Interval Arithmetic based Load Flow and Optimal Power Flow

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Certificate

This is to certify that the report titled **INTERVAL ARITHMETIC BASED LOAD FLOW AND OPTIMAL POWER FLOW**, submitted by **GADI SUNIL**, to the Indian Institute of Technology Madras, for the award of the degree BACHELOR OF TECHNOLOGY and MASTER OF TECHNOLOGY, is a bona fide record of the project work done by him under my supervision. The contents of this report, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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Abstract

KEYWORDS: Uncertainty, Load Flow, Optimal Power Flow, Interval Arithmetic, Krawczyk Method, Monte Carlo Simulation

Power Flow analysis is a fundamental tool for the study of power systems. This work presents a methodology to solve power flow with loads and generation being uncertain. In order to deal with the effect of uncertainties of loads and generation on power flow Interval Arithmetic (IA) techniques have been used in literature. In this present work Krawczyk method, an IA method, is applied to solve the nonlinear power balance equations of the system. The implementation was performed in Mathematica® environment. In order to assess the performance of the Krawczyk algorithm, the method was applied to 2-Bus, 4-Bus and 14-Bus systems and the results are compared with the results obtained by the Monte Carlo simulations. Later the same concept was extended to solve Optimal Power Flow with uncertainty, and the results are compared with the results obtained by the Monte Carlo simulations.

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Abbreviations

IA	**I**nterval **A**rithmetic
LF	**L**oad **F**low
OPF	**O**ptimal **P**ower **F**low
MC	**M**onte **C**arlo
DLF	**D**eterministic **L**oad **F**low
PLF	**P**robabilistic **L**oad **F**low
FLF	**F**uzzy **L**oad **F**low
NR	**N**ewton **R**aphson
PDF	**P**robability **D**istribution **F**unction
CDF	**C**umulative **D**istribution **F**unction
ELD	**E**conomic **L**oad **D**ispatch
KKT	**K**arush–**K**uhn–**T**ucker
pu	**p**er **u**nit
AC	**A**lternating **C**urrent
DC	**D**irect **C**urrent
IEEE	**I**nstitute of **E**lectrical and **E**lectronics **E**ngineers

Symbols

μ	Mean
σ	Deviation
$\|V_i\|$	Voltage magnitude at i^{th} bus
δ_i	Voltage angle at i^{th} bus
Y	Admittance
G	Conductance
B	Susceptance
P_G	Real Power generation
Q_G	Reactive Power generation
P_D	Real Power demand
Q_D	Reactive Power demand
N	Number of buses
N_g	Number of generator buses
N_v	Number of voltage controlled buses
$P_{g_i}^{min}$	Lower limit of generation of a generator
$P_{g_i}^{max}$	Upper limit of generation of a generator
$F(P_g)$	Operating fuel cost of a generator
λ	Incremental Fuel cost
λ_P	Lagrangian multiplier on active power equation
λ_Q	Lagrangian multiplier on reactive power equation

Chapter 1

Introduction

Power flow is the most important analysis in power systems and is a pre requisite for several other analysis like stability, state estimation etc. The Power flow problem is formulated as a set of precisely known nonlinear algebraic equations that must be solved simultaneously. From the solution of these equations, bus voltages can be determined precisely.

The load flow approach, referred as Deterministic Load Flow (DLF), requires precise values to be chosen for each input variable. The solution provides precise network voltages and flows through each line. The specified values rest upon assumptions about the operating condition derived from historical measurements or predictions about future conditions and thus, cannot be considered accurate. Even in the case where the inputs are based on measurements, inaccuracies arise from time-skew problems, three-phase unbalance, static modeling approximations of dynamic components (e.g., transformer tap changers), variations in line parameters, and so on[1]. Solutions obtained by DLF would be valid only for a single specific system configuration and operating condition. However, the system evolves through time. It appears that it would be more reasonable to ask not what the system looks like at a given instant, but rather to ask for the range of all plausible system conditions that might be encountered as a result of expected uncertainties in demand and other system parameters. Thus, loads and other parameters can be characterized not by a single number but by a range of values, an Interval.

1.1 Motivations and Objectives

The importance of these uncertainties should be recognized and the need for alternative approaches is needed. At a minimum, running numerous studies varying a few parameters or operating conditions to obtain a better feel for system performance should be done. More systematically, the probabilistic load flow (PLF) was proposed during 1970s [2] and has since been extended by several others [3]-[5].

The PLF considers load and generation as random variables with probability distributions. The output variables, i.e., voltages and power flows, are then random variables with the probability distributions obtained using probabilistic techniques. PLF solutions are typically obtained using a linearized model because of the complexity introduced by using random variables. Alternatively, one can employ Monte Carlo method (MC) as in [6] but this is generally too expensive computationally. Recently, a second family of load flow approaches has arisen based on fuzzy sets [7], termed here Fuzzy Load Flow (FLF). Later a related approach to FLF based on Interval Arithmetic(IA) was proposed in [8].

In this work Interval Arithmetic methods are understood and are applied to solve Load Flow with uncertainties. Similarly the topic is extended for solving OPF with uncertainties and the results are shown.

1.2 Organization of the Report

The report is organized into 5 chapters.

Chapter 1 gives an outline of the report, motivations and primary objectives of the work.

Chapter 2 briefly explains Load Flow and Optimal Power Flow and the the reasons for uncertainty in the power flow analysis are explained. Also MC method is explained briefly.

Chapter 3 explains the basics of Interval Mathematics and the different methods to solve Interval equations. Krawczyk Method is explained in detail. The Krawczyk algorithm for solving load flow is stated and is demonstrated with an example.

Chapter 4 Krawczyk method is applied to 2-Bus, 4-Bus and 14-Bus systems for solving probabilistic load flow. Krawczyk is also extended to optimal power flow and implemented on 3-Bus, 6-Bus and 14-Bus systems. The detailed simulation results are presented.

Chapter 5 presents the conclusion of this work.

The 14 Bus system data is given in the **Appendix**.

Chapter 2

Load Flow and Optimal Power Flow

2.1 Load Flow

Load flow studies are essential for power system planning and operation. Since the load is a static quantity and it is the power that flows through transmission lines, the purists prefer to call this Power Flow studies rather than load flow studies.

Through the load flow studies the voltage magnitudes and angles at each bus in the steady state can be obtained. This is rather important as the magnitudes of the bus voltages are required to be held within a specified limit. Once the bus voltage magnitudes and their angles are computed using the load flow analysis, the real and reactive power flows through each line can be computed. Also based on the difference between power flows in the sending and receiving ends, the losses in a particular line can also be computed. Furthermore, from the line flow we can also determine the over and under load conditions.

2.1.1 Real and Reactive Power Injected at a Bus

For the formulation of the real and reactive power injected at a bus, we need to define the following quantities. Let the voltage at the i^{th} bus be denoted by

$$V_i = |V_i| \angle\delta_i = |V_i| (\cos\delta_i + j\sin\delta_i) \tag{2.1}$$

Also let us define the admittance at bus i as

$$Y_{ii} = |Y_{ii}| \angle\theta_{ii} = |Y_{ii}| (\cos\theta_{ii} + j\sin\theta_{ii}) = G_{ii} + jB_{ii} \tag{2.2}$$

Similarly the admittance of the branch connected between the buses i and j can be written as

$$Y_{ij} = \left|Y_{ij}\right| \angle\theta_{ij} = \left|Y_{ij}\right| (\cos\theta_{ij} + j\sin\theta_{ij}) = G_{ij} + jB_{ij} \tag{2.3}$$

Let N be the total number of buses in the system. The current injected at bus i is given as

$$\begin{aligned} I_i &= Y_{i1}V_1 + Y_{i2}V_2 + ... + Y_{iN}V_N \\ &= \sum_{k=1}^{N} Y_{ik}V_k \end{aligned} \tag{2.4}$$

It is to be noted we shall assume the current injected into the bus to be positive and that leaving the bus to be negative. As a consequence the active power and reactive power entering a bus will also be assumed to be positive. The complex power at bus

i is then given by

$$
\begin{aligned}
P_i - jQ_i &= V_i^* I_i = V_i^* \sum_{k=1}^{N} Y_{ik} V_k \\
&= |V_i| (\cos\delta_i - j\sin\delta_i) \sum_{k=1}^{N} |Y_{ik} V_k| (\cos\theta_{ik} + j\sin\theta_{ik})(\cos\delta_k + j\sin\delta_k) \\
&= \sum_{k=1}^{N} |Y_{ik} V_i V_k| (\cos\delta_i - j\sin\delta_i)(\cos\theta_{ik} + j\sin\theta_{ik})(\cos\delta_k + j\sin\delta_k)
\end{aligned}
\tag{2.5}
$$

Note that

$$(\cos\delta_i - j\sin\delta_i)(\cos\theta_{ik} + j\sin\theta_{ik})(\cos\delta_k + j\sin\delta_k) = \cos(\theta_{ik} + \delta_k - \delta_i) + j\sin(\theta_{ik} + \delta_k - \delta_i) \tag{2.6}$$

Substituting (2.6) in (2.5) we get the real and reactive power injected at an i^{th} bus as

$$P_i = \sum_{k=1}^{N} |Y_{ik} V_i V_k| \cos(\theta_{ik} + \delta_k - \delta_i) \tag{2.7}$$

$$Q_i = -\sum_{k=1}^{N} |Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i) \tag{2.8}$$

2.1.2 Classification of Buses

The four unknown quantities associated with each bus i are P_i, Q_i, voltage angle δ_i and voltage magnitude $|V_i|$. The general practice in Power Flow studies is to identify 3 types of buses in the network. At each bus two of the four quantities P_i, Q_i, $|V_i|$ and δ_i are specified and the remaining two are calculated using the power flow equations (2.7) and (2.8). Depending on the quantity specified the buses are classified into three categories namely *generation bus, load bus* and *slack bus.*

Voltage Controlled Buses/PV Buses : These are the buses where generators are connected. Therefore the power generation at such buses is controlled through a prime mover while the terminal voltage is controlled through the excitation system. Keeping the input power constant through turbine-governor control and keeping the bus voltage constant using automatic voltage regulator, we can specify constant P_{g_i} and $|V_i|$ for these buses. This is why such buses are also referred to as PV buses. It is to be noted that the reactive power supplied by the generator Q_{g_i} depends on the system configuration and cannot be specified in advance. Furthermore we have to find the unknown angle δ_i of the bus voltage.

Load Buses/PQ Buses : At a PQ bus no generator is connected and hence the generated real power P_{g_i} and reactive power Q_{g_i} are taken as zero. The load drawn by these buses are defined by real power -P_{l_i} and reactive power -Q_{l_i} in which the negative sign accommodates for the power flowing out of the bus. This is why these buses are referred to as PQ bus. The objective of the load flow is to find the bus voltage magnitude $|V_i|$ and its angle δ_i at a PQ bus.

Slack/Swing Bus : Usually this bus is numbered 1 for the load flow studies. This bus sets the angular reference for all the other buses. Since it is the angle difference between two voltage sources that dictates the real and reactive power flow between them, the particular angle of the slack bus is not important. However it sets the reference against which angles of all the other bus voltages are measured. For this reason the angle of this bus is usually chosen as $0°$. Furthermore it is assumed that the magnitude of the voltage $|V_i|$ of this bus is known.

The unscheduled bus-voltage magnitudes and angles in the input data of the power flow study are called *state variables* or *dependent variables* since their values, which describe the state of system, depend on the quantities specified at all the buses.Hence the power flow problem is to determine values for all state variables by solving an equal number of power flow equations based on the input data specifications. If there are N_g number of PV buses in the system of N buses, there will be $(2N - N_g - 2)$ equations to be solved for $(2N - N_g - 2)$ state variables as shown in Table 2.1.

Once the state variables are calculated, the complete information about the system is known and all other quantities which depend on the state variables can be determined.

Bus Type	No. of Buses	Quantities Specified	No. of available equations	No. of state variables
Slack: $i = 1$	1	$\delta_1, \|V_1\|$	0	0
PV(i=2,....,N_g+1)	N_g	P_i , $\|V_i\|$	N_g	N_g
PQ(i=N_g+2,....,N)	$(N-N_g-1)$	P_i , Q_i	$2(N-N_g-1)$	$2(N-N_g-1)$
Total	N	$2N$	$(2N-N_g-2)$	$(2N-N_g-2)$

TABLE 2.1: Classification of Buses for Power Flow Analysis

P_1 , Q_1 at the slack bus , Q_i at each voltage controlled bus, the power loss P_L of the system can be easily found with the obtained results from the Load flow.

2.1.3 Methods to solve power flow equations

The functions P_i and Q_i of (2.7) and (2.8) are non linear functions of state variables δ_i and $|V_i|$. We therefore would require iterative methods for solving these non-linear equations. Several techniques are used to solve these non-linear equations like:

1. Gauss – Seidal Method
2. Newton Raphson Method
3. Fast Decoupled Power Flow etc.

The systems of equations are solved using the above iterative techniques[9]. Newton Raphson method has fast convergence and will be explained in the next section.

2.1.4 Newton Raphson(NR) Method

Taylor's series expansion for a function of two or more variables is the basis for the Newton Raphson Method of solving the power-flow problem.
Given a set of nonlinear equations

$$y_i = f_i(x_1, x_2, ..., x_n) \qquad (i = 1, 2, ..., n) \tag{2.9}$$

And the initial estimate for the solution vector be $x^{(0)} = x_1^{(0)}, x_2^{(0)}, ..., x_n^{(0)}$. Assuming $\Delta x_1, \Delta x_2, ..., \Delta x_n$ are the corrections required for $x_1^{(0)}, x_2^{(0)}, ..., x_n^{(0)}$ respectively, so that the equations (2.9) are solved i.e.

$$y_i = f_i(x_1^{(0)} + \Delta x_1, x_2^{(0)} + \Delta x_2, ..., x_n^{(0)} + \Delta x_n) \qquad (i = 1, 2, ..., n) \quad (2.10)$$

Each equation of set can be expanded by Taylor's series for a function of two or more variables.

$$\begin{aligned} y_i &= f_i(x_1^{(0)} + \Delta x_1, x_2^{(0)} + \Delta x_2, ..., x_n^{(0)} + \Delta x_n) \\ &= f_i(x_1^{(0)}, x_2^{(0)}, ..., x_n^{(0)}) + \Delta x_1 \left.\frac{\partial f_i}{\partial x_1}\right|_{x^{(0)}} + \Delta x_2 \left.\frac{\partial f_i}{\partial x_2}\right|_{x^{(0)}} + ... + \Delta x_n \left.\frac{\partial f_i}{\partial x_n}\right|_{x^{(0)}} + \Psi_i \end{aligned} \quad (2.11)$$

where Ψ_i a function of higher powers of $\Delta x_1, \Delta x_2, ..., \Delta x_n$ and 2^{nd}, 3^{rd}, ... derivatives of the function f_i. Neglecting Ψ_i, the linear set of equations resulting is as follows:

$$\begin{aligned} y_1 &= f_1(x_1^{(0)}, x_2^{(0)}, ..., x_n^{(0)}) + \Delta x_1 \left.\frac{\partial f_1}{\partial x_1}\right|_{x^{(0)}} + \Delta x_2 \left.\frac{\partial f_1}{\partial x_2}\right|_{x^{(0)}} + ... + \Delta x_n \left.\frac{\partial f_1}{\partial x_n}\right|_{x^{(0)}} \\ y_2 &= f_2(x_1^{(0)}, x_2^{(0)}, ..., x_n^{(0)}) + \Delta x_1 \left.\frac{\partial f_2}{\partial x_1}\right|_{x^{(0)}} + \Delta x_2 \left.\frac{\partial f_2}{\partial x_2}\right|_{x^{(0)}} + ... + \Delta x_n \left.\frac{\partial f_2}{\partial x_n}\right|_{x^{(0)}} \\ &\;\;... \\ y_n &= f_n(x_1^{(0)}, x_2^{(0)}, ..., x_n^{(0)}) + \Delta x_1 \left.\frac{\partial f_n}{\partial x_1}\right|_{x^{(0)}} + \Delta x_2 \left.\frac{\partial f_n}{\partial x_2}\right|_{x^{(0)}} + ... + \Delta x_n \left.\frac{\partial f_n}{\partial x_n}\right|_{x^{(0)}} \end{aligned} \quad (2.12)$$

$$\begin{bmatrix} y_1 - f_1(x_1^{(0)}, x_2^{(0)}, ..., x_n^{(0)}) \\ y_2 - f_2(x_1^{(0)}, x_2^{(0)}, ..., x_n^{(0)}) \\ \vdots \\ y_n - f_n(x_1^{(0)}, x_2^{(0)}, ..., x_n^{(0)}) \end{bmatrix} = \begin{bmatrix} \left.\frac{\partial f_1}{\partial x_1}\right|_{x^{(0)}} & \left.\frac{\partial f_1}{\partial x_2}\right|_{x^{(0)}} & \cdots & \left.\frac{\partial f_1}{\partial x_n}\right|_{x^{(0)}} \\ \left.\frac{\partial f_2}{\partial x_1}\right|_{x^{(0)}} & \left.\frac{\partial f_2}{\partial x_2}\right|_{x^{(0)}} & \cdots & \left.\frac{\partial f_2}{\partial x_n}\right|_{x^{(0)}} \\ \vdots & \vdots & \ddots & \vdots \\ \left.\frac{\partial f_n}{\partial x_1}\right|_{x^{(0)}} & \left.\frac{\partial f_n}{\partial x_2}\right|_{x^{(0)}} & \cdots & \left.\frac{\partial f_n}{\partial x_n}\right|_{x^{(0)}} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix} \quad (2.13)$$

or,

$$D \qquad = \qquad J \qquad\qquad M$$

Where J is the Jacobian for the functions f_i and M is the change vector Δx_i. In

Iterative form, it is written as:

$$\begin{aligned} D^{(k)} &= J^{(k)} M^{(k)} \\ M^{(k)} &= [J^{(k)}]^{-1} D^{(k)} \end{aligned} \tag{2.14}$$

The new values of x_i are calculated from

$$x_i^{(k+1)} = x_i^{(k)} + M^{(k)} \tag{2.15}$$

The process is repeated until two successive values for each x_i differ only by a specified tolerance. In this process J can be evaluated in each iteration or may be evaluated only once provided Δx_i are changing slowly. Because of quadratic convergence, Newton's method is mathematically superior to Gauss-Seidel method and is less prone to divergence with ill conditioned problems.

Newton-Raphson method is more efficient and practical for large power systems. Main advantage of this method is the number of iterations required to obtain a solution is independent of the size of the problem and computationally it is very fast.

The Power Flow equations given by (2.7) and (2.8) are

$$P_i = \sum_{k=1}^{N} | Y_{ik} V_i V_k | \cos(\theta_{ik} + \delta_k - \delta_i)$$

$$Q_i = - \sum_{k=1}^{N} | Y_{ik} V_i V_k | \sin(\theta_{ik} + \delta_k - \delta_i)$$

The above equations constitute a set of nonlinear algebraic equations in terms of the independent variables, voltage magnitude in per unit and phase angles in radians, it can be easily observed that there are two equations for each load bus given by (2.7) and (2.8) and there is one equation for each voltage controlled bus, given by (2.7). Expanding the above equations in Taylor-series and neglecting higher-order terms.

We obtain,

$$\begin{bmatrix} \Delta P_2^{(k)} \\ \vdots \\ \Delta P_N^{(k)} \\ \Delta Q_{N_g+1}^{(k)} \\ \vdots \\ \Delta Q_N^{(k)} \end{bmatrix} = \begin{bmatrix} \left(\frac{\partial P_2}{\partial \delta_2}\right)^{(k)} & \cdots & \left(\frac{\partial P_2}{\partial \delta_N}\right)^{(k)} & \left(\frac{\partial P_2}{\partial |V_2|}\right)^{(k)} & \cdots & \left(\frac{\partial P_2}{\partial |V_N|}\right)^{(k)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \left(\frac{\partial P_N}{\partial \delta_2}\right)^{(k)} & \cdots & \left(\frac{\partial P_N}{\partial \delta_N}\right)^{(k)} & \left(\frac{\partial P_N}{\partial |V_2|}\right)^{(k)} & \cdots & \left(\frac{\partial P_N}{\partial |V_N|}\right)^{(k)} \\ \left(\frac{\partial Q_{N_g+1}}{\partial \delta_2}\right)^{(k)} & \cdots & \left(\frac{\partial Q_{N_g+1}}{\partial \delta_N}\right)^{(k)} & \left(\frac{\partial Q_{N_g+1}}{\partial |V_2|}\right)^{(k)} & \cdots & \left(\frac{\partial Q_{N_g+1}}{\partial |V_n|}\right)^{(k)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \left(\frac{\partial Q_N}{\partial \delta_2}\right)^{(k)} & \cdots & \left(\frac{\partial Q_N}{\partial \delta_N}\right)^{(k)} & \left(\frac{\partial Q_N}{\partial |V_2|}\right)^{(k)} & \cdots & \left(\frac{\partial Q_N}{\partial |V_N|}\right)^{(k)} \end{bmatrix} \begin{bmatrix} \Delta \delta_2^{(k)} \\ \vdots \\ \Delta \delta_N^{(k)} \\ \Delta \left|V_{N_g+1}\right|^{(k)} \\ \vdots \\ \Delta |V_N|^{(k)} \end{bmatrix} \tag{2.16}$$

Where $\Delta P_2^{(k)}$,..., $\Delta P_N^{(k)}$ and $\Delta Q_{N_g+1}^{(k)}$,....., $\Delta Q_N^{(k)}$ are mismatches in active and reactive powers respectively. In the above equation, bus-1 is assumed to be the slack bus. Equation (2.16) can be written as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \tag{2.17}$$

The Jacobian matrix is inverted and the mismatch vector is found, it is then added to the initial guess to find the next iteration value. The process is continued until the values no longer vary. This interesting property of weak coupling between $P - \delta$ and $Q - V$ variables gave the necessary motivation in developing the Decoupled Load Flow method, in which $P - \delta$ and $Q - V$ problems are solved separately.

2.2 Uncertainty in Power Flows

The models used in Deterministic Load Flow analysis are only approximations. The available conventional methodology does not answer the presence of uncertainties in the mathematical modeling of power systems. Modeling and computation of engineering aspects of power systems is subject to many sources of uncertainty.

2.2.1 Sources of Uncertainty

There can be several reasons for uncertainty like:

1. Representation of the model.
2. Representation of various physical components.
3. Introduction of noise at the inputs.
4. Errors in the measured parameters of the system.
5. Errors in the magnitude of the demand assumed for the system.
6. Errors due to Numerical Modeling using finite arithmetic.

In addition, environmental, regulatory and technology change considerations often introduce uncertainties that are of greater significance, yet less quantifiable in nature.

2.2.2 Characterization of Uncertainty

The purpose of a formal characterization of uncertainty is to gain a greater understanding of a system or process. Single-solution answers, although pleasing in traditional engineering terms, often give an incomplete picture of the behavior of a system. A characterization that explicitly considers uncertainty allows us to create models and answer questions that are either impossible or difficult to answer with deterministic methods.

Several roles in the characterization of uncertainty are:

1. Uncertainty as an aid in the decision making process. Decision makers often consider the risk associated with a particular decision. The nature of the uncertainty also has an influence on the decision: overestimating a number may result in slightly higher costs of operation, but underestimating the same number could result in severe effects on a system, which will translate into considerably higher costs.

2. Deterministic solutions in the presence of uncertainty give deterministic answers which most probably will not be correct. Of greater value would be to bracket the solution and either give intervals guaranteed to contain the solution, or probabilistic measures guaranteed to contain the solution at a given level of confidence.

3. Certain types of solutions are often unacceptable: the design must be sure to exclude them. For example, designs that result in unstable eigenvalues are usually unacceptable.

4. Uncertainty is essential when reconciling mathematical models with measurements on physical systems. The classic example of this use of uncertainty is the state estimation problem in power systems, where more measurements than strictly needed are made on a system, and the state of the system is determined under the assumption that measurements are subject to error.

Methods for handling uncertainty can be applied to determine both engineering and economic parameters, such as current flows, voltages, cost and reliability (or security). Of increasing interest are methods capable of characterizing important externalities of a power system, such as environmental effects. These externalities are often associated with greater degrees of uncertainty than is customary within traditional engineering models. The fact that uncertainty exists is no reason, however, for simply ignoring an important concern. Rather, methods for capturing the inherent uncertainty must be used and incorporated into the more traditional ways of assessing the system.

2.2.3 Probabilistic Load Flow

Deterministic Load Flow(DLF) uses specific values of power generations and load demands of a selected network configuration to calculate system states and power flows. Therefore, DLF ignores uncertainties in the power systems, e.g. uncertainties as described above, the outage rate of generators, the change of network configurations and the variation of load demands. Furthermore, modern power systems with integration of Distributed Generation units, such as Wind Turbines and photo voltaic systems, introduce additional power fluctuations into the system due to their uncontrollable prime sources. Therefore, the deterministic approach is not sufficient for the analysis of modern power systems and the results from DLF may give an unrealistic assessment of the system performance. In order to take uncertainties into consideration, different mathematical approaches for uncertainty analysis can be used, such as the probabilistic approach, fuzzy sets [10]. Each method uses the notion of an "uncertain variable." An uncertain variable is a variable that can take more than one numeric value according to the point of view of the method. For probabilistic methods, uncertain variables are better known as random variables, for Interval methods they are known as Interval variables, and for fuzzy arithmetic methods are known as fuzzy or possibilitic variables. In this work Interval methods have been used to solve the Load flow with uncertainty and extend the topic to Optimal Power Flow.

The probabilistic approach has a solid mathematical background and has been applied to power systems in different areas. The Probability Load Flow(PLF) was first proposed in 1974 and has been further developed and applied into power system normal operation, short-term/long-term planning as well as other areas. The PLF requires inputs with Probability Distribution Function (PDF) or Cumulative Distribution Function (CDF) to obtain system states and power flows in terms of PDF or CDF, so that the system uncertainties can be included and reflected in the outcome. The PLF can be solved numerically, i.e. using a Monte Carlo method, or analytically, e.g. using a convolution method, or a combination of them. We discuss the Monte Carlo Method in the next section.

2.3 Monte Carlo(MC) Method

As we discussed above the uncertainty in inputs can be expressed as PDF and CDF. MC method is adopted for the PLF analysis. The two main features of MC simulation are random number generation and random sampling. Softwares such as MATLAB and MATHEMATICA provide algorithms for Pseudo Random Number generation. Refer to [11] for different techniques of random sampling, e.g. simple random sampling, stratified random sampling, etc. Although sampling techniques can be rather sophisticated, the PLF using MC is in principle doing Deterministic Load Flow for a large number of times with inputs of different combinations of nodal power values. Therefore, the exact nonlinear form of Load Flow equations can be used in the PLF analysis.

The capability to use the exact non-linear LF equations is the reason why results obtained from the PLF using MC are usually taken as a reference to the results obtained from other PLF algorithms with simplified LF equations, so as to check the accuracy of the algorithms. In spite of its relatively high accuracy, the MC method requires large amount of computation time due to the large number of LF calculations.

2.3.1 Example

Consider a 2 Bus system with a generator and a load as shown in Fig. 2.1.

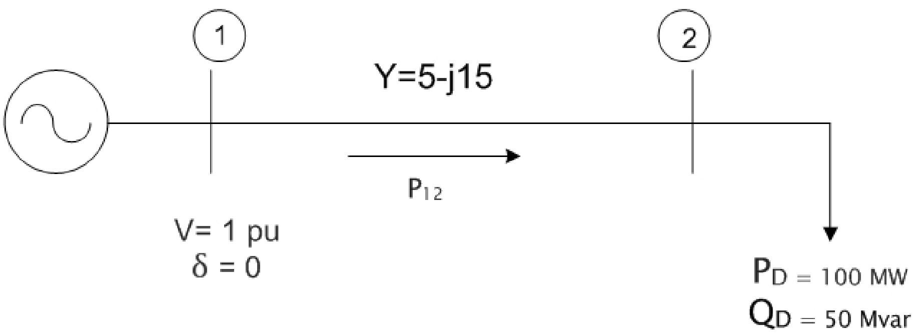

FIGURE 2.1: Single Line Diagram of a Two Bus System

With 100 MVA base, allowing the demand to vary by ±20%, we get the mean and deviation of the demands as

For P_D, $\mu = 1$ pu and $\sigma = 2/30$ pu.
For Q_D, $\mu = 0.5$ pu and $\sigma = 1/30$ pu.

Here we consider the fact that $\mu \pm 3\sigma$ covers 99.7% under the Normal Distribution Function.

The Power injection equations obtained for the given system are:

$$15.8V_2 \cos(108.43^\circ - \delta_2) + 5V_2^2 + P_D = 0 \tag{2.18}$$

$$15V_2^2 - 15.8V_2 \sin(108.43^\circ - \delta_2) + Q_D = 0 \tag{2.19}$$

Using a Random Number generator in Mathematica we take 1000 samples of both P_D and Q_D with mean as 1 and 0.5 and standard deviation as 2/30 and 1/30 respectively and solve the Load Flow equations for each such sample. The results obtained for those 1000 samples are gathered together and are formed into a Gaussian Distribution and the Mean and Deviation of those distributions are found. The obtained results are as follows:

For V_2, μ = 0.944927 pu and σ = 0.00282622 pu.
For δ_2, μ = -0.0530822 pu and σ = 0.00437375 pu.

Taking the range as $\mu \pm 3\sigma$, we get the outputs as:

V_2 = [0.935791 , 0.953406]
δ_2 = [-0.066204 , -0.039961]

Which mean that for a deviation of ±20% in the demands, the voltage V_2 lies between the values 0.935791 pu and 0.953406 pu and δ_2 lies between the values -0.066204 pu and -0.039961 pu.

MC method provides results with a high accuracy but the time required for the computation is large. So alternate methods like Interval Arithmetic is used which provides results with almost the same accuracy as MC but with less computational time. Interval Arithmetic based Load Flow is discussed in next Chapter.

2.4 Optimal Power Flow

2.4.1 Economic Dispatch

Electrical energy cannot be stored; it is generated from natural sources and delivered to the demands. A transmission system is used for delivery of electrical energy to the load points. In brief, an interconnected power system consists of three parts:

1. **Generators**, which produce the electrical energy.
2. **Transmission lines**, which transmits the produced energy to demands.
3. **Loads**, which consume the energy.

Since it is not possible to store electrical energy, the net energy generation in the system must be equal to the total system load and power losses. The main objective of power system is to supply the load continuously and as economic as possible. Planning the power generated by each generation unit and the system analysis is done in different steps from weeks until minutes before real time.

Economic Load Dispatch (ELD) is the process of allocating generation among different generating units; in such a way that the overall cost of generation is minimized. In ELD problem we do not consider the power losses in transmission lines and transmission limit constraints, so the total power generation must be equal to the total load. ELD is allocating loads to generation units with minimum cost while meeting the constraints. It is formulated as an optimization problem of minimizing the total costs of generation units. The total cost of generation includes fuel costs, costs of labor, supplies, maintenance. This cost depends on the amount of real power produced by the generator. Generation cost is considered as a quadratic function in terms of real power produced.

$$F_i(P_{g_i}) = a_i P_{g_i}^2 + b_i P_{g_i} + c_i \quad \text{Rs/h} \tag{2.20}$$

Thus, the total cost C is given by

$$C = \sum_{i=1}^{N_g} F_i(P_{g_i}) \tag{2.21}$$

The Economic Load Dispatch (ELD) problem can therefore be stated as:
Minimize

$$\sum_{i=1}^{N_g} F_i(P_{g_i})$$

Subjected to: the energy balance

$$\sum_{i=1}^{N_g} P_{g_i} = P_D \tag{2.22}$$

and the inequality constraints

$$P_{g_i}^{min} \leq P_{g_i} \leq P_{g_i}^{max} \qquad i = 1, 2, 3, ..., N_g \tag{2.23}$$

where,

P_{g_i} is the real power generation
P_D is the real power demand
N_g is the number of generator buses
$P_{g_i}^{min}$ is the lower limit of generation at i^{th} bus
$P_{g_i}^{max}$ is the upper limit of generation at i^{th} bus
$F_i(P_{g_i})$ is the operating fuel cost of the i^{th} generator

The above constrained optimization problem is converted into an unconstrained optimization problem. Using Lagrange multiplier method, a function L is defined as

$$L(P_{g_i}, \lambda) = F_i(P_{g_i}) + \lambda(P_D - \sum_{i=1}^{N_g} P_{g_i}) \tag{2.24}$$

where λ is the Lagrangian Multiplier.

Applying Karush–Kuhn–Tucker (KKT) conditions,

$$\frac{\partial L(P_{g_i}, \lambda)}{\partial P_{g_i}} = 0 \tag{2.25}$$

$$\frac{\partial L(P_{g_i}, \lambda)}{\partial \lambda} = 0 \tag{2.26}$$

By (2.25),

$$\frac{\partial L(P_{g_i}, \lambda)}{\partial P_{g_i}} = \frac{\partial F_i(P_{g_i})}{\partial P_{g_i}} - \lambda = 0 \implies \frac{\partial F_i(P_{g_i})}{\partial P_{g_i}} = \lambda \tag{2.27}$$

where, $\frac{\partial F_i(P_{g_i})}{\partial P_{g_i}}$ is the incremental fuel cost of the i^{th} generator.

By (2.26),

$$P_D - \sum_{i=1}^{N_g} P_{g_i} = 0 \tag{2.28}$$

Considering the cost function given by (2.20), the incremental cost can be defined as,

$$\frac{\partial F_i(P_{g_i})}{\partial P_{g_i}} = 2a_i P_{g_i} + b_i \tag{2.29}$$

Substituting the incremental cost into (2.27), the equation becomes

$$2a_i P_{g_i} + b_i = \lambda \qquad i = 1, 2, ..., N_g \tag{2.30}$$

Rearranging (2.30) to get P_{g_i}

$$P_{g_i} = \frac{\lambda - b_i}{2a_i} \qquad (i = 1, 2, ..., N_g) \tag{2.31}$$

Substituting the value of P_{g_i} in (2.28), we get

$$\lambda = \frac{P_D + \sum_{i=1}^{N_g} \frac{b_i}{2a_i}}{\sum_{i=1}^{N_g} \frac{1}{2a_i}} \tag{2.32}$$

If any particular generator either hits $P_{g_i}^{min}$ or $P_{g_i}^{max}$, its loading is held fixed at that value and the balance load is shared among other generators by running ELD again excluding the generator hitting the limit.

2.4.2 Optimal Power Flow

The optimal power flow or OPF has had a long history in its development. It was first discussed by Carpentier in 1962 [12] and took a long time to become a successful algorithm that could be applied in everyday use. Current interest in the OPF centers around its ability to solve for the optimal solution that takes account of the security of the system.

We can solve the OPF for the minimum generation cost and require that the optimization calculation also balance the entire power flow at the same time. Note also that the objective function can take different forms other than minimizing the generation cost. It is common to express the OPF as a minimization of the electrical losses in the transmission system, or to express it as the minimum shift of generation and other controls from an optimum operating point. We could even allow the adjustment of loads in order to determine the minimum load shedding schedule under emergency conditions. Regardless of the objective function, however, an OPF must guaranty that all security constraints are satisfied at the solution.

Highlights of OPF:

1. If the entire set of power flow equations are solved simultaneously with the generation cost minimization, the representation of incremental losses is exact. Further, with an objective function that minimizes the losses themselves, the power flow equations are quite necessary.

2. The Economic Load Dispatch solutions only observed the generation limits $P_{g_i}^{min} \leq P_{g_i} \leq P_{g_i}^{max}$. With all of the power flow constraints included in the formulation, many more of the power system limits can be included. These include limits on the generator reactive power($Q_{g_i}^{min} \leq Q_{g_i} \leq Q_{g_i}^{max}$), limits on the voltage magnitude at generation and load buses ($V_i^{min} \leq V_i \leq V_i^{max}$), and the flows in transmission lines or transformers expressed in either MW or MVA ($MVA_{ij}^{min} \leq MVA_{ij} \leq MVA_{ij}^{max}$). These operating constraints now allows the user to dispatch generation such that the constraints are not violated.

3. The OPF can also include constraints that represent operation of the system after contingency outages. These "security constraints" allow the OPF to dispatch the system in a defensive manner. That is, the OPF now forces the system to be operated so that if a contingency happened, the resulting voltages and flows would still be within limit. Thus, constraints such as the following might be incorporated:

$$V_i^{min} \leq V_i \text{ (with line nm out)} \leq V_i^{max}$$
$$MVA_{ij}^{min} \leq MVA_{ij} \text{ (with line nm out)} \leq MVA_{ij}^{max}$$

   which implies that the OPF would prevent the post-contingency voltage on bus k or the post-contingency flow on line ij from exceeding their limits for an outage of line nm. This special type of OPF is called a "security-constrained OPF".

4. In the ELD calculation developed, the only adjustable variables were the generator MW outputs themselves. In the OPF, there are many more adjustable or "control" variables that can be specified. Thus, the OPF gives us a framework to have many control variables adjusted in the effort to optimize the operation of the transmission system.

5. The ability to use different objective functions provides a very flexible analytical tool. The objective can be Minimization of generator cost, Minimization of losses, etc.

Mathematical Representation of OPF

Minimize operation cost of Generating stations

$$C = \sum_{i=1}^{N_g} F_i(P_{g_i}) = \sum_{i=1}^{N_g}(a_i P_{g_i}^2 + b_i P_{g_i} + c_i) \tag{2.33}$$

Subjected to:

1. active power balance in the network

$$P_i(V, \delta) - P_{g_i} + P_{d_i} = 0 \qquad (i = 1, 2, ..., N) \tag{2.34}$$

2. reactive power balance in the network

$$Q_i(V, \delta) - Q_{g_i} + Q_{d_i} = 0 \qquad (i = N_v + 1, N_v + 2, ..., N) \tag{2.35}$$

3. Security-related constraints called soft constraints.

   - limits on real power generation

$$P_{g_i}^{min} \leq P_{g_i} \leq P_{g_i}^{max} \qquad (i = 1, 2, ..., N_g) \tag{2.36}$$

   - limits on voltage magnitudes

$$V_i^{min} \leq V_i \leq V_i^{max} \qquad (i = N_v + 1, N_v + 2, ..., N) \tag{2.37}$$

   - limits on voltage angles

$$\delta_i^{min} \leq \delta_i \leq \delta_i^{max} \qquad (i = 2, 3, ..., N) \tag{2.38}$$

4. Functional constraint with a function of control variables

- limits on reactive power

$$Q_{g_i}^{min} \leq Q_{g_i} \leq Q_{g_i}^{max} \qquad (i = 1, 2, ..., N_g) \tag{2.39}$$

- limits on MVA rating of the transmission lines

$$MVA_{ij}^{min} \leq MVA_{ij} \leq MVA_{ij}^{max} \tag{2.40}$$

Real Power flow equations are:

$$P_i(V, \delta) = V_i \sum_{j=1}^{N} V_j(G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j)) \tag{2.41}$$

Reactive Power flow equations are:

$$Q_i(V, \delta) = -V_i \sum_{j=1}^{N} V_j(G_{ij} \cos(\delta_i - \delta_j) - B_{ij} \sin(\delta_i - \delta_j)) \tag{2.42}$$

where,

N is the number of buses
N_g is the number of generator buses
N_v is the number of voltage controlled buses
P_i is the active power injection into bus i
Q_i is the reactive power injection into bus i
P_{d_i} is the active power load demand on bus i
Q_{d_i} is the reactive power load demand on bus i
P_{g_i} is the real power generation on bus i
Q_{g_i} is the reactive power generation on bus i
$P_{g_i}^{min}$ is the lower limit of generator on bus i
$P_{g_i}^{max}$ is the upper limit of generator on bus i
V_i is the magnitude of voltage on bus i
δ_i is the phase of voltage on bus i
$Y_{ij} = G_{ij} + jB_{ij}$ are elements of admittance matrix

The load flow constraints are augmented into the objective function. The additional variables are known as Lagrange Multiplier functions or incremental cost functions. The cost function becomes:

$$L(P_g, V, \delta) = F(P_{g_i}) + \sum_{i=1}^{N} \lambda_{P_i}[P_i(V,\delta) - P_{g_i} + P_{d_i}] + \sum_{i=1}^{N} \lambda_{Q_i}[Q_i(V,\delta) - Q_{g_i} + Q_{d_i}] \quad (2.43)$$

Applying KKT conditions.

$$\frac{\partial L}{\partial P_{g_i}} = \frac{\partial F}{\partial P_{g_i}} - \lambda_{P_i} \quad (2.44)$$

$$\frac{\partial L}{\partial \delta_i} = \sum_{j=1}^{N} [\lambda_{P_j} \frac{\partial P_j}{\partial \delta_i}] + \sum_{j=N_v+1}^{N} [\lambda_{Q_j} \frac{\partial Q_j}{\partial \delta_i}] \qquad (i = 2, 3, ..., N) \quad (2.45)$$

$$\frac{\partial L}{\partial V_i} = \sum_{j=1}^{N} [\lambda_{P_j} \frac{\partial P_j}{\partial V_i}] + \sum_{j=N_v+1}^{N} [\lambda_{Q_j} \frac{\partial Q_j}{\partial V_i}] \qquad (i = N_v + 1, N_v + 2, ..., N) \quad (2.46)$$

$$\frac{\partial L}{\partial \lambda_{P_i}} = P_i(V,\delta) - P_{g_i} + P_{d_i} \quad (2.47)$$

$$\frac{\partial L}{\partial \lambda_{Q_i}} = Q_i(V,\delta) - Q_{g_i} + Q_{d_i} \quad (2.48)$$

Now the equations (2.44) - (2.48) are solved simultaneously using Newton Raphson Method to obtain the solution of the OPF.

Problem Name	Includes Voltage angle constraints?	Includes Voltage magnitude constraints?	Includes Transmission constraints?	Includes losses	Includes generator costs?	Includes contingency constraints?	Remarks
Load Flow	No, but can be added	Yes	No, but can be added	Yes	No	No	-
ELD	No	No	No	Depends	Yes	No	No Transmission Constraints
OPF	Yes	Depends	Yes	Yes	Yes	Yes	-

TABLE 2.2: Types of Power System Problems

2.4.3 Uncertainty in OPF

Very often OPF is addressed as a deterministic optimization problem with fixed model parameters and input variables. Such solutions provide no information regarding the degree of importance or likelihood of constraint violations. However, many random disturbances or uncertain factors exist during power system operations due to measurement errors, forecast inaccuracies or outages of system elements as discussed for the load flow. These uncertainties mainly come from bus loads, changes in network configuration and power supplies and so on. The dispatch scheduling for power system operations is highly related to load forecasting whose errors are inherently stochastic. Network parameters, such as the values of capacitances, resistances and inductances, will change unexpectedly due to weather changes or unexpected events. The availability of power generation is another uncertain factor in power systems. In particular, recent attention is paid to the increasing penetration of renewable generation in power systems such as wind and solar energy which necessitates the modeling of stochastic generation outputs. These uncertainties inevitably introduce errors in the optimal solutions and thus degenerate the decisions of a deterministic OPF[13].

Probabilistic methods have been applied to power systems, especially for the solution of probabilistic power flow problems(PLF). Borkowska [2] addressed an uncertain power flow problem by adopting a probabilistic linear transformation and normal random variables were considered to model the uncertainty. Several studies investigated how the power flow problem can be modeled and solved probabilistically instead of deterministically. In this work, we use Interval Arithmetic methods to solve OPF probabilistically by taking the uncertain variables as Intervals instead of specific values.

Summary

In this chapter, Load Flow and the methods to solve power flow equations are discussed. The reasons for uncertainty in the power flow analysis are explained. Monte Carlo simulations are the best way to take uncertainty into account. But the main drawback of MC method is the time taken. MC method is explained briefly taking an example. The importance of Optimal Power Flow is discussed and the mathematical representation of OPF is stated. Interval Methods are far more superior to MC method for solving Load Flow and OPF with uncertainties. Interval Arithmetic based Load Flow and OPF will be discussed in the next chapters.

Chapter 3

Interval Arithmetic

Interval Arithmetic, interval mathematics, interval analysis, or interval computation, is a method developed by mathematicians since the 1950s and 1960s, as an approach to putting bounds on rounding errors and measurement errors in mathematical computation and thus developing numerical methods that yield reliable results. Very simply put, it represents each value as a range of possibilities.

3.1 Definition of an Interval

The closed Interval denoted by [a , b] is the set of real numbers given by

$$[a, b] = \{x \in R : a \leq x \leq b\} \tag{3.1}$$

Endpoint Notation

We will adopt the convention of denoting intervals and their endpoints by capital letters. The left and right endpoints of an interval X will be denoted by X_1 and X_2, respectively. Thus,

$$X = [X_1, X_2] \tag{3.2}$$

Interval Equality

Two intervals X and Y are said to be *equal* if they are the same sets. Operationally, this happens if their corresponding endpoints are equal:

$$X = Y \text{ if } X_1 = Y_1 \text{ and } X_2 = Y_2 \tag{3.3}$$

Degenerate Intervals

We say that X=[X_1 , X_2] is degenerate if $X_1 = X_2$. Such an Interval contains a single real number X. By convention, we agree to identify a degenerate interval [X, X] with the real number X. In this sense, we may write such equations as

$$0 = [0, 0]$$

3.2 Operations

The four basic arithmetic operations (Addition, Subtraction, Multiplication, Division) are given as:

$$[X_1\ , X_2] \text{ op } [Y_1\ , Y_2] = [\text{Min}(X_1 \text{ op } Y_1\ , X_1 \text{ op } Y_2\ , X_2 \text{ op } Y_1\ , X_2 \text{ op } Y_2), \\ \text{Max}(X_1 \text{ op } Y_1\ , X_1 \text{ op } Y_2\ , X_2 \text{ op } Y_1\ , X_2 \text{ op } Y_2)]$$

Here, op is either $+, -, \times$ or $\div$

Provided that X op Y is allowed for all $X \in [X_1, X_2]$ and Y $\in [Y_1, Y_2]$.

For practical applications this can be simplified further:

1. Addition: $[X_1, X_2] + [Y_1, Y_2] = [X_1 + Y_1, X_2 + Y_2]$
2. Subtraction : $[X_1, X_2] - [Y_1, Y_2] = [X_1 - Y_2, X_2 - Y_1]$
3. Multiplication : $[X_1, X_2] * [Y_1, Y_2] = [\text{Min}(X_1 * Y_1\ , X_2 * Y_1\ , X_1 * Y_2\ , X_2 * Y_2)\ , \text{Max}(X_1 * Y_1\ , X_2 * Y_1\ , X_1 * Y_2\ , X_2 * Y_2)\ ]$
4. Division : $[X_1, X_2]/[Y_1, Y_2] = [X_1, X_2] * (1/[Y_1, Y_2])$
   Where, $\frac{1}{[Y_1, Y_2]} = [\frac{1}{Y_2}, \frac{1}{Y_1}]$ provided $0 \notin [Y_1, Y_2]$

For $Y_1 < 0 < Y_2$,

$$\frac{1}{[Y_1, Y_2]} = [-\infty, \frac{1}{Y_2}] \cup [\frac{1}{Y_1}, \infty] \tag{3.4}$$

This is called Extended Interval Arithmetic.

3.3 Properties of Intervals

Intersection and Union

For two intervals, $X = [X_1, X_2]$ and $Y = [Y_1, Y_2]$, we may define the intersection $X \cap Y$ as the interval

$$X \cap Y = Z : Z \in X \textbf{ and } Z \in Y = [\text{Max}(X_1, Y_1)\ ,\ \text{Min}(X_2, Y_2)] \tag{3.5}$$

Similarly in case of the union of X and Y is also an interval:

$$X \cup Y = Z : Z \in X \textbf{ or } Z \in Y = [\text{Min}(X_1, Y_1)\ ,\ \text{Max}(X_2, Y_2)] \tag{3.6}$$

Eq (3.5) and (3.6) are valid only for $X_2 < Y_1$. Which mean that there is an intersection for both the Intervals. Intersection plays a key role in Interval Analysis. If we have two intervals containing a result of interest regardless of how they were obtained ,then the intersection, which may be narrower, also contains the result. This is the basis for the iterations in our Iterative Methods.

Width of an Interval

The Width of an Interval $X = [X_1, X_2]$ is defined and denoted by

$$w(X) = X_2 – X_1 \tag{3.7}$$

Absolute Value of an Interval

The absolute value of $X = [X_1, X_2]$, denoted $\mid X \mid$, is the maximum of the absolute values of its endpoints:

$$\mid X \mid = \text{Max}\{\mid X_1 \mid, \mid X_2 \mid\} \tag{3.8}$$

Note that $\mid x \mid \leq \mid X \mid$ for every $x \in X$.

Mid Point of an Interval

The midpoint of $X = [X_1, X_2]$ is given by

$$m(X) = \frac{1}{2}(X_1 + X_2) \tag{3.9}$$

Any Interval X can be represented as:

$$\begin{aligned} X &= m(X) + \left[\frac{-1}{2}w(X), \frac{1}{2}w(X)\right] \\ &= m(X) + \left[\frac{1}{2}w(X)[-1, 1]\right] \end{aligned} \tag{3.10}$$

Commutativity and Associativity

Interval addition and Multiplication are commutative and associative. We have

$$X + Y = Y + X \tag{3.11}$$

$$XY = YX$$

For any 3 Intervals X , Y and Z, we have

$$X + (Y + Z) = (X + Y) + Z \tag{3.12}$$

$$X(YZ) = (XY)Z$$

Additive and Multiplicative Identity Elements

The degenerate intervals 0 and 1 are Additive and Multiplicative identity elements in the system of intervals. For ant Interval X

$$0 + X = X + 0 = X \tag{3.13}$$

$$1 \cdot X = X \cdot 1 = X$$

Subdistributivity

The distributive law $X(Y + Z) = XY + XZ$ of ordinary arithmetic **fails** to hold for Intervals. An easy counterexample can be obtained by taking
$X = [1, 2], Y = [1, 1]$ and $Z = [-1, -1]$

$$\begin{aligned} X(Y + Z) &= [1, 2] \cdot ([1, 1] + [-1, -1]) \\ &= [1, 2] \cdot [0, 0] \\ &= [0, 0] \end{aligned}$$

whereas,

$$\begin{aligned} XY + XZ &= [1, 2] \cdot [1, 1] - [1, 2] \cdot [1, 1] \\ &= [\min(1, 2), \max(1, 2)] - [\min(1, 2), \max(1, 2)] \\ &= [1, 2] - [1, 2] \\ &= [-1, 1] \end{aligned}$$

Clearly we observe that the Distributive law fails in case of Intervals. However, there is a Subdistributive law. For any three Intervals X, Y and Z

$$X(Y + Z) \subseteq XY + XZ \tag{3.14}$$

3.4 Interval Functions

Let f be a real-valued function of a single real variable x. The precise range of values taken by *f(x)* as x varies through a given interval X is the image of the set X under the mapping *f*.

$$f(X) = \{f(x) : x \in X\} \tag{3.15}$$

More generally, given a function $f = f(x_1, ..., x_n)$ of several variables, we have to find the image set

$$f(X_1, ..., X_n) = \{f(x_1, ..., x_n) : x_1 \in X_1, ..., x_n \in X_n\} \tag{3.16}$$

Interval Dependency
Consider the function

$$f(x) = x^2$$

If $X = [X_1, X_2]$, it is evident that the set

$$f(X) = \{x^2 : x \in X\} \tag{3.17}$$

can be expressed as

$$f(X) = \begin{cases} [X_1^2, X_2^2], & 0 \leq X_1 \leq X_2 \\ [X_2^2, X_1^2], & X_1 \leq X_2 \leq 0 \\ [0, \text{Max}\left\{X_1^2, X_2^2\right\}], & X_1 < 0 < X_2 \end{cases} \tag{3.18}$$

We will use (3.17) as the definition of X^2. This is not the same as $X \cdot X$. For instance $[-1, 1]^2 = [0,\ 1]$, whereas [-1, 1]·[-1, 1] = [-1, 1]

However, [-1, 1] does contain [0, 1]. The overestimation when we compute a bound on the range of X^2 as $X \cdot X$ is due to the phenomenon of *Interval Dependency.*

Namely, if we assume x is an unknown number known to lie in the interval X, then, when we form the product $x \cdot x$, the x in the second factor, although known only to lie in X must be the same as the x in the first factor, whereas, in the definition of the interval product $X \cdot X$, it is assumed that the values in the first factor and the values in the second factor vary independently.

Interval dependency is a crucial consideration when using interval computations. It is a major reason why simply replacing floating point computations by intervals in an existing algorithm is not likely to lead to satisfactory results

Monotonic Functions

For Monotonic functions i.e. either increasing or decreasing functions, the mapping is done straight forward.

- Let *f(x)* be an increasing function and $X = [X_1, X_2]$ be an Interval.
  The function maps the interval X into the interval

  $$f(X) = [f(X_1), f(X_2)] \tag{3.19}$$

  Let us take an example. As x varies through an interval $X = [X_1, X_2]$, the exponential function

  $$f(x) = e^x \qquad\qquad x \in R$$

  takes values from e^{X_1} to e^{X_2} since it is an Increasing function. That is, we can define

  $$e^X = [e^{X_1}, e^{X_2}] \tag{3.20}$$

- Let *f(x)* be an decreasing function and $X = [X_1, X_2]$ be an Interval.
  The function maps the interval X into the interval

  $$f(X) = [f(X_2), f(X_1)] \tag{3.21}$$

Let us take an example. As x varies through an interval $X = [X_1, X_2]$, the function

$$f(x) = e^{-x} \qquad\qquad x \in R$$

takes values from e^{-X_2} to e^{-X_1} since it is a Decreasing function. That is, we can define

$$e^{-X} = [e^{-X_2}, e^{-X_1}] \tag{3.22}$$

We should also note that some restrictions of a non-monotonic function could be monotonic. The function f given by

$$f(x) = \sin x \qquad\qquad x \in R$$

is not monotonic, but its restriction f_A to the
Set $A = [-\frac{\pi}{2}, \frac{\pi}{2}]$ is increasing, thus

$$\sin(X) = [\sin X_1, \sin X_2] \qquad \text{for} \quad X \subseteq [-\frac{\pi}{2}, \frac{\pi}{2}] \tag{3.23}$$

Similarly for Set $B = [\frac{\pi}{2}, \frac{3\pi}{2}]$ is decreasing, thus

$$\sin(X) = [\sin X_2, \sin X_1] \qquad \text{for} \quad X \subseteq [\frac{\pi}{2}, \frac{3\pi}{2}] \tag{3.24}$$

3.5 Methods to Solve Interval Functions

Functions with Interval variables and Interval numbers are known as Interval functions. There are several iterative methods to solve Interval functions like:

1. Interval Gauss – Seidal method
2. Interval Newton method
3. Krawczyk Method

The most widely used method to solve Interval Linear equations is Interval Gauss – Seidal method. While this may seem like a bad idea, the purpose of Gauss – Seidal iterations here is not to solve the power flow problem, but to solve the linear – equations that result from the Interval Newton Method and to do so with a little conservatism as possible.
Interval Newton method is the most popular one, but it requires to invert an Interval Matrix which is computationally expensive for higher orders. The next iteration of Interval Newton Method is given by (3.25).

$$X_{n+1} := \left(m(X_n) - \frac{f(m(X_n))}{f'(X_n)}\right) \cap X_n \tag{3.25}$$

Here calculation of $\frac{1}{f'(X_n)}$ is computationally expensive. So, therefore Krawczyk Method is widely used for solving non-linear equations especially for power flow equations. Krawczyk Method is described in the next section.

3.6 Krawczyk Method

We consider finite systems of non-linear equations of multiple variables.

$f_1(x_1, x_2, ..., x_n) = 0$
...
...
$f_n(x_1, x_2, ..., x_n) = 0$

Which we may write in vector notation as

$$f(x) = 0 \tag{3.26}$$

Suppose that f in (3.11) is continuously differentiable in an open domain D. Suppose that we can compute F and F' for f and f' defined on interval vectors $X \subseteq D$.

Let Y be a non-singular real matrix approximating the inverse of the real Jacobian matrix $F'(m(X))$ with elements

$$F'(m(X))_{ij} = \frac{\partial f_i(x)}{\partial x_j} \quad \text{at } x = m(X) \tag{3.27}$$

Let y be a real vector contained in the interval vector $X \subseteq D$. Define $g(y) = y - Yf(y)$. Then, since Y is nonsingular, $g(y) = y$ if and only if $f(y) = 0$. Thus, if there exists an $x \in X$ such that $g(x) = x$, i.e if there is a fixed point of g in x, then there is a solution to $f(x) = 0$ in x. However, if by $g'(y)$ we mean the Jacobian matrix of g at y, then

$$g'(y) = I - Yf'(y) \tag{3.28}$$

Therefore, the mean value extension of g over X about the point $y \in X$ is simply

$$y - Yf(y) + \{I - YF'(X)\}(X - y) = K(X) \tag{3.29}$$

I being Identity Matrix with appropriate index.
Thus, $K(X)$ must contain the range of g over X, i.e $g(X) \subseteq K(X)$. Thus, if $K(X) \subseteq X$, then $g(X) \subseteq X$, the hypotheses of the Schauder fixed-point theorem hold, so g has a fixed point in X, so $f(x) = 0$ has a solution in X. Since $K(X) \subseteq X$, if *f(x)* has a solution in X then it also has in *K(X)*.
In general for k^{th} iteration

$$X^{(k+1)} = X^{(k)} \cap K(X^{(k)}) \qquad (k = 1, 2, ...) \tag{3.30}$$

where,

$$K(X^{(k)}) = y^{(k)} - Y^{(k)} f(y^{(k)}) + \{I - Y^{(k)} F'(X^{(k)})\} Z^{(k)} \tag{3.31}$$

and

$$y^{(k)} = m(X^{(k)}), \qquad Z^{(k)} = X^{(k)} - m(y^{(k)}) \tag{3.32}$$

where $Y^{(k)}$ is chosen as an approximation to $[m(F'(X^{(k)}))]^{-1}$ i.e

$$Y^{(k)} = [m(F'(X^{(k)}))]^{-1} \tag{3.33}$$

Safe Starting Intervals

For any given trial box $X^{(0)}$, there are various things that can happen, including the following:

1. $K(X^{(0)}) \subset \text{int}(X^{(0)}) \implies$ convergence to the unique solution in $X^{(0)}$, where $\text{int}(X^{(0)})$ denotes the topological interior of the box $X^{(0)}$.
2. $K(X^{(0)}) \cap X^{(0)} = \emptyset \implies$ no solution in $X^{(0)}$.
3. $K(X^{(0)}) \cap X^{(0)} \neq \emptyset \implies$ no conclusion, but we can restart with $X^{(0)}_{NEW} = K(X^{(0)}) \cap X^{(0)}$.
4. $K(X^{(0)})$ is not defined. In this case, we can bisect $X^{(0)}$ and process each half separately.

$K(X^{(0)}) \subset \text{int}(X^{(0)}) \implies$ there exists a unique solution to *f(x)=0* in X only if f is continuous over X.
Therefore a Safe Starting Interval is to be chosen similar to initial guess to be chosen in Newton Raphson Method.

3.7 Load Flow Application

The Standard power flow equations can be expressed as

$$\begin{aligned} y &= f(x) \\ z &= g(x) \end{aligned} \tag{3.34}$$

Where, f and g are functions and y, z and x are Intervals.
In general,

y = real and reactive power injections (inputs)

x = voltage magnitude and angles (state variables)

z = line power flows (outputs)

The Krawczyk Method is applied to solve the set of equations $y=f(x)$ to find the states x and using these states line power flows are calculated.

Krawczyk Algorithm for solving Load Flow:

1. Create the admittance Y-Bus Matrix according to the line data given by the IEEE standard bus test systems.
2. Find the load and generator data in the form of Intervals. Here the generation and demands are taken as a variation of ±10% or ±20% depending upon the system.
3. Write the power flow equations (Power Injections) in term of our unknown variables.
4. Calculate the Jacobian $F'(X)$ of the power flow equations.
5. Take the initial starting intervals $X^{(0)}$ for the unknown variables. In general voltages starting Interval can be taken as [0.9 , 1.1] pu and angle starting interval can be taken as $\pm 10°$ to $\pm 15°$ depending upon the system.
6. After forming $X^{(0)}$, find the $y^{(0)} = \text{Mid}(X^{(0)})$ and $f(y^{(0)})$.
7. Calculate $Y^{(0)}$ as an approximate inverse of the matrix $m(F'(X^{(0)}))$. Here we are calculating inverse of an ordinary matrix but not an Interval matrix as needed for Interval Newton method.
8. Calculate $K(X^{(0)}) = y^{(0)} - Y^{(0)} f(y^{(0)}) + \{I - Y^{(0)} F'(X^{(0)})\} Z^{(0)}$.
9. Now find the next iteration value as $X^{(1)} = X^{(0)} \cap K(X^{(0)})$. If the Intersection is a null interval, then the starting Interval should be changed.
10. Repeat the iterations until the results no longer vary.
11. Use the obtained results find the required power flows and system details.

The algorithm will be demonstrated through an example as given below.

3.7.1 Example to Demonstrate Krawczyk Method of Load Flow

Consider a 2 Bus system with a generator and a load taken above in MC example. Refer Fig 2.1 for the single line diagram. With 100 MVA base,
Taking the demand as ±20% variation, we get

$$P_D = [0.8,\ 1.2] \text{ pu}$$
$$Q_D = [0.4,\ 0.6] \text{ pu}$$

The Power injection equations obtained for the given system are:

$$15.8V_2\cos(108.43° - \theta_2) + 5V_2^2 + [0.8, 1.2] = 0 \tag{3.35}$$

$$15V_2^2 - 15.8V_2\sin(108.43° - \theta_2) + [0.4, 0.6] = 0 \tag{3.36}$$

Taking the starting interval for V_2 and θ_2 as

$$V_2^{(0)} = [0.9,1.1] \text{ pu and } \theta_2^{(0)} = [-10° ,\ 10°] = [-0.1745 ,\ 0.1745] \text{ rad}$$

$$X^{(0)} = \begin{bmatrix} [0.9, 1.1] \\ [-0.1745, 0.1745] \end{bmatrix},\ y = m(X^{(0)}) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$F'(X) = \begin{bmatrix} 15.8\cos(108.43° - \theta_2) + 10V_2 & 15.8V_2\sin(108.43° - \theta_2) \\ 30V_2 15.8\sin(108.43° - \theta_2) & 15.8V_2\cos(108.43° - \theta_2) \end{bmatrix} \tag{3.37}$$

$$F'(X^{(0)}) = \begin{bmatrix} [1.47786, 8.6837] & [12.5051, 17.1922] \\ [11.3707, 19.1055] & [-8.27435, -2.08467] \end{bmatrix} \tag{3.38}$$

$$m(F'(X^{(0)})) = \begin{bmatrix} 5.08078 & 14.8486 \\ 15.2381 & -5.17951 \end{bmatrix} \tag{3.39}$$

$$Y^{(0)} = [m(F'(X^{(0)}))]^{-1} = \begin{bmatrix} 0.0205063 & 0.0587876 \\ 0.0603295 & -0.0201155 \end{bmatrix} \tag{3.40}$$

$$f(y) = \begin{bmatrix} [0.8, 1.2] \\ [0.4, 0.6] \end{bmatrix} \tag{3.41}$$

Substituting we get,

$$K(X^{(0)}) = \begin{bmatrix} [0.869862, 1.03034] \\ [-0.1294, 0.0288568] \end{bmatrix} \tag{3.42}$$

The next iteration value

$$X^{(1)} = X^{(0)} \cap K(X^{(0)}) = \begin{bmatrix} [0.9, 1.03034] \\ [-0.1294, 0.0288568] \end{bmatrix} \tag{3.43}$$

Repeating the process after updating the new voltage and angle for 7 iterations the results obtained are

$$X^{(7)} = \begin{bmatrix} [0.932196, 0.957736] \\ [-0.0685856, -0.0375379] \end{bmatrix} \tag{3.44}$$

Therefore, V_2 = [0.932196 , 0.957736] pu
and θ_2 = [-0.0685856 , -0.0375379] rad

Which mean that for a deviation of ±20% in the demands, the voltage V_2 lies between the values 0.932196 pu and 0.957736 pu and θ_2 lies between the values -0.0685856 pu and -0.0375379 pu.

Summary

In this chapter, the basics of Interval Mathematics are discussed and the different methods to solve Interval equations are stated. Krawczyk Method is explained detailedly. The Krawczyk algorithm for solving load flow is stated and is demonstrated with an example. The results for LF and OPF obtained on applying Krawczyk Method for different test systems are shown in the next chapter.

Chapter 4

Simulation Results

The Krawczyk Method is used for solving the Load Flow for 2, 4 and 14 Bus systems. The system is assumed to operate at normal conditions but all power demands may vary within certain ranges rather than have precise values. The problem is to find the voltages, angles and power flows in Intervals. The obtained results are compared with that of Monte Carlo simulations. To perform a meaningful comparison against interval methods, the solution results are characterized as an interval of $\pm 3\sigma$ around the mean.

Similarly the Krawczyk Method is used for solving the Optimal Power Flow for 3, 6 and 14 Bus systems. The problem is to find the voltages, angles and active/reactive locational marginal costs in Intervals. The obtained results are compared with that of Monte Carlo simulations.

4.1 Load Flow Results

4.1.1 2-Bus System

The single line diagram is shown in Fig. 3.1. The loads and generation are varied by $\pm 20\%$. The results obtained are tabulated in Table 4.1.

Variable	Interval Arithmetic	Monte Carlo
V_2 (pu)	[0.932196 , 0.957736]	[0.935791 , 0.953406]
θ_2 (rad)	[-0.0685856 , -0.0375379]	[-0.066204 , -0.039961]

TABLE 4.1: Comparison of Results of LF Obtained by IA and MC for 2-Bus System

Clearly we can see that the results obtained by Interval Arithmetic method are almost equal to that obtained from Monte Carlo simulations. The Power Flows in the lines are computed using Interval Arithmetic method and are tabulated in Table 4.2.

Line	P(pu)	Q(pu)
1-2	[0.739460 , 1.334510]	[0.315903 , 0.875014]
2-1	[-1.334641 , -0.698753]	[-0.811572 , -0.206201]

TABLE 4.2: Power Flows for the 2-Bus System

4.1.2 4-Bus System

Krawczyk method is implemented on 4-Bus system. The single line diagram of the 4-bus system is shown in Fig.4.1, line data is given in Table 4.3 and bus data is given in Table 4.4. The Q values of load are calculated from the corresponding P values assuming a power factor of 0.85. The results obtained by implementing Krawczyk are tabulated in Table 4.5. Table 4.5 also contains the results obtained by Monte Carlo method for sake of comparison. It can be seen from Table 4.5 that Krawczyk method give voltages and angles with slightly wider range than MC. However, the bounds of voltages and angles obtained by Krawczyk encapsulated the bounds obtained from MC. Hence, it is understood that Krawczyk will be overestimating the range. However, the computation time is very less with Krawczyk. The power flows in the lines are computed using Interval Arithmetic method and are tabulated in Table 4.6.

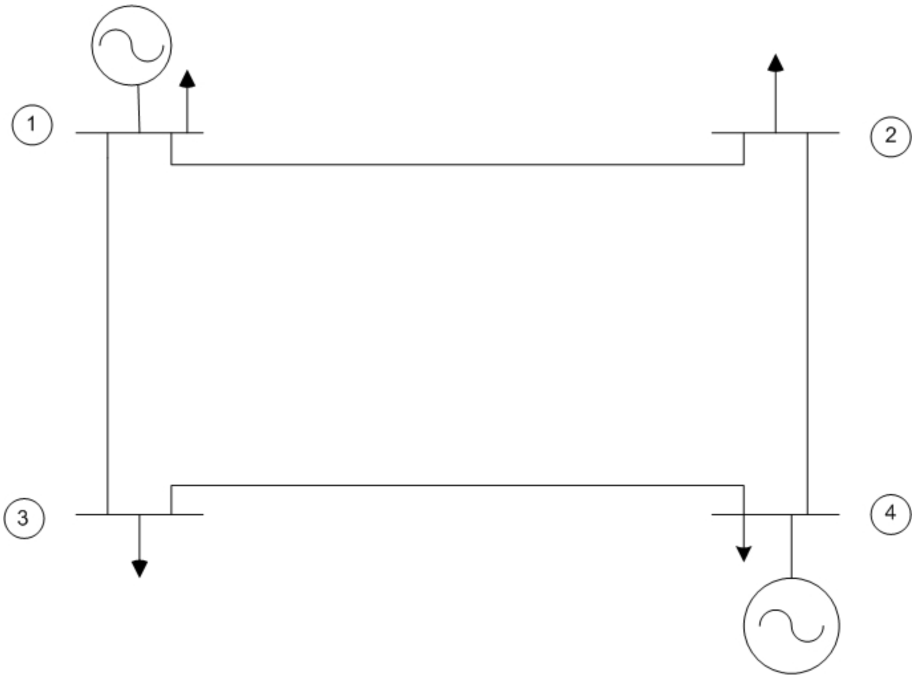

FIGURE 4.1: Single Line Diagram of Four Bus System

From Bus	To Bus	R(pu)	X(pu)	Y/2(pu)
1	2	0.01008	0.05040	0.05125
1	3	0.00744	0.03720	0.03875
2	4	0.00744	0.03720	0.03875
3	4	0.01272	0.06360	0.06375

TABLE 4.3: Line Data of the 4-Bus System

Bus	P_G MW	Q_G Mvar	P_D MW	Q_D Mvar	V pu	Remarks
1	-	-	50	30.9	$1.00\angle 0$	Slack Bus
2	0	0	170	105.35	-	Load Bus
3	0	0	200	123.94	-	Load Bus
4	318	-	80	49.58	$1.02\angle 0$	PV Bus

TABLE 4.4: Bus Data of the 4-Bus System

Variable	Interval Arithmetic	Monte Carlo
V_2 (pu)	[0.979095 , 0.985748]	[0.979810 , 0.984958]
V_3 (pu)	[0.964567 , 0.973441]	[0.965663 , 0.972337]
θ_2 (rad)	[-0.032858 , -0.001224]	[-0.026579 , -0.007441]
θ_3 (rad)	[-0.045563 , -0.019790]	[-0.040504 , -0.024794]
θ_4 (rad)	[0.006301 , 0.046847]	[0.013589 , 0.039643]

Table 4.5: Comparison of Results of LF Obtained by IA and MC for 4-Bus System

Line	P(pu)	Q(pu)
1-2	[0.077269 , 0.699591]	[0.097109 , 0.353055]
2-1	[-0.704288 , -0.074227]	[-0.448251 , -0.171603]
1-3	[0.631681 , 1.334366]	[0.423499 , 0.804274]
3-1	[-1.337, -0.616548]	[-0.845152 , -0.422257]
2-4	[-2.287821 , -0.359864]	[-1.053494 , -0.395703]
4-2	[0.326120 , 2.328652]	[0.412521 , 1.120482]
3-4	[-1.558144 , -0.511202]	[-0.805956 , -0.386738]
4-3	[0.496793 , 1.592599]	[0.356715 , 0.800234]

Table 4.6: Power Flows for the 4-Bus System

4.1.3 14-Bus System

Krawczyk method is implemented on 14-Bus system. The single line diagram, bus data and line data are given in **Appendix A**. The results obtained by implementing Krawczyk are tabulated in Table 4.7. Table 4.7 also contains the results obtained by Monte Carlo method for the sake of comparison.

Variable	Interval Arithmetic	Monte Carlo
V_4 (pu)	[1.023734 , 1.030410]	[1.025837 , 1.028223]
V_5 (pu)	[1.030334 , 1.036422]	[1.032197 , 1.034223]
V_7 (pu)	[1.041958 , 1.048523]	[1.043888 ,1.046512]
V_9 (pu)	[1.022803 , 1.033271]	[1.025614 , 1.030352]
V_{10} (pu)	[1.022587 , 1.033173]	[1.025695 , 1.029925]
V_{11} (pu)	[1.041932 , 1.048299]	[1.043935 , 1.046205]
V_{12} (pu)	[1.051237 , 1.054859	[1.052231 , 1.053849]
V_{13} (pu)	[1.043676 , 1.048911]	[1.045072 , 1.047488]
V_{14} (pu)	[1.011664 , 1.023722]	[1.013854 , 1.021446]
θ_2 (rad)	[-0.138105 , -0.074855]	[-0.110582 , -0.099734]
θ_3 (rad)	[-0.256852 , -0.193871]	[-0.252996 , -0.221153]
θ_4 (rad)	[-0.207151 , -0.160099]	[-0.205124 , -0.186692]
θ_5 (rad)	[-0.179311 , -0.138308]	[-0.177358 , -0.162210]
θ_6 (rad)	[-0.288904 , -0.230535]	[-0.283995 , -0.263257]
θ_7 (rad)	[-0.261926 , -0.207883]	[-0.260072 , -0.238830]
θ_8 (rad)	[-0.261926 , -0.207883]	[-0.260072 , -0.238830]
θ_9 (rad)	[-0.293250 , -0.233069]	[-0.289207 , -0.265961]
θ_{10} (rad)	[-0.298043 , -0.236902]	[-0.293267 , -0.270338]
θ_{11} (rad)	[-0.295699 , -0.235504]	[-0.290595 , -0.268843]
θ_{12} (rad)	[-0.305476 , -0.243300]	[-0.299182 , -0.277488]
θ_{13} (rad)	[-0.306062 , -0.243391]	[-0.299780 , -0.277638]
θ_{14} (rad)	[-0.319235 , -0.253200]	[-0.312578 , -0.288316]

Table 4.7: Comparison of Results of LF Obtained by IA and MC for 14-Bus System

The Power Flows in the lines are computed using Interval Arithmetic method and are tabulated in Table 4.8.

Line	P(pu)	Q(pu)
1-2	[1.35941 , 2.45973]	[-0.501965 , -0.040139]
2-1	[-2.41851 , -1.26332]	[0.081562 , 0.769372]
1-5	[0.674082 , 0.880017]	[-0.0783791 , 0.025209]
5-1	[-0.862523 , -0.635865]	[-0.012586 , 0.176351]
2-3	[0.302324 , 1.00282]	[-0.149546 , 0.218321]
3-2	[-0.985515 , -0.285886]	[-0.217662 , 0.208336]
2-4	[0.112877 , 0.808763]	[-0.268947 , 0.150098]
4-2	[-0.802773 , -0.100738]	[-0.175107 , 0.276407]
2-5	[-0.015633 , 0.649148]	[-0.248445 , 0.137313]
5-2	[-0.646655 , 0.020192]	[-0.17145 7, 0.234044]
3-4	[-0.610927 , 0.099544]	[-0.2721 , 0.254076]
4-3	[-0.096235 , 0.613722]	[-0.269076 , 0.239022]
4-5	[-1.85004 , 0.615864]	[-0.803126 , 0.909793]
5-4	[-0.605742 ,1.84689]	[-0.883226 , 0.782431]
4-7	[-0.00945391 , 0.537785]	[-0.226983 , 0.050145]
7-4	[-0.545379 , 0.016251]	[-0.049821 , 0.258843]
4-9	[0.042458 , 0.262303]	[-0.057995 , 0.062533]
9-4	[-0.265346 , -0.040922]	[-0.058526 , 0.079053]
5-6	[0.219503 , 0.663725]	[-0.232947 , -0.030082]
6-5	[-0.673936 , -0.209032]	[0.054276 , 0.312137]
6-11	[-0.278925 , 0.444631]	[-0.20029 , 0.390829]
11-6	[-0.437286 , 0.273616]	[-0.384176 , 0.198418]
6-12	[-0.19976 , 0.364748]	[-0.198105 , 0.260524]
12-6	[-0.36206 , 0.198181]	[-0.258887 , 0.199829]
6-13	[-0.367588 , 0.741915]	[-0.359313 , 0.559809]
13-6	[-0.733182 , 0.362398]	[-0.552545 , 0.36167]
7-8	[-0.363711 , 0.358552]	[-0.45367 , -0.081820]
8-7	[-0.372521 , 0.378099]	[0.086062 , 0.471616]
7-9	[-0.326813 , 0.895459]	[-0.196438 , 0.523567]
9-7	[-0.89135 , 0.319993]	[-0.523509 , 0.217368]
9-10	[-0.866513 , 0.963529]	[-0.766126 , 0.731221]
10-9	[-0.965147 , 0.868258]	[-0.734021 , 0.769383]

Line	P(pu)	Q(pu)
9-14	[-0.20188 , 0.38287]	[-0.254114 , 0.247175]
14-9	[-0.382812 , 0.203239]	[-0.247806 , 0.259137]
10-11	[-0.438871 , 0.348499]	[-0.400614 , 0.251878]
11-10	[-0.353361 , 0.444692]	[-0.254808 , 0.405873]
12-13	[-0.251434 , 0.290026]	[-0.263023 , 0.292585]
13-12	[-0.288519 , 0.250124]	[-0.291039 , 0.261661]
13-14	[-0.164895 , 0.295419]	[-0.142782 , 0.252186]
14-13	[-0.290385 , 0.161485]	[-0.247427 , 0.142089]

Table 4.8: Power Flows for the 14-Bus System

For better comparison, a plot is made for the voltages of 14-Bus System, obtained by Interval Arithmetic and Monte Carlo and is shown in Fig.4.2. The Simulation times are tabulated in Table 4.9. Calculations are performed in Mathematica using Intel(R) Core(TM) i5-3337U CPU @ 1.80 GHz.

Clearly by Fig.4.2 and Table 4.9 we can observe that the results of Interval method encloses the results of Monte Carlo method and the time required for Interval Arithmetic is much lesser than Monte Carlo. This demonstrates the validity of the krawczyk method. The Intervals obtained by both the methods are almost similar. This makes Interval Arithmetic methods superior than Probabilistic methods which require much computational time for real time calculations.

	Interval Arithmetic	Monte Carlo(10,000) samples
2 Bus	0.15 s	4.84 s
4 Bus	2.08 s	13.09 s
14 Bus	3.6 s	99.78 s

Table 4.9: Computational Time for LF taken by Different Methods for Different Systems

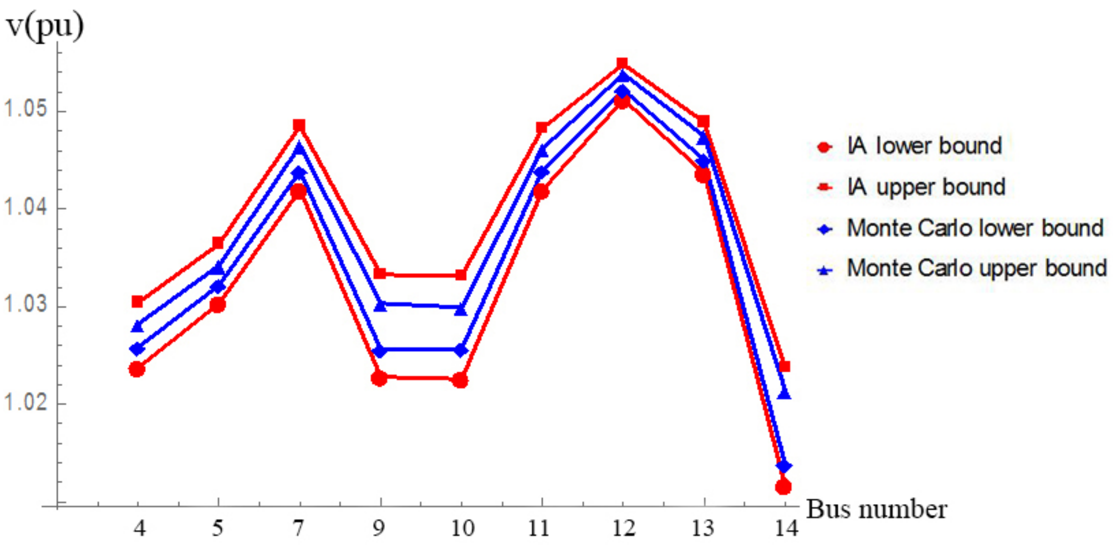

FIGURE 4.2: Comparison of Bus Voltages of LF using IA and MC Method for 14-Bus

4.2 OPF Results

4.2.1 3-Bus System

Consider a 3-Bus system given in Fig. 4.3. The series impedance of each line is given in Table 4.10. The cost functions of the generators are given by

$$\begin{aligned} F(P_{g_1}) &= 50P_{g_1}^2 + 351P_{g_1} + 44.4 \quad Rs/h \\ F(P_{g_2}) &= 50P_{g_1}^2 + 389P_{g_1} + 40.6 \quad Rs/h \end{aligned} \tag{4.1}$$

Line No	From Bus	To Bus	Z(pu)	Y_{SH}(pu)
1	1	2	0.02+j0.08	$j0.02$
2	1	3	0.02+j0.08	$j0.02$
3	2	3	0.02+j0.08	$j0.02$

TABLE 4.10: Line Data for 3-Bus System

Bus data for the system is given in Table 4.11.

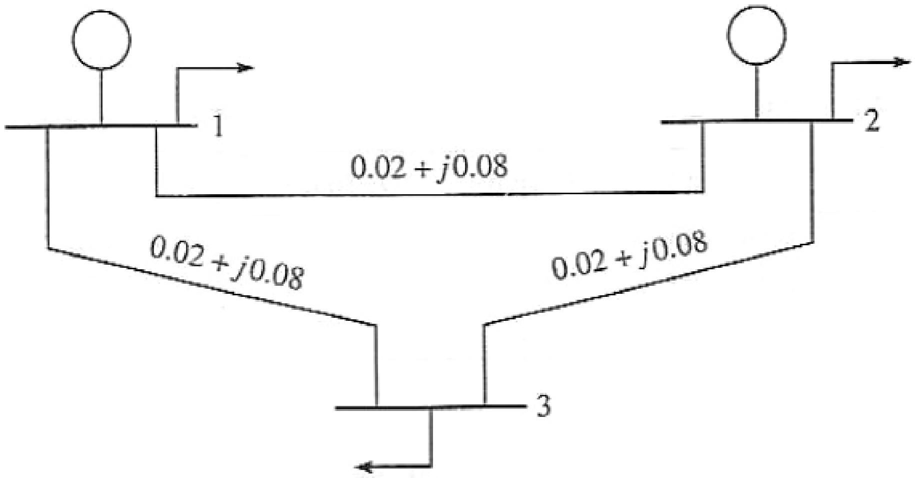

FIGURE 4.3: Single Line Diagram of 3-Bus System

Bus	P_G	Q_G	P_D	Q_D	V	θ	Type
1	-	-	0.2	0.1	1.04	$0°$	Slack
2	-	-	0	0	1.04	-	PV
3	0	0	1.5	0.6	-	-	PQ

TABLE 4.11: Bus Data for 3-Bus System in per unit

For the system given above, IA and MC methods are applied with the demand varying by ±10% from the nominal values given in Table 4.11. Obtained results are tabulated in Table 4.12. Clearly we can see that the results obtained by Interval Arithmetic method are almost equal to that obtained from Monte Carlo simulations.

Variable	Interval Arithmetic	Monte Carlo
P_{g_1}	[0.95951 , 1.13582]	[0.969955 , 1.12561]
P_{g_2}	[0.59078 , 0.76521]	[0.600268 , 0.755974]
V_3	[0.99686 , 1.00559]	[0.99811 , 1.00434]
θ_2	[-0.00499 , -0.00398]	[-0.0049883 , -0.00398246]
θ_3	[-0.06107 , 0.04795]	[-0.0604123 , -0.0486355]
λ_{P_1}	[446.95139 , 464.58173]	[447.996 , 463.561]
λ_{P_2}	[448.07845 , 465.52137]	[449.027 , 464.597]
λ_{P_3}	[459.64233 , 480.83314]	[460.769 , 479.744]
λ_{Q_3}	[5.14893 , 7.00213]	[5.39327 , 6.76161]

TABLE 4.12: Comparison of Results of OPF Obtained by IA and MC for 3-Bus System

4.2.2 6-Bus System

Krawczyk method is implemented for solving OPF on 6-Bus system. The single line diagram of the 6-bus system is shown in Fig.4.4, bus data and line data are given in Table 4.13 and Table 4.14. The generator cost function coefficients are given in Table 4.15. The results obtained by implementing Krawczyk method OPF are tabulated in Table 4.16. Table 4.16 also contains the results obtained by Monte Carlo method for sake of comparison.

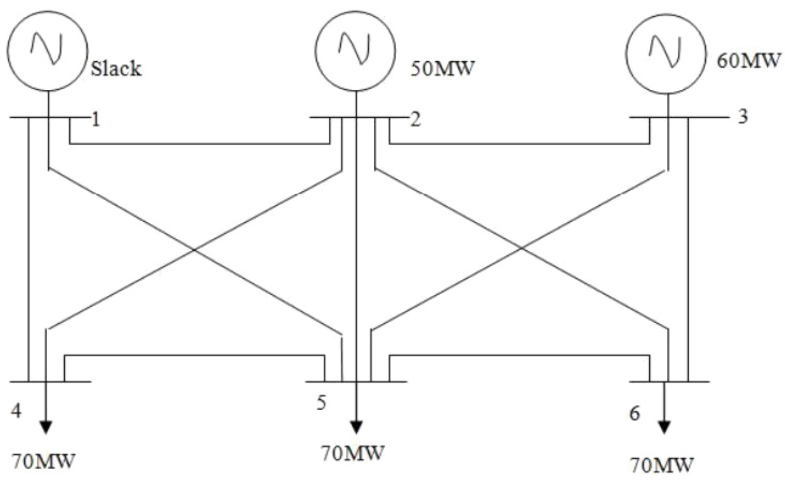

FIGURE 4.4: Single Line Diagram of 6-Bus System

Bus	P_G MW	Q_G Mvar	P_D MW	Q_D Mvar	V pu	Remarks
1	-	-	0	0	$1.05\angle 0$	Slack Bus
2	50	-	0	0	1.05	PV Bus
3	60	-	0	0	1.07	PV Bus
4	0	0	70	70	-	Load Bus
5	0	0	70	70	-	Load Bus
6	0	0	70	70	-	Load Bus

Table 4.13: Bus Data of the 6-Bus System

From Bus	To Bus	R(pu)	X(pu)	Y/2(pu)
1	2	0.1	0.2	0.02
1	4	0.05	0.2	0.02
1	5	0.08	0.3	0.03
2	3	0.05	0.25	0.03
2	4	0.05	0.1	0.01
2	5	0.10	0.3	0.02
2	6	0.07	0.2	0.025
3	5	0.12	0.26	0.025
3	6	0.02	0.1	0.01
4	5	0.2	0.4	0.04
5	6	0.1	0.3	0.03

Table 4.14: Line Data of the 6-Bus System

Bus No.	a_i (\$/MW^2-h)	b_i (\$/MW-h)	c_i (\$/h)
1	0.0107	11.669	213.1
2	0.0178	10.333	200
3	0.0148	10.833	240

Table 4.15: Generator cost functions of the 6-Bus System

A plot for λ_P is plotted for both IA and MC.

Variable	Interval Arithmetic	Monte Carlo
P_{g_1}	[0.53615 , 0.71146]	[0.57340 , 0.67428]
P_{g_2}	[0.70932 , 0.83436]	[0.73712 , 0.80664]
P_{g_3}	[0.69606 , 0.84684]	[0.72904 , 0.81392]
V_4	[0.98002 , 0.99507]	[0.98255 , 0.99249]
V_5	[0.97557 , 0.99399]	[0.97903 , 0.99013]
V_6	[0.99784 , 1.01162]	[1.00022 , 1.00902]
θ_2	[-0.02507 , -0.01595]	[-0.02291 , -0.01812]
θ_3	[-0.02589 , -0.01478]	[-0.02369 , -0.01699]
θ_4	[-0.05277 , -0.03531]	[-0.04928 , -0.03878]
θ_5	[-0.06674 , -0.04614]	[-0.06244 , -0.05039]
θ_6	[-0.06501 , -0.04602]	[-0.06212 , -0.04885]
λ_{P_1}	[1281.63668 , 1319.15183]	[1289.61 , 1311.2]
λ_{P_2}	[1285.81943 , 1330.33266]	[1295.71 , 1320.46]
λ_{P_3}	[1289.33319 , 1333.96565]	[1299.09 , 1324.22]
λ_{P_4}	[1329.41620 , 1385.26449]	[1341.27 , 1373.44]
λ_{P_5}	[1336.28161 , 1395.32266]	[1349.52 , 1382.24]
λ_{P_6}	[1318.32629 , 1372.17489]	[1330.18 , 1360.34]
λ_{Q_4}	[44.23977 , 61.01161]	[46.7915 , 58.5364]
λ_{Q_5}	[44.86809 , 63.77498]	[48.4752 , 60.622]
λ_{Q_6}	[27.35041 , 38.16358]	[29.348 , 36.3171]

TABLE 4.16: Comparison of Results of OPF Obtained by IA and MC for 6-Bus System

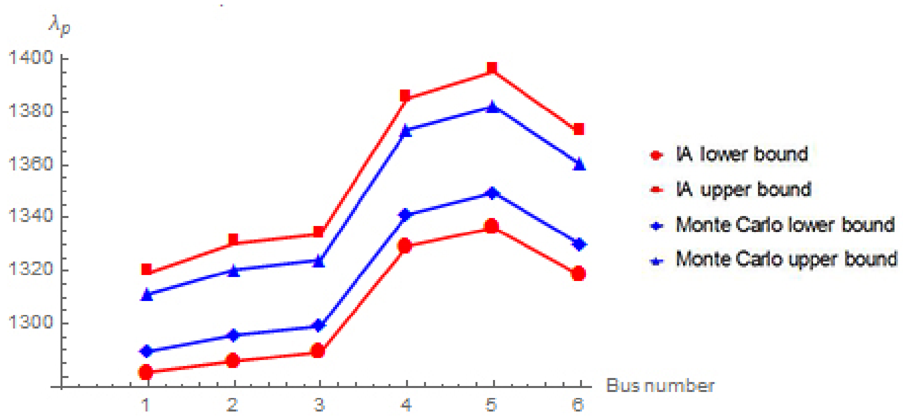

FIGURE 4.5: Comparison of λ_P of OPF using IA and Monte Carlo for 6-Bus System

4.2.3 14-Bus System

Similarly the method is applied to IEEE 14 Bus system with the demand as ±10% variation. The results obtained by implementing Krawczyk method OPF are tabulated in Table 4.17. Table 4.17 also contains the results obtained by Monte Carlo method for sake of comparison.The single line diagram, bus data and line data are given in **Appendix A**.

Variable	Interval Arithmetic	Monte Carlo
P_{g_1}	[2.04875 , 2.52592]	[2.18265 , 2.39393]
P_{g_2}	[0.38782 , 0.48422]	[0.41478 , 0.45764]
V_4	[1.02382 , 1.03033]	[1.02586 , 1.02823]
V_5	[1.03043 , 1.03635]	[1.03239 , 1.03434]
V_7	[1.04216 , 1.04833]	[1.04397 , 1.0465]
V_9	[1.02318 , 1.03291]	[1.02579 , 1.03029]
V_{10}	[1.02303 , 1.03275]	[1.02586 , 1.02993]
V_{11}	[1.04223 , 1.04801]	[1.04402 , 1.04623]
V_{12}	[1.05126 , 1.05483]	[1.05223 , 1.05387]
V_{13}	[1.04374 , 1.04886]	[1.04507 , 1.04752]
V_{14}	[1.01190 , 1.02349]	[1.01384 , 1.0215]
θ_2	[-0.09393 , -0.07537]	[-0.08897 , -0.08038]
θ_3	[-0.24307 , -0.19439]	[-0.23377 , -0.20371]
θ_4	[-0.19878 , -0.16058]	[-0.18805 , -0.17155]
θ_5	[-0.17123 , -0.13875]	[-0.16177 , -0.14841]
θ_6	[-0.28554 , -0.23118]	[-0.26785 , -0.24923]
θ_7	[-0.25849 , -0.20846]	[-0.24329 , -0.22404]
θ_8	[-0.25849 , -0.20846]	[-0.24329 , -0.22404]
θ_9	[-0.28978 , -0.23371]	[-0.27260 , -0.25132]
θ_{10}	[-0.29465 , -0.23748]	[-0.27673 , -0.25582]
θ_{11}	[-0.29229 , -0.23614]	[-0.27425 , -0.25458]
θ_{12}	[-0.30209 , -0.24396]	[-0.28301 , -0.26343]
θ_{13}	[-0.30267 , -0.24404]	[-0.28359 , -0.26354]
θ_{14}	[-0.31579 , -0.25385]	[-0.29618 , -0.27399]

Variable	Interval Arithmetic	Monte Carlo
λ_{P_1}	[3763.12317 , 4173.77052]	[3878.36 , 4060.18]
λ_{P_2}	[3939.08749 , 4421.11070]	[4073.93 , 4288.19]
λ_{P_3}	[4201.38478 , 4800.13347]	[4359.68 , 4643.95]
λ_{P_4}	[4123.41636 , 4684.90918]	[4283.39 , 4527.57]
λ_{P_5}	[4068.20498 , 4605.39246]	[4221.90 , 4454.14]
λ_{P_6}	[4074.25319 , 4617.37581]	[4230.55 , 4463.48]
λ_{P_7}	[4118.34825 , 4682.19340]	[4279.35 , 4523.87]
λ_{P_8}	[4118.34825 , 4682.19340]	[4279.35 , 4523.87]
λ_{P_9}	[4117.01964 , 4682.10932]	[4278.53 , 4523.30]
$\lambda_{P_{10}}$	[4130.07349 , 4701.24421]	[4294.71 , 4539.24]
$\lambda_{P_{11}}$	[4113.54080 , 4675.93529]	[4276.13 , 4515.88]
$\lambda_{P_{12}}$	[4131.93743 , 4699.81268]	[4297.93 , 4536.38]
$\lambda_{P_{13}}$	[4150.06837 , 4726.30709]	[4318.71 , 4560.31]
$\lambda_{P_{14}}$	[4209.05156 , 4814.76095]	[4387.56 , 4639.47]
λ_{Q_4}	[-8.69574 , 3.29258]	[-4.9014 , -0.476941]
λ_{Q_5}	[-6.89507 , 4.64372]	[-3.22041 , 1.01942]
λ_{Q_7}	[13.12509 , 26.22387]	[17.4024 , 21.9643]
λ_{Q_9}	[33.34188 , 55.82834]	[40.6085 , 48.5857]
$\lambda_{Q_{10}}$	[42.30293 , 68.06842]	[50.9364 , 59.432]
$\lambda_{Q_{11}}$	[28.92796 , 45.85289]	[34.8104 , 39.9701]
$\lambda_{Q_{12}}$	[17.30552 , 27.26385]	[20.571 , 23.9863]
$\lambda_{Q_{13}}$	[34.22336 , 51.82809]	[38.919 , 47.066]
$\lambda_{Q_{14}}$	[62.03185 , 95.77303]	[65.9238 , 91.9541]

Table 4.17: Comparision of Results of OPF Obtained by IA and MC for 14-Bus System

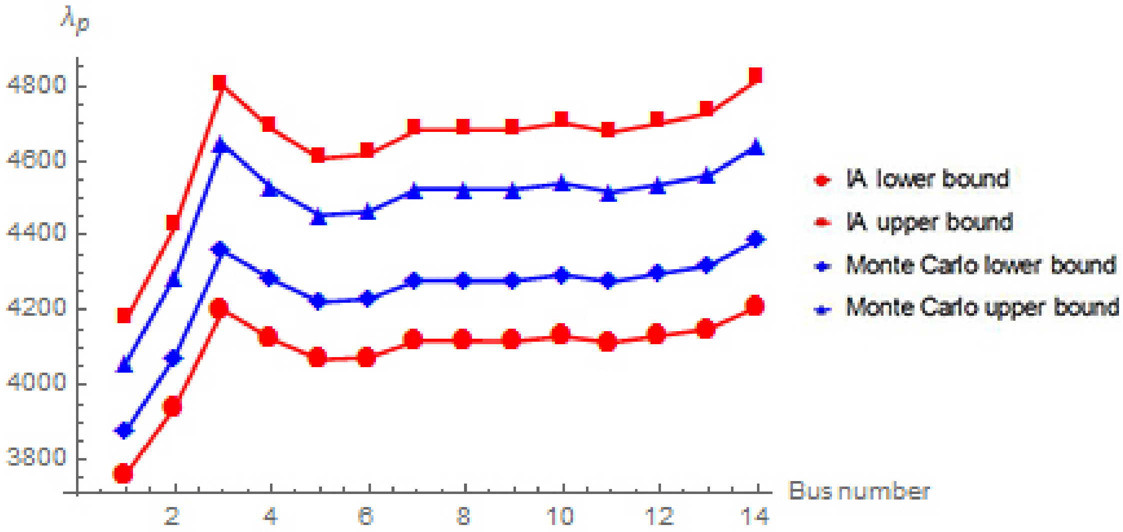

FIGURE 4.6: Comparison of λ_P of OPF using IA and Monte Carlo for 14-Bus System

	Interval Arithmetic	Monte Carlo (1000) samples
3 Bus	3.12 s	7.63 s
6 Bus	3.35 s	34.33 s
14 Bus	7.95 s	70.86 s

TABLE 4.18: Computational Time for OPF taken by Different Methods for Different Systems

For better comparison, a plot is made for the λ_P of 6-Bus System and 14-Bus System, obtained by Interval Arithmetic and Monte Carlo and is shown in Fig.4.5 and Fig.4.6 respectively. The Simulation times are tabulated in Table 4.18. Calculations are performed in Mathematica using Intel(R) Core(TM) i5-3337U CPU @ 1.80 GHz. Clearly by Fig.4.6 we can observe that the results of Interval method encloses the results of Monte Carlo method.

Summary

In this chapter, the results obtained on applying Interval Arithmetic and MC methods for different systems are shown. The results are compared using plots. The results obtained by IA are almost close to that obtained by the MC, which makes Interval Arithmetic method superior to MC method.

Chapter 5

Conclusions

Load Flow plays a major role in any power system study. The results obtained by load flow are used for power system analysis and planning. Load flow is used to solve only for fixed values of inputs. But in practical cases, the demands, generations and voltages are not certain. Many Probabilistic Methods like Monte Carlo Simulation methods are used to solve power flow problem with uncertainty. But the main drawback is the time required for the solution to converge. Interval methods can deal with uncertain input data in power flow problems. If input data vary within relatively small ranges, good results that contain all possible solutions are obtained. The results obtained by Interval methods are almost close to that obtained by the Monte Carlo and the time required is much less compared to that of Monte Carlo method's simulation time. This makes Interval Methods a powerful tool to solve power flow problems with uncertainty in practical situations. The Krawczyk Method (Interval Method) is tested successfully for solving Load Flow for 2, 4 and 14 Bus systems and the results are shown.

Similarly the topic is extended to Optimal Power Flow.The Krawczyk Method (Interval Method) is tested successfully for solving OPF for 3, 6 and 14 Bus systems comparing the results with those obtained by Monte Carlo simulations. One important conclusion of this work was that Interval analysis can substitute the repeated simulations required by Monte Carlo method.

Appendix A

14 Bus System data

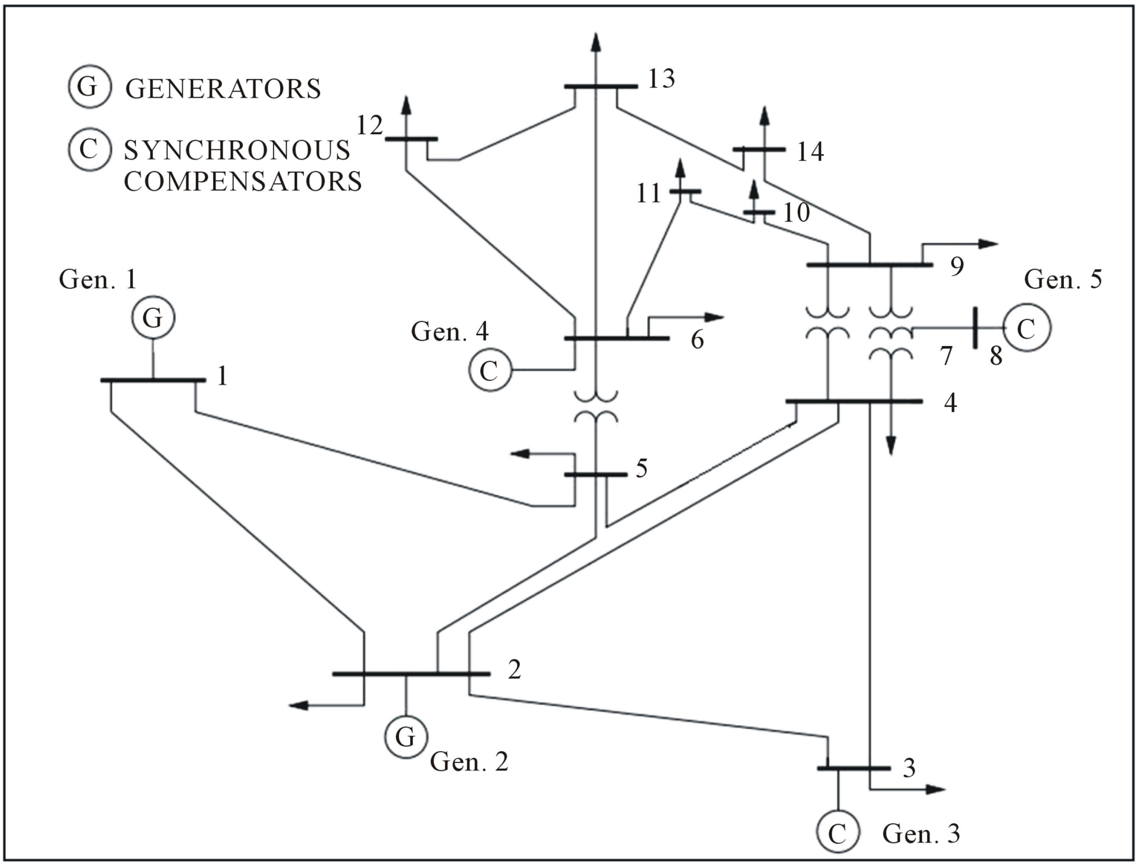

FIGURE A.1: Single Line Diagram of 14-Bus System

From Bus	To Bus	R(pu)	X(pu)	Y/2(pu)
1	2	0.01938	0.05917	0.0528
1	5	0.05403	0.22304	0.0492
2	3	0.04699	0.19797	0.0438
2	4	0.05811	0.17632	0.0374
2	5	0.05695	0.17388	0.034
3	4	0.06701	0.17103	0.0346
4	5	0.01335	0.04211	0.0128
4	7	0.00	0.20912	0.00
4	9	0.00	0.55618	0.00
5	6	0.00	0.25202	0.00
6	11	0.09498	0.1989	0.00
6	12	0.12291	0.25581	0.00
6	13	0.06615	0.13027	0.00
7	8	0.00	0.17615	0.00
7	9	0.00	0.11001	0.00
9	10	0.03181	0.08450	0.00
9	14	0.12711	0.27038	0.00
10	11	0.08205	0.19207	0.00
12	13	0.22092	0.19988	0.00
13	14	0.17093	0.34802	0.00

TABLE A.1: Line Data of 14-Bus System

Bus No.	a_i (\$/MW^2-h)	b_i (\$/MW-h)	c_i (\$/h)
1	0.0430293	20	0
2	0.2500	20	0

TABLE A.2: Generator cost functions of the 14-Bus System

Bus	P_G pu	Q_G pu	P_D pu	Q_D pu	V pu	Remarks
1	2.324	-0.169	0	0	$1.06\angle 0$	Slack Bus
2	0.4	-	0.217	0.127	1.045	PV Bus
3	0	-	0.942	0.19	1.01	PV Bus
4	0	0	0.478	-0.039	-	PQ Bus
5	0	0	0.076	0.016	-	PQ Bus
6	0	-	0.112	0.075	1.07	PV Bus
7	0	0	0	0	-	PQ Bus
8	0	-	0	0	1.09	PV Bus
9	0	0	0.295	0.166	-	PQ Bus
10	0	0	0.09	0.058	-	PQ Bus
11	0	0	0.035	0.018	-	PQ Bus
12	0	0	0.061	0.016	-	PQ Bus
13	0	0	0.135	0.058	-	PQ Bus
14	0	0	0.149	0.05	-	PQ Bus

TABLE A.3: Bus Data of the 14-Bus System

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