Near-Optimal Channel Estimation in Fast Fading Spatially Modulated Links

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THESIS CERTIFICATE

This is to certify that the thesis titled **Near-Optimal Channel Estimation in Fast Fading Spatially Modulated Links**, submitted by **Manideep Dunna**, to the Indian Institute of Technology, Madras, for the award of the degree of **Dual Degree(B.Tech + M.Tech)**, is a bona fide record of the research work done by him under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

KEYWORDS: **Spatial modulation, MIMO, Channel estimation, Maximum Likelihood, Kalman filter**

In this thesis, we propose a novel channel estimation method specially designed for fast fading spatially modulated (SM) MIMO links. For SM-MIMO, only one antenna is active at a time. In the pilot slot, we estimate the channel gain of the active antenna by least squares (LS) and for the inactive antenna, we exploit the spatial correlation between antennas by means of conditional maximum likelihood (ML) spatial estimate. We also leverage the temporal correlation to track the channel between two pilot slots by a third-order auto-regressive model. Finally, the Kalman filter is used to optimally combine the spatial and temporal estimates. We make use of a pilot arrangement that is specifically designed for the fast-fading scenario first discussed in Wu *et al.* (2014). Our approach to channel estimation on this pilot arrangement reveals more than two order of magnitude BER performance improvement at high SNR when compared to the approach used in Wu *et al.* (2014).

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ABBREVIATIONS

LTE	Long Term Evolution
MIMO	Multiple Input Multiple Output
SISO	Single Input Single Output
SM	Spatial Modulation
BER	Bit Error rate
LS	modified Least Squares
LMMSE	Linear Minimum Mean Squared Error
MSE	Mean Squared Error
ML	Maximum Likelihood
CE	Channel estimation
EM	Expectation-Maximization
AR	Autoregressive
AR(n)	n^{th} order Autoregressive model

NOTATION

N_t	Number of transmit antenna
N_r	Number of receive antenna
N_d	Data to Pilot ratio
f_c	Carrier frequency
f_D	Doppler shift
T_s	Symbol duration
$\mathbf{x}$	Bold face letters denote column vectors or matrices
$\mathbf{x}^H$	Hemitian of $\mathbf{x}$
$\mathbf{x}_n^{(k)}$	n^{th} element of k^{th} vector
$\|x\|$	absolute value of x
$diag(\mathbf{x})$	$N \times N$ diagonal matrix with diagonal elements given by $\mathbf{x}$
$\mathbf{I}_{N \times N}$	$N \times N$ Identity matrix
$\hat{\mathbf{q}}(\mathbf{n-1}\|\mathbf{n-1})$	Estimate of the state $\mathbf{q}(\mathbf{n-1})$ given the past n-1 observations
$\hat{\mathbf{e}}(\mathbf{n-1}\|\mathbf{n-1})$	Error in estimate of the state $\mathbf{q}(\mathbf{n-1})$ given the past n-1 observations
$\hat{\mathbf{q}}(\mathbf{n}\|\mathbf{n-1})$	Estimate of the state $\mathbf{q}(\mathbf{n})$ given the past n-1 observations
$\hat{\mathbf{e}}(\mathbf{n}\|\mathbf{n-1})$	Error in the estimate of the state $\mathbf{q}(\mathbf{n})$ given the past n-1 observations
$\mathbf{P}(\mathbf{n-1}\|\mathbf{n-1})$	State $\mathbf{q}(\mathbf{n-1})$ error covariance matrix given the past n-1 observations
$\mathbf{P}(\mathbf{n}\|\mathbf{n-1})$	State $\mathbf{q}(\mathbf{n})$ error covariance matrix given the past n-1 observations

CHAPTER 1

INTRODUCTION

Spatial modulation is an emerging modulation technique for MIMO systems, where additional data is encoded onto the antenna index that is being used for transmission. The primary motive behind the development of spatial modulation based MIMO systems is to reduce the hardware complexity and energy requirements at the base station. Systems with large number of antennae, raise the question of power consumption and spatial modulation combats this problem by employing limited number of RF chains to satisfy the energy requirements. In other words, this modulation scheme maximizes the Energy efficiency rather than the Spectral efficiency Di Renzo *et al.* (2014). Modulation in the spatial modulation can be thought of as selecting a subset from the available set of antenna for transmission, where the limited number fo RF chains available at the transmitter is greater than or equal to the size of the subset. MIMO systems typically have an RF chain per transmit antenna.

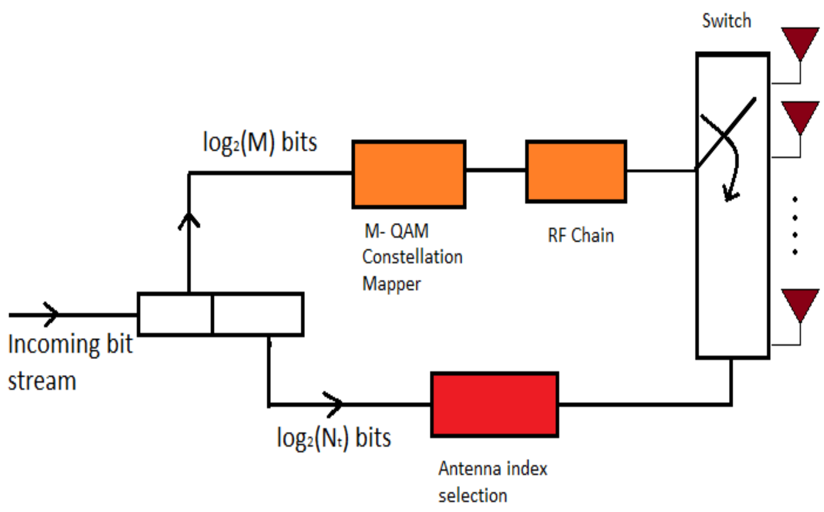

Figure 1.1: Spatial modulation

Consider a system with N_t transmit antennas, r RF chains using M-QAM constellation to map data bits to symbols. A group of incoming bits is split into two parts.

The bits in the first part are mapped to a M-QAM symbol and sent to the RF chain for transmission. Bits in the second part are used to select the antenna to be used for transmission. Figure 1.1 shows a typical SM-MIMO system with a single RF chain. Rate achieved by this technique equals to $\log_2 \binom{N_t}{r} + \log_2 M$ bits per channel use. In the presence of only one RF chain, rate turns out to be $\log_2 N_t + \log_2 M$.

The optimal decoding rule is given by Maximum likelihood principle. For a conventional N_t transmit antennae MIMO system, the number of possible transmitted symbol combinations varies exponentially with respect to the number of transmit antennae. So, the ML search complexity at the receiver equals to M^{N_t}. However, in case of N_t transmit antenna SM-MIMO setup, single antenna is active at any given time thus reducing the number of possible combinations to $M * N_t$. Hence, SM-MIMO is helpful in reducing the computational complexity at the receiver end. Sub-optimal and optimal approaches for data detection at the receiver had been presented in Jeganathan *et al.* (2008).

The above data decoding procedures assume perfect CSI at the receiver. Performance of SM-MIMO system is highly sensitive to the channel estimation error Sugiura and Hanzo (2012). Thus, accurate channel etimation plays an important role in the performance. In the classic MIMO configuration, channel estimation is done in a single shot by sending appropriate pilot symbols from all the antennas simultaneously. However, single stream MIMO limits us from sending simultaneous pilots because of the availability of only one RF chain. In the conventional method for SM, pilots are sent in successive time slots from each antenna one after the other during the channel estimation phase as in Figure 1.2. These channel estimates are used in the data phase for decoding.

Figure 1.2 depicts the pilot arrangement in a frame for a two transmit antennae SM-MIMO system. A frame is defined to include the training phase and estimation phase. In the conventional method, estimation is done at the beginning of the frame and these estimates are used to decode data for the rest of the frame duration. A single frame contains N_t consecutive pilots followed by $N_d * N_t$ data slots. Thus, channel estimation is done only once every $N_t(N_d + 1) * T_s$ time slots. However, these estimates become less reliable in fast fading scenarios. This issue is addressed in Wu *et al.* (2014) by spreading out the pilots throughout the frame, maintaining the same data to pilot ratio,

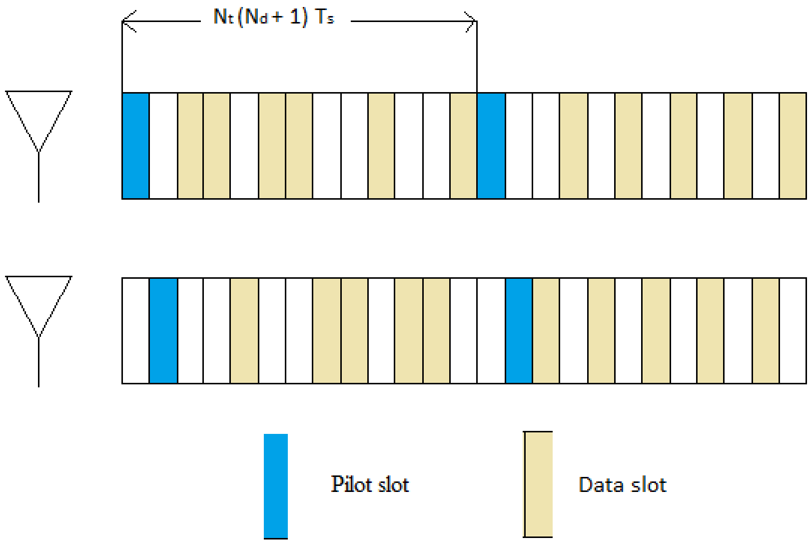

Figure 1.2: Conventional pilot arrangement for SM-MIMO

N_d. Each frame is divided into subframes each of which contains a pilot allowing us to estimate the channel more frequently i.e once every $(N_d + 1)$ time slots. My work is relevant to the pilot arrangement (which can accomodate higher Doppler) shown in Figure 1.3. However, this method is adhoc and does not explicitly take into account the channel statistics.

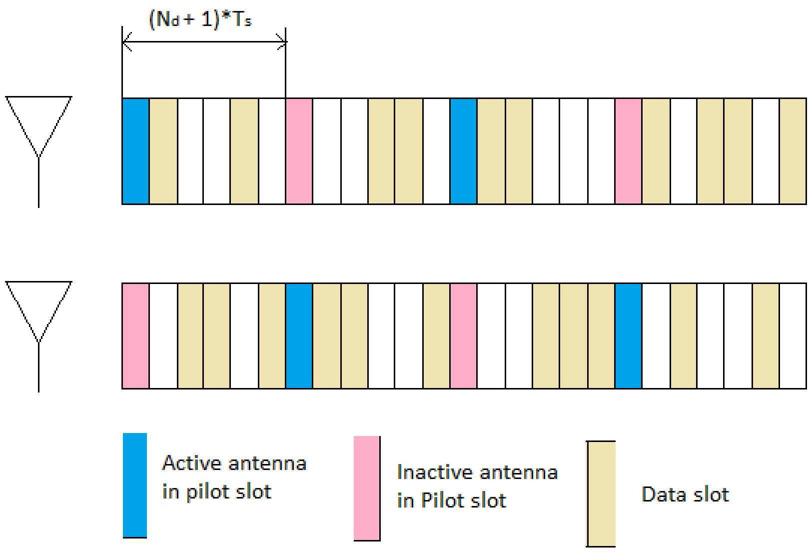

Figure 1.3: Pilots spread throughout the frame following the design in Wu *et al.* (2014)

In this thesis, we propose a novel method that explicitly incorporates the channel statistics such as spatial and temporal correlations into the estimation algorithm. The substantial improvement in error-rate performance over the approach in Wu *et al.* (2014)

is also presented.

The remainder of this thesis is organized as follows: Chapter 2 describes the system model. Chapter 3 describes the existing and proposed algorithms for channel estimation. Chapter 4 describes the simulation settings and the obtained results.

CHAPTER 2

SYSTEM MODEL

2.1 Introduction

There are many known well known models such as Jakes and Dent's models to simulate a SISO wireless channel. Here, we consider a single tap narrow band channel which has Bessel auto-correlation. A finite-order auto-regressive(AR) model can also generate channel gains following the Bessel auto-correlation. In a MIMO setting, there always exists some correlation between adjacent antenna. A particular structure for the antenna correlation matrix is considered and channel generation techniques to include Spatial and Temporal correlation are presented. Finally, the ML receiver for SM-MIMO system is also discussed.

2.2 Spatial Modulation

Consider an $N_r \times N_t$ antenna system using M-QAM constellation where only one antenna is active at a time. The incoming bit stream is divided into chunks of $log_2 M + log_2 N_t$ bits. The first segment of the bit chunk is used to determine the QAM symbol and the second segment also called as spatial symbol is used to select the active antenna. Thus a portion of the symbol is conveyed through the index of the active antenna(t_{act}). The transmitted signal is expressed by the vector **x** $= [x_1, x_2, x_3, ..., x_{N_t}]^T$, of which t_{act} antenna carries symbol S (where S is a symbol from M-QAM constellation) and the other elements are zero. Let $\mathcal{A}$ denote the set containing all possible vectors of **x**. Thus, we note that the cardinality of $\mathcal{A}$ is $N_t * M$.

2.3 Channel Model

For an $N_r \times N_t$ MIMO system, channel matrix is given by the $N_r \times N_t$ matrix

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & \dots & h_{1N_t} \\ h_{21} & h_{22} & h_{23} & \dots & h_{2N_t} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h_{N_r1} & h_{N_r2} & h_{N_r3} & \dots & h_{N_rN_t} \end{bmatrix}$$

where h_{mn} denotes the channel gain between the m^{th} receiver antenna and n^{th} transmitter antenna. Assuming only short-term fading effects, each of the elements of the matrix is a zero-mean complex Gaussian random variable and the magnitude follows the Rayleigh fading process.

2.3.1 Temporal correlation

Auto correlation of the Rayleigh fading process with unit average power is given by the Bessel function as follows:

$$E[h_{mn}(t)] = 0 \tag{2.1}$$

$$E[h_{mn}(t)h^*_{mn}(t + \Delta t)] = J_0(2\pi f_D \Delta t) \tag{2.2}$$

where J_0 is the Zeroth order Bessel function and f_D is Doppler shift

Fading process that follow the Bessel correlation can be simulated by the well-known Jakes model presented in Dent *et al.* (1993) as well as some finite-order models discussed in Baddour and Beaulieu (2005).

First-order AR model

A first-order model that generates samples with Bessel autocorrelation with doppler shift f_D is given by

$$g(n) = \sqrt{\alpha} g(n-1) + \sqrt{1-\alpha} p(n) \tag{2.3}$$

where $\alpha = J_0(2\pi f_D T_s)^2$ and $p(n) \sim$ i.i.d $\mathcal{CN}(0,1)$. We notice here that $p(n)$ and $g(n-1)$ are independent random processes. Hence, $E[p(n)g^*(n-1)] = 0$.

$$E[g(n)g^*(n)] = E[(\sqrt{\alpha}g(n-1) + \sqrt{1-\alpha}p(n))(\sqrt{\alpha}g^*(n-1) + \sqrt{1-\alpha}p^*(n))] \tag{2.4}$$

$$= E[\alpha g(n-1)g^*(n-1)] + \sqrt{1-\alpha}\sqrt{\alpha}E[g(n-1)p^*(n)] \tag{2.5}$$

$$+ \sqrt{1-\alpha}\sqrt{\alpha}E[g^*(n-1)p(n)] + (1-\alpha)E[p^*(n)p(n)]$$

$$= \alpha E[g(n)g^*(n)] + 0 + 0 + (1-\alpha) \tag{2.6}$$

From equation 2.6, it can be seen that $E[g(n)g^*(n)] = 1$. To verify that $g(n)$ follows rayleigh fading, compute the autocorrelation between two consecutive samples:

$$E[g(n)g^*(n-1)] = E[(\sqrt{\alpha}g(n-1) + \sqrt{1-\alpha}p(n))g^*(n-1)] \tag{2.7}$$

$$= \sqrt{\alpha}E[g(n-1)g^*(n-1)] + \sqrt{1-\alpha}E[p(n)g^*(n-1)] \tag{2.8}$$

$$= \sqrt{\alpha} \tag{2.9}$$

Hence, autocorrelation between two consecutive samples generated by first-order AR process will be $\sqrt{\alpha} = J_0(2\pi f_D T_s)$

2.3.2 Spatial correlation

Spatial correlation across the antenna is modeled according to equation [2.10] described in Oestges (2006).

$$\mathbf{H} = \mathbf{R_t}^{1/2}\mathbf{G}(\mathbf{R_r}^{1/2})^H \tag{2.10}$$

Here $\mathbf{G}$ is a matrix with all i.i.d entries. All the receive antenna are assumed to be independent. Thus the receive correlation matrix $\mathbf{R_r}$ is an identity matrix $\mathbf{I_{N_r \times N_r}}$. The exponential model in Loyka (2001) for the transmitter correlation matrix $\mathbf{R_t}$ is considered.

$$\mathbf{R_t} = \begin{bmatrix} 1 & \rho_{12} & \rho_{13} & \dots & \rho_{1N_t} \\ \rho_{21} & 1 & \rho_{23} & \dots & \rho_{2N_t} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_{N_t1} & \rho_{N_t2} & \rho_{N_t3} & \dots & 1 \end{bmatrix} \tag{2.11}$$

$$\begin{aligned} \text{where } \rho_{mn} &= \rho^{\frac{d_{mn}}{d}} \\ &= \rho^{|m-n|} \end{aligned} \tag{2.12}$$

Here $d_{mn} = (\mid m - n \mid d)$ is the separation between m^{th} and n^{th} transmit antenna, d is the reference distance in antenna array, and ρ is the correlation coefficient between antennas separated by a reference distance. The correlated channel gain matrices for different time instances are generated as follows:

(a) Generate the sequence of matrices G_n with entries i.i.d $\mathcal{CN}(0, 1)$ according to Jakes model.

(b) Introduce spatial correlation between the entries using the Kronecker model explained in (2.10)

$$\mathbf{H}_n = \mathbf{R_t}^{1/2}\mathbf{G}_n(\mathbf{R_r}^{1/2})^H \tag{2.13}$$

2.4 Optimal receiver

The vector of received samples denoted by $\mathbf{y} = [y_1, y_2, y_3, ..., y_{N_r}]^T$ is given by $\mathbf{y} = \mathbf{Hx} + \mathbf{w}$, where $\mathbf{w}$ is additive white noise vector. The channel gain matrix is $\mathbf{H} = [\mathbf{h_1}, \mathbf{h_2}, ..., \mathbf{h_{N_t}}]$. Considering all the possible symbols as equiprobable, the ML rule minimises the error probability.

$$\hat{\mathbf{x}} = \underset{\mathbf{x}\in\mathcal{A}}{\text{arg max}}\ P(\mathbf{y}|\mathbf{H}, \mathbf{x}) \tag{2.14}$$

$$= \underset{\mathbf{x}\in\mathcal{A}}{\text{arg min}}\ ||\mathbf{y} - \mathbf{Hx}||^{\mathbf{2}} \tag{2.15}$$

$$= \underset{\mathbf{x}\in\mathcal{A}}{\text{arg min}}\ \mathbf{x}\mathbf{H}^H\mathbf{Hx} - 2Re\left\{\mathbf{y}^H\mathbf{Hx}\right\} \tag{2.16}$$

$$[S, t_{act}] = \underset{t\in\{1,2,..N_t\},\ q\in M-QAM}{\text{arg min}}\ ||\mathbf{h_t}q||^2 - 2Re\left\{\mathbf{y}^H q\mathbf{h_t}\right\} \tag{2.17}$$

Maximum likelihood receiver is implemented by exhausting the whole search space i.e. compute $||\mathbf{y} - \mathbf{H}\mathbf{x}||^{\mathbf{2}}$ $\forall$ MN_t possible $\mathbf{x}$ vectors and choose the $\mathbf{x}$ for which it is minimized. The computational complexity of the receiver is $O(MN_t)$.

CHAPTER 3

CHANNEL ESTIMATION

3.1 Introduction

The channel state information (CSI) is required at the receiver for proper decoding of the symbols. In this chapter, some of the existing channel estimation method techniques along with the proposed technique for SM-MIMO systems are presented. The proposed technique makes use of Kalman filter and conditional ML estimation of a vector of varibles. Temporal channel variations are tracked with an AR(3) model and is described briefly here.

3.2 Conventional method

In conventional method, the pilots are placed in consecutive slots as shown in Figure 1.2. Let us say for a $N_r \times N_t$ system, the channel gain matrix be $\mathbf{H} = [\mathbf{h_1}, \mathbf{h_2}, ..., \mathbf{h_{N_t}}]$. The received vectors in the consecutive pilot slots are $\mathbf{y_1}, \mathbf{y_2}, ..., \mathbf{y_{N_t}}$. In every pilot slot, $e^{\frac{i\pi}{4}}$ is sent as the known symbol. Least Squares solution for the channel gains is given by

$$\hat{\mathbf{h}}_\mathbf{t} = \mathbf{y_t} e^{-\frac{i\pi}{4}} \quad \forall \ t \in \{1, 2, ..., N_t\} \tag{3.1}$$

These estimates are used to decode the symbols during the data transmission phase. But channel estimation being less frequent here makes this unsuitable for fast fading scenario. Hereafter, we refer to the conventional channel estimation method as CCE.

3.3 Interpolation method

Here, Pilot arrangement is modified as shown in Figure 1.3 to deal with the effects of fast fading. Each frame is divided into subframes, each of which contains a pilot slot.

Thus, pilots are spread throughout the frame allowing us to perform channel estimation much more frequenctly when compared to the conventional pilot arrangement. Figure 1.3 shows a series of subframes for a 2-transmit antenna MIMO system. The active antenna in the pilot slot changes every consecutive subframe. In a pilot slot, channel gain for the active antenna is given by

$$\mathbf{h}_{\mathbf{t_{act}}}(n) = \mathbf{y}(n)e^{-\frac{i\pi}{4}} \tag{3.2}$$

Channel gains for the inactive antennas ($t \neq t_{act}$) in each subframe are interpolated as presented in Wu *et al.* (2014). Hereafter, interpolation method is referred to as IM.

3.4 Proposed method

There are different kinds of time slots one might encounter in an SM-MIMO system. Either a time-slot can be a pilot slot or a data slot. An antenna can be active or inactive at a time which gives rise to 4 possible combinations as shown in 3.1.

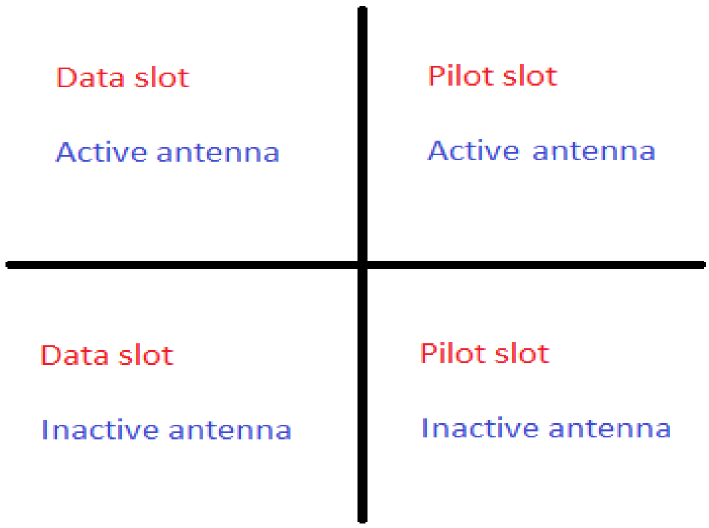

Figure 3.1: Classification of a time slot

3.4.1 Tracking channel variations with an AR(3) model

Channel variations across time follow Jakes' model and are modeled as a regression equation given in Baddour and Beaulieu (2005). We use an AR(3) model to fit Jakes fading. We are not going for higher order as it becomes computationally complex to implement them and the third order model provides a much better approximation than the first-order filter in Wu *et al.* (2014).

$$h_{r,t}(n) = a(1)h_{r,t}(n-1) + a(2)h_{r,t}(n-2) + a(3)h_{r,t}(n-3) + w(n) \tag{3.3}$$

where $h_{r,t}$ denotes (r,t) element from the channel gain matrix and $w(n) \sim$ i.i.d $\mathcal{CN}(0, \sigma_w^2)$. We notice here that $w(n)$ and $h_{r,t}(n-n_0)\ \forall\ n_0 > 0$ are independent random processes. Define $R_h(n) = J_0(2\pi f_D n T_s)$.

$$E[w(n)h^*_{r,t}(n-n_0)] = 0 \tag{3.4}$$

$$E[h^*_{r,t}(n-n_1)[h^(_{r,t}n-n_2)] = R_h(|n_1-n_2|) \tag{3.5}$$

The expression for σ_w^2, which is the variance of $w(n)$, is discussed in section 3.4.4. To obtain the coefficients $a(1), a(2), a(3)$ set up the Yule-walker equations as described in Orfanidis (2007).

$$\mathbf{N} = \begin{bmatrix} 1 & R_h(1) & R_h(2) \\ R_h(1) & 1 & R_h(1) \\ R_h(2) & R_h(1) & 1 \end{bmatrix} \tag{3.6}$$

$$\begin{bmatrix} a(1) & a(2) & a(3) \end{bmatrix} = \begin{bmatrix} R_h(1) & R_h(2) & R_h(3) \end{bmatrix} * \mathbf{N}^{-1} \tag{3.7}$$

Define

$$\mathbf{M} = \begin{bmatrix} R_h(1) & R_h(2) & R_h(3) \\ 1 & R_h(1) & R_h(2) \\ R_h(1) & 1 & R_h(1) \end{bmatrix} \tag{3.8}$$

3.4.2 Pilot slot

In a pilot slot, always a known symbol($e^{\frac{i\pi}{4}}$) is sent through the active antenna. The least squares(LS) estimate for channel gain is

$$\mathbf{h}_{\mathbf{t_{act}}}(n) = \mathbf{y}(n)e^{-\frac{i\pi}{4}} \tag{3.9}$$

where $\mathbf{y}(n)$ is the received vector.

Since there is only one active antenna in a pilot slot, it is required to make of the channel gains of the inactive transmit antennas as well. We estimate these gains using the maximum likelihood (ML) principle across the spatial dimension. Given the pilot based channel measurements for an active transmit antenna, channel estimation procedure for a 2×2 MIMO system is described below.

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{h_1} & \mathbf{h_2} \end{bmatrix}$$

Consider that h_{11} and h_{21} are obtained from pilots. Convert channel matrix into a column vector as

$$\text{vec}(\mathbf{H}) = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{12} \\ h_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{h_1} \\ \mathbf{h_2} \end{bmatrix} \tag{3.10}$$

and then the covariance matrix $\Sigma = \mathbf{E}[vec(\mathbf{H})vec(\mathbf{H})^H]$ is given by

$$\Sigma = \begin{bmatrix} \mathbf{\Sigma_{11}} & \mathbf{\Sigma_{12}} \\ \mathbf{\Sigma_{21}} & \mathbf{\Sigma_{22}} \end{bmatrix}$$

As the mean of the channel is "0", ML estimate for the unknown entries is discussed in

Kay and is given by

$$\hat{\mathbf{h}}_\mathbf{2} = argmax\ P(\mathbf{h_2} \mid \mathbf{h_1} = \hat{\mathbf{h}}_\mathbf{1}) \tag{3.11}$$

$$= \mathbf{\Sigma_{21}} \mathbf{\Sigma_{11}}^{-1} \hat{\mathbf{h}}_\mathbf{1} \tag{3.12}$$

Theorem 1. *The conditional ML estimate for* $\mathbf{h_2}$ *given by the equation (3.12) is equal to* $\rho\hat{\mathbf{h}}_\mathbf{1}$*, where* ρ *is the correlation coefficient.*

Proof. For a 2×2 MIMO system,

$$\mathbf{H} = \mathbf{R_r}^{1/2} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \mathbf{R_t}^{1/2} \tag{3.13}$$

$$\mathbf{R_t} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \tag{3.14}$$

$$\mathbf{R_r} = \mathbf{I_{2\times 2}} \tag{3.15}$$

where $g_{11}, g_{12}, g_{21}, g_{22}$ are i.i.d $\sim \mathcal{CN}(0,1)$ Using the properties of Kronecker product, we can write

$$\text{vec}(\mathbf{H}) = (\mathbf{R_t}^{1/2^T} \bigotimes \mathbf{R_r}^{1/2^T}) \begin{bmatrix} g_{11} \\ g_{12} \\ g_{21} \\ g_{22} \end{bmatrix} \tag{3.16}$$

$$\mathbf{\Sigma} = (\mathbf{R_t}^{1/2} \bigotimes \mathbf{R_r}^{1/2})^T E\left[\begin{bmatrix} g_{11} \\ g_{12} \\ g_{21} \\ g_{22} \end{bmatrix} \begin{bmatrix} g_{11}^* & g_{12}^* & g_{21}^* & g_{22}^* \end{bmatrix} \right] (\mathbf{R_t}^{1/2^T} \bigotimes \mathbf{R_r}^{1/2}) \tag{3.17}$$

$$= (\mathbf{R_t}^{1/2^T} \bigotimes \mathbf{R_r}^{1/2^T})\ \mathbf{I_{4\times 4}}\ (\mathbf{R_t}^{1/2^T} \bigotimes \mathbf{R_r}^{1/2^T}) \tag{3.18}$$

$$= \mathbf{R_t}^T \bigotimes \mathbf{R_r}^T \tag{3.19}$$

$$= \begin{bmatrix} \mathbf{I_{2\times 2}} & \rho\mathbf{I_{2\times 2}} \\ \rho\mathbf{I_{2\times 2}} & \mathbf{I_{2\times 2}} \end{bmatrix} \tag{3.20}$$

From equation (3.12),

$$\hat{\mathbf{h}}_{\mathbf{2}} = \rho \mathbf{I}_{\mathbf{2}\times\mathbf{2}}(\mathbf{I}_{\mathbf{2}\times\mathbf{2}})^{-1}\hat{\mathbf{h}}_{\mathbf{1}} \quad (3.21)$$

$$= \rho\hat{\mathbf{h}}_{\mathbf{1}} \quad (3.22)$$

■

In the general case, it can be simplified to

$$\hat{\mathbf{h}}_{\mathbf{t}} = \rho^{|t-t_{act}|}\mathbf{h}_{\mathbf{t_{act}}} \quad (3.23)$$

Covariance matrix estimation

In the beginning of the transmission, send N pilots on each transmit antenna one after the other. A time average of the estimated channel gain matrices is used to get an ergodic estimate for the Covariance matrix.

$$\mathbf{\Sigma} = E[vec(\mathbf{H})vec(\mathbf{H})^H] \quad (3.24)$$

$$\approx \frac{1}{N}\sum_{k=1}^{N} vec(\mathbf{H}_k)vec(\mathbf{H}_k)^H \quad (3.25)$$

3.4.3 Data slot

During the data phase, we perform Yule-walker propagation as described below for each transmit antenna regardless of whether is active or inactive during its time slot.

$$\hat{h}_{r,t}(n) = a(1)\hat{h}_{r,t}(n-1) + a(2)\hat{h}_{r,t}(n-2) + a(3)\hat{h}_{r,t}(n-3) \quad (3.26)$$

$$\forall\, t \in \{1, 2, ..., N_t\},\ r \in \{1, 2, ..., N_r\}$$

where $a(1), a(2), a(3)$ are given in section 3.4.4. The channel gain $\hat{h}_{r,t}$ is used to decode data according to the equation (2.16).

3.4.4 Kalman filter updation at pilot location

State equation:

$$\mathbf{q}(n) = \mathbf{A}\mathbf{q}(n-1) + \mathbf{w}(n) \tag{3.27}$$

$$\mathbf{A} = \begin{bmatrix} a(1) & a(2) & a(3) \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{3.28}$$

$$\mathbf{q}(n) = \begin{bmatrix} h_{r,t}(n) \\ h_{r,t}(n-1) \\ h_{r,t}(n-2) \end{bmatrix} \tag{3.29}$$

$$\mathbf{w}(n) = \begin{bmatrix} w(n) \\ 0 \\ 0 \end{bmatrix} \tag{3.30}$$

$\mathbf{q}(n)$ is the state vector. $w(n)$ is white noise with variance σ_w^2. Denote the covariance matrix of $\mathbf{w}(n)$ as $\mathbf{Q_w}$.

$$\mathbf{Q_w} = E[\mathbf{w}(n)\mathbf{w}^H(n)] \tag{3.31}$$

$$= E[(\mathbf{q(n)} - \mathbf{Aq(n-1)})(\mathbf{q(n)} - \mathbf{Aq(n-1)}^H)] \tag{3.32}$$

$$= \mathbf{N} + \mathbf{A}\mathbf{N}\mathbf{A}^H - \mathbf{A}\mathbf{M}^H - \mathbf{M}\mathbf{A}^H \tag{3.33}$$

where the expresssions for matrices **M, A, N** are given in section 3.4.1.

Observation equation:

In case of active antenna, received vector is going to be the observation. On the other hand, for an inactive antenna ML estimate described in section 3.4.2 serves as the observation.

$$\rho^{|t-t_{act}|}\hat{h}_{r,t_{act}}(n) = h_{r,t}(n) + p(n) \tag{3.34}$$

where $p(n)$ is observation noise. Equation (3.34) can be re-written as the observation equation:

$$\rho^{|t-t_{act}|}\hat{h}_{r,t_{act}}(n) = Cq(n) + p(n) \tag{3.35}$$

$$C = \begin{bmatrix}1 & 0 & 0\end{bmatrix} \tag{3.36}$$

Theorem 2. *Variance of observation noise, $Q_p = \sigma_p^2 = 1 - (1-\sigma_v^2)\rho^{2|t-t_{act}|}$ where σ_v^2 is the noise power at the receiver antenna.*

Proof. For $t = t_{act}$, observation noise is nothing but the receiver noise which has variance $E[|\hat{h}_{r,t_{act}}(n) - h_{r,t_{act}}(n)|^2] = \sigma_v^2$.

For $t \neq t_{act}$,

$$\sigma_p^2 = E[|\rho^{|t-t_{act}|}\hat{h}_{r,t_{act}}(n) - h_{r,t}(n)|^2] \tag{3.37}$$

$$= E[|\rho^{2|t-t_{act}|}|\hat{h}_{r,t_{act}}(n)|^2] - E[\rho^{|t-t_{act}|}\hat{h}_{r,t_{act}}(n)h^*_{r,t}(n)]$$
$$- E[\rho^{|t-t_{act}|}\hat{h}^*_{r,t_{act}}(n)h_{r,t}(n)] + E[|h_{r,t}(n)|^2] \tag{3.38}$$

$$= \rho^{2|t-t_{act}|}\sigma_v^2 - \rho^{2|t-t_{act}|} - \rho^{2|t-t_{act}|} + 1 \tag{3.39}$$

$$= 1 - (1-\sigma_v^2)\rho^{2|t-t_{act}|} \tag{3.40}$$

■

Optimal combining:

$$\hat{\mathbf{q}}(n-1|n-1) = \mathbf{A}^{N_d}\hat{\mathbf{q}}(n-N_d-1) \tag{3.41}$$

$$\hat{\mathbf{q}}(n|n-1) = \mathbf{A}\hat{\mathbf{q}}(n-1|n-1) \tag{3.42}$$

Define $\mathbf{k}$ as the Kalman filter gain vector given in Hayes (1996). Updated channel estimate will be:

$$\hat{\mathbf{q}}(n|n) = \hat{\mathbf{q}}(n|n-1) + \mathbf{k}(\rho^{|t-t_{act}|}\hat{h}_{r,t_{act}}(n) - \hat{\mathbf{q}}(n|n-1)) \tag{3.43}$$

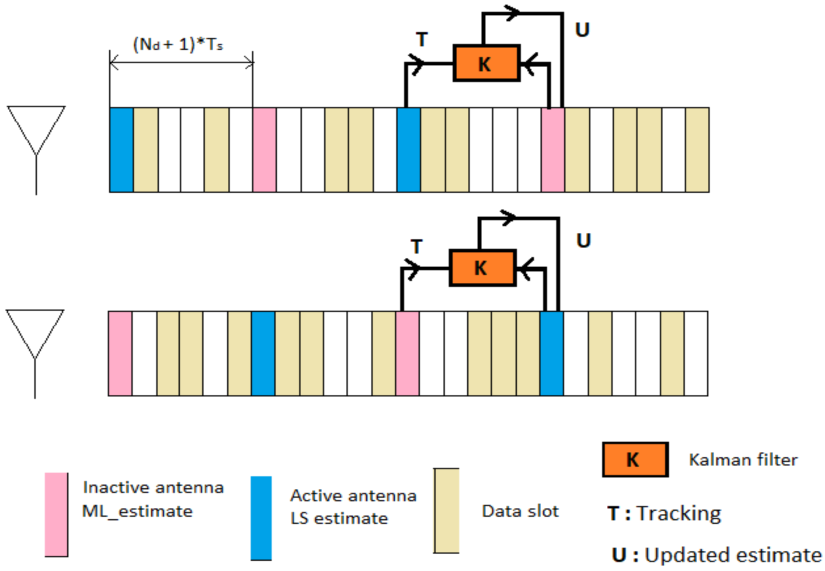

Figure 3.2: Kalman filter combining of spatio-temporal estimates

$$\mathbf{P}(n-1|n-1) = E[\mathbf{e}(n-1|n-1)\mathbf{e}^H(n-1|n-1)] \tag{3.44}$$

$$= E[(\mathbf{q}(n-1) - \hat{\mathbf{q}}(n-1|n-1))(\mathbf{q}(n-1) - \hat{\mathbf{q}}(n-1|n-1))^H] \tag{3.45}$$

$$\mathbf{P}(n|n-1) = \mathbf{A}\mathbf{P}(n-1|n-1)\mathbf{A}^H + \mathbf{Q}_\mathbf{w} \tag{3.46}$$

Kalman gain vector $\mathbf{k}$ is given by

$$\mathbf{k} = \mathbf{P}(n|n-1)\mathbf{C}^H[\mathbf{C}\mathbf{P}(n|n-1)\mathbf{C}^H + Q_p]^{-1} \tag{3.47}$$

Figure 3.2 illustrates the Kalman filtering at a pilot location.

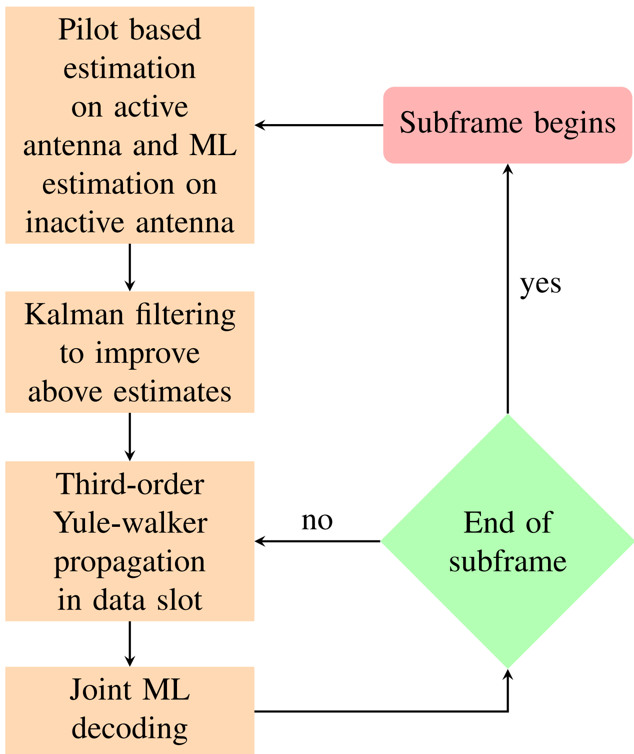

Figure 3.3: Proposed channel estimation method

CHAPTER 4

RESULTS

4.1 Simulation

In this section, the performance of the proposed channel estimation(PCE) method explained in section 3.4, is verified through simulations. We compare the BER performance of PCE with CCE and IM discussed in section 3. In the simulations, we set $N_t = 4,\ N_r = 2,\ N_d = 10,\ \rho = 0.5\ \&\ 0.8,$ velocity $= 5ms^{-1},\ T_s = 0.07ms,\ f_c = 2GHz$. Rayleigh fading channel is generated according to the Jakes Model.

BER performance: In figure 4.1 we have plotted the SNR vs BER performance for PCE as well as for CCE and IM methods for $\rho = 0.5$. We observe that up to 16 dB all the methods have similar BER performance and beyond 16 dB PCE outperforms CCE and IM and at 40 dB the BER of PCE floors. PCE achieved more than 2 orders improvement in BER over CCE and IM at high SNR. At high SNR, channel estimation error will be more prominent than the noise power and that is why BER of PCE floors. Here, $f_D * T_s = 0.002$ which is considered as high Doppler for the SM-MIMO model. BER performance of IM is worse compared to CCE though the pilot arrangements for IM is specifically designed to outperform CCE in high Doppler environment. This happens because of low spatial correlation. However PCE outperforms CCE even with such low spatial correlation because unlike in IM, PCE optimally captures the spatio-temporal effect of SM-MIMO channel.

In Figure 4.2 we repeated the simulation for $\rho = 0.8$. Here the BER performance of IM is better than CCE which is because of good spatial correlation. As before, PCE performs similar to CCE and IM till 15 dB. Beyond 15dB PCE outperforms CCE and IM and even achieves more than 2 orders BER improvement at high SNR.

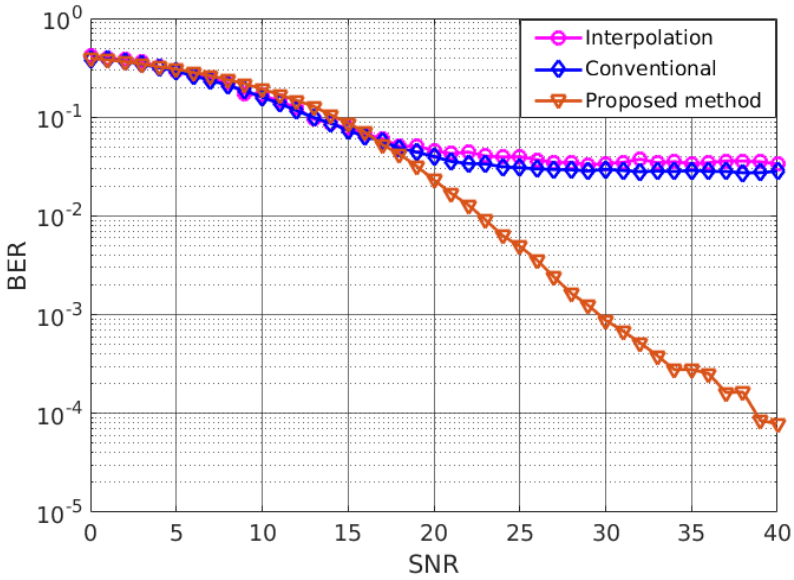

Figure 4.1: BER of SM with QPSK for $\rho = 0.5$

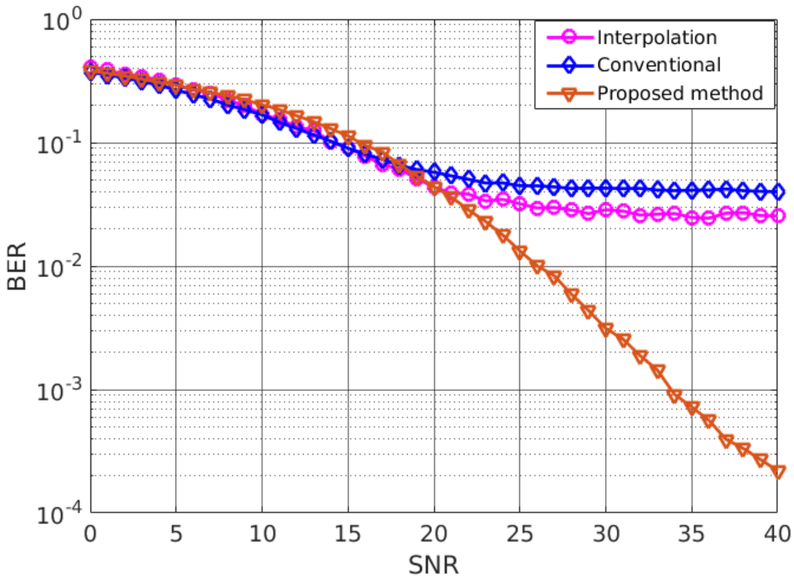

Figure 4.2: BER of SM with QPSK for $\rho = 0.8$

4.2 Conclusion

Coventional Channel Estimation for SM-MIMO is done by sequentially activating all the antenna to send pilot followed by the data portion. This process is very much affected by high Doppler shifts. In Wu *et al.* (2014) the authors proposed a pilot design that is very effective in fast fading scenarios. However, the estimation method described in Wu *et al.* (2014) is quite adhoc. In this thesis, we adopted the pilot arrangement from Wu *et al.* (2014), but significantly improved the channel estimation by properly modeling the spatio-temporal correlation of SM-MIMO system. In the simulation results, we have seen a great improvement of the proposed method over other methods in terms of BER performance. Even with low spatial correlation the proposed method outperforms CCE.

4.3 Future Work

In this thesis, channel estimation for SM-MIMO systems with a single active antenna is explored. This can be further extended to the situation where a subset of antennas (> 1) are active.

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LIST OF PAPERS BASED ON THESIS

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1. Near-Optimal Channel Estimation in Fast Fading Spatially Modulated Links
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