Generalized frequency division multiplexing

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THESIS CERTIFICATE

This is to certify that the thesis titled **Generalized frequency division multiplexing**, submitted by **Sriram Vasudevan**, to the Indian Institute of Technology, Madras, for the award of the degree of **Bachelor of Technology**, is a bona fide record of the research work done by him under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

The cellular systems of 4th generation have been optimized to provide high data rates and robust mobile coverage. The next generation cellular systems will face even larger challenges like even higher data rates, ultra low power consumption. A few promising control centric areas like machine to machine communication, cognitive radio require extremely short response times which is infeasible with 4th generation techniques. This thesis describes a new technique called Generalized frequency division multiplexing which is a block based multicarrier modulation scheme. After a solid introduction to GFDM, two transmitter models are described each with its own advantages and disadvantages. Based on these transmitters, three standard ways of receiving a signal are derived, and their performances are compared in terms of symbol error rate in AWGN and Rayleigh multipath fading channels. OFDM, though a reliable multicarrier modulation technique, is riddled with its own problems like high out of band radiation and high Peak to average power ratio(PAPR). These parameters are also measured for GFDM, and compared to those obtained with OFDM. Finally the thesis concludes that GFDM is a novel method, that is a generalization of the existing OFDM and SCFDM techniques, performs better than all of them in terms of OOB and PAPR.

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CHAPTER 1

INTRODUCTION

Almost all of present day communication systems rely on multicarrier systems for their obvious advantages over single carrier systems in frequency selective fading channels. Several utilities like machine to machine communication require very low power consumption, very low latency, and require the architecture to support a large number of users. GFDM is block based digital multicarrier modulation scheme, whose advantages lie in its flexiblilty. The data is spread across time and frequency axis which enables flexibility to either prioritize latency or bandwidth usage, depending on the application. The cyclic prefix of OFDM is retained to perform simple equalization when data is transmitted through multipath channels. Unlike OFDM where the transmitter just employs an FFT block, GFDM uses circular convolution to perform pulse shape filtering. This has the added advantage of tail biting, thereby preventing rate loss, but it also causes the carriers to become non-orthogonal. This induces self-interference within the GFDM block.

GFDM turns out to be a linear matrix model, as all the byprocesses like pulse shaping and upsampling are linear. Hence, the transmitter complexity is comparable to that of OFDM, the analysis of which is done in further sections. Different impulse responses can be used for pulse shaping, which will affect the symbol error rate and OOB emissions differently. GFDM also gives us better spectral efficiency compared to OFDM owing to less frequent Cyclic prefixes.

1.1 Background and Formulation

Orthogonal Frequency division multiplexing

OFDM employs several orthogonal carriers instead of a single wide band carrier. The data is in the form of complex numbers that represent the points on the constellation. Each symbol is loaded onto a particular frequency.

$$x(u) = \sum_{i=-\frac{N}{2}}^{\frac{N}{2}} X_i e^{j2\pi \frac{i}{N} u}$$

This is the equation of the inverse discrete fourier transform. Hence OFDM uses a single time-slot to modulate N symbols onto N carriers. OFDM employs many low bitrate carriers compared to the conventional single high bitrate carrier. Such kind of multicarrier modulation ensures performance under frequency selective channels. A cyclic prefix with length greater than that of channel delay spread is used to prevent inter-symbol inference. Hence, one only has to deal with intrasymbol interference(between the symbols transmitted in one time slot), which can be reduced by using appropriate pulse shaping filters.

Single carrier frequency division multiplexing

Another technique Single carrier frequency division multiplexing(SC-FDMA) which differs from OFDM in the fact that a single carrier does not contain all of any particular symbol. The symbols are pre-coded using a discrete fourier transform and a only a portion of each symbol is sent into any particular carrier. This is controlled using a sub-carrier mapping block.

CHAPTER 2

GFDM Transmitters:

2.1 Time domain transmitter:

Consider the same complex data matrix **D** where $d_{m,k}$ is the symbol to be loaded onto kth sub carrier in the mth time-slot.The time domain equation for modulation is given by:

$$\sum_{m=0}^{M-1}\sum_{k=0}^{K-1} d_{m,k} g_{tx}[n - mK] e^{j2\pi \frac{kn}{K}}$$

To speed up the transmission process, a modulation matrix is pre-calculated for the given sub carriers and time-slots. Incoming data is broken down according to symbol size, allocated to a point on the constellation and segregated into blocks of size MK. Each block is then modulated linearly according to $y = Ad$. First, upsampling is done by a factor N to satisfy Nyquist criterion. The upsampling matrix is given by

$$S_N^M = \{s_{n,m}\}, \quad where\ s_{n,m} = 1 \ \ if\ n = (M-1)N + 1, and\ 0\ otherwise$$

$$upsampled\ matrix\ \ X_D \ = \ S_N^M \mathbf{D}$$

Next, pulse shaping filter is applied in time-domain. The filter used is a raised cosine pulse. The continuous time expression of raised cosine pulse is given by:

$$h(t) \ = \ sinc\left(\frac{t}{T}\right) \frac{\cos(\frac{\pi\beta t}{T})}{1 - \frac{4\beta^2 t^2}{T^2}}$$

where β is the roll-off factor.

is sampled MN times to obtain the discrete form of the impulse response. This is to be circularly convolved with the upsampled signal. This is achieved my generating a matrix elements cyclically shifted according to impulse response. If $g_{tx} = \{g_n\}_{MNx1}$,

$$G_{tx} = \begin{pmatrix} g_1 & g_{MN} & \cdots\cdot & g_2 \\ g_2 & g_1 & \cdots\cdot & g_3 \\ . & & & \\ . & & & \\ . & & & \\ g_{MN} & g_{MN-1} & \cdots & g_1 \end{pmatrix} Hence, \;\; X_G = G_{tx}X_D$$

These MN samples have to be translated to their respective frequencies and transmitted. This can be done in a way similar to the way OFDM achieves it. From OFDM, we know that IFFT operation was used to translate symbols to carrier frequencies. Similarly, an MN dimensional IFFT operation is done on X_G matrix. Since we require only N carriers, we down-sample IFFT output to N samples.

$$X_W = X_G S_M^N W^H$$

where W^H is the inverse fourier transformation matrix. The diagonal elements of X_W matrix is the transmitted signal.

$$x = diag(G_{tx}(S_N^M)D(S_M^N)W^H)$$

$$x = diag((G'_{tx})D(W'_{tx}))$$

An nth diagonal element depends only on nth row of G'_{tx} and nth column of W'_{tx}.

$$[X_W]_{n,n} = K\left([g_{tx}]_n, [W_{tx}]_n^T\right) vec(\mathbf{D})$$

where $K(A, B)$ denotes the kronecker product of A and B; and vec(**D**) denotes the MK symbols in **D** arranged in a single column.

Assembling these kronecker products into each row of an MKxMK matrix, gives the modulation matrix A.

$$y = \begin{bmatrix} K\left([g_{tx}]_1, [W_{tx}]_1^T\right) \\ K\left([g_{tx}]_2, [W_{tx}]_2^T\right) \\ . \\ . \\ . \\ K\left([g_{tx}]_n, [W_{tx}]_n^T\right) \end{bmatrix} vec(\mathbf{D})$$

where y is output time-domain transmitted signal.

This way of interpreting the transmit signal by linear transform allows us to implement known receivers like Zero forcing receiver, MMSE receiver and Matched filter receiver.

2.2 Frequency domain transmitters:

signal model

Given a set of complex data symbols $d_k[m]$ which correspond to points on constellation, where $k = 0, 1, ...K - 1$ and $m = 0, 1, ...M - 1$.Each subcarrier has its own pulse shaping filter which are time and frequency shifted

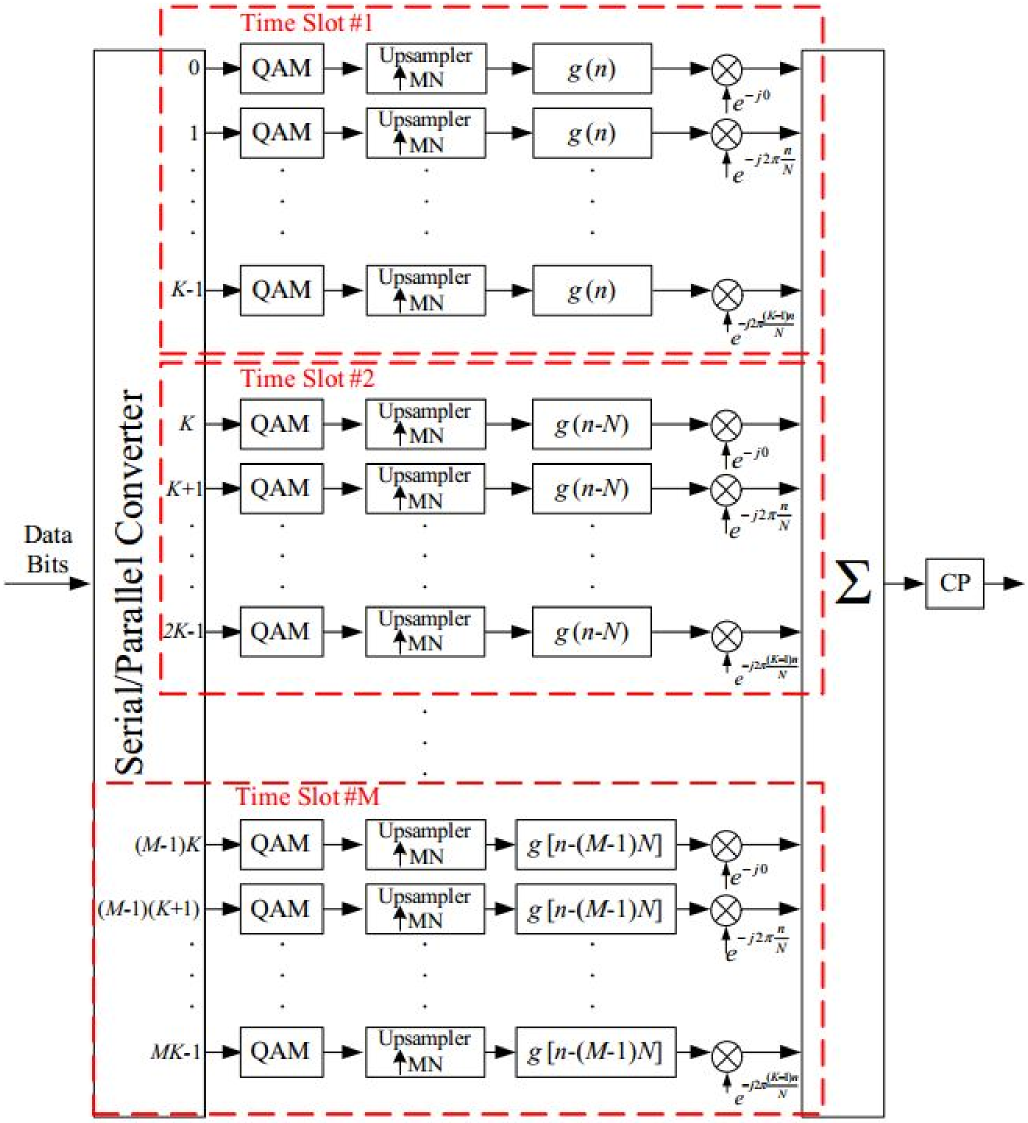

Figure 2.1: GFDM Transmitter Block

appropriately. To avoid aliasing due to filtering operation, each time slot is upsampled by N times, resulting in MN samples per subcarrier. All processing is done is baseband for each subcarrier, then is shifted to the required frequency and summed to obtain the transmitted vector.ie

$$\sum_{m=0}^{M-1}\sum_{k=0}^{K-1} d_k[m]g_{tx}[n-mN]e^{j2\pi\frac{kn}{N}}$$

Further, the data symbols ought to be upsampled by a a factor of N to satisfy the nyquist criterion before filtering. Using impulse train to perform upsampling the trasmitter equation becomes

$$x_k[n] = [[d_k[m]\delta(n-mN)] \star g_t x[n]]e^{j2\pi\frac{kn}{N}}$$

and

$$x_{tx}[n] = \sum_{k=0}^{K-1} x_k[n]$$

This requires computations of the order NKM^2. Hence all this process was converted into the frequency domain where one could leverage the properties of the fourier transform. Hence the expression is equivalent to

$$x_k[n] = IDFT_{MN}[DFT_{MN}(d_k[m]\delta(n-mN)).DFT_M N(g_{tx}[n]) \star DFT_M N(e^{j2\pi\frac{kn}{N}})]$$

But, not all of the above MN point transforms are needed. $DFT_{MN}(d_k[m]\delta(n-mN))$ can be interpreted as repetition of $DFT_M(d_k[m])$ N times in a sequence. Hence number of computations is reduced. Also the $DFT_{MN}(e^{j2\pi\frac{kn}{N}})$ is the impulse $\delta(f-\frac{k}{N})$. These computations are simplified when done with matrices.

Matrix model

Consider a $M\ X\ K$ matrix **D** containing the complex data symbols, where $d_k[m]$ corresponds to the kth column of **D**. First, an M point DFT is performed on each column, given by $X_D\ =\ W_M\mathbf{D}$. To perform upsampling in time domain, the samples have to repeated periodically in the frequency domain. Hence, upsampled signal is given by

$$X_G\ =\ RW_M\mathbf{D}$$

where R is the matrix $\{I_M, I_M, ..., I_M\}^T$ ie a concatenation of N identity matrices of size M. The pulse shaping filter is to be applied on this signal. In time domain, a circular convolution is done for filtering,but here, multiplication with a diagonal matrices containing elements of $W_M NG_{tx}$ on the principal diagonal would suffice.

$$X_F\ =\ LRW_M\mathbf{D}$$

This signal currently is in baseband and is to be translated to corresponding carrier frequency to be transmitted. Hence,

$$X_k\ =\ P^{(k)}LRW_M\mathbf{D}$$

where $P^{(k)} is \{0_{LM},\ 0_{LM},\ ..(k-1\ times)\ I_{LM},\ 0_{LM},\ 0_{LM}...\ (K-k\ times)\}$ and 0_{LM} is an all zero matrix of size LM and I_{LM} is an identity matrix of size LM. Final transmitted signal is given by the $IDFT_{MN}$ of this signal

$$x[n]\ =\ W_{MN}^H \sum_{k=0}^{K-1} P^{(k)}LRW_M\mathbf{D}$$

2.3 Which is better?

The transmitter computation is easier in frequency domain and hence is done this way. Assuming $Mlog_2(M)$ computations for an M point DFT, we require:

1. K times $Mlog_2(M)$ multiplications to convert given data symbols into frequency domain.
2. Pulse shaping requires MN complex multiplications per sub carrier, and hence KMN computations.
3. MN point inverse DFT requires $MNlog_2(MN)$ computations.

In total, GFDM complexity = $KMlog_2(M) \ + \ KMN \ + \ MNlog_2(MN)$ whereas, OFDM required only $MKlog_2(K)$. In most practical cases, N need not be of the order of K because the pulse shaping filter can be chosen such that it spans only a few symbols. For example, for raised cosine pulses, the pulse shaping filter even with worst case roll-off factor can only span 2 symbols in the frequency domain and hence N=2 would be sufficient.

CHAPTER 3

Receiver designs:

The GFDM signal is a linear transformation of the MK data symbols. Hence the usual methods of linear estimation can be employed here. The channel model is described below: Let y be the vector which contains the time samples $y[n]$, that are obtained at the receiver after low-noise amplification downmixing to baseband and analog-to-digital conversion. Further let n $N(0, \sigma_n^2)$ denote a noise vector containing AWGN samples with variance σ_n^2 . Assuming the analog processing is ideal, the received signal can be expressed as

$$y = Hx + n$$

, where H denotes the channel matrix whose dimensions are $N + N_{cp} + N_{ch} - 1$ cross $N + N_{cp}$, where N is signal length, N_{cp} is the length of cyclic prefix and N_{ch} is the length of channel delay spread. H has a toeplitz structure based on channel impulse response length N_{cp}. In this case, the length of the cyclic prefix is ensured to be longer than channel delay spread so that there is no interference among the GFDM frames. After removal of cyclic prefix at the receiver, the effective output signal is given by $y = Hx + n$, where H is an NXN matrix. After channel equalization, the received symbols are given by $z = H^{-1}HAd + H^{-1}w$. The transmitted symbols can be estimated by a linear estimator of the form $\hat{d} = Bz$. Different receiver designs estimate B in different ways.

3.0.1 Zero Forcing receiver:

This type of equaliser applies the inverse of channel response. Hence the zero forcing receiver is given by multiplying with Pseudo inverse of the transmission matrix.

3.0.2 Matched filter receiver:

Matched filter receiver can be seen as a combination of MK parallel correlators processing the received signal $r(n)$. Hence it can be implemented as a correlator receiver as shown in the figure below.
The symbol received in any subcarrier and at any time-slot is given by:

$$s_{\hat{k',m'}} = \sum_{n=0}^{MN-1} r(n)[g'_m(n) * p'_k(n)]$$

After multipath channel estimation and equalization, we can analytically find the decoded symbol.

$$s_{\hat{k',m'}} = s_{k',m'} + \sum_{m=0,m\neq m'}^{M-1} \left(s_{k',m} \sum_{n=0}^{NM-1} g_m(n) g^*_{m'}(n) \right) + \sum_{k=0,m\neq k'}^{K-1} \left(s_{k,m'} \sum_{n=0}^{NM-1} |g_{m'}(n)|^2 p_{k-k'}(n) \right)$$

$$+ \sum_{m=0,m\neq m'}^{M-1} \left(\sum_{k=0,m\neq k'}^{K-1} \left(s_{k,m} \sum_{n=0}^{NM-1} g_m(n) g^*_{m'}(n) p_{k-k'}(n) \right) \right)$$

$$where p_{k-k'}(n) = p_k(n) p^*_{k'}(n) = \exp^{-j2\pi n \frac{k-k'}{N}}$$

It has been assumed that the transmit pulse had unit energy. The first term is the actual symbol transmitted. The second term is the Inter Symbol interference. The third term corresponds to ICI caused by symbols from the same time slot. The fourth term corresponds to ICI caused by symbols from other time slots.

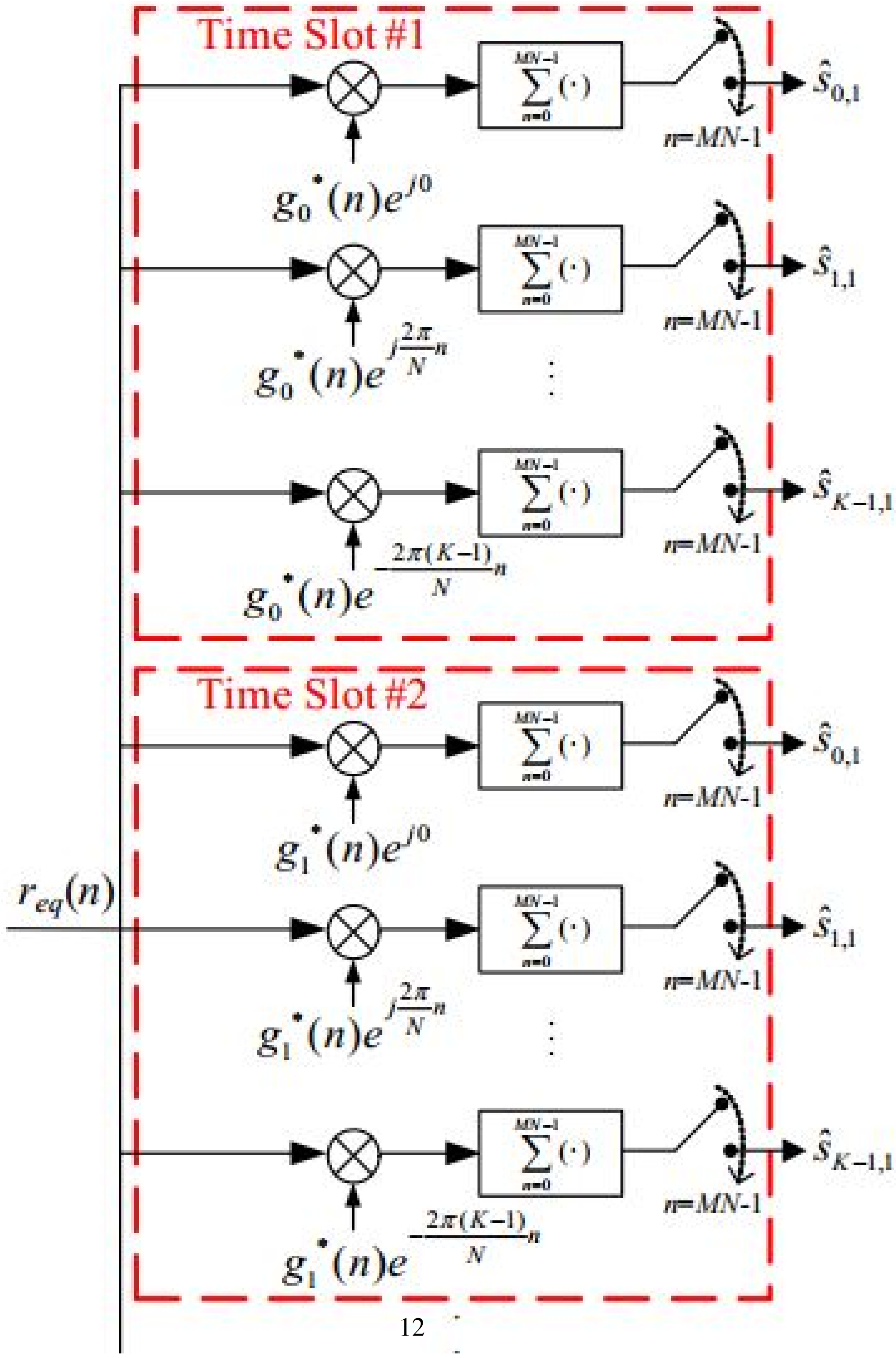

Figure 3.1: Block diagram of Matched filter receiver. There are MK correlator blocks.

Representing this in matrix form, it is same as multiplying the received samples by A^H, the hermitian of the transmission matrix.

$$\hat{d_{MF}} = A^H r(n)$$

The interference patterns can also be analysed by examining the matrix $A^H A$.

3.0.3 MMSE Receiver:

The MMSE receiver balances the noise enhancement and self interference properties of ZF and MFR receivers and churns out an optimal receiver. We know that received symbols can be written as $z = Ad + n$, where A is a combination of both channel matrix and the transmission matrix. Hence, both channel estimation and symbol estimation can be done in one shot. The linear MMSE estimator of the transmitted symbols is given by $\hat{d} = Bz$, and we know that the best linear estimator is given by $\hat{d} = R_z^{-1} R_{dz} Z$. Therefore,

$$\hat{d} = E\left[(Ad+n)(Ad+n)^H\right]^- 1 E\left[d(Ad+n)^H\right]$$

$$\hat{d} = \left(AA^H + R_n\right)^- 1 A^H$$

where R_n denotes the covariance matrix of noise.

CHAPTER 4

Comparison of Parameters:

GFDM uses both of these techniques in 2 different axes. A single data frame consists of data modulated in M different time slots and K different carrier frequencies. Each subcarrier is individually shaped with a filter typically RC or RRC. Another advantage of GFDM is its spectral efficiency. GFDM employs a cyclic prefix after every frame as the interference between the time slots is minimized using the pulse shaping filter, whereas OFDM uses a cyclic prefix for every time slot. The bit-rates obtained through GFDM and OFDM can therefore be compared as

$$\begin{aligned} Bit-rate_{OFDM} &= log_2(J)\tfrac{K}{T+T_{CP}} \\ Bit-rate_{GFDM} &= log_2(J)\tfrac{MK}{MT+T_{CP}} \end{aligned}$$

4.1 Out of band emissions:

GFDM allows for flexible design of transmit pulse. This section derives an expression for the power spectral density of a gfdm signal. Some techniques which are commonly used to reduce oob emissions is also mentioned. The power spectral density of the baseband signal is

$$P(f) = \lim_{T\to\inf}\left(\frac{1}{T}E(|FT(x_T(t))|^2)\right)$$

where $x_T(t)$ is the transmit signal truncated to $(-T/2, T/2)$.According to GFDM, $x_T(t)$ is the concatenation of multiple GFDM blocks.

$$x_T(t) = \sum_{v,m,k} d_{v,m,k} g_{0m}(t - vMT_s) \exp^{-j2\pi \frac{k}{T_s} t}$$

whose fourier transform is given by

$$X_T(f) = \sum_{v,m,k} d_{v,m,k} G_m \left(f - \frac{k}{T_s} \right) e^{-j2_s f}$$

where T_s is duration of one subsymbol, v ranges from $(-T/2MT_s, +T/2MT_s)$ and k, m are subcarrier and time slot indices respectively. Therefore, the PSD of GFDM signal is given by

$$P(f) = \frac{1}{MT_s} \sum_{k,m} \left| G_m \left(f - \frac{k}{T_s} \right) \right|^2$$

This value was integrated with $f \in OOB$ and $f \in B$, and the OOB emissions of OFDM and GFDM were compared and plotted below.

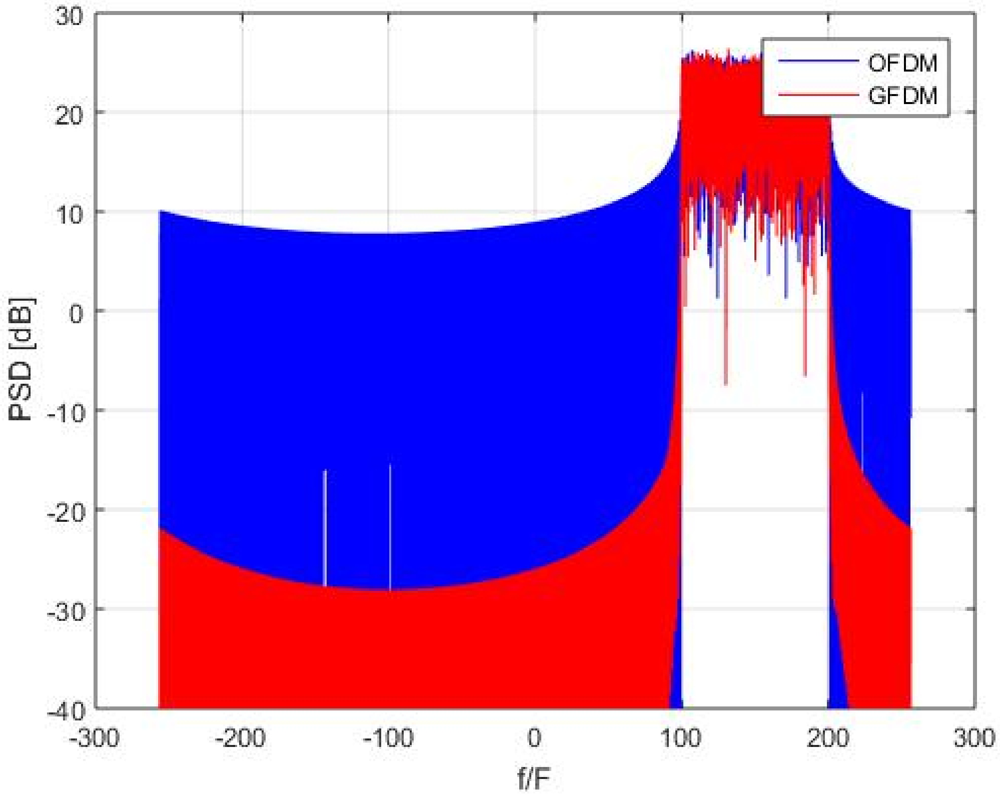

Figure 4.1: Out of band emissions for roll-off factor=0.5, M=5, K=128

4.2 Peak to average power ratio comparison:

High PAPR is one of the major drawbacks of OFDM systems. It drives the transmitter's power amplifier into its nonlinear region, thus causing nonlinear distortions. Only statistical properties of PAPR can be calculated. The plot below plots the probability that the PAPR is greater than a certain dB. The presence of GFDM curve below OFDM one shows that the probability of PAPR being greater than certain value is more for OFDM.

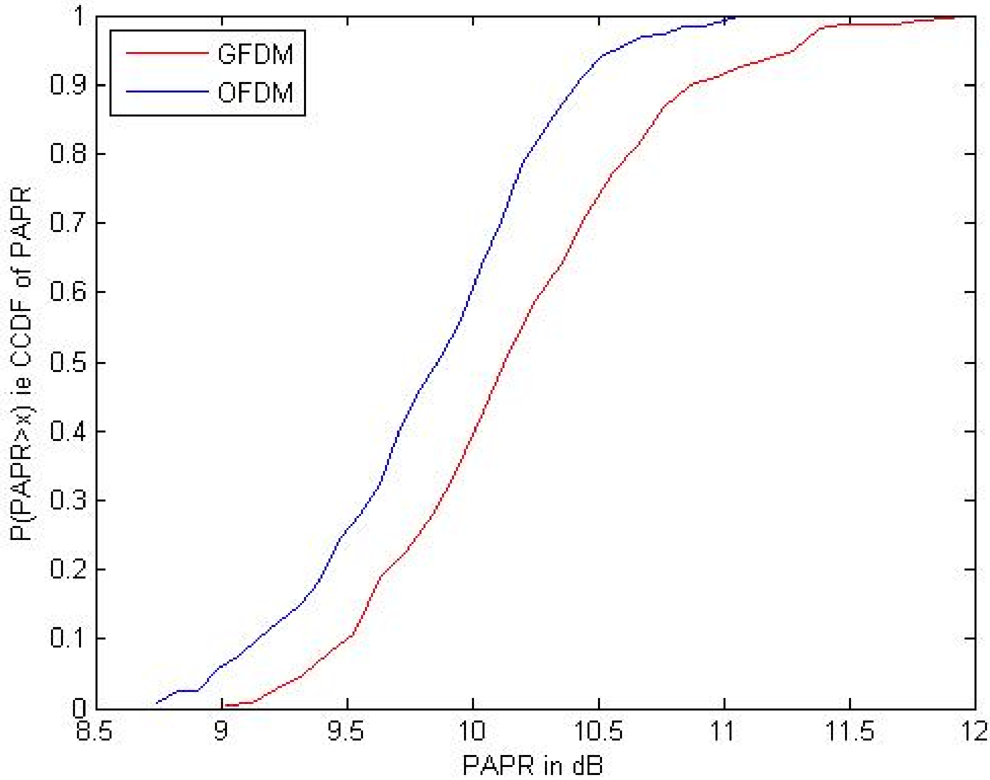

Figure 4.2: Peak to average power ratio plot for GFDM and OFDM

4.3 Spectral Efficiency:

Another advantage of GFDM is its spectral efficiency. GFDM employs a cyclic prefix after every frame as the interference between the time slots is minimized using the pulse shaping filter, whereas OFDM uses a cyclic prefix for every time slot. The bit-rates obtained through GFDM and OFDM can therefore be compared as

$$\begin{aligned} Bit - rate_{OFDM} &= log_2(J)\tfrac{K}{T+T_{CP}} \\ Bit - rate_{GFDM} &= log_2(J)\tfrac{MK}{MT+T_{CP}} \end{aligned}$$

One can use OFDM with MK subcarriers to ensure the same spectral efficiency, but that would limit the bandwidth available to each of the individual subcarriers. Hence, for the same spectral efficiency, GFDM offers a better bitrate compared to OFDM.

4.4 Symbol Error Rate:

GFDM is a very flexible scheme, which is heavily dependent on the parameters of the model used ie the pulse shape, the roll-off factor, the type of receiver used. The following plots summarize the results observed. The following table summarises the values of the parameters used:

Description	parameter	value
Total Bandwidth	B	20MHz
subcarrier bandwidth	B_{SC}	156.25KHz
Number of sub-carriers	K	128
Number of time-slots	M	5
Pulse shaping filter	g	raised cosine
roll-off factor	β	{0.5 , 0.9}
modulation order	μ	QPSK

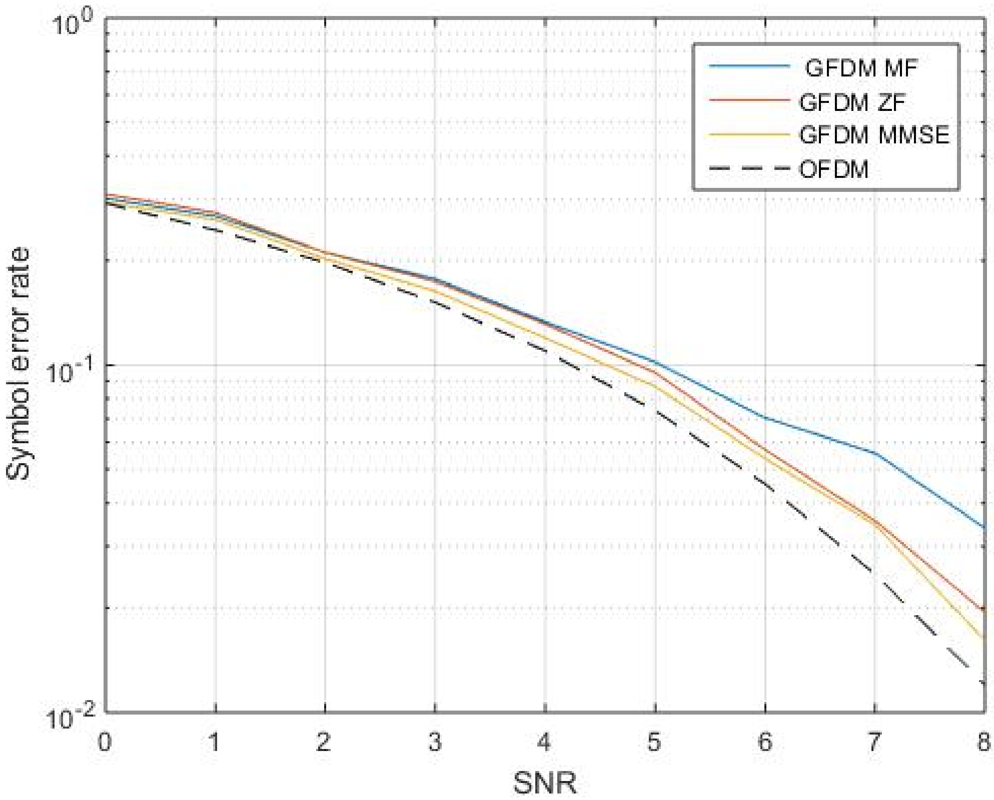

Figure 4.3: Symbol Error Rate of GFDM for rolloff factor=0.5

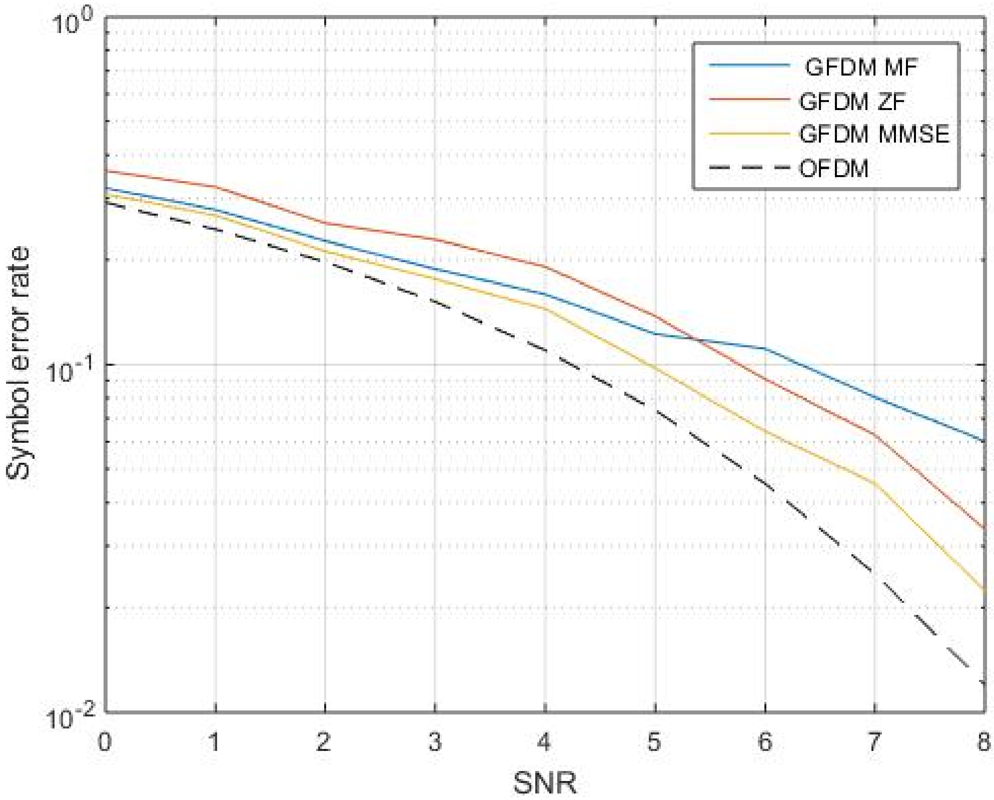

Figure 4.4: Symbol Error Rate of GFDM for rolloff factor=0.9

4.5 Conclusions

GFDM presents a more novel modulation method. It ensures a more stringent usage of bandwidth, and corrects issues like PAPR, OOB emissions. It is more spectrally efficient compared to existing OFDM systems, but all of it at the cost of a slight increase of Symbol error rate. It is a more flexible modulation scheme, as one can set M and K at will to prioritize latency or bandwidth.