B.Tech Final Year Project Report
IITM-SAT's Control Algorithm, and the Effect of errors in measured parameters

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Abstract

In this project, the performance of the control algorithm employed in the IITM-SAT is evaluated through simulation. The algorithm attempts to align the satellite's z-axis along the Earth's magnetic field by varying the current through three mutually perpendicular coils, called torque-rods, based on the readings of the Gyroscope and magnetometers. Also, the extent to which errors in these measurements affect the control algorithm is measured based on the simulation results.

1 Theory:

1.1 Target Specifications and tolerences:

The satellite is to be brought to as low an angular velocity as possible. After this the angle our payload's axis makes with the Earths magnetic field must be reduced. The initial angular velocity is expected to be 5-8 rpm (about 30-50 degrees per second), and sampling time is currently 5 secs.

1.2 Representation of Co-ordinate system for measuring parameters:

Without loss of generality, assume that the z- axis of the satellite is to be aligned with the Earth's magnetic field. Let us consider a co-ordinate attached to the satellite for convenience, as shown in Fig. 1.1

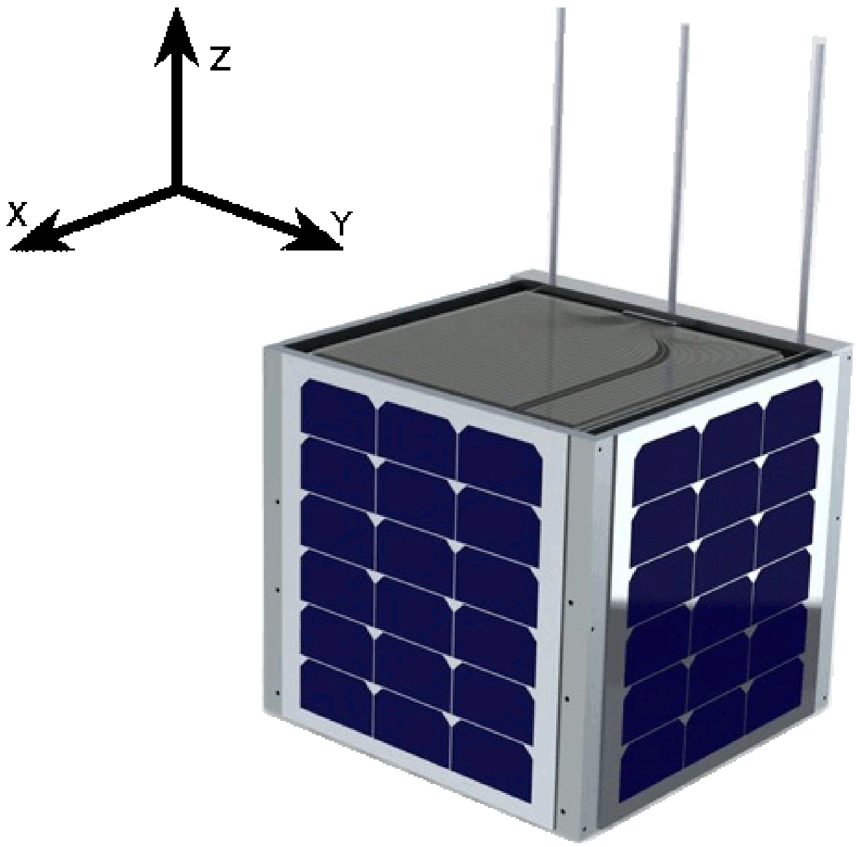


Figure 1.1: Representation of the coordinate system adopted. The axes rotate with the satellite

1.3 Basic Control Algorithm:

The IITM-SAT controls its own magnetic moment via torque rods. This allows it to control the net external torque on itself (as it is moving through Earth's magnetic field). It employs two control modes, namely:

1.3.1 De-tumbling Control Mode:

De-tumbling is a procedure to reduce the angular velocity of the satellite to zero, along all axes. This is done to facilitate the nominal control algorithm, which requires low angular speeds (at most a few degrees per second). The two algorithms adopted by IITM-SAT are:

1. **B-dot algorithm:**

   Uses the readings from only the magnetometer to determine the current to be supplied to the torque-rods. It has a very low convergence range forsampling time of 5 secs.

2. **B-Omega algorithm:**

   Uses the readings from both the gyroscope and magnetometer to determine the current to be supplied to the torque-rods. Converges for a much larger range of initial velocities, but requires more time

1.3.2 Nominal Control Mode:

Nominal Control basically orients the satellite until the measured magnetic field along all axes except the z-axis (which is to be aligned along the Earth's magnetic field) vanish. This results in the required satellite axis(here, without loss of generality, the z-axis) being aligned along the Earth's magnetic field. The current algorithm works under the assumption that the satellite is at a very low $\boldsymbol{\omega}(t)$, hence the need for detumbling. The algorithm uses a method called back-stepping[2].

1.4 Errors due to offset on algorithms:

Due to hysterisis, the torque-rod coils will produce some non zero value of magnetic field despite having zero current. This can affect the readings in the magentometer. Let us assume a random offset is introduced in the gyroscope and magnetometer, called $\boldsymbol{\omega}_{offset}$ and $\vec{B}_{offset}$ respectively. These offsets are expected are slowly changing and so, for most calculations, can be considered a constant. **Expected values of** $|\boldsymbol{\omega}_{offset}|$ and $|\vec{B}_{offset}|$ are in the orders of 1-2rpm and 100nT respectively.

1.4.1 De-tumbling Control Mode:

The main error induced here happens to be due to the fact that the switch between the De-tumbling algorithm and the Nominal control mode occur when the satellite's measured angular velocity $\boldsymbol{\omega}_{meas}(t)$ is close to zero, not when $\boldsymbol{\omega}(t)$ approaches zero. The switch therefore happens when $\boldsymbol{\omega}(t)$ approaches $\boldsymbol{\omega}_{offset}$

1. **B-dot:**

   Uses change in the measured value of magnetic field as an indicator of the $\boldsymbol{\omega}(t)$. This is due to the fact that in the satellites orbit, the rate of change of the magnitude of the Earth's magnetic field is negligible compared to the effect of the satellites own $\boldsymbol{\omega}(t)$. Since only use rate of change of magnetic field is considered, there is no effect due to $\vec{B}_{offset}$. There is no use for the gyroscope's readings, so $\boldsymbol{\omega}_{offset}$ also doesnt play a role in its convergence.

2. **B-Omega:**

   Uses the readings from both the gyroscope and magnetometer to determine the current to be supplied to the torque-rods. In this case, there is a severe offset due to both $\boldsymbol{\omega}_{offset}$ and $\vec{B}_{offset}$. However, the size of the convergence region(ie, the range of initial $\boldsymbol{\omega}(t)$ for which convergence happens) is largely unaffected

1.4.2 Nominal Control Mode:

Nominal Control basically orients the axis of the satellite to point along the Earth's magnetic field. The fact that there is a non-zero $\boldsymbol{\omega}_{offset}$ means that in the current algorithm, when control is sitched to nominal mode of operation, $\boldsymbol{\omega}(t)$ is non-zero. This causes the algorithm to fail. Also, the measured magnetic field is used as the Earths magnetic field, so $\vec{B}_{offset}$ will lead to a fixed pointing error.

2 Equations:

2.1 Control using torque-rods:

When a magnetic moment $\vec{m}$ travels through an external magnetic field $\vec{B}(t)$, it experiences a net torque given by

$$\boldsymbol{\tau}(t) = \vec{m}(t) \times \vec{B}(t) \tag{1}$$

This affects the angular velocity $\boldsymbol{\omega}(t)$ of the system according to the equation

$$\boldsymbol{\tau}(t) = \boldsymbol{J} * \frac{d}{dt}(\boldsymbol{\omega}(t)) \tag{2}$$

From Eqns. 1 and 2,

$$\vec{m}(t) \times \vec{B}(t) = \boldsymbol{J} * \frac{d}{dt}(\boldsymbol{\omega}(t)) \tag{3}$$

$$\boldsymbol{J}^{-1} * \vec{m}(t) \times \vec{B}(t) = \frac{d}{dt}(\boldsymbol{\omega}(t)) \tag{4}$$

$$\boldsymbol{\omega}(t) = \int (\boldsymbol{J}^{-1}\vec{m}(t) \times \vec{B}(t))dt \tag{5}$$

When a torque $\boldsymbol{\tau}(t)$ proportional to $\boldsymbol{\omega}(t)$ is applied, but in the opposite sense, such that it always counteracts $\boldsymbol{\omega}(t)$, the system keeps slowing down(equivalent to damping component in a mechanical system). Now, let us resolve $\vec{B}(t)$ along and perpendicular to $\boldsymbol{\omega}(t)$, whose direction is along axis of rotation. Consider an axis in the frame of Earth, centered on the satellite, with $\hat{z}$‘ along axis of rotation at this instant, and $\hat{x}$‘ making an angle θ with $\vec{B}_{\perp}(t)$(as in Fig. 2.1). Then

$$\vec{B}_{||}(t) = \frac{(\vec{B}(t).\boldsymbol{\omega}(t))}{|\boldsymbol{\omega}(t)|} * \hat{z}‘ \tag{6}$$

$$\vec{B}_{\perp}(t) = |\vec{B}_{\perp}(t)| * (cos\theta\hat{x}` + sin\theta\hat{y}`) \tag{7}$$

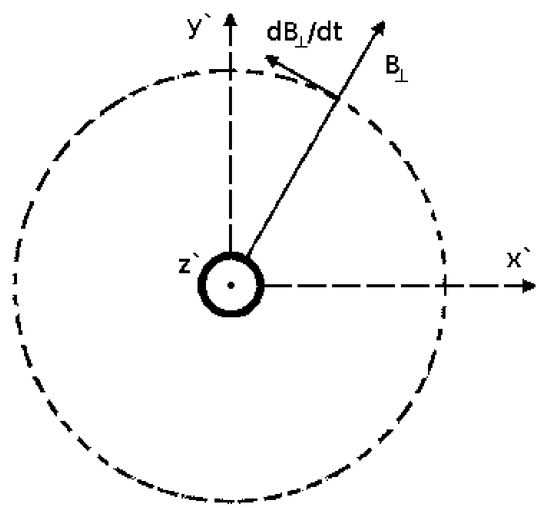


Figure 2.1: Representation of coordinate system, where $\hat{z}$`is along axis of rotation, i.e, $\omega(t)$

Now, if $\boldsymbol{\omega}(t)$ is along z` , and Earth's magnetic Field is roughly a constant in the orbit. Then,$\vec{B}_{||}(t)$ will be unaffected. Thus,

$$\frac{d}{dt}(\vec{B}_{\perp}(t)) = |\vec{B}_{\perp}(t)| * (-sin\theta\hat{x}` + cos\theta\hat{y}`) * \frac{d}{dt}(\theta) \tag{8}$$

$$\frac{d}{dt}(\vec{B}_{||}(t)) = 0 \tag{9}$$

$$\dot{B}(t) = \frac{d}{dt}(\vec{B}(t)) = |\vec{B}_{\perp}(t)| * (-sin\theta\hat{x}` + cos\theta\hat{y}`) * |\boldsymbol{\omega}(t)| \tag{10}$$

Note that

$$\boldsymbol{\omega}(t) \times \vec{B}(t) = |\vec{B}_{\perp}(t)| * (-sin\theta\hat{x}` + cos\theta\hat{y}`) * |\boldsymbol{\omega}(t)| \tag{11}$$

$$\boldsymbol{\omega}(t) \times \vec{B}(t) = \dot{B}(t) \tag{12}$$

2.2 Moment calculation for different algorithms:

2.2.1 De-tumbling Control Mode:

1. **B-dot[$\dot{B}(t) = \frac{d}{dt}(\vec{B}(t)$]:**
   From Eqn10, setting the magnetic moment of the satellite

$$\vec{m}(t) = k_{detumble}\dot{B}(t) \tag{13}$$

   will set $\vec{m}(t) \times \vec{B}(t)$, the net external torque applied on the satellite, to:

$$\boldsymbol{\tau}(t) = k_{detumble}\dot{B}(t) \times \vec{B}(t) \tag{14}$$

$$\boldsymbol{\tau}(t) = k_{detumble}((\boldsymbol{\omega}(t) \times \vec{B}(t)) \times \vec{B}(t) \tag{15}$$

$$= k_{detumble}[((\boldsymbol{\omega}(t).\vec{B}(t))\vec{B}(t) - (\boldsymbol{\omega}(t)(\vec{B}(t).\vec{B}(t))]$$

$$= k_{detumble}([|\vec{B}_{||}(t)||\boldsymbol{\omega}(t)|\vec{B}(t) - \boldsymbol{\omega}(t)|\vec{B}(t)|^2]$$

$$= \boldsymbol{\tau}_{||} + \boldsymbol{\tau}_{\perp}$$

   Where, $\boldsymbol{\tau}_{||} = -k_{detumble} * \boldsymbol{\omega}(t) * (|\vec{B}(t)|^2 - |\vec{B}_{||}(t)|^2$and $\boldsymbol{\tau}_{\perp} = k_{detumble} * |\vec{B}_{||}(t)| * |\boldsymbol{\omega}(t)| * \vec{B}_{\perp}(t)$. The first term counteracts $\boldsymbol{\omega}(t)$ (as it is along the axis of rotation, but negative), while the second changes the axis of rotation a little.

2. **B-Omega:**
   From Eqn10, clearly, $\boldsymbol{\omega}(t) \times \vec{B}(t)$ is basically a measure of $\dot{B}(t)$

$$\vec{m}(t) = k_{detumble}(\boldsymbol{\omega}(t) \times \vec{B}(t)) \tag{16}$$

   will then follow all equations as in B-dot.

2.2.2 Nominal Control Mode:

The switch to nominal mode occurs when $\boldsymbol{\omega}_{meas}(t)$ is below 0.8 degrees per second and return to detumbling if $\boldsymbol{\omega}_{meas}(t)$ exceeds 1 degree per second. This mode works to align the z-axis of the satellite with the magnetic field of the earth, i.e, reduce the B_x and B_y components to zero by varying the satellites orientation

$$v(t) = -k_{\mu}[B_x \; B_y]^T \tag{17}$$

$$u(t) = -k_z z(t) + \dot{v}(t) - \frac{1}{\gamma}[B_x \; B_y]^T) \tag{18}$$

$$\vec{m}(t) = -\frac{1}{|\vec{B}(t)|}\vec{B}(t) \times (\mathbf{N^{-1}}[0, u^T - d_{12}^T]^T) \tag{19}$$

3 Changes made to the exsisting algorithm to account for error:

For no error in either magnetometer or gyroscope, running either B-dot or B-omega followed by the nominal control algorithm will bring the satellite to point along the Earth's magnetic field. However, our satellite has errors, namely $\boldsymbol{\omega}_{offset}$ and $\vec{B}_{offset}$.

Thus simulations were run for the following two methods, and both provided satisfactory results:

3.1 Using B-dot to estimate $\boldsymbol{\omega}_{offset}$:

By reducing sampling time, the B-dot algorithm can be improved to converge for a larger range of initial velocities. For our required range of 5-8rpm, it is necessary to use a sampling time of 2-3 secs. This can be obtained as initially after satellite launch, the payload will not be run till the alignment is done, so the exta processing power can be used for the satellite's de-tumbling. After running for sufficient time, the satellites $\boldsymbol{\omega}(t)$ will be zero. At this point, the $\boldsymbol{\omega}_{meas}(t)$ is stored in a variable $\boldsymbol{\omega}_{estimate}$, and is a simple estimate of the $\boldsymbol{\omega}_{offset}$. From here on, $\boldsymbol{\omega}(t)$ will be given by $\boldsymbol{\omega}_{meas}(t) - \boldsymbol{\omega}_{estimate}$. It is only necessary to return to de-tumbling when either $\boldsymbol{\omega}(t)$ becomes too large or $\boldsymbol{\omega}_{estimate}$ changes too much in either $\boldsymbol{\omega}_{meas}(t) - \boldsymbol{\omega}_{estimate}$ will indicate the amount of change. So if $\boldsymbol{\omega}_{meas}(t) - \boldsymbol{\omega}_{estimate}$ exceeds a certain value, say 1rpm, then it is necessary to go back to de-tumbling. Due to the small values of$\boldsymbol{\omega}(t)$ involved, sampling time of 5 secs is sufficient for B-dot to converge in these cases.

3.2 Running B-Omega first then switching to B-dot:

If B-Omega algorithm is run first, it will converge to $\boldsymbol{\omega}_{offset}$ given enough time. Since $\boldsymbol{\omega}_{offset}$ is not too large, it will be within the convergence region of B-dot. This will allow us to proceed using the algorithm dicribed above, but at sampling time of 5 seconds throughout.

4 Results and Observations:

4.1 Performance in absence of error:

4.1.1 De-tumbling Control Mode:

1. **B-dot:**

   For T_{samp}=5 secs, converges for inital $\boldsymbol{\omega}(t)$ <15 degrees per second

2. **B-Omega:**

   Convergence takes from a little under 2 -3 hours to in excess of a day as initial $\boldsymbol{\omega}(t)$ is varied

4.1.2 Nominal Control Mode:

Converges normally, as shown in Fig.4.1. Notice how the pointing error fluctuates rapidly during the de-tumbling phase before converging about zero in the nominal control phase. Fig.4.2 and Fig 4.3 shows how the new algorithm performs detumbling. Note how B-omega is run before switching to b-dot.

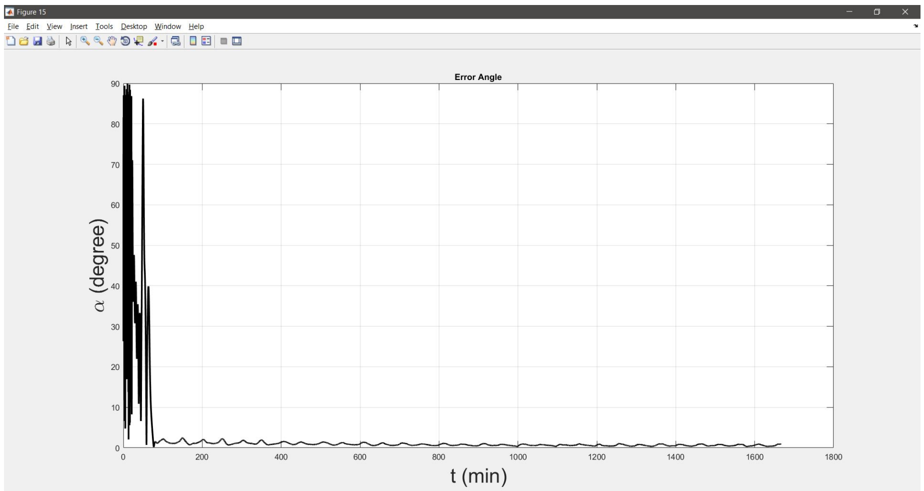


Figure 4.1: Plot of the angle made by the satellite's z-axis with the Earth's magnetic field vs time

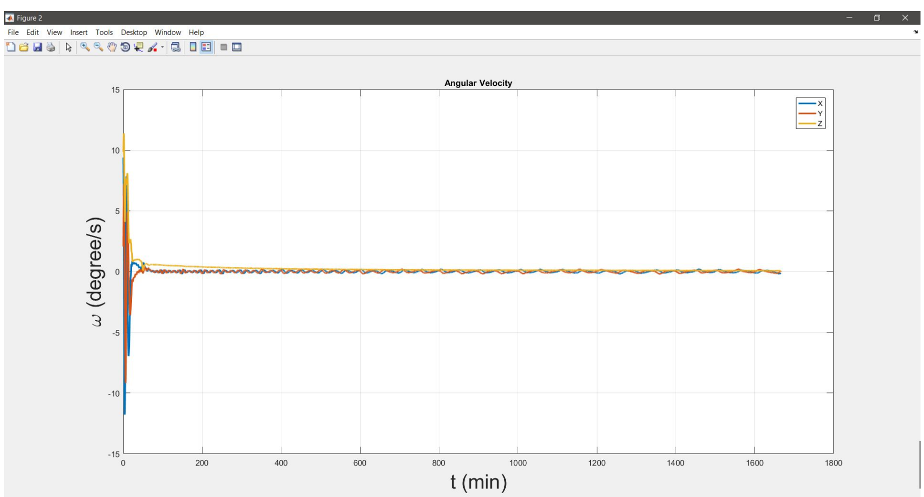


Figure 4.2: Performance of old algorithm's detumbling phase

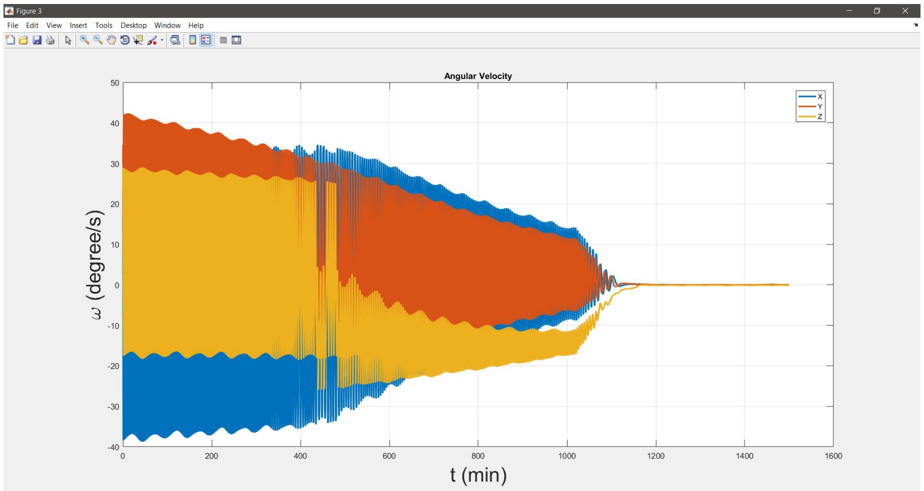


Figure 4.3: Performance of new algorithm's detumbling phase

4.2 Performance in the presence of error in gyroscope alone:

4.2.1 De-tumbling Control Mode:

1. **B-dot:**

   Conveges to $\boldsymbol{\omega}_{offset}$, and for or T_{samp}=5 secs, converges for inital $\boldsymbol{\omega}(t)$ <15 degrees per second

2. **B-Omega:**

   Conveges to $\boldsymbol{\omega}_{offset}$, and takes longer time to converge than in the case without error. If error is too large, there will not be convergence

4.2.2 Nominal Control Mode:

Control is never transferred to Nominal mode as everytime nominal mode acts on the system, omega changes drastically if the omega is non-zero, and the control is handed back to detumbling

4.3 Performance in the presence of error in magnetometer alone:

4.3.1 De-tumbling Control Mode:

1. **B-dot:**

   For T_{samp}=5 secs, converges for inital $\boldsymbol{\omega}(t)$ <15 degrees per second

2. **B-Omega:**

   Conveges but takes longer time to converge than in the case without error. Again, if the error is too large, there is no convergence.

4.3.2 Nominal Control Mode:

Error reflects in a final pointing error in the algorithm

4.4 Performance in the presence of error in magnetometer as well as gyroscope:

4.4.1 With $\boldsymbol{\omega}_{offset}$ estimation:

1. **B-dot method:**

   (a) T_{samp}=2secs: Converges for inital $\boldsymbol{\omega}(t)$ <60 degrees per second

   (b) T_{samp}=3secs: Converges for inital $\boldsymbol{\omega}(t)$ <40 degrees per second

   (c) T_{samp}=5secs: Converges for inital $\boldsymbol{\omega}(t)$ <15 degrees per second

2. **B-Omega into B-dot:**

   for T_{samp}=5secs, Takes a day to ensure all inital $\boldsymbol{\omega}(t)$ upto 60 degrees per second converge

4.4.2 Nominal Control Mode:

Still the pointing error exsists, the error induced due to b-offset has not been accounted for yet. However, omega error is accounted for (in Fig.**??**). Clearly, the offset is removed from all values of omega after t˜1440 mins, or 1 day), as we can see that the $\boldsymbol{\omega}_{meas}(t)$ starts to match with the actual $\boldsymbol{\omega}(t)$ (from Fig.**??**) of the satellite. Also, the convergence region, plotted here against $\boldsymbol{\omega}_{offset}$(on the x-axis) and$\vec{B}_{offset}$(on y axis), is much larger in the case where $\boldsymbol{\omega}_{offset}$ is corrected, i.e in the modified algorithm(See Fig. 4.7) as compared to original algorithm(Fig. 4.6)

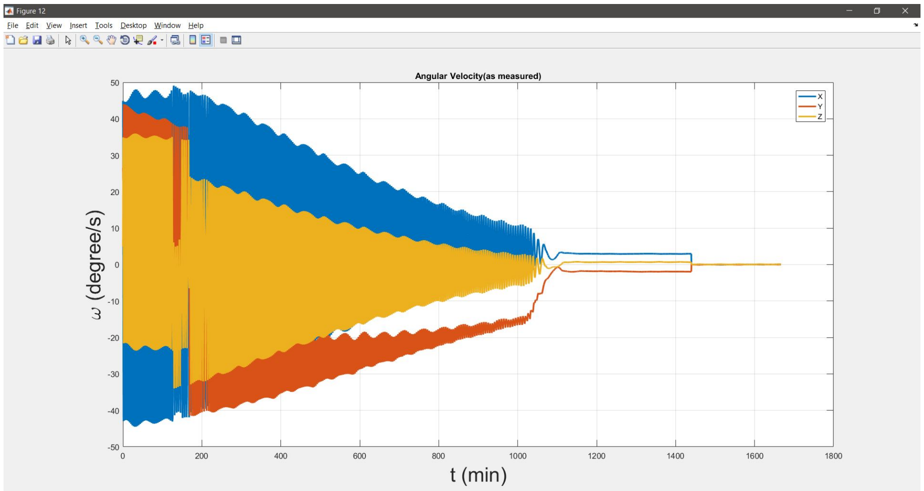


Figure 4.4: Plot of the $\omega_{meas}(t)$ vs time in order to track the performance of new algorithm's detumbling phase

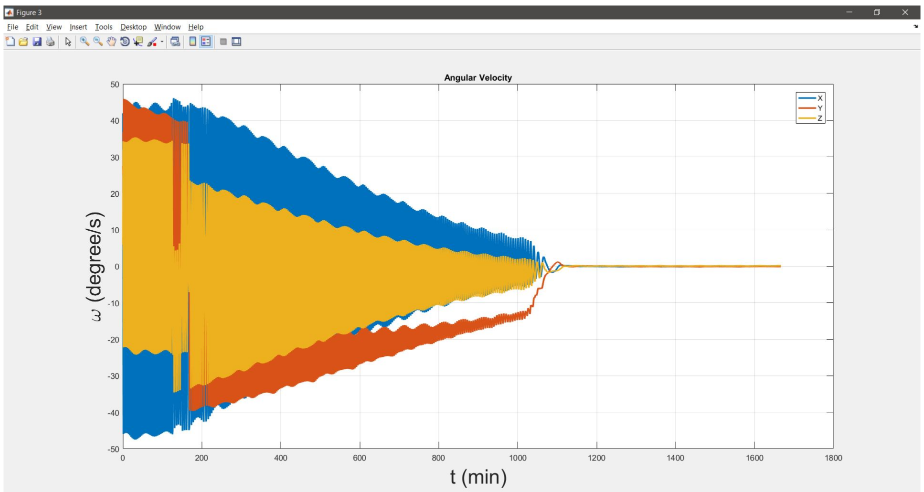
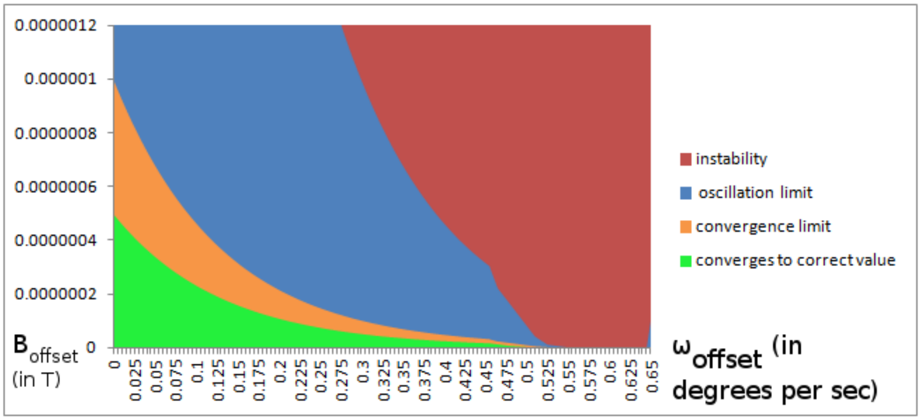
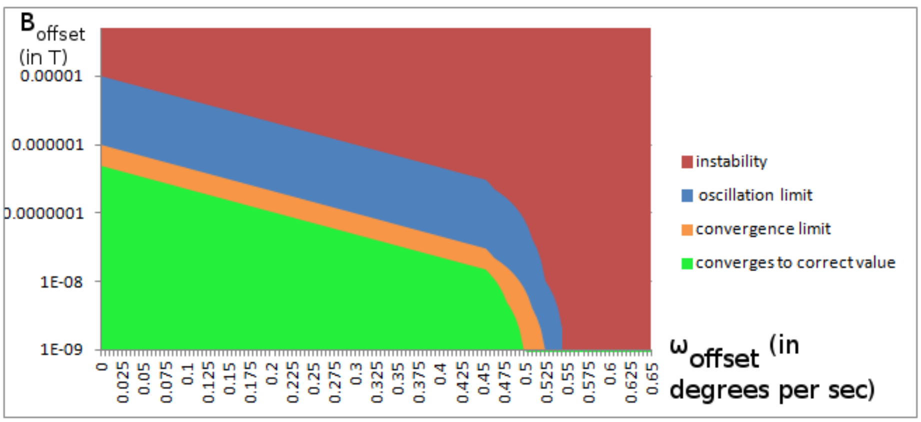


Figure 4.5: Plot of the $\omega(t)$ of the satellite vs time of the satellite for comparison



(a) Region plotted in linear scale



(b) Region plotted in log scale

Figure 4.6: Plot of the Convergence region (both convergence to the correct and incorrect value), oscillation region, and instability region

5 Frequently used terms:

- t - An index representative of the discrete time, where the seperation between two consecutive indices is the sampling time.
- T_{samp}- The sampling time used in the satellite. Tthe curren is updated in the torque-rods once every T_{samp}, based on the reading in the magnetometer (and gyro)[Currently 5 secs]
- $\vec{B}(t)$ - Magnetic Field of the Earth at the satellites location at time t.
- $\boldsymbol{\omega}(t)$ - Angular velocity of the satellite at time t

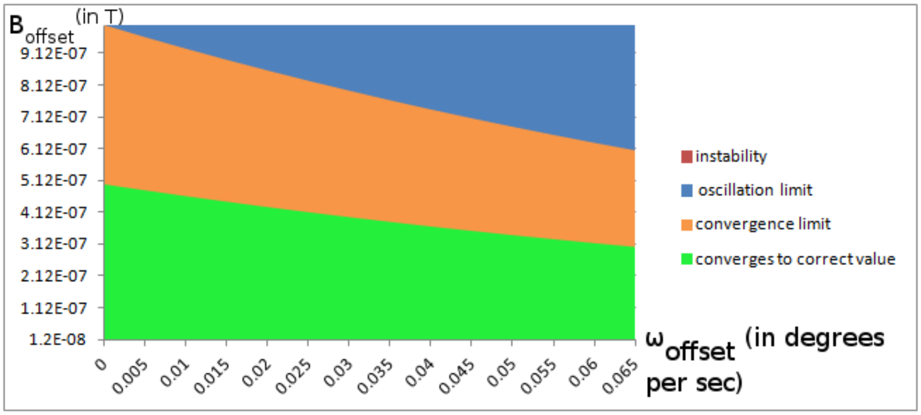
- $\vec{m}(t)$ - Magnetic Moment of the satellite at time t
- $I(t)$ - Current provided to the torque-rods at time t
- J - reprents the inertia matrix of the satellite, with the diagonal elements of this 3x3 matrix being the moments of inertia along x,y and z axes respectively
- $\tau(t)$ - Net external torque acting on the satellite as measured at time t
- The subscripts x,y,z when attached to a vector's name, refer to components of that vector along the respective axes. For example, $\vec{B}(t)$ has the components B_x, B_y and B_z
- $\vec{B}_{meas}(t)$ - Magnetic Field as measured by the magnetometer at time t
- $\dot{B}(t)$ - Rate of change of Magnetic Field as measured at time t, roughly calculated by $(\vec{B}(t) - \vec{B}(t-1))/T_{samp}$
- $\boldsymbol{\omega}_{meas}(t)$ - Angular velocity of the satellite as measured by the gyroscope at time t
- $\boldsymbol{\omega}_{offset}$ - Offest induced in the gyroscope.
- $\vec{B}_{offset}$ - Offest induced in the magnetometer.
- $\vec{A} \times \vec{B}$ - reprents the vector product, aka cross product of $\vec{A}$ and $\vec{B}$
- $v(t)$ - virtual control input at time t, expected value of b-dot in x and y axes
- $u(t)$ - inermediate control parameter, at time t
- $z(t)$ - error between actual value of b-dot in x and y axes at time t, and $v(t)$

References

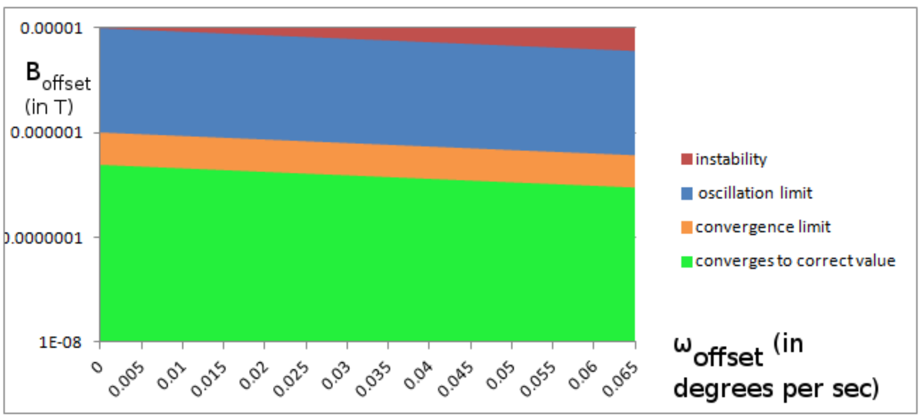
[1] Design of the Attitude Control Subsystem of IITMSAT, A Geomagnetic-field-pointing satellite - Deepti Kannapan, Akshay Gulati, Gourav Saha,Sruteesh Kuma

[2] Chapter 2, Control Design by Backstepping Technique for Satellite Line-of-Sight Control and Robotics -Deepti Kannapan

[3] Testbench software provided by Gourav Saha, which simulates and returns the satellites orbit parameters



(a) Region plotted in linear scale



(b) Region plotted in log scale

Figure 4.7: Plot of the Convergence region (both convergence to the correct and incorrect value), oscillation region, and instability region of the modified algorithm(using b-omega and b-dot to estimate ω_{offset}).