we take the work presented by Björnson and Jorswieck (2013) where the concept of resource allocation is defined as allocating the transmit power among the user equipments(UEs) and spatial directions, while satisfying a set of power constraints having regulatory, physical and economic significance. In our case, the process of resource allocation becomes fairly complex in nature when more than one antennas are installed at each base station, and it becomes really necessary to deeply understand the nature of multi-cell system resource allocation and come up with efficient ways to exploit the spatial domain resources in order to improve the system throughput, user satisfaction and revenue of the multi-cell communication systems to the maximum extent. Talking mathematically, the resource allocation corresponds to the selection of a signal correlation matrix for each of the UEs and this facilitates the calculation of the SINR corresponding to each UE. While formulating the problem for the resource allocation in multi-cell systems, we face the conundrum of choosing one of the very different ways to measure the performance experienced by each of the UEs and the inherent conflict between maximizing the performance experienced by different UEs. For this purpose, we define the concept of performance region (PR). This performance region acts a brigde between the individual performance and the system utility. And hence, the resource allocation problem can be formulated as a multi objective optimization problem where the boundary of the performance region represents all the efficent solutions to the above mentioned optimization problem.

The above discussed usage of the multiple antennas at each base station also gives rise to high directivity gains, directional communication and possible noise limited operations. This enhancement in the directivity gains and the ability of communicating with the users in fully and semi directional modes gives birth to a new concept called dynamic cell. In contrast to the traditional definition of the cell where the user association is decided using the simple association metric derived from a rule called minimuml distance rule based on reference signal received power (RSRP) and RSSI, the dynamic cell presents a new user centric user association paradigm. The traditional RSRP/RSSI based association leads to an unbalanced number of UEs in a cell, and hence limits the amount of resources available for each UE in densely populated cells while a lot of unused resources going to waste in case of sparsely populated cells. This is even infuriated by the use of directionality in communication feature of the multi-cell systems. As a result of the directionality, the whole system becomes a noise-limited system instead of interference-limited system. Hence, it is totally meaningless to use the minimum distance rule derived association metric, which is suited for interference-limited systems and definitely not for the noise-limited systems. This results in a number of disadvantages of the current static cell definition over the definition of dynamic cell. The main disadvantage is that the static cell formation is independent of the load of the cell as well as the capabilities of the UEs because of the predetermined coverage area of a BS.

The three main parameters which should affect the formation of the cell in a cellular network are:

- UE traffic demand
- channel between UE and BSs
- BSs loads

The RSRP/RSSI based minimum distance association metric only considers the second parameter, and a re-association is needed whenever this parameter is changed in the network, which is highly inefficient in the mmWave systems, as the mmWave technology enables the systems to incorporate a large number of antennas both at the BS as well as UE. So, clearly the definition of a dynamic cell seems very appropriate for the scenario in the discussion, which takes care of all the three parameters while deciding the user association in the multi-cell systems.

We start by discussing in detail about the work done by Björnson and Jorswieck (2013) on the topic of resource allocation in the multi-cell systems, clearly present the formulation of the constrained optimization problem for the allocation of the resources as presented in the research paper and then try to solve the stated optimization problem using the Polyblock Outer Approximation Algorithm(PA), and present the results. In the later part of this thesis work, we dig into the concept of dynamic cell as presented by Shokri-Ghadikolaei *et al.* (2015) and then demonstrate dynamic cell defined system clearly outperforming the traditional static cell defined system. We realize that the problem presented in the research paper[2] can not be solved as it is due to high computational complexity, so we make some necessary modifications keeping this fact in mind, and then determine the solution.

CHAPTER 2

SYSTEM MODEL

2.1 Introduction

Here in this chapter, we aim to gradually build the system model presented in Björnson and Jorswieck (2013) used for the purpose of resource allocation. This section presents conceptual and theoretical insights regarding the optimization of general multi-cell systems. For a good understanding, we first start with the single cell scenario and present the mathematical system model for the single cell communication system. Then, we aim to extend this single cell mathematical system model to present the mathematical model for the multi-cell communication system. To sum up the section, the necessary foundation work for the resource allocation is laid which is to be discussed and solved in the next chapter.

2.2 System Model: Single Cell Scenario

Let us consider a single cell scenario in which a base station operating with N antennas communicates with K_r UEs. We denote the k^{th} user by the symbol MS_k and we assume that it has only one antenna. We do not consider the scenario where the UEs are equipped with multiple antennas in this work. We assume single antenna UEs because it reduces the hardware complexity to a significant extent, it requires less knowledge of the channel at the transmitter end and is quite close to the optimal scenario in realistic situations.

We assume the channel to MS_k to be flat fading and the dimensionless vector $\mathbf{h}_k \in C^N$ represents the channel in the complex baseband. The complex valued element $[\mathbf{h}_k]_n$ represents from n^{th} transmit antenna, the channel gain is represented by the magnitude of the element while the argument represents the phase-shift caused by the channel in description. The whole collection of all the channel vectors is know as the *channel state*

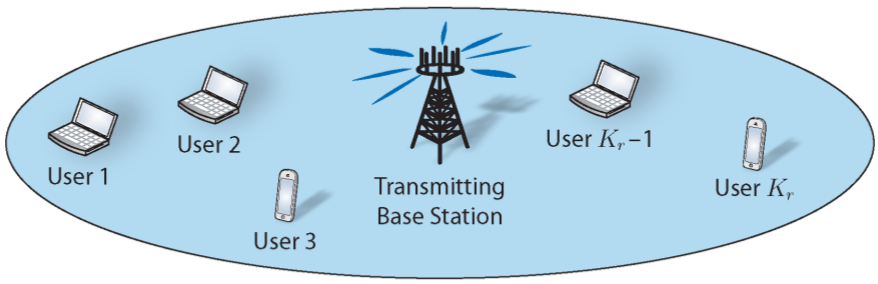


Figure 2.1: Illustration of multi user system with a base station (equipped with N antennas) and K_r users. [Source: Björnson and Jorswieck (2013)]

information (CSI) and we assume the the base station knows about the CSI perfectly. We take a few more assumptions into consideration such as the hardware is ideal and without any impairment so as to simplify the conceptual understanding of the problem that will be presented in the subsequent sections. It is a well known fact that it is generally impossible to model a real system perfectly. So, our goal in this work is to formulate a model that helps us to analyze the system but also at the same time is correct enough to provide valuable insights.

With respect to the linear input-output model taking these assumptions into consideration, the received signal at MS_k is y_k which is given by

$$y_k = \mathbf{h}_k^H \mathbf{x} + n_k \tag{2.1}$$

where $n_k \in C$ is the combined vector of additive noise and interference from the neighbouring systems. n_k is modeled as circularly symmetric complex Gaussian distributed, $n_k \quad \mathcal{CN}(0, \sigma^2)$, where σ^2 is the noise power. The input-output model is depicted in Figure 2.2.

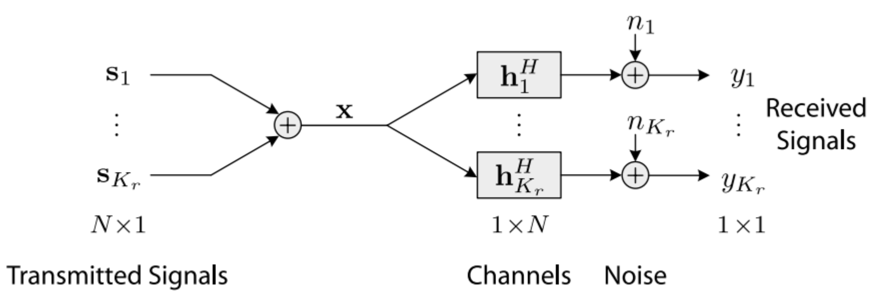


Figure 2.2: Block diagram for of the basic single cell model with N antennas serving K_r single-antenna UEs. [Source: Björnson and Jorswieck (2013)]

The data meant for each of the UEs is contained by the transmit signal $\mathbf{x} \in C^N$ and is given by

$$\mathbf{x} = \sum_{k=1}^{K_r} \mathbf{s}_k \tag{2.2}$$

where $\mathbf{s}_k$ is the signal meant for the UE MS_k. These signals are modeled as zero-mean with signal correlation matrices given by

$$\mathbf{S}_k = E\{\mathbf{s}_k \mathbf{s}_k^H\} \in C^{NXN} \tag{2.3}$$

The selection of the signal correlation matrices $\mathbf{S}_1$,....,$\mathbf{S}_{K_r}$ is called as the transmit strategy. The average power allocated to MS_k is tr($\mathbf{S}_k$). Now in the next subsection, we describe the power constraints that are imposed to the system model and play a crucial role in determing the transmit strategies.

2.2.1 Power Constraints

We very well know that the power resources available to us for the purpose of transmission are limited and need to be used judiciously. This motivates us to define constraints in such a form that it is taken well into the consideration under the system model. We assume that there are L power constraints, which are described by the model as

$$\sum_{k=1}^{K_r} tr(\mathbf{Q}_{lk} \mathbf{S}_k) \leq q_l \qquad l = 1, ..., L \tag{2.4}$$

where $\mathbf{Q}_{lk} \in C_{NXN}$ are Hermitian positive semi-definite weighing matrices and $q_l \geq 0$ are the limits for all l, k We also need to ensure that the power is constrained in all the spatial direction, and for this purpose the matrices should satisfy the condition

$$\sum_{l=1}^{L} \mathbf{Q}_{lk} > \mathbf{0}_N$$

Now, there might a number of reasons these constraints or limitations come into existence in our current system model such as

- **physical limitations** e.g. to protect the dynamic range of the power amplifiers;
- **regulatory constraints** e.g. to limit the power radiated in specific directions;
- **interference constraints** e.g. to control the interference caused to some UEs;

- **economic decisions** e.g. to manage the long term cost and revenue of running a BS

We can present two examples that can be viewed as the two extremes in the practical systems.

- Total power constraint i.e. $L = 1$ and $\mathbf{Q}_{1k} = \mathbf{I}_N$ for all k;
- Per-antenna constraint i.e. $L = N$ and $\mathbf{Q}_{1k}$ is only nonzero at the lth diagonal element.

The L linear power constraints described in (2.4) can also be broken down in order of describe per-user constraints as

$$tr(\mathbf{Q}_{lk}\mathbf{S}_k) \leq q_{lk} \qquad k = 1, ...K_r, l = 1, ..., L \tag{2.5}$$

for some limits $q_{lk} \geq 0$ for all l, k. In order to satisfy the constraint (2.4), the per-user power limits have to satisfy the following conditions

$$\sum_{k=1}^{K_r} q_{lk} \leq q_l \qquad l = 1, ..., L \tag{2.6}$$

The above mentioned decomposed representation of the linear power constraints turns out to be very helpful in deriving structural results on optimal transmit strategies. The selection of the limits q_{lk} is a very important part of the optimization process where it represents per-user power allocation in the system.

2.3 System Model: Multi-Cell Scenario

Now that we have defined the system model for the single cell scenario, it will be quite easy for us to just extend that model for the multi-cell scenario. In case of the multi-cell scenario, the channel from all the BSs to MS_k can be denoted by $\mathbf{h}_k = [\mathbf{h}_{1k}^T ... \mathbf{h}_{K_t k}^T]^T \in C^N$ where $\mathbf{h}_{jk} \in C^{N_j}$ denotes the channel from the BS_j. We consider a the following assumptions that are very crucial for defining the model of multi cell system:

- BS_j has the channel estimates to UEs in $C_j \in 1, 2, ..., K_r$, while the interference generated to the UEs $i \notin C_j$ is negligible and can be considered as Gaussian background noise;

- BS_j serves the users in the set $D_j \subset C_j$ with the data.

In the multi-cell scenario, only certain channel elements of $\mathbf{h}_k$ will carry the data. And these can be selected with the help of the diagonal matrices $\mathbf{D}_k \in C^{NXN}$ and $\mathbf{C}_k \in C^{NXN}$, which are defined as follows

$$\mathbf{D}_k = \begin{bmatrix} \mathbf{D}_{1k} & \dots & 0 \\ \vdots & \ddots & \\ 0 & & \mathbf{D}_{K_t k} \end{bmatrix} \quad \text{where} \mathbf{D}_{jk} = \begin{cases} \mathbf{I}_{N_j}, & if k \in D_j, \\ \mathbf{0}_{N_j}, & otherwise \end{cases} \tag{2.7}$$

$$\mathbf{C}_k = \begin{bmatrix} \mathbf{C}_{1k} & \dots & 0 \\ \vdots & \ddots & \\ 0 & & \mathbf{C}_{K_t k} \end{bmatrix} \quad \text{where} \mathbf{C}_{jk} = \begin{cases} \mathbf{I}_{N_j}, & if k \in C_j, \\ \mathbf{0}_{N_j}, & otherwise \end{cases} \tag{2.8}$$

Hence, one can easily notice that the channel thata carries the data to MS_k is $\mathbf{h}_k^H \mathbf{D}_k$ and the channel that carries the non-negligible interference is $\mathbf{h}_k^H \mathbf{C}_k$. The purpose of the matrices $\mathbf{C}_k$ and $\mathbf{D}_k$ are to ensure that the correct BS transmit to the MS_k while optimizing the resource allocation.

Hence, the complex baseband signal received at MS_k is given by

$$y_k = \mathbf{h}_k^H \mathbf{C}_k \sum_{i=1}^{K_r} \mathbf{D}_i \mathbf{s}_i + n_k \tag{2.9}$$

where the additive term n_k is assumed to model both the noise and weak uncoordinated interference from all the BS_j with $k \notin C_j$. This scenario is illustrated in the Figure 2.3

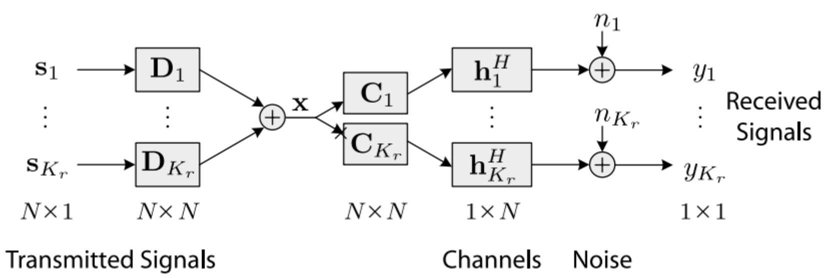


Figure 2.3: Block diagram of the system model for the multi-cell scenario. [Source: Björnson and Jorswieck (2013)]

Similar to the single cell scenario, the transmission of data is limited by a set of power constraints with size L. However, there is an important fact that has to be noted that in case of the multi cell scenario, the transmitted signals are $\mathbf{D}_k \mathbf{s}_k$ and not just $\mathbf{s}_k$, and thus each of the weighing matrix $\mathbf{Q}_{lk}$ should satisfy an additional constraint that $\mathbf{Q}_{lk} - \mathbf{D}_k^H \mathbf{Q}_{lk} \mathbf{D}_k$ is diagonal for all l, k. This additional constraint on the weighing matrices makes sure that the power cannot be allocated to unallowed subspaces for the purpose of reducing the measured power in the subspaces used for transmission, which is only possible when the weighing matrices $\mathbf{Q}_{lk}$ is nondiagonal.

CHAPTER 3

RESOURCE ALLOCATION IN MULTI CELL SYSTEMS

3.1 Introduction

Now that we have presented the whole system model of the multi-cell systems keeping the practical power constraints into consideration, its time that we proceed forward to the resource allocation in the system. We start by defining a general means of measuring the performance of the multi-cell system. There are two separate aspects that have to be considered when we talk about the measurement of the performance of the system:

- the performance experience by each user in the system;
- the system utility which is a collection of simultaneously achievable user performances

We briefly describe these two modes of measuring the system performance as these play a very important role in the formulation of the resultant optimization problem.

3.2 User Performance

Each user k in the system has its own quality measure which is represented by the user performance function g_k of the SINR experienced by the user while communicating in the system. This function describes the satisfaction that the system provides to the user and it generally is dependent on the factors such as throughput and delay constraints. For the sake of low complexity, we make an assumption that single user detection takes place i.e. a UE is not decoding and subtracting interfering signals while decoding its own signals. The SINR experienced by the user MS_k is given by

$$
\begin{aligned}
\text{SINR}_k(\mathbf{S}_1, ..., \mathbf{S}_{K_r}) &= \frac{\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k \mathbf{S}_k \mathbf{D}_k^H \mathbf{C}_k^H \mathbf{h}_k}{\mathbf{h}_k^H \mathbf{C}_k (\sum_{i \neq k} \mathbf{D}_i \mathbf{S}_i \mathbf{D}_i^H) \mathbf{C}_k^H \mathbf{h}_k + \sigma_k^2} \\
&= \frac{\mathbf{h}_k^H \mathbf{D}_k \mathbf{S}_k \mathbf{D}_k^H \mathbf{h}_k}{\mathbf{h}_k^H \mathbf{C}_k (\sum_{i \in \mathcal{I}_k} \mathbf{D}_i \mathbf{S}_i \mathbf{D}_i^H) \mathbf{C}_k^H \mathbf{h}_k + \sigma_k^2}
\end{aligned} \tag{3.1}
$$

where the simplification in the second equality is a result of $\mathbf{C}_k \mathbf{D}_k = \mathbf{D}_k$ and $\mathbf{C}_k \mathbf{D}_i \neq 0$ for the UEs i in

$$
\mathcal{I}_k = \bigcup_{\{j \in \mathcal{J} : k \in \mathcal{C}_j\}} \mathcal{D}_j \backslash \{k\} \tag{3.2}
$$

The set of UEs $\mathcal{I}_k$ is the set of co-UEs that are being served by the same BSs that contribute to the interference toward the MS_k. For our convenience, instead of writing $\text{SINR}_k(\mathbf{S}_1, ..., \mathbf{S}_{K_r})$, we just write SINR_k for the purpose of talking about SINR experienced by UE k.

User Performance Function : The user performance function can be defined as an arbitrary continuous, differentiable and strictly monotonically increasing function $g_k(\text{SINR})$ of the SINR which is used as a metric to measure the performance of MS_k. For notational convinience, the user performance function satisfies $g_k(0) = 0$

Information Rate : The information rate corresponds to the achievable mutual information/ achievable rate describing the number of bits that can be conveyed to a user per channel use with an arbitrarily low probability of decoding error. The achievable information rate is given by $g_k(\text{SINR}) = \log_2(1 + \text{SINR}_k)$

3.3 Multi Objective Resource Allocation

Whenever a situation comes where one has to maximize the performance of multiple users working simultaneously in a system, there always arise conflicts and tradeoffs between the individual performance and the systems performance as a whole. Each of the UEs has its own objective function $g_k(\text{SINR}_k)$ that has to be optimized for maximizing the user satisfaction, and hence there are K_r different objective functions that are fighting inherently for their optimization. This conflict gives rise to the tradeoff and these problems are formulated mathematically as *multi-objective optimization problems*. With everything that we have defined, the resource allocation problem can be formulated as follows:

$$\begin{aligned} \underset{\mathbf{S}_1 \geq \mathbf{0}_N, \ldots, \mathbf{S}_{K_r} \geq \mathbf{0}_N}{maximize} \; & \{g_1(\text{SINR}_1), \ldots, g_{K_r}(\text{SINR}_{K_r})\} \\ \text{subject to} \quad & \sum_{k=1}^{K_r} tr(\mathbf{Q}_{lk}\mathbf{S}_k) \leq q_l \qquad \forall l \end{aligned} \tag{3.3}$$

The multi-objective problem defined above can be seen as looking for a transmit strategy $\mathbf{S}_1, \ldots, \mathbf{S}_{K_r}$ which successfully satisfies the constraints and also maximizes the performance $g_k(\text{SINR}_k)$ of all the UEs. But, as the performance of all the UEs in the system is coupled by the constraints, generally there is not a single optimal solution to transmit strategies that simultaneously maximizes the performance of all the UEs. And, in order to study this conflicting situation of different objective functions fighting for their optimization, we define the set of all the feasible operating points $\mathbf{g} = [g_1 \ldots g_{K_r}]]^T$ and call this set as the achievable performance region denoted by the symbol $\mathcal{R}$. The performance region represents the performance that can be guaranteed to be simultaneously achievable by the UEs in the system. The region $\mathcal{R}$ is K_r dimensional and is non-empty as $\{\mathbf{0}_{K_r X 1}\} \in \mathcal{R}$. The shape of the performance region depends on a number of factors such as the constraints, cooperation in clusters and the channel vectors.

Utopia Point : The utopia point **u** is the unique solution to the optimization problem (3.3) in degenerate scenarios i.e. when the optimization decouples and all the users can achieve their maximal performance simultaneously. The utopia point describes an unattainable upper bound on the performance and in general, $\mathbf{u} \notin \mathcal{R}$.

We can only achieve a set of tentative solutions to the problem described in (3.3) and all of these solutions are operating points in $\mathcal{R}$ and are definitely not dominated by any other feasible point. These operating points are called as Pareto optimal and set of all the Pareto optimal points is known as the **Pareto Boundary**. The Pareto boundary is such that the performance cannot be enhanced for any user without degrading the performance for at least one of the other UEs.

One of the basic properties of the optimal resource allocation problem described in (3.3) is sufficiency of single-stream beamforming , that is having the signal correlation matrices $\mathbf{S}_k$ that are rank one. This can be seen as intuitive because each of the UEs have a single receive antenna. This property reformulates the problem defined in (3.3) by stating that all the tentative solutions can be achieved by $\mathbf{S}_k = \mathbf{v}_k\mathbf{v}_k^H$ for some beamforming vectors $\mathbf{v}_k$. The reformulated problem is given by

$$\begin{aligned} &\underset{\mathbf{S}_1 \geq \mathbf{0}_N, \ldots, \mathbf{S}_{K_r} \geq \mathbf{0}_N}{maximize} \{g_1(\text{SINR}_1), \ldots, g_{K_r}(\text{SINR}_{K_r})\} \\ \text{subject to} \quad &\text{SINR}_k = \frac{|\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k \mathbf{v}_k|^2}{\sum_{i \neq k} |\mathbf{h}_k^H \mathbf{D}_i \mathbf{C}_k \mathbf{v}_i|^2 + \sigma_k^2} \quad \forall k \\ &\sum_{k=1}^{K_r} tr(\mathbf{Q}_{lk}\mathbf{S}_k) \leq q_l \quad \forall l \end{aligned} \tag{3.4}$$

The reformulation of the problem in (3.3) reduces the search space for optimal points to a great extent and makes the implementation a lot easier.

In most of the cases, there are many of the Pareto optimal points and there is no specific point better than the others objectively. So, in order to compare the merits and demerits of these Pareto optimal points, the system designer needs to pitch in and describe his own preference as an aggregate system utility function $f : \mathcal{R} \to \mathbb{R}$. A system utility function is denoted by $f(g_1(\text{SINR}_1), \ldots, g_{K_r}(\text{SINR}_{K_r}))$ and is monotonically increasing on $[\mathbf{0}, \mathbf{u}]$

Now, since we have a concept of a system utility function, the multi-objective optimization problem in (3.4) can be reformulated as a single-objective optimization problem given by

$$\begin{aligned} &\underset{\mathbf{S}_1 \geq \mathbf{0}_N, \ldots, \mathbf{S}_{K_r} \geq \mathbf{0}_N}{maximize} f(g_1(\text{SINR}_1), \ldots, g_{K_r}(\text{SINR}_{K_r})) \\ \text{subject to} \quad &\text{SINR}_k = \frac{|\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k \mathbf{v}_k|^2}{\sum_{i \neq k} |\mathbf{h}_k^H \mathbf{D}_i \mathbf{C}_k \mathbf{v}_i|^2 + \sigma_k^2} \quad \forall k \\ &\sum_{k=1}^{K_r} tr(\mathbf{Q}_{lk}\mathbf{S}_k) \leq q_l \quad \forall l \end{aligned} \tag{3.5}$$

Also, a very important point to be noted is that, if f is an increasing function, then the global optimum is attained on the Pareto boundary. We will use this fact a lot in the future sections to provide very useful insights we receive from the simulations.

3.4 Single-Objective Resource Allocation

In this section, refering to the work presented in the research paper[1], we present the solution of the multi-cell resource allocation problem for any system utility $F(.)$ and any general user performance function $g_k(.)$. Further in the next section, we present the results of the simulations carried by us in order to solve the resource allocation problem. Since all the functions that we refer to and take into consideration are increasing in nature, hence we can the this problem as monotonic optimization problem. The standard form of the monotonic optimization problem we are trying to solve is given as follows:

$$\begin{aligned} &\underset{\mathbf{g}}{maximize}\, f(\mathbf{g}) \\ &\text{subject to} \quad \mathbf{g} \in \mathcal{R} \end{aligned} \tag{3.6}$$

Taking a careful look at the above presented problem, its quite evident that now we have come into different domain. Now, we know that g that we are taking into consideration already satisfies previous constraints, and this can be used to find the optimal point. This section basically describes a very useful algorithm presented in the research paper[1] for solving the monotonic optimization problems: *polyblock outer approximation algorithm (PA)*.

3.5 Polyblock Outer Approximation(PA) Algorithm

As presented in the research paper[1], we solve the monotonic optimization problem given in (3.6) by using the Polyblock Outer Approximation (PA) Algorithm. This algorithm is designed to improve the bounds on the optimal value of the objective function f, and this converges to the final optimal value with an accuracy ϵ.

$$f_{max} - f_{min} < \epsilon \tag{3.7}$$

The algorithm in question also finds and ϵ-optimal solution g* to the problem, which is also the feasible point with $f_{min} = f(\text{g}^*)$. The complexity of this algorithm is NP hard, as the number of iteration taken by the algorithm increases exponentially with the increase in number of UEs.

Determination of the bounds in a box : Let $\mathcal{M} = [\mathbf{a}, \mathbf{b}]$ be a box with $\mathcal{M} \cap \mathcal{M} \neq \phi$ and $\mathbf{r}(\tau)$ is a strictly increasing curve with $\mathbf{r}(0) = \mathbf{a}$ and $\mathbf{r}(\tau^{upper}) = \mathbf{b}$ for some $\tau^{upper} > 0$. Then the bounds on f can be given by

$$\begin{aligned} f_{min} &= f(\mathbf{n}); \\ f_{max} &= f(\mathbf{z}_k) \end{aligned} \tag{3.8}$$

where $\mathbf{z}_k = f(\mathbf{b} - [\mathbf{b} - \mathbf{m}]_k \mathbf{e}_k)$ with $\mathbf{e}_k$ denoting the kth column of $\mathbf{I}_{K_r}$, $\mathbf{n} = \mathbf{r}(\tau^{lower}_{final})$ and $m = \mathbf{r}(\tau^{upper}_{final})$. $[\mathbf{r}(\tau^{lower}_{final}), \mathbf{r}(\tau^{upper}_{final})]$ are the final resultant interval obtained when we solve the following problem using any of the line-search algorithms for some accuracy $\delta > 0$ of the line search.

$$\begin{aligned} & \underset{\mathbf{v}_1,\ldots,\mathbf{v}_{K_r},\tau}{maximize} \; f(\tau) \\ \text{subject to} \quad & r_k(\tau) = g_k(SINR_k) \quad \forall k \\ & \sum_{k=1}^{K_r} tr(\mathbf{Q}_{lk}\mathbf{S}_k) \leq q_l \qquad \forall l \\ & \tau \in [0, \tau^{upper}] \end{aligned} \tag{3.9}$$

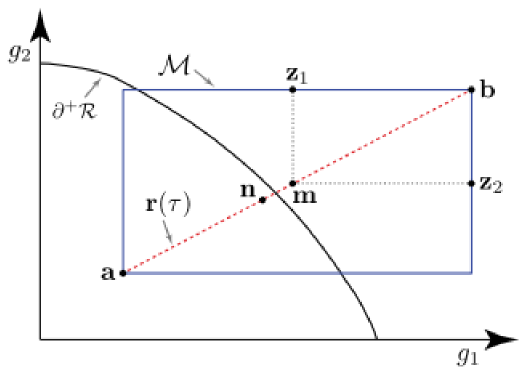


Figure 3.1: Illustration of the bounding procedure in the Polyblock algorithm. [Source: Björnson and Jorswieck (2013)]

As stated in the previous section, the optimal solution to the problem presented in (3.6) lies on the Pareto boundary of the performance region $\mathcal{R}$. So, what the PA algorithm does is that it looks for a solution to the problem by approximating the region itself and iteratively redefines the approximation. The PA algorithm is not directly applied to the original problem but to the slightly perturbed problem as stated in the research paper Björnson and Jorswieck (2013) given by

$$\begin{aligned} \underset{\mathbf{g}}{maximize}\, \tilde{f}(\mathbf{g}) = f([\mathbf{g}) - \mathbf{s}]_{+} + \mathbf{s}) \\ \text{subject to} \quad \mathbf{g} \in \mathcal{R} \end{aligned} \tag{3.10}$$

The reason we consider this perturbed problem and not the original problem is to prevent numerical convergenve issues like searching close to the axis.

Polyblock : A set $\mathcal{P} \subset \mathbb{R}_{+}$ is known as a polyblock if it is the union of a finite number of boxes with lower corners in the origin. There are a number of ways to write a polyblock using different set of vertices, but here we are concerned only about minimal set known as proper vertices, where no vertex is dominated by another vertex. A polyblock constructed with the finite set of vertices $\mathcal{V} = \{\mathbf{b}_1, ..., \mathbf{b}_{|\mathcal{V}|}\}$ is denoted by $\mathcal{P}(\mathcal{V})$.

The basic idea behind the working of the PA algorithm is that that the optimal maximum point of the problem in (3.10) is achieved at a proper vertex as the system utility function is increasing. So, if the region $\mathcal{R}$ is approximated by a polyblock, then the resultant Pareto boundary is approximated by the vertices of this polyblock.

Bounding Procedure : Let us consider the box $\mathcal{M}^{(n)} = [\mathbf{0}, \mathbf{g}^{(n)}]$ using

$$\mathbf{r}(\tau) = \tau \frac{\mathbf{g}^{(n)}}{||\mathbf{g}^{(n)}||} \qquad \tau \in [0, ||\mathbf{g}^{(n)}||] \tag{3.11}$$

Then we can use the rule for the determination of the bounds in the box as described earlier ot generate a feasible point $\mathbf{n}^{(n)}$ and a set of points $\{\mathbf{z}_k\}$ which upper bounds the performance in the box taken as $\mathcal{M}^{(n)}$ Then

$$\mathcal{V}_{n+1} = (\mathcal{V}_n \backslash \{\mathbf{g}^{(n)}\}) \bigcup_{k:[\mathbf{g}^{(n)}]_k > 0} \{\tilde{z}_k\} \tag{3.12}$$

The rule that is used to remove the improper vertices is described as:
For every $\mathbf{g} \in \mathcal{V}_n \backslash \{\mathbf{g}^{(n)}\}$ such that $\mathbf{g} \geq \mathbf{m}$ while $[\mathbf{g}]_k < [\mathbf{g}^{(0)}]_k$ for exactly one element k, then $\tilde{z}_k$ is to be removed from $\mathcal{V}_{n+1}$

Now that we have the sufficient background for the algorithm, the exact algorithm can be summarised as follows

Algorithm 1 Polyblock Outer Approximation (PA) Algorithm

Result: Solves the monotonic optimization problem (3.6)

Input: Feasible solution $\mathbf{g}_{feasible}$ on (3.6)

Input: Solution accuracy $\epsilon > 0$

and line search accuracy $\delta > 0$ **Input:** Initial vertex set $\mathcal{V}_1$ such that $\mathcal{P}(\mathcal{V}_1) \supset \mathcal{R}$

Initialization

Set $\mathbf{n}^{(0)} = \mathbf{g}_{feasible}$, $\mathbf{s} = \frac{\delta}{K_r}\mathbf{I}_{K_r}$, $n = 1$

Set $f_{min} = f(\mathbf{n}^{(0)})$ and $f_{max} = max_{\mathbf{b}\in\mathcal{V}_1}\,\tilde{f}(\mathbf{b})$

while $f_{max} - f_{min} > \epsilon$ **do**

Set $\mathbf{g}^{(0)} = argmax_{\mathbf{b}\in\mathcal{V}_n}\,\tilde{f}(\mathbf{b})$

Compute $\mathcal{V}_{n+1}$ using the bounding procedure defined earlier using

$\mathcal{M}^{(n)} = [\mathbf{0}, \mathbf{g}^{(n)}]$. Obtain the resulting feasible point $\mathbf{n}^{(n)}$

if $f(\mathbf{n}^{(n)}) > f_{min}$ **then**

Set $f_{min} = f(\mathbf{n}^{(n)})$

Set $f_{feasible} = \mathbf{n}^{(n)}$

end

Set $f_{max} = max_{\mathbf{b}\in\mathcal{V}_{n+1}}\,\tilde{f}(\mathbf{b})$

Remove all $\mathbf{b} \in \mathcal{V}_{n+1}$ with $\tilde{f}(\mathbf{b}) \leq f_{min} + \epsilon$

Set $n = n + 1$

end

Output: Final Interval $[f_{min}, f_{max}]$ on the optimal value

Output: Feasible point $\mathbf{g}_\epsilon^* = \mathbf{g}_{feasible}$ with $f_{min} = f(\mathbf{g}_\epsilon^*)$

3.6 Simulations and Results

In this section, we carry out the simulations for resource allocation using the PA algorithm and then present the results obtained. We maximize the sum information rate by approximation of the region around the optimal point using the polyblock. The algorithm improves the approximation of the region iteratively by just removing the points which are not able to contain the optimal point. The simulation setup contains a scenario with number of transmitters, $K_t = 2$, serving $K_r = 2$ receivers, where each of the transmitter is equipped with 2 antennas and the per-array power constraint on the transmitter is $q_l = 10dB$. The information rate is taken as $g_k(SINR_k) = log_2(1+SINR_k)$, which is considered as the user performance function. We take a realization of random channel in which the channel between a BS and UE is uncorrelated Rayleigh fading channel. We consider the line search accuracy, $\delta = 0.5$. We also show the Pareto boundary of the problem for illustration purposes, and also the optimal point which maximizes the sum rate in the network, denoted by the symbol $*$ marker in the plots .The progress of the algorithm under various iterations is shown in the figures presented in the Figure 3.2. From the below presented figures from the simulations carried out in the scenario explained earlier, we see that we start by initialising the polyblock to be have the vertices as origin and the utopia point. As the PA algorithm progresses, we get to see that the algorithm modifies the polyblock vertices to nearly approximate the performance region. And as a result of the algorithm, we get the ϵ - optimal solution of the problem. The optimal solution from the simulations under the above mentioned scenario is:

- The optimal network sum rate achieved in the system :

  11.2769 bits/channel use

- Optimal User Performance experienced by User 1 :

  6.1565 bits/channel use

- Optimal User Performance experienced by User 2 :

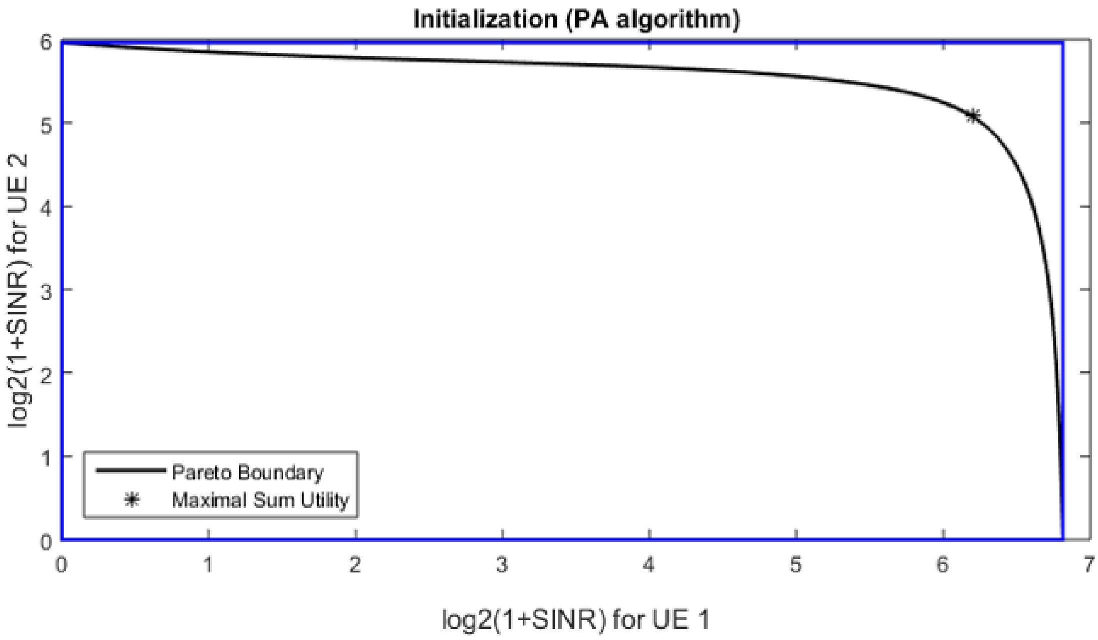
  5.1204 bits/channel use

We also observe that the PA algorithm takes more iterations to converge to an optimal value as the value of δ is increased. The observed values of the number of iterations taken corresponding to some values of δ are presented as follows:

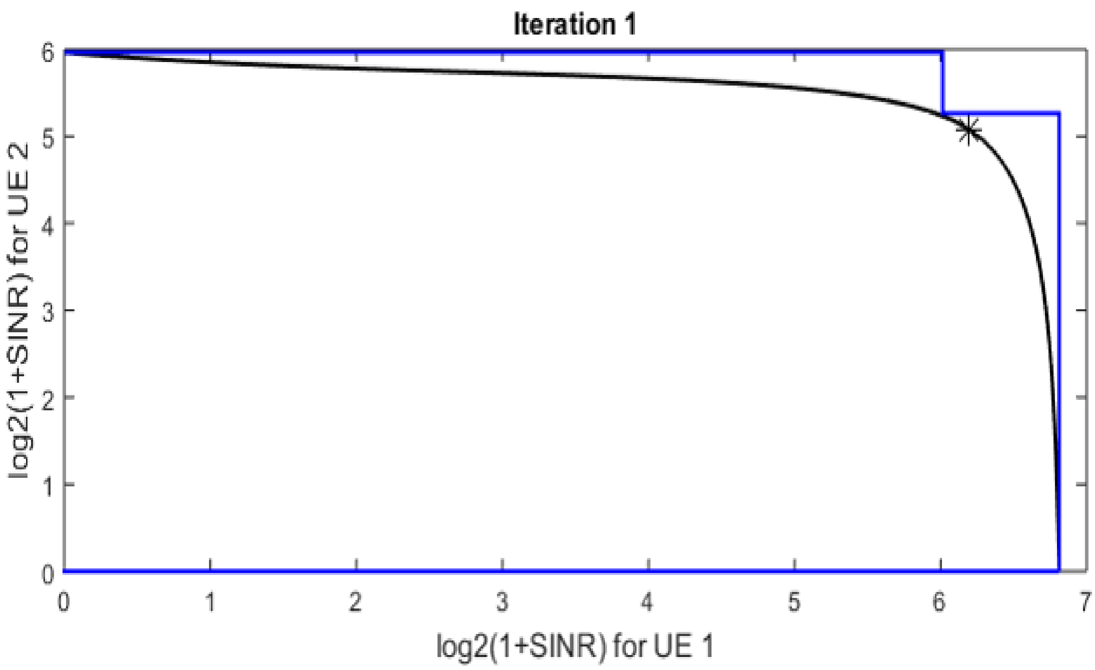
- For $\delta = 0.005$, number of iterations taken = 44
- For $\delta = 0.05$, number of iterations taken = 76
- For $\delta = 0.5$, number of iterations taken = 144

A possible explanation of this can be that as the value of the line search accuracy increases, the algorithm takes more number of outer iterations to reduce the volume of the polyblock.

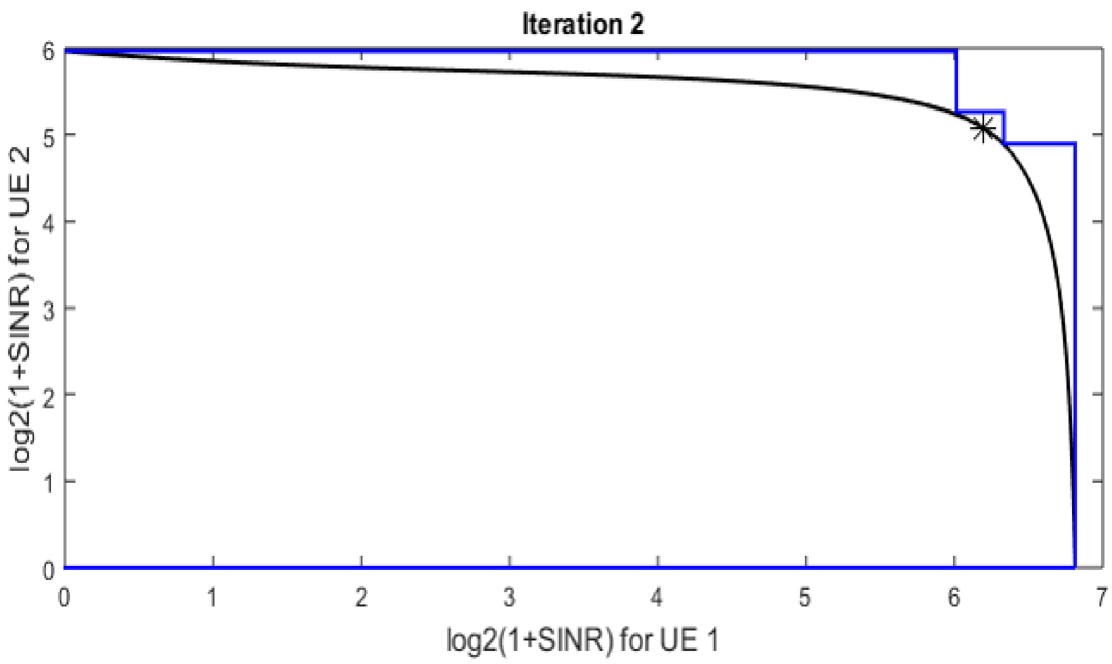
Figure 3.2: Simulation Results illustrating the progress of the Polyblock algorithm



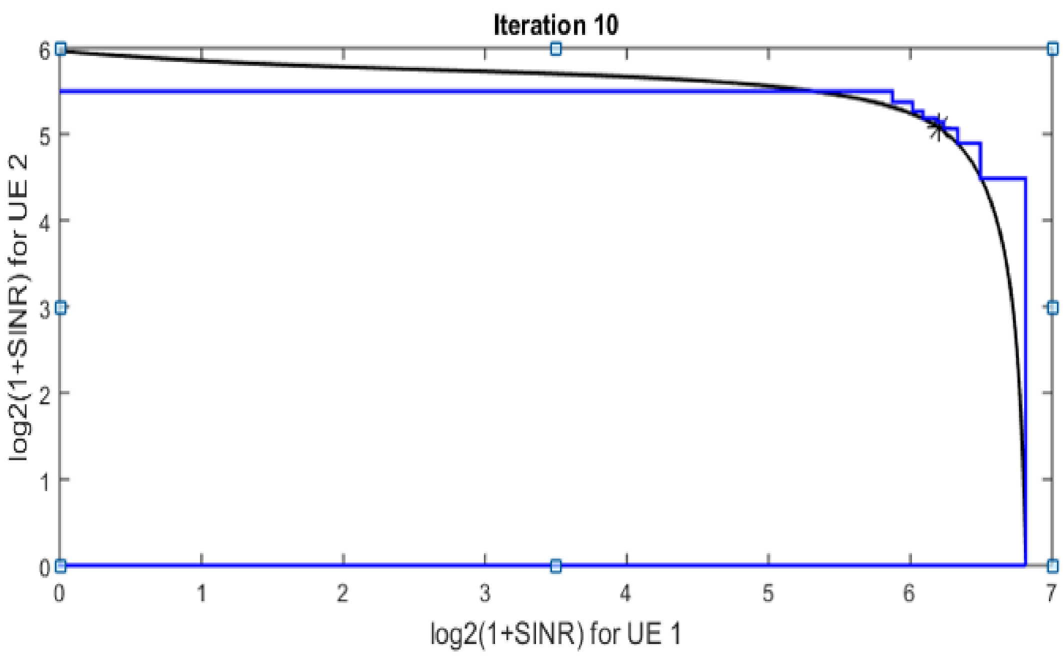
(a) Illustration of initialization iteration in the Polyblock algorithm



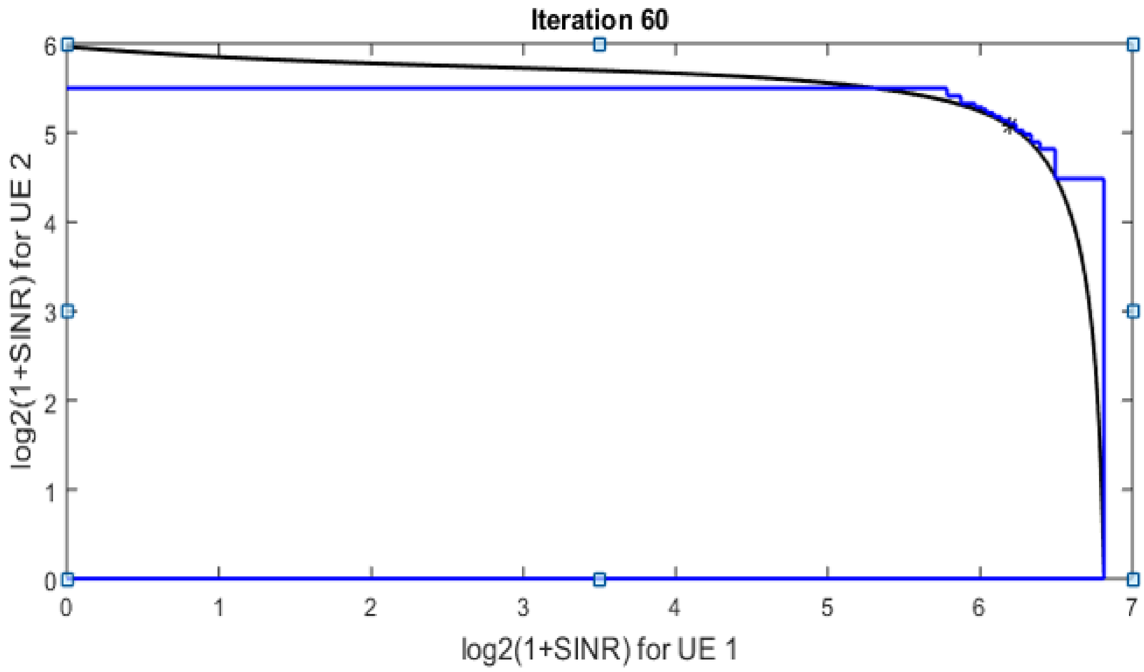
(b) Illustration of 1^{st} iteration in the Polyblock algorithm



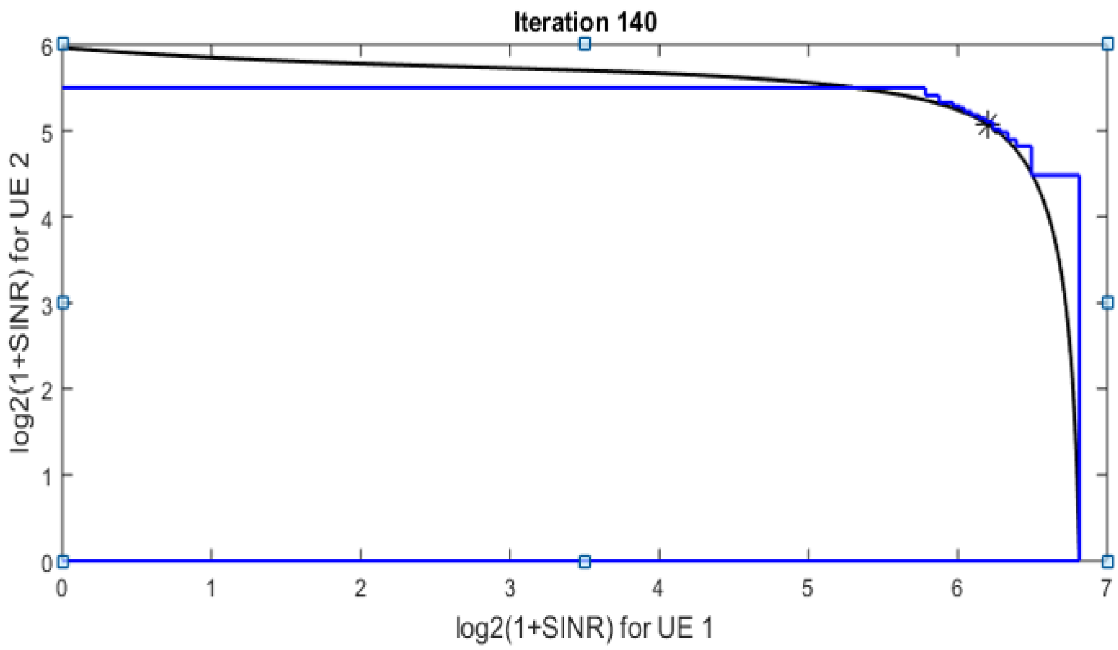
(c) Illustration of 2^{nd} iteration in the Polyblock algorithm



(d) Illustration of 10^{th} iteration in the Polyblock algorithm



(e) Illustration of 60^{th} iteration in the Polyblock algorithm



(f) Illustration of 140^{th} iteration in the Polyblock algorithm

We can easily observe that the region of the boundary around the optimal point has been approximated by the polyblock vertices successfully.

CHAPTER 4

DYNAMIC CELL

We know that the mmWave networks provide lots of degrees of freedom due to the directionality as compared to the traditional networks. With this massive number of degrees of freedom that the fully-directional communication offers and possibly MAC layer analog beamforming, a concept of dynamic cell can be defined as a set of UEs that are served by the same analog beamformer of the base station which are not necessarily co-located. This association/selection of the UEs to the BSs is done dynamically to improve some objective function which is maximum network sum rate in this case. When there are compelling fluctuations in the three parameters discussed above, there should be a requirement of a redefinition of the dynamic cell. There are mainly three criteria according to which the microcell BSs group the UEs together dynamically and form the new cells. These can be listed as follows:

- individual demands of the UEs must be met (Quality of Service, QOS provisioning)
- the trade off between the macro-level fairness and the spectral efficiency is improved through fair allocation of the resources also known as maximum utility maximization
- in order to guarantee the robustness due to blockage, every UE is grouped into at least two groups.In this proposed structure, any two colocated UEs may belong to different cells in case their demands are not met with resources available inside a cell and there exist proper spatial channels to form two independent cells in the cellular network.
  In addition to the above differences over the traditional cell definition, a new UE is not necessarily bound to be served by a BS which is geographically close to the UE, if this violates the Quality of Service of a UE that is already been associated with that particular BS. We know that serving a UE with farther BS which are less loaded as compared to the geographically close ones, is not a very good choice in the interference-limited traditional cellular networks, but it is quite feasible in the fully directional communication which is heavily based on the concept of proper resource allocation in the cellular network. We know that the microwave systems with omnidirectional operations are interference-limited systems, so the directionality feature in the mmWave systems with pencil-beam operations proves to be a very unique advantage in favor of the mmWave systems over the microwave networks.

4.1 Optimization Model for Cell Formation

In this section, we present the formulation of an optimization problem to get the optimal cell formation in cellular networks. We first present the formulation of the optimal cell for the fully-directional communications, and then later we will see that by some minor simplifications, we will be able to present the formulation for the semi-directional and omnidirectional communications.

Let the number of RF chains i.e the number of analog beams at the i^{th} BS be n_i. So basically what we intend to do is replace the BS i with n_i virtual BSs at the same position where each of there new virtual BS have one RF chain. These virtual BS are treated in the same way as the other BSs are treated. Some of the notations that we use throughout the report are mentioned below:

- $\mathcal{B}$: set of UEs
- $\mathcal{U}$: set of BSs
- $\boldsymbol{p}$: transmission power of a BS
- $\boldsymbol{\sigma}$: power of white Gaussian noise
- $\boldsymbol{g^c_{ij}}$: channel gain between BS i and UE j capturing both the path loss and and shadowing effects
- $\boldsymbol{\theta^b_i}$: operating bandwidth of the BS i
- $\boldsymbol{\theta^u_j}$: operating bandwidth of UE j
- $\boldsymbol{\zeta^b_{ij}}$: angle between the positive x-axis and the direction in which BS i sees UE j
- $\boldsymbol{\zeta^u_{ij}}$: angle between the positive x-axis and the direction in which UE j sees BS i
- $\boldsymbol{\phi^b_i}$: boresight angle of BS i relative to the x-axis
- $\boldsymbol{\phi^u_j}$: boresight angle of UE j relative to the x-axis
- $\boldsymbol{g^b_{ij}}$: directivity gain BS i adds to the link between UE j and BS i (transmission gain)
- $\boldsymbol{g^u_{ij}}$: directivity gain UE j adds to the link between BS i and UE j (reception gain)

Using the sectored antenna model presented in the paper by the authors, we have

$$g_{ij}^{b} = \begin{cases} \epsilon, & \text{if } \frac{\theta_i^b}{2} < |\phi_i^b - \zeta_{ij}^b| < 2\pi - \frac{\theta_i^b}{2} \\ \dfrac{2\pi - (2\pi - \theta_i^b)\epsilon}{\theta_i^b}, & \text{otherwise} \end{cases}$$

$$g_{ij}^{u} = \begin{cases} \epsilon, & \text{if } \frac{\theta_j^u}{2} < |\phi_j^u - \zeta_{ij}^u| < 2\pi - \frac{\theta_j^u}{2} \\ \dfrac{2\pi - (2\pi - \theta_j^u)\epsilon}{\theta_j^u}, & \text{otherwise} \end{cases}$$

Also, here in this case we make an assumption that because the association will act on a relatively larger time scale as compared to the instantaneous fluctuations, the SINR is averaged out. This use of a long term SINR model is very effective for long term resource allocation.

Coming back to the model, we can say that the power received by UE j from BS i is $pg_{ij}^b g_{ij}^c g_{ij}^u$.

So, the SINR at the UE j due to the transmission of BS i can be written as

$$\frac{pg_{ij}^b g_{ij}^c g_{ij}^u}{\sum_{k \in \mathcal{B} \setminus i} pg_{kj}^b g_{kj}^c g_{kj}^u + \sigma} \tag{4.1}$$

Each of the UE has its own performance or quality measure represented by some user performance function of the SINR. This function describes the satisfaction of the user. Here in this case, we are looking at the performance of the system as a whole. So, to achieve this goal, we consider the net achievable information rate (or mutual information) in the system as the performance criterion of the system as a whole. We denote the achievable rate of the link between BS i and UE j by $\boldsymbol{c_{ij}}$. Also let us denote the fraction of the resources used by the BS i to serve the UE j by $\boldsymbol{y_{ij}}$. Hence, we can say that the long term rate that UE j will receive from BS i should be $r_j = \sum_{k \in B} y_j c_{ij}$. Let U_j be a general utility function of r_j. Also let x_{ij} be a binary association variable, which is UE j is being served by the BS i and 0 otherwise. Let $\boldsymbol{\theta_i^b, min}$ and $\boldsymbol{\theta_j^u, min}$ denote the minimum possible bandwidth of BS i and UE j respectively.

Now we know many of the variables such as $\zeta_{ij}^b, \zeta_{ij}^u$ and g_{ij}^c for every BS i and UE j as we already know the network topology. we can collect all the control variables x_{ij} and y_{ij} in the matrices **X** and **Y**, and the others, namely θ_i^b, θ_j^u, ϕ_i^b and ϕ_j^u in the vectors $\boldsymbol{\theta^b}$, $\boldsymbol{\theta^u}$, $\boldsymbol{\phi^b}$ and $\boldsymbol{\phi^u}$ respectively. The optimal cell formation can be posed as below:

$$
\begin{aligned}
\underset{\mathbf{X},\mathbf{Y},\boldsymbol{\theta^b},\boldsymbol{\theta^u},\boldsymbol{\phi^b},\boldsymbol{\phi^u}}{maximize} & \sum_{j \in U} (\sum_{i \in B} y_{ij} c_{ij}) \\
\text{subject to} & \sum_{j \in U} y_{ij} \leq 1, \quad \forall i \in \mathcal{B}, \\
& \sum_{i \in B} x_{ij} = 1, \quad \forall j \in \mathcal{U}, \\
& 0 \leq y_{ij} \leq x_{ij}, \quad \forall i \in \mathcal{B}, \\
& x_{ij} \in \{0, 1\}, \quad \forall i \in \mathcal{B}, \\
& 0 \leq \phi_i^b \leq 2\pi, \quad \forall i \in \mathcal{B}, \\
& 0 \leq \phi_j^u \leq 2\pi, \quad \forall j \in \mathcal{U}, \\
& \theta_{i,min}^b \leq \theta_i^b \leq 2\pi, \quad \forall i \in \mathcal{B}, \\
& \theta_{j,min}^u \leq \theta_j^u \leq 2\pi, \quad \forall j \in \mathcal{U},
\end{aligned}
$$

4.1.1 Implementation Issues with the previous model and the Modifications

In order to solve the problem formulated in the previous section, we need to choose a system utility function U_j. Hence, to start with, we choose the identity function to be the system utility function. This case can be considered to be the simplest case of the **weighted arithmetic mean** system utility function. System utility functions such as the well known weighted arithmetic or geometric means usually give rise to non-convex monotonic optimization problems and their computational complexity scales exponentially with the number of UEs. These problems can be considered to be NP-hard. The NP-hard problems have a very major characteristic that there are no known algorithms to solve these problems in polynomial time, and also it is very widely believed that there exist no such algorithm to solve them in polynomial time. the weighted arithmetic mean utility function scenario that we are considering here, gives rise to an NP-hard optimization problem for any number of UEs.

We have stated that the number of variables rise exponentially with the increase in the number of UEs. Hence, we need to make some modifications in the above mentioned model in order to solve the optimization problem with the limited amount of resources that we have at our disposal. There are mainly two major modifications that we propose here:

1. Instead of carrying out the simulations for a network topology consisting of a large number of UEs, we consider a network topology with relatively less number of UEs. We take this suitable value to be equal to 10.
2. We observe that imposing the integer constraints on the variables x_{ij} just increases the computational complexity by a large factor, and this effect is directly reflected in the time taken for a single run of the simulation. Hence, we relax the integer constraints on the variables x_{ij} and then try to round off the values obtained for these variables to 0 or 1 in order to get near optimal association in the network.

4.1.2 Implementation of the Modified Model

We implement the above discussed optimal cell formation model along with the mentioned modifications in the MATLAB environment and then analyze the results hence obtained to draw some key observations.

Here, we use the Genetic Algorithm (GA) to solve the constrained relaxed optimization problem we have. A major reason to go for the Genetic Algorithm in order to solve our problem is that this algorithm gives the global optimal solutions and allows us to solve Mixed Integer Programming problems, which in case, can be used for less number of UEs as GA can't handle large number of variables while solving mixed integer optimization problems. We also tried to solve the problem by some very famous local optimization algorithms such as **fmincon** but we observed that the algorithm got stuck in the local optima, and better results were obtained by using the Genetic Algorithm.

Genetic Algorithm is an algorithm to solve optimization problems which may be constrained, unconstrained or may have mixed integer restrictions for some of the variables and is based on the process of natural selection, the process that derives biological evolution. The GA repeatedly modifies a population of individual solutions. At each successive iteration, GA randomly selects individuals from the current population who are to become the parents of the children for the next generation. In this way, over successive generations, the population evolves towards an optimal solution.

4.1.3 Simulations and Results

It is quite evident that a considerable number of new degrees of freedom are introduced to the system by the mmWave networks, that should in turn obviously enhance the performance of the network. In order to evaluate the performance gain due to the presence of these new degrees of freedom, we simulate a network with various topologies with 2 BSs and 10 UEs, which are distributed in an area of 1 square kilometer. The key parameters considered for our simulation are mentioned below:

- We consider a mmWave wireless channel with path-loss exponent $\alpha = 3$.
- The control channel bandwidth is $50kHz$, hence the noise power is $-127dB$.
- The SNR threshold of the typical UE is $0dB$.
- All the BSs adopt a transmission power of $30dBm$.

The results of the simulations are presented below. There are 8 experiments carried out in total. In the experiments 1 - 4, we solve the optimization problem for the fully directional communication mode by varying the number of RF chains per BS. Similarly, the experiments 5 - 7 are for carried to solve the problem for the semi directional communication mode by varying the number of RF chains per BS. And finally, the last experiment shows the result for the omni directional communication mode. The performance gain results presented in the table are averaged over 5 random topologies.

Table 4.1: PERFORMANCE OF RESOURCE ALLOCATION IN VARIOUS MODES OF COMMUNICATION WITH ONE RF CHAIN PER UE. ALL RATES ARE MEASURED IN bits/s/Hz

Experiment	Communication Mode	RF Chains per BS	Network Sum Rate	Minimum Rate	Jain's Fairness Index
1	Fully Directional	1	39.904	2.03	0.84
2		2	54.080	2.86	0.897
3		3	79.796	3.17	0.851
4		4	92.454	4.24	0.874
5	Semi Directional	1	23.991	1.11	0.743
6		2	50.385	1.52	0.775
7		3	66.681	1.96	0.859
8	Omni-Directional	1	5.964	0.13	0.724

4.2 Conclusions

From the results presented in Table 1, we can say that the fully directional mode outperforms the other modes, especially the traditional omni-directional mode by a large factor, as the directionality feature enhances the link budget and also reduces the interference. Not only the fully directional mode, but the semi directional mode also performs very well when compared to the traditional mode of communication. Particularly, we can see that as compared to the omni directional mode, there is a significant enhancement in the network sum rates in the fully-directional and semi-directional modes of communication. Also, we cannot forget about the fact that the increase in the number of RF chains introduces new degrees of freedom in the system, which further leads to the improvement in the sum rates and the minimum rates. Under ideal conditions, the network data throughput increases approximately linearly with the number of transmitting antennas. And finally, we can also observe that the fully directional mode of communication clearly outperforms the other modes in terms of fairness in resource allocation in the system which is verified by the Jain's Fairness Index.

CHAPTER 5

SUMMARY

In order to be able to solve the resource allocation problem, we convert the multi-objective resource allocation optimization problem to a constrained single-objective optimization problem by the means of a system utility function that is decided by the system designer in the chapter IV. We, as system designers, choose the sum information rate as the utility function and then solve the earlier presented single-objective optimization problem using Polyblock Outer Approximation (PA) algorithm, which basically approximates the performance region of the system, and exploiting the fact that the optimal point lies on the proper vertices of the polyblock, we determine the ϵ-optimal point in the system and present the information rate experienced by each of the users when the resources have been allocated optimally. We support our claims by demonstrating the progress of the algorithm which clearly shows the process of approximation of the performance region alongside the illustration of the Pareto boundary in each of the iterations.

In the later part of the work, we discuss about the concept of dynamic cell in a little depth. We take the optimal cell formation optimization problem proposed in Shokri-Ghadikolaei *et al.* (2015) and carry out the simulations. Since the complexity of the proposed problem increases exponentially with the increase in the number of variables in the system, we make some important modifications in the proposed problem during the implementation, such as relaxing the integer constraints on the user association variables. We carry multiple experiments under different communication modes, namely fully, semi and omni-directional communication modes with 2 BS and 10 UEs varying the number of RF chains per BS to determine the network sum rate achieved in the system, minimum rate of UE and Jain's Fairness Index to show the fairness in the system. The results of each experiment is determined by averaging out the results obtained in 10 sample experiments with varying network topologies. From the simulations conducted in various situation, we clearly infer that the resultant fully directional mode of communication outperforms the traditional omni-directional mode of communication in terms of achieved network sum rate, minimum rate and fairness in resource allocation in the system.

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