Analysis of Mean Delay in Signalized Road Intersections

A Project Report

submitted by

THAMMISETTY THARUN

in partial fulfilment of the requirements
for the award of the degree of

BACHELOR OF TECHNOLOGY

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY MADRAS.

May 2016

CERTIFICATE

This is to certify that the report titled **Analysis of Mean Delay in Signalized Road Intersections**, submitted by **Thammisetty Tharun**, to the Indian Institute of Technology, Madras, towards the partial fulfilment of **B. Tech** Degree requirements, is a bona fide record of the work done by him under my supervision. The contents of this report, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Dr. Krishna Jagannathan
Assistant Professor
Project Guide
Dept. of Electrical Engineering
IIT-Madras, 600 036

Place: Chennai

Date: 4th May 2016

ACKNOWLEDGEMENTS

I would first like to thank my advisor Dr.Krishna Jagannathan for his invaluable support and for steering me in the right direction whenever I needed help.

I would also like to thank Samrat Mukhophadhyay for giving me his valuable insights into the project.

Finally, I must express my gratitude to my family and my friends for their unfailing support and encouragement without which none of this would have been possible. Thank You.

Thammisetty Tharun

ABSTRACT

KEYWORDS: Queuing Theory; Traffic Intersections ; Tandem Queues ; Webster's formula ; Interrupted Queues.

A road network carrying vehicular traffic can be viewed as a packet network where roads are communication links and signalized intersections are the servers. Many people have adopted the models of congestion control theory to model and analyze the mean delay experienced in a vehicular network.

In this report we study the previous models employed to analyze the mean delay in a signalized intersection. A traffic intersection can be viewed as a queue with an interrupted server. This report discusses about three different queuing models used to model the traffic intersection. We also study the model proposed by Anurag Kumar et al. which accounts for the lane indiscipline seen on roads by allowing vehicles to form batches. Webster's delay formula is one of the most popular results in the traffic literature which gives an approximation for waiting time in an interrupted M/D/1 queue. This report interprets the terms in Webster's formula and the quality of approximations is illustrated by numerical examples.

This project aims to propose a model to analyze the mean queuing delay of signalized intersections in tandem. We give a reasonable approximation to a tandem interrupted M/D/1 queue. This report also provides a assembly queue model similar to that of the one provided by Anurag Kumar et al. to analyze lane indisciplined queues in tandem.

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NOTATION

λ	Arrival rate into the queue
λ_c	Arrival rate of cars
λ_m	Arrival rate of motorcycle pairs
C_0	Batch consisting of car alone
C_2	A car and motorcycle batch
M_2	Batch consisting of a single motorcycle pair
M_4	A motorcycle-motorcycle batch
$\tilde{\mathcal{V}}$	Set of all possible batches formed
ν_x	stationary probability that the markov chain is in state x
$a_i(x)$	Rate of departure of batch type i from state x
f_{ij}	probability that a batch j follows a batch i
q_0	probability that the interrupted queue is empty
q_1	probability that the interrupted queue has exactly one batch
μ	mean service rate of vehicles at the interrupted queue

CHAPTER 1

INTRODUCTION

With Intelligent Transport Systems (ITS) becoming a possibility there is an increased interest in modeling, analysis, inference and control of traffic in road networks. ITS can help to make on-line decisions and adjust the traffic signal timings to minimize the congestion in a road network. The analysis of signalized intersections has been an area of interest to researchers of applied probability, operations research and control. While this is a well-studied topic the models used by the researchers do not give an accurate description of road traffic in Indian contexts where the traffic is indisciplined and dominated by two-wheelers. Anurag *et al.* (2015) paper presents and analyses a model in this context.

A signalized intersection is a junction where two or more roads cross, and is generally controlled by traffic signals. The congestion phenomenon at an intersection is governed by parameters such as the duration of the red and green signals, the arrival rates of the vehicles, the way vehicles may form batches and the rate at which the batched vehicles exit the intersection during the green time. An understanding of such congestion phenomena, and the design of signal timing as to optimize the delay at an intersection, are basic questions in any road network analysis and design.

If the departure process of a signalized intersection is fed as an input to an another signalized intersection the two intersections are said to be in tandem. The congestion in the second intersection depends on the duration of red and green signals at both the intersections, the signal offset between the two intersections and the extent of batching (in case of indisciplined traffic). We study such queues in Chapter 4.

1.1 Contributions

1. We will discuss the model presented by Anurag *et al.* (2015) which accounts for the lane indiscipline in the Indian roads by allowing the motorcycles to form batches. We will support the conclusions made with appropriate simulations.

2. We will interpret and analyze the classical webster's delay formula for interrupted M/D/1 queue term by term and support the conclusions with the help of simulations.
3. In chapter 4 we analyze the mean delay of a tandem queue where both the first and second queues are interrupted with deterministic service process.We also provided the simulations and analytical results for such a queue.
4. We further provide a discrete time assembly queue model for the case where the tandem intersections allow batching using the following probabilities obtained in chapter 2.

1.2 Literature Survey

The problem of analyzing the mean delay at a signalized intersection is well-studied in traffic engineering literature. The poisson point process is one of the most widely used approximation to model the arrival of traffic flow into the interrupted queue. The reason for the wide use of this model in the literature can be attributed to its mathematical tractability. Webster came up with the first analytical approximation for the mean delay in an interrupted M/D/1 queue in 1958.

Samrat (2014) has proposed three successively simplistic models to analyze the mean delay in interrupted queues.

- M/SM/1 model: The arrival process is poisson and the initial vehicle is sampled from the stationary probability distribution and the subsequent vehicles are sampled from the following probability distribution of the previous vehicle. The service time of the current vehicle depends on the length of the vehicle if it is the first vehicle in the queue else the service time depends on the tail to tail distance between the current vehicle and the vehicle preceding it.
- M/G/1 model: It's a simplification of the above model. The inter arrivals are exponentially distributed. All the vehicles are sampled from stationary probability distribution. The service times can be calculated from the precedence probability distribution given by the formula.

$$\pi_i pij = \pi_j f_{ji} \tag{1.1}$$

  Now from the precedence probability distribution we will sample a vehicle type. We will compute the service time for the current vehicle, depending on the current vehicle type and the preceding vehicle type sampled from the precedence probability distribution.
- M/D/1 model: The inter arrivals are exponentially distributed and this model consists of a single vehicle type and the lagging headway for the vehicle type would be the mean of all lagging headways.

Federgruen and Green (1986) analyses queues with interruption, with Poisson arrival process and general i.i.d. service time distributions.For the case that Federgruen and Green analyses, where the arrival rate is same during both green times and red times, and the service time distributions are same for vehicles arriving during green times and red times. Sengupta (1990) analyses the general case of a queue with alternating green times and red times with different arrival distributions and service distributions during green time and red time.Federgruen and Green use the concept of completion time introduced by Gaver (1962) where as Sengupta uses the concept of residual service times.

CHAPTER 2

Analysis of Delay with Indisciplined Traffic

In this chapter, we will briefly explain the model presented by Anurag *et al.* (2015) and also provide the numerical simulations to support the conclusions arrived at in the paper.

2.1 Overview of the model

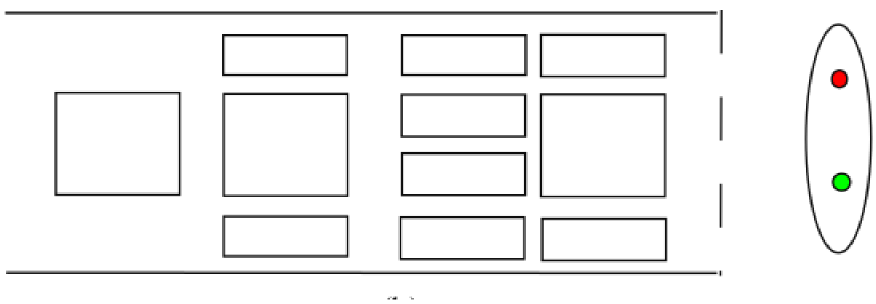


Figure 2.1: Depicting the batch formation in Indisciplined traffic queues.

In this model, there are two types of vehicles, cars and motorcycles. Cars arrive according to a Poisson process with rate λ_c and motorcycles always arrive in pairs with arrival rate λ_m. To account for the lane indiscipline vehicles are allowed to form batches. The motorcycle pairs can always move up to the head of the line (HOL) position to form a batch. It should be noted that the cars cannot do the same. The batching process is depicted in Figure 2.1.

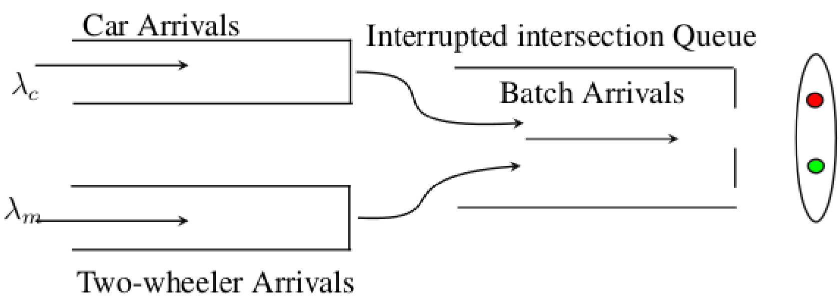
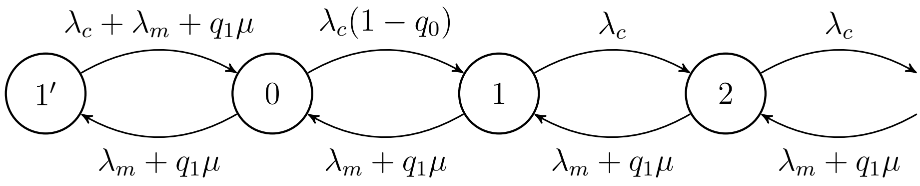


Figure 2.2: Assembly queue model.

The approximate analytical model in the paper contains two assembly queues one for each vehicle type and an interrupted batching queue in which the batches exit the queue. All the vehicles enter their corresponding assembly queues when the interrupted batch queue is non empty and when the assembly queue has a car and motorcycle pair they exit the assembly queue and join the batch queue. In case the interrupted queue is empty the car directly joins the interrupted queue. The transitions of such an assembly queue forms a birth death markov chain which is given below where each node corresponds to the length of the assembly queue.



2.2 Analytical expressions for q_0, q_1 and μ

Finding q_0 and q_1 requires finding the stationary queue length distribution of an interrupted queue where the arrival process is the batch departure process of the assembly queues which is not poisson so the exact analysis is intractable and hence these quantities are obtained from an interrupted M/G/1 model.

2.2.1 Expression for q_0

From the analysis of interrupted M/G/1 queues by Sengupta (1990) the approximate expression for q_0 can be shown as

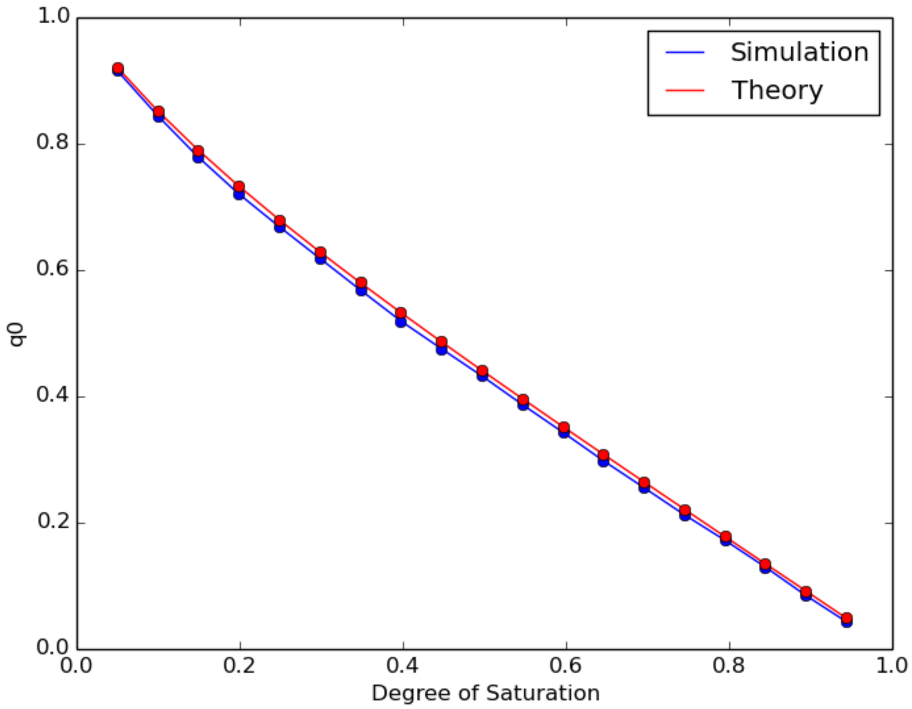
$$q_0 = \frac{g}{c} - \lambda\tau + \frac{u_0}{\lambda c}(1 - e^{-r\lambda}) \tag{2.1}$$

Where u_0 is $(1 - \lambda\tau)w_0$ which is also shown by Sengupta.

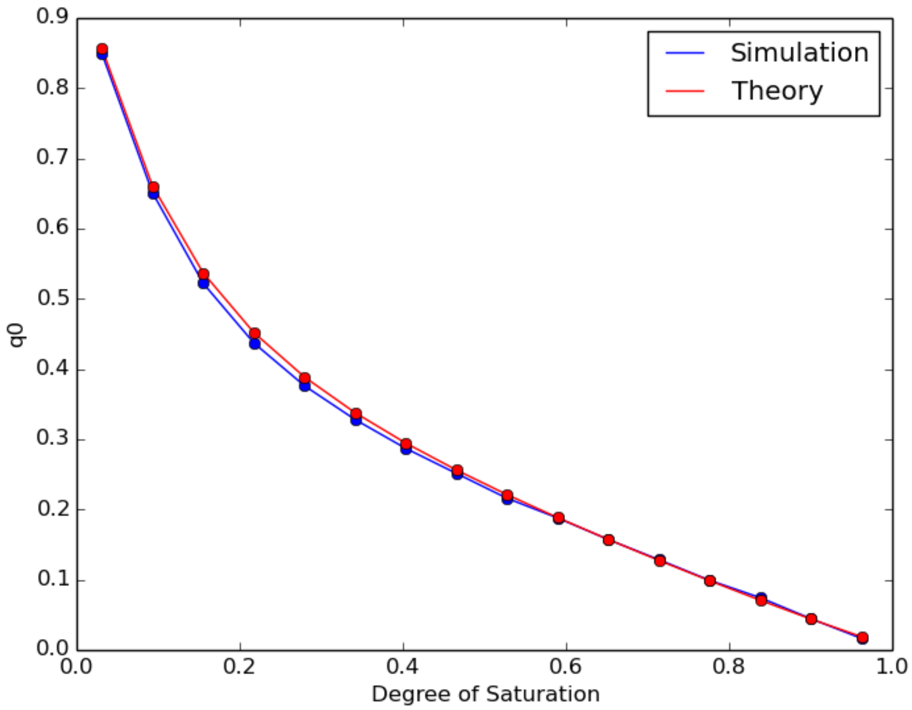
$w_0 = 1 - r_0$ where r_0 is the solution for the the equation and $\rho = \frac{\lambda\tau r}{(1-\lambda\tau)}$

$$z = e^{\frac{-g(1-z)}{\rho}} \tag{2.2}$$

The simulations below show the accuracy of the approximation.



(a) g=50s and c=60s



(b) g=20s and c=60s

Figure 2.3: Plots (a) and (b) show the simulated and approximated estimates foe q_0 for different values of green time and cycle time

2.2.2 Expressions for q_1 and μ

From the analysis of uninterrupted M/G/1 queue we have the equation

$$\Pi(z) = \frac{(1-\rho)(1-z)(C^*(\lambda(1-z)))}{(C^*(\lambda(1-z))) - z} \tag{2.3}$$

Since $\Pi(z) = \sum_k \pi_k z^k$ where π_k is the stationary probability of queue length equal to k.

$$q_1 = \Pi'(0) \tag{2.4}$$

C^* is LS transform of completion time. From the above equations q_1 can be obtained as

$$q_1 = q_0 \frac{1 - C^*(\lambda)}{C^*(\lambda)} \tag{2.5}$$

Completion time is defined as the time for which a customer remains in the head-of-the-line position. This can be rigorously obtained by the analysis of Federgruen and Green (1986). But, we present a simple proof using Little's theorem.

$$q_0 = 1 - \lambda E(C) \tag{2.6}$$

From Eq 2.6

$$E(C) = \frac{r}{c} + \tau - \frac{u_0}{\lambda^2 c}(1 - e^{-r\lambda}) \tag{2.7}$$

The simulations below show the accuracy of approximations.

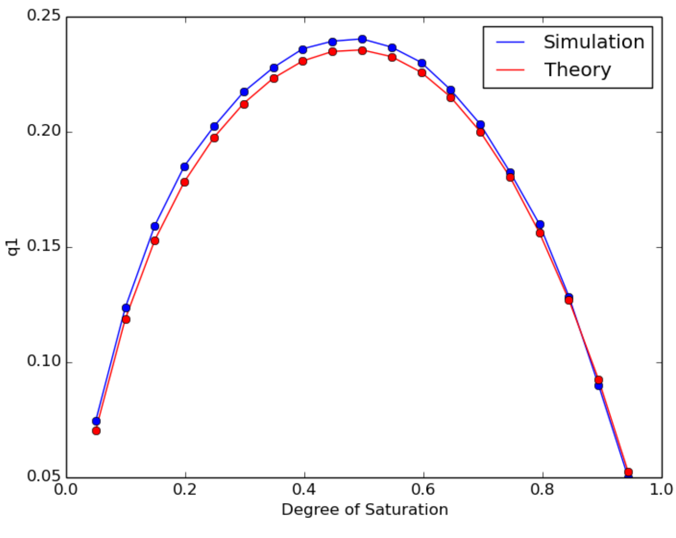


Figure 2.4: g=50s and c=60s

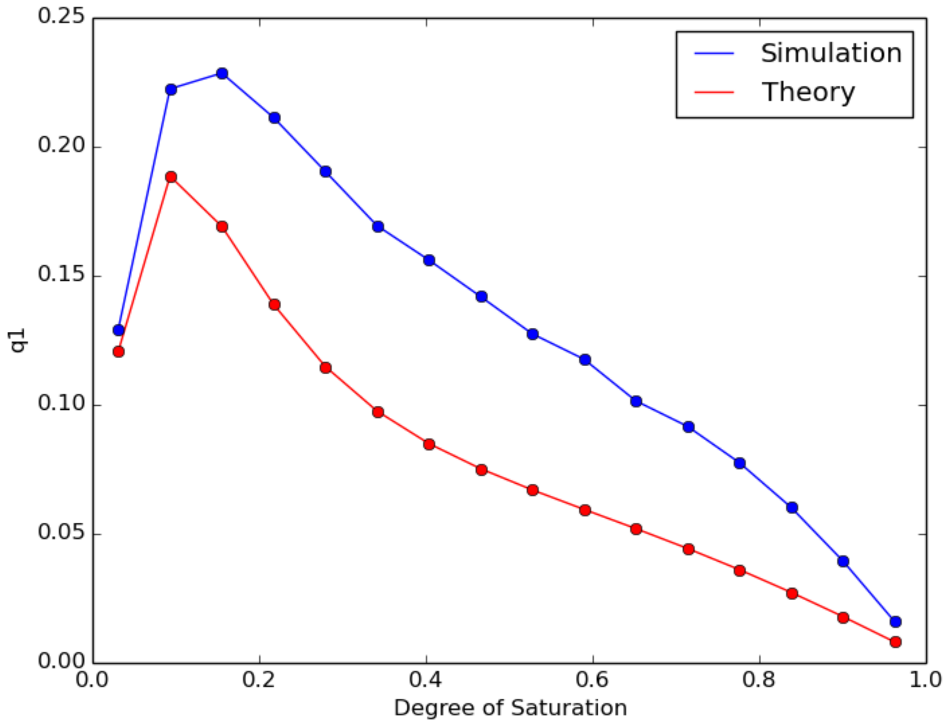


Figure 2.5: g=20s and c=60s

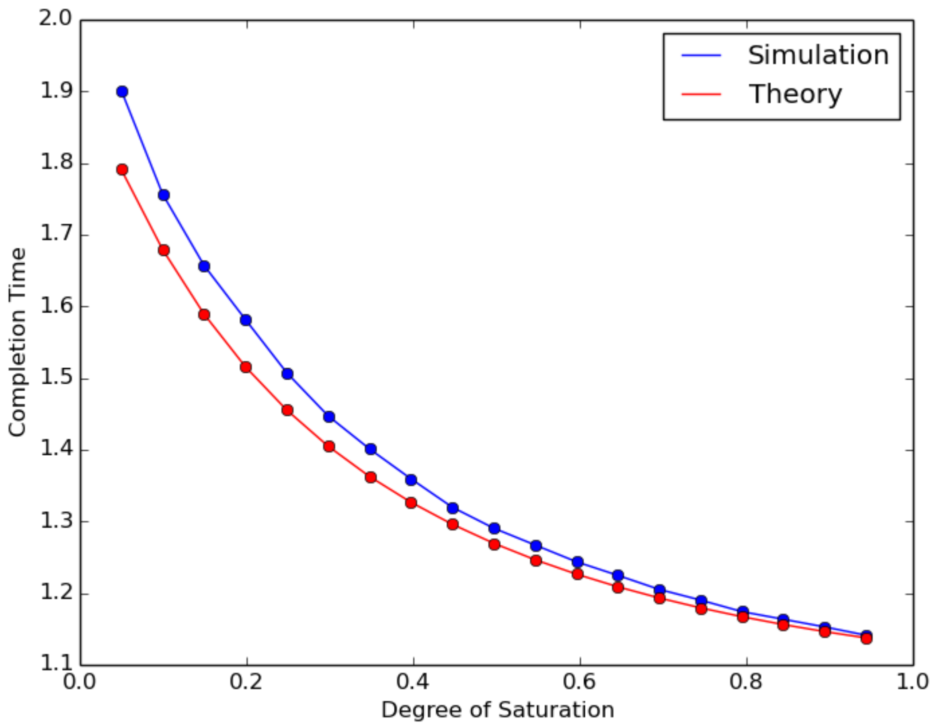


Figure 2.6: g=50s and c=60s

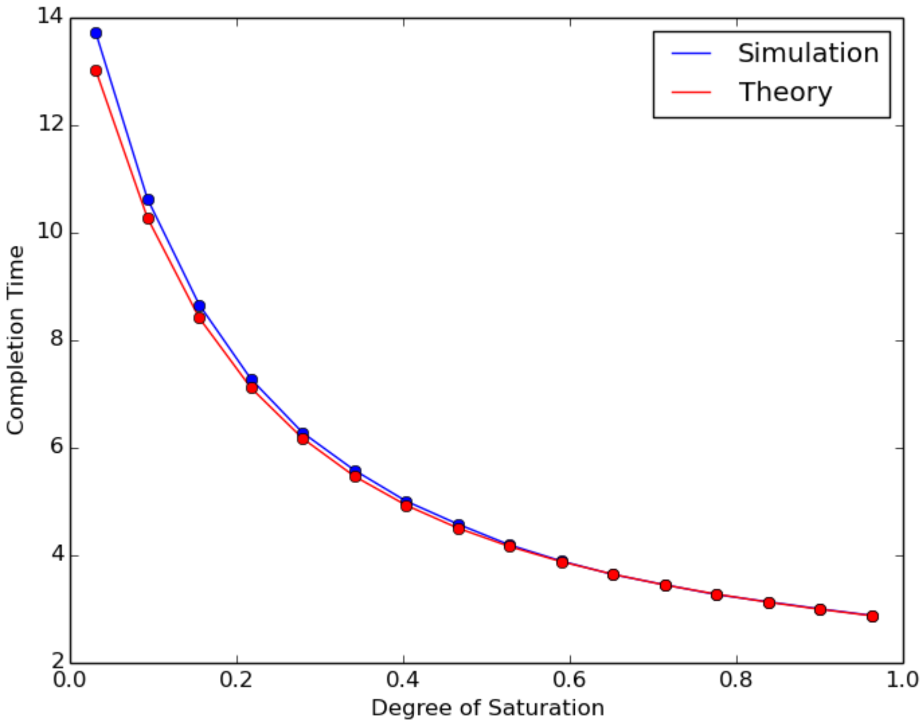


Figure 2.7: g=20s and c=60s

2.3 Analysis of Delay

The model considered has two delay components the delay experienced in the assembly queue and the delay experienced in the interrupted batch queue.

2.3.1 Delay in Assembly queuing system

The expected number of motorcycle pairs in the assembly queuing system

$$EM_p = \nu_{1'} \tag{2.8}$$

The delay experienced by the motorcycle pairs is given by

$$EWM_p = \frac{EM_p}{\lambda_m} \tag{2.9}$$

The expected number of cars in the assembly queuing system

$$EC_p = \sum_k k\nu_k \tag{2.10}$$

The delay experienced by the motorcycle cars is given by

$$EWC_p = \frac{EC_p}{\lambda_c} \tag{2.11}$$

The total delay is given by

$$d_{Assembly} = \frac{2\lambda_m}{\lambda_c + 2\lambda_m} EWM_p + \frac{\lambda_c}{\lambda_c + 2\lambda_m} EWC_p \tag{2.12}$$

The total delay experienced in the system is

$$d = d_{Assembly} + \frac{c(1-\frac{g}{c})^2}{2(1-\frac{g}{c}x)} + \frac{\overline{q}}{\lambda} - \tau + 0.65\left(\frac{c}{\lambda^2}\right)^{\frac{1}{3}} x^{2+5\frac{g}{c}} \tag{2.13}$$

CHAPTER 3

Interpreting Webster's delay formula

In this chapter, we will provide an intuitive interpretation of every individual term present in the interrupted M/D/1 delay formula.

3.1 Interrupted queue with fluid arrival

Consider an interrupted queue with fluid arrival with arrival rate λ and service rate μ. In a situation where the arrival rate is low compared to the arrival rate at the degree of saturation of such a queue we can assume that fluid arriving in the red time is cleared during the green time. The queue occupancy in such a case can be calculated as the area under the graph in Figure 3.1.

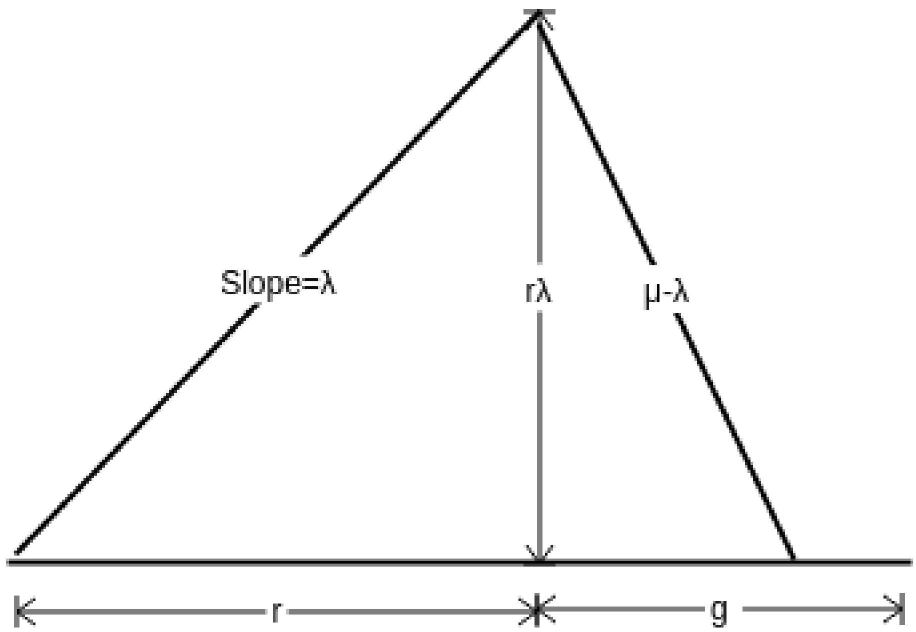


Figure 3.1: Queue occupancy when the arrival is fluid with low arrival rate

The total area under the graph is given by

$$= \frac{1}{2}(r+g)r\lambda$$
$$= \frac{1}{2}(r+\frac{r\lambda}{\mu-\lambda})r\lambda$$
$$= \frac{1}{2}\frac{r^2\mu\lambda}{\mu-\lambda}$$

The average queue occupancy can be obtained by dividing the the above equation with c.

$$= \frac{1}{2}\frac{r^2\mu\lambda}{c(\mu-\lambda)}$$

From Little's theorem mean delay can be obtained by dividing it with arrival rate λ.

$$d = \frac{c(1-\frac{g}{c})^2}{2(1-\frac{g}{c}x)} \tag{3.1}$$

So, the equation 3.1 gives the delay experienced in an interrupted fluid queue when the traffic arriving in the red time is cleared in the green time. This equation is the first term in Webster's delay formula.

3.2 M/D/1 queue with no interruptions

In an M/D/1 queue with no service interruptions the delay experienced is given as a function of utilization $\rho = \frac{\lambda}{\mu}$ where λ is the arrival rate and μ is the service rate.

$$d = \frac{\rho^2}{2\lambda(1-\rho)} \tag{3.2}$$

The utilization in interrupted queue is measured as degree of saturation which is $\lambda\tau\frac{c}{g}$. Therefore second term in Webster's delay formula can be viewed as the delay experienced in an uninterrupted M/D/1 queue when the utilization is inflated by $\frac{c}{g}$.

$$d = \frac{x^2}{2\lambda(1-x)} \tag{3.3}$$

CHAPTER 4

Signalized intersections in tandem

In this chapter, we study signalized intersections in tandem and try to provide some results which will help in analyzing their mean delay. The delay experienced in such a system has four components the delay in first queue, service time in second queue, time taken to reach from the first queue to second queue and the delay experienced in second queue.

4.1 Interrupted M/D/1 intersections

In this section, we will consider a simple system where the first queue is a M/D/1 with service interruption and the second queue has a deterministic service process with interruptions whose input is the output process of the first queue.

In the following analysis, we make the following assumptions. Vehicles will not overtake the vehicles in front, all the vehicles take the same time to reach the second queue in this case (30s), new vehicles will not join the vehicle stream going to the second queue and the signal offset between the two queues is zero that is the signals are synchronized.

In chapter 3 we showed that the first term in the Webster's formula comes from fluid approximation of the process when the arrival rate is low. When the first queue is very close to saturation the departure process of the first queue will have $\frac{1}{\tau}$ rate and since the service process of the second queue is deterministic the mean service time of the second queue will be τ. This is a model very similar to the one studied by Clayton(1941) and he has approximated the mean delay of such a queue only with the fluid approximation.

Assuming the fluid model to study the second queue. In one cycle the traffic gets accumulated during red time and during green time the traffic gets serviced at rate $\frac{1}{\tau}$ for a time say y and during the remaining time $g-y$ traffic arrives at rate λ. The traffic gets serviced at rate $\frac{1}{\tau}$ during the green time of second queue. The time y can be calculated as follows

$$r\lambda + y\lambda = \frac{y}{\tau}$$
$$y = \frac{r\lambda}{\frac{1}{\tau}-\lambda}$$

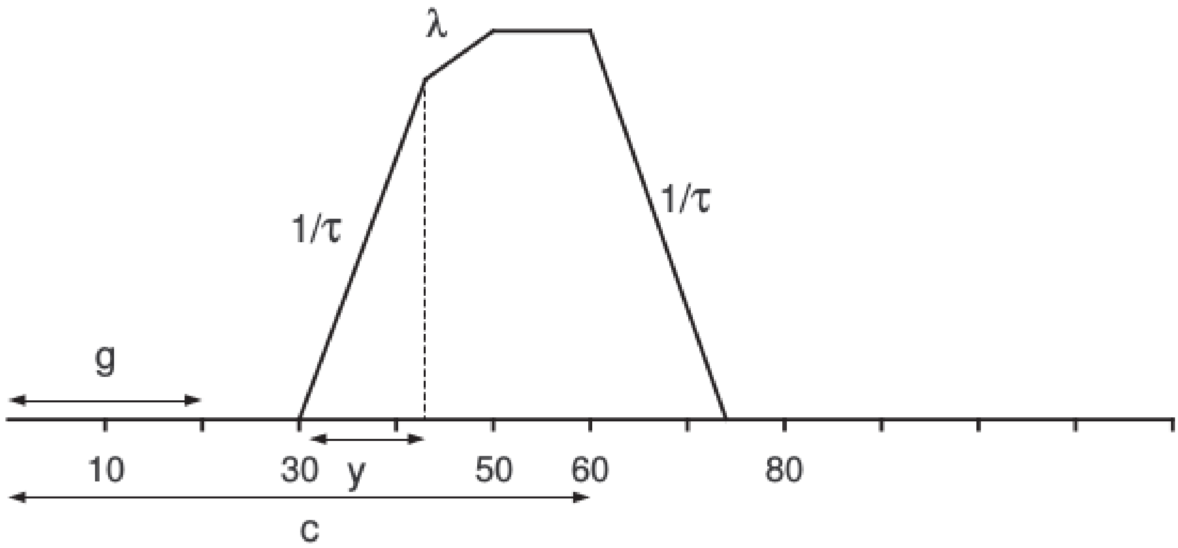


Figure 4.1: Queue occupancy plot for the second queue when g=20s and c=60s

Let the area under the graph be A, then the mean delay van be obtained by applying Little's law.

$$\overline{d} = \frac{A}{c\lambda}$$

Therefore the total delay in the system can be approximated as

$$d_{total} = d_1 + \tau + 30 + \overline{d}$$

4.2 Assembly Queue modeling of tandem batched queue

Analyzing a tandem queuing system where vehicular batching is allowed is extremely complex. In this section we provide an assembly queue model similar to that provided by Anurag *et al.* (2015) but with a DTMC. The reasons for adopting a DTMC approach is

1. The departure process from the first queue is not poisson and hence cannot be modeled as a CTMC.
2. The transition rates of the CTMC that represent the batching process in the assembly queue is dependent on the queue length distribution of the intersection queue and hence the batch departure process i.e. the batch arrival process into the intersection queue is dependent upon queue length. Hence the batch departure process is not renewal.

To get the transition probabilities of the DTMC we need the following probabilities of the batch departure process from the first queue. Let X_k denote the kth batch departing the queue then the following probability f_{ij} is

$$f_{ij} = P(X_{k+1} = j | X_k = i)$$

From Anurag *et al.* (2015) this f_{ij} is

$$f_{ij} = \frac{\sum_{x,y \in \chi} \nu_x a_x p_{xy} \alpha_i(x,y) \phi_j(y)}{\sum_{x \in \chi} \nu_x a_i(x)} \tag{4.1}$$

where

$\alpha_i(x, y)$: is probability that there is a departure of type i and goes from state x to state y, χ is set of all states in markov chain in chapter 2 and p_{xy} is probability of departure from state x to state y.

ν_x can be obtained from the balance equations from the markov chain in Chapter 2 as shown below.

$$(\lambda_c + \lambda_m + q_1\mu)\nu_{1'} = \lambda_m(1 - q_0)\nu_0$$
$$\lambda_c(1 - q_0)\nu_0 = (\lambda_m + q_1\mu)\nu_1$$
$$\lambda_c\nu_x = (\lambda_m + q_1\mu)\nu_{x+1}, \, x \geq 1$$

From the above equations,

$$\nu_{1'} = \frac{1}{1 + (\frac{\lambda_c+\lambda_m+q_1\mu}{\lambda_m-\lambda_c+q_1\mu})(\frac{\lambda_m+q_1\mu-\lambda_c q_0}{\lambda_m(1-q_0)})} \tag{4.2}$$

$$\nu_0 = \frac{\lambda_c + \lambda_m + q_1\mu}{\lambda_m(1-q_0)}\nu_{1'} \tag{4.3}$$

$$\nu_1 = \frac{\lambda_c(1-q_0)}{\lambda_m + q_1\mu}\nu_0 \tag{4.4}$$

$$\nu_x = (\frac{\lambda_c}{\lambda_m + q_1\mu})^{x-1}\nu_1, x \geq 1 \tag{4.5}$$

From the CTMC in Chapter 2 we get the other quantities

‘

$x, y \in \chi$	p_{xy}
$x = 1', y = 0$	1
$x = 0, y = 1'$	$\frac{\lambda_m}{\lambda_c+\lambda_m}$
$x = 0, y = 1$	$\frac{\lambda_c}{\lambda_c+\lambda_m}$
$x = k, y = k+1 (k \geq 1)$	$\frac{\lambda_c}{\lambda_c+\lambda_m+q_1\mu}$
$x = k, y = k-1 (k \geq 1)$	$\frac{\lambda_m}{\lambda_c+\lambda_m+q_1\mu}$
else	0

Table 4.1: Transition Probabilities of assembly queue markov chain in chapter 2

$i \in \mathcal{V}$	$\alpha_i(x, y)$
$i = C_0$	$\frac{q_1\mu}{\lambda_m+q_1\mu}$ $x = k, y = k-1$ $(k \geq 1)$ $else$ 0
$i = C_2$	$\frac{\lambda_c}{\lambda_c+\lambda_m+q_1\mu}$ $x = 1', y = 0$ $\frac{\lambda_m}{\lambda_m+q_1 mu}$ $x = k, y = k-1$ $(k \geq 1)$ $else$ 0
$i = M_2$	$\frac{q_1\mu}{\lambda_c+\lambda_m+q_1\mu}$ $x = 1', y = 0$ $else$ 0
$i = M_4$	$\frac{\lambda_m}{\lambda_c+\lambda_m+q_1\mu}$ $x = 1', y = 0$ $else$ 0

Table 4.2: $\alpha_i(x, y)$ for the markov chain in chapter 2

$x \in \chi$	$\phi_i(x)$
$x = 0$	$(\frac{\lambda_c}{\lambda_c+\lambda_m})(\frac{q_1\mu}{\lambda_m+q_1\mu}),\ i = C_0$ $(\frac{\lambda_c\lambda_m}{\lambda_c+\lambda_m})(\frac{1}{\lambda_m+q_1\mu} + \frac{1}{\lambda_c+\lambda_m+q_1\mu}),\ i = C_2$ $(\frac{\lambda_m}{\lambda_c+\lambda_m})(\frac{q_1\mu}{\lambda_c+\lambda_m+q_1\mu})\ i = M_2$ $(\frac{\lambda_m}{\lambda_c+\lambda_m})(\frac{\lambda_m}{\lambda_c+\lambda_m+q_1\mu})\ i = M_4$
$x = 1'$	$0, ß = C_0$ $\frac{\lambda_c}{\lambda_c+\lambda_m+q_1\mu}, ß = C_2$ $\frac{q_1\mu}{\lambda_c+\lambda_m+q_1\mu}, ß = M_2$ $\frac{\lambda_m}{\lambda_c+\lambda_m+q_1\mu}, ß = M_4$
$x = 1, 2, ...$	$\frac{q_1\mu}{\lambda_m+q_1\mu}, ß = C_0$ $\frac{\lambda_m}{\lambda_m+q_1\mu} mß = C_2$ $0,\ i = M_2, M_4$

Table 4.3: $\phi_i(x)$ for markov chain in chapter 2

$x \in \chi$	$a_i(x)$
$x = 0$	$0,\ \forall i \in \nu$
$x = 1'$	$0,\ i = C_0$ $\lambda_c,\ i = C_2$ $q_1\mu,\ i = M_2$ $\lambda_m,\ i = M_4$
$x = 1, 2, ..$	$q_1\mu,\ i = C_0$ $\lambda_m,\ i = C_2$ $0,\ i = M_2, M_4$

Table 4.4: $a_i(x)$ for markov chain in chapter 2

From the above tables following probabilities can be obtained by plugging them into equation 4.1. From the following probabilities probability of occurrence of each batch can be obtained and given by table 4.5.

$x \in \mathcal{V}$	P_x
C_0	$P_{C_0} = \sum_{i \in \mathcal{V}} f_{iC_0}$
C_2	$P_{C_2} = \sum_{i \in \mathcal{V}} f_{iC_2}$
M_2	$P_{M_2} = \sum_{i \in \mathcal{V}} f_{iM_2}$
M_4	$P_{M_4} = \sum_{i \in \mathcal{V}} f_{iM_4}$

Table 4.5: Probabilities of occurrence of batches from Q1

From table 4.5 and using $\overline{q_0}$ which is the probability of second queue being empty and P_1 the probability that the second queue contains zero batches once the current batch leaves the queue, we can construct a DTMC for the assembling process where the states in the markov chain correspond to the number of vehicles in the assembly system as below.

The state $1'$ represents the instance where there is exactly one motorcycle pair in the system. In this state, when either a single car or a motorcycle pair arrives they form batch and leave the queue and the assembly queue goes to state 0, if M_4 batch arrives it remains in the same state and if C_2 batch arrives we assume either C_2 or M_4 can be formed with equal probability and correspondingly remain in the same state or go to state 1.

In the state 0, if either a C_2 or a M_4 batch comes they directly join the second queue, if a motorcycle pair comes it goes to state $1'$ and if a car arrives it goes to state 1 if the batch queue is non empty.

In the state 1, on the arrival of C_0 goes to state 2, remains in the same state on the arrival of C_2 or goes to state $1'$ on the arrival of M_4.

For the states i $(\forall i \geq 2)$ on the arrival of C_0 goes to state $i + 1$, remains in the same state on the arrival of a C_2 or goes to state $i - 2$ on the arrival of M_4.

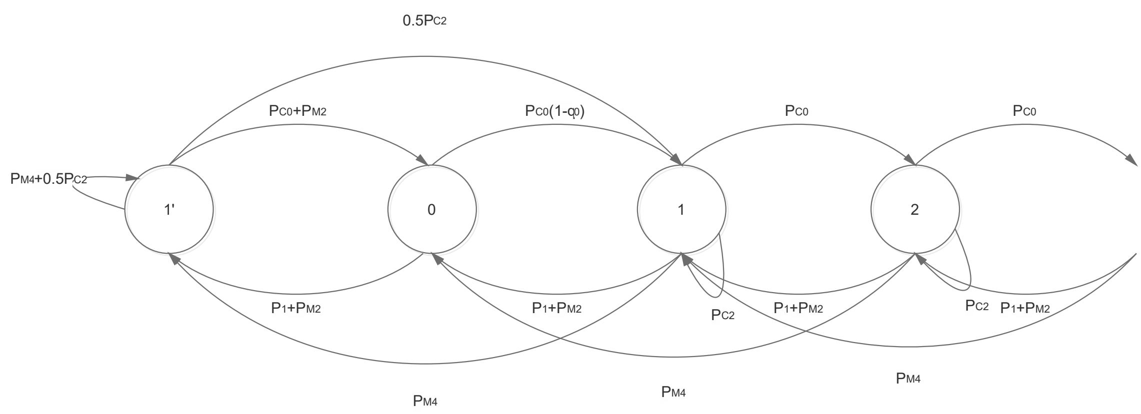


Figure 4.2: Assembly queue markov chain for the second queue

CHAPTER 5

Future Work

The assembly queuing model presented in this report can be extended to obtain the delay in the tandem queuing system when batching is present. Alternatively, customer metamorphosis models can be employed to model the batching process and to obtain the mean delay. Optimization methods can be used to adjust the signal timings and the signal offsets between the tandem queues to minimize the mean queuing delay experienced in the system. The models presented in the report analyze the delay in case of single lane traffic this can be extended to study a traffic system having multiple lanes. The batching model presented can be extended to include multiple vehicle types.

CHAPTER 6

References

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