Resource allocation and Decode-and-forward relaying for the diamond relay channel with multicarrier transmission

A Project Report

submitted by

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THESIS CERTIFICATE

This is to certify that the thesis titled **Resource allocation and on capacity of Half-Duplex MIMO Gaussian Diamond Channel**, submitted by **Thejaswi Havisha Jamisetty**, to the Indian Institute of Technology, Madras, for the award of the degree of **Master of Technology (Dual Degree)**, is a bonafide record of the research work done by her under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

KEYWORDS: Capacity; Decode-and-Forward; Power Allocation; Greedy Algorithm; MIMO

A dual hop communication system comprising a source, destination that are connected by a pair of noninterfering relays is called a Gaussian Diamond channel. There are many ways to allocate the resources between the relays connecting the source to the destination. Different states and scheduling schemes are studied to achieve a constant gap from the maximum achievable rate possible. We analyze a special case of the system where we define a parameter Δ and prove that when $\Delta > 0$ a MDF-MAC protocol gives rates that are within a constant gap from capacity. We consider a system having mutliple parallel gaussian subchannels between the source and each relay, also between each relay and destination. We investigate an algorithm that allocates resouces optimally with an objective to maximise end-to-end rate. Simulations are done to check if the algorithm is achieving rates that are close to the optimum allocation. The above methodologies are used to analyse the Average rate vs Power for different positions of the relays in each of the protocols discussed.

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ABBREVIATIONS

MAC	Multiple-Access State
MDF	Multihopping Decode-and-Forward
MIMO	Multiple Input Multiple Output
OFDM	Orthogonal Frequency Division Multiplexing
SNR	Signal-to-Noise Ratio
BC	Broadcast State
S-D	Source-Destination
S-R	Source-Relay
R-D	Relay-Destination

NOTATION

Bold face lower case letters	Vectors
Bold face upper case letters	Matrices
$\mathbb{N}$	Set of natural numbers
$\mathcal{N}(\mu, \sigma^2)$	Gaussian distribution with Mean μ and Variance σ^2
$\mathcal{CN}(\mu, \sigma^2)$	Circularly symmetric complex Gaussian distribution with Mean μ and Variance σ^2
$(.)^H$	Hermitian Operator
$(.)^T$	Transpose Operator
$Q(.)$	Q-Function

CHAPTER 1

Introduction

1.1 Motivation

In the recent years, with the increase in the demand of users in wireless networks, extensive research has been happening in the field of wireless communication. High-quality broadband services, voice and video applications need Fourth Generation(4G) cellular architecture. However, the bandwidth available limits the performance and research is aimed at enhancing the performance with the limited resourced available. . Multiple input multiple output (MIMO), orthogonal frequency division multiplexing (OFDM), and advanced error control codes do not reduce the corruption that creeps in because of the interference. Fixed relays can be used to enhance the coverage, Fig. 1.1

The relay channel was first introduced by van der Muelen in 1971. A two-way or bidirectional relay channel consists of two nodes exchanging messages through one or more relays. Relay networks find applications in multi-hop wireless networks, sensor networks with transmitter power limitations etc.. Relay networks of different topologies have been studied under different relaying schemes like Amplify and Forward (AF), Decode and Forward (DF), Compress and Forward (CF) and Lattice forward. Among

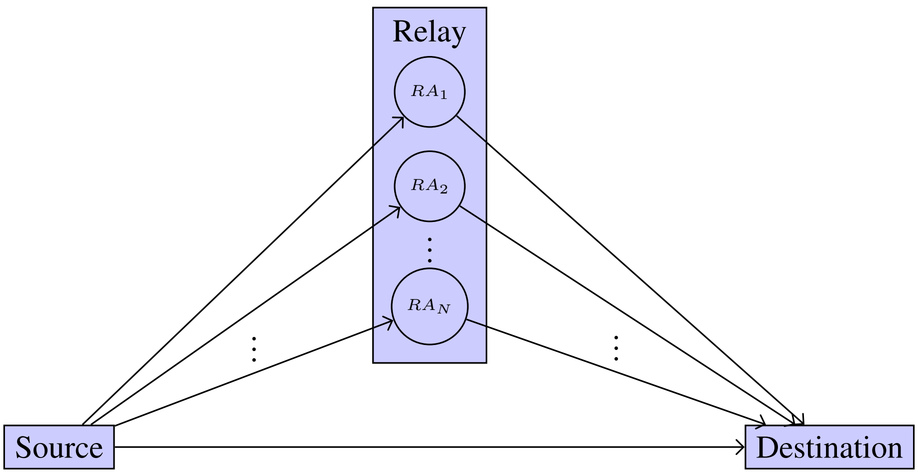

Figure 1.1: Relay channel with multi-antenna relay

the different relay channels, the diamond relay channel has attracted significant interest. We restrict our attention to this channel in the current thesis. We consider half-duplex relays since they are more practical and cost-efficient than full-duplex relays.

Several challenges are encountered while designing a wireless communication system. Transmission of data from the Source and the Destination is interfered with many obstacles like hills, buildings,etc. A possibility of multiple reflections from the ground and other objects also adds to the corruption of data. This is called multipath fading of the data. Transmission of data over long distances suffers reduced signal strength along with fading.

To reduce the impact of fading, various techniques like diversity in time, frequency and space are employed in cellular networks. Multi-Input-Multi-Output (MIMO) systems' broadcast nature is exploited and are designed to reduce error probability. Cooperative relaying is one such type of spatial diversity scheme that benefits itself of antennas distributed across the nodes. By transmitting identical messages in multiple independent paths having different channel conditions we can achieve spatial diversity. Processing is done at an intermediate node, like relays, and the information is forwarded to the destination through another set of independent channels. This way, we get multiple copies of the message at the receiver. Spatial diversity achieves gain that proportionally increases with the product of transmitting and receiving antennas.The number of antennas in a node maybe limited by space constraints.

In cooperative relaying, idle users and active users corporate and compromise on their resources to allocate more to the active user. Which leads to multiple channels for data transmission that are obtained by sharing the available resources. There are two ways to approach cooperative transmission, based on the role the relays play in the system: the amplify and forward (AF) scheme and the decode and forward scheme (DF). The AF approach is mostly simple which is a non-regenerative approach where the relay amplifies and forwards the signal received from the source. It is also non as non-regenerative relay. AF relaying protocols are popular for the ease of implementation at the relay. The DF scheme is more complex in which the relay station decodes the received data and forwards the decoded and regenerated symbols. It is also known as regenerative approach. The DF relaying introduces more delay in transmission than AF relaying, but it gives better performance than the counter one.

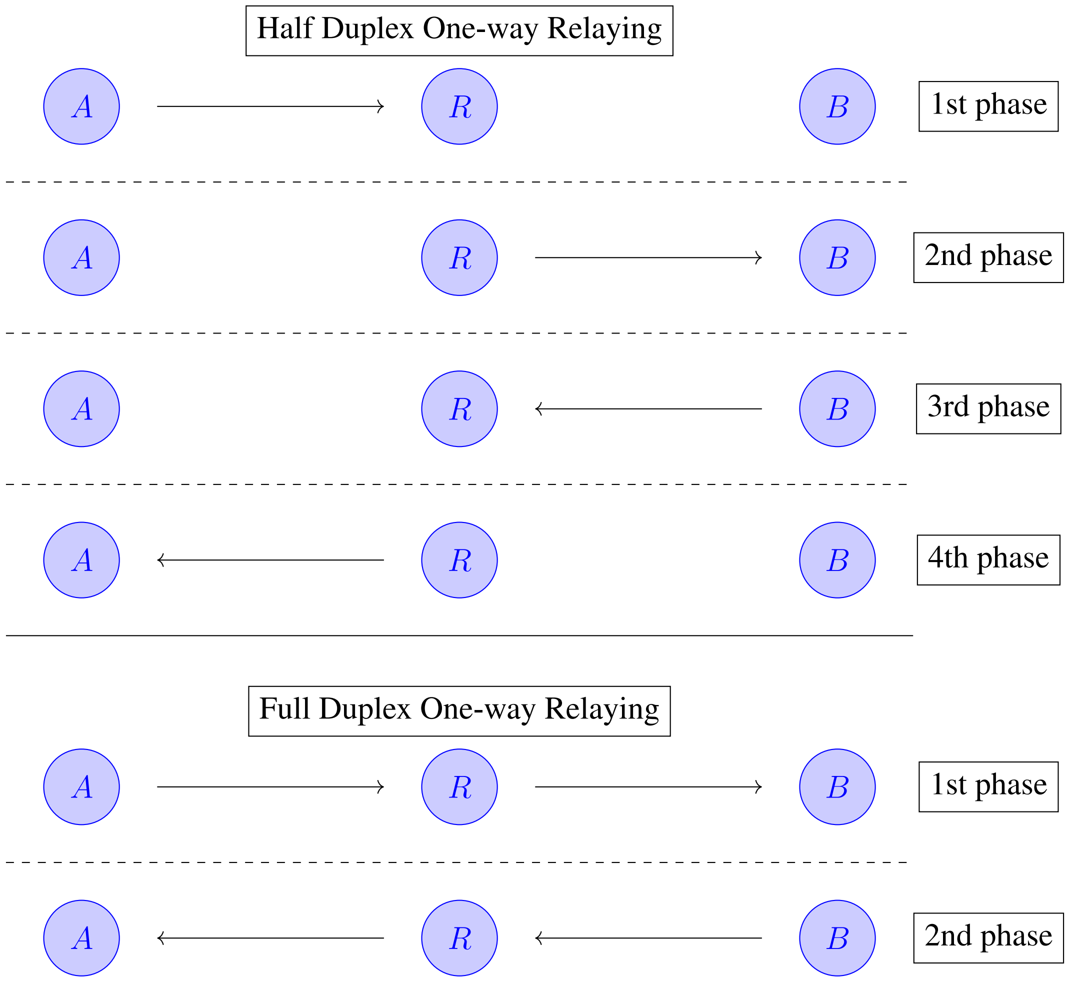

Figure 1.2: Network model for HD-OW and FD-OW relaying

Relays may also differ based on their transmit-receive protocol as Half duplex and Full duplex. In Half-Duplex, each node either transmits or receives at a point of time. In Full duplex, both receiving and transmission can be done simultaneously. Naturally, FD systems are more complex than HD systems. Half-duplex one way (HD-OW) and full-duplex one-way (FD-OW) relaying have been considered. Fig.1.2.

In HD-OW relaying, all nodes are in half-duplex mode and it will take four channel uses to achieve bi-directional communication. In the first channel phase, node A transmits its data signal to the relay node R in only one direction at the time. Subsequently, in the second channel phase, the relay node R transmits the processed signal to node B. In the third and fourth channel phases, node B transmits the signal to node A by relaying node R. Such HD-OW relaying suffers from a significant loss of spectral efficiency because of the pre-log factor 1/2, which dominates the capacity at highsignal-to-noise ratio (SNR).

FD-OW relaying also has been proposed in where all the nodes operate in full-duplex mode. Therefore, it will take two channel uses or each bi-directional communication. Although the number of required channel uses for FD-OW relaying is the same as that of HD-TW relaying, the spectral efficiency of FD-OW relaying may decrease due to self-interfering signal at node R.

To meet the growing demand for high data rate services, one solution being employed in next generation cellular systems is to deploy low-cost relay stations in each cell. A relay station positioned closer to the cell experiences low received SNR on the Base station-Relay Station link. This will lead to higher interference to neighbouring cells. Alternatively, placing RS at a distance from the cell leads to low SNR on RS-MS link. This makes the cell edge uses more vulnerable to outage. Hence, to acieve maximum efficiency in throughput and coverage, we need to determine optimal positioning of the relays and also it is important to assess the Optimal resource allocatiion across the relays for best performance of the system.

OFDM is viewed as an interesting technique in broadband wireless networks that has the potential to reduce the negative impacts of multipath fading. This can be achieved by transmitting information over multiple narrowband channels each having different fading levels. A prominent improvement in the performance can be observed by dynamic power allocation, when subchannel gains are known at the source. In a wireless system, the communication between the Source and destination is established by intermediate nodes, relays, when there is no direct connection. In such a system, signals are passed by multi-hopping. Such transmission helps in enhancing coverage and throughput of wireless networks.

1.2 Problem Setting

In this thesis, we primarily focus on a relay channel where multiple antennas are available at the relay. A dual-hop configuration with two parallel half-duplex relays with no direct link established between the Source and Destination is a simple system to begin with. Despite being simplified, this system helps us in understanding the basic difficulty in finding optimal scheme in the system.(see Fig.1.3)

We consider the MDF scheme and try to generalize its optimal condition by defining a fundamental parameter, Δ. We extend this result to a 2-relay MIMO Gaussian diamond channel. We inspect if a constant gap can be achieved under certain constraints on Δ. We also identify the transmit covariance matrices to be used by each relay in the multiple- access (MAC) state. While considering a wireless network having multiple subchannels, we assume that the transmission is done in two hops where the source forwards the data to the relays and then the relays retransmit the decoded data to the destination. We investigate resource allocation, i.e, subchannel and power allocation to maximize the end-to-end rate. We then use a greedy algorithm for end-to-end rate maximization problem that simultaneously allocates subchannels and power. Simulations are done to compare the Total rate vs. Node power for different locations of the relays.

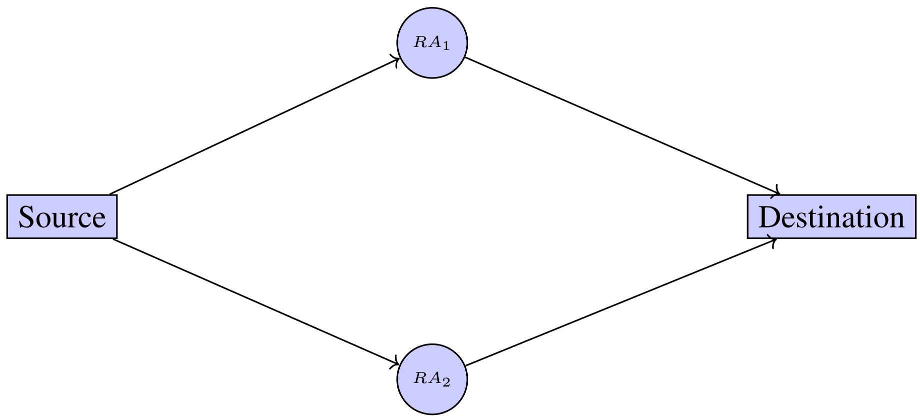

Figure 1.3: Two-Relay MIMO Gaussian Diamond channel with multi-antenna relay

1.3 Thesis Outline

The rest of the thesis has been organized as follows.

- Chapter 2 - Broadly review the existing literature related to half-duple Gaussian MIMO channels, decode-and-forward protocols and resource allocation techniques used in relaying systems.
- Chapter 3 - Establish the system model and protocols used in the thesis. The MIMO equivalent model of the relay communication system and the system model used to simulate the greedy algorithm is presented here.
- Chapter 4 - Coding and scheduling schemes achieving within constant gap from the maximum achievable rate possible in MDF-MAC. Extending it to a 2-relay multiple-input multiple-output (MIMO) Gaussian diamond channel.
- Chapter 5 - Formulate the optimization problem for joint power and subchannel allocation and compare it with other protocols.
- Chapter 6 - Simulation results obtained regarding the behaviour of greedy algorithm and compared with other protocols for different positions of the pair of relays.
- Chapter 7 - The summary, conclusion of the work done and future research possibilities in this area are also mentioned as a final note.

CHAPTER 2

Literature Survey

Multihop systems are introduced in [1], these systems enhance the coverage and throughput of wireless systems. They are also used in framing wireless communication standards [2].

Multi-hop systems, are employed to increase the coverage and the throughput of wireless systems [1]. These systems play a crucial role in developing wireless communication standards, such as IEEE 802.16j (also known as WiMAX) [2]. The single relay channel communication was first studied in [3]. in [4], two of the most important schemes, decode-and-forward and compress-and-forward were introduced. Multi-relay networks were investigated by several researchers. after [4]. A comprehensive analysis of the progress in this field can be found in [5].

The network with two parallel relays is a simplified model to understand key aspects of multi-relay networks. This is introduced in [6] and [7]. For full-duplex relays, papers [6] and [7] show upper and lower bounds on the capacity of the diamond channel. Amplify- and-forward, and the decode-and-forward schemes, and also a hybrid of both the schemes based on time-sharing are also considered.

Kochman et al. in [8] came up with a rematch-and-forward scheme in which bandwidth can be assigned to the first and second hops in varying proportions. in [9], a combined amplify-and-decode-forward strategy which proved to always perform better than the scheme discussed in [8] is introduced. In [10], a combination of the decode-and-forward and compress-and-forward schemes are proposed to achieve the capacity for a special case of the diamond channel with a noiseless relay.

Although the relay channel has been studied extensively, the exact capacity of the channel is still unknown. The approximate capacity of a single-antenna Gaussian relay channel to within one bit was found in [13]. The multi-input multi-output (MIMO) Gaussian relay channel was studied in [14] and the C012 approximate capacity of the MIMO Gaussian relay channel to within a finite number of bits was recently found in

[15]. We consider the 2-relay MIMO Gaussian diamond channel i.e., the multiantenna generalization of the diamond channel considered in [16].

OFDM can reduce the negative impact of multipath fading by employing a method of transmitting data over multiple narrowband channels that have different fading levels. Significant performance improvement can be achieved by dynamic power allocation [17], when he subchannel gains are known at the transmitter,

Resource allocation techniques employed in relay systems are provided in [18]. In the amplify and forward (AF) scheme, optimal power allocation is studied in [21]. Optimal time and power allocation for a decode and forward (DF) method is discussed in [22], with a constraint on the average total power, and the objective being, maximizing the capacity or to minimizing the outage probabiilty.

In [23], a broadband relay channel comprising many parallel, independent Rayleigh fading channels is investigated. [23] assumes that a direct link is present between the source and the destination. Research in done papers [24], [25],[26], [27] separately assess downlink and uplink problems, however, rate matching, is not considered either of those.

CHAPTER 3

System Model

The half-duplex relay channel in Fig. 4.1 is considered. We assume that S, R_1, R_2 and D have n_s, n_1, n_2 and n antennas, respectively. The received signals at relays R_1, R_2 and destination D are given by:

$$\mathbf{y_1} = \mathbf{H_{01}} x_0 + \mathbf{z_1},$$
$$\mathbf{y_2} = \mathbf{H_{02}} x_0 + \mathbf{z_2},$$
$$\mathbf{y} = \mathbf{H_{13}} x_1 + \mathbf{H_{23}} x_2 + \mathbf{z_3} \quad (3.1)$$

respectively, where $\mathbf{x_0}$, $\mathbf{x_1}$, $\mathbf{x_2}$ are the transmit signals from S, R_1, and R_2 respectively, $\mathbf{H}_{01}$, $\mathbf{H}_{02}$, $\mathbf{H}_{13}$, and $\mathbf{H}_{23}$ are the real n_1 x n_s, n_2 x n_s, n x n_1 and n x n_2 MIMO channel matrices corresponding to the $S - R_1$, $S - R_2$, $R_1 - D$, and $R_2 - D$ channels, and $\mathbf{z}_1$, $\mathbf{z}_2$, and $\mathbf{z}_3$ are the n x 1 Gaussian noise vectors with distribution N(0,I) at R_1, R_2, and D, respectively.

We assume constant power constraints for each node across all states. For nodes S, R_1, and R_2, without loss of generality, the power constraints are taken to be 1, i.e., P_0 = P_1 = P_2 = 1. Let

$$C(H,P) = \max_{0 \le Q; tr(Q) \le P} 0.5 * logdet(I + HQH^T),$$

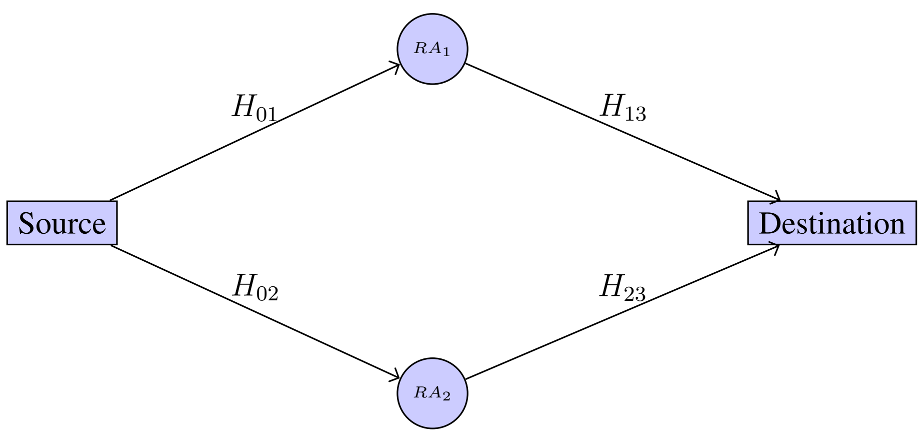

Figure 3.1: Channel coefficients for multi-antenna relay

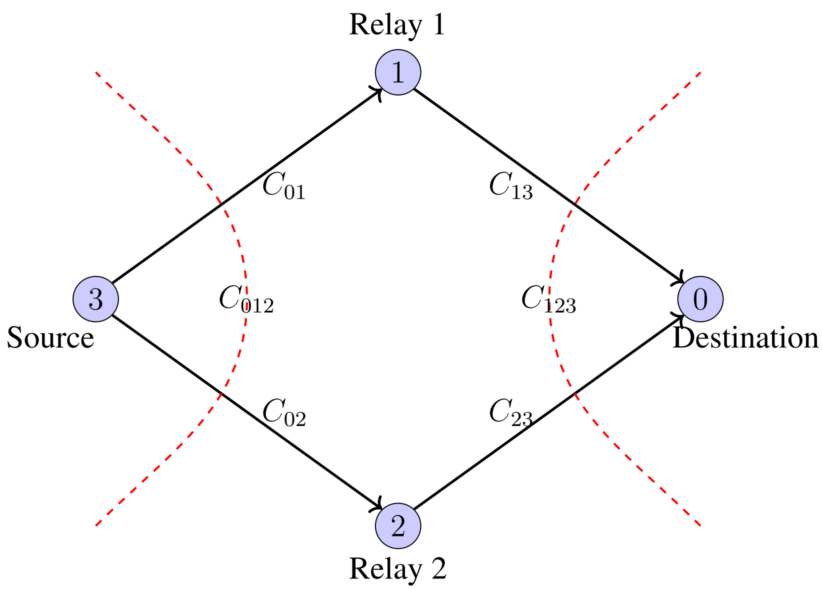

Figure 3.2: The diamond channel with its fundamental parameterΔ

The channel parameters are defined as follows: $C_{01} = C(\mathbf{H_{01}}, \mathbf{1})$, $C_{02} = C(\mathbf{H_{02}}, \mathbf{1})$, $C_{13} = C(\mathbf{H_{13}}, \mathbf{1})$, $C_{23} = C(\mathbf{H_{23}}, \mathbf{1})$, $C_{012} = C(\mathbf{H_{012}}, \mathbf{1})$, and $C_{123} = C(H_{123}, 2)$, where $H_{012}^T = [H_{01}^T H_{02}^T]$and $H_{123} = [H_{13}H_{23}]$. The optimal covariance matrix Q corresponding to each of these capacities are denoted $\mathbf{K_{01}}$, $\mathbf{K_{02}}$, $\mathbf{K_{13}}$, $\mathbf{K_{23}}$, $\mathbf{K_{012}}$, and $\mathbf{K_{123}}$, respectively. For example, we have

$$C_{13} = 0.5 * logdet(I + H_{13}K_{13}H_{13}^T)$$

3.1 Transmission Modes

There are four transmission modes as shown in Fig. 3.3 -

1. Broadcast Mode: Source transmits independent data to Relays 1 and 2, in t_1 fraction of the transmission time, using the superposition coding technique.
2. Forward Mode I: Source transmits new data to Relay 1,in t_2 fraction of the transmission time.Simultaneously, Relay 2 forwards the re-encoded version of the data that it might have received during Broadcast Mode and/or Forward Mode II to Destination.
3. Forward Mode II: Source transmits new data to Relay 2, in t_3 fraction of the transmission time.Simultaneously, Relay 1 forwrards the re-encoded version of the data that it might have received during Broadcast Mode and/or Forward Mode I to Destination.

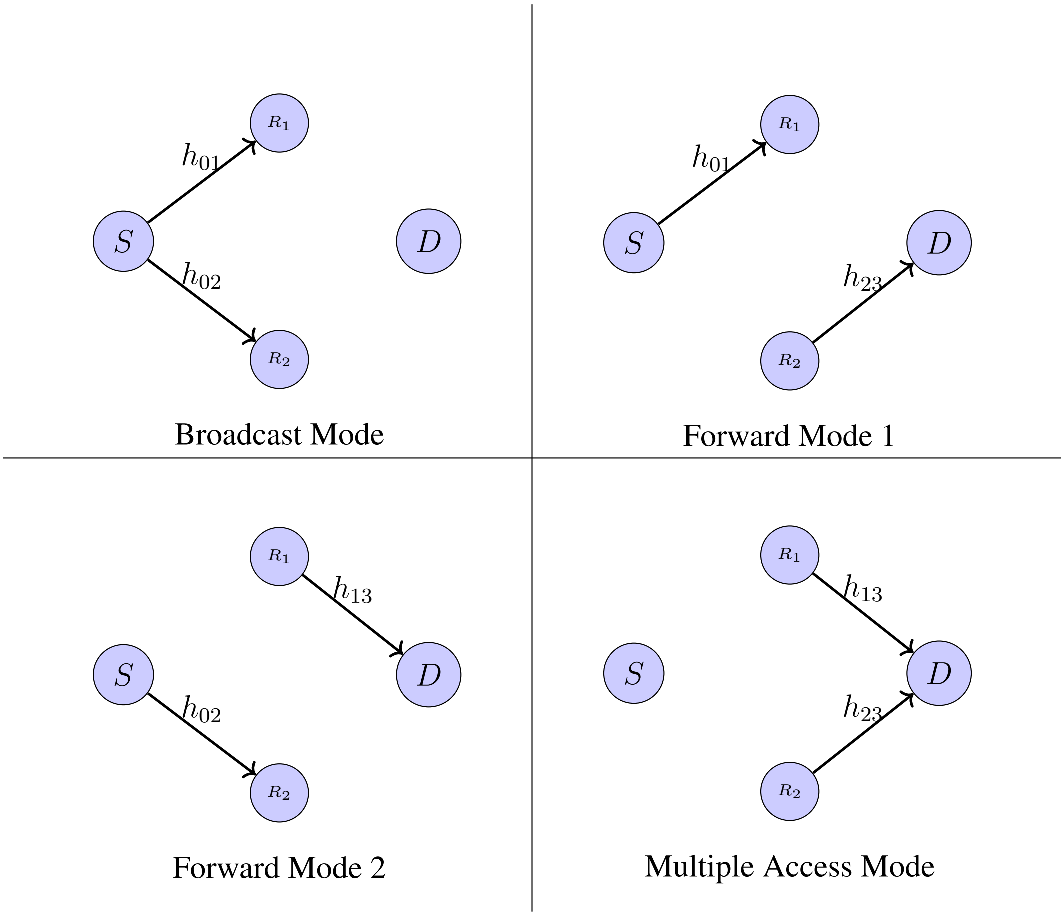

Figure 3.3: Transmission modes for the diamond channel

4. Multiple-Access Mode: Relays 1 and 2 simultaneously transmit the residual information to Destination, in the remaining t_4 fraction of the transmission time. Joint decoding is performed at the destination to decode the received data.

3.2 Resource Allocation

Decode-and-Forward is employed to a relay system having a Source S, Destination D and say M relays , RL_M, where m = (1,2,..M). Assuming no direct link between $S - D$, and all transmissions are done with the help of relays in two hops. We assume the N parallel subchannels in the system to be interfered by independent, unit variance AWGN (Fig.3.4). For each subchannel n, the complex channel gain of $S - RL_m$ is denoted by $h_{m,n}$ and the channel gain of $RL_m - D$ is given by $\bar{h}_{m,n}$). Hence, the power gains are given by $H_{m,n}$ = $|h_{m,n}|^2$ and $H_{m,n}$ = $|\bar{h}_{m,n}|^2$. S AND RL_m are constrained by the power resources available, P_S anf P_R.

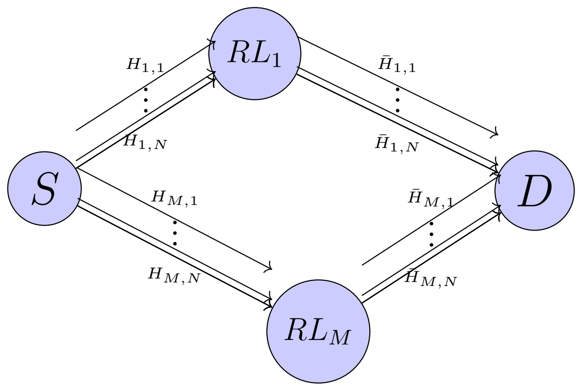

Figure 3.4: System Model

As we are working with half-duplex relays, the transmission is done over 2 phases of equal time slots : Phase 1 and Phase 2. In Phase 1, the information transmission from the Source to each relay is considered. In Phase 2, the passing on of information from each relay to the Destination is considered. All the subchannels may be used in both phase1 and phase 2. Key assumptions are made that each subchannel can be allocated to only one relay in both the phases. In phase 1, source power allocated to n is P_n. In phase 2, RL_m power allocated to n is $\bar{R}_m$

The total rate of $S - RL_m$ in phase 1 is R_m and the total rate of $RL_m - D$ phase 2 is $\bar{R}_m$, then we get

$$R_m = \frac{1}{2} \sum_{n \in E_m} log_2(1 + H_{m,n} P_n) \tag{3.2}$$

$$\bar{R}_m = \frac{1}{2} \sum_{n \in E_m} log_2(1 + \bar{H}_{m,n} \bar{P}_n) \tag{3.3}$$

As relays are operating in decode-and-forward, the contribution of RL_m to the end-to-end rate is constrained by the minimum of R_m and $\bar{R}_m$. We try to find optimum subchannel allocation, E_m and $\bar{E}_m$ and P_n and $\bar{P}_{m,n}$, with the objective to maximise the end-to-end achievable rate, R_{total}. The optimization problem can be derived as

$$\max_{P_n, \bar{P}_{m,n}, E_m, \bar{E}_{m,n}} R_{total} = \max_{P_n, \bar{P}_{m,n}, E_m, \bar{E}_{m,n}} \sum_{m=1}^{M} min(R_m, \bar{R}_m) \tag{3.4a}$$

subject to

$$\sum_{n=1}^{N} P_n \leq P_S \tag{3.4b}$$

$$\sum_{n=1}^{N} \bar{P}_{m,n} \leq P_m \forall m \in (1, ...M) \tag{3.4c}$$

$$P_n, \bar{P}_{m,n} \geq 0 \forall m, n \tag{3.4d}$$

$$E_1, E_2 .... E_M aredisjoint \tag{3.4e}$$

$$\bar{E}_1, \bar{E}_2 ... \bar{E}_M aredisjoint \tag{3.4f}$$

$$E_1 \cup E_2 \cup .... \cup E_M \subset (1, 2...N) \tag{3.4g}$$

$$\bar{E}_1 \cup \bar{E}_2 \cup .... \cup \bar{E}_M \subset (1, 2...N) \tag{3.4h}$$

In the optimization above, the sum of the limiting relay rates is maximized. This is achieved by matching rate of each relay in both the phases. Moreover, the optimization includes decision of E_m and $\bar{E}_m$ and the objective function is minimizing the two functions, therefore it not a convex problem anymore.

CHAPTER 4

Capacity of the Half-Duplex MIMO Gaussian Diamond Channel

In this chapter, we consider a dual-hop communication system, as shown in Fig.4.1. The system comprises a source (S), two parallel half-duplex relays (R1, R2), and a destination (D), indexed as 0, 1, 2, and 3 respectively, as given in Fig. 4.1. Asuume no interference between Source and Destination,and between the relays. The channel gain is assumed to be constant and it is known to all nodes.

As we are considering half-duplex relays, 4 states of transmission exist (see Fig. 3.3). Each state is allocated t_1 , t_2 , t_3 , t_4 proportion of the total transmission time respectively. The total transmission time is normalized to 1. Hence the constraint on time is, $\sum_{i=1}^{4} t_i$ = 1. We assume fixed scheduling and it is known to all nodes prior to transmission. To find communication protocols that achieve rates close to the channel capacity, we define a fundamental parameter Δ of the channel as:

$$\Delta = C_{01}C_{02} - C_{13}C_{23}$$

The MDF-MAC protocol is a multihopping decode-and- forward protocol which uses the Forward Modes and MAC. The total transmission time is normalized to 1 and States 1, 2, and 3 of Fig.5.2 are used for t_1, t_2, and t_3 fractions of the total transmission time. Let R_1 and R_2 be the rates of transmission from relays R_1 and R_2 to the destination in the multiple access state (State 3). Then, the maximum achievable rate R_{MAC} from S to D of the MDF- MAC scheme is given by

$$R_{MAC} = \max_{0\leq t_i, \sum_i t_i=1} min(t_1C_{01}, t_2C_{13} + R_1) + min(t_2C_{02}, t_1C_{23} + R_2) \quad (4.1)$$

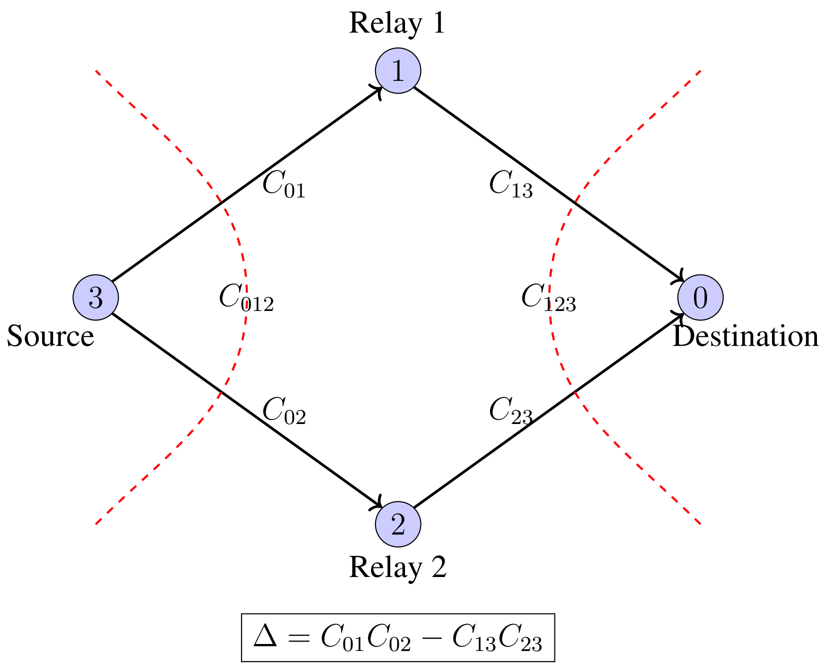

Figure 4.1: The diamond channel with its fundamental parameterΔ

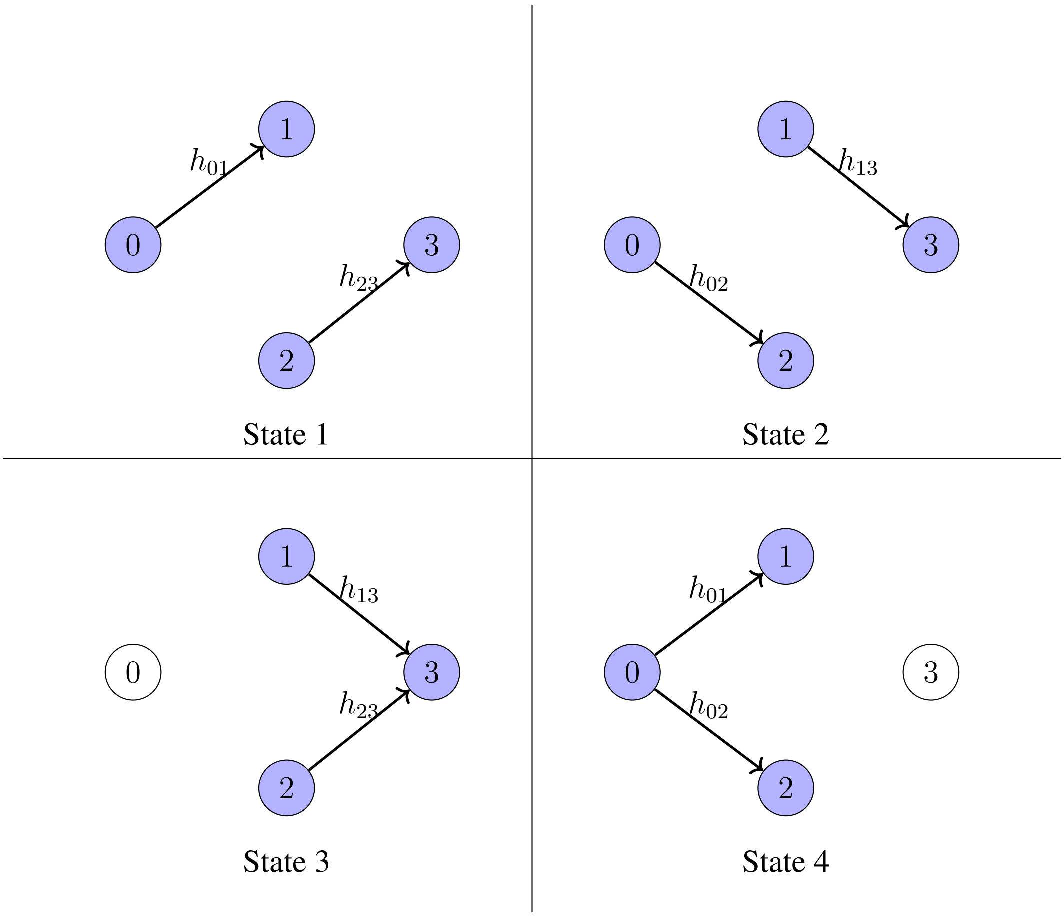

Figure 4.2: States of the diamond channel

We consider the MDF-MAC protocol. In [16], the MDF-MAC was shown to be within 0.71 bits of capacity for the single- antenna Gaussian diamond channel. In [16], the following methodoogy is applied to compute the gap result

- Firstly, obtain an upper bound on the capacity by solving a linear program that is associated with half duplex cutset bound.
- Secondly, formulate a linear program to compute the achievable rate R_{MAC} using the MDF-MAC protocol.
- Then, solve the LP to obtain an achievable R_{MAC}
- Analyse the gap between the achievable rate and the upper bound and it is shown to be bounded if C_{123} - C_{MAC} and C_{123} - $(C_{13} + C_{23})$ are both bounded by a finite constant, where C_{MAC} is the sum rate in the MAC state of MDF-MAC.
- C_{123} - C_{MAC} and C_{123} - $(C_{13} + C_{23})$ are shown to be bounded by finite constants.

We expand this methodology to a MIMO case and observe how the system behaves. We encounter difficulty in step 2 and step 5. In step 2, it is easier to formulate a linear program to compute the achievable rate R_{MAC} using the MDF-MAC protocol in a single-antenna case. This is because, the linear program gives a rate region for (R_1,R_2) in the MAC state which is a pentagon specified by a finite number of linear inequalities. In the MIMO setting, the rate region for (R_1 , R_2) is an infinite union of such pentagons and cannot be exactly described by a finite number of linear inequalities. This capacity region for the MAC state is given by

$$C_{MAC} = \bigcup_{Q_1,Q_2:Q_i\geq 0, tr(Q_i)\leq 1} C'_{MAC}(Q_1, Q_2) \tag{4.2}$$

where $C'_{MAC}(Q_1, Q_2)$ is a pentagon obtained by choosing the covariance matrices at the relays R_1 and R_2 to be Q_1 and Q_2, respectively, i.e., $C'_{MAC}(Q_1, Q_2)$ is the set of all (R_1, R_2) satisfying

$$R_1 \leq \tfrac{1}{2} logdet(I + H_{13}Q_1H_{13}^T) = C'_{13}$$
$$R_2 \leq \tfrac{1}{2} logdet(I + H_{23}Q_2H_{23}^T) = C'_{23}$$

$$R_1 + R_2 \leq \frac{1}{2} logdet(I + H_{13}Q_1H_{13}^T + H_{23}Q_2H_{23}^T) = C'_{MAC} \tag{4.3}$$

By fixing Q_1 and Q_2, we can get a linear program for R_{MAC} even in the MIMO case. However, this should be done carefully since we should to be able to bound C_{123} - C_{MAC} in Step 5. Thus, this choice affects Steps 3, 4, and 5. We study this problem and provide such an appropriate choice for Q_1 and Q_2.

4.1 Cut Set Upper Bound and the Dual Program

The methodology proposed by Khojastepour et al. to compute cut-set type upper bound for general half-duplex networks with K relays is as follows :

1. Firstly, the input distribution and scheduling is fixed, i.e., $p(X_0, X_1, X_2)$, and t_1, t_2, t_3, t_4 such that $\sum_{i=1}^{4} t_i = 1$.
2. Then, $R_{i,j}$ which is the rate of the cut j for each transmission mode i is calculated, where i, j ∈ 1,...,2K.
3. $R_{i,j}$ is multiplied by the corresponding time interval t_i .
4. $\sum_{i=1}^{2K} t_i R_{i,j}$ is computed and minimized over all cuts.
5. Supremum is taken over all input distributions and schedulings

The upper bound, called R_{up} is given by the following LP -

$$\text{maximise } R_{up}$$
$$\text{subject to : } R_{up} \leq t_1 C_{012} + t_2 C_{01} + t_3 C_{02} + t_4.0$$
$$R_{up} \leq t_1 C_{01} + t_2 (C_{01} + C_{23}) + t_3.0 + t_4 C_{23}$$
$$R_{up} \leq t_1 C_{02} + t_2.0 + t_3 (C_{02} + C_{13}) + t_4 C_{13}$$
$$R_{up} \leq t_1.0 + t_2 C_{23} + t_3 C_{13} + t_4 C_{123}$$
$$\sum_{i=1}^{4} t_1 = 1$$

Every feasible point in the dual program provides an upper bound on the primal. We use this fact to get single-equation upper bounds on the capacity. The dual of the linear program is as follows:

$$\text{minimise } R_{up}$$
$$\text{subject to : } R_{up} \geq \tau_1 C_{012} + \tau_2 C_{01} + \tau_3 C_{02} + \tau_4.0$$
$$R_{up} \geq \tau_1 C_{01} + \tau_2 (C_{01} + C_{23}) + \tau_3.0 + \tau_4 C_{23}$$

$$R_{up} \geq \tau_1 C_{02} + \tau_2 .0 + \tau_3 (C_{02} + C_{13}) + \tau_4 C_{13}$$
$$R_{up} \geq \tau_1 .0 + \tau_2 C_{23} + \tau_3 C_{13} + \tau_4 C_{123}$$
$$\sum_{i=1}^{4} \tau_1 = 1, \tau_i \geq 0.$$

τ_i , in the dual LP, where i in (1, . . . , 4) maps to the i^{th} rate constraint in the primal LP. The primal LP is always feasible. From the weak duality property of LP, we benefit

The duality of linear programming makes sure that there is no gap between the primal and the dual solutions. The advantage of using the dual problem is that any feasible choice of the τ gives an upper bound to the rate obtained by solving the original LP. This property is known as the weak duality property of LP . Appropriate (i.e., τ) are chosen in the dual program to get fairly tight upper bounds. Using such vectors simplifies the gap analysis.

4.2 MDF-MAC Scheme

MAC mode paired with MDF with independent messages sent from the relays to Destination. In this scheme, the relays enjoy an increased transmission time. Three transmission modes, i.e., Multiple-Access Mode and Forward Modes I and II are employed. By setting t_1 = 0, the achievable rates for Δ > 0 together with their corresponding scheduling are as follows, respectively for $\Gamma' \leq 0$and Γ' > 0:

$$R^1_{MDF-MAC} = \frac{C_{01}(C_{02} + C_{13})}{C_{01} + C_{13}} - \frac{C_{02}\Delta}{(C_{01} + C_{13})(C_{MAC} - C_{23} + C_{02})} \tag{4.4}$$

$$R^2_{MDF-MAC} = \frac{C_{02}(C_{01} + C_{23})}{C_{02} + C_{23}} - \frac{C_{01}\Delta}{(C_{02} + C_{23})(C_{MAC} - C_{23} + C_{01})} \tag{4.5}$$

where,

$$\Gamma' = C_{02}[C_{123} - C_{23}] - C_{01}[C_{123} - C_{13}] \tag{4.6}$$

If Δ = 0, t_4 becomes zero and the scheme is now nothing but the MDF scheme.

The upper bound for $\Gamma' \leq 0$ and $\Gamma' > 0$ becomes :

$$R_{up}^{3} = \frac{C_{01}(C_{02}+C_{13})}{C_{01}+C_{13}} - \frac{C_{02}\Delta}{(C_{01}+C_{13})(C_{123}-C_{13}+C_{02})} + \delta \tag{4.7}$$

$$R_{up}^{4} = \frac{C_{02}(C_{01}+C_{23})}{C_{02}+C_{23}} - \frac{C_{01}\Delta}{(C_{02}+C_{23})(C_{123}-C_{23}+C_{01})} + \delta \tag{4.8}$$

The gaps for $\Gamma' \leq 0$ and $\Gamma' > 0$ becomes :

$$K_{MAC}^{1} = R_{up}^{3} - R_{MDF-MAC}^{1}$$
$$K_{MAC}^{2} = R_{up}^{4} - R_{MDF-MAC}^{2}$$

And, from [16] we know that

$$C_{123} - C_{MAC} \leq \tfrac{1}{2}$$

Therefore, it is clearly shown that the gap is at most 1 + δ bits. Therefore, adding MAC to MDF scheme ensures the gap of less than 0.71 bits from the upper bounds for $\Delta > 0$.

4.3 Linear program for MAC in MIMO case

For a given $\mathbf{Q_1}$ and $\mathbf{Q_2}$, we defined C'_{13}, C'_{23} and C'_{MAC} in (4.2) . For this $\mathbf{Q_1}$ and $\mathbf{Q_2}$, we can formulate a linear program for R_{MAC} as follows using (4.1) and (4.2).

maximise R_{MAC}

subject to:

$$R_{MAC} \leq t_1 C_{01} + t_2 C_{02}$$
$$R_{MAC} \leq t_2(C_{02}+C_{13}) + R_1$$
$$R_{MAC} \leq t_1(C_{01}+C_{23}) + R_2$$
$$R_{MAC} \leq t_1 C_{23} + t_2 C_{13} + R_1 + R_2$$
$$R_1 \leq t_3 C'_{13}$$
$$R_2 \leq t_3 C'_{23}$$
$$R_1 + R_2 \leq t_3 C'_{MAC}$$
$$\textstyle\sum_{i=1}^{3} t_i = 1, t_i \leq 0$$

Using Fourier-Motzkin elimination to eliminate variables R_1 and R_2, the above optimization problem can be reduced to:

$$\begin{gathered}\text{maximise } R_{MAC}\\ \text{subject to:}\\ R_{MAC} \leq t_1 C_{01} + t_2 C_{02}\\ R_{MAC} \leq t_2(C_{02} + C_{13}) + t_3 C'_{13}\\ R_{MAC} \leq t_1(C_{01} + C_{23}) + t_3 C'_{23}\\ R_{MAC} \leq t_1 C_{23} + t_2 C_{13} + t_3 C'_{MAC}\\ \textstyle\sum_{i=1}^{3} t_i = 1, t_i \leq 0\end{gathered}$$

For $\Gamma' \leq 0$, we choose the pentagon $C_{MAC}(K_{13}, K'_{23})$ given by :

$$(R_1, R_2) : R_1 \leq C_{13}, R_2 \leq C'_{23}, R_1 + R_2 \leq C'MAC1 \tag{4.9}$$

For $\Gamma' > 0$, we choose the pentagon $C_{MAC}(K'_{13}, K_{23})$ given by :

$$(R_1, R_2) : R_1 \leq C'_{13}, R_2 \leq C_{23}, R_1 + R_2 \leq C'MAC2 \tag{4.10}$$

After one iteration of iterative water filling C'_{MAC1} (or C'_{MAC2}) the gap between the sum rate and the sum capacity of the MIMO MAC can be bounded by a finite constant.

We can show the difference between the achievable rate and the upper bound to be:

$$K^1_{MAC} = R^1_{up} - R^1_{MAC-MDF} \tag{4.11}$$

or

$$K^2_{MAC} = R^2_{up} - R^2_{MAC-MDF} \tag{4.12}$$

depending on whether C'_{MAC} = C'_{MAC1} or C'_{MAC2}. This gap can be bounded if we can bound $C_{123} - C'_{MAC}$ and δ.

CHAPTER 5

Resource Allocation and Decode-and-forward relaying for the diamond relay channel with multicarrier transmission

In this chapter, we study the optimal resource allocation to maximize end-to-end rate for a given subchannel allocation. The we extend by jointly optimizing the subchannel and power allocation. The optimization problem (5.1a) for optimal power allocation for a given subchannel assignment together with power constraints, (5.1b), (5.1c) and (5.1d) , when the sets E_m and $\bar{E}_m$ are given for all $m \in (1, .., M)$ is as follows :

$$\max_{P_n, \bar{P}_{m,n}, E_m, \bar{E}_{m,n}} R_{total} = \max_{P_n, \bar{P}_{m,n}, E_m, \bar{E}_{m,n}} \sum_{m=1}^{M} min(R_m, \bar{R}_m) \tag{5.1a}$$

subject to

$$\sum_{n=1}^{N} P_n \leq P_S \tag{5.1b}$$

$$\sum_{n=1}^{N} \bar{P}_{m,n} \leq P_m \forall m \in (1, ...M) \tag{5.1c}$$

$$P_n, \bar{P}_{m,n} \geq 0 \forall m, n \tag{5.1d}$$

$$E_1, E_2....E_M aredisjoint \tag{5.1e}$$

$$\bar{E}_1, \bar{E}_2...\bar{E}_M aredisjoint \tag{5.1f}$$

$$E_1 \cup E_2 \cup .... \cup E_M \subset (1, 2...N) \tag{5.1g}$$

$$\bar{E}_1 \cup \bar{E}_2 \cup .... \cup \bar{E}_M \subset (1, 2...N) \tag{5.1h}$$

5.1 Optimal Resource Allocation For A Given Subchannel Assignment

To optimize the power allocation, it can be formulated as follows :

$$\max_{P_n, \bar{P}_{m,n}} \sum_{m=1}^{M} min(R_m, \bar{R}_m) \tag{5.2a}$$

$$\sum_{n=1}^{N} P_n \leq P_S \tag{5.2b}$$

$$\sum_{n=1}^{N} \bar{P}_{m,n} \leq P_m \forall m \in (1, ..., M) \tag{5.2c}$$

$$P_n, \bar{P}_{mn,} \geq 0 \forall m, n \tag{5.2d}$$

here, R_m and $\bar{R}_m$ are the rates of each RL_m in phase 1 and phase 2, respectively and they are as follows:

$$R_m = \frac{1}{2} \sum_{n \in E_m} log_2(1 + H_{m,n} P_n) \tag{5.3}$$

$$\bar{R}_m = \frac{1}{2} \sum_{n \in E_m} log_2(1 + \bar{H}_{m,n} \bar{P}_n) \tag{5.4}$$

When subchannel allocation is known, E_m and $\bar{E}_m$ for all $m \in (1, .., M)$, the optimal power allocation that solves (5.2a) - (5.2d) for any RL_m is found by waterfilling

$$\bar{P}^*_{m,n} = (\frac{1}{\bar{\lambda}_m} - \frac{1}{\bar{H}_{m,n}})^+ \forall n \in \bar{E}_m \tag{5.5}$$

where $\frac{1}{\bar{\lambda}_m}$ is such that it satisfies RL_m power budget, P_R. The optimal power allocation of the source is given by

P^*_n =$(\frac{1}{\lambda} - \frac{1}{H_{m,n}})^+$, $R^*_m < \bar{R}^*_m$ for $n \in E_m$
P^*_n =$(\frac{1+\mu}{\lambda} - \frac{1}{H_{m,n}})^+$, $R^*_m = \bar{R}^*_m$ for $n \in E_m$

where R^*_m = $\frac{1}{2} \sum_{n \in E_m} log_2(1 + H_{m,n} P^*_n)$ and $\bar{R}^*_m$ = $\frac{1}{2} \sum_{n \in \bar{E}_m} log_2(1 + H_{m,n} P^*_{m,n})$

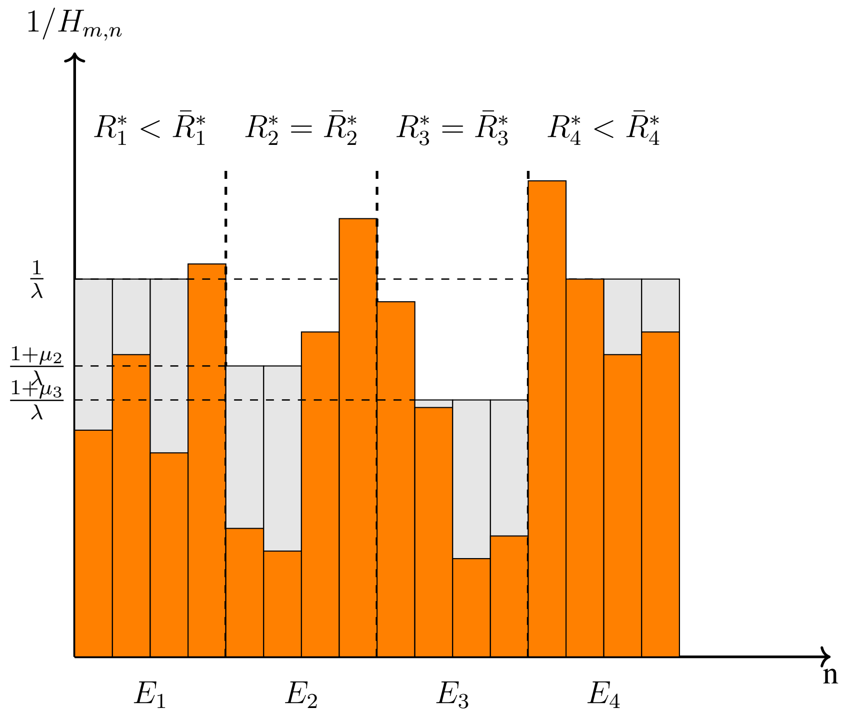

Figure 5.1: Optimal source power allocation when subchannel assignment is known for phase 1, E_m and known optimal phase 2 rates, $\bar{R}_m^*$, m= 1,2,...M with M= 4 relays and N = 16 subchannels

are the rates corresponding to RL_m in phase 1 and phase 2, respectively, with optimal power allocation and λ, μ_m chosen to satisfy source power budget, P_S.

5.2 Greedy Algorithm For a Joint Subchannel And Power Allocation

We have now discussed how to find R_{opt}, the optimal power allocation once we know the subchannel allocation in each phase. Now, we try to optimize the subchannel allocation along with the power optimizing problem. We employ a greedy approach to proceed with the optimization. The bottleneck condition of the maximum rate at which the relays can decode and forward is the minimum of the transmission rates in phase 1 and phase 2. It is not a good condition if there is a mismatch between the rates of the phase 1 and phase 2 as this is simply leading to waste of valuable resources. Therefore, our algorithm should strive to decrease the mismatch by allocating more power to that phase which is choking the rate of transmission. A greedy algorithm is proposed that simultaneously allocated subchannels and powers in each phase. The focus and main objective at every step is to maximise the end-to-end transmission rate by minimising the mismatch. Let R_{opt}(E) denote the end-to-end rate obtained by optimal power allocation for a given subchannel allocation $E = [E_1, ..., E_M; \bar{E}_1, ..., \bar{E}_M]$ as described in the earlier section. The following algorithm is from the paper "Resource Allocation in Wireless Networks with Multiple Relays" by Kag ?an Bakanog ?lu et al.

1. Initialization
   (a) Set A = (1,...,N) and $\bar{A}$ = (1,...,N) which are the available subchannels in the phase 1 and phase 2, respectively.
   (b) Set $E_m = \phi$ and $\bar{E}_m = \phi$
2. Until $A = \phi$ and $\bar{A} = \phi$
   (a) Set $S_m = \phi$ and (or) $\bar{S}_m = \phi$
   (b) For m=1 to M
      i. Find $n^* = argmax H_{m,n} \forall n \in A$ and $\bar{n}^* = argmax \bar{H}_{m,n} \forall \bar{n} \in \bar{A}$
      ii. Find R^1 using R_{opt} when n^* is tentatively allocated to RL_m in phase 1, that is $E_m = E_m \cup (n^*)$
      iii. Find R^2 using R_{opt} when $\bar{n}^*$ is tentatively allocated to RL_m in phase 2, that is $\bar{E}_m = \bar{E}_m \cup (n^*)$
      iv. Find R_3 using R_{opt} when both n^* and $\bar{n}^*$ are tentatively allocated to RL_m in phase 1 and phase 2, respectively, that is $E_m = E_m \cup (n^*)$ and $\bar{E}_m = \bar{E}_m \cup (\bar{n}^*)$
      v. Find $R_m = max(R_1, R_2, R_3)$. The maximum suggests which phase(s) to allocate an additional subchannel to RL_m
      vi. Based on the maximum in step 2(b)v above, set $S_m = (n^*)$ and (or) $\bar{S}_m = (\bar{n}^*)$
   (c) Find m^* = $argmax(R_m)$
   (d) Update $E_{m^*} = E_{m^*} \cup S_{m^*}$ and (or) $\bar{E}_{m^*} = \bar{E}_{m^*} \cup \bar{S}_{m^*}$
   (e) Update $A = A - S_{m^*}$ and (or) $\bar{A} = \bar{A} - \bar{S}_{m^*}$

We set $A, \bar{A}, E_m and \bar{E}_m \forall m = (1, ..., M)$ where A and $\bar{A}$ are the available subchannel sets in each phase and $E_m, \bar{E}_m$ denote the subchannels that are already allocated. Initially, $A, \bar{A}$ are the full set of the Number of subchannels in the system, the greedy algorithm iterates till $A, \bar{A}$ are empty, i.e., each subchannel is allocated to one relay in both the phases. Initially, $A, \bar{A}$ are the full set of the Number of subchannels in the system, the greedy algorithm iterates till $A, \bar{A}$ are empty, i.e., each subchannel is allocated to one relay in both the phases. $E_m, \bar{E}_m$ are initially null sets, by the end of the iterations, E_m are allocated such that they satisfy (5.1g) and $\bar{E}_m$ are allocated such that they satisfy (5.1h). For each relay, the best subchannel among available subchannels (given by A and $\bar{A}$) are tentatively chosen in phase 1 and phase 2. In steps 2(b)ii-2(b)v, phase(s) to allocate subchannels is decided. The respective subchannel allocation for each relay is defined as S_m in phase 1 and(or) in $\bar{S}_m$ in phase 2. Once these steps are iterated for all the relays in the system, we find the relay,RL^*_m, which had the largest end-to-end rate increase. The subchannels stored in the sets S^*_m and (or) $\bar{S}^*_m$ are allocated to RL^*_m and the available subchannel sets (A and $\bar{A}$) are then updated accordingly. This algorithm continues until all subchannels are allocated. The algorithm improves the rates at each step when additional subchannels are allocated, hence it is called greedy.

5.3 DF protocols with multi carrier transmission

In this section, we formulate various DF protocols for the diamond relay channel with parallel subchannels motivated by the capacity gap results above. These DF protocols differ in the number of network states used, the allocation of subcarriers and power in each state, and the resulting operating rate vector for each state. The diamond relay network we are considering is given in the Fig. 5.2

5.3.1 Greedy allocation

A DF protocol that uses only states 3 and 4 in Fig. 5.2 is considered. A greedy algorithm for joint subchannel and power allocation was proposed to optimize this DF protocol with only states 3 and 4. In this method, each subchannel was allocated only for transmission to one relay in state 4 (BC), and for transmission from only one relay in state 3 (MAC). Therefore, frequency division is used in both states.

5.3.2 MDF protocol with multicarrier transmission

The MDF protocol is a multihopping decode-and-forward protocol using states 1 and 2. The states 1 and 2 are used for t_1 and t_2 fractions of the total transmission time. For the multicarrier case, optimal waterfilling power allocation is used for each link in states 1 and 2 to achieve the corresponding parallel Gaussian channel capacities. Therefore, the

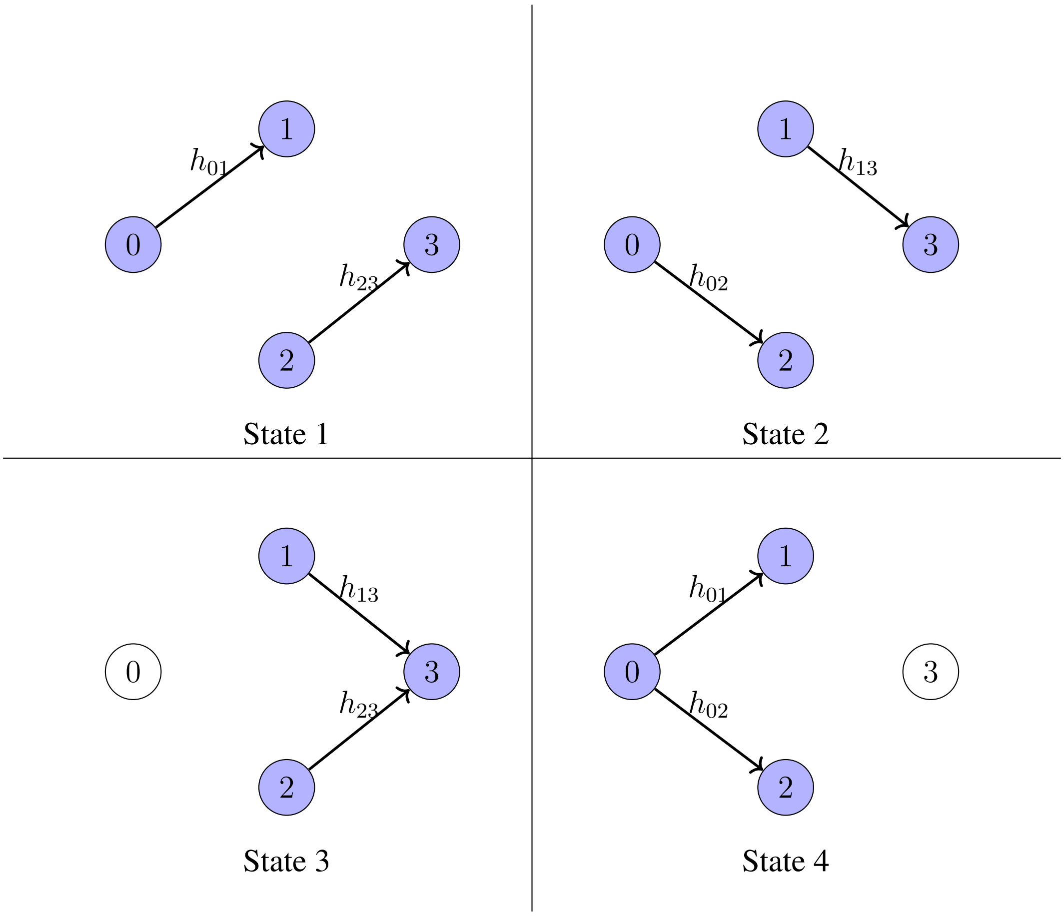

Figure 5.2: States of the diamond channel

maximum achievable rate R_{MDF} from S to D is given by

$$R_{MDF} = \max_{\sum_i t_i=1; t_i \geq 0} min(t_1 C_{01}, t_2 C_{13}) + min(t_2 C_{02}, t_1 C_{23}) \tag{5.6}$$

or, equivalently, by the linear program given below:

$$\begin{gathered} \text{maximize } R_{MDF} \\ \text{subject to :} \\ R_{MDF} \leq t_1 C_{01} + t_2 C_{02} \\ R_{MDF} \leq t_1 (C_{01} C_{23}) \\ R_{MDF} \leq t_2 (C_{13} C_{02}) \\ R_{MDF} \leq t_1 C_{23} + t_2 C_{13} \\ \textstyle\sum_{i=1}^{2} t_i = 1, t_i = 1 \end{gathered}$$

5.3.3 MDF-MAC protocol with multicarrier transmission

The MDF-MAC protocol is a multi-hopping decode-and-forward protocol that uses states 1, 2 and 3. The states 1, 2 and 3 are used for t_1, t_2 and t_3 fraction of the total transmission time. In states 1 and 2, each link is used at its capacity corresponding to the optimal waterfilling power allocation. Suppose R_1 and R_2 are the rates at which D receives messages from R_1 and R_2 in the MAC state of the channel. Then, the maximum achievable rate $R_{MDF-MAC}$ from S and D (for this choice of R_1 and R_2)is given by

$$R_{Mac} = \max_{\sum_i t_i=1; t_i \geq 0} min(t_1 C_{01}, t_2 C_{13} + t_3 R_1) + min(t_2 C_{02}, t_1 C_{23} + t_3 R_2) \tag{5.7}$$

The rate pair (R1,R2) can be chosen to be any pair from the capacity region of the MAC channel in state 3. Optimizing over this (R1,R2) can be reduced to a linear program if we fix a power allocation at each relay for the MAC state, i.e., if we restrict (R1 , R2) to the MAC constraints, we get the linear program which we have discussed in section 4.3

5.3.4 Cutset Bound

The cutset upper bound provides an upper bound on the achievable rate from source to destination using any relaying protocol (not restricted to DF). Therefore, comparison with the cutset upper bound gives us an idea of how close any proposed protocol is with respect to capacity.

CHAPTER 6

Simulation results

In this chapter, we present the results for simulations to study the behaviour of greedy algorithm as a function of Power. Also, different protocols MDF-MAC, MAC, Greedy allocation are all compared to the cutset bound as a function of Power under various scenarios considering different positions of the relays.

6.1 Simulation Set-up

We set up relay system as shown in Fig. 3.4. We set M=2 and N=8, that is, a 2-relay system with 8 sub-channels. The distance of S and D from their center is taken to be 1. For simplicity, we assume that the transmitted power from the Source (P_S) is equal to the transmitted power of the Relays (P_R). As we vary P from 0dB to 20dB, we plot the Total Rate vs Power when the Relays are at a particular position between the Source and Relay. Then, we see how the position of the relays effect the Total rate while keeping the Power transmitted constant.

6.1.1 Greedy Algorithm

Greedy algorithm is applied to obtain a subchannel assignment with an optimal power allocation. Approach to implement the Greedy algorithm is as follows:

- Initialise the number of relays,M, and number of subchannels,N, Transmitting power of the Source, P_S, and Transmitting power of the Relays P_R that you are using in your relay system. In our case, we assume P_S = P_R
- Define distance between the $S - R_1$, $S - R_2$, $R_1 - D$, $R_2 - D$ as a function,d, the horizontal measure of the position of relays from the midpoint of Source and Destination. Because, here we assume that both relays are moving on an imaginary vertical line that is passing through the horizontal line between $S - D$.
- Define the channel gains H_{01}, H_{02},H_{13},H_{23} as a normal random variable with mean 0 and corresponding σ which is defined as $(1/d^2)$

- P is used in the waterfilling algortihm that is used to calculated the end-to-end rate in the following steps.
- Set A and $\bar{A}$ as One vectors with the length of the number of subchannels. As we optimally allocate subchannels to relays, we replace the allocated subchannels' position in the vector with a Zero.
- Set E and $\bar{E}$ as a zero 3D vector with the dimensions (M,M,N). This vector updates itself as we go on optimally allocating the subchannels to the relays in Phase 1 and Phase 2.
- We iterate the greedy algorithm till all the subchannels are allocated to relays in both the phases. In other words, till A and $\bar{A}$ become zero vectors.
- Set S_m and $\bar{S}_m$ as null vectors. These vectors will be used to store the subchannel allocation for a particular relay in one iteration. Once the end-to-end rates of two vectors are compared, the S_m and $\bar{S}_m$ of the relay having max rate will be used to update the corresponding E_m and $\bar{E}_m$.
- For each relay, RL_m, we find the subchannel that has the maximum channel gain of those belonging to A in phase 1, n^* and $\bar{A}$ in phase 2, $\bar{n}^*$
- We compute the end-to-end rate R_1 when we assume $n^?$ is tentatively allocated to RL_m in phase 1, that is $E_m = E_m \cup n^*$
- We compute the end-to-end rate R_2 when we assume $\bar{n}^?$ is tentatively allocated to RL_m in phase 2, that is $\bar{E}_m = \bar{E}_m \cup \bar{n}^*$
- We compute the end-to-end rate R_3 when we assume both n^* and $\bar{n}^?$ are allocated to RL_m, that is $E_m = E_m \cup n^*$ and $\bar{E}_m = \bar{E}_m \cup \bar{n}^*$
- Then, we find which of the above three assumptions are giving the maximum rate,R_m, that is, find maximum of R_1, R_2, R_3
- Depending on which allocation is giving R_m, we update S_m =n^* and(or) $\bar{S}_m$= $\bar{n}^*$.
- After repeating the process for all the relays, we find which relay has the highest end-to-end rate, m^*
- Update the $E_{m^*} = E_{m^*} \cup S_{m^*}$ and (or) $\bar{E}_{m^*} = \bar{E}_{m^*} \cup \bar{S}_{m^*}$ and $A = A - S_{m^*}$ and (or) $\bar{A} = \bar{A} - \bar{S}_{m^*}$
- Repeat till A and $\bar{A}$ become zero vectors

6.1.2 Analysis

We plot the Total rate vs Power(in dB) for three values d, horizontal distance of the relays from the center of $S - D$. The plot is given in the Fig. 6.1. It shows that the best rate is observed when the relays are equidistant from the S and D.

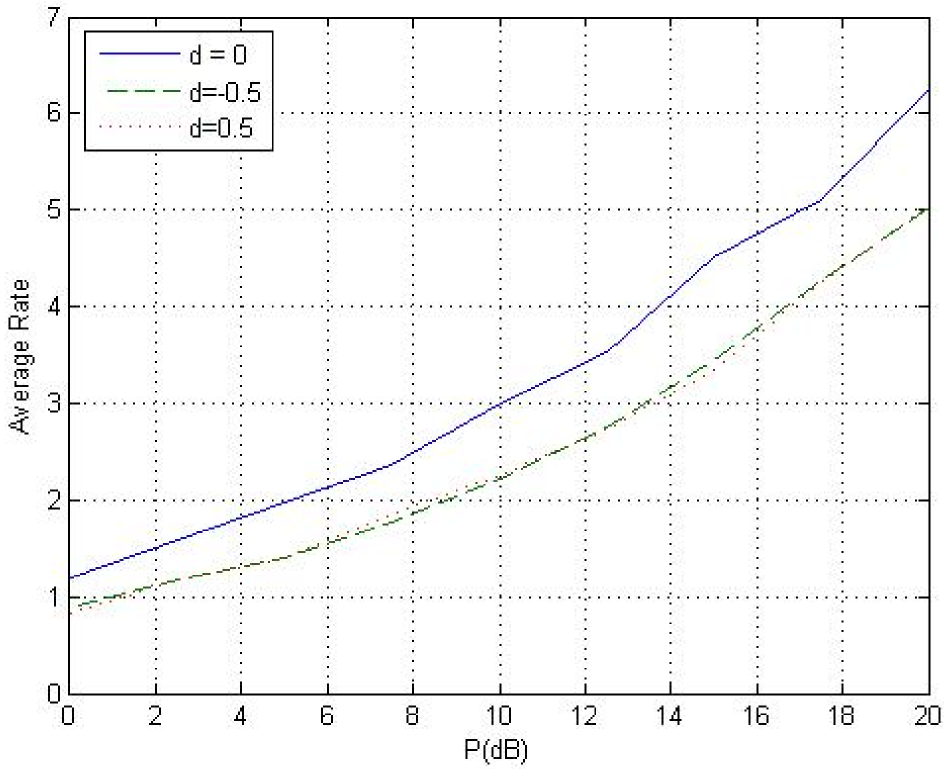

Figure 6.1: Total rate vs node power where P_S=P_R= P for M = 2, N = 8

6.2 Simulation Results

We compare the achieved rates of the MDF, MDF-MAC, with the rate achieved by the greedy algorithm and the cutset upper bound using simulations. We assume the links in the diamond relay channel are all Rayleigh faded. The number of subchannels considered is 16 and the subchannels are assumed to be either i.i.d. or correlated. For the i.i.d. subchannels case, the channel distributions are given by h_{01} $N(0, \frac{1}{d_{01}^2})$, h_{02} $N(0, \frac{1}{d_{02}^2})$, h_{13} $N(0, \frac{1}{d_{13}^2})$, h_{23} $N(0, \frac{1}{d_{23}^2})$ where d_{01}, d_{02}, d_{13}, d_{23} , are the $S - R_1, S - R_2, R_1 - D, R_2 - D$ distances, respectively. Various scenarios with different relay locations are studied to provide insight on the performance of the different protocols. The scenarios considered are described in Fig. 6.1.

Scenario	d_{01}	d_{02}	d_{13}	d_{23}	Nature of subchannels
1.	1	1	1	1	i.i.d
2.	1	1	2	2	i.i.d
3.	2	2	1	1	i.i.d
4.	1	2	1	2	i.i.d

Figure 6.2: Scenarios considered

6.2.1 Scenario 1

In this scenario, each of the relays is equidistant from S and D,i.e.,$d_{01} = d_{02} = d_{13} = d_{23}$ =1. In Fig. 6.3 we compare the cut-set upper bound with the average rates achieved by the MAC, MDF, and greedy allocation protocols. In each case, the average rates are obtained by averaging over 100 channel realizations.

With respect to Fig. 6.3:

Observations :

1. The MDF-MAC protocol achieves rates very close to the cut-set upper bound
2. MDF scheme also gives rates close to the MDF-MAC protocol.
3. MDF-MAC and MDF protocols perform significantly better than greedy allocation in this scenario.
4. The slope of the MDF-MAC protocol rate R_{MAC} and MDF rate R_{MDF} follows the slope of the cutset bound and is more than the slope of the greedy allocation rate R_{greedy}

Analysis :

- Observation 3 is because MDF-MAC and MDF use states 1 and 2 which are important in this scenario, while the greedy allocation uses states 3 and 4.
- Observation 5 mplies that the gap between the MDF-MAC protocol and greedy allocation will increase with increasing P.

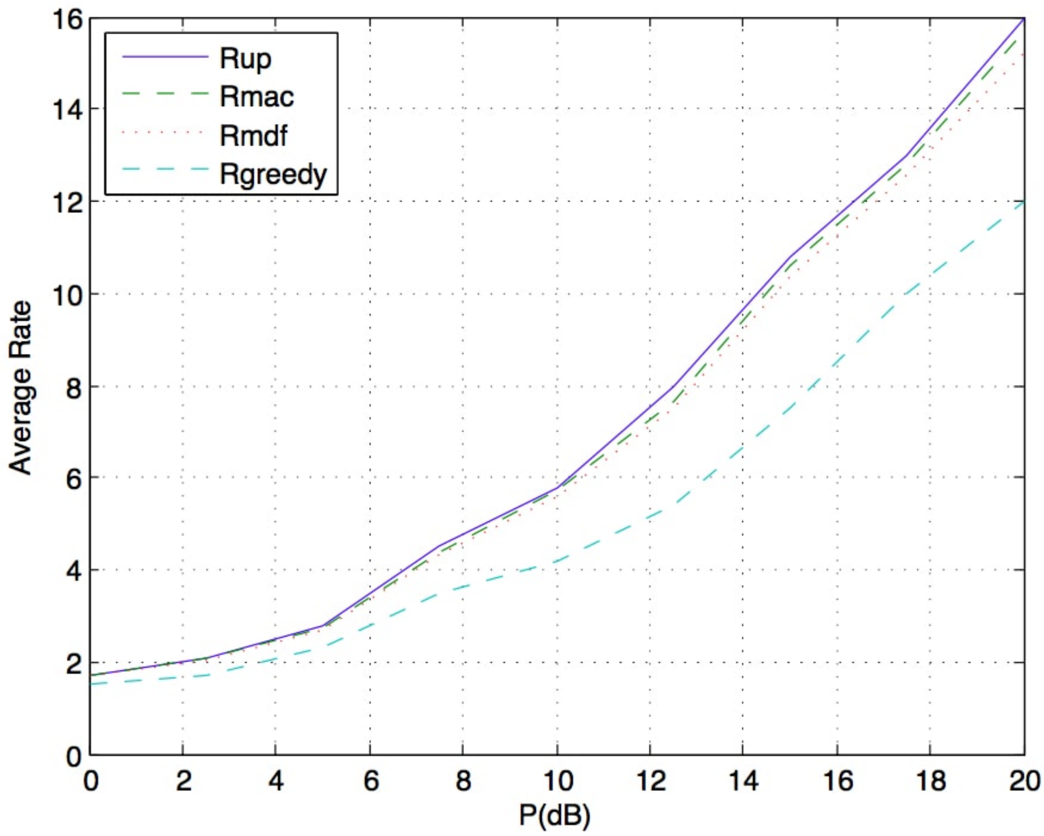

Figure 6.3: Scenario 1

6.2.2 Scenario 2

In this scenario, we consider the relays to be twice as close toS as they are to D,i.e.,d_{01} = d_{02} =1 and d_{13} = d_{23} = 2. Fig. 6.4 compares the cut-set upper bound, MDF-MAC, MDF and greedy allocation rates for this scenario.

With respect to Fig.6.4

Observations :

1. MDF-MAC protocol gives the best rates and the rates are close to cut-set upper bound
2. MDF-MAC protocol is better than the greedy allocation scheme
3. The greedy allocation is better than the MDF protocol in this scenario.

Analysis :

- Observation 1 is because in this scenario for most of the realizations of the channel, Δ is positiveis because in this scenario for most of the realizations of the channel, Δ is positive

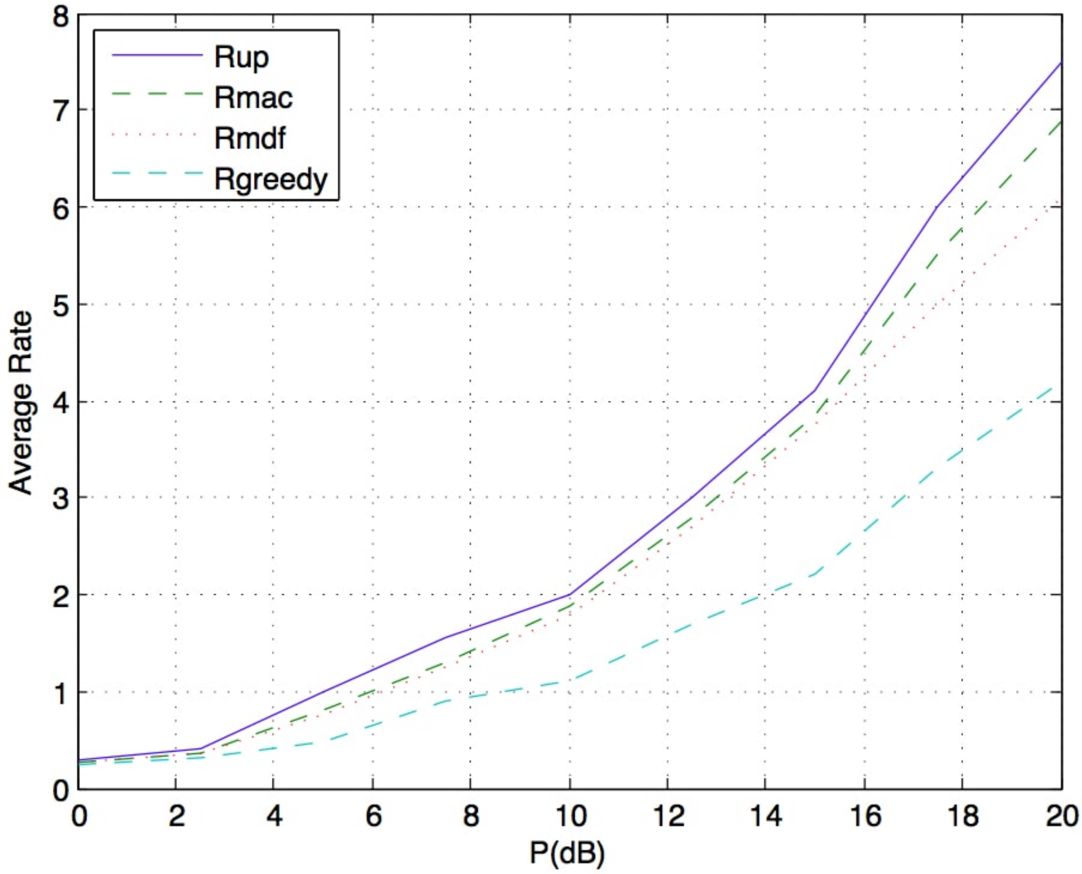

Figure 6.4: Scenario 2

- Observation 3 is because the MAC state is important in this scenario and is not used in the MDF protocol, but used in the greedy allocation.

6.2.3 Scenario 4

In this scenario, R_1 is closer to S while R_2 is closer to D,i.e.,d_{01} = d_{23} =1 and d_{02} = d_{13} =2. Fig.6.5 gives the average rates for 100 realizations of the channel for this scenario. This figure shows that MDF-MAC protocol is the best scheme in this setting. The other comparisons are similar to scenario 1.

6.2.4 Scenario 3

In this scenario, we consider the distance of each relay from S to be double that from D,i.e.,$d_{01} = d_{02} = 2$ and $d_{13} = d_{23}$ = 1.MDF-BC is ideal for this scenario because $\Delta < 0$ for most of the realizations of the channel.

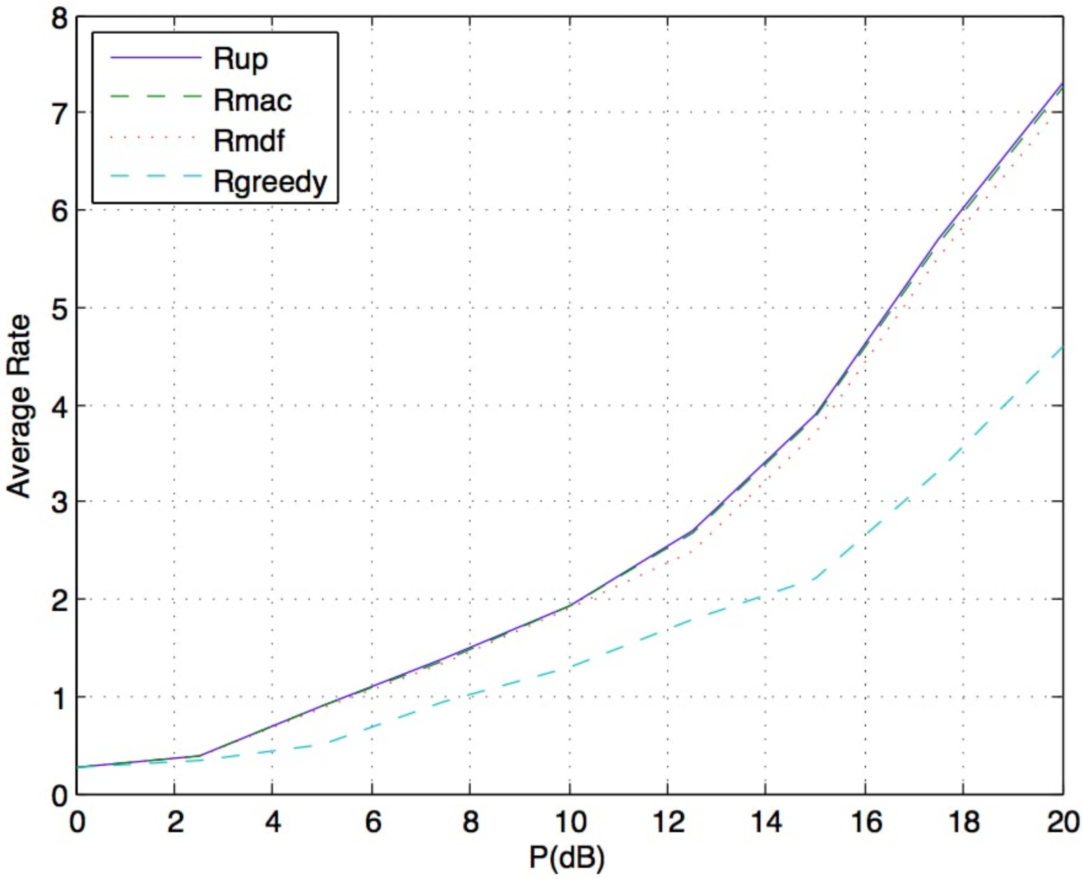

Figure 6.5: Scenario 4

CHAPTER 7

Conclusions and Future Scope

7.1 Conclusions

The capacity of the Half-duplex diamond channel is considered for the Decode-and-Forward protocol. This idea is extended to a MIMO case when the defined parameter $\Delta \geq 0$ is satisfied. Linear programming is done to compute R_up, R_{MDF_MAC}, R_{MAC}. In the case of Multiple relays and subchannels, optimal allocation is achieved by greedy allocation of the resources. These rates are evaluated and compared in various scenarios depending on the position of the relays and varying Power.

From the simulation results, we could compare R_{up}, $R_{MDF-MAC}$ and R_{greedy} based on the mostly probable value of Δ.

7.2 Future Scope

- We can further investigate by extending the system model to more than two-relay case
- Considering the relays in a 3-dimensional space, and evaluate how their 3D position effects the Rate. The question of formulating a parameter similar to Δ in the case of more than 2-relay system should be resolved.
- Similar efforts could be made in a scenario having direct link between $S - D$

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