On the Sum-Rate Capacity of the Gaussian X Channel

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THESIS CERTIFICATE

This is to certify that the thesis titled **On the Sum-Rate Capacity of the Gaussian X Channel**, submitted by **Praneeth Kumar V**, to the Indian Institute of Technology, Madras, for the award of the degree of **Master of Technology**, is a bona fide record of the research work done by him under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

KEYWORDS: Sum capacity, Interference channel, MIMO, X channel, MAC, TIN.

The two-user Gaussian X channel is similar to the Gaussian interference channel consisting of two transmitters and two receivers, but with each transmitter having an independent message to each receiver. Sum-rate capacity of Gaussian X channel is investigated in this work. The sum-rate capacity of scalar Gaussian X channel is determined to within constant number of bits in a sub-region of the *mixed* interference regime. It is shown that using Gaussian codebooks and Multiple Access channel(MAC) scheme at either receiver achieves the sum rate within constant number of bits to the sum capacity of scalar Gaussian X channel in this sub-region of the mixed interference regime. In mixed interference regime, regions corresponding to n bits gap from MAC scheme are also determined.

The sum capacity of the two-user MIMO Gaussian X channel is determined in the *noisy* interference regime. This sum capacity is achieved by using Gaussian codebooks for the messages on both the direct links (or both the cross links) and treating the interference from the cross links (or direct links) as noise. The sufficient conditions for a MIMO Gaussian X channel to be in noisy interference regime are obtained. The *dual channel* of a GXC is defined and used in proving the results. The sum capacity for MISO and SIMO GXC's, which are special cases of MIMO GXC, are also obtained.

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ABBREVIATIONS

IC	Interference Channel
MIMO	Multi-Input Multi-Output
GIC	Gaussian Interference Channel
HK	Han & Kobayashi
GZIC	Gaussian Z Interference Channel
GZC	Gaussian Z Channel
GXC	Gaussian X Channel
DOF	Degrees of Freedom
SNR	Signal to Noise Ratio
TIN	Treating Interference as Noise
MAC	Multiple Access Channel
DM	Discrete Memoryless
MISO	Multi-Input Single-Output
SIMO	Single-Input Multi-Output

NOTATION

X, a	Scalars
$\boldsymbol{x}$	Column vectors
$\mathbf{H}$	Matrix
X^n	sequence of scalars
$\boldsymbol{x}^n$	sequence of column vectors
$E(.)$	Expectation of a random variable
$H(.)$	Entropy of a discrete random variable
$h(.)$	Differential entropy of a continuous random variable
$H(.\|.), h(.\|.)$	Conditional entropy , conditional differential entropy
$I(.;.)$	Mutual information
$I(.;.\|.)$	Conditional mutual information
$\mathcal{N}(0, \sigma_i^2)$	Gaussian distribution with zero mean and σ_i^2 variance
$\mathcal{CN}(\mathbf{0}, \mathbf{I})$	Circularly symmetric complex Gaussian distributed with zero mean and covariance matrix $\mathbf{I}$
$\mathbf{M}^T$	Transpose of $\mathbf{M}$
$\mathbf{M}^\dagger$	Conjugate transpose of $\mathbf{M}$
$\mathbf{M}^{-1}$	Inverse of $\mathbf{M}$
$\|\mathbf{M}\|$	Determinant of $\mathbf{M}$
$\mathcal{Y}$	Alphabet
C_{sum}	Sum-rate capacity
$\mathbf{A} \preceq \mathbf{B}$	$\mathbf{B} - \mathbf{A}$ is positive semidefinite
$\mathbf{A} \succ 0$	$\mathbf{A}$ is positive definite
$\gamma(x)$	$\frac{1}{2}\log(1+x)$

CHAPTER 1

Introduction

In multi-user wireless networks, nodes are distributed spatially and share the same communication medium. When the nodes send information to the intended receivers, they cause interference to the unintended receivers. As a result the performance of the entire network is limited by the interference. So, interference networks have become the subject of interest and many efforts have been made trying to characterize and to improve the performance of the network by establishing different optimal strategies.

A simple interference network is an interference channel (IC), whose capacity region is still an open problem. The capacity region is known only in few special cases. Interestingly, [1] showed that the interference does not affect the capacity region of Gaussian interference channel (GIC), if it is sufficiently high. The best known achievable inner bound is the Han-Kobayashi (HK) region [19], whose calculation is practically impossible. It uses message splitting (common and private) and joint decoding at the receivers. Chong *et al.,* [7] simplified the description of the HK region. Various outer bounds on IC and GIC were proposed by [2], [4], [15], [20]. Sato [10] determined the capacity region of a GIC in strong interference regime and defined the strong and very strong interference regimes for a general IC. Costa and El Gamal [13] established the capacity region in strong interference regime for a general IC. Sato [8] described the outer bound on degraded GIC. In [12], Costa showed the equivalence between the class of Gaussian Z interference channel (GZIC) and the degraded GIC. Kramer [5] obtained two outer bounds to the GIC, one by using genie aided method and optimizing over it and the other by using [8], [12] results. Etkin *et al.,* [15] showed that the HK inner bound is to within 1 bit to the capacity region of GIC and also obtained the Generalized Degrees of Freedom (GDOF), which is defined for SNR approaching infinity. Sum-rate optimality of treating interference as noise for a GIC in low interference regime was shown by [20], [25]. Sason [11] obtained an achievable rate region and proved that in high power regime TDMA/FDMA is a suboptimal strategy for sum-rate.

Shang *et al.,* [23] have generalized the known capacity results of scalar GIC to vector (MIMO) GIC. [26] introduced the concept of generally strong interference where the capacity is achieved by jointly decoding the signal and the interference at receivers and it is only required for the capacity achieving input distribution. Optimality of treating interference as noise in the low interference regime for MIMO GIC was discussed in [21].

"Z" channel, which was introduced in [18], is another simple interference network. An achievable rate region of Gaussian Z Channel (GZC) with very strong crossover link gain was established in [18]. The inner and outer bounds for GZC with weak crossover link gain and the capacity region of GZC with unity crossover link gain were derived in [14]. Chong *et al.,* in [6], derived the capacity region of GZC with moderately strong crossover link gain and obtained outer bounds for strong crossover link gain. Degrees of freedom results on MIMO IC, ZC, X channel under certain conditions were obtained in [17], [3], [16]. In [3], Huang *et al.,* showed the achievability of the sum capacity by treating interference as noise for scalar Gaussian X channel (GXC) in noisy interference regime, which is same as for GIC.

In this work, we obtain sum capacity of vector Gaussian X channel in the noisy interference by using the similar genie chosen in [3] along with the proof technique used in [21] for the MIMO Gaussian IC. Results for the SIMO and MISO GXC in the noisy interference are obtained. We also show the sum capacity of scalar GXC to be within a 1 bit gap to the sum rate achieved by the MAC at either receiver in a sub-region of the mixed interference regime. Genie aided method is used in proving these results. The main motive for considering simple networks is that intuition gained from their analysis can be generalized later to multi-terminal networks.

1.1 Organization of Thesis

The organization of thesis is as follows.

- Chapter 2 summarizes the known capacity results of Gaussian interference channels, Z interference channel and Z channel.
- Chapter 3 describes the sum capacity of scalar X channel to be within constant

gap from sum-rate achieved by the MAC at either receiver in a sub-region of the mixed interference regime.

- Chapter 4 describes the sum capacity of MIMO X channel in the noisy interference regime.
- Chapter 5 presents the simulation and numerical results of sum capacity of X channel.
- Chapter 6 concludes the thesis by discussing the scope for future work.

CHAPTER 2

Gaussian Interference Channel and “Z” Channel

In this chapter we discuss the known capacity results of two user Gaussian interference channel, Z interference channel and Z channel. We also provide few results on Degrees of Freedom (DOF) of MIMO channels.

2.1 System Model of Gaussian Interference Channel

Let us consider a two user scalar Gaussian IC given by (2.1), (2.2). It is shown in the Fig. 2.1.

$$Y_1^{'} = h_{11}X_1^{'} + h_{12}X_2^{'} + V_1, \tag{2.1}$$

$$Y_2^{'} = h_{21}X_1^{'} + h_{22}X_2^{'} + V_2, \tag{2.2}$$

where $X_i^{'}$ is the transmitted signal of transmitter i, h_{ij} is the channel coefficient from transmitter j to receiver i and $Y_i^{'}$ is the received signal at receiver i. $V_i \sim \mathcal{N}\left(0, \sigma_i^2\right)$ and is i.i.d. across time. Assume $X_i^{'}, Y_i^{'}, h_{ij}$ and V_i to be real valued scalars. The average power constraint on transmitter i over an n length block is $\frac{1}{n}\sum_{k=1}^{n} E\left[{X_{ik}^{'}}^2\right] \leq P_i^{'}$.

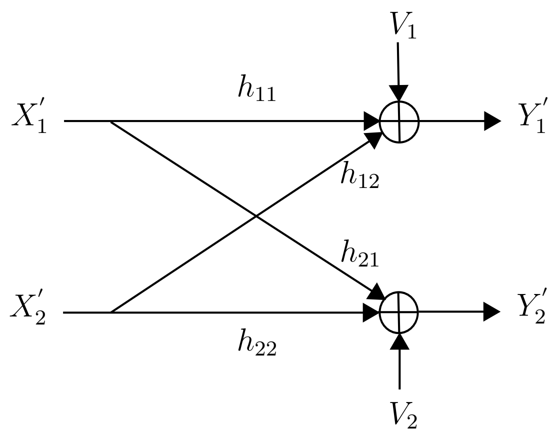

Figure 2.1: Two user Gaussian interference channel

2.1.1 Standard GIC

In [1], it is shown that a GIC can be transformed into a standard GIC of the form (2.3), (2.4). Both channels are equivalent in the sense of achieving same capacity region. This can be achieved using scaling transformations.

$$Y_1 = X_1 + aX_2 + Z_1, \tag{2.3}$$

$$Y_2 = bX_1 + X_2 + Z_2, \tag{2.4}$$

where $a = \frac{h_{12}\sigma_2}{h_{22}\sigma_1}$, $b = \frac{h_{21}\sigma_1}{h_{11}\sigma_2}$ and $h_{11}, h_{22} \neq 0$ with the power constraints on each transmitter are given by $P_1 = \frac{h_{11}^2}{\sigma_1^2}P_1'$, $P_2 = \frac{h_{22}^2}{\sigma_2^2}P_2'$. X_i is the transmitted signal of transmitter i, a, b are channel coefficients and Y_i is the received signal at receiver i. $Z_i \sim \mathcal{N}(0, 1)$ and is i.i.d. across time. In the rest of discussion GIC will refer only to the standard form.

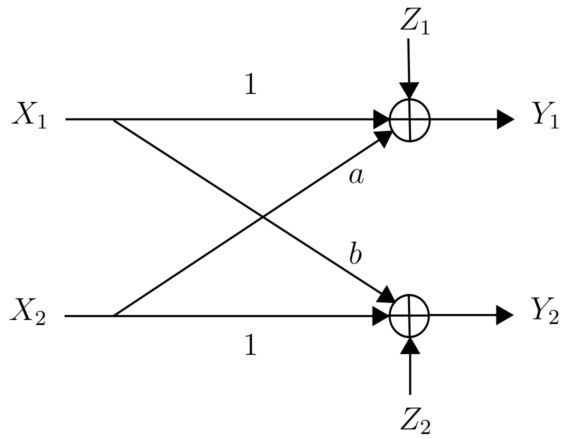

Figure 2.2: Two user standard Gaussian interference channel

Messages are uniformly generated at independent sources. Rate R_i is defined as $R_i = \frac{\log_2 M_i}{n}$, where M_i is the cardinality of message set at the encoder of transmitter i and n is the symbol duration. A code $(2^{nR_1}, 2^{nR_2}, n)$ has two encoding and two decoding functions. Encoder of transmitter i has an encoding function f_i that maps the message into a codeword. The decoder of receiver i has a decoding function g_i that maps the received codeword into a message.

$$f_i : \{1, 2, ..., 2^{nR_i}\} \mapsto \mathcal{X}_i^n$$

$$g_i : \mathcal{Y}_i^n \mapsto \{1, 2, ..., 2^{nR_i}\}$$

The probability of error in decoding the transmitted message at receivers is

$$P_{e1}^{(n)} = \frac{1}{M_1 M_2} \sum_{j=1}^{M_1} \sum_{k=1}^{M_2} Pr\left\{g_1(Y_1^n) \neq j | W_1 = j, W_2 = k\right\},$$

$$P_{e2}^{(n)} = \frac{1}{M_1 M_2} \sum_{j=1}^{M_1} \sum_{k=1}^{M_2} Pr\left\{g_2(Y_2^n) \neq k | W_1 = j, W_2 = k\right\}.$$

Define $\lambda^{(n)} = \max\{P_{e1}^{(n)}, P_{e2}^{(n)}\}$. A rate pair (R_1, R_2) is achievable if there exists a sequence of codes $(2^{nR_1}, 2^{nR_2}, n)$ such that $\lambda^{(n)} \to 0$ as $n \to \infty$. Capacity region is defined as the closure of all achievable rate pairs.

2.2 Interference Regimes

The capacity results of GIC are broadly divided based on the parameter values (a, b).

2.2.1 Very Strong Interference Regime

A discrete memoryless IC is said to be in very strong interference regime if it satisfies the conditions

$$I(X_1; Y_2) \geq I(X_1; Y_1 | X_2), \tag{2.5}$$

$$I(X_2; Y_1) \geq I(X_2; Y_2 | X_1), \tag{2.6}$$

for all input product distributions $p(x_1)p(x_2)$. The capacity region is given by,

$$R_1 \leq I(X_1; Y_1 | X_2, Q), \tag{2.7}$$

$$R_2 \leq I(X_2; Y_2 | X_1, Q), \tag{2.8}$$

for some pmf $p(q)p(x_1|q)p(x_2|q)$, where Q is a time sharing random variable . A similar condition for GIC (but not equivalent) is $a^2 \geq 1 + P_1, b \geq 1 + P_2$ and the capacity region is given by

$$R_1 \leq \frac{1}{2}\log(1+P_1), \tag{2.9}$$

$$R_2 \leq \frac{1}{2}\log(1+P_2). \tag{2.10}$$

The capacity is achieved using the scheme successive decoding of interference and cancelling it out. Carleial proved, in [1, Theorem 7], that interference doesnot reduce the capacity if it is sufficiently high, then the capacity region is same as that of no interference case.

Remark 1 *The GIC with very strong interference may not satisfy the conditions (2.5), (2.6).*

2.2.2 Strong Interference Regime

A discrete memoryless IC is said to be in strong interference regime if it satisfies the conditions

$$I(X_1;Y_2|X_2) \geq I(X_1;Y_1|X_2), \tag{2.11}$$

$$I(X_2;Y_1|X_1) \geq I(X_2;Y_2|X_1), \tag{2.12}$$

for all input product distributions $p(x_1)p(x_2)$. Costa *et al.*, in [13], proved the capacity region of DMIC with strong interference to be

$$R_1 \leq I(X_1;Y_1|X_2,Q), \tag{2.13}$$

$$R_2 \leq I(X_2;Y_2|X_1,Q), \tag{2.14}$$

$$R_1+R_2 \leq \min\left\{I(X_1,X_2;Y_1|Q), I(X_1,X_2;Y_2|Q)\right\}, \tag{2.15}$$

for some pmf $p(q)p(x_1|q)p(x_2|q)$, where Q is a time sharing random variable. It is the intersection region of two MAC capacity regions at receiver 1 and 2. An equivalent condition for GIC is $a^2 \geq 1, b \geq 1$ and the capacity region is given by

$$R_1 \leq \frac{1}{2}\log(1+P_1), \tag{2.16}$$

$$R_2 \leq \frac{1}{2}\log(1+P_2), \tag{2.17}$$

$$R_1 + R_2 \leq \min\left\{\frac{1}{2}\log(1+P_1+a^2P_2), \frac{1}{2}\log(1+b^2P_1+P_2)\right\}. \tag{2.18}$$

The capacity is achieved using the scheme joint decoding of both interference and signal at each receiver. Sato proved, in [10], the above result on GIC.

Remark 2 *The very strong interference is the subregime of strong interference. It can be observed that any channel that satisfies the very strong interference conditions also satisfies strong interference conditions.*

2.2.3 Mixed Interference Regime

A GIC is said to be in mixed interference if $a^2 > 1, b^2 < 1$ or $a^2 < 1, b^2 > 1$. Consider the case $a^2 < 1, b^2 > 1$ without loss of generality and the sum-rate capacity is known in this regime which is given by

$$C_{sum} = \min\left\{\frac{1}{2}\log\left(1+\frac{P_1}{1+a^2P_2}\right) + \frac{1}{2}\log(1+P_2), \frac{1}{2}\log\left(1+b^2P_1+P_2\right)\right\} \tag{2.19}$$

The sum-rate capacity is achieved by the scheme joint decoding of interference and signal at receiver 2 and treating interference as noise at receiver 1. Converse part of the result is proved using Gaussian Z interference channel (GZIC). Refer to section 2.3 for results of GZIC. Two GZIC's can be formed by removing interference links. Each GZIC will perform better than the underlying GIC because of the lack of interference link. So the sum-rate capacity of each GZIC is an outer bound (2.19) to GIC.
For the case $a^2 > 1, b^2 < 1$, the sum capacity result is obtained by interchanging a, b and also the indices.

2.2.4 Noisy Interference Regime

It is the sub-regime of weak interference regime ($a^2 < 1, b^2 < 1$). It is defined as the regime in which treating interference as noise is sum-rate optimal. The sufficient condition is given by

$$\left|a\left(1+b^2P_1\right)\right| + \left|b\left(1+a^2P_2\right)\right| \leq 1, \tag{2.20}$$

and the sum capacity is given by

$$C_{sum} = \frac{1}{2}\log\left(1+\frac{P_1}{1+a^2P_2}\right) + \frac{1}{2}\log\left(1+\frac{P_2}{1+b^2P_1}\right). \tag{2.21}$$

This result was proved in [21] by giving the appropriate side information to the receivers. The genie is chosen to satisfy usefulness and smartness conditions given in [21] and showed that (2.21) is the outer bound if (2.20) is satisfied. The same result was also proved independently in [25] by deriving the outer bound.

2.2.5 Other Known Results

- In [5], Kramer derived two outer bounds for the Gaussian interference channel. First outer bound is derived by giving genie to receiver 1 and optimizing it. Second one is derived using GZIC outer bound, results of [9], [12]. Proving it requires transmitter cooperation with power constraint $P_1 + P_2$. It also showed that second bound is better than the first in weak interference regime.

- Etkin *et al.*, in [15], showed that capacity region of Gaussian interference channel is within *one bit* to the simple Han-Kobayashi scheme. In this genie aided outer bound was derived and showed the gap between outer and the inner bound is within one bit.

- A few other improved outer bounds were derived in [25], [20].

- Sason, in [11], proved that in high power regime with moderate interference TDM/FDM scheme is sum capacity sub optimal.

Table 2.1: Sum capacity of Gaussian interference channel in various regimes. We will denote $\gamma(x) = \frac{1}{2}\log(1+x)$.

Parameter range		Sum capacity	Achievable scheme of sum capacity
$a^2 \geq 1 + P_1, b^2 \geq 1 + P_2$		$\gamma(P_1) + \gamma(P_2)$	Successive decoding of interference and signal
$a^2 \geq 1, b^2 \geq 1$		$\min \left\{ \begin{array}{c} \gamma(P_1 + a^2 P_2) \\ \gamma(b^2 P_1 + P_2) \\ \gamma(P_1) + \gamma(P_2) \end{array} \right\}$	Joint decoding of both signal and interference
$a^2 \leq 1, b^2 \geq 1$ (or $a^2 \geq 1, b^2 \leq 1$)		$\min \left\{ \begin{array}{c} \gamma\left(\frac{P_1}{1+a^2P_2}\right) + \gamma(P_2) \\ \gamma\left(b^2 P_1 + P_2\right) \end{array} \right\}$	Treating interference as noise at receiver 1 and joint decoding at receiver 2
$a^2 \leq 1, b^2 \leq 1$	$\lvert a\left(1 + b^2 P_1\right)\rvert + \lvert b\left(1 + a^2 P_2\right)\rvert \leq 1$	$\gamma\left(\frac{P_1}{1+a^2P_2}\right) + \gamma\left(\frac{P_2}{1+b^2P_1}\right)$	Treating interference as noise at both receivers
	$\lvert a\left(1 + b^2 P_1\right)\rvert + \lvert b\left(1 + a^2 P_2\right)\rvert \geq 1$	Inner and outer bounds are known	——

2.3 Gaussian Z Interference Channel

The system model of Gaussian Z interference channel is same as the Gaussian interference channel with $b = 0$ and is given by

$$Y_1 = X_1 + aX_2 + Z_1, \tag{2.22}$$

$$Y_2 = X_2 + Z_2. \tag{2.23}$$

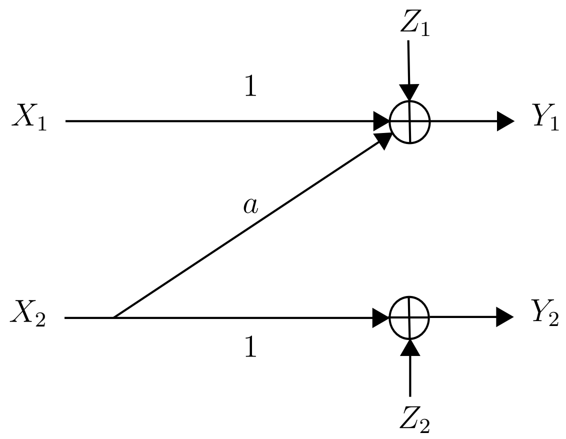

Figure 2.3: Two user Gaussian Z interference channel

Table 2.2: Summarized capacity results of Gaussian Z interference channel.

	Parameter range	Capacity region	Sum-rate capacity
1	$a^2 \geq 1 + P_1$	$R_1 \leq \frac{1}{2}\log(1 + P_1)$ $R_2 \leq \frac{1}{2}\log(1 + P_2)$	$\frac{1}{2}\log(1 + P_1) + \frac{1}{2}\log(1 + P_2)$
2	$1 \leq a^2 \leq 1 + P_1$	$R_1 \leq \frac{1}{2}\log(1 + P_1)$ $R_2 \leq \frac{1}{2}\log(1 + P_2)$ $R_1 + R_2 \leq$ $\frac{1}{2}\log(1 + P_1 + a^2P_2)$	$\frac{1}{2}\log(1 + P_1 + a^2P_2)$
3	$a^2 \leq 1$	—	$\frac{1}{2}\log\left(1 + \frac{P_1}{1+a^2P_2}\right) + \frac{1}{2}\log(1 + P_2)$

First two cases in the table 2.2 are similar to GIC very strong interference and strong interference respectively. So the same arguments of GIC can be applied to GZIC in deriving the results. For the case $a^2 < 1$, Costa , in [12], showed the equivalence between

GZIC and degraded GIC. Sato [9] determined the outer bound and sum capacity of degraded GIC. So GZIC with $a^2 < 1$ should have the same sum capacity as degraded GIC.

2.4 MIMO Gaussian IC

The system model for MIMO Gaussian IC is given as following.

$$\boldsymbol{y}_1 = \mathbf{H}_{11}\boldsymbol{x}_1 + \mathbf{H}_{12}\boldsymbol{x}_2 + \boldsymbol{z}_1, \tag{2.24}$$

$$\boldsymbol{y}_2 = \mathbf{H}_{21}\boldsymbol{x}_1 + \mathbf{H}_{22}\boldsymbol{x}_2 + \boldsymbol{z}_2. \tag{2.25}$$

where $\boldsymbol{x}_i$ is a $t_i \times 1$ vector, $\boldsymbol{y}_i$, $\boldsymbol{z}_i$ are $r_i \times 1$ vectors, $\mathbf{H}_{ij}$ is $r_i \times t_j$ channel matrix and t_i, r_j are the number of antennas at the transmitter i, receiver j respectively. Noise vector $\boldsymbol{z}_i \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{I}_{r_i \times r_i}\right)$ and is i.i.d. across time. The average covariance constraint on the i^{th} transmitter over an n symbol duration is

$$\frac{1}{n}\sum_{k=1}^{n} E\left[\boldsymbol{x}_{ik}\boldsymbol{x}_{ik}^{\dagger}\right] \preceq \mathbf{S}_i. \tag{2.26}$$

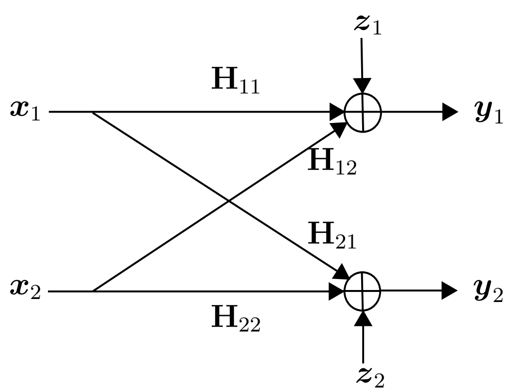

Figure 2.4: MIMO Gaussian interference channel

The following results on MIMO GIC were stated and proved in [23]. Let us define

$$\mathcal{B}_i = \{\mathbf{B} | \text{all columns of } \mathbf{B}^{\dagger} \text{are in the null space of } \mathbf{S}_i\}.$$

2.4.1 Very Strong Interference

The conditions for very strong interference MIMO IC are derived in the same way as scalar GIC . If a MIMO IC with $\mathbf{H}_{12} \neq \mathbf{0}, \mathbf{H}_{21} \neq \mathbf{0}$ satisfies

$$\log \left|\mathbf{I} + \mathbf{H}_{11}\mathbf{S}_1\mathbf{H}_{11}^\dagger + \mathbf{H}_{12}\mathbf{S}_2\mathbf{H}_{12}^\dagger\right| \geq \log \left|\mathbf{I} + \mathbf{H}_{11}\mathbf{S}_1\mathbf{H}_{11}^\dagger\right| + \log \left|\mathbf{I} + \mathbf{H}_{12}\mathbf{S}_2\mathbf{H}_{12}^\dagger\right| \quad (2.27)$$

$$\log \left|\mathbf{I} + \mathbf{H}_{21}\mathbf{S}_1\mathbf{H}_{21}^\dagger + \mathbf{H}_{22}\mathbf{S}_2\mathbf{H}_{22}^\dagger\right| \geq \log \left|\mathbf{I} + \mathbf{H}_{21}\mathbf{S}_1\mathbf{H}_{21}^\dagger\right| + \log \left|\mathbf{I} + \mathbf{H}_{22}\mathbf{S}_2\mathbf{H}_{22}^\dagger\right| \quad (2.28)$$

then the capacity region is given by

$$R_1 \leq \log \left|\mathbf{I} + \mathbf{H}_{11}\mathbf{S}_1\mathbf{H}_{11}^\dagger\right|, \quad (2.29)$$

$$R_2 \leq \log \left|\mathbf{I} + \mathbf{H}_{22}\mathbf{S}_2\mathbf{H}_{22}^\dagger\right|. \quad (2.30)$$

The capacity region is achieved by successive decoding of interference and signal at receivers.

2.4.2 Aligned Strong Interference

The conditions for aligned strong interference are that $\mathbf{H}_{11}, \mathbf{H}_{22}$ should be linear transformations of $\mathbf{H}_{21}, \mathbf{H}_{12}$. The capacity region is similar to scalar GIC with strong interference, where joint decoding at receivers is used to achieve the capacity. If there exist matrices $\mathbf{A}_1, \mathbf{A}_2, \mathbf{B}_1$ and $\mathbf{B}_2$ such that

$$\mathbf{H}_{11} = \mathbf{A}_1\mathbf{H}_{21} + \mathbf{B}_1, \quad (2.31)$$

$$\mathbf{H}_{22} = \mathbf{A}_2\mathbf{H}_{12} + \mathbf{B}_2, \quad (2.32)$$

where $\mathbf{A}_i\mathbf{A}_i^\dagger \preceq \mathbf{I}$ and $\mathbf{B} \in \mathcal{B}_i, i = 1, 2$ then the capacity region of a MIMO IC is

$$R_1 \leq \log \left|\mathbf{I} + \mathbf{H}_{11}\mathbf{S}_1\mathbf{H}_{11}^\dagger\right|, \quad (2.33)$$

$$R_2 \leq \log \left|\mathbf{I} + \mathbf{H}_{22}\mathbf{S}_2\mathbf{H}_{22}^\dagger\right|, \quad (2.34)$$

$$R_1 + R_2 \leq \log \left|\mathbf{I} + \mathbf{H}_{11}\mathbf{S}_1\mathbf{H}_{11}^\dagger + \mathbf{H}_{12}\mathbf{S}_2\mathbf{H}_{12}^\dagger\right|, \quad (2.35)$$

$$R_1 + R_2 \leq \log \left|\mathbf{I} + \mathbf{H}_{21}\mathbf{S}_1\mathbf{H}_{21}^\dagger + \mathbf{H}_{22}\mathbf{S}_2\mathbf{H}_{22}^\dagger\right|. \quad (2.36)$$

2.4.3 Noisy Interference

A MIMO IC is said to have noisy interference if sum-rate capacity is achieved as by treating interference as noise. If there exist matrices $\mathbf{A}_i, \mathbf{B} \in \mathcal{B}_i$ and Hermitian positive definite matrices $\Sigma_i, i = 1, 2$ such that

$$\mathbf{A}_1^\dagger \mathbf{A}_1 \preceq \mathbf{\Sigma}_1 \preceq \mathbf{I} - \mathbf{A}_2 \mathbf{\Sigma}_2^{-1} \mathbf{A}_2^\dagger, \tag{2.37}$$

$$\mathbf{A}_2^\dagger \mathbf{A}_2 \preceq \mathbf{\Sigma}_2 \preceq \mathbf{I} - \mathbf{A}_1 \mathbf{\Sigma}_1^{-1} \mathbf{A}_1^\dagger, \tag{2.38}$$

$$\mathbf{H}_{21} = \mathbf{A}_1^\dagger \left(\mathbf{I} + \mathbf{H}_{12}\mathbf{S}_2\mathbf{H}_{12}^\dagger\right)^{-1} \mathbf{H}_{11} + \mathbf{B}_1, \tag{2.39}$$

$$\mathbf{H}_{12} = \mathbf{A}_2^\dagger \left(\mathbf{I} + \mathbf{H}_{21}\mathbf{S}_1\mathbf{H}_{21}^\dagger\right)^{-1} \mathbf{H}_{22} + \mathbf{B}_2, \tag{2.40}$$

then the sum capacity of a MIMO IC with noisy interference is

$$\log\left|\mathbf{I} + \mathbf{H}_{11}\mathbf{S}_1\mathbf{H}_{11}^\dagger \left(\mathbf{I} + \mathbf{H}_{12}\mathbf{S}_2\mathbf{H}_{12}^\dagger\right)^{-1}\right| + \log\left|\mathbf{I} + \mathbf{H}_{12}\mathbf{S}_2\mathbf{H}_{12}^\dagger \left(\mathbf{I} + \mathbf{H}_{21}\mathbf{S}_1\mathbf{H}_{21}^\dagger\right)^{-1}\right| \tag{2.41}$$

Similar conditions on MIMO IC with noisy interference under average power constraint were derived in [21, Theorem 1], by using useful and smart genie for finding an outer bound on sum capacity.

2.4.4 Mixed Aligned Interference

In mixed aligned interference MIMO IC one link satisfies strong aligned interference condition and other satisfies weak interference condition. If there exist matrices $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{B}_1$ and $\mathbf{B}_2$ such that

$$\mathbf{H}_{11} = \mathbf{A}_1\mathbf{H}_{21} + \mathbf{B}_1, \tag{2.42}$$

$$\mathbf{H}_{12} = \mathbf{A}_2^\dagger\mathbf{H}_{22} + \mathbf{B}_2, \tag{2.43}$$

where $\mathbf{A}_i\mathbf{A}_i^\dagger \preceq \mathbf{I}$ and $\mathbf{B} \in \mathcal{B}_i, i = 1, 2$ then the sum capacity of a MIMO IC with mixed aligned interference is

$$\min \left\{ \begin{array}{l} \log \left| \mathbf{I} + \mathbf{H}_{21}\mathbf{S}_1\mathbf{H}_{21}^\dagger + \mathbf{H}_{22}\mathbf{S}_2\mathbf{H}_{22}^\dagger \right|, \\ \log \left| \mathbf{I} + \mathbf{H}_{11}\mathbf{S}_1\mathbf{H}_{11}^\dagger \left(\mathbf{I} + \mathbf{H}_{12}\mathbf{S}_2\mathbf{H}_{12}^\dagger \right)^{-1} \right| + \log \left| \mathbf{I} + \mathbf{H}_{22}\mathbf{S}_2\mathbf{H}_{22}^\dagger \right|. \end{array} \right\} \quad (2.44)$$

This result is proved similar to scalar GIC by bounding the sum capacity of MIMO GIC by two MIMO GZIC's and showing the achievability of it.

2.4.5 Generally Strong Interference

As we discussed in the section 2.3, the very strong interference is the subregime of strong interference for scalar Gaussian IC but this is not the case with MIMO IC. There exists channels that satisfy very strong interference conditions but not strong aligned interference conditions. The very strong interference condition for GIC is more general than DMIC condition, which is stringent requiring all input distributions to satisfy. But very strong interference condition is to be satisfied only by the capacity achieving distribution. This kind of condition also relaxes the strong interference condition, where jointly decoding interference and signal achieves the capacity. With the above motivation Shang *et al.*,, in [26], introduced the concept of generally strong interference, which includes both strong and very strong interference cases as subcases.

Definition 1 *An IC is said to have generally strong interference if the capacity region is achieved by jointly decoding the signal and the interference at each receiver or equivalently if its capacity region is given as follows*

$$R_1 \leq I(X_1; Y_1 | X_2, Q), \quad (2.45)$$

$$R_2 \leq I(X_2; Y_2 | X_1, Q), \quad (2.46)$$

$$R_1 + R_2 \leq \min \{ I(X_1, X_2; Y_1 | Q), I(X_1, X_2; Y_2 | Q) \}, \quad (2.47)$$

where Q is a time sharing random variable

Definition 2 *An IC is said to have generally strong interference at (R_1, R_2) if (R_1, R_2)*

is on the boundary of the capacity region and it satifies (2.45),(2.46),(2.47) for some input distributions of X_1 and X_2 or equivalently if (R_1, R_2) is achieved by jointly decoding the signal and the interference at each receiver.

For any further reading refer to [26], [22]

2.5 Gaussian Z Channel

The channel model of Gaussian Z channel is same as GZIC (2.22), (2.23) but the receiver 2 has independent message to each receiver. Fig. 2.5 indicates the GZC. W_{ij} is

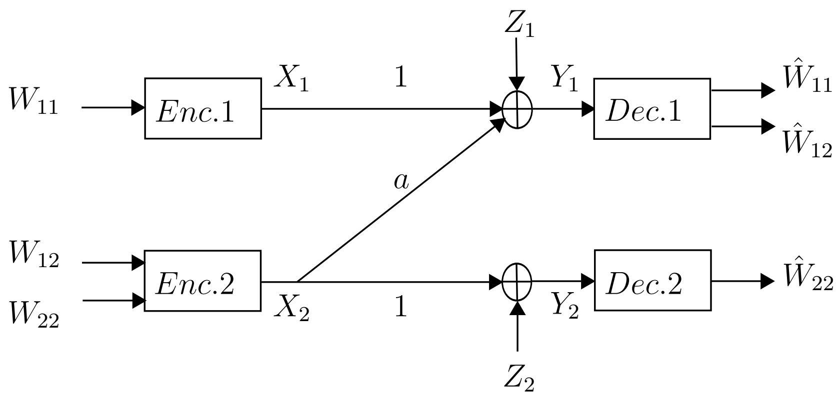

Figure 2.5: Gaussian Z channel

the message from transmitter j to receiver i.The rate triplet (R_{11}, R_{12}, R_{22}) is said to be achievable if there is a reliable transmission of all messages. The capacity region is defined as closure of achievable rate triplets . This channel was first introduced and studied in [18]. Inner bounds, outer bounds and sum capacity are known for this channel. The capacity results are classified based on the crossover link gain and are given as follows. We will define $\gamma(x) = \frac{1}{2}\log(1 + x)$

2.5.1 Weak Crossover Link

GZC's with crossover link gain $a^2 < 1$ are defined to have weak crossover link. Liu, Ulukus studied this channel in [14] and derived inner and outer bounds. The achievable region, which is an inner bound, is given as follows. For any $0 \leq \beta \leq 1$

$$R_{11} \leq \gamma\left(\frac{P_1}{a^2\beta P_2 + 1}\right), \tag{2.48}$$

$$R_{12} \leq \gamma\left(\frac{a^2(1-\beta)P_2}{a^2\beta P_2 + 1}\right), \tag{2.49}$$

$$R_{22} \leq \gamma(\beta P_2), \tag{2.50}$$

$$R_{11} + R_{12} \leq \gamma\left(\frac{P_1 + a^2(1-\beta)P_2}{a^2\beta P_2 + 1}\right). \tag{2.51}$$

This inner bound is derived using superposition coding at transmitter 2 with Gaussian codebooks and βP_2 power allocated to W_{22} and also using successive decoding at both the receivers. The outer bound is given by proving the converse. For some $0 \leq \beta \leq 1$, achievable rate triplet (R_{11}, R_{12}, R_{22}) has to satisfy the following conditions

$$R_{12} \leq \gamma\left(\frac{a^2(1-\beta)P_2}{a^2\beta P_2 + 1}\right), \tag{2.52}$$

$$R_{22} \leq \gamma(\beta P_2), \tag{2.53}$$

$$R_{11} + R_{12} \leq \gamma\left(\frac{P_1 + a^2(1-\beta)P_2}{a^2\beta P_2 + 1}\right). \tag{2.54}$$

The sum capacity of GZC with weak crossover link is $\frac{1}{2}\log(1+P_2)+\frac{1}{2}\log(1+\frac{P_1}{1+a^2P_2})$. It is achieved by setting $R_{12} = 0$ and treating interference as noise at receiver 1.

2.5.2 Unity Crossover Link Gain

This is the case when $a^2 = 1$. The capacity region of GZC with unity crossover link gain is

$$R_{11} \leq \frac{1}{2}\log(1 + P_1), \tag{2.55}$$

$$R_{12} + R_{22} \leq \frac{1}{2}\log(1 + P_2), \tag{2.56}$$

$$R_{11} + R_{12} + R_{22} \leq \frac{1}{2}\log(1 + P_1 + P_2). \tag{2.57}$$

This was also proved in [14] by finding an equivalent channel and using the arguments of [12]. In the equivalent channel the receivers are able to decode all the messages forming MAC.

2.5.3 Strong Crossover Link

A GZC is said to have a strong crossover link if $a^2 > 1$. Chong *et al.*, in [6], derived the inner and outer bounds for this case. The achivable rate region is given as the convex closure of all rate triplets satisfying

$$R_{11} \leq \gamma\left(P_1\right), \tag{2.58}$$
$$R_{12} \leq \gamma\left(a^2\beta P_2\right), \tag{2.59}$$
$$R_{22} \leq \gamma\left(\frac{(1-\beta)P_2}{1+\beta P_2}\right), \tag{2.60}$$
$$R_{11} + R_{12} \leq \gamma\left(a^2\beta P_2 + P_1\right), \tag{2.61}$$
$$R_{11} + R_{12} + R_{22} \leq \gamma\left(a^2 P_2 + P_1\right). \tag{2.62}$$

for any $0 \leq \beta \leq 1$. This is derived in [6] by showing the correspondence between GZC with strong crossover link and degraded ZC of type II. Rate splitting, superposition coding at transmitters and joint decoding at receivers is used in deriving the achievable rate region. The outer bound is given by proving the converse. For some $0 \leq \beta \leq 1$, achievable rate triplet (R_{11}, R_{12}, R_{22}) has to satisfy the following conditions

$$R_{11} \leq \gamma\left(P_1\right), \tag{2.63}$$
$$R_{12} \leq \gamma\left(a^2\beta P_2\right), \tag{2.64}$$
$$R_{22} \leq \gamma\left(\frac{(1-\beta)P_2}{1+\beta P_2}\right), \tag{2.65}$$
$$R_{11} + R_{12} + R_{22} \leq \gamma\left(a^2 P_2 + P_1\right). \tag{2.66}$$

The sum capacity for GZC with strong crossover link is $\frac{1}{2}\log(1 + P_1 + a^2 P_2)$. This is achieved by setting $R_{22} = 0$, which forms a MAC at receiver 1. Then the sum capacity achieved by MAC is the sum capacity of GZC.

2.5.4 Moderately Strong Crossover Link

The condition for moderately strong crossover link GZC is $1 < a^2 < 1 + P_1$. It is a special case of strong crossover link GZC $(a^2 > 1)$ for which the capacity region is known. The capacity region is the convex closure of rate triplets satisfying

$$R_{11} \leq \gamma\left(P_1\right), \tag{2.67}$$

$$R_{12} \leq \gamma\left(a^2 \beta P_2\right), \tag{2.68}$$

$$R_{22} \leq \gamma\left(\frac{(1-\beta)P_2}{1+\beta P_2}\right), \tag{2.69}$$

$$R_{11} + R_{12} + R_{22} \leq \gamma\left(a^2 P_2 + P_1\right). \tag{2.70}$$

for some $0 \leq \beta \leq 1$. It is derived in [6, Theorem 8] by showing that all rate triplets in the outer bound of strong crossover link GZC are achievable. If $a^2 > 1 + P_1$ then GZC is known as very strong crossover link GZC. The achievable rate region is given by the achievable rate region of strong crossover link GZC without (2.62), which is redundant. This region was determined in [18] using superposition coding and successive decoding.

2.6 Degrees of Freedom

Degrees of Freedom (DOF) of a channel is defined as

$$\eta = \lim_{\rho \to \infty} \frac{C_\Sigma(\rho)}{\log \rho}, \tag{2.71}$$

where $C_\Sigma(\rho)$ is the sum capacity for a signal to noise ratio (SNR) (ρ). The basic idea behind degrees of freedom is that how the sum capacity scales with SNR as it tends to infinity. Etkin *et al.,* in [15] determined the DOF of scalar Gaussian interference channel. For various MIMO systems DOF results can be found in [17], [16] and they are summarized below. Assume full rank channel matrices.

For point to point channel with M inputs and N outputs the DOF is

$$\eta(PTP) = \min(M, N).$$

For MAC channel with M_1, M_2 inputs at respective transmitters and N outputs at the receiver the DOF is

$$\eta(MAC) = \min(M_1 + M_2, N).$$

Similarly for a broadcast channel with M inputs at the transmitter and N_1, N_2 outputs at respective receivers the DOF is

$$\eta(BC) = \min(M, N_1 + N_2).$$

For an interference channel with M_1, M_2 inputs at respective transmitters and N_1, N_2 outputs at respective receivers the DOF is

$$\eta(IC) = \min\{M_1 + M_2, N_1 + N_2, \max(M_1, N_2), \max(M_2, N_1)\}.$$

Outer bound on the DOF of MIMO Z channel was derived in [17]. Consider a MIMO Z channel with M_1, M_2 inputs at transmitters 1, 2 respectively and N_1, N_2 outputs at receivers 1, 2 respectively , the bound on DOF is given by

$$\eta(Z) \leq \min(N_1, M_2).$$

In [17], Jafar and Shamai determined the DOF of MIMO X channel for a special case of all nodes having equal number of antennas M, $M \neq 1$, to be $\frac{4}{3}M$. Interference alignment was used in proving the result. Degree of freedom was generalized to degrees of freedom region in [15],[17].

CHAPTER 3

Sum Capacity of Scalar "X" Channel to within a Constant Gap

" *How close is the sum rate achieved by the MAC at either receiver 1 or 2 to the sum capacity of the Gaussian X channel?* " is the question we will answer in this chapter. We will prove the results for a sub-region of the mixed interference regime

3.1 System Model

A scalar Gaussian X channel in the standard form is given by

$$Y_1 = X_1 + aX_2 + Z_1, \tag{3.1}$$

$$Y_2 = bX_1 + X_2 + Z_2, \tag{3.2}$$

where $Z_1, Z_2 \sim \mathcal{N}(0,1)$ and i.i.d in time. Transmitter i has an average power constraint P_i i.e., $\frac{1}{n}\sum_{k=1}^{n} E\left[X_{ik}^2\right] \leq P_i$. In the X channel each transmitter is having separate and independent messages for receivers. See Fig. 3.1.

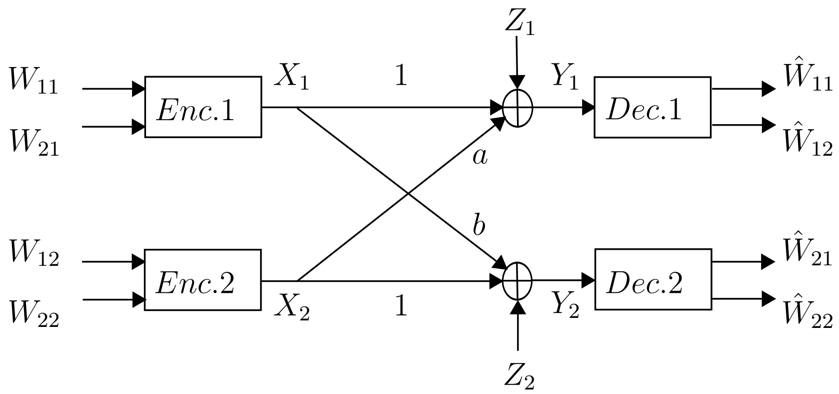

Figure 3.1: Two user scalar Gaussian X channel

3.1.1 MAC Strategy

The MAC strategy at a receiver i is obtained by setting $W_{j1}, W_{j2} = \phi$, where $j \neq i$, at transmitters 1, 2 and decoding W_{i1}, W_{i2} at receiver i. MAC strategy at receiver 1 is sum rate optimal for Gaussian Z channel with strong crossover link gain. So it's natural to think whether MAC strategy is sum-rate optimal or not.

3.2 On the Sum Rate Achieved by MAC at Receiver 1

Theorem 1 *For a Gaussian X channel with $a^2 \geq 1$, $\eta^2 \leq 1$ and $\eta\rho = b(a^2 P_2 + 1)$ the sum-rate capacity will be within the the gap $\frac{1}{2}\log_2\left(\frac{1-\frac{\rho^2}{a^2P_2+1}}{1-\rho^2}\right)$ bits to the sum rate achieved by using MAC at receiver 1.*

Proof: Consider the genie-aided X channel with the side information S_1 given to the receiver 1. See Fig. 3.2. Let the side information be $S_1 = bX_1 + \eta W$ where $W \sim \mathcal{N}(0,1)$ is correlated with Z_1 and ρ is the correlation coefficient. We will find the outer bound on the sum capacity of the genie-aided channel which in turn is an outer bound to the original X channel.

$$n(R_{11} + R_{12} + R_{21} + R_{22}) = H(W_{11}, W_{12}, W_{21}, W_{22}) \tag{3.3}$$

$$= I(W_{11}, W_{12}, W_{21}, W_{22}; Y_1^n, S_1^n) + H(W_{11}, W_{12}, W_{21}, W_{22}|Y_1^n, S_1^n) \tag{3.4}$$

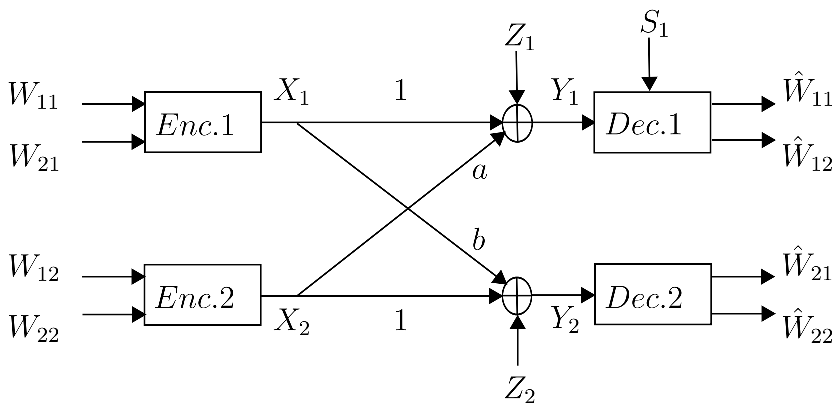

Figure 3.2: Genie-aided scalar Gaussian X channel

Let us bound the term $H(W_{11}, W_{12}, W_{21}, W_{22}|Y_1^n, S_1^n)$ using Fano's inequality and also derive the required conditions.

$$\begin{aligned} H(W_{11}, W_{12}, W_{21}, W_{22}|Y_1^n, S_1^n) &= H(W_{11}|Y_1^n, S_1^n) + H(W_{12}|Y_1^n, S_1^n, W_{11}) \\ &+ H(W_{21}|Y_1^n, S_1^n, W_{11}, W_{12}) \\ &+ H(W_{22}|Y_1^n, S_1^n, W_{11}, W_{12}, W_{21}) \qquad (3.5) \\ &\overset{(a)}{\leq} H(W_{11}|Y_1^n, S_1^n) + H(W_{12}|Y_1^n, S_1^n) + H(W_{21}|S_1^n) \\ &+ H(W_{22}|Y_1^n, W_{11}, W_{21}) \qquad (3.6) \end{aligned}$$

Step (a) follows from the fact that conditioning reduces the entropy. The first two terms in (3.6) can be bounded by $n(\epsilon_1 + \epsilon_2)$. The next two terms can be bounded if certain conditions are met. Let us consider the fourth term, as we know $H(W_{22}|Y_2^n) \leq n\epsilon_4$ at receiver 2.

$$\begin{aligned} H(W_{22}|Y_2^n) \leq n\epsilon_4 &\Rightarrow H(W_{22}|Y_2^n, W_{11}, W_{21}) \leq n\epsilon_4 \qquad (3.7) \\ &\Rightarrow H(W_{22}|bX_1^n + X_2^n + Z_2^n, W_{11}, W_{21}) \leq n\epsilon_4 \qquad (3.8) \\ &\Rightarrow H(W_{22}|X_2^n + Z_2^n, W_{11}, W_{21}) \leq n\epsilon_4 \qquad (3.9) \end{aligned}$$

$$\begin{aligned} H(W_{22}|Y_1^n, W_{11}, W_{21}) &= H(W_{22}|X_1^n + aX_2^n + Z_2^n, W_{11}, W_{21}) \qquad (3.10) \\ &= H(W_{22}|X_2^n + \frac{Z_2^n}{a}, W_{11}, W_{21}) \qquad (3.11) \end{aligned}$$

If $a^2 \geq 1$, $X_2^n + \frac{Z_2^n}{a}$ is less noisy version than $X_2^n + Z_2^n$, then (3.11) is bounded by $n\epsilon_4$. Now consider the third term, as we know $H(W_{21}|Y_2^n) \leq n\epsilon_3$ at receiver 2.

$$\begin{aligned} H(W_{21}|Y_2^n) \leq n\epsilon_3 &\Rightarrow H(W_{21}|Y_2^n, W_{12}, W_{22}) \leq n\epsilon_3 \qquad (3.12) \\ &\Rightarrow H(W_{21}|bX_1^n + X_2^n + Z_2^n, W_{12}, W_{22}) \leq n\epsilon_3 \qquad (3.13) \\ &\Rightarrow H(W_{21}|bX_1^n + Z_2^n) \leq n\epsilon_3 \qquad (3.14) \end{aligned}$$

$$H(W_{21}|S_1^n) = H(W_{21}|bX_1^n + \eta W^n) \qquad (3.15)$$

If $\eta^2 \leq 1$, $bX_1^n + \eta W^n$ is less noisy version than $bX_1^n + Z_2^n$, then (3.15) is bounded by $n\epsilon_3$. As a result $H(W_{11}, W_{12}, W_{21}, W_{22}|Y_1^n, S_1^n) \leq n(\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4)$.

$$
\begin{aligned}
n(R_{11} + R_{12} + R_{21} + R_{22} - \epsilon) &\leq I(W_{11}, W_{12}, W_{21}, W_{22}; Y_1^n, S_1^n) && (3.16)\\
&= h(Y_1^n, S_1^n) - h(Y_1^n, S_1^n|W_{11}, W_{12}, W_{21}, W_{22})\\
&\overset{(b)}{\leq} nh(Y_{1G}, S_{1G}) - nh(Z_1, W) && (3.17)\\
&= nI(X_{1G}, X_{2G}; Y_{1G}, S_{1G}) && (3.18)\\
&= nI(X_{1G}, X_{2G}; Y_{1G}) + nI(X_{1G}, X_{2G}; S_{1G}|Y_{1G}) && (3.19)
\end{aligned}
$$

Step (b) is due to Gaussian distribution maximizes the entropy.
Now consider $I(X_{1G}, X_{2G}; S_{1G}|Y_{1G}) = I(X_{1G}; S_{1G}|Y_{1G}) + I(X_{2G}; S_{1G}|Y_{1G}, X_{1G})$, where first term on R.H.S can be made zero iff $\eta\rho = b(a^2P_2 + 1)$.

$$
\begin{aligned}
I(X_{1G}; bX_{1G} + \eta W|X_{1G} + aX_{2G} + Z_1) = 0 && (3.20)\\
\Leftrightarrow E\left[\frac{\eta W(aX_{2G} + Z_1)}{b}\right] = E\left[(aX_{2G} + Z_1)^2\right] && (3.21)
\end{aligned}
$$

The second term on R.H.S $I(X_{2G}; S_{1G}|Y_{1G}, X_{1G})$ corresponds to the gap between the sum capacity and sum rate achieved by MAC at receiver 1.

$$
\begin{aligned}
I(X_{2G}; S_{1G}|Y_{1G}, X_{1G}) &= I(X_{2G}; bX_{1G} + \eta W|X_{1G} + aX_{2G} + Z_1, X_{1G}) && (3.22)\\
&= h(bX_{1G} + \eta W|X_{1G} + aX_{2G} + Z_1, X_{1G}) && (3.23)\\
&\quad - h(bX_{1G} + \eta W|X_{1G} + aX_{2G} + Z_1, X_{1G}, X_{2G}) && (3.24)\\
&= h(\eta W|aX_{2G} + Z_1) - h(\eta W|Z_1) && (3.25)\\
&= \frac{1}{2}\log_2\left(\frac{1 - \frac{\rho^2}{a^2P_2+1}}{1-\rho^2}\right) \text{ bits} && (3.26)
\end{aligned}
$$

The sum capacity of genie-aided X channel, which is the outer bound on the sum capacity of original X channel, is given by (3.27). $I(X_{1G}, X_{2G}; Y_{1G})$ is the achievable sum rate of original X channel using the MAC strategy at receiver 1 and $\frac{1}{2}\log_2\left(\frac{1-\frac{\rho^2}{a^2P_2+1}}{1-\rho^2}\right)$

is the atmost gap between the sum capacity and sum rate achieved by MAC at receiver 1.

$$C_{sum}^{GA-X} = I(X_{1G}, X_{2G}; Y_{1G}) + \frac{1}{2}\log_2\left(\frac{1-\frac{\rho^2}{a^2P_2+1}}{1-\rho^2}\right) \tag{3.27}$$

■

3.2.1 Region Corresponding to n bits Gap from MAC at Receiver 1 by Theorem 1

ρ correspondindg to the n bits gap is given as follows

$$\frac{1}{2}\log_2\left(\frac{1-\frac{\rho^2}{a^2P_2+1}}{1-\rho^2}\right) \leq n \tag{3.28}$$

$$\rho^2 \leq \frac{(2^{2n}-1)}{(2^{2n}-\frac{1}{a^2P_2+1})} \tag{3.29}$$

The region corresponding to the n bits gap is given by $a^2 \geq 1, b^2 \leq \frac{(2^{2n}-1)}{(2^{2n}(a^2P_2+1)-1)(a^2P_2+1)}$

3.2.2 Another Set of Conditions by Giving Different S_1

Theorem 2 *For a Gaussian X channel with* $b^2 \leq 1$, $\eta^2 \leq 1$ *and* $\eta\rho = \frac{P_1+1}{a}$ *the sum-rate capacity of X channel will be within the the gap* $\frac{1}{2}\log_2\left(\frac{1-\frac{\rho^2}{P_1+1}}{1-\rho^2}\right)$ *bits to the sum rate achieved by using MAC at receiver 1.*

Proof: Consider the side information $S_1 = X_2 + \eta W$, where $W \sim \mathcal{N}(0,1)$ is correlated with Z_1 and ρ is the correlation coefficient. Let us expand the term $H(W_{11}, W_{12}, W_{21}, W_{22}|Y_1^n, S_1^n)$ little differently from (3.5) and bound it using Fano's inequality by deriving the required conditions.

$$\begin{aligned} H(W_{11}, W_{12}, W_{21}, W_{22}|Y_1^n, S_1^n) &= H(W_{11}|Y_1^n, S_1^n) + H(W_{12}|Y_1^n, S_1^n, W_{11}) \\ &\quad + H(W_{22}|Y_1^n, S_1^n, W_{11}, W_{12}) \\ &\quad + H(W_{21}|Y_1^n, S_1^n, W_{11}, W_{12}, W_{22}) \qquad (3.30) \\ &\overset{(a)}{\leq} H(W_{11}|Y_1^n, S_1^n) + H(W_{12}|Y_1^n, S_1^n) + H(W_{22}|S_1^n) \\ &\quad + H(W_{21}|Y_1^n, W_{12}, W_{22}) \qquad (3.31) \end{aligned}$$

Step (a) follows from the fact that conditioning reduces the entropy. The first two terms in (3.31) can be bounded by $n(\epsilon_1 + \epsilon_2)$. The next two terms can be bounded if certain conditions are met. Let us consider the fourth term, as we know $H(W_{21}|Y_2^n) \leq n\epsilon_4$ at receiver 2.

$$\begin{aligned} H(W_{21}|Y_2^n) \leq n\epsilon_4 &\Rightarrow H(W_{21}|Y_2^n, W_{12}, W_{22}) \leq n\epsilon_4 \qquad (3.32) \\ &\Rightarrow H(W_{21}|bX_1^n + X_2^n + Z_2^n, W_{12}, W_{22}) \leq n\epsilon_4 \qquad (3.33) \\ &\Rightarrow H(W_{21}|X_1^n + \frac{Z_2^n}{b}, W_{12}, W_{22}) \leq n\epsilon_4 \qquad (3.34) \end{aligned}$$

$$\begin{aligned} H(W_{21}|Y_1^n, W_{12}, W_{22}) &= H(W_{21}|X_1^n + aX_2^n + Z_2^n, W_{12}, W_{22}) \qquad (3.35) \\ &= H(W_{21}|X_1^n + Z_2^n, W_{12}, W_{22}) \qquad (3.36) \end{aligned}$$

If $b^2 \leq 1$, $X_1^n + Z_2^n$ is less noisy version than $X_1^n + \frac{Z_2^n}{b}$, then (3.36) is bounded by $n\epsilon_4$. Now consider the third term, as we know $H(W_{22}|Y_2^n) \leq n\epsilon_3$ at receiver 2.

$$\begin{aligned} H(W_{22}|Y_2^n) \leq n\epsilon_3 &\Rightarrow H(W_{22}|Y_2^n, W_{11}, W_{21}) \leq n\epsilon_3 \qquad (3.37) \\ &\Rightarrow H(W_{22}|bX_1^n + X_2^n + Z_2^n, W_{11}, W_{21}) \leq n\epsilon_3 \qquad (3.38) \\ &\Rightarrow H(W_{22}|X_2^n + Z_2^n) \leq n\epsilon_3 \qquad (3.39) \end{aligned}$$

$$H(W_{22}|S_1^n) = H(W_{22}|X_2^n + \eta W^n) \qquad (3.40)$$

If $\eta^2 \leq 1$, $X_2^n + \eta W^n$ is less noisy version than $X_2^n + Z_2^n$, then (3.40) is bounded by $n\epsilon_3$. As a result $H(W_{11}, W_{12}, W_{21}, W_{22}|Y_1^n, S_1^n) \leq n(\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4)$.

$$
\begin{aligned}
n(R_{11} + R_{12} + R_{21} + R_{22} - \epsilon) &\leq I(W_{11}, W_{12}, W_{21}, W_{22}; Y_1^n, S_1^n) && (3.41)\\
&= h(Y_1^n, S_1^n) - h(Y_1^n, S_1^n|W_{11}, W_{12}, W_{21}, W_{22})\\
&\overset{(b)}{\leq} nh(Y_{1G}, S_{1G}) - nh(Z_1, W) && (3.42)\\
&= nI(X_{1G}, X_{2G}; Y_{1G}, S_{1G}) && (3.43)\\
&= nI(X_{1G}, X_{2G}; Y_{1G}) + nI(X_{1G}, X_{2G}; S_{1G}|Y_{1G}) && (3.44)
\end{aligned}
$$

Step (b) is due to Gaussian distribution maximizes the entropy.
Consider $I(X_{1G}, X_{2G}; S_{1G}|Y_{1G}) = I(X_{2G}; S_{1G}|Y_{1G}) + I(X_{1G}; S_{1G}|Y_{1G}, X_{2G})$, where first term on R.H.S can be made zero iff $\eta\rho = \frac{P_1+1}{a}$.

$$
\begin{aligned}
I(X_{2G}; X_{2G} + \eta W|X_{1G} + aX_{2G} + Z_1) = 0 && (3.45)\\
\Leftrightarrow E\left[\frac{\eta W(X_{1G} + Z_1)}{a}\right] = E\left[\left(\frac{X_{1G} + Z_1}{a}\right)^2\right] && (3.46)
\end{aligned}
$$

The second term on R.H.S $I(X_{2G}; S_{1G}|Y_{1G}, X_{1G})$ corresponds to the gap between the sum capacity and sum rate achieved by MAC at receiver 1.

$$
\begin{aligned}
I(X_{1G}; S_{1G}|Y_{1G}, X_{2G}) &= I(X_{1G}; X_{2G} + \eta W|X_{1G} + aX_{2G} + Z_1, X_{2G}) && (3.47)\\
&= h(X_{2G} + \eta W|X_{1G} + aX_{2G} + Z_1, X_{2G}) && (3.48)\\
&\quad - h(X_{2G} + \eta W|X_{1G} + aX_{2G} + Z_1, X_{1G}, X_{2G}) && (3.49)\\
&= h(\eta W|X_{1G} + Z_1) - h(\eta W|Z_1) && (3.50)\\
&= \frac{1}{2}\log_2\left(\frac{1 - \frac{\rho^2}{P_1+1}}{1-\rho^2}\right) \text{ bits} && (3.51)
\end{aligned}
$$

The sum capacity of genie-aided X channel, which is the outer bound on the sum capacity of original X channel, is given by (3.52). $I(X_{1G}, X_{2G}; Y_{1G})$ is the achievable sum rate of original X channel using MAC strategy at receiver 1 and $\frac{1}{2}\log_2\left(\frac{1-\frac{\rho^2}{P_1+1}}{1-\rho^2}\right)$ is the atmost gap between the sum capacity and sum rate achieved by MAC at receiver 1.

$$C_{sum}^{GA-X} = I(X_{1G}, X_{2G}; Y_{1G}) + \frac{1}{2}\log_2\left(\frac{1-\frac{\rho^2}{P_1+1}}{1-\rho^2}\right) \tag{3.52}$$

■

3.2.3 Region Corresponding to n bits Gap from MAC at Receiver 1 by Theorem 2

ρ correspondindg to the n bits gap is given as follows

$$\frac{1}{2}\log_2\left(\frac{1-\frac{\rho^2}{P_1+1}}{1-\rho^2}\right) \leq n \tag{3.53}$$

$$\rho^2 \leq \frac{(2^{2n}-1)}{(2^{2n}-\frac{1}{P_1+1})} \tag{3.54}$$

The region corresponding to the n bits gap is given by $b^2 \leq 1$, $a^2 \geq \frac{(2^{2n}(P_1+1)-1)(P_1+1)}{(2^{2n}-1)}$

3.3 On the Sum Rate Achieved by MAC at Receiver 2

Theorem 3 *For a Gaussian X channel with* $b^2 \geq 1$, $\eta^2 \leq 1$ *and* $\eta\rho = a(b^2P_1+1)$ *the sum capacity of X channel will be within the the gap* $\frac{1}{2}\log_2\left(\frac{1-\frac{\rho^2}{b^2P_1+1}}{1-\rho^2}\right)$ *bits to the sum rate achieved by using MAC at receiver 2.*

Proof: The side information $S_2 = aX_2 + \eta W$ is given to receiver 2 instead of S_1 to receiver 1, where $W \sim \mathcal{N}(0,1)$ is correlated with Z_2 and ρ is the correlation coefficient. By following the same approach of section 3.2 but at receiver 2 will prove the theorem. ■

3.3.1 Region Corresponding to n bits Gap from MAC at Receiver 2 by Theorem 3

ρ correspondindg to the n bits gap is given as follows

$$\frac{1}{2}\log_2\left(\frac{1-\frac{\rho^2}{b^2P_1+1}}{1-\rho^2}\right) \leq n \tag{3.55}$$

$$\rho^2 \leq \frac{(2^{2n}-1)}{(2^{2n}-\frac{1}{b^2P_1+1})} \tag{3.56}$$

The region corresponding to the n bits gap is given by $b^2 \geq 1$, $a^2 \leq \frac{(2^{2n}-1)}{(2^{2n}(b^2P_1+1)-1)(b^2P_1+1)}$

Theorem 4 *For a Gaussian X channel with $a^2 \leq 1$, $\eta^2 \leq 1$ and $\eta\rho = \frac{P_2+1}{b}$ the sum capacity of X channel will be within the the gap $\frac{1}{2}\log_2\left(\frac{1-\frac{\rho^2}{P_2+1}}{1-\rho^2}\right)$ bits to the sum rate achieved by using MAC at receiver 2.*

Proof: The side information $S_2 = X_1 + \eta W$ is given to receiver 2 instead of S_1 to receiver 1, where $W \sim \mathcal{N}(0,1)$ is correlated with Z_2 and ρ is the correlation coefficient. By following the same approach of section 3.2.2 but at receiver 2 will prove the theorem. ■

3.3.2 Region Corresponding to n bits Gap from MAC at Receiver 2 by Theorem 4

ρ correspondindg to the n bits gap is given as follows

$$\frac{1}{2}\log_2\left(\frac{1-\frac{\rho^2}{P_2+1}}{1-\rho^2}\right) \leq n \tag{3.57}$$

$$\rho^2 \leq \frac{(2^{2n}-1)}{(2^{2n}-\frac{1}{P_2+1})} \tag{3.58}$$

The region corresponding to the n bits gap is given by $a^2 \leq 1$, $b^2 \geq \frac{(2^{2n}(P_2+1)-1)(P_2+1)}{(2^{2n}-1)}$

Remark 3 *Both theorem 3 and 4 can be proved by applying theorem 1 and 2 respectively to the standardized dual channel. Refer to chapter 4 for the definition of dual channel.*

CHAPTER 4

Noisy Interference Regime: Sum Capacity of MIMO "X" Channel

In this chapter we will derive sufficient conditions for sum-rate optimality of treating interference as noise for MIMO X channel.

4.1 System Model

We consider the two user MIMO Gaussian XC in Fig. 4.1, which is same as the MIMO Gaussian IC except that each transmitter has separate independent messages for both receivers. The MIMO Gaussian XC is described by the following equations:

$$\boldsymbol{y}_1 = \mathbf{H}_{11}\boldsymbol{x}_1 + \mathbf{H}_{12}\boldsymbol{x}_2 + \boldsymbol{z}_1, \tag{4.1}$$

$$\boldsymbol{y}_2 = \mathbf{H}_{21}\boldsymbol{x}_1 + \mathbf{H}_{22}\boldsymbol{x}_2 + \boldsymbol{z}_2, \tag{4.2}$$

where $\boldsymbol{x}_i$ is a $t_i \times 1$ vector, $\boldsymbol{y}_i$, $\boldsymbol{z}_i$ are $r_i \times 1$ vectors, $\mathbf{H}_{ij}$ is $r_i \times t_j$ channel matrix and t_i, r_j are the number of antennas at the transmitter i, receiver j respectively. Noise vector $\boldsymbol{z}_i \sim \mathcal{N}\left(\underline{0}, I_{r_i \times r_i}\right)$ and is i.i.d. across time. The average power constraint on the i^{th} transmitter over an n symbol duration is

$$\frac{1}{n}\sum_{k=1}^{n} E\left[\boldsymbol{x}_{ik}\boldsymbol{x}_{ik}^T\right] \in \mathcal{Q}_i, \tag{4.3}$$

where

$$\mathcal{Q}_i = \left\{\mathbf{Q}_i : \mathbf{Q}_i \succeq 0, \mathrm{tr}(\mathbf{Q}_i) \leq P_i\right\}. \tag{4.4}$$

Rate R_{ij} is the rate of reliable transmission from transmitter j to receiver i. An achievable rate over the MIMO Gaussian XC is charaterized by $\left(R_{11}, R_{21}, R_{12}, R_{22}\right)$.

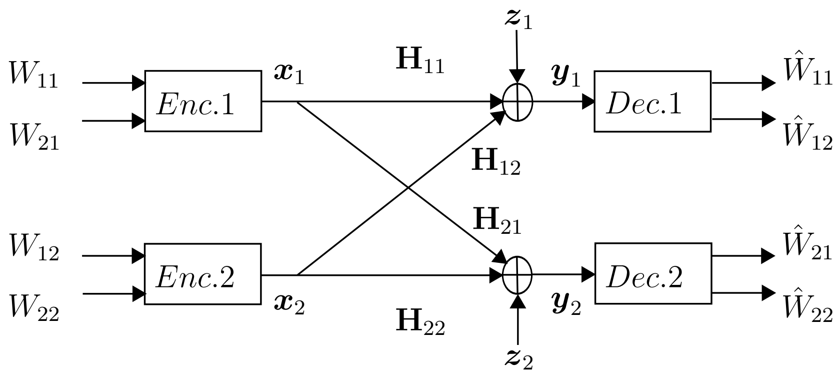

Figure 4.1: Two user MIMO Gaussian X channel

The capacity region is defined as the closure of all achievable rate tuples, and the sum capacity is the maximum achievable sum rate $R_{11} + R_{21} + R_{12} + R_{22}$.

4.2 Sum Capacity in the Noisy Interference Regime

The MIMO Gaussian X channel is defined by the parameters $\{\mathbf{H}_{11}, \mathbf{H}_{21}, \mathbf{H}_{12}, \mathbf{H}_{22}\}$ and the power constraints $\{P_1, P_2\}$. Let us denote the channel in Fig. 4.1 by GXC($\mathbf{H}_{11}, \mathbf{H}_{21}, \mathbf{H}_{12}, \mathbf{H}_{22}, P_1, P_2$).

Definition 3 (Dual Channel) *GXC($\mathbf{H}_{21}, \mathbf{H}_{11}, \mathbf{H}_{22}, \mathbf{H}_{12}, P_1, P_2$) is the dual channel of GXC($\mathbf{H}_{11}, \mathbf{H}_{21}, \mathbf{H}_{12}, \mathbf{H}_{22}, P_1, P_2$) and is shown in Fig. 4.2. It is obtained by* ***interchanging the receivers*** *and is described by the following equations:*

$$\boldsymbol{y}_1^{'} = \mathbf{H}_{21}\boldsymbol{x}_1^{'} + \mathbf{H}_{22}\boldsymbol{x}_2^{'} + \boldsymbol{z}_1^{'}$$
$$\boldsymbol{y}_2^{'} = \mathbf{H}_{11}\boldsymbol{x}_1^{'} + \mathbf{H}_{12}\boldsymbol{x}_2^{'} + \boldsymbol{z}_2^{'}.$$

Messages $W_{11}^{'}, W_{21}^{'}, W_{12}^{'}, W_{22}^{'}$ in the dual channel are the same as messages $W_{21}, W_{11}, W_{22}, W_{12}$ respectively, for the original channel.

Remark 4 *$(R_{11}, R_{21}, R_{12}, R_{22})$ is achievable in the channel GXC($\mathbf{H}_{11}, \mathbf{H}_{21}, \mathbf{H}_{12}, \mathbf{H}_{22}, P_1, P_2$) if and only if $(R_{11}^{'}, R_{21}^{'}, R_{12}^{'}, R_{22}^{'}) = (R_{21}, R_{11}, R_{22}, R_{12})$ is achievable in the channel GXC($\mathbf{H}_{21}, \mathbf{H}_{11}, \mathbf{H}_{22}, \mathbf{H}_{12}, P_1, P_2$).*

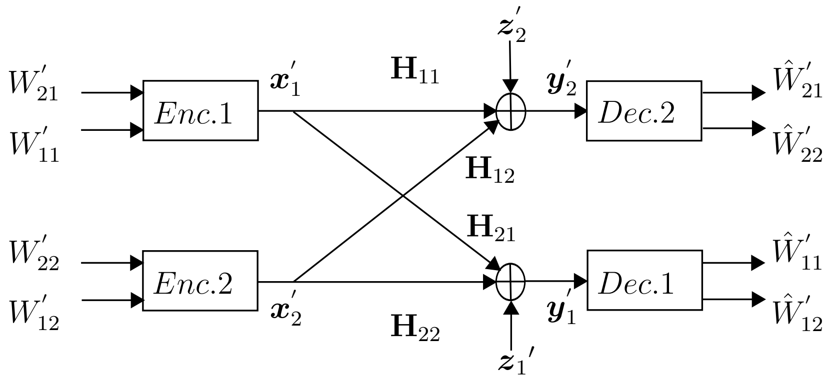

Figure 4.2: GXC $(\mathbf{H}_{21}, \mathbf{H}_{11}, \mathbf{H}_{22}, \mathbf{H}_{12}, P_1, P_2)$

Remark 5 *GXC(*$\mathbf{H}_{11}, \mathbf{H}_{21}, \mathbf{H}_{12}, \mathbf{H}_{22}, P_1, P_2$*) and the dual channel GXC(*$\mathbf{H}_{21}, \mathbf{H}_{11}, \mathbf{H}_{22}, \mathbf{H}_{12}, P_1, P_2$*) have the same sum capacity.*

4.2.1 Treating Interference as Noise (TIN) scheme

In the Treating Interference as Noise (TIN) scheme for GXC($\mathbf{H}_{11}, \mathbf{H}_{21}, \mathbf{H}_{12}, \mathbf{H}_{22}, P_1, P_2$), only the two direct messages W_{11}, W_{22} are sent using Gaussian codebooks and the interference is treated as noise at both receivers. Let R^X_{TIN} be the achievable sum rate by treating interference as noise for given $\mathbf{Q}_1, \mathbf{Q}_2$ and

$$(\mathbf{Q}_1^*, \mathbf{Q}_2^*) = \underset{\mathbf{Q}_i \in \mathcal{Q}_i, i \in \{1,2\}}{\arg\max} \; R^X_{TIN}(\mathbf{Q}_1, \mathbf{Q}_2). \tag{4.5}$$

Another TIN scheme would be to send only the two cross messages W_{12}, W_{21} using Gaussian codebooks and treat interference as noise at the receivers. This is the same as using the TIN scheme with direct messages for the dual channel GXC($\mathbf{H}_{21}, \mathbf{H}_{11}, \mathbf{H}_{22}, \mathbf{H}_{12}, P_1, P_2$).

4.2.2 Noisy interference regime

First, we determine the noisy interference regime where the TIN scheme with direct messages is sum capacity optimal. The noisy interference regime where the TIN scheme with cross mesages is sum capacity optimal follows directly by applying the result for direct messages to the dual channel.

Theorem 5 *If there exist matrices* $\mathbf{A}_1$*,* $\mathbf{A}_2$*,* $\mathbf{\Sigma}_1 \succ 0$*,* $\mathbf{\Sigma}_2 \succ 0$*, full rank matrices* $(\mathbf{Q}_1^*, \mathbf{Q}_2^*)$ *that solve problem (4.5), and they satisfy the following conditions:*

$$\mathbf{\Sigma}_1 \preceq I - \mathbf{A}_2\mathbf{\Sigma}_2^{-1}\mathbf{A}_2^T \tag{4.6}$$

$$\mathbf{\Sigma}_2 \preceq I - \mathbf{A}_1\mathbf{\Sigma}_1^{-1}\mathbf{A}_1^T \tag{4.7}$$

$$(\mathbf{A}_1^T(\mathbf{H}_{12}\mathbf{Q}_2^*\mathbf{H}_{12}^T)^{-1}\mathbf{H}_{11} - \mathbf{H}_{21})\mathbf{Q}_1^* = 0 \tag{4.8}$$

$$(\mathbf{A}_2^T(\mathbf{H}_{21}\mathbf{Q}_1^*\mathbf{H}_{21}^T)^{-1}\mathbf{H}_{22} - \mathbf{H}_{12})\mathbf{Q}_2^* = 0, \tag{4.9}$$

the sum capacity of the MIMO Gaussian X channel GXC($\mathbf{H}_{11}, \mathbf{H}_{21}, \mathbf{H}_{12}, \mathbf{H}_{22}, P_1, P_2$*) is achieved by the TIN scheme with direct messages only, and is given by:*

$$C_{sum}^X = R_{TIN}^X(\mathbf{Q}_1^*, \mathbf{Q}_2^*) \tag{4.10}$$

$$= \max_{\mathbf{Q}_i \in \mathcal{Q}_i, i \in \{1,2\}} \frac{1}{2}\log\left|I + \mathbf{H}_{11}\mathbf{Q}_1\mathbf{H}_{11}^T\left(I + \mathbf{H}_{12}\mathbf{Q}_2\mathbf{H}_{12}^T\right)^{-1}\right|$$
$$+ \frac{1}{2}\log\left|I + \mathbf{H}_{22}\mathbf{Q}_2\mathbf{H}_{22}^T\left(I + \mathbf{H}_{21}\mathbf{Q}_1\mathbf{H}_{21}^T\right)^{-1}\right|. \tag{4.11}$$

Proof: Consider the genie-aided channel in Fig. 4.3 with a genie providing side information $\boldsymbol{s}_1, W_{21}$ to receiver 1 and $\boldsymbol{s}_2, W_{12}$ to receiver 2, where

$$\boldsymbol{s}_1 = \mathbf{H}_{21}\boldsymbol{x}_1 + \boldsymbol{w}_1$$
$$\boldsymbol{s}_2 = \mathbf{H}_{12}\boldsymbol{x}_2 + \boldsymbol{w}_2$$

and $\begin{bmatrix} \boldsymbol{z}_i \\ \boldsymbol{w}_i \end{bmatrix} \sim \mathcal{N}\left(\underline{0}, \begin{bmatrix} I & \mathbf{A}_i \\ \mathbf{A}_i^T & \mathbf{\Sigma}_i \end{bmatrix}\right)$.

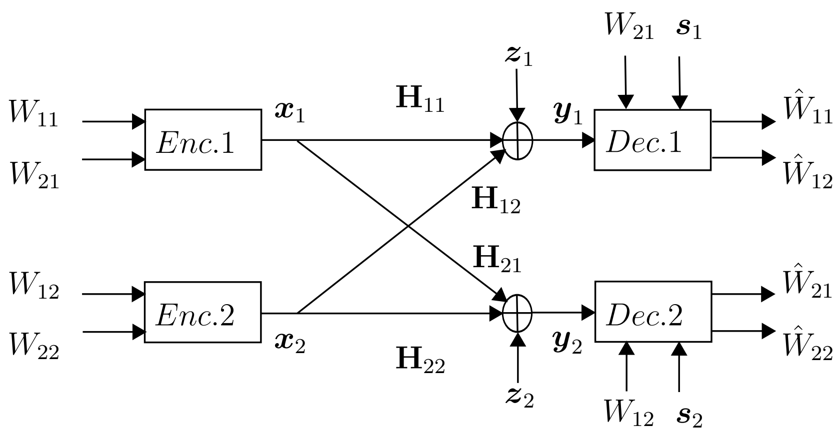

Figure 4.3: Genie-aided MIMO Gaussian X channel

Here, $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ are chosen as for the MIMO-IC in [21], and W_{21} and W_{12} are provided to receivers 1 and 2 respectively as in [3] for the SISO XC. Using Fano's inequality, we have

$$
\begin{aligned}
& n(R_{11} + R_{12} - \epsilon) \\
& \leq I(W_{11}, W_{12}; \boldsymbol{y}_1^n, \boldsymbol{s}_1^n, W_{21}) && (4.12) \\
& = I(W_{11}, W_{12}; \boldsymbol{y}_1^n, \boldsymbol{s}_1^n | W_{21}) + I(W_{11}, W_{12}; W_{21}) && (4.13) \\
& \overset{(a)}{=} I(W_{11}, W_{12}; \boldsymbol{s}_1^n | W_{21}) + I(W_{11}, W_{12}; \boldsymbol{y}_1^n | \boldsymbol{s}_1^n, W_{21}) && (4.14) \\
& = h(\boldsymbol{s}_1^n | W_{21}) - h(\boldsymbol{s}_1^n | W_{11}, W_{12}, W_{21}) \\
& \quad + h(\boldsymbol{y}_1^n | \boldsymbol{s}_1^n, W_{21}) - h(\boldsymbol{y}_1^n | \boldsymbol{s}_1^n, W_{11}, W_{12}, W_{21}) && (4.15) \\
& = h(\boldsymbol{s}_1^n | W_{21}) - h(\mathbf{H}_{21} \boldsymbol{x}_1^n + \boldsymbol{w}_1^n | W_{11}, W_{12}, W_{21}) \\
& \quad + h(\boldsymbol{y}_1^n | \boldsymbol{s}_1^n, W_{21}) \\
& \quad - h(\mathbf{H}_{11} \boldsymbol{x}_1^n + \mathbf{H}_{12} \boldsymbol{x}_2^n + \boldsymbol{z}_1^n | \boldsymbol{s}_1^n, W_{11}, W_{12}, W_{21}) && (4.16) \\
& \overset{(b)}{=} h(\boldsymbol{s}_1^n | W_{21}) - h(\boldsymbol{w}_1^n) + h(\boldsymbol{y}_1^n | \boldsymbol{s}_1^n, W_{21}) \\
& \quad - h(\mathbf{H}_{12} \boldsymbol{x}_2^n + \boldsymbol{z}_1^n | \boldsymbol{w}_1^n, W_{12}) && (4.17) \\
& \overset{(c)}{\leq} h(\boldsymbol{s}_1^n | W_{21}) - h(\boldsymbol{w}_1^n) + h(\boldsymbol{y}_1^n | \boldsymbol{s}_1^n) \\
& \quad - h(\mathbf{H}_{12} \boldsymbol{x}_2^n + \boldsymbol{z}_1^n | \boldsymbol{w}_1^n, W_{12}) && (4.18) \\
& = h(\boldsymbol{s}_1^n | W_{21}) - nh(\boldsymbol{w}_1) + h(\boldsymbol{y}_1^n | \boldsymbol{s}_1^n) \\
& \quad - h(\mathbf{H}_{12} \boldsymbol{x}_2^n + \boldsymbol{z}_1^n | \boldsymbol{w}_1^n, W_{12}) && (4.19) \\
& \overset{(d)}{\leq} h(\boldsymbol{s}_1^n | W_{21}) - nh(\boldsymbol{w}_1) + nh(\boldsymbol{y}_{1G} | \boldsymbol{s}_{1G}) \\
& \quad - h(\mathbf{H}_{12} \boldsymbol{x}_2^n + \boldsymbol{z}_1^n | \boldsymbol{w}_1^n, W_{12}) && (4.20)
\end{aligned}
$$

where step (a) follows from the independence of the messages, step (b) follows from the deterministic encoding of W_{11}, W_{21} to $\boldsymbol{x}_1^n$, step (c) follows from the fact that conditioning reduces the entropy, and step (d) is because the Gaussian distribution maximizes the conditional entropy [21, Lemma 7].

Similarly, we have

$$
\begin{aligned}
n(R_{21} + R_{22} - \epsilon) \leq\ & h(\boldsymbol{s}_2^n | W_{12}) - nh(\boldsymbol{w}_2) + nh(\boldsymbol{y}_{2G} | \boldsymbol{s}_{2G}) \\
& - h(\mathbf{H}_{21} \boldsymbol{x}_1^n + \boldsymbol{z}_2^n | \boldsymbol{w}_2^n, W_{21}). && (4.21)
\end{aligned}
$$

Adding (4.20) and (4.21), we have

$$\begin{aligned} n(C_{sum}^{GA-X} - 2\epsilon) \leq & [h(\boldsymbol{s}_1^n|W_{21}) - h(\mathbf{H}_{21}\boldsymbol{x}_1^n + \boldsymbol{z}_2^n|\boldsymbol{w}_2^n, W_{21})] \\ & + [h(\boldsymbol{s}_2^n|W_{12}) - h(\mathbf{H}_{12}\boldsymbol{x}_2^n + \boldsymbol{z}_1^n|\boldsymbol{w}_1^n, W_{12})] \\ & - nh(\boldsymbol{w}_1) + nh(\boldsymbol{y}_{1G}|\boldsymbol{s}_{1G}) \\ & - nh(\boldsymbol{w}_2) + nh(\boldsymbol{y}_{2G}|\boldsymbol{s}_{2G}), \end{aligned} \tag{4.22}$$

where C_{sum}^{GA-X} is the sum capacity of the genie-aided MIMO XC. Now, consider the first term in (4.22).

$$\begin{aligned} & h(\boldsymbol{s}_1^n|W_{21}) - h(\mathbf{H}_{21}\boldsymbol{x}_1^n + \boldsymbol{z}_2^n|\boldsymbol{w}_2^n, W_{21}) \\ & = h(\mathbf{H}_{21}\boldsymbol{x}_1^n + \boldsymbol{w}_1^n|W_{21}) - h(\mathbf{H}_{21}\boldsymbol{x}_1^n + \boldsymbol{z}_2^n|\boldsymbol{w}_2^n, W_{21}) \\ & \stackrel{(e)}{=} h(\mathbf{H}_{21}\boldsymbol{x}_1^n + \boldsymbol{w}_1^n|W_{21}) - h(\mathbf{H}_{21}\boldsymbol{x}_1^n + \boldsymbol{v}_1^n|W_{21}) \\ & \stackrel{(f)}{=} h(\mathbf{H}_{21}\boldsymbol{x}_1^n + \boldsymbol{w}_1^n|W_{21}) - h(\mathbf{H}_{21}\boldsymbol{x}_1^n + \boldsymbol{w}_1^n + \tilde{\boldsymbol{v}}_1^n|W_{21}) \\ & = -I(\tilde{\boldsymbol{v}}_1^n; \mathbf{H}_{21}\boldsymbol{x}_1^n + \boldsymbol{w}_1^n + \tilde{\boldsymbol{v}}_1^n|W_{21}) \\ & \leq -I(\tilde{\boldsymbol{v}}_1^n; \mathbf{H}_{21}\boldsymbol{x}_1^n + \boldsymbol{w}_1^n + \tilde{\boldsymbol{v}}_1^n) \\ & \stackrel{(g)}{\leq} -nI(\tilde{\boldsymbol{v}}_1; \mathbf{H}_{21}\boldsymbol{x}_{1G} + \boldsymbol{w}_1 + \tilde{\boldsymbol{v}}_1) \\ & = nh(\boldsymbol{s}_{1G}) - nh(\mathbf{H}_{21}\boldsymbol{x}_{1G} + \boldsymbol{z}_2|\boldsymbol{w}_2) \end{aligned}$$

where $\boldsymbol{v}_1^n$ is the MMSE error in estimating $\boldsymbol{z}_2^n$ given $\boldsymbol{w}_2^n$ and $\boldsymbol{v}_1^n \sim \mathcal{N}\left(\underline{0}, I - \mathbf{A}_2\mathbf{\Sigma}_2^{-1}\mathbf{A}_2^T\right)$, and step (e) follows from [21, Lemma 9]. In step (f), $\tilde{\boldsymbol{v}}_1^n \sim \mathcal{N}\left(\underline{0}, I - \mathbf{A}_2\mathbf{\Sigma}_2^{-1}\mathbf{A}_2^T - \mathbf{\Sigma}_1\right)$ and is independent of $\boldsymbol{w}_1^n$. Since the covariance matrix has to be positive semidefinite, we get the condition (4.6). Step (f) follows since $\boldsymbol{w}_1^n + \tilde{\boldsymbol{v}}_1^n$ has the same marginal as $\boldsymbol{v}_1^n$. Step (g) follows from the worst case noise result [21, Lemma 8]. Similarly, the second term in (4.22) can also be simplified and we get condition (4.7) in the process. Overall, we get

$$\begin{aligned} n(C_{sum}^{GA-X} - 2\epsilon) \leq & [nh(\boldsymbol{s}_{1G}) - nh(\mathbf{H}_{21}\boldsymbol{x}_{1G} + \boldsymbol{z}_2|\boldsymbol{w}_2)] \\ & + [nh(\boldsymbol{s}_{2G}) - nh(\mathbf{H}_{12}\boldsymbol{x}_{2G} + \boldsymbol{z}_1|\boldsymbol{w}_1)] \\ & - nh(\boldsymbol{w}_1) + nh(\boldsymbol{y}_{1G}|\boldsymbol{s}_{1G}) \\ & - nh(\boldsymbol{w}_2) + nh(\boldsymbol{y}_{2G}|\boldsymbol{s}_{2G}) \end{aligned}$$

$$= \sum_{i=1}^{2} nI(\boldsymbol{x}_{iG}; \boldsymbol{y}_{iG}, \boldsymbol{s}_{iG}) \triangleq nR_{TIN}^{GA-X}(\mathbf{Q}_1, \mathbf{Q}_2) \tag{4.23}$$

Therefore, we have

$$\begin{aligned} C_{sum}^{GA-X} &\leq R_{TIN}^{GA-X}(\mathbf{Q}_1, \mathbf{Q}_2) \quad \text{for some } \mathbf{Q}_i \in \mathcal{Q}_i \\ &\leq \max_{\mathbf{Q}_i \in \mathcal{Q}_i, i \in \{1,2\}} R_{TIN}^{GA-X}(\mathbf{Q}_1, \mathbf{Q}_2), \end{aligned}$$

resulting in

$$C_{sum}^{GA-X} = \max_{\mathbf{Q}_i \in \mathcal{Q}_i, i \in \{1,2\}} R_{TIN}^{GA-X}(\mathbf{Q}_1, \mathbf{Q}_2). \tag{4.24}$$

At this point, for a given $\mathbf{Q}_1$, $\mathbf{Q}_2$ the genie-aided outer bound $R_{TIN}^{GA-X}(\mathbf{Q}_1, \mathbf{Q}_2)$ is the same as the genie-aided outer bound in [21] for the MIMO-IC. Therefore, the remaining steps that involve proving that the genie does not increase sum capacity, i.e., proving the smart genie conditions (4.8), (4.9) and $R_{TIN}^{GA-X}(\mathbf{Q}_1^*, \mathbf{Q}_2^*) = R_{TIN}^{X}(\mathbf{Q}_1^*, \mathbf{Q}_2^*)$, where $\mathbf{Q}_1^*, \mathbf{Q}_2^*$ are full rank matrices, are similar to [21, Theorem 1] and are not repeated here.

■

Theorem 6 *If there exist matrices $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{\Sigma}_1 \succ 0$, $\mathbf{\Sigma}_2 \succ 0$, full rank matrices ($\mathbf{Q}_1^*, \mathbf{Q}_2^*$) which solve problem (4.5), and they satisfy the following conditions:*

$$\mathbf{\Sigma}_1 \preceq I - \mathbf{A}_2 \mathbf{\Sigma}_2^{-1} \mathbf{A}_2^T \tag{4.25}$$

$$\mathbf{\Sigma}_2 \preceq I - \mathbf{A}_1 \mathbf{\Sigma}_1^{-1} \mathbf{A}_1^T \tag{4.26}$$

$$(\mathbf{A}_1^T (\mathbf{H}_{22} \mathbf{Q}_2^* \mathbf{H}_{22}^T)^{-1} \mathbf{H}_{21} - \mathbf{H}_{11}) \mathbf{Q}_1^* = 0 \tag{4.27}$$

$$(\mathbf{A}_2^T (\mathbf{H}_{11} \mathbf{Q}_1^* \mathbf{H}_{11}^T)^{-1} \mathbf{H}_{12} - \mathbf{H}_{22}) \mathbf{Q}_2^* = 0, \tag{4.28}$$

the sum capacity of GXC($\mathbf{H}_{11}, \mathbf{H}_{21}, \mathbf{H}_{12}, \mathbf{H}_{22}, P_1, P_2$) is achieved by using the TIN scheme with cross messages only, and is given by

$$C^X_{sum} = R^X_{TIN}(\mathbf{Q}_1^*, \mathbf{Q}_2^*) \tag{4.29}$$

$$= \max_{\mathbf{Q}_i \in \mathcal{Q}_i, i\in\{1,2\}} \frac{1}{2}\log\left| I + \mathbf{H}_{21}\mathbf{Q}_1\mathbf{H}_{21}^T \left(I + \mathbf{H}_{22}\mathbf{Q}_2\mathbf{H}_{22}^T\right)^{-1}\right|$$

$$+ \frac{1}{2}\log\left| I + \mathbf{H}_{12}\mathbf{Q}_2\mathbf{H}_{12}^T \left(I + \mathbf{H}_{11}\mathbf{Q}_1\mathbf{H}_{11}^T\right)^{-1}\right|. \tag{4.30}$$

Proof: Apply Theorem 1 to the dual channel GXC $(\mathbf{H}_{21}, \mathbf{H}_{11}, \mathbf{H}_{22}, \mathbf{H}_{12}, P_1, P_2)$ and use Remark 5. ■

4.3 Symmetric MISO and SIMO Gaussian XCs

Now, we state the results for the symmetric MISO and SIMO Gaussian XCs. The main difference between the MISO and MIMO cases is in proving $R^{GA-X}_{TIN}(\mathbf{Q}_1^*, \mathbf{Q}_2^*) = R^X_{TIN}(\mathbf{Q}_1^*, \mathbf{Q}_2^*)$. However, given that the genie-aided outer bound $R^{GA-X}_{TIN}(\mathbf{Q}_1, \mathbf{Q}_2)$ in Theorem 1 is the same as the bound in [21], this part remains the same as in [21, Theorem 2] and [21, Theorem 3]. Therefore, the proofs are not included.

4.3.1 Symmetric MISO Gaussian XC

A symmetric MISO Gaussian XC can be simplified to the following standard form [21] [22, Section 4.1]:

$$y_1 = \boldsymbol{d}^T\boldsymbol{x}_1 + h\boldsymbol{c}^T\boldsymbol{x}_2 + z_1 \tag{4.30a}$$

$$y_2 = h\boldsymbol{c}^T\boldsymbol{x}_1 + \boldsymbol{d}^T\boldsymbol{x}_2 + z_2, \tag{4.30b}$$

where $\boldsymbol{d} = [\cos\theta \, \sin\theta]^T, \theta \in (0, \frac{\pi}{2}), \boldsymbol{c} = [1\,0]^T$. Note that $(\mathbf{Q}_1^*, \mathbf{Q}_2^*)$ are unit rank matrices [24] in the MISO case and not full rank as required in Theorems 5 and 6 for the MIMO case. Here, we need to find sufficient conditions on h for the TIN scheme to achieve sum capacity in the noisy interference regime. Following an approach similar to [21, Theorem 2], we get the following result. We will define $(x)_+ = \max\{0, x\}$

Theorem 7 *The sum capacity of the MISO Gaussian X channel GXC($\boldsymbol{d}^T, h\boldsymbol{c}^T, h\boldsymbol{c}^T, \boldsymbol{d}^T, P, P$) is given by*

$$C_{sum}^{X} = \begin{cases} \log\left(1 + \dfrac{h^2 P\cos^2\theta}{1+P} + h^2 P\sin^2\theta\right), & h \geq h_0(\theta, P) \\ \log\left(1 + \dfrac{P\cos^2\theta}{1+h^2 P} + P\sin^2\theta\right), & h \leq h_1(\theta, P) \end{cases} \tag{4.31}$$

where $h_0(\theta, P)$ and $h_1(\theta, P)$ are the positive solutions to the equations (4.32) and (4.33), respectively.

$$\left(\frac{1}{h_0^2} - \sin^2\theta\right) = \left(\frac{\cos\theta}{1+P} - \frac{1}{h_0}\right)_+^2 \tag{4.32}$$

$$\left(h_1^2 - \sin^2\theta\right) = \left(\frac{\cos\theta}{1+h^2P} - \frac{1}{h_1}\right)_+^2. \tag{4.33}$$

Equation (4.33) specifies the condition under which sum capacity is achieved using the TIN scheme with direct messages only. Equation (4.32), which is obtained using the dual channel, specifies the condition under which sum capacity is achieved using the TIN scheme with cross messages only.

4.3.2 Symmetric SIMO Gaussian XC

A symmetric SIMO Gaussian XC can be simplified to the following standard form:

$$\boldsymbol{y}_1 = \boldsymbol{d}x_1 + h\boldsymbol{c}x_2 + \boldsymbol{z}_1 \tag{4.33a}$$

$$\boldsymbol{y}_2 = h\boldsymbol{c}x_1 + \boldsymbol{d}x_2 + \boldsymbol{z}_2, \tag{4.33b}$$

where $\boldsymbol{d} = [\cos\theta \ \sin\theta]^T, \theta \in (0, \frac{\pi}{2}), \boldsymbol{c} = [1\,0]^T$. Following an approach similar to [21, Theorem 3], we get this result.

Theorem 8 *The sum capacity of the SIMO Gaussian X channel GXC($\boldsymbol{d}, h\boldsymbol{c}, h\boldsymbol{c}, \boldsymbol{d}$, P, P) is also given by equation (4.31) where $h_0(\theta, P)$, and $h_1(\theta, P)$ are the positive solutions to the equations (4.32) and (4.33), respectively. Under these conditions SIMO XC achieves the same sum capacity as MISO XC.*

CHAPTER 5

Simulation

We will analyze and graphically represent the results obtained in chapters 3, 4 here.

5.1 Mixed Interference Regime of Scalar GXC: 1 bit Gap from MAC at Receiver

Fig. 5.1 is the plot between crosslink channel coefficients a^2 and b^2 with power constraints $P1 = -3$ dB, $P2 = -3$ dB. This plot indicates four regions corresponding to theorems in chapter 3 . In these regions sum rate achieved by MAC strategy at corresponding receiver will be within $n = 1$ bit gap from the sum capacity of Gaussian X channel.

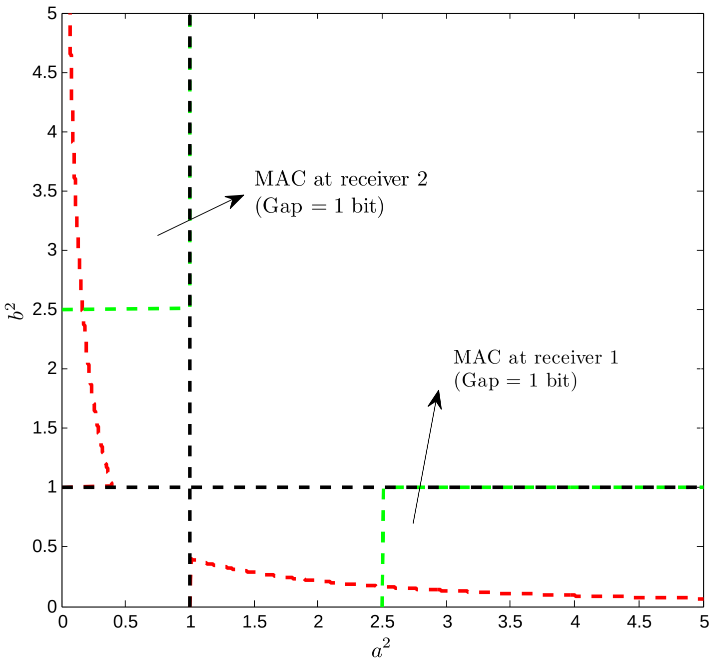

Figure 5.1: Regions indicating MAC at either receivers achieve within $n = 1$ bit to the sum capacity. $P1 = -3$ dB, $P2 = -3$ dB

If the power constraint P_1 or P_2 are very low, these regions cover most of the mixed interference regime i.e., $a^2 \leq 1, b^2 \geq 1$ or $a^2 \geq 1, b^2 \leq 1$.

5.2 Noisy Interference Regime of MIMO GXC

The noisy interference regime for the MISO and SIMO Gaussian XCs is illustrated in Figure 5.2. If h is greater than $h_0(\theta, P)$, or lesser than $h_1(\theta, P)$, the sum capacity is

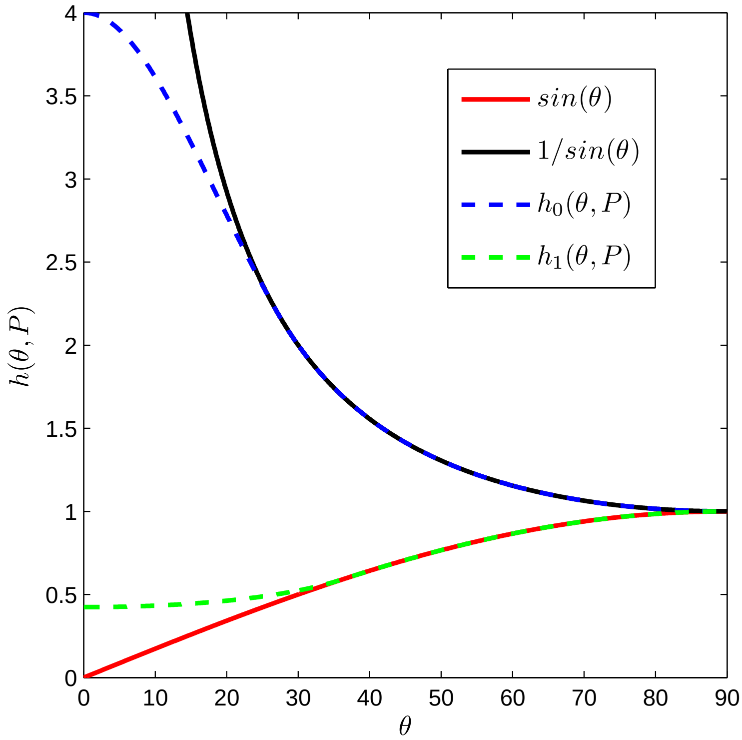

Figure 5.2: Lower and upper bounds on h for MISO and SIMO Gaussian symmetric X channel with $P = 0$ dB

achieved using the TIN scheme. This noisy interference regime comprises a larger set of channels when compared to the MIMO IC result, which has only the $h_1(\theta, P)$ bound. The bounds for SISO X channel, a special case of MISO X channel with $\theta = 0$, are:

$$|h(1 + h^2 P)| < 0.5, \left|\frac{(1 + P)}{h}\right| < 0.5 \tag{5.1}$$

and were derived in [3, Theorem 6.1]. As $P \to \infty$, $h_0(\theta, P)$ and $h_1(\theta, P)$ curves approach $1/\sin(\theta)$ and $\sin(\theta)$ respectively.

CHAPTER 6

Conclusion

Sum-rate capacity results of Gaussian X channel in mixed and noisy interference regime are derived. In mixed interference regime the sum-rate capacity of scalar Gaussian X channel is determined to within constant number of bits from sum rate achieved by MAC scheme. We used genie aided channel in deriving the outer bound which is in turn outer bound to the original GXC. By varying the genie we derived another outer bound. All these bounds together give set of conditions on the channel parameters for n bit gap from the MAC scheme. We also observed that as powers are sufficiently low, the set of conditions cover most of the mixed interference regime.

We also established the sum capacity of the two-user MIMO Gaussian X channel in the *noisy* interference regime. Once again the genie aided technique is used to derive the result. We also defined dual channel of a GXC which achieves same sum capacity and made use of it in proving the conditions. The sum capacity is achieved by using Gaussian codebooks for the messages on both the direct links (or both the cross links) and treating the interference from the cross links (or direct links) as noise. Moreover we derived the sum capacity for MISO and SIMO GXC's. We also showed the existing SISO GXC results as a special case.

We summarized known results on Gaussian interference channels and Z channels, as some techniques known for these channels are used in proving our results.

6.1 Scope for Future Work

We can further investigate the optimal sum rate (sum capacity) achieving scheme in the mixed interference regime. The question on the optimality of MAC scheme should also be resolved. Characterizing the capacity region of GXC is a daunting task as it was in the case of GIC. But efforts towards it may lead to interesting schemes, techniques like genie aided approach and also information theoretic inequalities. Neverthless the intuition developed for these networks can be applied to physical networks and also the concepts can be extended to the large multi-terminal (a general scenerio) networks.

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