On the Many-to-one and One-to-many Gaussian Interference Channels

A Project Report

submitted by

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in partial fulfilment of the requirements
for the award of the degree of

MASTER OF TECHNOLOGY

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY MADRAS.

MAY 2017

THESIS CERTIFICATE

This is to certify that the thesis titled **On the Many-to-one and One-to-many Gaussian Interference Channels**, submitted by **Abhiram Gnanasambandam**, to the Indian Institute of Technology, Madras, for the award of the degree of **Master of Technology**, is a bonafide record of the research work done by her under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ACKNOWLEDGEMENTS

I owe my sincere gratitude to all those people who are responsible for the successful completion of this project and because of whom my graduate experience has been one that I would cherish forever.

First and foremost, I offer my earnest gratitude to my guide, Prof. Srikrishna Bhashyam whose knowledge and dedication has inspired me to work efficiently on the project and I thank him for motivating me, and allowing me freedom and flexibility while working on the project. I would like to extend my gratitude to all the professors whose invaluable guidance and expertise has helped me figure out my area of interest.

My special thanks and deepest gratitude to Antony and Ragini for helping me out whenever I had a problem. They have enriched the project experience with their knowledge of the subject matter and invaluable suggestions. I thank all my lab-mates at ESB 215C whose acquaintance and support helped me in one way or other, throughout this learning process.

I thank my friends at IIT Madras for a great experience. I would like to thank my parents for their valuable support and encouragement which made my life wonderful and I dedicate this project to my family and friends.

ABSTRACT

KEYWORDS: Gaussian Interference channels(IC), Many-to-one IC, One-to-many IC, HK scheme, Nested Lattice codes.

In this work, we obtain new sum capacity results for the Gaussian many-to-one and one-to-many interference channels. *Simple Han-Kobayashi (HK) schemes*, i.e., HK schemes with Gaussian signaling, no time-sharing, and no common-private power splitting, achieve sum capacity under the channel conditions for which the new results are obtained. First, by careful Fourier-Motzkin elimination, we obtain the HK achievable rate region for the K-user Gaussian many-to-one and one-to-many channels in simplified form, i.e., only in terms of the K rates R_1, R_2, …, R_K. We also obtain the achievable sum rate using Fourier-Motzkin elimination. Then, to obtain sum capacity results, we derive genie-aided upper bounds that match the achievable sum rate of simple HK schemes under certain channel conditions. We then show how the already existing nested lattice code results for symmetric many-to-one ICs can be extended to asymmetric cases also. We also discuss a little about separability in multi-antenna case.

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ABBREVIATIONS

IC	Interference Channels
GIC	Gaussian Interference Channels
HK	Han and Kobayashi
GZIC	Gaussian Z Interference Channels
MAC	Multiple Access Channels
BC	Broadcast Channels

NOTATION

Bold face lower case letters	Vectors
Bold upper case letters	Matrices
$\mathcal{N}(\mu, \sigma)$	Gaussian distribution with Mean μ and variance σ^2
$(.)^T$	Transpose Operator
$I(a; b)$	Mutual information between a and b
E	Expectation Operator
p	Probabaility operator

CHAPTER 1

Introduction

In a multi-user wireless network where all the users share the same communication medium interference is common. The signal from a transmitter acts as an interference at all the receivers other than its intended receivers. The existence of interference affects the performance of the system greatly. The users should strategize in the best way to possible to achieve the best results. A lot of effort has been put into identifying strategies that achieve capacity in the presence of interference.

The K-user Interference channel (IC) is a special case which has K distinct transmit-receive pairs that interfere with each other. The capacity region or even the sum capacity are not known in general. In this thesis we studytwo special cases of K - user interference channels namely Many-to-one interference channels (Fig. 3.1) and One-to-many interference channels(Fig. 4.1). In a many-to-one interference channel the interference is seen only at one receiver from all the transmitters, while in a one-to-many interference channel, signal from a single transmitter interferes with all the other receivers.

1.1 Literature review

The capacity region or even the sum capacity are not known in general. The sum capacity of the Gaussian IC is known under some channel conditions Carleial (1975); Shang *et al.* (2008); Motahari and Khandani (2009); Shang *et al.* (2009); Annapureddy and Veeravalli (2009). In Carleial (1975), the capacity region and sum capacity for the 2-user IC were determined under strong interference conditions. In Shang *et al.* (2008); Motahari and Khandani (2009); Shang *et al.* (2009); Annapureddy and Veeravalli (2009), the sum capacity of the K-user Gaussian IC was obtained under *noisy* interference conditions. Under these conditions, Gaussian signaling and treating interference as noise at each receiver achieves sum capacity. In Motahari and Khandani (2009), the sum capacity of the 2-user Gaussian IC under mixed interference conditions was also obtained.

The many-to-one Gaussian IC and one-to-many Gaussian IC are special cases of the Gaussian IC where only one receiver experiences interference or only one transmitter causes interference. Even for these simpler topologies, exact capacity results are hard to obtain. The one-to-many IC and many-to-one IC were studied in Jovicic *et al.* (2010); Bresler *et al.* (2010); Annapureddy and Veeravalli (2009); Cadambe and Jafar (2009); Prasad *et al.* (2016, 2014). In Jovicic *et al.* (2010); Bresler *et al.* (2010), approximate capacity and degrees of freedom results are obtained for the many-to-one and one-to-many ICs. The sum capacity under *noisy* interference conditions is obtained for the many-to-one and one-to-many Gaussian ICs in Annapureddy and Veeravalli (2009); Cadambe and Jafar (2009). The same results can also be obtained as a special case of the result in Shang *et al.* (2008). Recently, for the many-to-one Gaussian IC, channel conditions under which Gaussian signalling and a combination of treating interference as noise and interference decoding is sum rate optimal were obtained in Prasad *et al.* (2016).

For the symmetric many-to-one IC, structured lattice codes were shown to achieve sum capacity under some strong interference conditions in Zhu and Gastpar (2015). Other special cases of the Gaussian IC, namely the cyclic IC and cascade IC were studied in Zhou and Yu (2013); Liu and Erkip (2011).

1.2 Overview

First we look at a motivating example that directed us towards this work in Chapter 2. Then we obtain new sum capacity results for Gaussian many-to-one and one-to-many ICs in chapters 3 and 4 . Then we look at how lattice coding may help us achieve sum-capacity in some region in chapter 5 for a many-to-one IC. And we discuss about seperability in a multi-antenna case and the scope for future work in chapter 6.

We consider the work done in chapters 3 and 4 to be our major contribution. First, by careful Fourier-Motzkin elimination, we obtain the Han-Kobayashi (HK) achievable rate region for the K-user Gaussian many-to-one and one-to-many channels in simplified form, i.e., only in terms of the K rates R_1, R_2, ..., R_K. Then, we focus on *simple* HK schemes with Gaussian signaling, no timesharing, and no common-private power splitting. We show that genie-aided sum capacity upper bounds *match* the achievable sum rates of simple HK schemes under some channel conditions. We also discuss how the genie-aided bounds used in this paper differ from the bounds in Nam (2015*b*,*a*) for the K-user many-to-one Gaussian IC. Overall, we obtain new sum capacity results for a larger subset of possible channel conditions than currently known in exisiting literature in Annapureddy and Veeravalli (2009); Cadambe and Jafar (2009); Prasad *et al.* (2016); Zhu and Gastpar (2015); Tuninetti (2011). In Annapureddy and Veeravalli (2009); Cadambe and Jafar (2009) only the case when all the interference is treated as noise was considered. In Zhu and Gastpar (2015), only the symmetric many-to-one IC was considered. In Tuninetti (2011); Prasad *et al.* (2016), only a successive decoding strategy was considered. Furthermore, the conditions under which sum capacity is achieved in Tuninetti (2011) are not obtained explicitly in terms of the channel parameters. We allow joint decoding of the desired and interfering signals as well and obtain conditions explicitly in terms of the channel parameters. In the simple HK schemes considered in our paper, either the interference from a particular transmitter is decoded fully or gets treated as noise. For the many-to-one case, we consider schemes where k out of K-1 interfering signals are decoded at receiver 1. For the one-to-many case, we consider schemes where k out of K-1 receivers decode the interfering signal.

CHAPTER 2

The motivating example - GZIC

In this chapter we look at the capacity results for the a Gaussian Z-interference channel, which will motivate us towards the results obtained in Chapters 3 and 4.

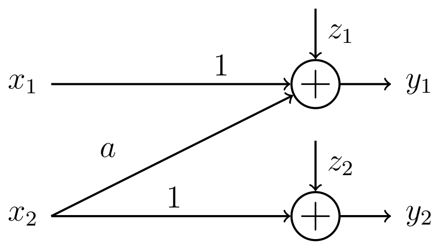

Figure 2.1: Standard form Many-to-one ZIC

2.1 Channel Model

The Z-IC is a special case of a two user IC, where interference happens only at once receiver. he channel model in standard form for the Gaussian Z-IC are shown in Fig. 2.1. The received signals in the Gaussian many-to-one IC in standard form are given by:

$$y_1 = x_1 + ax_2 + z_1 \tag{2.1}$$

$$y_2 = x_2 + z_2, \tag{2.2}$$

where x_i is transmitted from transmitter i, $z_i \sim \mathcal{N}(0, 1)$ for each i. The average power constraint at transmitter i is P_i. We take a look at the the sum capacity results available for the GZIC channel.

2.2 Sum Capacity results

Based on the results in Han and Kobayashi (1981), Sato (2006), Annapureddy and Veeravalli (2009) Shang *et al.* (2008), we give the following sum-capacity results for the GZIC. The sum-rate capacity of a GZIC is given by

$$\begin{cases} \frac{1}{2}\log(1+P_1)+\frac{1}{2}\log(1+P_2), & \text{if } a^2 \geq 1+P_1 \\ \frac{1}{2}\log(1+P_1+a^2P_2) & \text{if } 1 \leq a^2 < 1+P_1 \\ \frac{1}{2}\log(1+\frac{P_1}{1+a^2P_2})+\frac{1}{2}\log(1+P_2) & \text{if } a^2 < 1 \end{cases}$$

and is achieved by using Gaussian inputs, decoding and subtracting interference X_2 for $a^2 \geq 1+P_1$, jointly decoding X_1 and X_2 at Receiver 1 for $1 \leq a^2 < 1+P_1$, and treating X_2 as noise for $a^2 < 1$.

2.3 Comparison with the sum-capacity results available for Many-to-one and One-to-many ICs

For the many-to-one Gaussian IC, channel conditions under which Gaussian signalling and a combination of treating interference as noise and interference decoding is sum rate optimal were obtained in Prasad *et al.* (2016). In Tuninetti (2011), sum capacity was obtained for K-user Gaussian Z-like interference channels under some channel conditions. In both Prasad *et al.* (2016) and Tuninetti (2011), a *successive decoding* strategy where interference is decoded before decoding the desired signal is considered. They don't consider a case where the interfering signals get jointly decoded along with the required signal.Prasad *et al.* (2016) doesn't even consider jointly decoding a set of interfering signals. Similarly for the One-to-many ICs also don't have results that consider jointly decoding the interference and the required signal. In this work, we obtain some region where sum-capacity is achieved for both the one-to-many and Many-to-one interference channels if we also consider jointly decoding the interference and signal.

CHAPTER 3

Many-to-one IC

3.1 Channel Model in standard form

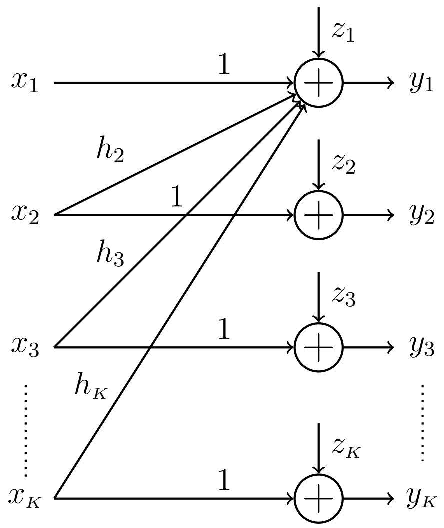

Figure 3.1: Standard form Many-to-one IC

Many-to-one IC is a special case of Gaussian IC where interference occurs at only one receiver. The channel model in standard form for the Gaussian many-to-one are shown in Fig. 3.1. The received signals in the Gaussian many-to-one IC in standard form are given by:

$$y_1 = x_1 + \sum_{j=2}^{K} h_i x_i + z_1 \tag{3.1}$$

$$y_i = x_i + z_i, i = 2, 3, ..., K, \tag{3.2}$$

where x_i is transmitted from transmitter i, $z_i \sim \mathcal{N}(0, 1)$ for each i. The average power constraint at transmitter i is P_i.

3.2 Achievable rate region for Han-Kobayashi (HK) scheme in simplified form

Let W_i be the message at transmitter i. For each $i = 2, 3, ..., K$, the message is split into two parts $W_i = \{W_{i0}, W_{i1}\}$, where W_{i0} is common message that gets decoded at receiver i and also at receiver 1, and W_{i1} is the private message that gets decoded only at receiver i. The HK achievable rate region in simplified form in the Theorem below is stated for the discrete memoryless channel, and can be readily extended to the Gaussian many-to-one IC with average power constraints using standard approaches Han and Kobayashi (1981); Gamal and Kim (2011).

Theorem 1. *For the discrete memoryless K-user many-to-one IC, the HK achievable rate region is given by the set of all $(R_1, R_2, \ldots, R_K)$ that satisfy:*

$$R_1 + \sum_{j\in\mathcal{N}} R_j \leq \sum_{j\in\mathcal{N}} I(X_j; Y_j|Q, U_j) + I(U_{\mathcal{N}} X_1; Y_1|U_{\mathcal{F}-\mathcal{N}}, Q), \forall\mathcal{N} \subseteq \mathcal{F} \quad (3.3)$$

$$R_i \leq I(X_i; Y_i|Q), i \in [2 : K] \quad (3.4)$$

where $U_{\mathcal{A}} = \{U_i, i \in \mathcal{A}\}$, $\mathcal{F} = \{2, 3, ..., K\}$ and $(Q, U_2, U_3, \ldots, U_K, X_1, X_2, \ldots X_K)$ is distributed as

$$p(q, u_2, \ldots, u_K, x_1, \ldots, x_K) = p(q)p(x_1|q)\prod_{i=2}^{K}(p(u_i|q)p(x_i|u_i, q). \quad (3.5)$$

Proof. Let S_i, $i = 2, 3, \ldots, K$, denote the rates for private messages W_{i1}, $i = 2, 3, \ldots, K$, respectively. Let T_i, $i = 2, 3, \ldots, K$, denote the rates for common messages W_{i0}, $i = 2, 3, \ldots, K$, respectively. Note that $R_i = S_i + T_i$, $i = 2, 3, \ldots, K$. Using standard analysis of HK schemes, we get the following achievable rate region in terms of $\{R_i\}$ and $\{T_i\}$:

$$R_i - T_i \leq I(X_i; Y_i|Q, U_i) \quad (3.6)$$

$$R_i \leq I(X_i; Y_i|Q), \quad (3.7)$$

for $i = 2, 3, \ldots, K$, and

$$R_1 + \sum_{i \in \mathcal{N}} T_i \leq I(U_{\mathcal{N}}, X_1; Y_1 | Q, U_{\mathcal{F}-\mathcal{N}}) \tag{3.8}$$

for all possible $\mathcal{N} \subseteq \mathcal{F}$ and $\mathcal{F} = \{2, 3, \ldots, K\}$. We also add the trivial constraints

$$T_i \geq 0, T_i \leq R_i. \tag{3.9}$$

The simplified rate region in (3.3) and (3.4) in terms of only the R_i's can be obtained using Fourier-Motzkin elimination. The main steps of the Fourier-Motzkin elimination are provided below.

We eliminate the variables in the following sequence: $T_2, T_3, \ldots, T_K$. After eliminating $T_2, T_3, \ldots, T_k$, the set of inequalities is given by:

$$R_1 + \sum_{i \in \mathcal{N}} R_i + \sum_{i \in \mathcal{S}} T_i \leq \sum_{i \in \mathcal{N}} I(X_j; Y_j | Q, U_j) + I(U_{\mathcal{N}}, U_{\mathcal{S}}, X_1; Y_1 | U_{\mathcal{F}-(\mathcal{S} \bigcup \mathcal{N})}, Q),$$
$$\forall \mathcal{N} \subseteq \{2, 3, \ldots, k\}, \mathcal{S} \subseteq \{k+1, \ldots, K\}.$$

For $k + 1 \leq i \leq K$

$$R_i - T_i \leq I(X_i; Y_i | Q, U_i),$$
$$T_i \geq 0, T_i \leq R_i.$$

For $2 \leq i \leq K$

$$R_i \leq I(X_i; Y_i | Q). \tag{3.10}$$

This can be proved by induction.

Setting $k = K$, we get the required inequalities in (3.3) and (3.4) after elimination of $T_2, T_3, \ldots, T_K$. □

Corollary 1. *The achievable sum rate S for a discrete memoryless many-to-one IC satisfies:*

$$S \leq \sum_{i \in \mathcal{N}} I(X_i; Y_i|Q, U_i) + \sum_{i \in \mathcal{F}-\mathcal{N}} I(X_i; Y_i|Q) + I(U_{\mathcal{N}} X_1; Y_1|U_{\mathcal{F}-\mathcal{N}}, Q), \forall \mathcal{N} \subseteq \mathcal{F}, \quad (3.11)$$

where $\mathcal{F} = \{2, 3, \ldots, K\}$.

Proof. First, we substitute $R_1 = S - \sum_{i=2}^{K} R_i$. Then, we eliminate the variables in the following sequence: $R_2, R_3, \ldots, R_K$. After eliminating $R_2, R_3, \ldots, R_k$, the set of inequalities is given by:

$$S - \sum_{i \in \mathcal{B}} R_i \leq \sum_{i \in (\mathcal{S}-\mathcal{B})} I(X_i; Y_i|Q, U_i) + \sum_{i \in \mathcal{N}} I(X_i; Y_i|Q, U_i)$$
$$\sum_{i \in \mathcal{M}-\mathcal{N}} I(X_i; Y_i|Q) + I(U_{\mathcal{S}-\mathcal{B}}, U_{\mathcal{N}}, X_1; Y_1|U_{\mathcal{M}-\mathcal{N}}, U_{\mathcal{B}}, Q),$$

$\forall \mathcal{B} \subseteq \mathcal{S}$ and $\mathcal{S} = \{k+1, \ldots, K\}$ and $\forall \mathcal{N} \subseteq \mathcal{M}$ and $\mathcal{M} = \{2, 3, \ldots, k\}$, and for $k+1 \leq i \leq K$

$$R_i \leq I(X_i; Y_i|Q). \quad (3.12)$$

This can be proved by induction.

Setting $k = K$, we get the required result in (3.11) after elimination of $R_2, R_3, \ldots, R_K$. □

Simple HK schemes: Consider HK schemes with Gaussian signaling, no timesharing, and no common-private power splitting, i.e., $X_i \sim \mathcal{N}(0, P_i), \ \forall \ 1 \leq i \leq K$, Q is constant, and $U_i = X_i, i \in \mathcal{B}$ and $U_i = \phi, i \notin \mathcal{B}$ for a fixed $\mathcal{B} \subseteq \{2, 3, \ldots, K\}$. The set $\mathcal{B}$ denotes the indices of the set of transmit messages decoded at receiver 1. For simple HK schemes, we get the following sum rate result directly from Corollary 1.

Corollary 2. *The achievable sum rate of a simple HK scheme over the Gaussian many-to-one IC satisfies:*

$$S \leq \frac{1}{2} \sum_{i \notin \mathcal{B}} \log(1 + P_i) + \frac{1}{2} \sum_{i \in \mathcal{M}} \log(1 + P_i) + \frac{1}{2} \log \left(1 + \frac{P_1 + \sum_{i \in \mathcal{B}-\mathcal{M}} h_i^2 P_i}{1 + \sum_{i \notin \mathcal{B}} h_i^2 P_i} \right), \forall \mathcal{M} \subseteq \mathcal{B} \quad (3.13)$$

for a fixed $\mathcal{B} \subseteq \{2, 3, \ldots, K\}$.

3.3 Sum capacity results

Consider the simple HK scheme with $\mathcal{B} = \{2, 3, \ldots, k\}$, i.e., interference from transmitters 2 to k are decoded at receiver 1. We choose successive indices 2 to k only for notational convenience, and the results can be generalized to any set of $k-1$ indices by just relabeling the transmitters. For this case, from (3.13), we have the following 2^{k-1} sum rate constraints:

$$S \leq \frac{1}{2}\sum_{i=k+1}^{K}\log(1+P_i)+\frac{1}{2}\sum_{i\in\mathcal{M}}\log(1+P_i)+\frac{1}{2}\log\left(1+\frac{P_1+\sum\limits_{i\in\mathcal{B}-\mathcal{M}}h_i^2P_i}{1+\sum\limits_{i=k+1}^{K}h_i^2P_i}\right), \forall\mathcal{M}\subseteq\mathcal{B}. \tag{3.14}$$

The least of these 2^{k-1} upper bounds will determine the maximum achievable sum rate for this simple HK scheme. We will now discuss two cases below where we can show that the simple HK scheme achieves sum capacity.

Case 1 ($\mathcal{MI}k_0$): Here we consider the case when the inequality corresponding to $\mathcal{M} = \mathcal{B}$ in (3.14) is the dominant inequality, i.e., its right hand side is the least.

Theorem 2. *For the K-user Gaussian many-to-one IC satisfying the following channel conditions:*

$$\prod_{i\in\mathcal{B}-\mathcal{N}}(1+P_i).(1+\sum_{j=k+1}^{K}h_j^2P_j+P_1) \leq 1+\sum_{i\notin\mathcal{N}}h_i^2P_i+P_1, \forall\mathcal{N}\subset\mathcal{B}, \mathcal{N}\neq\mathcal{B}, \tag{3.15}$$

$$\sum_{j=k+1}^{K}h_j^2 \leq 1, \tag{3.16}$$

where $\mathcal{B} = \{2, 3, \ldots, k\}$, $k \in \{1, 2, .., K\}$*, the sum capacity is given by*

$$S = \frac{1}{2}\log\left(1+\frac{P_1}{1+\sum_{j=k+1}^{K}h_j^2P_j}\right)+\sum_{i=2}^{K}\frac{1}{2}\log(1+P_i). \tag{3.17}$$

Proof. The converse or upper bound has already been proved in (Prasad *et al.*, 2016, Thm. 7) under the condition (3.16) using the genie-aided channel in Fig. 3.2. This sum rate can be achieved by the simple HK scheme if the inequality corresponding to the $\mathcal{M} = \mathcal{B}$ case is the dominant inequality in (3.14). This inequality is dominant if the conditions in (3.15) are satisfied. □

Remark 1. *The case of $k = 1$ is taken to be $\mathcal{B} = \phi$ resulting in condition (3.16) alone, thereby recovering the sum capacity result for treating all interference as noise in Annapureddy and Veeravalli (2009).*

Remark 2. *The achievability conditions in (3.15) are less stringent than the achievability conditions in Prasad* et al. *(2016) since joint decoding in the simple HK scheme is better than the successive interference cancellation decoding used in Prasad* et al. *(2016). This can be noted in Fig. 5.1 where the region obtained using this theorem includes an additional shaded region for the case $\mathcal{MI}3_0$ compared to the result in Prasad* et al. *(2016).*

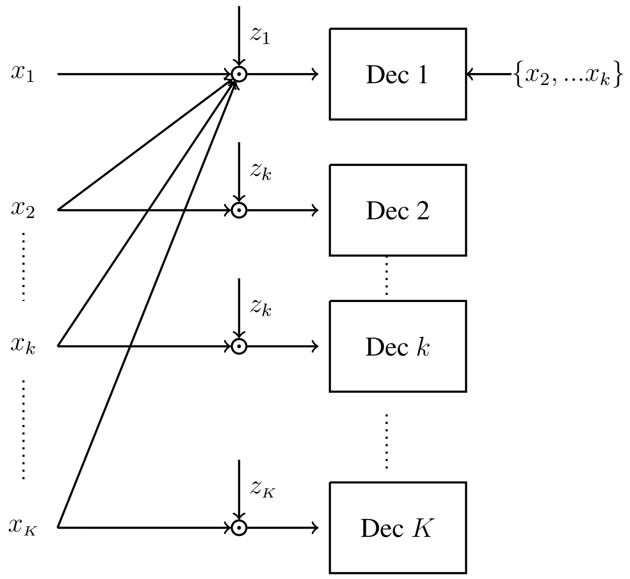

Figure 3.2: Side Information for $\mathcal{MI}k_0$

Case 2 ($\mathcal{MI}k_1$): Here we consider the case when the inequality corresponding to $\mathcal{M} = \mathcal{B}\backslash\{k\} = \{2, 3, \ldots, k-1\}$ in (3.14) is the dominant inequality.

Theorem 3. *For the K-user Gaussian many-to-one IC satisfying the following channel conditions:*

$$\prod_{i \notin \mathcal{B}-\mathcal{N}} (1+P_i)\left(1+P_1+\sum_{i=k+1}^{K} h_i^2 P_i + \sum_{i\in\mathcal{B}-\mathcal{N}} h_i^2 P_i\right) \geq \prod_{i=2, i\neq k}^{K} (1+P_i)(1+P_1+\sum_{j=k}^{K} h_j^2 P_j) \tag{3.18}$$

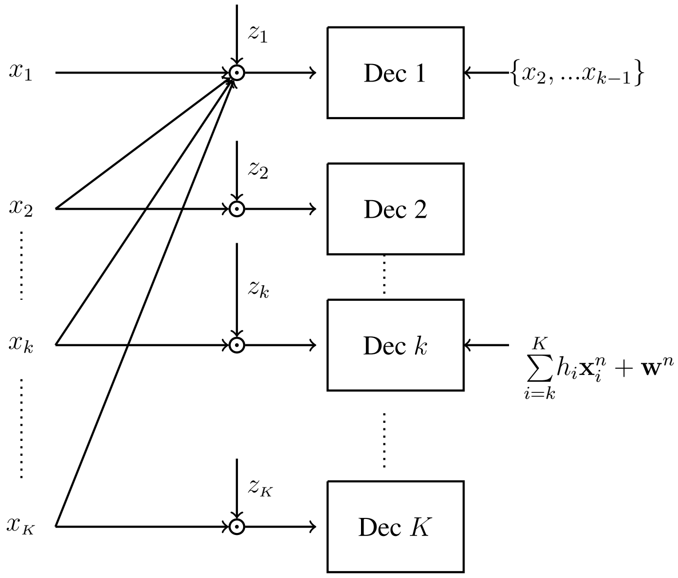

Figure 3.3: Side Information for $\mathcal{MI}k_1$

$$\forall \mathcal{N} \subseteq \mathcal{B}, \mathcal{N} \neq \{2,3,..,k-1\} \textit{ and } \mathcal{B} = \{2,3,...k\}$$
$$\sum_{i=k+1}^{K} h_i^2 \leq 1 - \rho^2, \quad \rho h_k = 1 + \sum_{i=k+1}^{K} h_i^2 P_i \tag{3.19}$$

the sum capacity is given by

$$S = \sum_{\substack{i=2 \\ i \neq k}}^{K} \frac{1}{2} \log(1 + P_i) + \frac{1}{2} \log \left(1 + \frac{P_1 + h_k^2 P_k}{1 + \sum_{i=k+1}^{K} h_i^2 P_i} \right).$$

Proof. The sum rate S in the theorem statement can be achieved by the simple HK scheme if the inequality corresponding to $\mathcal{M} = \mathcal{B} \backslash \{k\}$ is the dominant inequality in (3.13). This inequality is dominant if (3.18) is satisfied.

For the converse or upper bound, we consider the genie-aided channel in Fig. 3.3, where a genie provides the signal $\mathbf{s}_1^n = \{\mathbf{x}_2^n, \mathbf{x}_3^n, ..., \mathbf{x}_{k-1}^n\}$ to receiver 1 and the signal $\mathbf{s}_k^n = \sum_{i=k}^{K} h_i \mathbf{x}_i^n + \mathbf{w}^n$ to receiver k, where $\mathbf{w}^n$ is i.i.d. $\mathcal{N}(0,1)$, and w and z_k are jointly Gaussian with $E[wz_k] = \rho$. Now, we have

$$nS \leq I(\mathbf{x}_1^n; \mathbf{y}_1^n | \mathbf{s}_1^n) + \sum_{i=2, i \neq k}^{K} I(\mathbf{x}_i^n; \mathbf{y}_i^n) + I(\mathbf{x}_k^n; \mathbf{y}_k^n, \mathbf{s}_k^n)$$

$$
\begin{aligned}
&= h(\mathbf{y}_1^n|\mathbf{s}_1^n) - h(\mathbf{y}_1^n|\mathbf{s}_1^n,\mathbf{x}_1^n) + \sum_{i=2,i\neq k}^{K} (h(\mathbf{y}_i^n) - h(\mathbf{z}_i^n)) + h\left(\mathbf{s}_k^n\right) + h(\mathbf{y}_k^n|\mathbf{s}_k^n) - h(\mathbf{y}_k^n,\mathbf{s}_k^n|\mathbf{x}_k^n) \\
&\overset{(a)}{\leq} nh(y_{1G}|s_{1G}) - h(\mathbf{y}_1^n|\mathbf{s}_1^n,\mathbf{x}_1^n) + \sum_{i=2}^{k-1}(nh(y_{iG}) - nh(z_i)) \\
&+ \sum_{i=k+1}^{K} (h(\mathbf{y}_i^n) - nh(z_i)) + h\left(\mathbf{s}_k^n\right) + nh(y_{kG}|s_{kG}) - h\left(\sum_{i=k+1}^{K} h_i\mathbf{x}_i^n + \mathbf{w}^n|\mathbf{z}_k^n\right) - h(\mathbf{z}_k^n) \\
&\overset{(b)}{\leq} nh(y_{1G}|s_{1G}) + \sum_{i=2}^{k-1}(nh(y_{iG}) - nh(z_i)) + \sum_{i=k+1}^{K} (h(\mathbf{y}_i^n) - nh(z_i)) + nh(y_{kG}|s_{kG}) \\
&-h\left(\sum_{i=k+1}^{K} h_i\mathbf{x}_i^n + \mathbf{w}^n|\mathbf{z}_k^n\right) - nh(z_k) \\
&\overset{(c)}{\leq} nh(y_{1G}|s_{1G}) + \sum_{i=2}^{k-1}(nh(y_{iG}) - nh(z_i)) + \sum_{i=k+1}^{K} (nh(y_{iG}) - nh(z_i)) + nh(y_{kG}|s_{kG}) \\
&-nh(\sum_{i=k+1}^{K} h_i x_{iG} + w|z_k) - nh(z_k) \\
&\overset{(d)}{=} nI(x_{1G};y_{1G}|s_{1G}) + \sum_{i=2,i\neq k}^{K} nI(x_{iG};y_{iG}) + nI(x_{kG};s_{kG}) \\
&= nI(x_{1G},x_{kG};y_{1G}|s_{1G}) + \sum_{i=2,i\neq k}^{K} nI(x_{iG};y_{iG}),
\end{aligned}
$$

where $x_{iG} \sim \mathcal{N}(0, P_i)$, s_{iG} and y_{iG} represent the Gaussian side information and output that result when all the inputs are Gaussian as described in Annapureddy and Veeravalli (2009), (a) follows from the fact that Gaussian inputs maximize differential entropy and $h(\mathbf{y}_k^n,\mathbf{s}_k^n|\mathbf{x}_k^n) = h(\mathbf{z}_k^n) + h\left(\sum_{i=k+1}^{K} h_i\mathbf{x}_i^n + \mathbf{w}^n|\mathbf{z}_k^n\right)$, (b) follows from $h(\mathbf{y}_1^n|\mathbf{s}_1^n,\mathbf{x}_1^n) = h\left(\mathbf{s}_k^n\right)$, (c) follows from application of (Prasad *et al.*, 2016, Lemma 2) to $\sum_{i=k+1}^{K} h(\mathbf{y}_i^n) - h(\sum_{i=k+1}^{K} h_i\mathbf{x}_i^n + \mathbf{w}^n|\mathbf{z}_k^n)$ under (3.19), and (d) follows from the fact that $x_{kG} \to s_{kG} \to y_{kG}$ forms a Markov Chain (Annapureddy and Veeravalli, 2009, Lemma 8) for our choice of ρ in (3.19). □

In Tuninetti (2011), only a successive decoding strategy where the desired signal is always decoded after decoding the interfering signals, is considered. However, jointly decoding the interfering signal and the desired signal (Scheme $\mathcal{MI}k_1$) is required above to achieve capacity.

(Tuninetti, 2011, Theorem 2) identifies channel conditions where sum-capacity is achieved for a Z-like interference channel. By considering a appropriate H matrix for a many-to-one IC we can derive the channel constitions necessary for achieving sum-capacity for the $\mathcal{MI}1_0$ as shown in (Tuninetti, 2011, Example 1).

(Tuninetti, 2011, Theorem 3) extends the result to a general K-user IC by using a "successive decoding strategy". Even though the paper does not give the channel conditions explicitly, we can see that if we consider that only the receiver k receive the interference, we obtain the results we get for $\mathcal{MI}k_0$. The (Tuninetti, 2011, Theorem 2) gives the condition (3.16). And the (Tuninetti, 2011, Theorem 3) gives the condition (3.15). As the interference occurs only at receiver k, the constraints we get from (Tuninetti, 2011, Theorem 3) are nothing but the HK achievable region when $U_i = X_i$ for $i \in \{2, 3, \dots k\}$ and $U_i = \phi$ for $i \in \{k+1, \dots K\}$ in Theorem 1. This gives the constraints on sumrate given by (3.14) when $\mathcal{M} = \mathcal{B} = \{2, 3, ...k\}$. And we can easily see that the sum-rate capacity they mention also matches with our sum-rate capacity for $\mathcal{MI}k_0$.

The outer bound in (Nam, 2015*b*, Theorem 2) for the 3-user case matches our outer bound only for $\mathcal{MI}2_1$. Our K-user upper bounds are tighter than the K-user upper bounds in Nam (2015*a*) for the many-to-one setting. Furthermore, the genie signal used in Theorem 3 is different from the genie signals considered in Nam (2015*a*).

If we consider a many-to-one IC, we can see that according to (Nam, 2015*b*, Theorem), we get the upper bound on the sumrate as

$$R_1 + R_2 + R_3 \leq I(X_{1G}; Y_{1G}) + I(X_{2G}; Y_{2G}, S_{2G}) + I(X_{3G}; Y_{3G})$$

if $h_3^2 \leq \sigma^2_{V_{N_2}}$, where $V_{N_2} = (N_2|Z_2)$ and $E[Z_2 N_2] = \rho_{N_2}$, where $S_2 = h_2 X_2 + h_3 X_3 + N_2$. This upper bound matches with (*) in the proof of Theorem 3.

In Bresler *et al.* (2010), there is an example 3-user channel where the HK scheme does not achieve capacity, while a scheme based on interference alignment does. It can be verified that this 3-user example channel, when written in standard form, does not satisfy any of the conditions under which sum capacity is derived in this paper.

In (Bresler *et al.*, 2010, Section II.B), the authors consider a 3 user many-to-one IC with the channel conditions given below.

$$
\begin{aligned}
y_1 &= \beta\tilde{x}_1 + \beta\tilde{x}_2 + \beta\tilde{x}_3 + z_1 \\
y_2 &= \sqrt{\beta}\tilde{x}_2 + z_2 \\
y_3 &= \sqrt{\beta}\tilde{x}_3 + z_3,
\end{aligned}
$$

where $z_i \sim \mathcal{N}(0,1)$ for each $i \in \{1,2,3\}$. The average power constraint at transmitter i is $\tilde{P}_i$.

This can be converted to the standard form by considering

$$x_1 = \beta.\tilde{x}_1,\ x_2 = \sqrt{\beta}.\tilde{x}_2,\ x_2 = \sqrt{\beta}.\tilde{x}_2.$$

The equations in standard form will be

$$
\begin{aligned}
y_1 &= x_1 + \sqrt{\beta}x_2 + \sqrt{\beta}x_3 + z_1 \\
y_2 &= x_2 + z_2 \\
y_3 &= x_3 + z_3,
\end{aligned}
$$

with power constraints

$$P_1 = \beta^2.\tilde{P}_1,\ P_2 = \beta.\tilde{P}_2,\ P_3 = \beta.\tilde{P}_3.$$

We can see that $h_2 = h_3 = \sqrt{\beta}$. They consider a case where $\tilde{P}_1 = \tilde{P}_2 = \tilde{P}_3 = 1$ The authors prove that for $\beta \geq 2$, the capacity cannot be achieved by any HK-type scheme.

We can see that these channel conditions do not satisfy any of the conditions given in Table 3.1 or the conditions for $\mathcal{MI}1_0$ and $\mathcal{MI}2_0$ given in Prasad *et al.* (2016). We here explicitly see how it does not satisfy the conditions necessary for $\mathcal{MI}3_0$.

To satisfy the conditions for M̧I3_0, the following conditions must be satisfied.

$$
\begin{aligned}
&\beta \geq 1 + \beta^2 \\
&2\beta^2 \geq ((1+\beta)^2 - 1)(1+\beta^2)
\end{aligned}
$$

Strategy	Channel conditions
$\mathcal{MI}2_1$	(i) $h_2^2 \leq 1 + P_1 + h_3^2 P_3$, $h_3^2 \leq 1 - \left(\frac{1+h_3^2 P_3}{h_2}\right)^2, h_2^2 \geq 1$
	(ii) $h_3^2 \leq 1 + P_1 + h_2^2 P_2$, $h_2^2 \leq 1 - \left(\frac{1+h_2^2 P_2}{h_3}\right)^2, h_3^2 \geq 1$
$\mathcal{MI}3_0$	$h_2^2 \geq 1 + P_1, h_3^2 \geq 1 + P_1$ $h_2^2 P_2 + h_3^2 P_3 \geq ((1+P_2)(1+P_3) - 1)(1+P_1)$
$\mathcal{MI}3_1$	(i) $h_2^2 \geq 1 + P_1 + h_3^2 P_3, h_3^2 \leq 1 + P_1, h_3^2 \geq 1$, $\frac{1+P_3}{1+P_2} \geq \frac{1+P_1+h_3^2 P_3}{1+P_1+h_2^2 P_2}$
	((i) $h_3^2 \geq 1 + P_1 + h_2^2 P_2, h_2^2 \leq 1 + P_1, h_2^2 \geq 1$, $\frac{1+P_2}{1+P_3} \geq \frac{1+P_1+h_2^2 P_2}{1+P_1+h_3^2 P_3}$

Table 3.1: Channel conditions under which sum capacity is achieved using simple HK schemes in Theorems 2 and 3 for the 3-user Gaussian many-to-one IC. Conditions for $\mathcal{MI}1_0$ and $\mathcal{MI}2_0$ are already given in Prasad *et al.* (2016). These conditions are plotted in Fig. 5.1 for a given set of power constraints.

We can see that for $\beta \geq 2$ none of the conditions are satisfied.

The results in Theorems 2 and 3 for the Gaussian K-user many-to-one IC are now listed in Table 3.1 for the 3-user case.

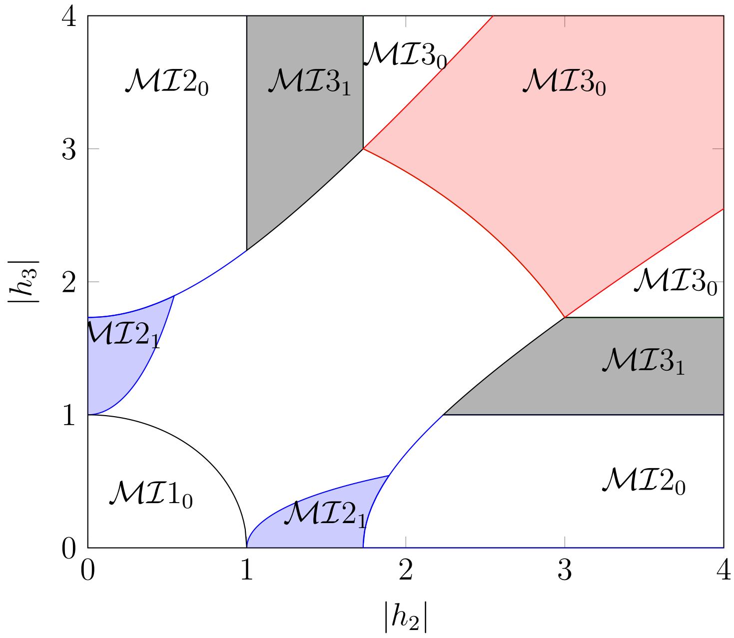

Figure 3.4: Channel conditions where sum capacity is obtained for the 3-user many-to-one IC using simple HK schemes, $P_1 = P_2 = P_3 = 2$.

CHAPTER 4

One-to-Many IC

4.1 Channel Model in Standard Form

The channel models (in standard form) for the Gaussian many-to-one and one-to-many ICs are shown in Fig. 4.1. The received signals in the Gaussian one-to-many IC in standard form are given by:

$$y_i = x_i + h_i x_K + z_i,\ i = 1, 2, 3, \cdots, K-1 \tag{4.1}$$

$$y_K = x_K + z_K. \tag{4.2}$$

where x_i is transmitted from transmitter i, $z_i \sim \mathcal{N}(0, 1)$ for each i. The average power constraint at transmitter i is P_i.

4.2 Achievable rate region for Han-Kobayashi (HK) scheme in simplified form

Let $\mathcal{I}$ denote the set of indices of the receivers at which interference is decoded, and $\mathcal{J}$ be the set of receivers at which interference is treated as noise, i.e., $\mathcal{J}$ = $\{1, 2, \cdots, K-1\} \backslash \mathcal{I}$. Let W_i be the message at transmitter i. The message W_K gets split into two parts $W_K = \{W_{K0}, W_{K1}\}$, where W_{K0} represents the common message that gets decoded at every receiver in $\mathcal{I}$ and W_{K1} is the private message that gets decoded only at receiver K.

Theorem 4. *For the discrete memoryless K-user one-to-many IC, the HK achievable rate*

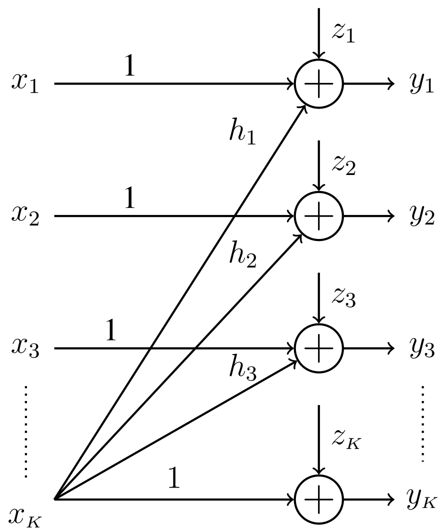

Figure 4.1: Standard form One-to-many IC

region is given by the set of all $(R_1, R_2, \ldots, R_K)$ *that satisfy*

$$\begin{aligned}
&R_i \leq I(X_i; Y_i|Q), i \in \mathcal{J} \\
&R_i \leq I(X_i; Y_i|Q, U), i \in \mathcal{I} \\
&R_i + R_K \leq I(X_i, U; Y_i|Q) + I(X_K; Y_K|Q, U), i \in \mathcal{I} \\
&R_K \leq I(X_K; Y_K|Q),
\end{aligned}$$

where $(Q, U, X_1, X_2, \ldots, X_K)$ *is distributed as*

$$p(q, u, x_1, x_2, ..x_K) = p(q) \prod_{i=1}^{K-1} p(x_i|q)p(u|q)p(x_K|u, q).$$

Proof. Let S denote the rate of the private message W_{K1} and T denote the rate of the common message W_{K0}. Note that $R_K = S + T$. Using standard analysis of HK schemes, we get the following achievable rate region in terms of $\{R_i\}$ and $\{T\}$:

$$\begin{aligned}
&R_i \leq I(X_i; Y_i|Q), i \in \mathcal{J} \\
&R_i \leq I(X_i; Y_i|U, Q), i \in \mathcal{I} \\
&R_i + T \leq I(X_i U; Y_i|Q), i \in \mathcal{I} \\
&R_K - T \leq I(X_K; Y_K|U, Q)
\end{aligned}$$

$$R_K \le I(X_K; Y_K|Q)$$

Using Fourier-Motzkin elimination to eliminate T, we get the rate region $\square$

Simple HK scheme: Let $X_i \sim \mathcal{N}(0, P_i),\ \ \forall\, 1 \le i \le K$, Q is constant, and $U = X_K$. From Theorem 4, we directly get the following result.

Corollary 3. *The achievable rate region for the simple HK scheme over the Gaussian one-to-many IC is given by:*

$$R_i \le \frac{1}{2}\ \log(1 + \frac{P_i}{1 + h_i^2 P_K}), i \in \mathcal{J}, \tag{4.3}$$

$$R_i \le \frac{1}{2}\ \log(1 + P_i), i \in \mathcal{I}, \tag{4.4}$$

$$R_i + R_K \le \frac{1}{2}\ \log(1 + P_i + h_i^2 P_K), i \in \mathcal{I}, \tag{4.5}$$

$$R_K \le \frac{1}{2}\ \log(1 + P_K). \tag{4.6}$$

Corollary 4. *The achievable sum rate S for the simple HK scheme over the Gaussian one-to-many IC when $\mathcal{J} = \phi$ satisfies*

$$S \le \sum_{j=1}^{K} \frac{1}{2}\ \log(1 + P_j), \tag{4.7}$$

$$S \le \sum_{\substack{j=1 \\ j \ne i}}^{K-1} \frac{1}{2}\ \log(1 + P_j) + \frac{1}{2}\ \log(1 + P_i + h_i^2 P_K),$$
$$\forall\, 1 \le i \le K - 1. \tag{4.8}$$

Proof. Given $\mathcal{J} = \phi$, we get the following rate constraints:

$$R_i \le \frac{1}{2}\ \log(1 + P_i), 1 \le i \le K - 1$$
$$R_i + R_K \le \frac{1}{2}\ \log(1 + P_i + h_i^2 P_K),\ 1 \le i \le K - 1$$
$$R_K \le \frac{1}{2}\ \log(1 + P_K).$$

First, we substitute $R_K = S - \sum_{i=1}^{K-1} R_i$. Then, we eliminate the variables in the following sequence: $R_1, R_2, \ldots, R_{K-1}$. After eliminating $R_1, R_2, \ldots, R_k$, the set of inequalities is given by:

$$R_i \leq \frac{1}{2}\log(1+P_i), k+1 \leq i \leq K-1$$
$$S - \sum_{j=k+1}^{K-1} R_j \leq \sum_{j=1}^{k}\frac{1}{2}\log(1+P_j) + \frac{1}{2}\log(1+P_K)$$
$$S - \sum_{j=k+1}^{K-1} R_j \leq \sum_{j=1,j\neq i}^{k}\frac{1}{2}\log(1+P_j) + \frac{1}{2}\log(1+P_i+h_i^2P_K), 1 \leq i \leq k$$
$$S - \sum_{j=k+1,j\neq i}^{K-1} R_j \leq \sum_{j=1}^{k}\frac{1}{2}\log(1+P_j) + \frac{1}{2}\log(1+P_i+h_i^2P_K),\ k+1 \leq i \leq K-1$$

This can be proved by induction.

Setting $k = K-1$, i.e., after elimination of $R_1, R_2, \ldots, R_{K-1}$, we get the required inequalities in (4.7) and (4.8). □

4.3 Sum capacity results

Consider the simple HK scheme where interference from transmitter K is decoded at k receivers. Without loss of generality, we can consider the set these k receivers to be $\mathcal{I} = \{1, 2, \ldots, k\}$ and $\mathcal{J} = \{k+1, k+2, \ldots, K-1\}$ (other choices can be easily handled by relabeling the receivers). We denote this scheme to be $\mathcal{OI}_k$.

Theorem 5. *For the K-user Gaussian one-to-many IC satisfying the following conditions:*

$$1 + P_i \leq |h_i|^2, 1 \leq i \leq k, \tag{4.9}$$
$$\sum_{j=k+1}^{K-1} \frac{|h_j|^2P_K + |h_j|^2}{|h_j|^2P_K + 1} \leq 1, \tag{4.10}$$

the sum capacity is given by

$$S = \frac{1}{2}\sum_{i=1}^{k}\log(1+P_i) + \frac{1}{2}\log(1+P_K) + +\frac{1}{2}\sum_{j=k+1}^{K-1}\log\left(1+\frac{P_j}{1+|h_j|^2P_K}\right). \tag{4.11}$$

Proof. For achievabililty, consider the achievable rate region in Corollary 3 for the simple HK

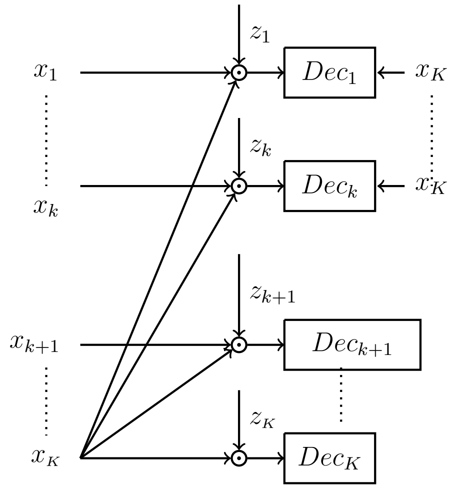

Figure 4.2: Side Information for $\mathcal{OI}_k$

scheme $\mathcal{OI}_k$. Under (4.9), constraint (4.5) is redundant. From the remaining constraints (4.3), (4.4), and (4.6), we get the achievable sum rate to be equal to the sum capacity in the theorem statement.

For the converse, consider the genie-aided channel in Fig. 4.2, where a genie provides x_K to receivers 1 to k. The first k receivers can now achieve the point-to-point channel capacities without any interference. The genie-aided channel can be considered to be a combination of these k point-to-point channels and a Gaussian one-to-many IC with users $k+1$ to K of the original channel. The sum capacity of the k point-to-point channels corresponds to the first term in the right hand side of (4.11). The sum capacity of the Gaussian one-to-many IC with users $k+1$ to K is upper bounded by the sum of the second and third terms in (4.11) under condition (4.10) (Annapureddy and Veeravalli, 2009, Thm. 5). Thus, we have the required sum capacity result.

□

Now, we consider the special case where $\mathcal{I} = \{1, 2, \ldots, K-1\}$ and $\mathcal{J} = \phi$, i.e., the interference gets decoded at all receivers. For this special case, we now have a sum capacity result for conditions not included in Theorem 5. We will denote this case $\mathcal{OI}_{K-1_1}$.

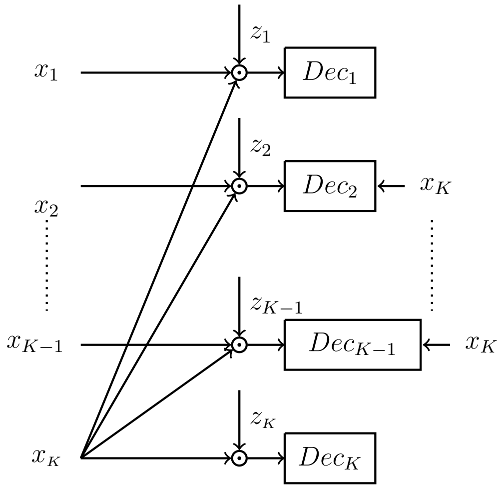

Figure 4.3: Side Information for $\mathcal{OI}_{K-1_1}$

Theorem 6. *For the K-user Gaussian one-to-many IC satisfying the following conditions:*

$$1 \le h_l^2 \le 1 + P_l \tag{4.12}$$

$$\frac{h_l^2}{1+P_l} \le \frac{h_i^2}{1+P_i}, 1 \le i \le K-1 \text{ and } i \ne l \tag{4.13}$$

for any $l \in \{1, 2, \ldots, K-1\}$, the sum capacity is

$$S = \frac{1}{2} \sum_{j=1, j\ne l}^{K-1} \log(1+P_j) + \frac{1}{2}\log(1+P_l+h_l^2 P_K). \tag{4.14}$$

Proof. For achievability, consider the achievable sum rate in corollary 4. The sum capacity in (4.14) is the right-hand side of the inequality corresponding to $i = l$ in Corollary 4. This inequality is the dominant inequality under conditions (4.12) and (4.13).

For the converse, consider the genie-aided channel (shown in Fig. 4.3 for $l = 1$), where a genie provides x_K to all receivers 1 to $K-1$ except receiver l. The genie-aided channel is a combination of $K-2$ point-to-point channels and a Gaussian one-sided IC with users l and K of the original channel. The sum capacity of the $K-2$ point-to-point channels corresponds to the first term in (4.14). The sum capacity of the Gaussian one-sided IC with users l and K is upper bounded by the second term in (4.14) under condition (4.12) (Sason, 2004, Thm. 2). Thus, we have the required result. □

The results in Theorems 5 and 6 for the Gaussian K-user one-to-many IC are now listed in

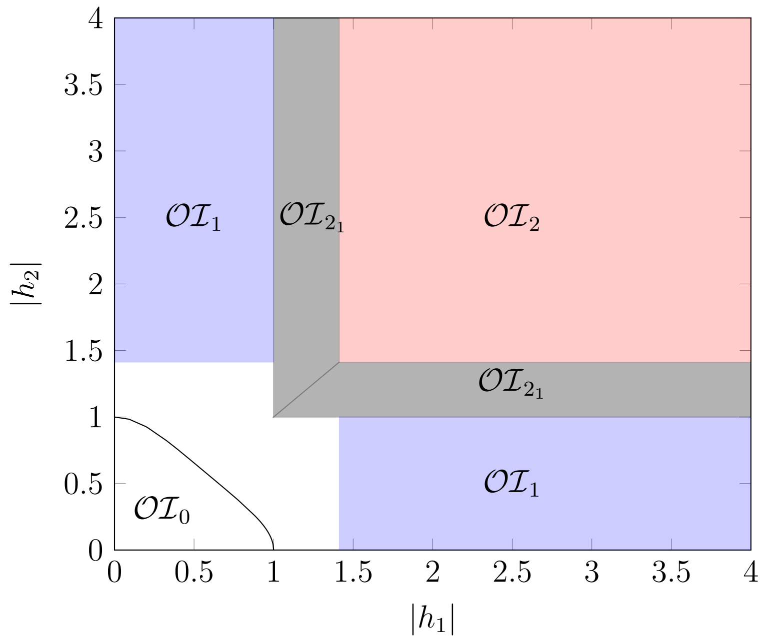

Figure 4.4: Channel conditions where sum capacity is obtained for the 3-user one-to-many IC using simple HK schemes, $P_1 = P_2 = P_3 = 1$.

Table 4.1 for the 3-user case.

Strategy	Channel conditions
OI_0	$\sum_{j=1}^{2} \frac{h_j^2 P_K + h_j^2}{h_j^2 P_K + 1} \leq 1$
OI_1	(i) $h_1^2 \geq 1 + P_1,\ h_2^2 \leq 1$
	(ii) $h_2^2 \geq 1 + P_1,\ h_1^2 \leq 1$
OI_2	$h_1^2 \geq 1 + P_1,\ h_2^2 \geq 1 + P_2$
OI_{2_1}	(i) $1 \leq h_2^2 \leq 1 + P_1, h_2^2 \geq \frac{1+P_2}{1+P_1} h_1^2$
	(ii) $1 \leq h_1^2 \leq 1 + P_2, h_1^2 \geq \frac{1+P_1}{1+P_2} h_2^2$

Table 4.1: Channel conditions under which sum capacity is achieved using simple HK schemes in Theorems 5 and 6 for the 3-user Gaussian one-to-many IC. These conditions are plotted in Fig. 5.1 for a given set of power constraints.

CHAPTER 5

Lattice Codes for Many-to-One Interference Channels

Zhu and Gastpar (2015) shows that a coding scheme based on the compute and forward approach on the lattice codes achieves capacity for a symmetric IC under some strong interference scheme. Here we extend the results to asymmetric channels. We first look at some of the results from Zhu and Gastpar (2015) which we will be using. We use the same model here that we used in Chapter 3.

5.1 Nested lattice codes(Zhu and Gastpar (2015))

For each user k we choose a lattice Λ_k which is good for AWGN channel. Let Λ_c denote the coarsest lattice. It has been shown that we can find another K simultaneously good nested lattice such that $\Lambda_k^s \subseteq \Lambda_c$ where

$$\sigma_k^2 = \sigma^2(\Lambda_k^s) = \beta_k^2 P,\ \beta_k > 0$$
$$x_k = \left(\frac{t_k}{\beta_k} + d_k\right) \mod \Lambda_k^s/\beta_k$$

where t_k is the lattice point corresponding to the message w_k that is to be sent.

For a given coefficient matrix $\mathbf{A}_{L\times K}$,

$$\mathbf{A} = \begin{pmatrix} a_1(1) & \dots & a_K(1) \\ \vdots & \vdots & \vdots \\ a_1(L) & \dots & a_K(L) \end{pmatrix}$$

where all entries are integers and there are L sets of coefficients.

$$a(l) = [a_1(l) \ ...\ a_k(l)]$$

. All the L integer sums can be reliably decoded at receiver 1 if

$$R_k \leq \min\ r_k(a_{l|1:l-1}, \underline{\beta}),\ a_k(l) \neq 0$$

$$r_k(a_{l|1:l-1}, \underline{\beta}) = \underset{\alpha_1'..\alpha_l \in R}{max}\ 1/2\ log^+(\frac{\sigma_k^2}{N_0(l)})$$

where $a_{l|1:l-1}$ means that when l^{th} sum is being decoded, we assume that all the previous $l-1$ sums are decoded.

$$N_0(l) = \alpha_l^2 + \sum_{k=2}^{K}(\alpha_l h_k - a_k(l)\beta_k - \sum_{j=1}^{l-1}\alpha_j a_k(j)\beta_k)^2 P_k\ +\ (\alpha_l - a_1(l)\beta_1 - \sum_{j=1}^{l-1}\alpha_j a_1(j)\beta_1)^2 P_1$$

5.2 Results

We take

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 & . & . & . & 1 \\ 1 & 0 & 0 & . & . & . & 0 \end{pmatrix}$$

and $\beta_k = h_k$. We try to minimise $N_0(l)$.

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 & . & . & . & 1 \end{pmatrix}$$

Zhu and Gastpar (2015) says that this A can be used to achieve capacity within a constant gap in a particular region if the channel is symmetric. We extend the result to non - symmetric channels too using the same A matrix.

$$N_0(1) = \alpha_1'^2 + \sum_{k=2}^{K}(\alpha_1' h_k - h_k)^2 P_k\ + \alpha_1'^2 P_1$$

We have to find the α_1' which maximises $N_0(1)$. So $\alpha_1' = \frac{\sum_{i=2}^{K} h_i^2 P_i}{1+P_1+\sum_{i=1}^{K} h_i^2 P_i}$

Therefore

$$N_0(1) = \frac{(1+P_1)(\sum\limits_{i=2}^{K} h_i^2 P_i)}{1+P_1+\sum\limits_{i=2}^{K} h_i^2 P_i}$$

$$\begin{aligned} r_k(a_1,\underline{\beta}) &= \frac{1}{2}\log^+(\frac{h_k^2 P_k(1+P_1+\sum\limits_{i=2}^{K} h_i^2 P_i)}{(1+P_1)(\sum\limits_{i=2}^{K} h_i^2)}) \\ &= \frac{1}{2}\log^+\left(\frac{h_k^2 P_k}{\sum_{i=2}^{K} h_i^2 P_i} + \frac{h_k^2 P_k}{1+P_1}\right) \\ &\geq \frac{1}{2}\log^+(\frac{h_k^2 P_k}{1+P_1}) \end{aligned}$$

Now

$$a_2 = (1\ 0\ 0\ ...\ 0)$$

.

$$N_0(2) = \alpha_2^2 + \sum_{k=2}^{K}(\alpha_2 h_k - \alpha_1 h_k)^2 P_k + (\alpha_2 - 1)^2 P_1$$

The α_1 and α_2, which maximises the value of $N_0(2)$ are $\alpha_1 = \alpha_2 = \frac{P_1}{1+P_1}$

Therefore we get

$$r_1(a_{2|1},\underline{\beta}) = 1/2\ log^+(1+P_1)$$

Since this is the only constraint on user 1, we can see that user 1 always achieves capacity.

Now for

$$h_k^2 \geq \frac{(1+P_k)(1+P_1)}{P_k}$$

every user achieves capacity. and for

$$h_k^2 \geq (1+P_1)$$

every user other than user 1 achieves rate within 0.5 bits of capacity.

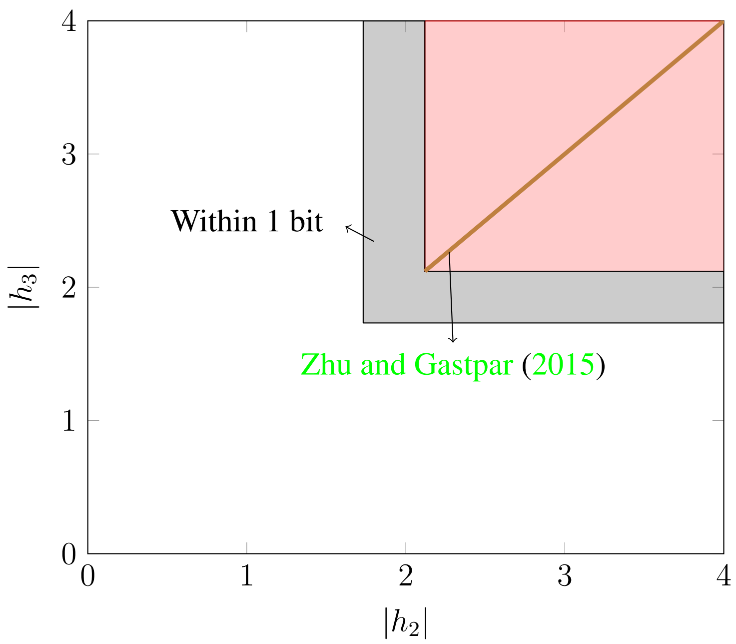

Figure 5.1: Channel conditions where sum capacity is obtained for the 3-user many-to-one IC using nested lattice codes, $P_1 = P_2 = P_3 = 2$.

CHAPTER 6

Multi antenna Channels

6.1 Separability

It is known that the capacity of parallel (e.g., multi- carrier) Gaussian point-to-point, multiple access and broadcast channels can be achieved by separate encoding for each sub-channel (carrier) subject to a power allocation across carriers.It has been shown that parallel interference channels are not separable, i.e., joint coding is needed to achieve capacity in general. Cadambe and Jafar (2010) studies the separability, from a sum-capacity perspective, of single hop Gaussian interference networks with independent messages and arbitrary number of transmitters and receivers. We provide the main results from the paper here.

6.1.1 MAC-Z-BC network

A network (S, D, E, M) (S = set of transmitters , D = Set of receivers , $E \subseteq D \times S$ = Set of edges in the network , $M \subseteq E$ = Message set,i.e. edges which carry messages) is called MAC-Z-BC if $M = E$ and E has the following properties:

$$\deg(T_i) > 1, i' \neq i \implies \deg(T_{i'}) = 1, \forall i', i \in S$$

$$\deg(R_j) > 1, j' \neq j \implies \deg(R_{j'}) = 1, \forall j', j \in D$$

where $\deg(T_i)$ is the degree of Transmitter i and $\deg(R_j)$ is the degree of Receiver j.

Given a network $N = (S, D, E, M)$, an instance of the network is uniquely identified by $(F, \bar{P}, \bar{\mathbf{H}})$ where F denotes the number of carriers, $\bar{P}$ denotes the power constraints and $\bar{\mathbf{H}}$ is a $DF \times SF$ dimensional channel gain matrix.

Theorem 7. *(Cadambe and Jafar, 2010, Theorem 1) A network N is separable if and only if it is the MAC-Z-BC network (or one of its sub-networks).*

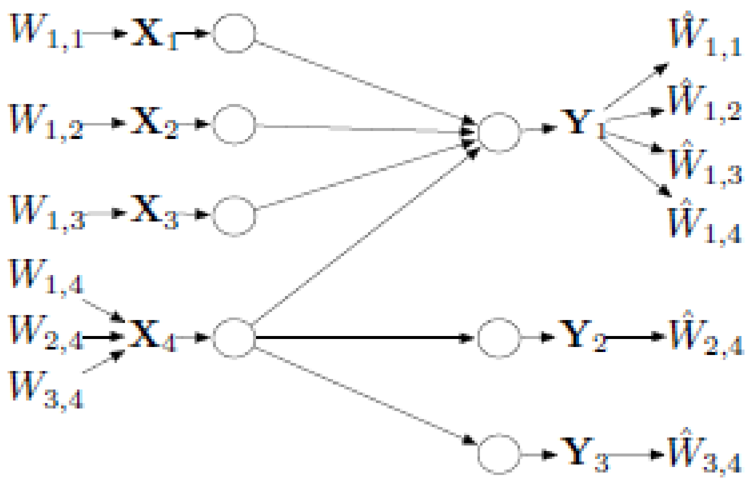

Figure 6.1: A MAC-Z-BC network with $S = 4$ transmitters and $D = 3$ receivers

Theorem 8. *(Cadambe and Jafar, 2010, Theorem 2) Consider a single-carrier MAC-Z-BC channel characterized by (1, $\bar{P}$,$\bar{H}$), where $\deg(T_I) > 1 < \deg(R_j) \implies j = 1, i = S$. Then its sum-capacity is*

$$C^{MAC\_Z\_BC}(1, \bar{P}, \bar{\boldsymbol{H}}) = \frac{1}{2}\log\left(1 + \sum_{j=1}^{S} |\boldsymbol{H}_{1,j}|^2 P_j\right) + \log\left(\frac{1 + |\boldsymbol{H}|^2 P_S}{1 + |\boldsymbol{H}_{1,S}|^2 P_S}\right), \quad (6.1)$$

where $H = \max_{j=1,2,...D} |\boldsymbol{H}_{j,S}|$

Let us consider X channel(XC). We obtain XC by allowing messages in all links of an IC. We can see that Many-to-one or One-to-many XC do not satisfy the conditions necessary for a MAC-Z-BC network, except for the 2 user case where both the Many-to-one and One-to-many XCs become a simple Z-network. So, we can say that except for the 2 user case, the multi-antenna Many-to-one and One-to-many XCs are not seperable. So, that means that Z-IC is also seperable, when regions where the strategy of decoding both the signal at the first receiver achieves capacity.

6.2 Future scope

The capacity region or region where sum-capacity is achieved for even a Multi-Antenna 2 user Gaussian Interference channel (GIC) does not have a closed form solution. We could extend the results for multi-antenna Many-to-one and One-to-many ICs as done in (Shang and Chen, 2013, Chapter 4).Cadambe and Jafar (2009) and Maddah-Ali *et al.* (2008) show that interference alignment is very useful for MIMO channels. So, we could try using interference alignment for the multi-antenna cases.

CHAPTER 7

Summary

We derived new sum capacity results for the K-user Gaussian many-to-one and one-to-many ICs. For both the many-to-one and one-to many IC, two new classes of channel conditions under which sum capacity is achieved were determined (cases $\mathcal{MI}k_0$,$\mathcal{MI}k_1$,$\mathcal{OI}_k$,$\mathcal{OI}_{K-1_1}$). In all these cases, simple HK schemes with Gaussian signaling, no time-sharing and no common-private power splitting achieve sum capacity. Then we we extended the results of Zhu and Gastpar (2015) from symmetric many-to-one IC to asymmetric many-to-one IC. We then looked at how a general many-to-one and one-to-many ICs are not separable.

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PAPER BASED ON THESIS

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