Capacity bounds of Gaussian Relay Channels

A Project Report

submitted by

KUNNATH BALAKRISHNAN RESHMA

in partial fulfilment of the requirements
for the award of the degree of

MASTER OF TECHNOLOGY

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY MADRAS.

MAY 2014

THESIS CERTIFICATE

This is to certify that the thesis titled **Capacity bounds of Gaussian Relay Channels**, submitted by **Kunnath Balakrishnan Reshma**, to the Indian Institute of Technology, Madras, for the award of the degree of **Master of Technology**, is a bona fide record of the research work done by her under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Prof. Srikrishna Bhashyam
Project Guide
Associate Professor
Dept. of Electrical Engineering
IIT-Madras, 600 036

Place: Chennai

Date: 6th May 2014

ACKNOWLEDGEMENTS

First and foremost, I am deeply indebted to my advisor Dr. Srikrishna Bhashyam for his guidance and encouragement throughout the course of this project. I would like to thank him profusely for his patient guidance even at times when the project was not going in the right direction.

I would like to thank the Department of Electrical Engineering for providing an excellent laboratory and computational resources for making this project possible. I would like to acknowledge all the professors of the department whose courses instilled my interest in various topics.

I would also like to thank my labmates and friends for their support and guidance. Finally, since a mere word of thanks would be really insufficient to express my gratitude to my parents and other family members, I would like to dedicate my work to them.

ABSTRACT

We consider two 2-way relay channels -a three-node Gaussian relay network where two nodes (say a, b) want to communicate with each other and the third node acts as a relay and a Gaussian diamond channel where 2 nodes act as relays. Relaying protocols and possible rate pairs (R_a , R_b) for the two-way communication are investigated. We obtain an outer bound for the rate region of all achievable (R_a , R_b) in the full duplex mode and compare it with outer bound for the half duplex mode. This outer bound is based on the full duplex and the half-duplex cut-set bound respectively and is applicable to any protocol. We also extend the results to the MIMO case for all the channels studied. Numerical results and comparisons of the achievable rate regions of some protocols and the outer bound are also presented. We also propose and analyze the scale and forward protocol for two way communication between nodes in a diamond channel with no direct link.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS **i**

ABSTRACT **ii**

LIST OF FIGURES **viii**

ABBREVIATIONS **ix**

NOTATION **x**

1 INTRODUCTION **1**

1.1 Organization of Thesis 3

2 GAUSSIAN TWO-WAY RELAY CHANNEL **4**

2.1 Gaussian 2 way relay channel without direct link 4

2.1.1 System model 4

2.1.2 Outer bounds for Gaussian 2 way relay channel with no direct link-Full duplex 5

2.1.3 Protocols for Two-way Relay Channel without Direct Link-Full Duplex 6

2.1.4 Outer bounds for Gaussian 2 way relay channel with no direct link-Half duplex 7

2.1.5 Protocols for Two-way Relay Channel without Direct Link . 8

2.2 Gaussian 2 way relay channel with direct link between A and B . . 9

2.2.1 System Model 9

2.2.2 Outer bounds for Gaussian 2 way relay channel with direct link between A and B-Full duplex 9

2.2.3 Protocols for Two-way Relay Channel with Direct Link between A and B-Full Duplex 11

2.2.4 Outer bounds for Gaussian 2 way relay channel with direct link between A and B-Half duplex 11

2.2.5 Protocols for Two-way Relay Channel With Direct Link . . 12

3 GAUSSIAN DIAMOND RELAY CHANNEL 16
3.1 Gaussian diamond relay channel with no direct link 16
3.1.1 System Model 16
3.1.2 Outer bounds for Gaussian diamond relay channel with no direct link-Full duplex 18
3.1.3 Outer bounds for Gaussian diamond relay channel with no direct link-Half duplex 19
3.2 Gaussian diamond relay channel with direct link between A and B . 21
3.2.1 System Model 21
3.2.2 Outer bounds for Gaussian diamond relay channel with direct link between A and B-Full duplex 21
3.2.3 Outer bounds for Gaussian diamond relay channel with direct link between A and B-Half duplex 23
3.3 Gaussian diamond relay channel with direct link between Relays . . 24
3.3.1 System Model 24
3.3.2 Outer bounds for Gaussian diamond relay channel with direct link between R1 and R2-Full duplex 25
3.3.3 Outer bounds for Gaussian diamond relay channel with direct link between R1 and R2-Half duplex 27
4 SCALE AND FORWARD PROTOCOL 29
4.1 Theorem 29
4.1.1 Assumptions: 29
4.1.2 Proof 30
4.1.3 Remarks 30
4.2 Other Protocols for half duplex diamond relay channel 31
4.2.1 MDF Protocol 31
4.2.2 CF-CMAC Protocol 32
4.2.3 CF-BC Protocol 33
5 OUTER BOUNDS FOR CAPACITY OF GAUSSIAN TWO-WAY RELAY CHANNEL AND GAUSSIAN DIAMOND RELAY CHANNEL FOR MIMO CASE 35
5.1 MIMO point to point channel 35
5.2 Reciprocity 35
5.3 MIMO Multiple Access channel 36

5.4 MIMO Broadcast channel 36
5.5 Multiple Access Broadcast channel duality 37
5.6 2 way relay channel with no direct link 38
5.6.1 System Model 38
5.6.2 Outer bound-Full Duplex 38
5.6.3 Outer bound-Half Duplex 38
5.7 2 way relay channel with direct link 39
5.7.1 System Model 39
5.7.2 Outer bound for Gaussian relay channel with direct link between A and B-Full Duplex 39
5.7.3 Outer bounds for Gaussian relay channel with direct link between A and B-Half duplex 40
5.8 Gaussian diamond relay channel with no direct link 40
5.8.1 System Model 40
5.8.2 Outer bound for Gaussian diamond relay channel with no direct link -Full Duplex 41
5.8.3 Outer bound for Gaussian diamond relay channel with no direct link -Half Duplex 41
5.9 Gaussian diamond relay channel with direct A-B link 42
5.9.1 System Model 42
5.9.2 Outer bound for Gaussian diamond relay channel with direct link between A and B-Full Duplex 42
5.9.3 Outer bound for Gaussian diamond relay channel with direct link between A and B-Half Duplex 43
5.10 Gaussian diamond relay channel with direct link between relays . . 44
5.10.1 System Model 44
5.10.2 Outer bound for Gaussian diamond relay channel with direct link between relays-Full Duplex 44
5.10.3 Outer bound for Gaussian diamond relay channel with direct link between relays-Half Duplex 45

6 NUMERICAL RESULTS 46

6.1 Comparison of Rate Regions for 2 way relay channel with no direct link 46
6.2 Comparison of Rate Regions for 2 way relay channel with direct link 48
6.3 Comparison of outer bounds for 2 way relay with and without direct links 51

6.4 Comparison of outer bounds for diamond relay with and without direct links 53

6.5 Scale and forward Protocol 54

6.5.1 Comparison of Scale and forward Protocol with other protocols and the outer bound 54

6.5.2 Ratio of scaling coefficients Vs Power at the relays 56

6.6 Outer bounds in full duplex and half duplex for 2 way relay and diamond relay channel-MIMO case 57

6.6.1 2 way relay 57

6.6.2 Diamond relay 58

7 CONCLUSION **60**

7.1 Contribution of this thesis 60

7.2 Future Work 60

LIST OF FIGURES

2.1 Two-way Gaussian Relay Channel with no direct Link 5

2.2 Cuts used for finding the bound 6

2.3 Multiple Access State-1 7

2.4 Broadcast-State-2 7

2.5 Two-way Gaussian Relay Channel with Direct Link 9

2.6 Cuts used for finding the bound 10

2.7 Possible states in a Half-duplex Two-way Relay Channel 12

2.8 States used in TDBC protocol 13

2.9 States used in HBC protocol 14

3.1 Two-way Gaussian Diamond Relay Channel with no direct Link . . 17

3.2 Cuts used for finding the bound 17

3.3 Possible states in a Half-duplex Two-way Diamond Relay Channel . 20

3.4 Two-way Gaussian Diamond Relay Channel with Direct A-B Link . 21

3.5 Cuts used for finding the bound 22

3.6 Possible states in a Half-duplex Two-way Diamond Relay Channel with direct A-B link 23

3.7 Two-way Gaussian Relay Channel with Direct Link between Relays 25

3.8 Cuts used for finding the bound 26

3.9 Possible states in a Half-duplex Two-way Diamond Relay Channel with direct link between relays 27

6.1 Comparison of Rate Regions for 2 way relay channel with no direct link $\gamma_1 = 3$ dB, $\gamma_2 = 3$ dB 46

6.2 Comparison of Rate Regions for 2 way relay channel with no direct link $\gamma_1 = 3$ dB, $\gamma_2 = 15$ dB 47

6.3 Comparison of Rate Regions for 2 way relay channel with no direct link $\gamma_1 = 25$ dB, $\gamma_2 = 25$ dB 47

6.4 Comparison of Rate Regions for 2 way relay channel with direct link-Full duplex $\gamma_1 = 3$ dB, $\gamma_2 = 15$ dB,$\gamma_3 = 3$ dB 48

6.5 Comparison of Rate Regions for 2 way relay channel with direct link-Half duplex $\gamma_1 = 3$ dB, $\gamma_2 = 15$ dB,$\gamma_3 = 3$ dB 48

6.6 Comparison of Rate Regions for 2 way relay channel with direct link-Full duplex $\gamma_1 = 10$ dB, $\gamma_2 = 15$ dB,$\gamma_3 = 3$ dB 49

6.7 Comparison of Rate Regions for 2 way relay channel with direct link-Half duplex $\gamma_1 = 10$ dB, $\gamma_2 = 15$ dB,$\gamma_3 = 3$ dB 49

6.8 Comparison of Rate Regions for 2 way relay channel with direct link-Full duplex $\gamma_1 = 20$ dB, $\gamma_2 = 25$ dB,$\gamma_3 = 20$ dB 50

6.9 Comparison of Rate Regions for 2 way relay channel with direct link-Half duplex $\gamma_1 = 20$ dB, $\gamma_2 = 25$ dB,$\gamma_3 = 20$ dB 50

6.10 Comparison of outer bound for 2 way relay channel with and without direct link $\gamma_1 = 20$ dB, $\gamma_2 = 20$ dB,$\gamma_3 = 3$ dB 51

6.11 Comparison of outer bound for 2 way relay channel with and without direct link $\gamma_1 = 20$ dB, $\gamma_2 = 25$ dB,$\gamma_3 = 10$ dB 52

6.12 Outer bounds for diamond relay channel with and without direct link $\gamma_{a1} = 15$ dB, $\gamma_{a2} = 10$ dB,$\gamma_{b1} = 15$ dB, $\gamma_{b2} = 10$ dB,$\gamma_d = 7$ dB . . 53

6.13 Outer bounds for diamond relay channel with and without direct link $\gamma_{a1} = 14$ dB, $\gamma_{a2} = 6$ dB,$\gamma_{b1} = 5$ dB, $\gamma_{b2} = 12$ dB,$\gamma_d = 8$ dB . . . 53

6.14 Rate region for Scale and Forward protocol: $\gamma_{a1} = 30$ dB, $\gamma_{a2} = 3$ dB, $\gamma_{b1} = 20$ dB $\gamma_{b2} = 4$ dB 54

6.15 Rate region for Scale and Forward protocol: $\gamma_{a1} = 10$ dB, $\gamma_{a2} = 12$ dB, $\gamma_{b1} = 14$ dB $\gamma_{b2} = 16$ dB 55

6.16 Rate region for Scale and Forward protocol,Outer bound and other protocols: $\gamma_{a1} = 10$ dB, $\gamma_{a2} = 10$ dB, $\gamma_{b1} = 10$ dB $\gamma_{b2} = 10$ dB 55

6.17 Rate region for Scale and Forward protocol,Outer bound and other protocols: $\gamma_{a1} = 30$ dB, $\gamma_{a2} = 20$ dB, $\gamma_{b1} = 3$ dB $\gamma_{b2} = 4$ dB 56

6.18 a/b ratio Vs Power at the relay 1 $h_{a1} = 1$, $h_{a2} = 2$,$h_{b1} = 5$, $h_{b2} = 1$ 56

6.19 Full duplex and half duplex outer bounds for MIMO 2-way relay with no direct link: 57

6.20 Full duplex and half duplex outer bounds for MIMO 2-way relay with direct A-B link: 58

6.21 Full duplex and half duplex outer bounds for MIMO diamond relay with no direct link: 58

6.22 Full duplex and half duplex outer bounds for MIMO diamond relay with direct A-B link: 59

6.23 Full duplex and half duplex outer bounds for MIMO diamond relay with direct link between relays: 59

ABBREVIATIONS

AF	Amplify and Forward
CF	Compress and Forward
DF	Decode and Forward
MF	Mixed Forward
LF	Lattice Forward
MABC	Multiple Access Broadcast
TDBC	Time Division Broadcast
HBC	Hybrid Broadcast
CF-CMAC	Compute and Forward - Compound Multiple Access Channel
CF-BC	Compute and Forward -Broadcast Channel
MDF	Multi-hopping Decode and Forward
MIMO	Multiple Input Multiple Output
SDP	Semi Definite Programming

NOTATION

a,b	Terminal Nodes
r	Relay Node
h	Channel Gain
G	Channel Gain Matrix
$\mathcal{C}$	Capacity
γ	Signal-to-noise ratio
N	Receiver Noise Variance
P	Transmit Power of each node
R_a	Data transmission rate from node a to node b
R_b	Data transmission rate from node b to node a
λ_i	Fraction of channel use in state i

CHAPTER 1

INTRODUCTION

A two-way or bidirectional relay channel consists of two nodes exchanging messages through one or more relays. Relay networks find applications in multi-hop wireless networks, sensor networks with transmitter power limitations etc. Among the different relay channels, the three node two-way relay channel and the diamond relay channel have attracted significant interest. We restrict our attention to these two channels in the current thesis. Two-way relaying without the direct link between nodes a and b is studied in [10; 3; 19; 16]. The more general two-way relaying with the direct link is studied in [7; 6; 14]. In [7], achievable rate regions are derived for three protocols, namely the multiple access broadcast (MABC) protocol, time division broadcast (TDBC) protocol, and the hybrid broadcast (HBC) protocol under the decode-and-forward (DF) relaying scheme. In [6], the TDBC and MABC protocols have been studied under other relaying schemes such as amplify-and-forward (AF), compress-and-forward (CF), mixed forward (MF) and Lattice forward (LF). A partial DF protocol, which is a superposition of DF and CF was studied in [13]. In [14], a three-phase cooperative MABC protocol (CoMABC), which outperforms the MABC and TDBC protocols in terms of sum rate in asymmetric channel conditions, was proposed. A transmission scheme based on doubly nested lattice codes was also proposed in [14], i.e., the CoMABC protocol uses the Lattice forward relaying strategy and is not a DF relaying protocol. A DF protocol using all states called All States (AS) protocol and a protocol called HBC-AS protocol that is an improvement to HBC protocol using side information is proposed in [2]. A generalized outer bound for the capacity region for all relaying protocols was derived in [2] using the half-duplex cut-set bound in [5]. This outer bound considers all possible states of the network.

The half-duplex Gaussian diamond relay channel where a source node and a destination node communicate with each other through two non-interfering relays has been studied in [12], [17] and [4]. In [17], the the AF, CF and DF relaying schemes have been generalized to the diamond channel and some hybrid schemes also have been proposed.

In [4], multi-hopping decode-and-forward (MDF) protocols for one-way communication were proposed to achieve rates within a constant gap of a capacity outer bound. Coding schemes based on nested lattice coding and compute and forward have been proposed in [8], [15]. These exploit the mixing of the two flows and improve the rates in both directions. Relaying protocols for the bidirectional communication over half duplex diamond relay channel have been proposed and their achievable rate regions have been determined in [11]. Two protocols- The CF-CMAC(Compute and Forward Multiple Access Channel) and the CF-BC(Compute and Forward-Broadcast), that use compute and forward scheme at the two relays are proposed for the diamond relay channel. The MDF protocol in [4] is also extended to two-way relaying by simple time sharing between the flows in the two directions. Outer bound and some achievable rate regions for half duplex diamond relay channel with direct links between relays and with direct links between sources are also discussed in [11]. Multiple-input and multiple-output, or MIMO is the use of multiple antennas at both the transmitter and receiver to improve communication performance. MIMO technology has attracted attention in wireless communications because it improves throughput, spectral efficiency and reliability. MIMO is an important part of modern wireless communication standards such as IEEE 802.11n (Wi-Fi), 4G, 3GPP Long Term Evolution, WiMAX and HSPA+.

In this work we compute and compare the half duplex and full duplex outer bound for the case of multiple input multiple output for all the channels discussed in [2] and [11]. We also derive an outer bound for the full duplex mode for the two way relay channel and the Gaussian 2 way diamond channel and compare with the half duplex case. This outer bound is based on the full duplex and the half-duplex cut-set bound respectively and is applicable to any protocol. Numerical results and comparisons of the achievable rate regions of some protocols and the outer bound are also presented. We also propose and analyze the scale and forward protocol for two way communication between nodes in a diamond channel with no direct link. Scale and Protocol for one way communication for diamond channel was proposed in [17]. We extend this protocol for the 2 way communication case and compare the achievable rate region with some protocols in [11] and the outer bound.

1.1 Organization of Thesis

The organization of this thesis is as follows.

- Chapter 2 deals with the derivation of an outer bound for capacity region of gaussian 2 way relay channel with and without direct links for the full duplex and half duplex modes. Some existing protocols of 2 way relay channel are also discussed.
- Chapter 3 deals with the derivation of an outer bound for capacity region of the full duplex mode of the diamond channel without direct links, diamond channel with direct link between sources and diamond channel with direct link between relays. Outer bound for capacity region of half duplex mode is also discussed.
- Chapter 4 discusses the scale and forward protocol proposed in this thesis for the gaussian two way diamond relay channel.
- Chapter 5 deals with the derivation of an outer bound for capacity region of gaussian 2 way relay channel and diamond channel discussed in the previous chapters for the full duplex mode and half duplex mode of the MIMO case.
- Chapter 6 presents some numerical results on the rate region for scale and forward protocol and also some results on outer bound for capacity region of different channels discussed in previous chapters and
- Chapter 7 concludes this thesis discussing possible future works.

CHAPTER 2

GAUSSIAN TWO-WAY RELAY CHANNEL

In this chapter, an outer bound for the capacity region of the two-way Gaussian relay channel is derived for the cases with and without direct $a-b$ link for both half-duplex and full-duplex mode. This is derived using the half-duplex cut-set bound and full-duplex cut set bound respectively. In the two-way relay network, there are two flows, one from a to b and another from b to a. The outer bound for the two flows is derived considering two cuts that separate nodes a and b and information flow in both directions across the cuts. For a full-duplex relay network, any achievable rate from source to destination R is upper bounded as :

$$R \leq \min_{S} \; I(X_S; Y_{S^c} | X_{S^c}), \tag{2.1}$$

For a half-duplex relay network with m states, any achievable rate from source to destination R is upper bounded as :

$$R \leq \sup_{\{\lambda_i\}: \sum_{i=1}^{m} \lambda_i \leq 1} \; \min_{S} \sum_{i=1}^{m} \lambda_i I(X_S; Y_{S^c} | X_{S^c}, i), \tag{2.2}$$

where a cut partitions the set of nodes into sets S and S^c such that the source node is in S, the destination is in S^c, and S^c is the complement of S. In order to bound R_a (rate of information flow from a to b), we consider the cuts defined by $S_1 = \{a\}$, and $S_2 = \{a, r\}$. Similarly to bound R_b, we use $S_3 = \{b\}$ and $S_4 = \{b, r\}$.

2.1 Gaussian 2 way relay channel without direct link

2.1.1 System model

Consider two nodes a and b communicating with each other in the network shown in Fig. 2.1. A relay node r assists this communication by receiving the information from these nodes and forwarding to the desired destination. There is no direct link between

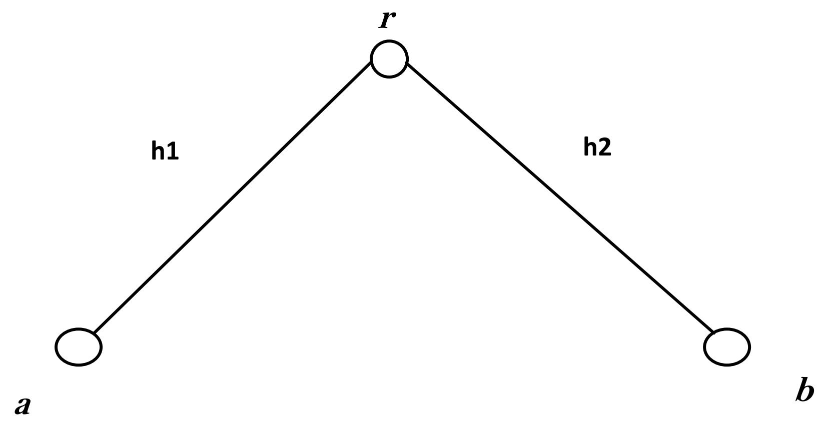


Figure 2.1: Two-way Gaussian Relay Channel with no direct Link

the nodes. For simplicity each node is assumed to have the same transmit power P and each receiver a noise variance of N for each state. Let h_1 and h_2 be the gains of channels a-r and b-r respectively. The SNRs of these channels are denoted by $\gamma_1 = \frac{h_1^2 P}{N}$ and $\gamma_2 = \frac{h_2^2 P}{N}$. We use R_a and R_b to denote the rate of data transmission (bits per channel use) from node a to node b and from node b to node a, respectively. In full duplex case all the nodes transmit and receive simultaneously all the time. In half duplex case, there are 2 states-Multiple Access state where both a and b transmit to relay and Broadcast state when relay transmits to a and b. Let λ_i, $i = 1, 2$ denote the fraction of channel uses in states $1, 2$ respectively. Let $\mathcal{C}(\gamma) \triangleq 0.5log_2(1+\gamma)$ represents the capacity of a gaussian channel with SNR of γ

2.1.2 Outer bounds for Gaussian 2 way relay channel with no direct link-Full duplex

Using the full duplex cut set bound we get :

$$\begin{aligned} R_a &\leq \min\{I\left(X_a; Y_r, Y_b | X_r, X_b\right), I\left(X_a, X_r; Y_b | X_b\right)\} \\ R_b &\leq \min\{I\left(X_b, X_r; Y_a | X_a\right), I\left(X_b; Y_r, Y_a | X_r, X_a\right)\} \end{aligned} \tag{2.3}$$

Solving the mutual information terms gives the following bound

$$\begin{aligned} R_a &\leq \min\{\mathcal{C}(\gamma_1), \mathcal{C}(\gamma_2)\} \\ R_b &\leq \min\{\mathcal{C}(\gamma_1), \mathcal{C}(\gamma_2)\} \end{aligned} \tag{2.4}$$

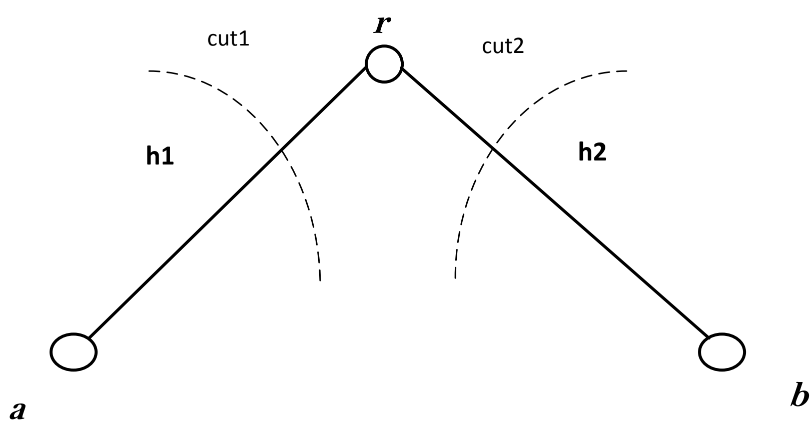


Figure 2.2: Cuts used for finding the bound

We solve the optimization problem of maximizing R_a, keeping the ratio of R_a and R_b fixed and satisfying the above constraints, to get the outer bound. The complete outer bound region can be evaluated by solving the above problem for different values of the ratio of R_a and R_b. The outer bound is valid for any relaying scheme (since it is based only on the cut-set bound and mutual information bounds) for the two-way relay channel.

2.1.3 Protocols for Two-way Relay Channel without Direct Link-Full Duplex

Lattice Coding

For a Gaussian TRC, we can achieve the following region [8]:

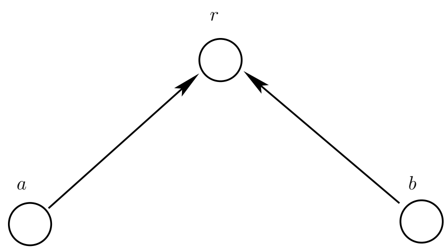
$$\begin{aligned} R_a &\le \mathcal{C}(\gamma_2) \\ R_a &\le [\mathcal{C}(\gamma_1 - \frac{\gamma_2}{\gamma_1+\gamma_2})]^+ \\ R_b &\le \mathcal{C}(\gamma_1) \\ R_b &\le [\mathcal{C}(\gamma_2 - \frac{\gamma_1}{\gamma_1+\gamma_2})]^+ \end{aligned} \tag{2.5}$$

where $[x]^+ \triangleq max(x, 0)$

To achieve the above region,nested lattice codes formed from a lattice partition chain are used for the uplink. For the downlink, a structured binning of messages at the relay is used, which is naturally introduced by the nested lattice codes. The destination nodes decode each other's message using this binning information and their own messages as side information.

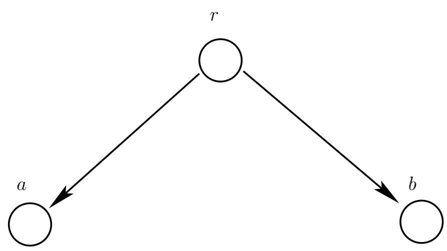
2.1.4 Outer bounds for Gaussian 2 way relay channel with no direct link-Half duplex

For a two-way relay channel without direct link, the possible states are as shown in the fig. 2.3 and fig. 2.4 Using S_1, and S_2 as defined above, we get :



$State - 1$

Figure 2.3: Multiple Access State-1



$State - 2$

Figure 2.4: Broadcast-State-2

$$R_a \leq \sup_{\{\lambda_i\}:\sum_{i=1}^{2}\lambda_i \leq 1} \min\{\lambda_1 I\left(X_a; Y_r, Y_b | X_r, X_b, i=1\right), \lambda_2 I\left(X_a, X_r; Y_b | X_b, i=2\right)\} \tag{2.6}$$

Similarly bounding R_b

$$R_b \leq \sup_{\{\lambda_i\}:\sum_{i=1}^{2}\lambda_i \leq 1} \min\{\lambda_2 I\left(X_b, X_r; Y_a | X_a, i=2\right), \lambda_1 I\left(X_b; Y_r, Y_a | X_r, X_b, i=1\right)\} \tag{2.7}$$

Solving the mutual information terms, we get the following outer bound

$$\begin{aligned} R_a &\leq \lambda_1 \mathcal{C}(\gamma_1) \\ R_a &\leq \lambda_2 \mathcal{C}(\gamma_2) \\ R_b &\leq \lambda_1 \mathcal{C}(\gamma_2) \\ R_b &\leq \lambda_2 \mathcal{C}(\gamma_1) \\ \sum_{i=1}^{2} \lambda_i &= 1, 0 \leq \lambda_i \leq 1 \end{aligned} \tag{2.8}$$

We maximize R_a over all possible λ_i's, keeping the ratio of R_a and R_b fixed and satisfying the above constraints to get the outer bound. The complete outer bound region can be evaluated by solving the above problem for different values of the ratio of R_a and R_b.

2.1.5 Protocols for Two-way Relay Channel without Direct Link

Multiple Access Broadcast (MABC) protocol

MABC is a two phase protocol in which states 1 and 2 shown in fig. 2.3 and fig. 2.4 are used. An achievable rate region for MABC with *decode-and-forward* scheme is discussed here. In the first phase,terminal nodes send messages to relay node. After first phase, relay decodes the received messages and forward it to destinations in second phase. The achievable rate regions for MABC is given in [7]. It is the closure of the set of all points (R_a, R_b) satisfying following constraints.

$$\begin{aligned} &R_a \leq \min\{\lambda_1 \mathcal{C}(\gamma_1), \lambda_2 \mathcal{C}(\gamma_2)\} \\ &R_b \leq \min\{\lambda_1 \mathcal{C}(\gamma_2), \lambda_2 \mathcal{C}(\gamma_1)\} \\ &R_a + R_b \leq \lambda_1 \mathcal{C}(\gamma_1 + \gamma_2) \\ &\lambda_1 + \lambda_2 = 1,\ 0 \leq \lambda_i \leq 1,\ i = 3, 4 \end{aligned} \tag{2.9}$$

2.2 Gaussian 2 way relay channel with direct link between A and B

2.2.1 System Model

The 2-way relay channel with direct link between the end nodes is shown in Figure 2.5. The system model is similar to Section 2.1, but for an additional link between A and B. The SNR of the direct link is denoted as $\gamma_3 = \frac{h_3^2 P}{N}$

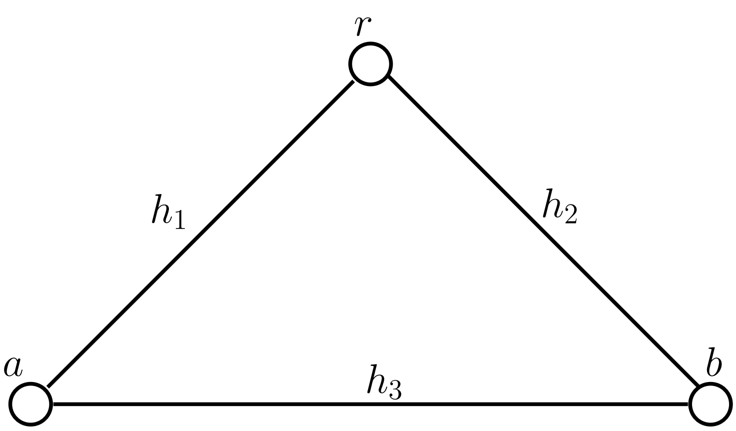


Figure 2.5: Two-way Gaussian Relay Channel with Direct Link

2.2.2 Outer bounds for Gaussian 2 way relay channel with direct link between A and B-Full duplex

We derive an outer bound for the capacity region of the full-duplex two-way Gaussian relay channel. This is derived using the full-duplex cut-set bound. The outer bound for the two flows is derived considering two cuts that separate nodes a and b and information flow in both directions across the cuts. Combining these four bounds (2 cuts $\times$ 2 directions), we get the rate region outer bound.

$$\begin{aligned} R_a &\leq \min\{I\left(X_a; Y_r, Y_b | X_r, X_b\right), I\left(X_a, X_r; Y_b | X_b\right)\} \\ R_b &\leq \min\{I\left(X_b, X_r; Y_a | X_a\right), I\left(X_b; Y_r, Y_a | X_r, X_a\right)\} \end{aligned} \tag{2.10}$$

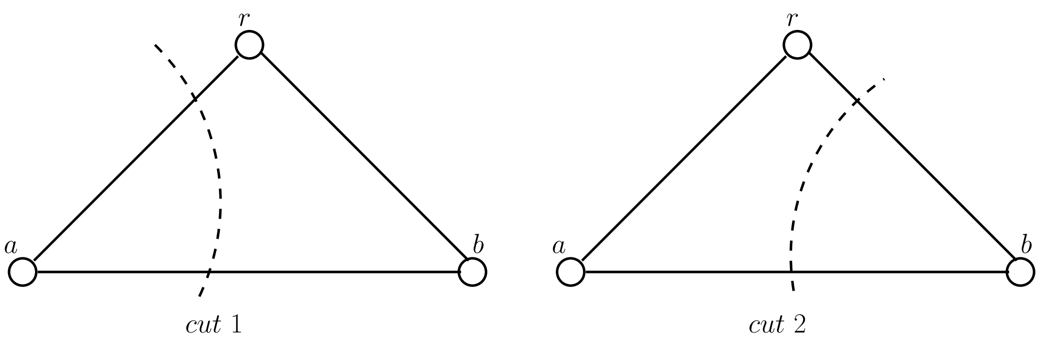


Figure 2.6: Cuts used for finding the bound

Now, we can upper bound the various mutual information terms as in [5] to get:

$$
\begin{aligned}
I\left(X_a; Y_r, Y_b | X_r, X_b\right) &\leq \mathcal{C}(\gamma_1 + \gamma_3) \\
I\left(X_b; Y_r, Y_a | X_r, X_a\right) &\leq \mathcal{C}(\gamma_2 + \gamma_3)
\end{aligned}
\tag{2.11}
$$

$$
\begin{aligned}
I\left(X_a, X_r; Y_b | X_b\right) &\leq \mathcal{C}\left(\gamma_2 + \gamma_3 + 2\sqrt{\gamma_2\gamma_3}\right) \\
I\left(X_b, X_r; Y_a | X_a\right) &\leq \mathcal{C}\left(\gamma_1 + \gamma_3 + 2\sqrt{\gamma_1\gamma_3}\right)
\end{aligned}
\tag{2.12}
$$

Finally, we can obtain each point on the outer bound of the rate region by solving the following optimization problem. Maximize R_a keeping the ratio of R_a and R_b fixed and satisfying the following constraints :

$$
\begin{aligned}
R_a &\leq \mathcal{C}(\gamma_1 + \gamma_3) \\
R_a &\leq \mathcal{C}\left(\gamma_2 + \gamma_3 + 2\sqrt{\gamma_2\gamma_3}\right) \\
R_b &\leq \mathcal{C}(\gamma_2 + \gamma_3) \\
R_b &\leq \mathcal{C}\left(\gamma_1 + \gamma_3 + 2\sqrt{\gamma_1\gamma_3}\right)
\end{aligned}
\tag{2.13}
$$

The complete outer bound region can be evaluated by solving the above problem for different values of the ratio of R_a and R_b.

2.2.3 Protocols for Two-way Relay Channel with Direct Link between A and B-Full Duplex

Decode and Forward Protocol

The following rate region is achievable for the full-duplex Gaussian two-way relay channel with pure decode forward scheme [9]:

$$\begin{aligned} R_a &\leq \mathcal{C}(\gamma_1) \\ R_a &\leq \mathcal{C}(\gamma_2 + \gamma_3) \\ R_b &\leq \mathcal{C}(\gamma_2) \\ R_b &\leq \mathcal{C}(\gamma_1 + \gamma_3) \\ R_a + R_b &\leq \mathcal{C}(\gamma_1 + \gamma_2) \end{aligned} \tag{2.14}$$

Lattice Coding

It is shown in [18] that the following rates are achievable for the two-way AWGN relay channel with direct links

$$\begin{aligned} &R_a \leq \mathcal{C}(\gamma_2 + \gamma_3) \\ &R_a \leq [\mathcal{C}(\gamma_1 - \frac{\gamma_2}{\gamma_1 + \gamma_2})]^+ \\ &R_b \leq \mathcal{C}(\gamma_1 + \gamma_3) \\ &R_b \leq [\mathcal{C}(\gamma_2 - \frac{\gamma_1}{\gamma_1 + \gamma_2})]^+ \end{aligned} \tag{2.15}$$

where $[x]^+ \triangleq max(x, 0)$

2.2.4 Outer bounds for Gaussian 2 way relay channel with direct link between A and B-Half duplex

An outer bound for the capacity region of the half-duplex two-way Gaussian relay channel is derived in [2]. This is derived using the half-duplex cut-set bound ans is as fol-

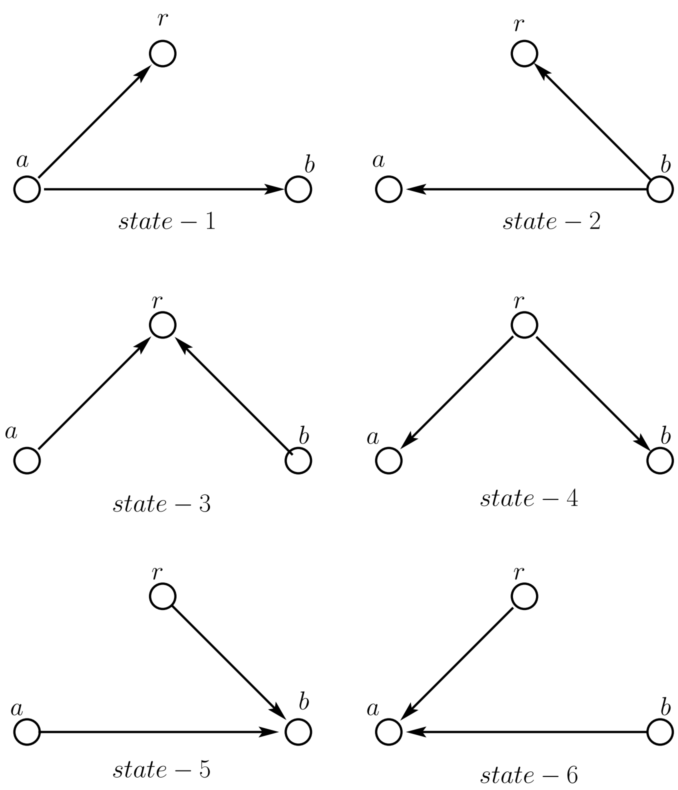


Figure 2.7: Possible states in a Half-duplex Two-way Relay Channel

lows:

$$\begin{aligned}
&R_a \leq \lambda_1\mathcal{C}(\gamma_1+\gamma_3)+\lambda_3\mathcal{C}(\gamma_1)+\lambda_5\mathcal{C}\left(\gamma_3\right)\\
&R_a \leq \lambda_1\mathcal{C}(\gamma_3)+\lambda_4\mathcal{C}(\gamma_2)+\lambda_5\mathcal{C}\left(\gamma_2+\gamma_3+2\sqrt{\gamma_2\gamma_3}\right)\\
&R_b \leq \lambda_2\mathcal{C}(\gamma_2+\gamma_3)+\lambda_3\mathcal{C}(\gamma_2)+\lambda_6\mathcal{C}\left(\gamma_3\right)\\
&R_b \leq \lambda_2\mathcal{C}(\gamma_3)+\lambda_4\mathcal{C}(\gamma_1)+\lambda_6\mathcal{C}\left(\gamma_1+\gamma_3+2\sqrt{\gamma_1\gamma_3}\right)\\
&\quad\sum_{i=1}^{6}\lambda_i = 1, 0 \leq \lambda_i \leq 1
\end{aligned} \tag{2.16}$$

The complete outer bound region can be evaluated by solving the above problem for different values of the ratio of R_a and R_b.

2.2.5 Protocols for Two-way Relay Channel With Direct Link

Time Division Broadcast (TDBC) protocol

TDBC [7] is a three phase protocol in which states 1, 2 and 4 are used. In first phase (state 1), a transmits at a rate equal to the capacity of link between a and r. At this time node b listens to this transmission and uses this information as *side information*

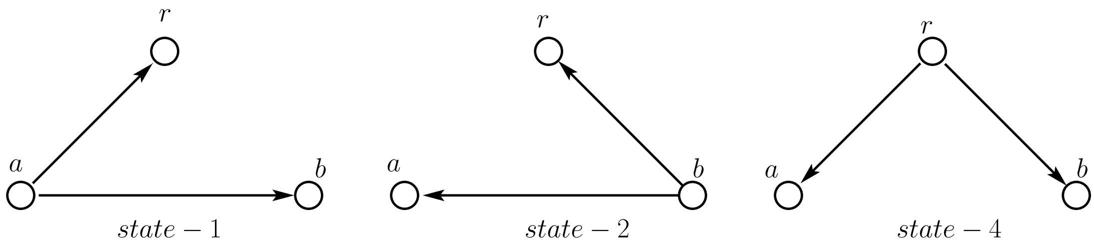


Figure 2.8: States used in TDBC protocol

for decoding after phase 3 (state 4). Phase 2 is similar to phase 1 in which b transmits at a rate equal to the capacity of link between b and r and a listens to this. At the end of each of these phases, relay node r decode the messages and does a binning operation on these messages. In phase 3, r transmits this binned information to a and b. Since a and b know the message meant for the other destination, relay node r can use XOR of these messages for broadcasting. Achievable rate region for TDBC protocol under decode-and-forward scheme is given in [7]. It is the closure of the set of all points (R_a, R_b) satisfying following constraints.

$$
\begin{aligned}
R_a &\leq \min\{\lambda_1\mathcal{C}(\gamma_1), \lambda_1\mathcal{C}(\gamma_3) + \lambda_4\mathcal{C}(\gamma_2)\} \\
R_b &\leq \min\{\lambda_2\mathcal{C}(\gamma_2), \lambda_2\mathcal{C}(\gamma_3) + \lambda_4\mathcal{C}(\gamma_1)\} \\
\lambda_1 &+ \lambda_2 + \lambda_4 = 1,\ 0 \leq \lambda_i \leq 1,\ i = 1, 2, 4
\end{aligned}
\tag{2.17}
$$

Hybrid Broadcast (HBC) protocol

HBC is a four phase protocol in which states 1, 2, 3 and 4 are used. State 1, 2 and 4 are used in the same way as in TDBC. State 3, where terminals a and b transmit simultaneously (MAC) to relay r, is also added. These messages are decoded at the relay and forwarded as such in state 4. In HBC, state 4 is used for forwarding binned messages from state 1 and 2 as well as for forwarding messages received in state 3. The HBC protocol is always better than TDBC and MABC protocols since they are special cases of the HBC protocol. The use of *side information* from one phase in decoding during another phase provides improvement in TDBC and HBC over MABC for some channel conditions. Achievable rate region for HBC protocol is given in [7]. It is the

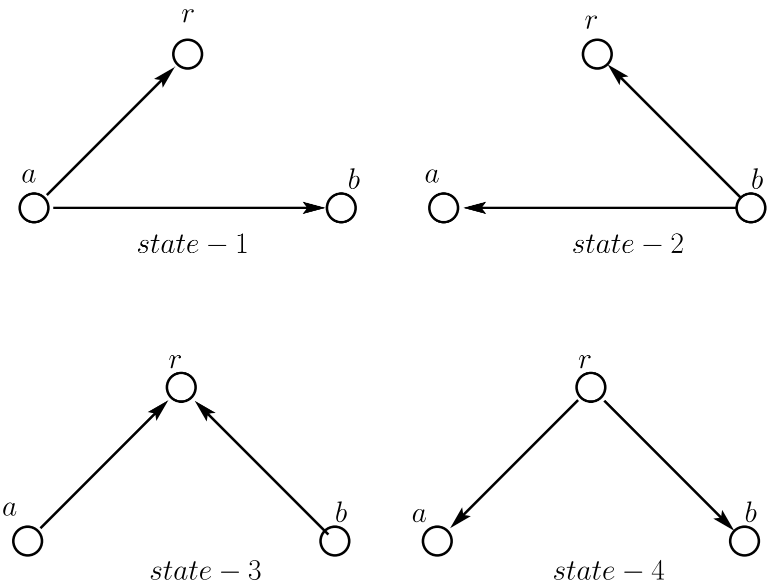


Figure 2.9: States used in HBC protocol

closure of the set of all points (R_a, R_b) satisfying following constraints.

$$\begin{aligned}
&R_a \leq \min\{(\lambda_1 + \lambda_3)\mathcal{C}(\gamma_1), \lambda_1\mathcal{C}(\gamma_3) + \lambda_4\mathcal{C}(\gamma_2)\}, \\
&R_b \leq \min\{(\lambda_2 + \lambda_3)\mathcal{C}(\gamma_2), \lambda_2\mathcal{C}(\gamma_3) + \lambda_4\mathcal{C}(\gamma_1)\}, \\
&R_a + R_b \leq \lambda_1\mathcal{C}(\gamma_1) + \lambda_2\mathcal{C}(\gamma_2) + \lambda_3\mathcal{C}(\gamma_1 + \gamma_2), \\
&\quad \sum_{i=1}^{4} \lambda_i = 1, 0 \leq \lambda_i \leq 1.
\end{aligned} \tag{2.18}$$

Cooperative MABC (CoMABC) protocol

CoMABC is a three phase protocol in which states 3,4 and 5 (or 6 depending on channel conditions) is used. Assume that $\gamma_1 \leq \gamma_2$. Since the link between b and r is better than the link between a and r, relay may receive more data from b than from a. Using MABC protocol, the relay may finish forwarding of messages from a before finishing forwarding of messages from b since $a \leftrightarrow r$ link is weaker than $b \leftrightarrow r$ link and more data have to be forwarded to a. Thus, node b remain idle for some time. This idle time may be long depending on the defference between the channel gains of two links. In CoMABC protocol, state 6 is added (for $\gamma_1 \leq \gamma_2$ case) in addition to state 3 and 4 of MABC protocol. So, when the forwarding of messages to b is finished, state 6 is used for further forwarding of messages to a. Node b joins the $r \rightarrow a$ transmission by using some cooperative technique to get either diversity where node b retransmits some data it already transmitted to relay node r in state 1 or multiplexing where node b

transmits new data. Similarly if $\gamma_2 \leq \gamma_1$, state 5 will be used in which node a assists the forwarding of data from r to b. Achievable rate region for CoMABC protocol is given in [14]. It is the closure of the set of all points (R_a, R_b) satisfying following constraints.

$$
\begin{aligned}
R_a &\leq \min\{\lambda_3 R^*_{ar}, \lambda_4 \mathcal{C}(\gamma_2)\} \\
R_b &\leq \min\{\lambda_3 R^*_{br} + \lambda_6 \mathcal{C}(\gamma_3), \lambda_4 \mathcal{C}(\gamma_1) \\
&\qquad + \lambda_6 \mathcal{C}(\gamma_1 + \gamma_3)\} \\
R^*_{ar} &= \left[\log\left(\frac{\gamma_1}{\gamma_1 + \gamma_2} + \gamma_1\right)\right]^+ \\
R^*_{br} &= \left[\log\left(\frac{\gamma_2}{\gamma_1 + \gamma_2} + \gamma_2\right)\right]^+
\end{aligned}
\tag{2.19}
$$

where $[x]^+ \triangleq max(x, 0)$.

CHAPTER 3

GAUSSIAN DIAMOND RELAY CHANNEL

In this chapter, an outer bound for the capacity region of the two-way Gaussian diamond relay channel is derived for the cases with and without direct links for both half-duplex and full-duplex modes. This is derived using the half-duplex cut-set bound and full-duplex cut set bound respectively.

3.1 Gaussian diamond relay channel with no direct link

3.1.1 System Model

Consider two nodes a and b whose bi-directional communication is assisted by two relays, r_1 and r_2 in diamond topology as shown in Fig. 3.1. No link is present between the two nodes A and B or between the two relays. So all communication is through the two non-interfering relays. For simplicity each node is assumed to have the same transmit power P and each receiver noise variance of N for each state. Let h_{a1} , h_{a2} , h_{b1} and h_{b2} be the gains of channels a-r_1 , a-r_2 , b-r_1 and b-r_2 respectively. The SNRs of these channels are denoted by $\gamma_{a1} = \frac{h_{a1}^2 P}{N}$, $\gamma_{a2} = \frac{h_{a2}^2 P}{N}$,$\gamma_{b1} = \frac{h_{b1}^2 P}{N}$ and $\gamma_{b2} = \frac{h_{b2}^2 P}{N}$ respectively. The channel is assumed to be known, reciprocal and constant. Channel state information is assumed to be available at all nodes. We use R_a and R_b to denote the rate of data transmission (bits per channel use) from node a to node b and from node b to node a, respectively. Let $\mathcal{C}(\gamma) \triangleq 0.5log_2(1+\gamma)$ represents the capacity of a gaussian channel with SNR of γ

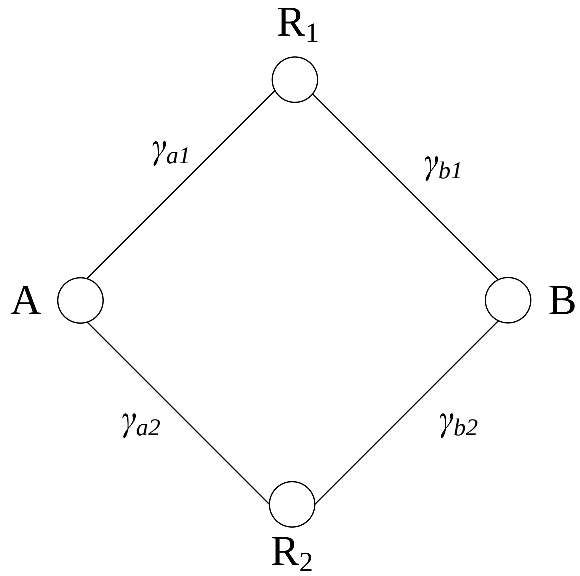


Figure 3.1: Two-way Gaussian Diamond Relay Channel with no direct Link

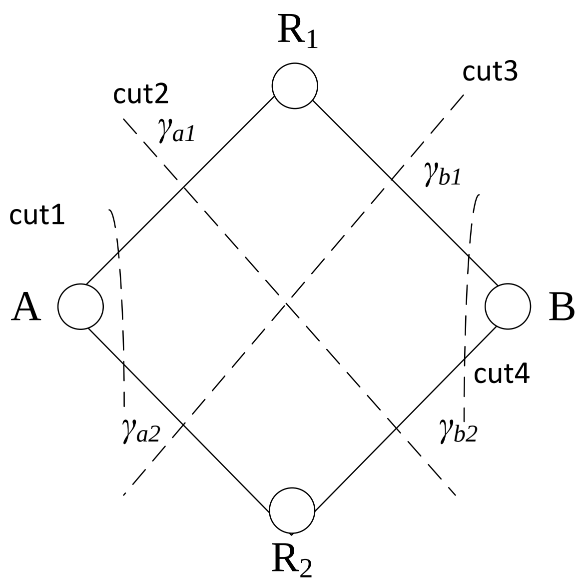


Figure 3.2: Cuts used for finding the bound

3.1.2 Outer bounds for Gaussian diamond relay channel with no direct link-Full duplex

Using full duplex cutset bound we get:

$$
\begin{aligned}
R_a \leq \min\{ & I\left(X_a; Y_{r1}, Y_{r2}, Yb | X_b, X_{r1}, X_{r2}\right), I\left(X_a, X_{r1}; Y_b, Y_{r2} | X_b, X_{r2}\right), \\
& I\left(X_a, X_{r2}; Y_b, Y_{r1} | X_b, X_{r1}\right), I\left(X_a, X_{r1}, X_{r2}; Y_b | X_b\right)\} \\
R_b \leq \min\{ & I\left(X_b; Y_{r1}, Y_{r2}, Y_a | X_a, X_{r1}, X_{r2}\right), I\left(X_b, X_{r1}; Y_a, Y_{r2} | X_a, X_{r2}\right), \\
& I\left(X_b, X_{r2}; Y_a, Y_{r1} | X_a, X_{r1}\right), I\left(X_b, X_{r1}, X_{r2}; Y_a | X_a\right)\}
\end{aligned} \tag{3.1}
$$

$$
\begin{aligned}
R_a \leq \min\{ & I\left(X_a; Y_{r1}, Y_{r2} | X_b\right), I\left(X_a, X_{r1}; Y_b, Y_{r2} | X_b, X_{r2}\right), \\
& I\left(X_a, X_{r2}; Y_b, Y_{r1} | X_b, X_{r1}\right), I\left(X_{r1}, X_{r2}; Y_b\right)\} \\
R_b \leq \min\{ & I\left(X_b; Y_{r1}, Y_{r2} | X_a\right), I\left(X_b, X_{r1}; Y_a, Y_{r2} | X_a, X_{r2}\right), \\
& I\left(X_b, X_{r2}; Y_a, Y_{r1} | X_a, X_{r1}\right), I\left(X_{r1}, X_{r2}; Y_a\right)\}
\end{aligned} \tag{3.2}
$$

Now, we can upper bound the various mutual information terms as in [5] to get:

$$
\begin{aligned}
I\left(X_a; Y_{r1}, Y_{r2} | X_b\right) &\leq \mathcal{C}(\gamma_{a1} + \gamma_{a2}) \\
I\left(X_b; Y_{r1}, Y_{r2} | X_a\right) &\leq \mathcal{C}(\gamma_{b1} + \gamma_{b2})
\end{aligned} \tag{3.3}
$$

$$
\begin{aligned}
I\left(X_a, X_{r1}; Y_b, Y_{r2} | X_b, X_{r2}\right) &\leq \mathcal{C}(\gamma_{a2}) + \mathcal{C}(\gamma_{b1}) \\
I\left(X_a, X_{r2}; Y_b, Y_{r1} | X_b, X_{r1}\right) &\leq \mathcal{C}(\gamma_{b2}) + \mathcal{C}(\gamma_{a1}) \\
I\left(X_b, X_{r1}; Y_a, Y_{r2} | X_a, X_{r2}\right) &\leq \mathcal{C}(\gamma_{a1}) + \mathcal{C}(\gamma_{b2}) \\
I\left(X_b, X_{r2}; Y_a, Y_{r1} | X_a, X_{r1}\right) &\leq \mathcal{C}(\gamma_{b1}) + \mathcal{C}(\gamma_{a2})
\end{aligned} \tag{3.4}
$$

$$
\begin{aligned}
I\left(X_{r1}, X_{r2}; Y_b\right) &\leq \mathcal{C}\left(\gamma_{b1} + \gamma_{b2} + 2\sqrt{\gamma_{b1}\gamma_{b2}}\right) \\
I\left(X_{r1}, X_{r2}; Y_a\right) &\leq \mathcal{C}\left(\gamma_{a1} + \gamma_{a2} + 2\sqrt{\gamma_{a1}\gamma_{a2}}\right)
\end{aligned} \tag{3.5}
$$

Finally, we can obtain each point on the outer bound of the rate region by solving the following optimization problem. Maximize R_a keeping the ratio of R_a and R_b fixed and satisfying the following constraints:

$$\begin{aligned}
R_a &\leq \mathcal{C}(\gamma_{a1} + \gamma_{a2}) \\
R_a &\leq \mathcal{C}\left(\gamma_{b1} + \gamma_{b2} + 2\sqrt{\gamma_{b1}\gamma_{b2}}\right) \\
R_a &\leq \mathcal{C}(\gamma_{a2}) + \mathcal{C}(\gamma_{b1}) \\
R_a &\leq \mathcal{C}(\gamma_{b2}) + \mathcal{C}(\gamma_{a1}) \\
R_b &\leq \mathcal{C}(\gamma_{b1} + \gamma_{b2}) \\
R_b &\leq \mathcal{C}\left(\gamma_{a1} + \gamma_{a2} + 2\sqrt{\gamma_{a1}\gamma_{a2}}\right) \\
R_b &\leq \mathcal{C}(\gamma_{a2}) + \mathcal{C}(\gamma_{b1}) \\
R_b &\leq \mathcal{C}(\gamma_{b2}) + \mathcal{C}(\gamma_{a1})
\end{aligned} \tag{3.6}$$

The complete outer bound region can be evaluated by solving the above problem for different values of the ratio of R_a and R_b.

3.1.3 Outer bounds for Gaussian diamond relay channel with no direct link-Half duplex

The diamond relay network has 16 possible states, because each of the four nodes can be either in transmit or receive state(half-duplex constraint). We can ignore the 2 states in which all nodes are in transmit or all are in receive states, as they do not serve any purpose. The remaining 14 useful states can be seen in Fig.3.3. An outer bound for the capacity region of the half-duplex two-way Gaussian relay channel is derived in [11] using the half-duplex cut-set bound. Only six states of Fig.3.3 are considered for writing the cut-set bound:14, 13, 2, 3, 6, 7 with the fraction of time the network is in

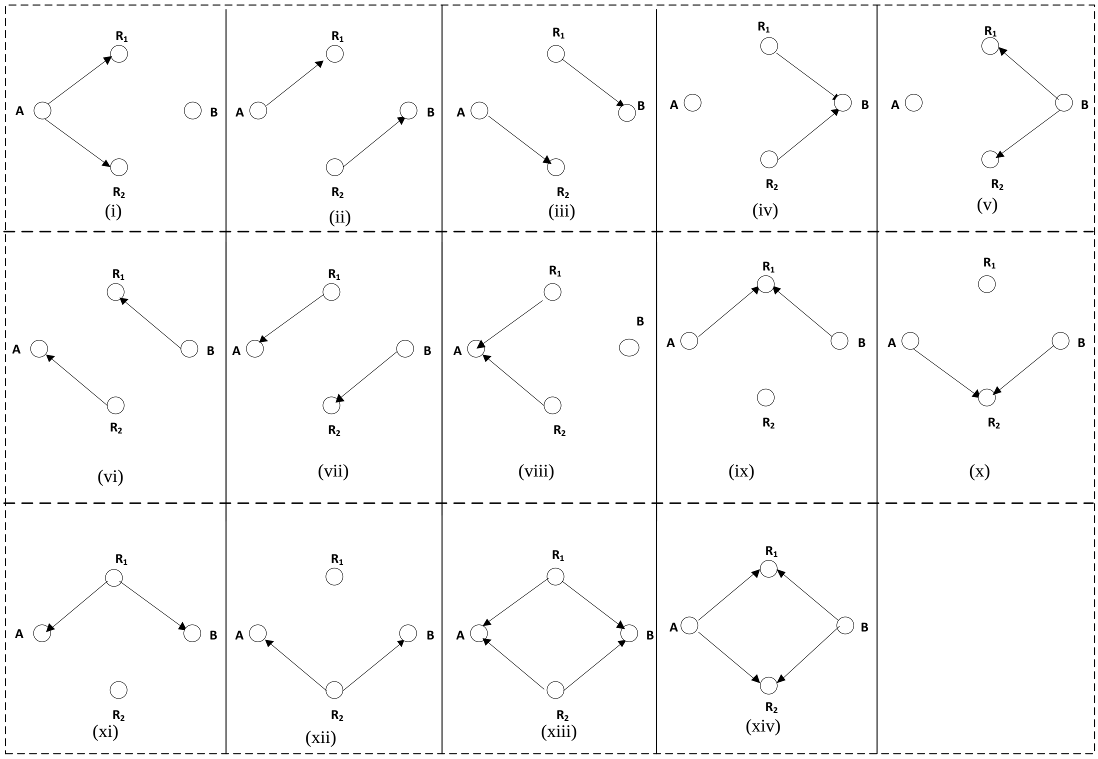


Figure 3.3: Possible states in a Half-duplex Two-way Diamond Relay Channel

these states being denoted as $\lambda_1; \lambda_2; \lambda_3; \lambda_4; \lambda_5; \lambda_6$, respectively.

$$\begin{aligned}
&R_a \leq \lambda_1 \mathcal{C}(\gamma_{a1} + \gamma_{a2}) + \lambda_3 \mathcal{C}(\gamma_{a1}) + \lambda_4 \mathcal{C}(\gamma_{a2}) \\
&R_a \leq \lambda_1 \mathcal{C}(\gamma_{a2}) + \lambda_2 \mathcal{C}(\gamma_{b1}) + \lambda_4 \mathcal{C}(\gamma_{a2}) + \lambda_4 \mathcal{C}(\gamma_{b1}) \\
&R_a \leq \lambda_1 \mathcal{C}(\gamma_{a1}) + \lambda_2 \mathcal{C}(\gamma_{b2}) + \lambda_3 \mathcal{C}(\gamma_{a1}) + \lambda_3 \mathcal{C}(\gamma_{b2}) \\
&R_a \leq \lambda_2 \mathcal{C}(\sqrt{\gamma_{b1}} + \sqrt{\gamma_{b2}})^2 + \lambda_3 \mathcal{C}(\gamma_{b2}) + \lambda_4 \mathcal{C}(\gamma_{b1}) \\
&R_b \leq \lambda_1 \mathcal{C}(\gamma_{b1} + \gamma_{b2}) + \lambda_5 \mathcal{C}(\gamma_{b1}) + \lambda_6 \mathcal{C}(\gamma_{b2}) \\
&R_b \leq \lambda_1 \mathcal{C}(\gamma_{b2}) + \lambda_2 \mathcal{C}(\gamma_{a1}) + \lambda_6 \mathcal{C}(\gamma_{b2}) + \lambda_6 \mathcal{C}(\gamma_{a1}) \\
&R_b \leq \lambda_1 \mathcal{C}(\gamma_{b1}) + \lambda_2 \mathcal{C}(\gamma_{a2}) + \lambda_5 \mathcal{C}(\gamma_{b1}) + \lambda_5 \mathcal{C}(\gamma_{a2}) \\
&R_b \leq \lambda_2 \mathcal{C}(\sqrt{\gamma_{a1}} + \sqrt{\gamma_{a2}})^2 + \lambda_5 \mathcal{C}(\gamma_{a2}) + \lambda_6 \mathcal{C}(\gamma_{a1}) \\
&\quad \sum_{i=1}^{6} \lambda_i = 1, 0 \leq \lambda_i \leq 1
\end{aligned} \tag{3.7}$$

The complete outer bound region can be evaluated by solving the above problem for different values of the ratio of R_a and R_b.

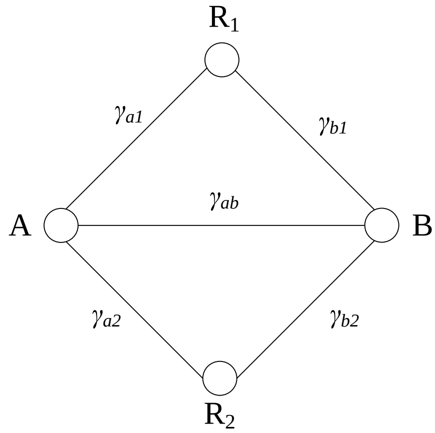


Figure 3.4: Two-way Gaussian Diamond Relay Channel with Direct A-B Link

3.2 Gaussian diamond relay channel with direct link between A and B

3.2.1 System Model

The diamond relay channel with direct link between the end nodes is shown in Fig 3.4. The system model is similar to Section 3.1, but for an additional link between A and B. The SNR of the direct link is denoted as $\gamma_{ab} = \frac{h_d^2 P}{N}$

3.2.2 Outer bounds for Gaussian diamond relay channel with direct link between A and B-Full duplex

Using the full duplex cutset bound we get:

$$\begin{aligned} R_a \leq \min\{ & I\left(X_a; Y_{r1}, Y_{r2}, Yb | X_b, X_{r1}, X_{r2}\right), I\left(X_a, X_{r1}; Y_b, Y_{r2} | X_b, X_{r2}\right), \\ & I\left(X_a, X_{r2}; Y_b, Y_{r1} | X_b, X_{r1}\right), I\left(X_a, X_{r1}, X_{r2}; Y_b | X_b\right)\} \\ R_b \leq \min\{ & I\left(X_b; Y_{r1}, Y_{r2}, Ya | X_a, X_{r1}, X_{r2}\right), I\left(X_b, X_{r1}; Y_a, Y_{r2} | X_a, X_{r2}\right), \\ & I\left(X_b, X_{r2}; Y_a, Y_{r1} | X_a, X_{r1}\right), I\left(X_b, X_{r1}, X_{r2}; Y_a | X_a\right)\} \end{aligned} \tag{3.8}$$

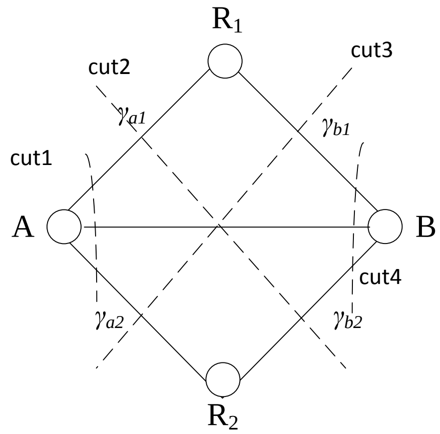


Figure 3.5: Cuts used for finding the bound

Now, we can upper bound the various mutual information terms to get:

$$\begin{aligned} I(X_a; Y_{r1}, Y_{r2}, Y_b | X_b, X_{r1}, X_{r2}) &\leq \mathcal{C}(\gamma_{a1} + \gamma_{a2} + \gamma_{ab}) \\ I(X_b; Y_{r1}, Y_{r2}, Y_a | X_a, X_{r1}, X_{r2}) &\leq \mathcal{C}(\gamma_{b1} + \gamma_{b2} + \gamma_{ab}) \end{aligned} \tag{3.9}$$

$$\begin{aligned} I\left(X_a, X_{r1}; Y_b, Y_{r2} | X_b, X_{r2}\right) &\leq \mathcal{C}(\gamma_{a2} + \gamma_{ab}) + \mathcal{C}(\gamma_{b1}) \\ I\left(X_a, X_{r2}; Y_b, Y_{r1} | X_b, X_{r1}\right) &\leq \mathcal{C}(\gamma_{b2}) + \mathcal{C}(\gamma_{a1} + \gamma_{ab}) \\ I\left(X_b, X_{r1}; Y_a, Y_{r2} | X_a, X_{r2}\right) &\leq \mathcal{C}(\gamma_{a1}) + \mathcal{C}(\gamma_{b2} + \gamma_{ab}) \\ I\left(X_b, X_{r2}; Y_a, Y_{r1} | X_a, X_{r1}\right) &\leq \mathcal{C}(\gamma_{b1} + \gamma_{ab}) + \mathcal{C}(\gamma_{a2}) \end{aligned} \tag{3.10}$$

$$\begin{aligned} I(X_a, X_{r1}, X_{r2}; Y_b | X_b) &\leq \mathcal{C}(\sqrt{\gamma_{b1}} + \sqrt{\gamma_{b2}} + \sqrt{\gamma_{ab}})^2 \\ I(X_b, X_{r1}, X_{r2}; Y_a | X_a) &\leq \mathcal{C}(\sqrt{\gamma_{a1}} + \sqrt{\gamma_{a2}} + \sqrt{\gamma_{ab}})^2 \end{aligned} \tag{3.11}$$

Finally, we can obtain each point on the outer bound of the rate region by solving the following optimization problem. Maximize R_a keeping the ratio of R_a and R_b fixed and satisfying the following constraints :

$$
\begin{aligned}
R_a &\le \mathcal{C}(\gamma_{a1} + \gamma_{a2} + \gamma_{ab}) \\
R_a &\le \mathcal{C}(\sqrt{\gamma_{b1}} + \sqrt{\gamma_{b2}} + \sqrt{\gamma_{ab}})^2 \\
R_a &\le \mathcal{C}(\gamma_{a2} + \gamma_{ab}) + \mathcal{C}(\gamma_{b1}) \\
R_a &\le \mathcal{C}(\gamma_{b2}) + \mathcal{C}(\gamma_{a1} + \gamma_{ab}) \\
R_b &\le \mathcal{C}(\gamma_{b1} + \gamma_{b2} + \gamma_{ab}) \\
R_b &\le \mathcal{C}(\sqrt{\gamma_{a1}} + \sqrt{\gamma_{a2}} + \sqrt{\gamma_{ab}})^2 \\
R_b &\le \mathcal{C}(\gamma_{a1}) + \mathcal{C}(\gamma_{b2} + \gamma_{ab}) \\
R_b &\le \mathcal{C}(\gamma_{b1} + \gamma_{ab}) + \mathcal{C}(\gamma_{a2})
\end{aligned} \tag{3.12}
$$

The complete outer bound region can be evaluated by solving the above problem for different values of the ratio of R_a and R_b.

3.2.3 Outer bounds for Gaussian diamond relay channel with direct link between A and B-Half duplex

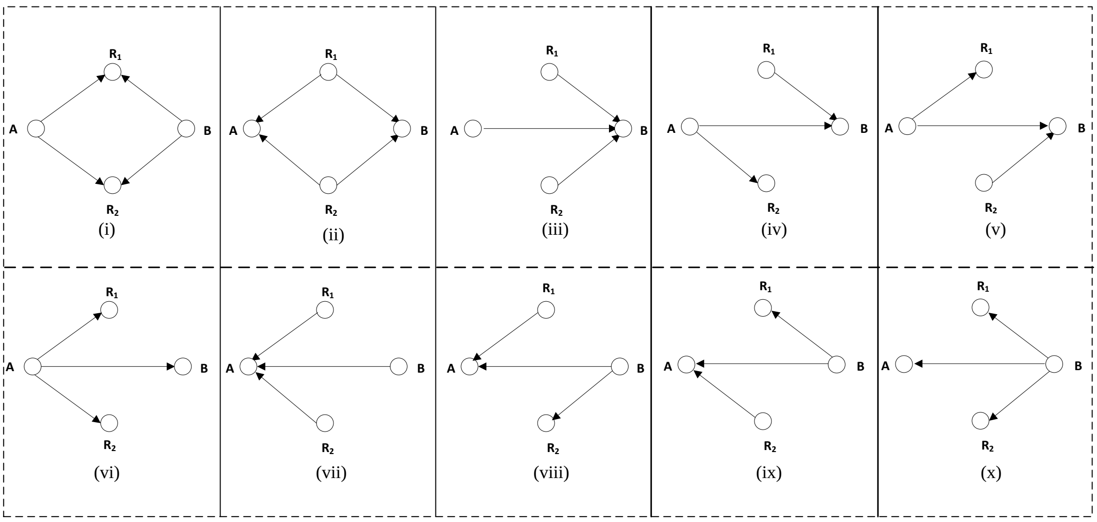


Figure 3.6: Possible states in a Half-duplex Two-way Diamond Relay Channel with direct A-B link

An outer bound for the capacity region of the half-duplex two-way Gaussian diamond relay channel is derived in [11] using the half-duplex cut-set bound and is as follows:

$$
\begin{aligned}
& R_a \leq \lambda_1 \mathcal{C}(\gamma_{a1} + \gamma_{a2}) + \lambda_3 \mathcal{C}(\gamma_{ab}) + \lambda_4 \mathcal{C}(\gamma_{a2} + \gamma_{ab}) \\
& \quad + \lambda_5 \mathcal{C}(\gamma_{a1} + \gamma_{ab}) + \lambda_6 \mathcal{C}(\gamma_{a1} + \gamma_{a2} + \gamma_{ab}) \\
& R_a \leq \lambda_1 \mathcal{C}(\gamma_{a2}) + \lambda_2 \mathcal{C}(\gamma_{b1}) + \lambda_3 \mathcal{C}(\sqrt{\gamma_{ab}} + \sqrt{\gamma_{b1}})^2 + \\
& \quad \lambda_4 \mathcal{C}(\gamma_{b1}) + \lambda_4 \mathcal{C}(\gamma_{a2} + \gamma_{ab}) + \lambda_6 \mathcal{C}(\gamma_{a2} + \gamma_{ab}) + \lambda_5 \mathcal{C}(\gamma_{ab}) \\
& R_a \leq \lambda_1 \mathcal{C}(\gamma_{a1}) + \lambda_2 \mathcal{C}(\gamma_{b2}) + \lambda_3 \mathcal{C}(\sqrt{\gamma_{ab}} + \sqrt{\gamma_{b2}})^2 + \\
& \quad \lambda_4 \mathcal{C}(\gamma_{ab} + \lambda_5 \mathcal{C}(\gamma_{b2}) + \lambda_5 \mathcal{C}(\gamma_{a1} + \gamma_{ab}) + \lambda_6 \mathcal{C}(\gamma_{a1} + \gamma_{ab}) \\
& R_a \leq \lambda_2 \mathcal{C}(\sqrt{\gamma_{b1}} + \sqrt{\gamma_{b2}})^2 + \lambda_3 \mathcal{C}(\sqrt{\gamma_{b1}} + \sqrt{\gamma_{b2}} + \sqrt{\gamma_{ab}})^2 + \\
& \quad \lambda_4 \mathcal{C}(\sqrt{\gamma_{b1}} + \sqrt{\gamma_{ab}})^2 + \lambda_5 \mathcal{C}(\sqrt{\gamma_{ab}} + \sqrt{\gamma_{b2}})^2 + \lambda_6 \mathcal{C}(\gamma_{ab}) \\
& R_b \leq \lambda_1 \mathcal{C}(\gamma_{b1} + \gamma_{b2}) + \lambda_7 \mathcal{C}(\gamma_{ab}) + \lambda_8 \mathcal{C}(\gamma_{b2} + \gamma_{ab}) + \\
& \quad \lambda_9 \mathcal{C}(\gamma_{b1} + \gamma_{ab}) + \lambda_{10} \mathcal{C}(\gamma_{b1} + \gamma_{b2} + \gamma_{ab}) \\
& R_b \leq \lambda_1 \mathcal{C}(\gamma_{b2}) + \lambda_2 \mathcal{C}(\gamma_{a1}) + \lambda_7 \mathcal{C}(\sqrt{\gamma_{ab}} + \sqrt{\gamma_{a1}})^2 + \\
& \quad \lambda_8 \mathcal{C}(\gamma_{a1}) + \lambda_8 \mathcal{C}(\gamma_{b2} + \gamma_{ab}) + \lambda_{10} \mathcal{C}(\gamma_{b2} + \gamma_{ab}) + \lambda_9 \mathcal{C}(\gamma_{ab}) \\
& R_b \leq \lambda_1 \mathcal{C}(\gamma_{b1}) + \lambda_2 \mathcal{C}(\gamma_{a2}) + \lambda_7 \mathcal{C}(\sqrt{\gamma_{ab}} + \sqrt{\gamma_{a2}})^2 + \\
& \quad \lambda_8 \mathcal{C}(\gamma_{ab}) + \lambda_9 \mathcal{C}(\gamma_{a2}) + \lambda_9 \mathcal{C}(\gamma_{b1} + \gamma_{ab}) + \lambda_{10} \mathcal{C}(\gamma_{b1} + \gamma_{ab}) \\
& R_b \leq \lambda_2 \mathcal{C}(\sqrt{\gamma_{a1}} + \sqrt{\gamma_{a2}})^2 + \lambda_7 \mathcal{C}(\sqrt{\gamma_{a1}} + \sqrt{\gamma_{a2}} + \sqrt{\gamma_{ab}})^2 + \\
& \quad \lambda_8 \mathcal{C}(\sqrt{\gamma_{a1}} + \sqrt{\gamma_{ab}})^2 + \lambda_9 \mathcal{C}(\sqrt{\gamma_{ab}} + \sqrt{\gamma_{a2}})^2 + \lambda_{10} \mathcal{C}(\gamma_{ab}) \\
& \quad \sum_{i=1}^{10} \lambda_i = 1, 0 \leq \lambda_i \leq 1
\end{aligned} \tag{3.13}
$$

The complete outer bound region can be evaluated by solving the above problem for different values of the ratio of R_a and R_b.

3.3 Gaussian diamond relay channel with direct link between Relays

3.3.1 System Model

The diamond relay channel with direct link between the relays is shown in Fig3.7. The system model is similar to Section 3.1, but for an additional link between R_1 and R_2.

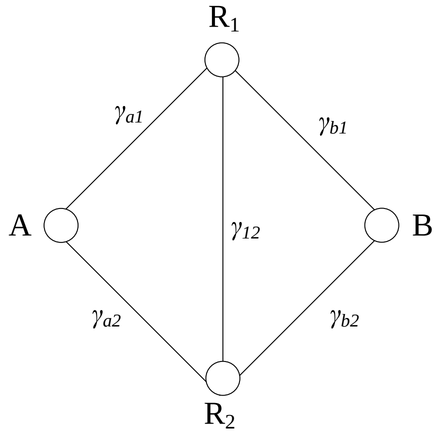


Figure 3.7: Two-way Gaussian Relay Channel with Direct Link between Relays

The SNR of the direct link is denoted as $\gamma_d = \frac{h_d^2 P}{N}$. The two relays are no longer non-interfering. A relay, say R_2 can treat the information from other relay R_1 in two ways. One way is to consider it as interference, and the other way is to consider it as a message to be decoded and forwarded to the destination.

3.3.2 Outer bounds for Gaussian diamond relay channel with direct link between R1 and R2-Full duplex

The outer bound for the two flows is derived considering four cuts as shown in fig 3.8that separate nodes a and b and information flow in both directions across the cuts. Combining these eight bounds (4 cuts $\times$ 2 directions), we get the rate region outer bound.

$$\begin{aligned} R_a \leq \min\{ & I\left(X_a; Y_{r1}, Y_{r2}, Yb | X_b, X_{r1}, X_{r2}\right), I\left(X_a, X_{r1}; Y_b, Y_{r2} | X_b, X_{r2}\right), \\ & I\left(X_a, X_{r2}; Y_b, Y_{r1} | X_b, X_{r1}\right), I\left(X_a, X_{r1}, X_{r2}; Y_b | X_b\right)\} \\ R_b \leq \min\{ & I\left(X_b; Y_{r1}, Y_{r2}, Ya | X_a, X_{r1}, X_{r2}\right), I\left(X_b, X_{r1}; Y_a, Y_{r2} | X_a, X_{r2}\right), \\ & I\left(X_b, X_{r2}; Y_a, Y_{r1} | X_a, X_{r1}\right), I\left(X_b, X_{r1}, X_{r2}; Y_a | X_a\right)\} \end{aligned} \tag{3.14}$$

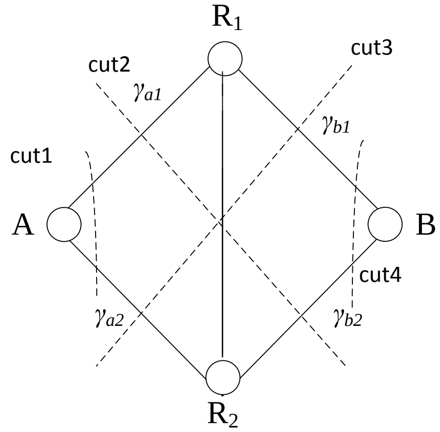


Figure 3.8: Cuts used for finding the bound

Now, we can upper bound the various mutual information terms to get:

$$I\left(X_a; Y_{r1}, Y_{r2}, Y_b | X_b, X_{r1}, X_{r2}\right) \leq \mathcal{C}(\gamma_{a1} + \gamma_{a2})$$
$$I\left(X_b; Y_{r1}, Y_{r2}, Y_a | X_a, X_{r1}, X_{r2}\right) \leq \mathcal{C}(\gamma_{b1} + \gamma_{b2}) \tag{3.15}$$

$$I\left(X_a, X_{r1}; Y_b, Y_{r2} | X_b, X_{r2}\right) \leq \mathcal{C}(\gamma_{a2}) + \mathcal{C}(\gamma_{b1} + \gamma_d)$$
$$I\left(X_a, X_{r2}; Y_b, Y_{r1} | X_b, X_{r1}\right) \leq \mathcal{C}(\gamma_{b2} + \gamma_d) + \mathcal{C}(\gamma_{a1})$$
$$I\left(X_b, X_{r1}; Y_a, Y_{r2} | X_a, X_{r2}\right) \leq \mathcal{C}(\gamma_{a1} + \gamma_d) + \mathcal{C}(\gamma_{b2})$$
$$I\left(X_b, X_{r2}; Y_a, Y_{r1} | X_a, X_{r1}\right) \leq \mathcal{C}(\gamma_{b1}) + \mathcal{C}(\gamma_{a2} + \gamma_d) \tag{3.16}$$

$$I\left(X_a, X_{r1}, X_{r2}; Y_b | X_b\right) \leq \mathcal{C}(\sqrt{\gamma_{b1}} + \sqrt{\gamma_{b2}})^2$$
$$I\left(X_b, X_{r1}, X_{r2}; Y_a | X_a\right) \leq \mathcal{C}(\sqrt{\gamma_{a1}} + \sqrt{\gamma_{a2}})^2 \tag{3.17}$$

We can obtain each point on the outer bound of the rate region by solving the following optimization problem. Maximize R_a keeping the ratio of R_a and R_b fixed and satisfying the following constraints :

$$\begin{aligned}
R_a &\le \mathcal{C}(\gamma_{a1} + \gamma_{a2}) \\
R_a &\le \mathcal{C}(\sqrt{\gamma_{b1}} + \sqrt{\gamma_{b2}})^2 \\
R_a &\le \mathcal{C}(\gamma_{a2}) + \mathcal{C}(\gamma_{b1} + \gamma_d) \\
R_a &\le \mathcal{C}(\gamma_{b2} + \gamma_d) + \mathcal{C}(\gamma_{a1}) \\
R_b &\le \mathcal{C}(\gamma_{b1} + \gamma_{b2}) \\
R_b &\le \mathcal{C}(\sqrt{\gamma_{a1}} + \sqrt{\gamma_{a2}})^2 \\
R_b &\le \mathcal{C}(\gamma_{a1} + \gamma_d) + \mathcal{C}(\gamma_{b2}) \\
R_b &\le \mathcal{C}(\gamma_{b1}) + \mathcal{C}(\gamma_{a2} + \gamma_d)
\end{aligned} \tag{3.18}$$

The complete outer bound region can be evaluated by solving the above problem for different values of the ratio of R_a and R_b.

3.3.3 Outer bounds for Gaussian diamond relay channel with direct link between R1 and R2-Half duplex

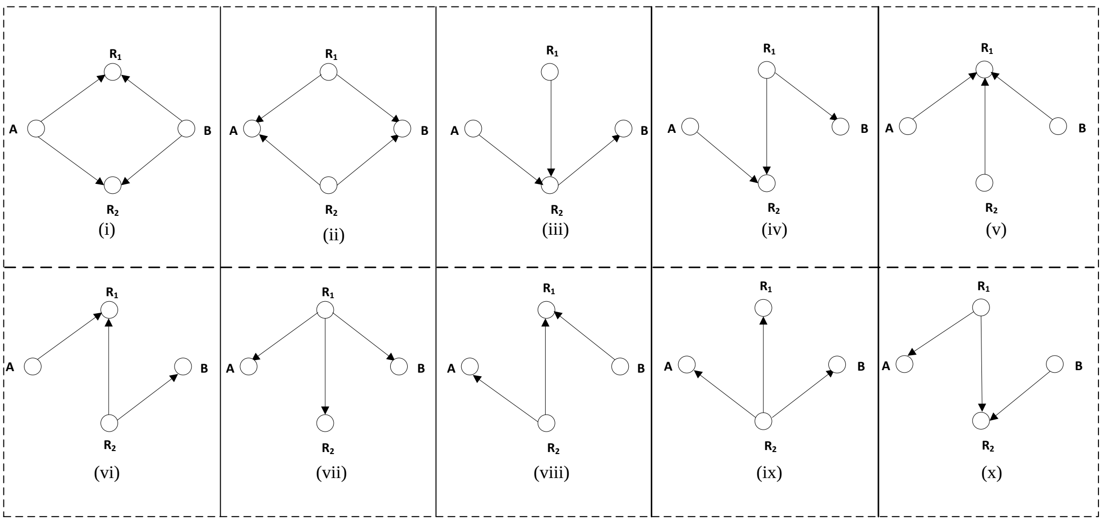


Figure 3.9: Possible states in a Half-duplex Two-way Diamond Relay Channel with direct link between relays

An outer bound for the capacity region of the half-duplex two-way Gaussian relay channel is derived in [11] and is as follows:

$$
\begin{aligned}
&R_a \leq \lambda_1\mathcal{C}(\gamma_{a1}+\gamma_{a2}) + \lambda_3\mathcal{C}(\gamma_{a2}) + \lambda_4\mathcal{C}(\gamma_{a2}) + \lambda_5\mathcal{C}(\gamma_{a1}) + \lambda_6\mathcal{C}(\gamma_{a1})\\
&R_a \leq \lambda_1\mathcal{C}(\gamma_{a2}) + \lambda_2\mathcal{C}(\gamma_{b1}) + \lambda_3\mathcal{C}(\sqrt{\gamma_d}+\sqrt{\gamma_{a2}})^2 + \lambda_4\mathcal{C}(\gamma_{a2}) + \lambda_4\mathcal{C}(\gamma_{b1}+\gamma_d) +\\
&\quad \lambda_7\mathcal{C}(\gamma_{b1}+\gamma_d) + \lambda_{10}\mathcal{C}(\gamma_d)\\
&R_a \leq \lambda_1\mathcal{C}(\gamma_{a1}) + \lambda_2\mathcal{C}(\gamma_{b2}) + \lambda_5\mathcal{C}(\sqrt{\gamma_d}+\sqrt{\gamma_{a1}})^2 + \lambda_6\mathcal{C}(\gamma_{a1}) + \lambda_6\mathcal{C}(\gamma_{b2}+\gamma_d) +\\
&\quad \lambda_9\mathcal{C}(\gamma_{b2}+\gamma_d) + \lambda_8\mathcal{C}(\gamma_d)\\
&R_a \leq \lambda_2\mathcal{C}(\sqrt{\gamma_{b1}}+\sqrt{\gamma_{b2}})^2 + \lambda_4\mathcal{C}(\gamma_{b1}) + \lambda_7\mathcal{C}(\gamma_{b1}) + \lambda_9\mathcal{C}(\gamma_{b2}) + \lambda_6\mathcal{C}(\gamma_{b2})\\
&R_b \leq \lambda_1\mathcal{C}(\gamma_{b1}+\gamma_{b2}) + \lambda_3\mathcal{C}(\gamma_{b2}) + \lambda_{10}\mathcal{C}(\gamma_{b2}) + \lambda_5\mathcal{C}(\gamma_{b1}) + \lambda_8\mathcal{C}(\gamma_{b1})\\
&R_b \leq \lambda_1\mathcal{C}(\gamma_{b2}) + \lambda_2\mathcal{C}(\gamma_{a1}) + \lambda_3\mathcal{C}(\sqrt{\gamma_d}+\sqrt{\gamma_{b2}})^2 + \lambda_4\mathcal{C}(\gamma_d) + \lambda_7\mathcal{C}(\gamma_{a1}+\gamma_d) +\\
&\quad \lambda_{10}\mathcal{C}(\gamma_{a1}+\gamma_d) + \lambda_{10}\mathcal{C}(\gamma_{b2})\\
&R_b \leq \lambda_1\mathcal{C}(\gamma_{b1}) + \lambda_2\mathcal{C}(\gamma_{a2}) + \lambda_5\mathcal{C}(\sqrt{\gamma_d}+\sqrt{\gamma_{b1}})^2 + \lambda_6\mathcal{C}(\gamma_d) + \lambda_9\mathcal{C}(\gamma_{a2}+\gamma_d) +\\
&\quad \lambda_8\mathcal{C}(\gamma_{a2}+\gamma_d) + \lambda_8\mathcal{C}(\gamma_{b1})\\
&R_b \leq \lambda_2\mathcal{C}(\sqrt{\gamma_{a1}}+\sqrt{\gamma_{a2}})^2 + \lambda_{10}\mathcal{C}(\gamma_{a1}) + \lambda_7\mathcal{C}(\gamma_{a1}) + \lambda_9\mathcal{C}(\gamma_{a2}) + \lambda_8\mathcal{C}(\gamma_{a2})\\
&\quad \sum_{i=1}^{10}\lambda_i = 1, 0 \leq \lambda_i \leq 1
\end{aligned}
\tag{3.19}
$$

The complete outer bound region can be evaluated by solving the above problem for different values of the ratio of R_a and R_b.

CHAPTER 4

SCALE AND FORWARD PROTOCOL

We consider the half duplex gaussian diamond relay channel with no direct link as in Section 3.1. Let h_{a1} , h_{a2} , h_{b1} and h_{b2} be the gains of channels a-r_1 , a-r_2, b-r_1 and b-$r_2$respectively. The relays R_1 and R_2 receive signals from the two sources A and B during the first half. In the second half each relay node scales sum of its received signals by multiplying with a complex constant, and sends it out in Stage 2. The constants are chosen such that the signals from the two relays add up coherently at the destination, maximizing the end-to-end SNR. Since each sender knows the signal that it has sent and also knows the channel coefficients, it is possible at the destination to remove the signal sent by it in the first half. This scheme can be described as

$$\begin{aligned} & X_1 = a_1 Y_1, X_2 = a_2 Y_2 \\ st \qquad & E||X_1||^2 \leq P, E||X_2||^2 \leq P \end{aligned} \tag{4.1}$$

4.1 Theorem

For a bidirectional gaussian diamond relay channel, the rate under scale and forward is

$$\begin{aligned} Ra &= 0.5\log(1 + \frac{(a||h_{b1}h_{a1}|| + b||h_{b2}h_{a2}||)^2)}{a^2(||h_{b1}||)^2 + b^2(||h_{b2}||)^2 + 1}) \\ Rb &= 0.5\log(1 + \frac{(a||h_{b1}h_{a1}|| + b||h_{b2}h_{a2}||)^2)}{a^2(||h_{a1}||)^2 + b^2(||h_{a2}||)^2 + 1}) \\ st \qquad & a^2(||h_{a1}||^2 + ||h_{b1}||^2)P + \sigma^2) \leq P \\ & b^2(||h_{a2}||^2 + ||h_{b2}||^2)P + \sigma^2) \leq P \end{aligned} \tag{4.2}$$

4.1.1 Assumptions:

All the nodes are half duplex. The channels are assumed to be reciprocal. Channel coefficients are known at all the nodes. All nodes transmit with same power P and have same noise variance. However the protocol can be generalized to the case of arbitrary powers at the nodes.

4.1.2 Proof

Let Y_a be the received signal at A and let Y_b be the received signal at B. It is easy to show that Y_a=($h_{a1}a_1h_{a1}+h_{a2}a_2h_{a2}$) X_a+($h_{a1}a_1h_{b1}+h_{a2}a_2h_{b2}$) X_b+($h_{a1}a_1N_1$+ $h_{a2}a_2N_2$+ N_3) and Y_b=($h_{b1}a_1h_{b1} + h_{b2}a_2h_{b2}$) X_b+($h_{a1}a_1h_{b1} + h_{a2}a_2h_{b2}$) X_b+($h_{b1}a_1N_1$+ $h_{b2}a_2N_2$+ N_3) Since each sender knows the signal that it has sent and also knows the channel coefficients, it is possible at the destination to remove the signal sent by it in the first half. In order that the two relay signals add up coherently, a1, a2 should be of the form $a_1 = a(h_{a1}h_{b1})^*/||(h_{a1}h_{b1})||; a_2 = b(h_{a2}h_{b2})^*/||(h_{a2}h_{b2})||$ with a, b being positive. After cancelling the message sent by itself, we have $Ya = (||h_{a1}h_{b1}||a + ||h_{a2}h_{b2}||b)X_b + (h_{a1}a_1N_1 + h_{a2}a_2N_2 + N_3)$. By Shannon's formula, the end-to-end achievable rate is as given in the above theorem where the factor 0.5 is due to the equal time-sharing.

4.1.3 Remarks

Consider R_a. Denote $\alpha = a||h_{b1}||$ and $\beta = b||h_{b2}||$. The maximum achievable rate thus becomes

$$\max_{\alpha,\beta} 0.5\log(1 + \frac{(\alpha||h_{a1}|| + \beta||h_{a2}||)^2P}{(\alpha^2 + \beta^2 + 1)\sigma^2}) \tag{4.3}$$

We note that

$$\begin{aligned}&(\alpha||h_{a1}|| + \beta||h_{a2}||)^2/(\alpha^2 + \beta^2 + 1)\\ &\leq (\alpha^2 + \beta^2)(||h_{a1}||^2 + ||h_{a2}||)^2)/(\alpha^2 + \beta^2 + 1))\\ &\leq (||h_{a1}||^2 + ||h_{a2}||)^2\end{aligned} \tag{4.4}$$

with the first inequality becoming equality only if

$$\alpha/\beta = ||h_{a1}||/||h_{a2}|| \;\; i.e \;\; a/b = ||h_{a1}||||h_{b2}||/||h_{a2}||||h_{b1}|| \tag{4.5}$$

That is, the effective SNR from a relay to the destination should be proportional to its received SNR from the source. Similarly for R_b we have

$$a/b = ||h_{b1}||||h_{a2}||/||h_{a1}||||h_{b2}|| \tag{4.6}$$

We thus have the following observation. In a single source single destination case when the multi-access part of the network has better channel condition, i.e., in the case of R_a, when $P||h_{b1}||^2$ and $P||h_{b2}||^2$ are much larger than $P||h_{a1}||^2$ and $P||h_{a2}||^2$, the optimal strategy in scale-forward is to scale the signals so that $a/b = ||h_{a1}||||h_{b2}||/||h_{a2}||||h_{b1}||$. In the two source case if the power at the relay P_1 is such that $a^2(||h_{a1}||^2 + ||h_{b1}||^2)P + \sigma^2)$ and $b^2(||h_{a2}||^2+||h_{b2}||^2)P+\sigma^2)$ is much less than P_1 then Ra is maximized by $a/b = ||h_{a1}||||h_{b2}||/||h_{a2}||||h_{b1}||$ and Rb is maximized by $a/b = ||h_{b1}||||h_{a2}||/||h_{a1}||||h_{b2}||$

We compare the achievable rates of this protocol with other known protocols in Chapter 7 which are described briefly below:

4.2 Other Protocols for half duplex diamond relay channel

4.2.1 MDF Protocol

Two-way MDF is a simple two-way protocol that uses the MDF protocol of [4] for both direction flows ($A \rightarrow B$ and $B \rightarrow A$) in a time-sharing manner. Thus, States 1-4 in Figure 3.3 will be used for communication from A to B, and States 5-8 will be used for communication from B to A. The two-way MDF uses Decode and Forward scheme at the relays. The maximum achievable rate for the one-way MDF protocol can be computed as in [4]. Suppose this rate for communication from A to B is R_{a-mdf} and for communication from B to A is R_{b-mdf} , then the achievable rate region for the two-way MDF protocol is the triangular region enclosed by the three straight lines: $(1)Ra = 0, (2)Rb = 0$, and (3) the line joining $(0; R_{b-mdf})$ and $(R_{a-mdf}; 0)$.

To find R_{a-mdf} we use

$$
\begin{aligned}
maximize\ \ R_1 + R_2 \\
subject\ \ to: \\
R_1 &\leq \lambda_1 u + \lambda_2 \mathcal{C}(\gamma_{a1}) \\
R_1 &\leq \lambda_3 \mathcal{C}(\gamma_{b1}) + Z_1 \\
R_2 &\leq \lambda_1 v + \lambda_3 \mathcal{C}(\gamma_{a2}) \\
R_2 &\leq \lambda_2 \mathcal{C}(\gamma_{b2}) + Z_2 \\
Z_1 &\leq \lambda_4 \mathcal{C}(\gamma_{b1}) \\
Z_2 &\leq \lambda_4 \mathcal{C}(\gamma_{b2}) \\
Z_1 + Z_2 &\leq \lambda_4 \mathcal{C}(\gamma_{b1} + \gamma_{b2}) \\
u &= \mathcal{C}(\eta\gamma_{a1}) \quad if \quad \gamma_{a2} \leq \gamma_{a1} \\
&= \mathcal{C}(\gamma_{a1}) - \mathcal{C}(\eta\gamma_{a1}) \quad if \quad \gamma_{a1} < \gamma_{a2} \\
v &= \mathcal{C}(\eta\gamma_{a2}) \quad if \quad \gamma_{a1} < \gamma_{a2} \\
&= \mathcal{C}(\gamma_{a2}) - \mathcal{C}(\eta\gamma_{a2}) \quad if \quad \gamma_{a2} < \gamma_{a1} \\
&\sum_{i=1}^{4} \lambda_i = 1, 0 \leq \lambda_i \leq 1, 0 \leq \eta \leq 1
\end{aligned}
\tag{4.7}
$$

Here u and v are rates associated with relays 1 and 2 respectively in broadcast state and η is power allocation parameter.

4.2.2 CF-CMAC Protocol

The Compute-and-forward-Compound MAC (CF-CMAC) protocol is a three state protocol using States 9, 10 and 13 of Figure 3.2. States 9 and 10 are Multiple Access Channels in which both the nodes A and B transmit to one of the relays. State 13 is an interference channel in which both the relays simultaneously transmit to both nodes A and B. States 9 and 10 employ nested lattice coding scheme as in [18]. This protocol employs a compute and forward strategy at the relays, making use of the fact that a relay need not decode all the information received by it, it only needs to forward sufficient information to enable the receiving nodes to decode correctly. The relays attempt to decode the sum of the messages received from A and B, instead of decoding the individual messages. This sum is forwarded to the two nodes in State 13.The achievable rate for this protocol is derived in [11] by solving the following optimization problem. Maximize R_a keeping the ratio of R_a and R_b fixed and satisfying the following constraints :

$$\begin{aligned}
R_a &= Z^9_{ar_1} + Z^{10}_{ar_2} \\
R_b &= Z^9_{br_1} + Z^{10}_{br_2} \\
Z^9_{ar_1} &= Z^{13}_{r_1 b} \\
Z^9_{ar_2} &= Z^{13}_{r_2 b} \\
Z^9_{br_1} &= Z^{13}_{r_1 a} \\
Z^9_{br_2} &= Z^{13}_{r_2 a} \\
Z^9_{ar_1} &= \lambda_9[\mathcal{C}(\gamma_{a1} - \frac{\gamma_{b1}}{\gamma_{a1} + \gamma_{b1}})]^+ \\
Z^9_{br_1} &= \lambda_9[\mathcal{C}(\gamma_{b1} - \frac{\gamma_{a1}}{\gamma_{a1} + \gamma_{b1}})]^+ \\
Z^{10}_{ar_2} &= \lambda_{10}[\mathcal{C}(\gamma_{a2} - \frac{\gamma_{b2}}{\gamma_{a2} + \gamma_{b2}})]^+ \\
Z^{10}_{br_2} &= \lambda_{10}[\mathcal{C}(\gamma_{b2} - \frac{\gamma_{a2}}{\gamma_{a2} + \gamma_{b2}})]^+ \\
Z^{13}_{r_1 b} &= \lambda_{13}\mathcal{C}(\gamma_{b1}) \\
Z^{13}_{r_1 a} &= \lambda_{13}\mathcal{C}(\gamma_{a1}) \\
Z^{13}_{r_2 a} &= \lambda_{13}\mathcal{C}(\gamma_{a2}) \\
Z^{13}_{r_2 b} &= \lambda_{13}\mathcal{C}(\gamma_{b2}) \\
\sum_{i=9,10,13} \lambda_i &= 1, 0 \leq \lambda_i \leq 1
\end{aligned} \tag{4.8}$$

where $[x]^+ \triangleq max(x, 0)$

4.2.3 CF-BC Protocol

The CF-BC (Compute and Forward-Broadcast Channel) protocol uses States 9-12. Only one relay is used in each state. States 9 and 10 are used the same way as in the CF-CMAC protocol, i.e, A and B simultaneously transmit to R1 in State 9 and to R2 in

State 10. The relays R1 and R2 compute the sum of messages received by them in states 9 and 10 and forward them to the end nodes in Broadcast States 11 and 12 respectively. The nodes can decode the messages meant from them using their apriori knowledge of the messages sent by them to the relays. This scheme is basically a time sharing of two-way relaying with one relay. The achievable rate for this protocol is derived in [11] by solving the following optimization problem. Maximize R_a keeping the ratio of R_a and R_b fixed and satisfying the following constraints :

$$\begin{aligned}
R_a &= Z_{ar_1}^9 + Z_{ar_2}^{10} \\
R_b &= Z_{br_1}^9 + Z_{br_2}^{10} \\
Z_{ar_1}^9 &= Z_{r_1b}^{11} \\
Z_{ar_2}^9 &= Z_{r_2b}^{12} \\
Z_{br_1}^9 &= Z_{r_1a}^{11} \\
Z_{br_2}^9 &= Z_{r_2a}^{12} \\
Z_{ar_1}^9 &= \lambda_9[\mathcal{C}(\gamma_{a1} - \frac{\gamma_{b1}}{\gamma_{a1}+\gamma_{b1}})]^+ \\
Z_{br_1}^9 &= \lambda_9[\mathcal{C}(\gamma_{b1} - \frac{\gamma_{a1}}{\gamma_{a1}+\gamma_{b1}})]^+ \\
Z_{ar_2}^{10} &= \lambda_{10}[\mathcal{C}(\gamma_{a2} - \frac{\gamma_{b2}}{\gamma_{a2}+\gamma_{b2}})]^+ \\
Z_{br_2}^{10} &= \lambda_{10}[\mathcal{C}(\gamma_{b2} - \frac{\gamma_{a2}}{\gamma_{a2}+\gamma_{b2}})]^+ \\
Z_{r_1b}^{13} &= \lambda_{11}\mathcal{C}(\gamma_{b1}) \\
Z_{r_1a}^{13} &= \lambda_{11}\mathcal{C}(\gamma_{a1}) \\
Z_{r_2a}^{13} &= \lambda_{12}\mathcal{C}(\gamma_{a2}) \\
Z_{r_2b}^{13} &= \lambda_{12}\mathcal{C}(\gamma_{b2}) \\
\sum_{i=9,10,11,12} \lambda_i &= 1, 0 \leq \lambda_i \leq 1
\end{aligned} \tag{4.9}$$

where $[x]^+ \triangleq max(x, 0)$

CHAPTER 5

OUTER BOUNDS FOR CAPACITY OF GAUSSIAN TWO-WAY RELAY CHANNEL AND GAUSSIAN DIAMOND RELAY CHANNEL FOR MIMO CASE

5.1 MIMO point to point channel

We model the MIMO point-to-point communication channel as a Gaussian vector channel, where the output of the channel Y corresponding to the input X is Y = GX+Z. Here Y is an r-dimensional vector, X is a t-dimensional vector , $Z \sim N(0, K_Z), K_Z > 0$, is an r-dimensional noise vector, and G is an $r \times t$ constant channel gain matrix with its element G_{j_k} representing the gain of the channel from transmitter antenna k to receiver antenna j. The channel is discrete-time and the noise vector process Z(i) is i.i.d. with $Z(i) \sim N(0, K_Z)$ for every transmission $i \epsilon [1 : n]$. We assume average transmission power constraint P on every codeword. We can assume without loss of generality that $K_Z = Ir$, since the channel Y = GX+Z with a general $K_Z > 0$ can be transformed into the channel $W = K_z^{-\frac{1}{2}} Y = K_z^{-\frac{1}{2}} GX + Z_1$ where $Z_1 = K_z^{-\frac{1}{2}} Z$and vice versa. Here $Z_1 \sim N(0, I_r)$.
The capacity of the point to point channel is [1]

$$\max_{\{K_z\}: trace(K_z) \leq P} \frac{1}{2} log(G K_z G^T + I_r) \tag{5.1}$$

5.2 Reciprocity

Since the channel gain matrices G and G^T have the same set of (nonzero) singular values, the channels with gain matrices G and G^T have the same capacity. This is evident from the following lemma For every $r \times t$ channel matrix G and $t \times t$ matrix $K_z > 0$, there exists an $r \times r$ matrix $\tilde{K} > 0$ such that $trace(\tilde{K}) \leq trace(K)$,

$$|G^T \tilde{K} G + I_t| = |G K G^T + I_r| \tag{5.2}$$

5.3 MIMO Multiple Access channel

Consider the MIMO multiple access communication system where each sender wishes to communicate an independent message to the receiver. Assume a Gaussian vector multiple access channel (GV-MAC) model $Y = G_1X_1 + G_2X_2 + Z$, where Y is an r-dimensional output vector, X_1 and X_2 are t-dimensional input vectors, G_1 and G_2 are $r \times t$ channel gain matrices, and $Z \sim N(0, K_Z), K_Z > 0$ is an r-dimensional noise vector. As before, we assume without loss of generality that $K_Z = I_r$. We further assume average power constraint P on each of X_1 and X_2. The capacity region of the multiple access channel is

$$\begin{aligned} R_1 &\leq \frac{1}{2} log(G_1K_1G_1^T + I_r) \\ R_2 &\leq \frac{1}{2} log(G_2K_2G_2^T + I_r) \\ R_1 + R_2 &\leq \frac{1}{2} log(G_1K_1G_1^T + G_2K_2G_2^T + I_r) \end{aligned} \tag{5.3}$$

The sum-capacity of the GV-MAC can be found by solving the maximization problem

$$\begin{aligned} & maximize \frac{1}{2} log(G_1K_1G_1^T + G_2K_2G_2^T + I_r) \\ & subject\ \ to \\ & trace(K_j) \leq P \\ & \qquad K_j \geq 0, j = 1, 2 \end{aligned} \tag{5.4}$$

5.4 MIMO Broadcast channel

Consider the MIMO Broadcast communication system where a sender wishes to communicate with more than one receiver. For the simple case of one sender, two receivers, assume a Gaussian vector broadcast channel model $Y_1 = G_1X_1 + Z_1, Y_2 = G_2X_2 + Z_2$ where Y_1 and Y_2 are r-dimensional output vectors at receiver 1 and 2 respectively, X is t-dimensional input vector, G_1 and G_2 are $r \times t$ channel gain matrices, and $Z_1 \sim N(0, K_1), K_1 > 0$ and $Z_2 \sim N(0, K_2), K_2 > 0$ are r-dimensional noise vectors. As before, we assume without loss of generality that $K_1 = K_2 = I_r$. We further assume average power constraint P on X_1 and X_2. Unlike the scalar Gaussian BC, the

Gaussian vector BC is not in general degraded, and the capacity region is known only in several special cases. Here,we use the Multiple Access Broadcast channel duality to compute the rate region of MIMO Broadcast channel.

5.5 Multiple Access Broadcast channel duality

Given the GV-BC (referred to as the original BC) with channel gain matrices G_1 and G_2 and power constraint P, consider a GV-MAC with channel gain matrices G_1^T and G_2^T(referred to as the dual MAC). Broadcast Multiple Access Duality Lemma states that the rate region of broadcast channel is the same as that of the dual MAC but under the sum-power constraint. The capacity region of the broadcast channel is thus

$$\begin{aligned} R_1 &\leq \frac{1}{2}log(G_1^T K_1 G_1 + I_r) \\ R_2 &\leq \frac{1}{2}log(G_2^T K_2 G_2 + I_r) \\ R_1 + R_2 &\leq \frac{1}{2}log(G_1^T K_1 G_1 + G_2^T K_2 G_2 + I_r) \\ Trace(K_1) + Trace(K_2) &\leq P \end{aligned} \tag{5.5}$$

The sum-capacity of the GV-MAC can be found by solving the maximization problem

$$\begin{aligned} & maximize \ \frac{1}{2}log(G_1^T K_1 G_1 + G_2^T K_2 G_2 + I_r) \\ & subject \ to \\ & trace(K_1) + trace(K_2) \leq P \\ & K_j \geq 0, j = 1, 2 \end{aligned} \tag{5.6}$$

For the MIMO case we use the cut-set bounds as in the single antenna case. The capacities are computed using the above results for point to point, broadcast and multiple access channels by SDP optimization. As an example, for a point to point channel capacity C,

$$
\begin{aligned}
C &\leq \sup_{\{F(x):E(XX^T)\leq P\}} I(X;Y) \\
&\leq \sup_{\{F(x):E(XX^T)\leq P\}} h(Y)-h(Z) \\
&\leq \max_{\{K_x\geq 0:trace(K_x)\leq P\}} \frac{1}{2} log(G_1^T K_x G_1 + I_r)
\end{aligned}
\tag{5.7}
$$

5.6 2 way relay channel with no direct link

5.6.1 System Model

The system is similar to section 2.1 except the fact that all nodes are MIMO. Let G_1 and G_2 be the $r \times t$ gain matrices of channels a-r and b-r respectively.
Let $\mathcal{C}(G_1) \triangleq \max_{\{K_x\geq 0:trace(K_x)\leq P\}} \frac{1}{2} log(G_1^T K_x G_1 + I_r)$ represents the capacity of a gaussian channel with channel gain matrix of G.

5.6.2 Outer bound-Full Duplex

$$
\begin{aligned}
R_a &\leq \min\{\mathcal{C}(G_1), \mathcal{C}(G_2)\} \\
R_b &\leq \min\{\mathcal{C}(G_1), \mathcal{C}(G_2)\}
\end{aligned}
\tag{5.8}
$$

5.6.3 Outer bound-Half Duplex

$$
\begin{aligned}
R_a &\leq \lambda_1 \mathcal{C}(G_1) \\
R_a &\leq \lambda_2 \mathcal{C}(G_2) \\
R_b &\leq \lambda_1 \mathcal{C}(G_2) \\
R_b &\leq \lambda_2 \mathcal{C}(G_1) \\
\sum_{i=1}^{2} \lambda_i &= 1, 0 \leq \lambda_i \leq 1
\end{aligned}
\tag{5.9}
$$

5.7 2 way relay channel with direct link

5.7.1 System Model

The system is similar to section 2.2 except the fact that all nodes are MIMO. Let G_1 and G_2 be the $r \times t$ gain matrices of channels a-r and b-r respectively. Let G_3 be the $t \times t$ gain matrix of channels a-b.
Let $\mathcal{C}(G_1) \triangleq \max_{\{K_x \geq 0: trace(K_x) \leq P\}} \frac{1}{2} log(G_1^T K_x G_1 + I_r)$ represents the capacity of a gaussian channel with channel gain matrix of G.
Let $\mathcal{C}_{\mathcal{BC}}(G_1, G_2) \triangleq \max_{\{K_i \geq 0, i=1,2: trace(K_1)+trace(K_2) \leq P\}} \frac{1}{2} log(G_1^T K_1 G_1 + G_2^T K_2 G_2 + I_r)$ represents the capacity of a gaussian broadcast channel with channel gain matrix of G_1 and G_2.
Let $\mathcal{C}_{\mathcal{MAC}}(G_1, G_2) \triangleq \max_{\{K_i \geq 0: trace(K_i) \leq P, i=1,2\}} \frac{1}{2} log(G_1 K_1 G_1^T + G_2 K_2 G_2^T + I_r)$ represents the capacity of a gaussian multiple access channel with channel gain matrix of G_1 and G_2.

5.7.2 Outer bound for Gaussian relay channel with direct link between A and B-Full Duplex

$$
\begin{aligned}
R_a &\leq \mathcal{C}_{\mathcal{BC}}(G_1, G_3) \\
R_a &\leq \mathcal{C}_{\mathcal{MAC}}(G_2^T, G_3) \\
R_b &\leq \mathcal{C}_{\mathcal{BC}}(G_2, G_3^T) \\
R_b &\leq \mathcal{C}_{\mathcal{MAC}}(G_1^T, G_3^T)
\end{aligned}
\tag{5.10}
$$

The complete outer bound region can be evaluated by solving the above problem for different values of the ratio of R_a and R_b.

5.7.3 Outer bounds for Gaussian relay channel with direct link between A and B-Half duplex

$$\begin{aligned}
R_a &\leq \lambda_1 \mathcal{C}_{\mathcal{BC}}(G_1, G_3) + \lambda_3 \mathcal{C}(G_1) + \lambda_5 \mathcal{C}(G_3) \\
R_a &\leq \lambda_1 \mathcal{C}(G_3) + \lambda_4 \mathcal{C}(G_2) + \lambda_5 \mathcal{C}_{\mathcal{MAC}}(G_2^T, G_3) \\
R_b &\leq \lambda_2 \mathcal{C}_{\mathcal{BC}}(G_2, G_3^T) + \lambda_3 \mathcal{C}(G_2) + \lambda_6 \mathcal{C}(G_3) \\
R_b &\leq \lambda_2 \mathcal{C}(G_3) + \lambda_4 \mathcal{C}(G_1) + \lambda_6 \mathcal{C}_{\mathcal{MAC}}(G_1^T, G_3^T) \\
&\sum_{i=1}^{6} \lambda_i = 1, 0 \leq \lambda_i \leq 1
\end{aligned} \tag{5.11}$$

The complete outer bound region can be evaluated by solving the above problem for different values of the ratio of R_a and R_b.

5.8 Gaussian diamond relay channel with no direct link

5.8.1 System Model

The system is similar to section 3.1 except the fact that all nodes are MIMO. Let G_{a1}, G_{a2}, G_{b1} and G_{b2} be the $r \times t$ gain matrices of channels a-r_1 ,a-r_2, b-r_1 and b-r_2 respectively.
Let $\mathcal{C}(G_1) \triangleq \max_{\{K_x \geq 0: trace(K_x) \leq P\}} \frac{1}{2} log(G_1^T K_x G_1 + I_r)$ represents the capacity of a gaussian channel with channel gain matrix of G_1.
Let $\mathcal{C}_{\mathcal{BC}}(G_1, G_2) \triangleq \max_{\{K_i \geq 0, i=1,2: trace(K_1)+trace(K_2) \leq P\}} \frac{1}{2} log(G_1^T K_1 G_1 + G_2^T K_2 G_2 + I_r)$ represents the capacity of a gaussian broadcast channel with channel gain matrix of G_1 and G_2.
Let $\mathcal{C}_{\mathcal{MAC}}(G_1, G_2) \triangleq \max_{\{K_i \geq 0: trace(K_i) \leq P, i=1,2\}} \frac{1}{2} log(G_1 K_1 G_1^T + G_2 K_2 G_2^T + I_r)$ represents the capacity of a gaussian multiple access channel with channel gain matrix of G_1 and G_2.

5.8.2 Outer bound for Gaussian diamond relay channel with no direct link -Full Duplex

$$\begin{aligned}
R_a &\leq \mathcal{C}_{\mathcal{BC}}(G_{a1}, G_{a2}) \\
R_a &\leq \mathcal{C}_{\mathcal{MAC}}(G_{b1}^T, G_{b2}^T) \\
R_a &\leq \mathcal{C}(G_{a2}) + \mathcal{C}(G_{b1}) \\
R_a &\leq \mathcal{C}(G_{b2}) + \mathcal{C}(G_{a1}) \\
R_b &\leq \mathcal{C}_{\mathcal{BC}}(G_{b1}, G_{b2}) \\
R_b &\leq \mathcal{C}_{\mathcal{MAC}}(G_{a1}^T, G_{a2}^T) \\
R_b &\leq \mathcal{C}(G_{a2}) + \mathcal{C}(G_{b1}) \\
R_b &\leq \mathcal{C}(G_{b2}) + \mathcal{C}(G_{a1})
\end{aligned} \tag{5.12}$$

The complete outer bound region can be evaluated by solving the above problem for different values of the ratio of R_a and R_b.

5.8.3 Outer bound for Gaussian diamond relay channel with no direct link -Half Duplex

$$\begin{aligned}
R_a &\leq \lambda_1 \mathcal{C}_{\mathcal{BC}}(G_{a1}, G_{a2}) + \lambda_3 \mathcal{C}(G_{a1}) + \lambda_4 \mathcal{C}(G_{a2}) \\
R_a &\leq \lambda_1 \mathcal{C}(G_{a2}) + \lambda_2 \mathcal{C}(G_{b1}) + \lambda_4 \mathcal{C}(G_{a2}) + \lambda_4 \mathcal{C}(G_{b1}) \\
R_a &\leq \lambda_1 \mathcal{C}(G_{a1}) + \lambda_2 \mathcal{C}(G_{b2}) + \lambda_3 \mathcal{C}(G_{a1}) + \lambda_3 \mathcal{C}(G_{b2}) \\
R_a &\leq \lambda_2 \mathcal{C}_{\mathcal{MAC}}(G_{b1}^T, G_{b2}^T) + \lambda_3 \mathcal{C}(G_{b2}) + \lambda_4 \mathcal{C}(G_{b1}) \\
R_b &\leq \lambda_1 \mathcal{C}_{\mathcal{B}} \mathcal{C}(G_{b1}, G_{b2}) + \lambda_5 \mathcal{C}(G_{b1}) \lambda_6 \mathcal{C}(G_{b2}) \\
R_b &\leq \lambda_1 \mathcal{C}(G_{b2}) + \lambda_2 \mathcal{C}(G_{a1}) + \lambda_6 \mathcal{C}(G_{b2}) + \lambda_6 \mathcal{C}(G_{a1}) \\
R_b &\leq \lambda_1 \mathcal{C}(G_{b1}) + \lambda_2 \mathcal{C}(G_{a2}) + \lambda_5 \mathcal{C}(G_{b1}) + \lambda_5 \mathcal{C}(G_{a2}) \\
R_b &\leq \lambda_2 \mathcal{C}_{\mathcal{MAC}}(G_{a1}^T, G_{a2}^T) + \lambda_5 \mathcal{C}(G_{a2}) + \lambda_6 \mathcal{C}(G_{a1}) \\
&\sum_{i=1}^{6} \lambda_i = 1, 0 \leq \lambda_i \leq 1
\end{aligned} \tag{5.13}$$

The complete outer bound region can be evaluated by solving the above problem for different values of the ratio of R_a and R_b.

5.9 Gaussian diamond relay channel with direct A-B link

5.9.1 System Model

The system is similar to section 3.1 except the fact that all nodes are MIMO. Let G_{11}, G_{12}, G_{21} and G_{22} be the $r \times t$ gain matrices of channels a-r_1 ,a-r_2 b-r_1 and b-r_2 respectively. Let G_d be the $t \times t$ gain matrix of channels a-b .
Let $\mathcal{C}(G_1) \triangleq \max_{\{K_x \geq 0: trace(K_x) \leq P\}} \frac{1}{2} log(G_1^T K_x G_1 + I_r)$ represents the capacity of a gaussian channel with channel gain matrix of G.
Let $\mathcal{C}_{\mathcal{BC}}(G_1, ..., G_n) \triangleq \max_{\{K_i \geq 0, i=1..n: \sum_{i=1}^{n} trace(K_i) \leq P\}} \frac{1}{2} log(\sum_{i=1}^{n} G_i^T K_i G_i + I_r)$ represents the capacity of a gaussian broadcast channel with channel gain matrix of G_1 ,G_2 to G_n.
Let $\mathcal{C}_{\mathcal{MAC}}(G_1, .., G_n) \triangleq \max_{\{K_i \geq 0: trace(K_i) \leq P, i=1,..n\}} \frac{1}{2} log(\sum_{i=1}^{n} G_i K_i G_i^T + I_r)$ represents the capacity of a gaussian multiple access channel with channel gain matrix of G_1,G_2 to G_n

5.9.2 Outer bound for Gaussian diamond relay channel with direct link between A and B-Full Duplex

$$\begin{aligned}
R_a &\leq \mathcal{C}_{\mathcal{BC}}(G_{a1}, G_{a2}, G_{ab}) \\
R_a &\leq \mathcal{C}_{\mathcal{MAC}}(G_{b1}^T, G_{b2}^T, G_{ab}) \\
R_a &\leq \mathcal{C}_{\mathcal{BC}}(G_{a2}, G_{ab}) + \mathcal{C}(G_{b1}) \\
R_a &\leq \mathcal{C}(G_{b2}) + \mathcal{C}_{\mathcal{BC}}(G_{a1}, G_{ab}) \\
R_b &\leq \mathcal{C}_{\mathcal{BC}}(G_{b1}, G_{b2}, G_{ab}^T) \\
R_b &\leq \mathcal{C}_{\mathcal{MAC}}(G_{a1}^T, G_{a2}^T, G_{ab}^T) \\
R_b &\leq \mathcal{C}(G_{a1}) + \mathcal{C}_{\mathcal{BC}}(G_{b2}, G_{ab}^T) \\
R_b &\leq \mathcal{C}_{\mathcal{BC}}(G_{b1}, G_{ab}^T) + \mathcal{C}(G_{a2})
\end{aligned} \tag{5.14}$$

The complete outer bound region can be evaluated by solving the above problem for different values of the ratio of R_a and R_b.

5.9.3 Outer bound for Gaussian diamond relay channel with direct link between A and B-Half Duplex

$$
\begin{aligned}
&R_a \leq \lambda_1 \mathcal{C}_{\mathcal{BC}}(G_{a1}, G_{a2}) + \lambda_3 \mathcal{C}(G_{ab}) + \lambda_4 \mathcal{C}_{\mathcal{BC}}(G_{a2}, G_{ab}) \\
&\quad + \lambda_5 \mathcal{C}_{\mathcal{BC}}(G_{a1}, G_{ab}) + \lambda_6 \mathcal{C}_{\mathcal{BC}}(G_{a1}, G_{a2}, G_{ab}) \\
&R_a \leq \lambda_1 \mathcal{C}(G_{a2}) + \lambda_2 \mathcal{C}(G_{b1}) + \lambda_3 \mathcal{C}_{\mathcal{MAC}}(G_{ab}, G_{b1}^T) + \\
&\quad \lambda_4 \mathcal{C}(G_{b1}) + \lambda_4 \mathcal{C}_{\mathcal{BC}}(G_{a2}, G_{ab}) + \lambda_6 \mathcal{C}_{\mathcal{BC}}(G_{a2}, G_{ab}) + \lambda_5 \mathcal{C}(G_{ab}) \\
&R_a \leq \lambda_1 \mathcal{C}(G_{a1}) + \lambda_2 \mathcal{C}(G_{b2}) + \lambda_3 \mathcal{C}_{\mathcal{MAC}}(G_{ab}, G_{b2}^T) + \\
&\quad \lambda_4 \mathcal{C}(G_{ab} + \lambda_5 \mathcal{C}(G_{b2}) + \lambda_5 \mathcal{C}_{\mathcal{BC}}(G_{a1}, G_{ab}) + \lambda_6 \mathcal{C}(G_{a1}, G_{ab}) \\
&R_a \leq \lambda_2 \mathcal{C}_{\mathcal{MAC}}(G_{b1}^T, G_{b2}^T) + \lambda_3 \mathcal{C}_{\mathcal{MAC}}(G_{b1}^T, G_{b2}^T, G_{ab}) + \\
&\quad \lambda_4 \mathcal{C}_{\mathcal{MAC}}(G_{b1}^T, G_{ab}) + \lambda_5 \mathcal{C}_{\mathcal{MAC}}(G_{ab}, G_{b2}^T) + \lambda_6 \mathcal{C}(G_{ab}) \\
&R_b \leq \lambda_1 \mathcal{C}_{\mathcal{BC}}(G_{b1}, G_{b2}) + \lambda_7 \mathcal{C}(G_{ab}) + \lambda_8 \mathcal{C}_{\mathcal{BC}}(G_{b2}, G_{ab}^T) + \\
&\quad \lambda_9 \mathcal{C}_{\mathcal{BC}}(G_{b1}, G_{ab}^T) + \lambda_{10} \mathcal{C}_{\mathcal{BC}}(G_{b1}, G_{b2}, G_{ab}^T) \\
&R_b \leq \lambda_1 \mathcal{C}(G_{b2}) + \lambda_2 \mathcal{C}(G_{a1}) + \lambda_7 \mathcal{C}_{\mathcal{MAC}}(G_{ab}^T, G_{a1}^T) + \\
&\quad \lambda_8 \mathcal{C}(G_{a1}) + \lambda_8 \mathcal{C}_{\mathcal{BC}}(G_{b2}, G_{ab}^T) + \lambda_{10} \mathcal{C}_{\mathcal{BC}}(G_{b2}, G_{ab}^T) + \lambda_9 \mathcal{C}(G_{ab}) \\
&R_b \leq \lambda_1 \mathcal{C}(G_{b1}) + \lambda_2 \mathcal{C}(G_{a2}) + \lambda_7 \mathcal{C}_{\mathcal{MAC}}(G_{ab}^T, G_{a2}^T) + \\
&\quad \lambda_8 \mathcal{C}(G_{ab}) + \lambda_9 \mathcal{C}(G_{a2}) + \lambda_9 \mathcal{C}_{\mathcal{BC}}(G_{b1}, G_{ab}^T) + \lambda_{10} \mathcal{C}_{\mathcal{BC}}(G_{b1}, G_{ab}^T) \\
&R_b \leq \lambda_2 \mathcal{C}_{\mathcal{MAC}}(G_{a1}^T, G_{a2}^T) + \lambda_7 \mathcal{C}_{\mathcal{MAC}}(G_{a1}^T, G_{a2}^T, G_{ab}^T) + \\
&\quad \lambda_8 \mathcal{C}_{\mathcal{MAC}}(G_{a1}^T, G_{ab}^T) + \lambda_9 \mathcal{C}_{\mathcal{MAC}}(G_{ab}^T, G_{a2}^T) + \lambda_{10} \mathcal{C}(G_{ab}) \\
&\quad \sum_{i=1}^{10} \lambda_i = 1, 0 \leq \lambda_i \leq 1
\end{aligned}
\tag{5.15}
$$

The complete outer bound region can be evaluated by solving the above problem for different values of the ratio of R_a and R_b.

5.10 Gaussian diamond relay channel with direct link between relays

5.10.1 System Model

The system is similar to section 3.1 except the fact that all nodes are MIMO. Let G_{11} , G_{12} , G_{21} and G_{22} be the $r \times t$ gain matrices of channels a-r_1, a-r_2, b-r_1 and b-r_2 respectively. Let G_d be the $r \times r$ gain matrix of channels r_1-r_2.
Let $\mathcal{C}(G_1) \triangleq \max_{\{K_x \geq 0: trace(K_x) \leq P\}} \frac{1}{2} log(G_1^T K_x G_1 + I_r)$ represents the capacity of a gaussian channel with channel gain matrix of G.
Let $\mathcal{C}_{\mathcal{BC}}(G_1, G_2) \triangleq \max_{\{K_i \geq 0, i=1,2: trace(K_1)+trace(K_2) \leq P\}} \frac{1}{2} log(G_1^T K_1 G_1 + G_2^T K_2 G_2 + I_r)$ represents the capacity of a gaussian broadcast channel with channel gain matrix of G_1 and G_2.
Let $\mathcal{C}_{\mathcal{MAC}}(G_1, G_2) \triangleq \max_{\{K_i \geq 0: trace(K_i) \leq P, i=1,2\}} \frac{1}{2} log(G_1 K_1 G_1^T + G_2 K_2 G_2^T + I_r)$ represents the capacity of a gaussian multiple access channel with channel gain matrix of G_1 and G_2.

5.10.2 Outer bound for Gaussian diamond relay channel with direct link between relays-Full Duplex

$$\begin{aligned}
R_a &\leq \mathcal{C}_{\mathcal{BC}}(G_{a1}, G_{a2}) \\
R_a &\leq \mathcal{C}_{\mathcal{MAC}}(G_{b1}^T, G_{b2}^T) \\
R_a &\leq \mathcal{C}(G_{a2}) + \mathcal{C}_{\mathcal{BC}}(G_{b1}^T, G_d) \\
R_a &\leq \mathcal{C}_{\mathcal{BC}}(G_{b2}^T, G_d^T) + \mathcal{C}(G_{a1}) \\
R_b &\leq \mathcal{C}_{\mathcal{BC}}(G_{b1}, G_{b2}) \\
R_b &\leq \mathcal{C}_{\mathcal{MAC}}(G_{a1}^T, G_{a2}^T) \\
R_b &\leq \mathcal{C}_{\mathcal{BC}}(G_{a1}^T, G_d) + \mathcal{C}(G_{b2}) \\
R_b &\leq \mathcal{C}(G_{b1}) + \mathcal{C}_{\mathcal{BC}}(G_{a2}^T, G_d^T)
\end{aligned} \tag{5.16}$$

The complete outer bound region can be evaluated by solving the above problem for different values of the ratio of R_a and R_b

5.10.3 Outer bound for Gaussian diamond relay channel with direct link between relays-Half Duplex

$$
\begin{aligned}
&R_a \leq \lambda_1 \mathcal{C}_{\mathcal{BC}}(G_{a1}, G_{a2}) + \lambda_3 \mathcal{C}(G_{a2}) + \lambda_4 \mathcal{C}(G_{a2}) + \lambda_5 \mathcal{C}(G_{a1}) + \lambda_6 \mathcal{C}(G_{a1}) \\
&R_a \leq \lambda_1 \mathcal{C}(G_{a2}) + \lambda_2 \mathcal{C}(G_{b1}) + \lambda_3 \mathcal{C}_{\mathcal{MAC}}(G_d, G_{a2}) + \lambda_4 \mathcal{C}(G_{a2}) + \lambda_4 \mathcal{C}_{\mathcal{BC}}(G_{b1}^T, G_d) + \\
&\quad \lambda_7 \mathcal{C}_{\mathcal{BC}}(G_{b1}^T, G_d) + \lambda_{10} \mathcal{C}(G_d) \\
&R_a \leq \lambda_1 \mathcal{C}(G_{a1}) + \lambda_2 \mathcal{C}(G_{b2}) + \lambda_5 \mathcal{C}_{\mathcal{MAC}}(G_d^T, G_{a1}) + \lambda_6 \mathcal{C}(G_{a1}) + \lambda_6 \mathcal{C}_{\mathcal{BC}}(G_{b2}^T, G_d^T) + \\
&\quad \lambda_9 \mathcal{C}_{\mathcal{BC}}(G_{b2}^T, G_d^T) + \lambda_8 \mathcal{C}(G_d) \\
&R_a \leq \lambda_2 \mathcal{C}_{\mathcal{MAC}}(G_{b1}^T, G_{b2}^T) + \lambda_4 \mathcal{C}(G_{b1}) + \lambda_7 \mathcal{C}(G_{b1}) + \lambda_9 \mathcal{C}(G_{b2}) + \lambda_6 \mathcal{C}(G_{b2}) \\
&R_b \leq \lambda_1 \mathcal{C}_{\mathcal{BC}}(G_{b1}, G_{b2}) + \lambda_3 \mathcal{C}(G_{b2}) + \lambda_{10} \mathcal{C}(G_{b2}) + \lambda_5 \mathcal{C}(G_{b1}) + \lambda_8 \mathcal{C}(G_{b1}) \\
&R_b \leq \lambda_1 \mathcal{C}(G_{b2}) + \lambda_2 \mathcal{C}(G_{a1}) + \lambda_3 \mathcal{C}_{\mathcal{MAC}}(G_d, G_{b2}) + \lambda_4 \mathcal{C}(G_d) + \lambda_7 \mathcal{C}_{\mathcal{BC}}(G_{a1}^T, G_d) + \\
&\quad \lambda_{10} \mathcal{C}_{\mathcal{BC}}(G_{a1}^T, G_d) + \lambda_{10} \mathcal{C}(G_{b2}) \\
&R_b \leq \lambda_1 \mathcal{C}(G_{b1}) + \lambda_2 \mathcal{C}(G_{a2}) + \lambda_5 \mathcal{C}_{\mathcal{MAC}}(G_d^T, G_{b1}) + \lambda_6 \mathcal{C}(G_d) + \lambda_9 \mathcal{C}_{\mathcal{BC}}(G_{a2}^T, G_d^T) + \\
&\quad \lambda_8 \mathcal{C}_{\mathcal{BC}}(G_{a2}^T, G_d^T) + \lambda_8 \mathcal{C}(G_{b1}) \\
&R_b \leq \lambda_2 \mathcal{C}_{\mathcal{MAC}}(G_{a1}^T, G_{a2}^T) + \lambda_{10} \mathcal{C}(G_{a1}) + \lambda_7 \mathcal{C}(G_{a1}) + \lambda_9 \mathcal{C}(G_{a2}) + \lambda_8 \mathcal{C}(G_{a2}) \\
&\quad \sum_{i=1}^{10} \lambda_i = 1, 0 \leq \lambda_i \leq 1
\end{aligned}
\tag{5.17}
$$

The complete outer bound region can be evaluated by solving the above problem for different values of the ratio of R_a and R_b

CHAPTER 6

NUMERICAL RESULTS

In this chapter, we compare the outer bounds for half duplex and full duplex case for all the channels discussed above. We also make comparisons between the outer bound and achievable rate for a few of the channels. In addition we compare the scale and forward protocol with other protocols and also with the half duplex outer bound of the diamond channel. We also plot the variation of the ratio of optimal scaling constants against power at the relay. Comparisons are made between half duplex and full duplex outer bounds for the MIMO case for all the channels for the cases 1. MIMO at relay alone and 2. all nodes being MIMO.

6.1 Comparison of Rate Regions for 2 way relay channel with no direct link

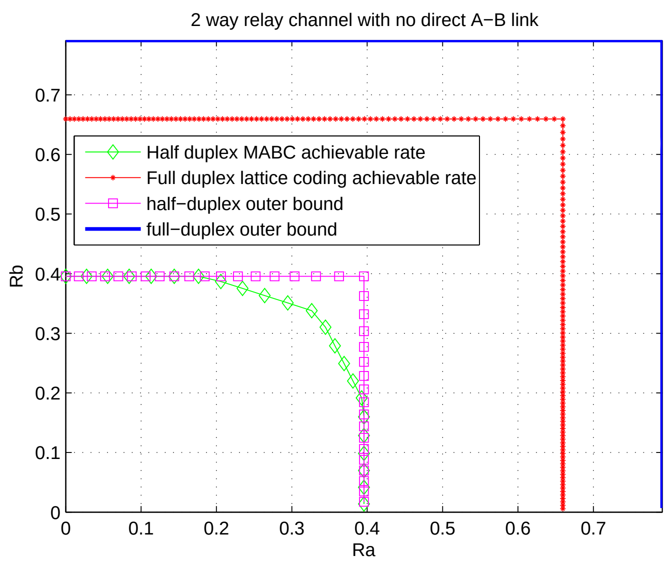


Figure 6.1: Comparison of Rate Regions for 2 way relay channel with no direct link $\gamma_1 = 3$ dB, $\gamma_2 = 3$ dB

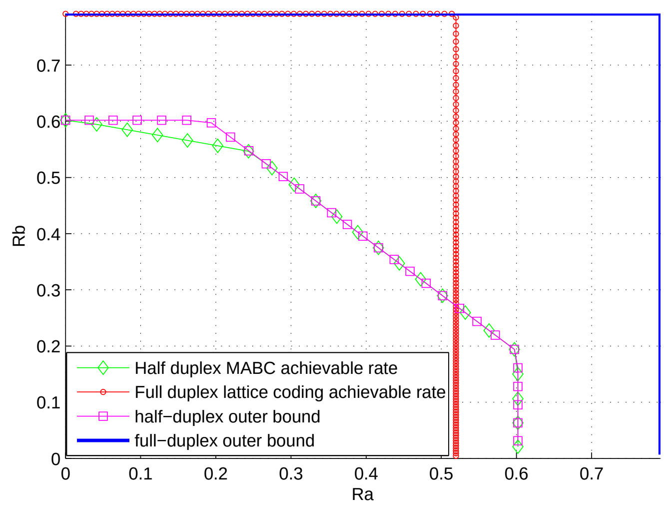


Figure 6.2: Comparison of Rate Regions for 2 way relay channel with no direct link $\gamma_1 = 3$ dB, $\gamma_2 = 15$ dB

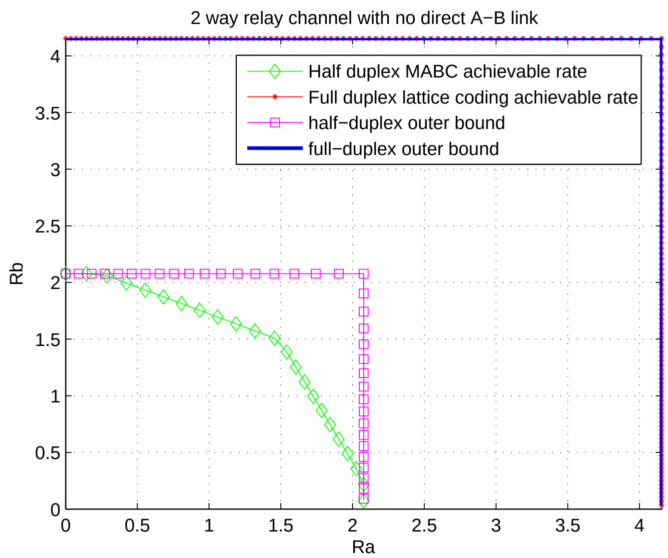


Figure 6.3: Comparison of Rate Regions for 2 way relay channel with no direct link $\gamma_1 = 25$ dB, $\gamma_2 = 25$ dB

We observe in the above cases that full duplex outer bound is greater than the half duplex outer bound. Lattice coding achieves rates close to the full duplex outer bound at high SNR. MABC protocol achieves rates close to outer bound in some regions. MABC also achieves some rate points that the full duplex lattice coding cannot achieve as in Fig. 6.2. This is because in the case of Fig. 6.2 one link is much weaker than the other making the term $\gamma_2/\gamma_2 + \gamma_1$ significant enough to cause the maximum achievable rate R_a to be much below the outer bound and also below the rate achievable by MABC protocol.

6.2 Comparison of Rate Regions for 2 way relay channel with direct link

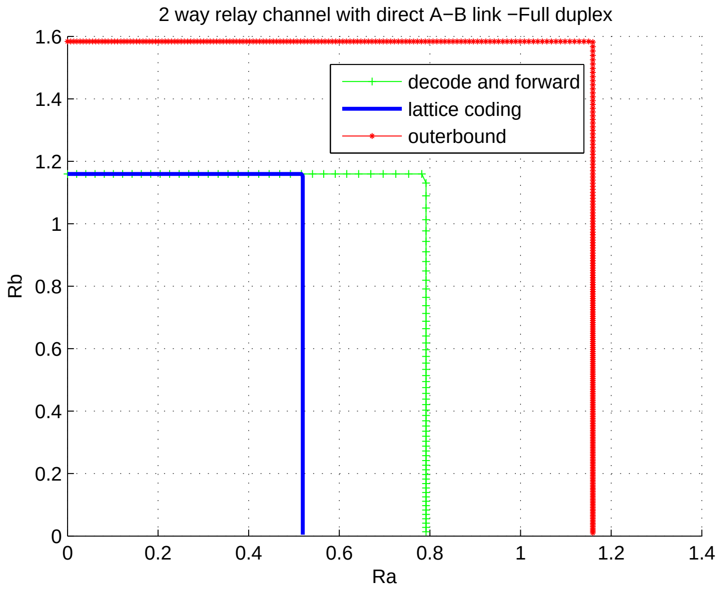


Figure 6.4: Comparison of Rate Regions for 2 way relay channel with direct link-Full duplex $\gamma_1 = 3$ dB, $\gamma_2 = 15$ dB,$\gamma_3 = 3$ dB

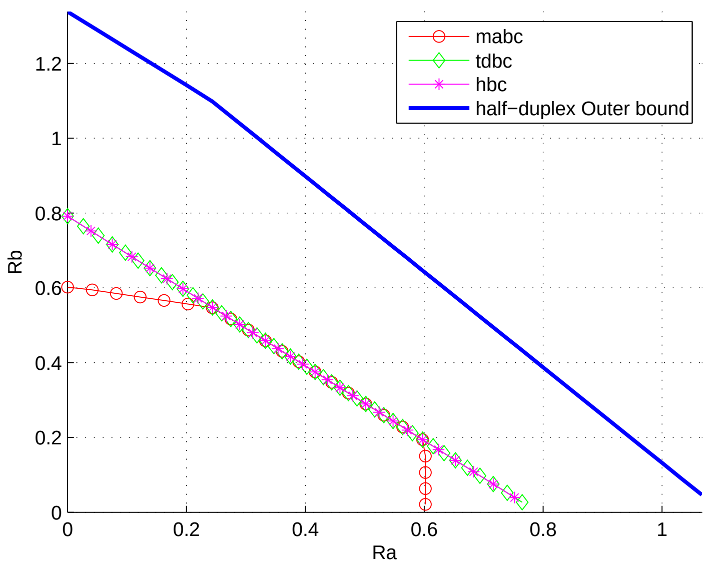


Figure 6.5: Comparison of Rate Regions for 2 way relay channel with direct link-Half duplex $\gamma_1 = 3$ dB, $\gamma_2 = 15$ dB,$\gamma_3 = 3$ dB

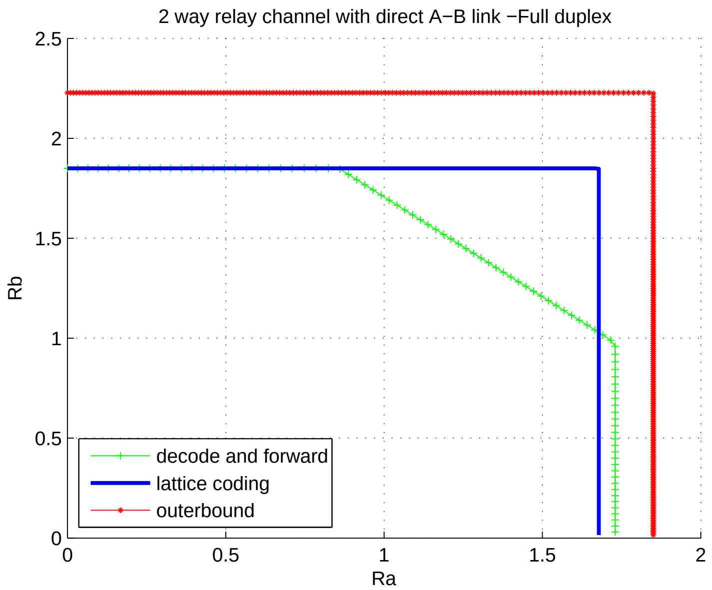


Figure 6.6: Comparison of Rate Regions for 2 way relay channel with direct link-Full duplex $\gamma_1 = 10$ dB, $\gamma_2 = 15$ dB,$\gamma_3 = 3$ dB

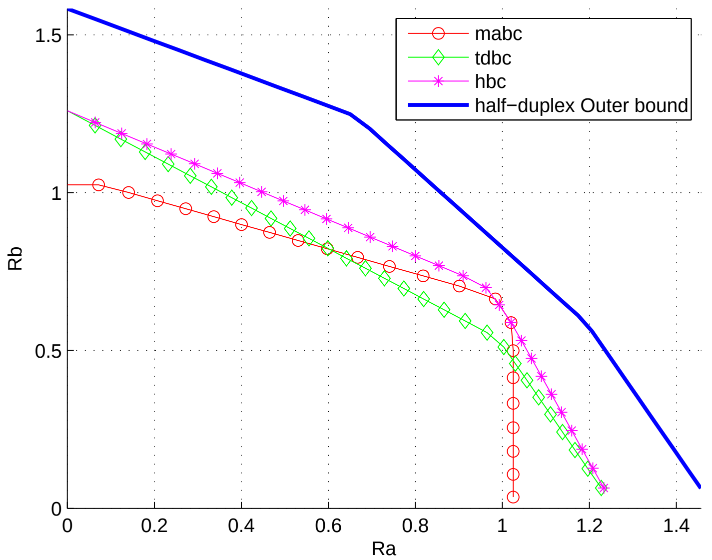


Figure 6.7: Comparison of Rate Regions for 2 way relay channel with direct link-Half duplex $\gamma_1 = 10$ dB, $\gamma_2 = 15$ dB,$\gamma_3 = 3$ dB

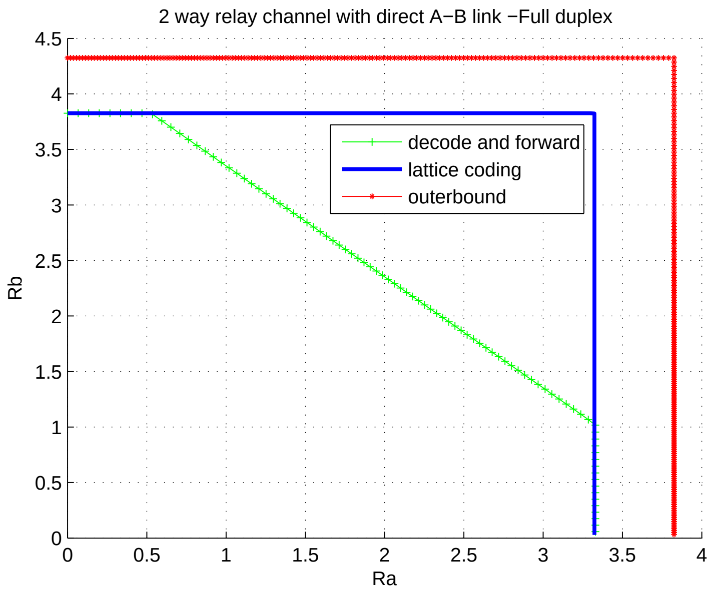


Figure 6.8: Comparison of Rate Regions for 2 way relay channel with direct link-Full duplex $\gamma_1 = 20$ dB, $\gamma_2 = 25$ dB,$\gamma_3 = 20$ dB

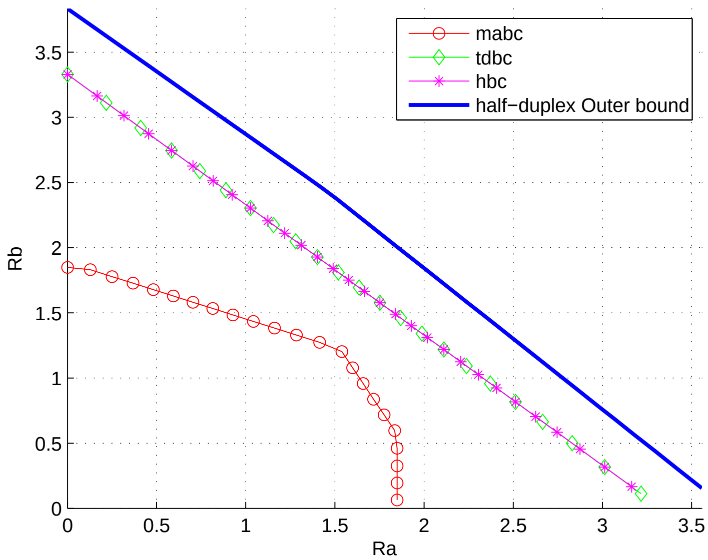


Figure 6.9: Comparison of Rate Regions for 2 way relay channel with direct link-Half duplex $\gamma_1 = 20$ dB, $\gamma_2 = 25$ dB,$\gamma_3 = 20$ dB

We observe in the above cases that full duplex outer bound is greater than the half duplex outer bound. Lattice coding achieves rates close to the full duplex outer bound at high SNR. At low SNR decode and forward has better rates. In Fig. 6.6 we observe that some rate pairs achieved by lattice coding are not achievable by decode and forward and vice versa. In half duplex case, HBC rate region is higher and includes the rate region of MABC and TDBC as expected since MABC and TDBC are special cases of HBC. When direct link is very weak, MABC is close to rate region of TDBC and HBC. This is because MABC does not use the direct link. Also similar to the case of 2 way relay with no direct link, we can observe in Fig.6.4 and Fig.6.5 that half duplex protocols can achieve better rate for R_a than full duplex lattice coding when link $a - r$ is much weaker than $b - r$.

6.3 Comparison of outer bounds for 2 way relay with and without direct links

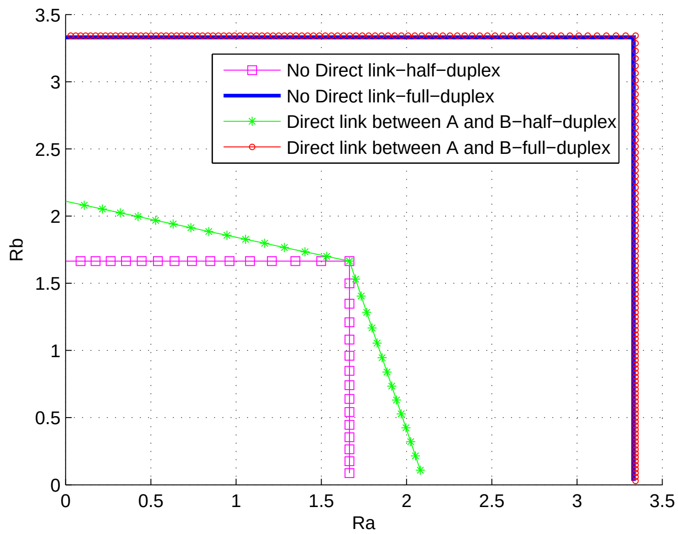


Figure 6.10: Comparison of outer bound for 2 way relay channel with and without direct link $\gamma_1 = 20$ dB, $\gamma_2 = 20$ dB,$\gamma_3 = 3$ dB

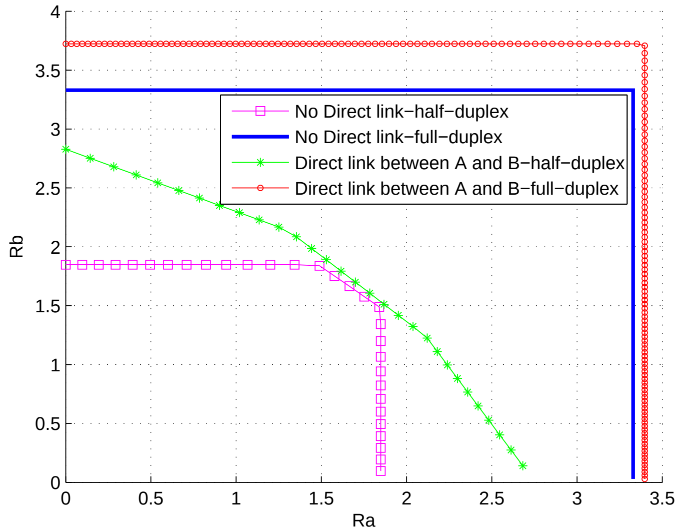


Figure 6.11: Comparison of outer bound for 2 way relay channel with and without direct link $\gamma_1 = 20$ dB, $\gamma_2 = 25$ dB,$\gamma_3 = 10$ dB

We observe that when the direct link between the terminal nodes is very weak compared to the link between a and r and the link between b and r, it will not make any difference if we use the direct link or not. In Fig. 6.10 we observe the full duplex outer bounds overlap. In the half duplex case, the rate for 2 way relay with direct link coincides with the one without direct link in the case of equal rates$(Ra = Rb)$.

6.4 Comparison of outer bounds for diamond relay with and without direct links

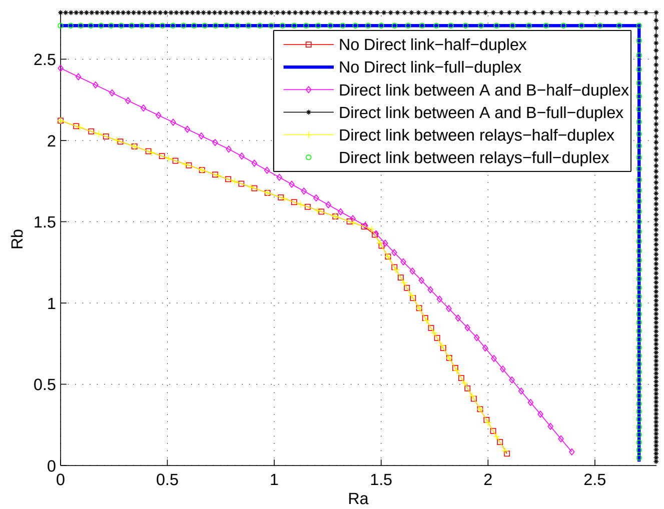


Figure 6.12: Outer bounds for diamond relay channel with and without direct link $\gamma_{a1} = 15$ dB, $\gamma_{a2} = 10$ dB,$\gamma_{b1} = 15$ dB, $\gamma_{b2} = 10$ dB,$\gamma_d = 7$ dB

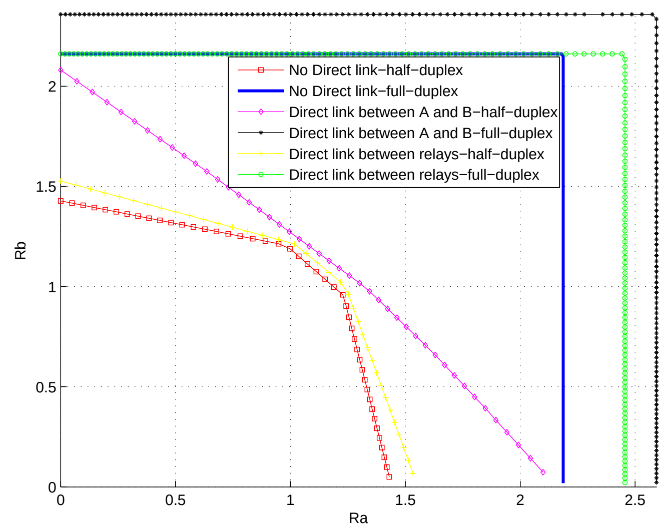


Figure 6.13: Outer bounds for diamond relay channel with and without direct link $\gamma_{a1} = 14$ dB, $\gamma_{a2} = 6$ dB,$\gamma_{b1} = 5$ dB, $\gamma_{b2} = 12$ dB,$\gamma_d = 8$ dB

We observe that the outer bound for diamond relay with direct link between A and B is better than the other two cases for both full duplex and half duplex mode. The link between the nodes A and B provides a direct path of communication between them, so this outer bound is better than the one without direct link in all channel conditions. The link between the relays does not provide any improvement for the symmetric channel case. The direct link between relays is observed to increase the achievable region only when the three hop path (through the $R_1 - R_2$ link) is better than the other two in terms of SNR as in Fig 6.13. In other cases, this bound is found to overlap with the outer bound for the diamond channel without direct links.

6.5 Scale and forward Protocol

6.5.1 Comparison of Scale and forward Protocol with other protocols and the outer bound

Comparison of achievable rate regions of Scale and forward Protocol with different protocols and the outer bound is shown for different channel conditions: Asymmetric channel (as in Fig 6.15), Symmetric channel (as in Fig 6.16), Weaker Channels from both sources to one of the relays (as in Fig 6.14), Weaker channels from both relays to one of the sources (as in Fig 6.17)

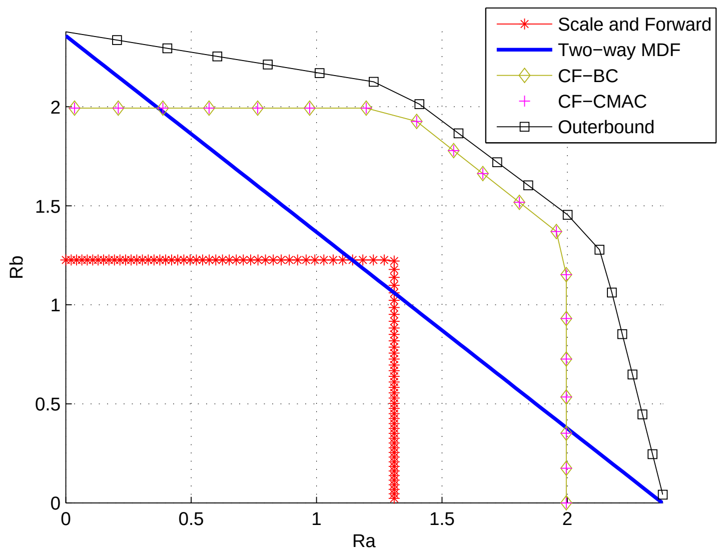


Figure 6.14: Rate region for Scale and Forward protocol: $\gamma_{a1} = 30$ dB, $\gamma_{a2} = 3$ dB, $\gamma_{b1} = 20$ dB $\gamma_{b2} = 4$ dB

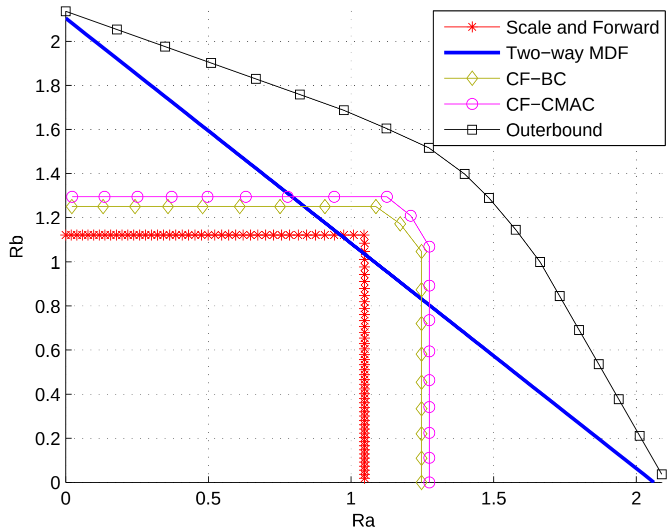


Figure 6.15: Rate region for Scale and Forward protocol: $\gamma_{a1} = 10$ dB, $\gamma_{a2} = 12$ dB, $\gamma_{b1} = 14$ dB $\gamma_{b2} = 16$ dB

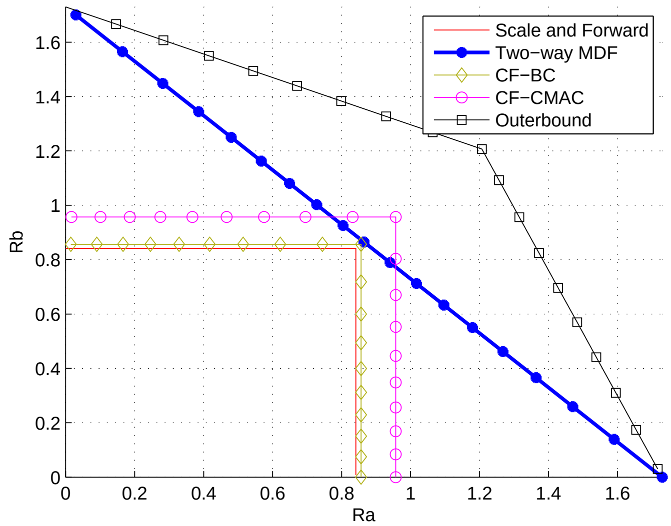


Figure 6.16: Rate region for Scale and Forward protocol,Outer bound and other protocols: $\gamma_{a1} = 10$ dB, $\gamma_{a2} = 10$ dB, $\gamma_{b1} = 10$ dB $\gamma_{b2} = 10$ dB

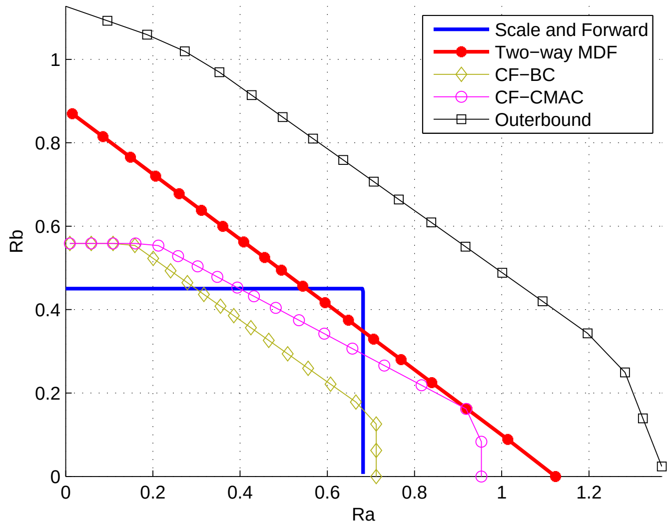


Figure 6.17: Rate region for Scale and Forward protocol,Outer bound and other protocols: $\gamma_{a1} = 30$ dB, $\gamma_{a2} = 20$ dB, $\gamma_{b1} = 3$ dB $\gamma_{b2} = 4$ dB

We observe that the scale and forward protocol achieves rates close to the other protocols and in some cases rate points outside the achievable rate region of other protocols. In the case in Fig 6.17 where channels from source B to the relays is weak, scale and forward achieves better rate than all the other protocols discussed in the region of R_a approximately equal to R_b

6.5.2 Ratio of scaling coefficients Vs Power at the relays

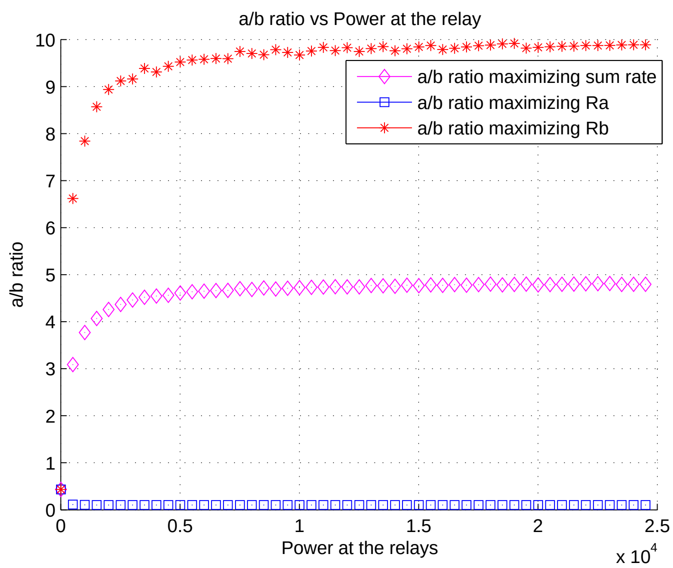


Figure 6.18: a/b ratio Vs Power at the relay 1 $h_{a1} = 1$, $h_{a2} = 2$,$h_{b1} = 5$, $h_{b2} = 1$

We observe that the a/b ratio that maximizes R_a approaches $||h_{a1}||||h_{b2}||/||h_{a2}||||h_{b1}||$ which is 0.1 in this case .

Also the a/b ratio that maximizes R_b approaches $||h_{b1}||||h_{a2}||/||h_{a1}||||h_{b2}||$ which in this case is 10.

6.6 Outer bounds in full duplex and half duplex for 2 way relay and diamond relay channel-MIMO case

The outer bounds evaluated as in Chapter 5 are plotted below for the cases of relays alone being MIMO and the case of all nodes being MIMO.

6.6.1 2 way relay

The channel matrix for 2 way relays for the below two cases have values as :$a - r$ has value 3 for all transmitter receiver pairs, $b - r$ has value 5 for all transmitter receiver pairs, $a - b$ has value 4 for all transmitter receiver pairs. For example, channel gain matrix G_1 $(a - r)$ for 3×1 case would be $[3; 3; 3]$. We observe that the outer bound is larger for the case with all nodes being MIMO.

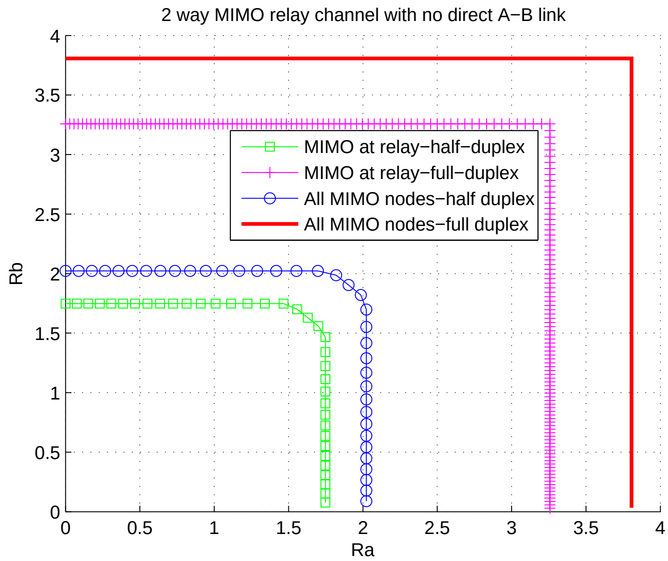


Figure 6.19: Full duplex and half duplex outer bounds for MIMO 2-way relay with no direct link:

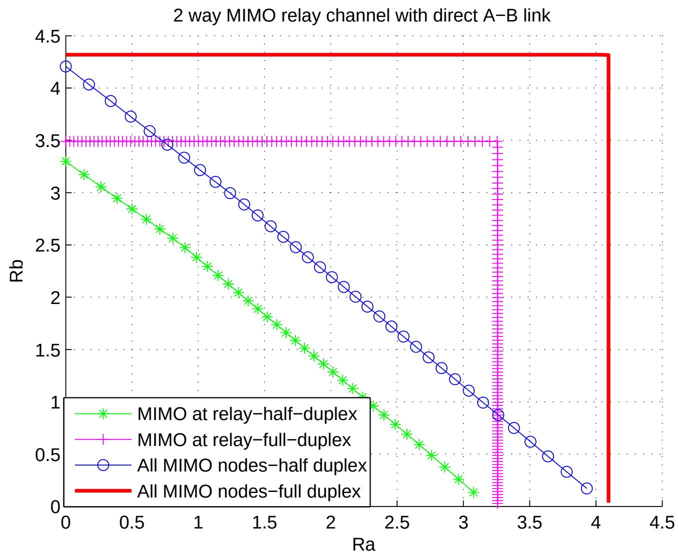


Figure 6.20: Full duplex and half duplex outer bounds for MIMO 2-way relay with direct A-B link:

6.6.2 Diamond relay

The channel matrix for the following cases have values as :$a - r_1$ has value 3 for all transmitter receiver pairs, $b - r_1$ has value 4 for all transmitter receiver pairs, $a - r_2$ has value 6 for all transmitter receiver pairs, $b - r_2$ has value 5 for all transmitter receiver pairs, $a - b$ has value 1 for all transmitter receiver pairs. For example channel gain matrix G_{12} $(a - r_1)$ for 3×1(r $\times$ t) case would be$[3; 3; 3]$

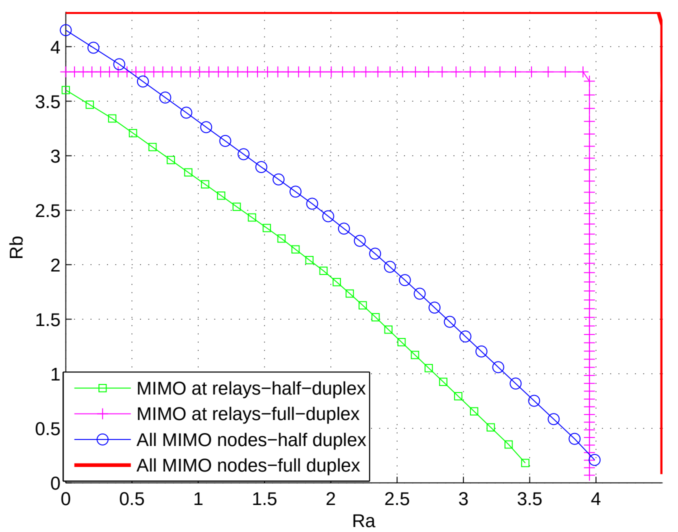


Figure 6.21: Full duplex and half duplex outer bounds for MIMO diamond relay with no direct link:

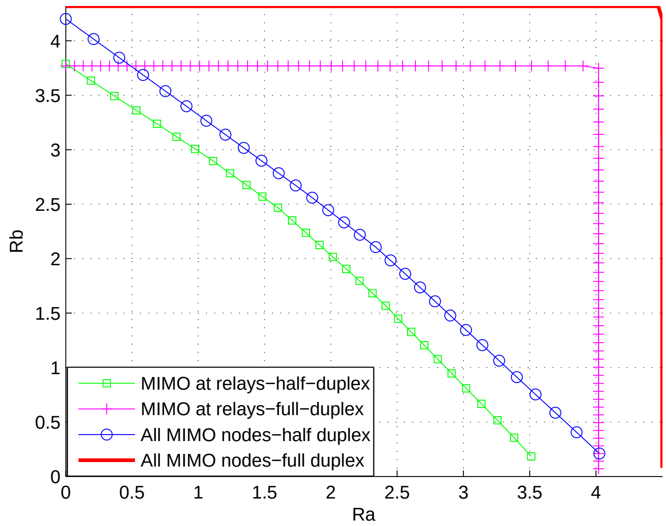


Figure 6.22: Full duplex and half duplex outer bounds for MIMO diamond relay with direct A-B link:

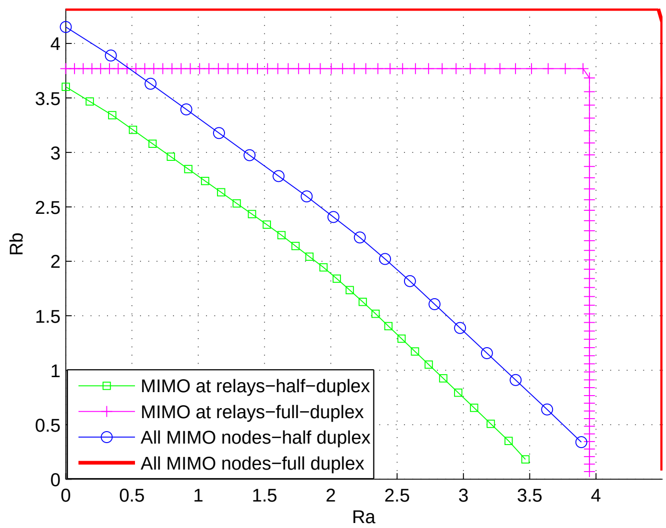


Figure 6.23: Full duplex and half duplex outer bounds for MIMO diamond relay with direct link between relays:

Comparison of outer bounds for the case of relays alone being MIMO, and the case of all nodes being MIMO shows that the case of all nodes being MIMO has a larger outer bound and thus is better than the case of relays alone being MIMO.

CHAPTER 7

CONCLUSION

7.1 Contribution of this thesis

The two-way Gaussian relay channel and the diamond channel with and without direct link has been studied for full duplex and half duplex cases. Outer bound is derived for each of the cases. This gives an insight to the maximum rates that can be achieved in either of the cases and thus provides a new benchmark for comparing further improvements to two-way relaying protocols. Scale and forward protocol is proposed and analyzed for the 2 way diamond channel. We also look into the MIMO case for the channels discussed above and make comparative studies between the half duplex and full duplex cases.We also compare the outer bounds for the case of only the relays being MIMO with the case of all nodes being MIMO.

7.2 Future Work

New protocols for the different channels can be proposed as there is quite a gap between existing protocols and the outer bound. The 2X2 interference network of State 14 of diamond relay channel can be exploited to improve the half duplex achievable rate region. Achievable rate regions for 2 way full duplex diamond channel can be found. Gap between the achievable rate region of the existing protocols and the outer bound, for different channels discussed in this thesis can be analyzed to know the conditions under which a protocol can be used to achieve rates close to outer bound. New protocols for MIMO relay channel can be proposed for both full duplex and half duplex modes.

REFERENCES

[1] **Abbas El Gamal, Y.-H. K.**, *Network information theory*. Cambridge University Press, 2011.

[2] **Ashar, P. V.-B. S. T. A., K.I.**, Outer bounds for the capacity region of a gaussian two-way relay channel. *In Communication, Control, and Computing (Allerton), Annual Allerton Conference on*. IEEE, 2012.

[3] **Baik, I.** and **S. Chung**, Network coding for two-way relay channels using lattices. *In Communications, 2008. ICC'08. IEEE International Conference on*. IEEE, 2008.

[4] **H. Bagheri, A. M.** and **A. Khandani**, On the capacity of the half-duplex diamond channel. IEEE, 2010.

[5] **Khojastepour, M.**, **A. Sabharwal**, and **B. Aazhang**, On capacity of gaussian'cheap'relay channel. *In Global Telecommunications Conference, 2003. GLOBECOM'03. IEEE*, volume 3. IEEE, 2003.

[6] **Kim, S.**, **N. Devroye**, **P. Mitran**, and **V. Tarokh** (2011). Achievable rate regions and performance comparison of half duplex bi-directional relaying protocols. *Information Theory, IEEE Transactions on*, **57**(10), 6405–6418.

[7] **Kim, S.**, **P. Mitran**, and **V. Tarokh** (2008). Performance bounds for bidirectional coded cooperation protocols. *Information Theory, IEEE Transactions on*, **54**(11), 5235–5241.

[8] **Nam, W.**, **S. Chung**, and **Y. Lee** (2010). Capacity of the gaussian two-way relay channel to within $\frac{1}{2}$ bit. *Information Theory, IEEE Transactions on*, **56**(11), 5488–5494.

[9] **Peng Zhong, M., Vu**, Partial decode-forward coding schemes for the gaussian two-way relay channel. *In Communications (ICC), IEEE International Conference on*. IEEE, 2012.

[10] **Popovski, P.** and **H. Yomo**, Physical network coding in two-way wireless relay channels. *In Communications, 2007. ICC'07. IEEE International Conference on*. IEEE, 2007.

[11] **Prathyusha, B. S. T. A., V.**, The gaussian two-way diamond channel. *In Communication, Control, and Computing (Allerton), Annual Allerton Conference on*. IEEE, 2013.

[12] **Schein, B.** and **R. Gallager**, The gaussian parallel relay network. *In Information Theory, 2000. Proceedings. IEEE International Symposium on*. IEEE, 2000.

[13] **Schnurr, C.**, **S. Stanczak**, and **T. Oechtering**, Achievable rates for the restricted half-duplex two-way relay channel under a partial-decode-and-forward protocol. *In Information Theory Workshop, 2008. ITW'08. IEEE*. IEEE, 2008.

[14] **Tian, Y.**, **D. Wu**, **C. Yang**, and **A. Molisch** (2012). Asymmetric two-way relay with doubly nested lattice codes. *Wireless Communications, IEEE Transactions on*, 694 – 702.

[15] **W. Nam, S.-Y. C.** and **Y. Lee**, Nested lattice codes for gaussian relay networks with interference. *In Information Theory, IEEE Transactions on*, volume 57. IEEE, 2011.

[16] **Wilson, M.**, **K. Narayanan**, **H. Pfister**, and **A. Sprintson** (2010). Joint physical layer coding and network coding for bidirectional relaying. *Information Theory, IEEE Transactions on*, **56**(11), 5641 –5654. ISSN 0018-9448.

[17] **Xue, F.** and **S. Sandhu**, Cooperation in a half-duplex gaussian diamond relay channel. *In Information Theory, IEEE Transactions on*, volume 53.

[18] **Yiwei Song, N., Devroye**, Lattice codes for the gaussian relay channel:decode-and-forward and compress-and-forward. *In Information Theory, IEEE Transactions on*, volume 59. IEEE, 2013.

[19] **Zhang, S.** and **S. Liew** (2009). Channel coding and decoding in a relay system operated with physical-layer network coding. *Selected Areas in Communications, IEEE Journal on*, **27**(5), 788–796.