IMPLEMENTATION OF PROPORTIONAL FAIR SCHEDULING ALGORITHM USING SOFTWARE DEFINED RADIO

A Project Report

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THESIS CERTIFICATE

This is to certify that the thesis titled **IMPLEMENTATION OF PROPORTIONAL FAIR SCHEDULING ALGORITHM USING SOFTWARE DEFINED RADIO** , submitted by **P LAKSHMA NAIK** , to the Indian Institute of Technology, Madras, for the award of the degree of **Master of Technology**, is a bona fide record of the research work done by him under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

KEYWORDS: Software Defined Radio ; Universal Software Radio Peripheral; Orthogonal Frequency Division Multiplexing; Proportional Fair Scheduling

We consider implementation of Proportional Fair Scheduling (PFS) and Heuristic Scheduling Algorithms for the downlink of a cellular OFDM system using Software Defined Radio . As in opportunistic scheduling schemes, Scheduler will always allocates the channel resources to users who have good channel conditions .The overall Throughput of the system may be high but some users who are far away will get channel resources very less times and throughout of that users will also be very less. To overcome this we are implementing PFS for allocating channel resources fairly for all users in downlink using software defined radio.

Proportional fair is a compromise based scheduling algorithm. It's based upon maintaining a balance between two competing interests. Trying to maximize total [wired/wireless network] throughput while at the same time allowing all users at least a minimal level of service. This is done by assigning each data flow a data rate or a scheduling priority (depending on the implementation) that is inversely proportional to its anticipated resource consumption.

TABLE OF CONTENTS

LIST OF FIGURES

NOTATION

T_u	Time period of the OFDM symbol
N_c	No. of Sub carriers of OFDM symbol
N_g	Cyclic prefix length
P_{av}	Average power of OFDM symbol
α	Fairness parameter of scheduling
$U_i(.)$	i'th user utility function
c_i	i'th user quality of service parameter
$w_{i,t}$	time-varying weight assigned to the i'th user at time t tied to the QoS requirements of th
$W(i,k)$	Average throughput of user i up to time slot k
$e_{i,j}$	Received SNR of i'th user on j'th tone per unit power
$x_{i,j}$	Fraction of time j'th tone allocated to i'th user
$s_{i,j}$	a maximum SNR constraint on tone j for user i
$\jmath$	Jain's fairness index

CHAPTER 1

Introduction

We consider the implementation of Proportional Fair scheduling(PFS) algorithm for the downlink OFDM wireless system using SDR . We assume a single base station transmitting to a set of mobile users. At each time slot the base station can transmit to some of the mobile users. The purpose of the scheduler is to choose, at each time slot, the mobile users to which the base station should transmit.

A key feature of this problem is that in a wireless environment, the rate at which the base station can transmit to the different mobiles is both mobile-dependent and time-dependent. If a mobile is close to the base station then the energy required to transmit a bit to the mobile is small and hence the transmission can be at high rates(Andrews (2004)). In contrast, if the mobile is far from the base station then the energy required to transmit each bit is large and so only low transmission rates are possible. This accounts for the mobile-dependent nature of the transmission rates.

The time dependent nature of the transmission rates is due to mobility and fading. As a mobile moves away from the base station the feasible transmission rates decrease. Also, when the channel for a mobile enters a fade, the feasible transmission rates decrease. We shall sometimes refer to the rate at which the base station can transmit to a mobile as the channel condition between the base station and the mobile.

Motivated by the above properties of wireless networks, we require a scheduling algorithm for a slotted system with user dependent and time dependent channel conditions. We assume that the scheduler is aware of the channel conditions for each slot before it has to make the scheduling decision for that slot. In a wireless system this is feasible as long as the mobile users are continually measuring their channel condition and reporting this information back to the base station. A desirable feature of any scheduling algorithm is that it tends to serve a user when its channel is good. However, we cannot simply always serve the user with the best channel since then some other users may be starved.

In (D.Tse) it is shown that under certain conditions Proportional Fair maximizes the sum of the algorithms of the long-run average data rates provided to the users. The reason that this is a desirable objective is that we then achieve fairness in the sense that increasing some user's rate by a multiplicative factor has the same effect on the objective as multiplying another user's rate by the same multiplicative factor.

Most of the analysis of wireless schedulers, and in particular the analysis of proportional fair, assumes infinitely backlogged queues. That is, we assume that at all times all mobiles have data queued at the base station that needs to be transmitted to them.

In my project i am using PFS for OFDMA downlink scheduling. orthogonal frequency division multiple access (OFDMA) is an attractive multiple access technique for packet-based broadband wireless access. Radio resource allocation in OFDMA can exploit both multiuser diversity and frequency diversity to increase the system throughput. To achieve the throughput gain and guarantee fairness at the same time, we can utilize the proportional fair (PF) scheduler for OFDMA. There are some researches on the PF scheduler for OFDMA with the assumption that a base station (BS) has the perfect knowledge of every user's channel state information as per-sub band (or per-sub carrier) signal-to-noise-ratio (SNR). However, the assumption is not valid any more in practical systems.

Flow of thesis:

The thesis is organized as follows.
Chapter 2 discusses some of the basic concepts of OFDM and limitations of OFDM,Software Defined Radio.
Chapter 3 discusses about Concepts Proportional Fair Scheduling(PFS) and implementation of PFS and Heuristic Scheduling algorithm.
Chapter 4 discussed about results which are obtained using matlab and using USRP.
Finally chapter 5 gives the conclusion and future work.

CHAPTER 2

Basics and Background

2.1 OFDM

OFDM signal is a combination of several sub-carriers with very small sub-carrier spacing, in the order of KHz, equal to the smallest possible case $|\Delta f| = \pm\frac{1}{T}$. Multiplexing such hundreds of symbols (BPSK/QPSK/16-QAM) into different sub-carriers in the same time interval is possible. This Hyper-symbol is referred to as one OFDM symbol.

2.1.1 Modulation

Baseband equivalent of an OFDM signal is given by

$$x(t) = \sum_{k=0}^{N_C-1} a_k^{(m)} e^{j2\pi\Delta f t} \tag{2.1}$$

defined in the interval $mT_u < t \leq (m+1)T_u$. Where T_u is time period of the OFDM symbol, N_C is number of sub-carriers that are to be multiplexed, $a_k^{(m)}$ is set of the sub-symbols in m^{th} OFDM symbol and $|\Delta f| = \frac{1}{T_u}$. It is important to note that the information that is modulated from an OFDM signal is in frequency domain.

One of the biggest advantages of the OFDM signal is that it can be implemented easily by using IFFT module. Signal shown in the equation 2.1 can be constructed by the following discrete time signal and then giving it to an Digital to Analog converter.

$$x(n) = \sum_{k=0}^{N-1} a'_k e^{j2\pi kn/N} \tag{2.2}$$

Where

$$a'_k = \begin{cases} a_k, & 0 < k < N_c \\ 0, & N_c \leq k < N \end{cases} \tag{2.3}$$

and N is chosen as a nearest greater integer to N_c and also can be expressed in the form 2^k. Sampling rate of the system is given by $f_s = N\Delta f$. Note that superscript m, which indicates the symbol number, is left out for convenience.

Figure(2.1) shows a baseband transmitter structure for OFDM utilizing the Fourier transform for modulation. Here the serial data stream is mapped to complex data sym-

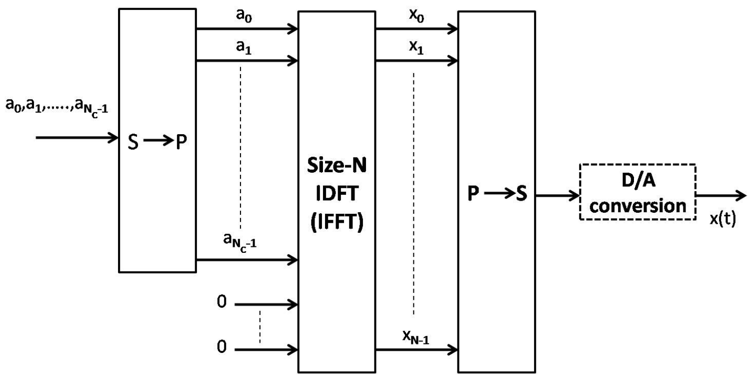

Figure 2.1: The functional structure of OFDM Modulator by means of IFFT processing

bols (PSK, QAM, etc) with a symbol rate of $\frac{1}{T}$. The data is then demultiplexed by a serial to parallel converter resulting in a block of N complex symbols, a_0 to $a_{N_c}-1$. The parallel samples are then passed through an N point IFFT (in this case no oversampling is assumed) with a rectangular window of length $N.\ T_s$, resulting in complex samples x_0 to x_{N-1}. Assuming the incoming complex data is random it follows that the IFFT is a set of N independent random complex sinusoids summed together. The samples, x_0 to x_{N-1} are then converted back into a serial data stream producing a baseband OFDM transmit symbol of length $T = N.T_s$.

A Cyclic Prefix (CP), which is a copy of the last part of the samples is appended to the front of the serial data stream before Radio Frequency (RF) up conversion and transmission. The CP combats the disrupting effects of the channel which introduce Inter Symbol Interference (ISI) and is discussed in more detail in section (2.4.2).

2.1.2 Demodulation

Figure(2.2) shows a baseband receiver structure for OFDM utilizing the Fourier transform for modulation.In the receiver the whole process is reversed to recover the transmitted data, the CP is removed prior to the serial to parallel converter. After serial to parallel conversion N point FFT is applied for reversing the effect of the IFFT. The complex symbols at the output of the FFT, $\widehat{a}_0, \widehat{a}_1, ..., \widehat{a}_{N_c-1}$ are then decoded and the original bit steam recovered.

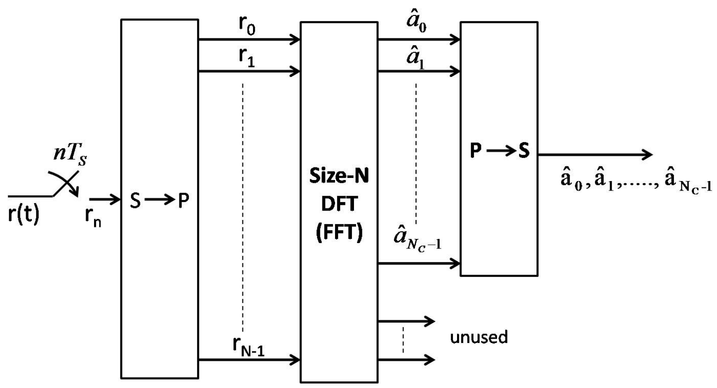

Figure 2.2: The functional structure of OFDM Demodulator by means of FFT processing

2.2 Orthogonality in OFDM

Orthogonal Frequency Division Multiplexing (OFDM) is a special case of multi carrier transmission which is highly attractive for implementation. In OFDM, the non-frequency selective narrow band sub channels into which the frequency-selective wide band channel is divided are overlapping but orthogonal. This avoids the need to separate the carriers by means of guard-bands, and therefore makes OFDM highly spectrally efficient. The spacing between the sub channels in OFDM is such they can be perfectly separated at the receiver. This allows for a low-complexity receiver implementation, which makes OFDM attractive for high-rate mobile data transmission such as the LTE downlink.

Two signals $x_1(t)$ and $x_2(t)$ are said to be orthogonal if their inner product, $< x_1(t), x_2(t) > \equiv \int_{-\infty}^{\infty} x_1(t)x_2^*(t)dt$, is zero. Using this definition one can prove that any two complex sinusoids e^{j2f_1t} and e^{j2f_2t} , which are defined for all time, are always orthogonal but for the trivial case $f_1 = f_2$. In contrast the orthogonality condition for two complex sinusoids that are defined for finite interval of time $T$and zero otherwise is more stricter than the former case. They are orthogonal only if $\Delta f = f_2 - f_1 = \pm\frac{n}{T}$, where n is an integer or $|\Delta f| \gg \frac{1}{T}$. This property of the complex sinusoids allows one to construct a signal which is an array of closely spaced and frequency overlapping sub-carriers and still maintain the orthogonality.

2.3 Use of a Cyclic Prefix

In order to protect successive OFDM symbols from multi path a CP of length N_g is used which is a copy of the last part of the samples of a OFDM transmit block appended to the front before transmission as depicted in Figure (2.3). The transmitted signal is therefore $N + N_g$ samples. Provided that the length of the CP is chosen so that it is longer than the longest expected delay path successive OFDM symbols will be free of ISI.

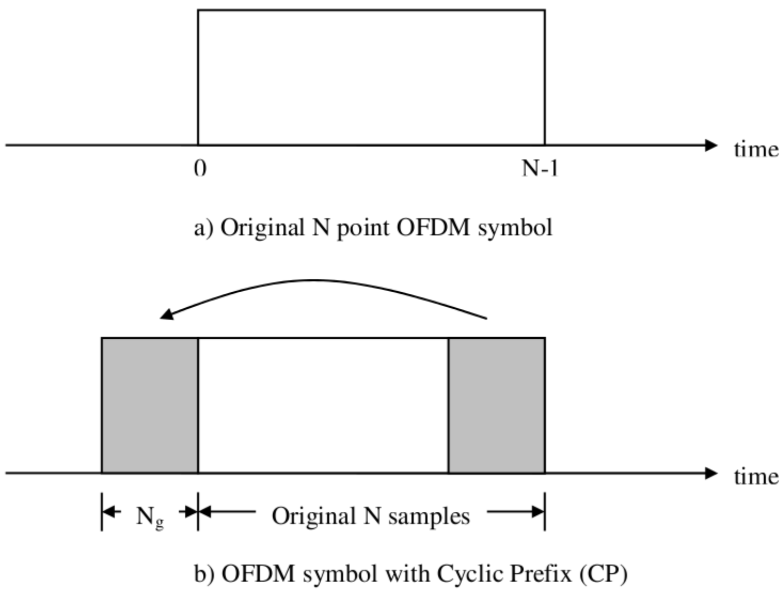

Figure 2.3: OFDM symbol a) without cyclic prefix, and b) with cyclic prefix.

At the receiver a window of N samples is chosen from the $N + Ng$ length block for maximum power, the rest of the repeated samples are discarded. After cyclic shifting to get the samples back into the original order a FFT is performed to demodulate the data.

Obviously the use of a CP decreases the data rate by a factor of

$$\frac{N}{N+N_g}$$

as the repeated samples are discarded in the receiver so it is important to keep the length of the CP as short as possible with respect to the rms delay spread. A loss in the SNR of the received signal is also incurred due to the lost energy in the CP. However there are techniques which use the CP for both frequency offset estimation and symbol synchronization. Also when filtering the signal there is a delay before the filter is at full power, by using a CP the delay will occur in CP so that N of the samples will be at full power. The CP (with repeated samples) retains the cyclic nature of the symbol by creating a periodic received signal for processing, eliminating ICI.

2.4 Limitations in OFDM

Previous sections have detailed the advantages of OFDM, however the advantages are offset by some problems that are unique to OFDM, namely time and frequency synchronization problems and non linearities.

2.4.1 Synchronization

Both time and frequency synchronization are a major drawback in OFDM, the following sections detail the problem and provide a basic introduction into solutions for these problems.

Timing errors

Timing synchronization is the process of finding the start of a symbol in the receiver. If the timing mismatch is within the CP the demodulation produces a linear phase rotation at the output of the FFT which can be corrected with a channel estimator.

$$\tilde{x}_b(t) = \sum_{k=0}^{N-1} \widehat{a}_{\delta t}(k) e^{j2\pi(k\Delta f t)}$$

Where

$$\widehat{a}_{\delta t}(k) = a(k)e^{j2\pi(f_c + k\Delta f)\delta t}$$

is the phase shift.

frequency errors

Frequency offset errors are caused by mismatch between the RF oscillators, Doppler shifts, and phase noise introduced by non linear channels. Frequency offset causes the received signal to not be sampled at the peak. Power from adjacent sub carriers is also sampled as well. OFDM is more sensitive to frequency offset than single carrier systems due to the tight orthogonal packing of the sub carriers. Suggested solutions to frequency synchronization (like symbol synchronization) based on pilot symbols and the cyclic prefix are used. where it is noted that time and frequency synchronization. The PAPR is the relation between the maximum power of a sample in a given OFDM transmit symbol divided by the average power of that OFDM symbol. Frequency sensitivity can be made more robust by reducing the number of sub carriers within a set bandwidth thereby increasing the frequency distance between sub carriers. However this shortens the symbol time which increases the demand on timing synchronization, therefore a trade off must be made.

2.4.2 Non linearities

Another limiting aspect of multi carrier and OFDM modulation is the high instantaneous signal peak with respect to the signals average power. Large peaks are due to the superposition of N random phase sine waves in the IFFT. Hardware components such as the Digital-to-Analog Converter (DAC), IFFT/FFT with limited word length and most importantly the High Power Amplifier (HPA) will be driven into saturation unless they are designed to operate over large dynamic ranges. If the signal is allowed to go into saturation both in band noise (which degrades the BER) and out of band radiation ICI will result.

2.4.3 Peak to Average Power Ratio

The PAPR is the relation between the maximum power of a sample in a given OFDM transmit symbol divided by the average power of that OFDM symbol. The mean envelope power of the baseband expression (assuming same constellation on each sub carrier) is defined as

$$P = \frac{1}{T}\int_{t=0}^{T} |x_m(t)|^2 dt = \frac{1}{N}\sum_{k=0}^{N-1} |X_{m,k}| \tag{2.4}$$

where $x_m(t)$ is output signal after IFFT operation, $X_{m,k}$ are assumed to be complex Quadrature Amplitude Modulated (QAM) data which are statistically independent, identically distributed (i.i.d) random variables with 0 mean and variance $\sigma^2 \triangleq E[|X_{m,k}|^2]$. The average power is defined as (2.5)

$$P_{av} = E[P] = E[|x_m(t)|^2] \tag{2.5}$$

The PAPR can then be defined

$$\zeta = \frac{\max_{0\leq t\leq T} |x_m(t)|^2}{P_{av}} \tag{2.6}$$

Where $\max_{0\leq t\leq T} |x_m(t)|^2$ is the maximum instantaneous power within the period of $0 \leq t \leq T$.

For the OFDM system of N-sub-channels, the worst case PAPR is N.

$$PAPR[x_m(t)] \leq N$$

2.5 SOFTWARE DEFINED RADIO

A software-defined radio system, or SDR, is a radio communication system where components that have been typically implemented in hardware (e.g. mixers, filters, amplifiers, modulators/demodulators, detectors, etc.) are instead implemented by means of software on a personal computer or embedded system . While the concept of SDR is not new, the rapidly evolving capabilities of digital electronics render practical many processes which used to be only theoretically possible.

Block Diagram of the SDR is given by the figure 2.4. A basic SDR system may consist

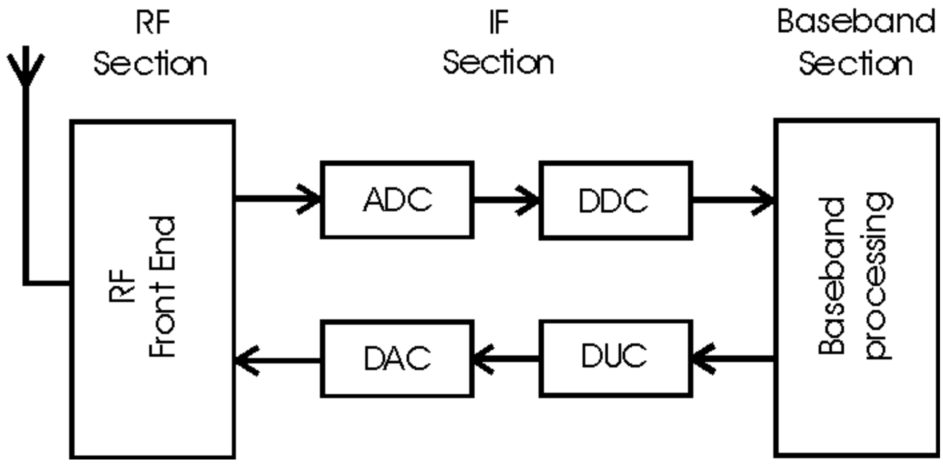

Figure 2.4: Block Diagram of SDR

of a personal computer equipped with a sound card, or other analog-to-digital converter, preceded by some form of RF front end. Significant amounts of signal processing are handed over to the general-purpose processor, rather than being done in special-purpose hardware. Such a design produces a radio which can receive and transmit widely different radio protocols (sometimes referred to as waveforms) based solely on the software used.

Software radios have significant utility for the military and cell phone services, both of which must serve a wide variety of changing radio protocols in real time.In the long term, software-defined radios are expected by proponents like the SDRForum (now The Wireless Innovation Forum) to become the dominant technology in radio communications. SDRs, along with software defined antennas are the enablers of the cognitive radio.

A software-defined radio can be flexible enough to avoid the "limited spectrum" assumptions of designers of previous kinds of radios, in one or more ways including spread spectrum and ultrawideband techniques allow several transmitters to transmit in the same place on the same frequency with very little interference, typically combined with one or more error detection and correction techniques to fix all the errors caused by that interference. software defined antennas adaptively "lock onto" a directional signal, so that receivers can better reject interference from other directions, allowing it to detect fainter transmissions.
cognitive radio techniques: each radio measures the spectrum in use and communicates that information to other cooperating radios, so that transmitters can avoid mutual interference by selecting unused frequencies. dynamic transmitter power adjustment, based on information communicated from the receivers, lowering transmit power to the minimum necessary, reducing the near-far problem and reducing interference to others. wireless mesh network where every added radio increases total capacity and reduces the power required at any one node. Each node only transmits loudly enough for the message to hop to the nearest node in that direction, reducing near-far problem and reducing interference to others.
An SDR (Software Defined Radio) system is a technique of how to use a universal front end to receive or transmit RF signal whose waveform is defined by software applications. An USRP (Universal Software Radio Peripheral) is a one such universal hardware front end used to build complete SDR platform.

2.5.1 USRP

USRP is a hardware front end, which can use to Translate RF signals to Baseband and vice versa.By interfacing it with host PC through Gigabit Ethernet/USB we can generate desire waveforms through uhd and transmit it over channel and to do reverse operation as well. The job of USRP is to get the complex baseband samples from host PC generated by signal processing software tools and translate it to analog RF signal. The Block diagram of the USRP is figure4.3

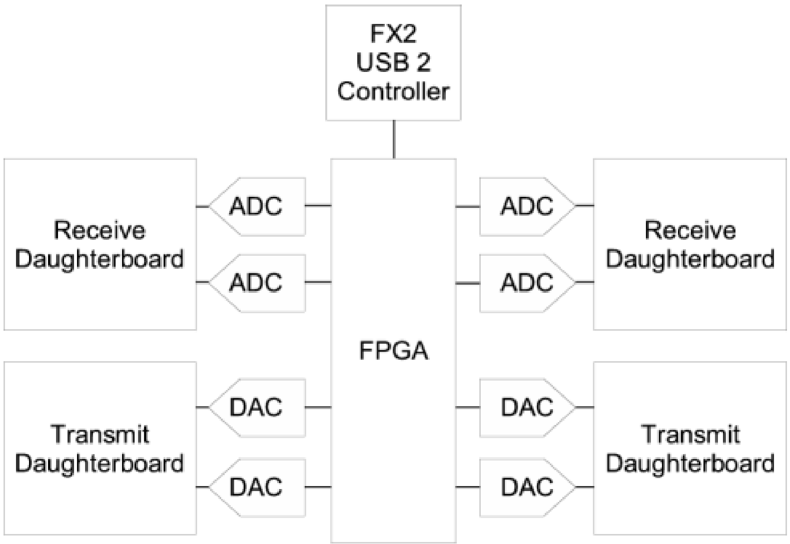

Figure 2.5: USRP Block Diagram

USRP mainly contains two parts .

1.Mother Board

2.Daughter Board

Mother Board: Mother board contains ADCs, DACs, FPGA for Processing of complex Baseband Signals. The USRP has 4 high-speed analog to digital converters (ADCs), each at 12 bits per sample, 64MSamples/sec. There are also 4 high-speed digital to analog converters (DACs), each at 14 bits per sample, 128MSamples/sec. These 4 input and 4 output channels are connected to an Altera Cyclone EP1C12 FPGA. The FPGA, in turn, connects to a USB2 interface chip, the Cypress FX2, and on to the computer. The USRP connects to the computer via a high speed USB2 interface only, and will not work with USB1.1.

Daughter Board: On the mother board there are four slots, where you can plug in up to 2 RX basic daughter boards and 2 TX basic daughter boards or 2 RFX boards. The daughter boards are used to hold the RF receiver interface or tuner and the RF transmitter. There are slots for 2 TX daughter boards, labeled TXA and TXB, and 2 corresponding RX daughter boards, RXA and RXB. Each daughter board slot has access to 2 of the 4 high-speed AD / DA converters (DAC outputs for TX, ADC inputs for RX).

2.5.2 UHD

The USRP Hardware Driver software (UHD) is the hardware driver for all USRP devices. It works on all major platforms (Linux, Windows, and Mac) and can be built with GCC, Clang compilers. The goal of the UHD software is to provide a host driver and API for current and future Ettus Research products.

CHAPTER 3

Proportional Fair Scheduling

3.1 Concept of Proportional Fair Scheduling

Proportional fair is a compromise-based scheduling algorithmHaider and Harris (2007). It's based upon maintaining a balance between two competing interests: Trying to maximize total [wired/wireless network] throughput while at the same time allowing all users at least a minimal level of service. This is done by assigning each data flow a data rate or a scheduling priority (depending on the implementation) that is inversely proportional to its anticipated resource consumption.

3.1.1 PFS for single carrier case

Let us suppose there are $\mathcal{K}$ users $\{i = 1, 2, ...K\}$ in the downlink system. In the k th time slot let the acheivable rates of users is given by r_i where i ranges from 1 to K and $W_i(k)$ are the average throughputs of users up to k time slot . Let $\bar{r}_{ei}(k)$ be the weighting factors of users used for Scheduling in k'th time slot. The average weighting factors for k+1 are given by the equation

$$\bar{r}_{ei}(k+1) = (1-\alpha)W_i(k) + \alpha r_i(k); \tag{3.1}$$

In each time slot j th user is selected using the formula given below.The PFS algorithm, as originally proposed in Jalali *et al.* (2000), selects the user using equation (3.2)

$$j = \arg \max_{1 \le i \le n} \frac{r_i^{c_i}(k)}{\bar{r}_{ei}(k)} \tag{3.2}$$

where $0 < \alpha < 1$ is a weighting factor, whose value may be chosen as 0.001. $c_i \ge 1$ is a constant.In order to further enhance the performance of (3.2), the exponent c_i has

been made adaptive (after every 50 TTIs) in Aniba and Aissa (2004) . However, it is mentioned that as $\alpha \to 0$, will provide equal air time to all mobile users. In such cases, the denominator in equation (3.2) is computed by a simple arithmetic mean of instantaneous rates , i.e. $\bar{r}_{ei}(k) = \sum_{n=1}^{k-1} \frac{r_i(n)}{k-1}$,Yang *et al.* (2006).
For $\alpha = 1$ case it will become maximum throughput scheduling in which scheduler just consider the present achievable data rates of the users.

To gain further insight into scheduling operations, we can consider any Scheduling Algorithms as Gradient based scheduling framework. A feasible rate is selected so that it has maximum projection onto gradient of the system utility function. The utility functions of RR, Max throughput and PFS algorithms can be computed by using equation(3.3) and are given in table 3.1 for comparison purposes, Hosein (2002). Here the utility function $U(\bar{W}_i(k))$ are appropriate functions of average throughput of user up to k th time slot

$$j = \arg \max_{1 \le i \le K} \left\{ r_i(k). \frac{dU_i(W_i(k))}{dW_i(k)} \right\} \tag{3.3}$$

where j is the index of the selected user to be allocated the channel.

Table 3.1: Utility functions for RR, Max Throughput and PFS

Scheduling algorithm	Utility function	$r_i(k).\frac{dU_i(W_i(k))}{dW_i(k)}$
RR	1	0
MAX Throughput	$W_i(k)$	$r_i(k)$
PFS	$\log W_i(k)$	$\frac{r_i(k)}{W_i(k)}$

3.1.2 PFS for OFDM downlink system

We can extend PFS scheduling for OFDM case ,Huang *et al.* (2009). Let us consider The downlink OFDM system case . In OFDM there will be multi carriers which have to be allocated among the users. consider downlink transmissions in an OFDM cell from a base station to a set $\mathcal{K} = \{1, ..., K\}$ of mobile users.

In each time-slot, the scheduling and resource allocation decision can be viewed as selecting a rate vector $\mathbf{r}_t = (r_{1,t}, ..., r_{K,t})$ from the current feasible rate region

$R(e_t) \subseteq R_K^+$, where e_t indicates the time-varying channel state information available at the scheduler at time t. Following the gradient-based scheduling framework, an $r_t \in R(e_t)$ is selected that has the maximum projection onto the gradient of a system utility function $U(\mathbf{W}_t) = \sum_1^K U_i(W_{i,t})$ where $U_i(W_{i,t})$ is an increasing concave utility function of user i's average throughput $W_{i,t}$ up to time t. In other words, the scheduling and resource allocation decision is the solution to

$$\max_{r_t \in R(e_t)} \bigtriangledown U(\mathbf{W}_t)^T.\mathbf{r}_t = \max_{r_t \in R(e_t)} \sum_i U_i^{'}(W_{i,t}) r_{i,t}, \tag{3.4}$$

Where $U_i^{'}(.)$ is the derivative of $U_i(.)$. For example one class of Utility function if given by

$$U_i(W_{i,t}) = \begin{cases} \frac{c_i}{\alpha}(W_{i,t})^{\alpha} & \alpha \leq 1, \alpha \neq 0, \\ c_i log(W_{i,t}) & \alpha = 0. \end{cases} \tag{3.5}$$

where $\alpha \leq 1$ is a fairness parameter and c_i is a QOS(quality of service) weight. With equal class weights, $\alpha = 1$ results in the scheduling rule that maximizes the sum-rate during each slot; $\alpha = 0$ results in the proportionally fair rule.
In general, we consider the problem of

$$\max_{r_t \in R(e_t)} \sum_i w_{i,t} r_{i,t}, \tag{3.6}$$

where $w_{i,t}$ is called weight of user i which is calculated using the derivative of Utility function of user i up to time t. $w_{i,t}$ is a time-varying weight assigned to the i'th user at time t tied to the QOS requirements of the user , We note that (3.6) must be re solved at each scheduling instance because of changes in both the channel state and the weights (e.g., the gradients of the utilities). While the former changes are due to the time varying nature of wireless channels, the latter changes are due to new arrivals and past service decisions.

Rate regions:

Let $\mathcal{N} = \{1, ..., N\}$ denote the set of tones. For each $j \in N$ and user $i \in K$, let e_{ij} be the received signal-to-noise ratio (SNR) per unit power. We denote the power allocated to user i on tone j by p_{ij} and the fraction of that tone allocated to user i by x_{ij} . The total power allocation must satisfy $\sum_{i,j} p_{ij} \leq P$, and the total allocation for each tone j must satisfy $\sum_i x_{ij} \leq 1$. For a given allocation, with perfect channel estimation, user i's feasible rate on tone j is

$$r_{ij} = x_{ij} Blog(1 + \frac{p_{ij} e_{ij}}{x_{ij}}), \tag{3.7}$$

which corresponds to the shannon capacity of a gaussian noise channel with bandwidth $x_{ij}B$ and received SNR $\frac{p_{ij}e_{ij}}{x_{ij}}$. This SNR arises from viewing p_{ij} as the energy per time-slot user i uses on tone j; the corresponding transmission power becomes $p_{ij}x_{ij}$ when only a fraction part x_{ij} of the tone is allocated.

Rate region can be defined as

$$\textit{R(e)} = \Big\{ \mathbf{r} : r_i = \sum_j x_{ij} Blog\big(1 + \frac{p_{ij} e_{ij}}{x_{ij}}\big), \tag{3.8}$$

$$\sum_{i,j} p_{i,j} \leq P, \sum_i x_{ij} \leq 1 \forall j, \mathbf{x,p} \in \chi \Big\}$$

where $\chi := \prod_{j=1}^{N} \chi_j$, and for all j∈ N,

$$\chi_j := \Big\{ (x^j, p^j) \geq 0 : x_{ij} \leq 1, p_{ij} \leq \frac{x_{ij}\tilde{s}_{ij}}{e_{ij}} \forall i \Big\}, \tag{3.9}$$

where $\mathbf{x}^j := (x_{ij}, \forall i \in \mathcal{K})$ and $\mathbf{p}^j := (p_{ij}, \forall i \in \mathcal{K})$ Here $\tilde{s}_{ij} = \frac{Z_{ij}}{1 - Z_{ij}\beta}$, where $Z_{ij} \leq 1/\beta$ is a maximum SNR constraint on tone j for user i and β is channel estimation factor.

3.2 Sub channelization

With many tones and users, providing pilots and/or feedback per tone can require excessive overhead; e.g., in IEEE 802.16e , a channel with bandwidth 1.25Mhz to 20Mhz is divided from 128 to 2048 tones. One way to reduce this overhead is for feedback and resource allocation to be done at the granularity of sub channels of disjoint sets of tones, i.e., constant power is used and coding is done across the tones in the same sub channel.

Our model can be adapted to this setting by viewing *N* as the set of sub channels and e_{ij} as the effective SNR per unit power for user i on the jth sub channel. Specifically, assuming that k tones are bundled into one sub channel.
Basically three types of sub channelizations are there.They can be characterized as (i) adjacent channelization, where adjacent tones are grouped together as in the optional "band AMC mode" in IEEE 802.16d/e ; (ii) interleaved channelization, where tones are (perfectly) interleaved as in the interleaved channelization in IEEE 802.16d/e ; and (iii) random channelization, where tones are randomly assigned as in systems that employ frequency hopping as in the Flash OFDM system . Adjacent channelization enables the resource allocation to better exploit frequency diversity. Interleaved or random channelization reduces the variance of the effective SNR across sub channels for each user; when the variance is small, user i can simply feed back a single ei(SNR per unit power) value. Random channelizations also aid in managing inter-cell interference.

In my project i am dealing with OFDM symbol of 512 tones. In that 390 data carriers and others are used for channel estimation and frequency and timing synchronization . I have divided 390 tones to 39 sub channels using adjacent sub channelization . In my case x_{ij}=1. So in one time slot a whole sub channel is allocated to only one user . And also assuming the perfect channel estimation .

3.3 OPTIMAL AND SUBOPTIMAL SCHEDULING ALGORITHMS

From Equation equation (3.6) the scheduling and resource allocation problem can be stated as

$$\max_{x,p\ \in \chi} V(\mathbf{x,p}) = \sum_{i,j} w_i x_{ij} log(1 + \frac{p_{ij} e_{ij}}{x_{ij}}) \tag{3.10}$$

subject to : $\sum_{i,j} p_{ij} \leq P$and$\sum_i x_{ij} \leq 1, \forall j \in N$. In the equation ((3.10)) i am taking $x_{ij} = 1$ because i am allocating whole sub channel to one one user at a time. The above equations can be viewed as optimization problem. From (Huang *et al.* (2009)) scheduling Algorithm for the optimal resource allocation to the optimization problem is is as follows .

3.3.1 Optimal scheduling algorithm

(1).First compute the optimal lagrangian mutliplier(λ) of the equation((3.11)) using golden section search method . The function is given by

$$L(\lambda) = \lambda P + \sum_j \mu_j^*(\lambda). \tag{3.11}$$

where for every sub channel $\mu_j^*(\lambda)$ is given by

$$\mu_j^*(\lambda) = \max_i \mu_{ij}(\lambda) \tag{3.12}$$

μ_{ij} is an element of the matrix μ . Elements of μ matrix are computed using below equation

$\mu_{ij}(\lambda) = w_i h(\frac{w_i e_{ij}}{\lambda}, \widetilde{s}_{ij})$, and

$h(w, \widetilde{s}_{ij}) = log(1 + (w-1)^+ \wedge \widetilde{s}_{ij}) - \frac{1}{w}(w-1)^+ \wedge \widetilde{s}_{ij})$

where $a \wedge b$ = min(a,b) ,$\widetilde{s}$ is a large number .

(2). After finding optimal lambda value we can evaluate the power allocation matrix using the equation below.

$$p_{ij}(\lambda) = \frac{1}{e_{ij}}\Big[(\frac{w_i e_{ij}}{\lambda} - 1)^{+} \wedge \widetilde{s}_{ij}\Big] \tag{3.13}$$

(3). Next step is allocation of sub channels to the the users . It is achieved by computing μ matrix for the optimal lambda value. Each column of the matrix represents sub channel j.

we will find the maximum value in each column and the index i of the maximum value will give the user which we have to allocate the sub channel j with the power p_{ij}.

$$\begin{array}{c c} & \begin{array}{cccc} sc_1 & sc_2 & \dots & sc_N \end{array} \\ \begin{array}{c} user_1 \\ user_2 \\ user_3 \\ user_K \end{array} & \begin{pmatrix} \mu_{11} & \mu_{12} & \dots & \mu_{1N} \\ \mu_{21} & \mu_{22} & \dots & \mu_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{K1} & \mu_{K2} & \dots & \mu_{KN} \end{pmatrix} \end{array}$$

(4). If in some columns there are more than one elements which have maximum value we choose the extreme point corresponding to the sub gradient with the smallest non-negative value.

i.e., Combination of the users , for which $\sum_{j=1}^{N} p_{ij}$ is closest to P , without exceeding it. Other heuristic rules for choosing an extreme point can also be used. Note that this requires searching over all extreme points, which has a worst case complexity of $O(K^N)$ (if all users were tied on every tone).

However, typically there are only two users tied on one tone and so this has almost constant complexity.

3.3.2 Single sort suboptimal algorithm

Now we introduce sub-optimal algorithm that do not require finding the optimal λ iteratively. Instead, a carrier allocation is determined by a single sort on each tone based on some easily calculated metric.
These heuristic algorithms are much faster than the previous algorithm, although it's complexity is again $O(K^N)$.

$$\begin{array}{c c} & \begin{array}{cccc} sc_1 & sc_2 & \dots & sc_N \end{array} \\ \begin{array}{c} user_1 \\ user_2 \\ user_3 \\ user_K \end{array} & \begin{pmatrix} R_{11} & R_{12} & \dots & R_{1N} \\ R_{21} & R_{22} & \dots & R_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ R_{K1} & R_{K2} & \dots & R_{KN} \end{pmatrix} \end{array}$$

Each sub channel j is allocated to the user with the largest value of $w_i R_{ij}$, where w_i is weighting factor of user i and

$$R_{ij} = log\Big[1 + \Big(\widetilde{s}_{ij} \wedge \big(e_{ij} P/N\big)\Big)\Big]$$

the user i could achieve on sub channel j under power allocation P/N . Any ties are broken arbitrarily, and power allocation P/N is used.

3.4 Implementation of PFS,Heuristic Scheduling Algorithms on USRP

The system i have considered is downlink OFDM system with the two, three users at the Receiver side . As the First step of implementation I have to feedback channel information from each receiver to Base station. For this purpose i am using user datagram programming(UDP). This is one of socket programming concepts. Base station and receivers will be communicated using sendto() , recvform() functions of the UDP socket programming . I am considering all Receivers as the servers in the socket programming and i am binding them to different ports.

TRANSMITTER SIDE

In each iteration of transmission the Base station will request for the channel information using sendto() to each receivers independently. If the channel information is available at the receiver then it sends channel information to Base station. The base station will receive this channel information using the recvfrom() function.

The Base station will wait at recvform() function for 30 ms and if it does not get any channel information from the receiver it will use the previous channel information for the present iteration of scheduling. We are considering channel with 11 taps. The channel taps are converted to frequency domain channel informations(gains) using fast fourier transforms which are defined in FFTW c++ library. The output of the fast fourier transform will be an array of 512 elements. Then i am taking the channel information of the data carrier sub channels.

The next step is use this channel information for the present iteration of scheduling . I am assuming that all users data is infinitively back logged. That means i am generating some random data for each and every user of length of whole OFDM Block. The scheduling is done for nine OFDM symbols each time. All the nine OFDM symbols combined called OFDM block. In each OFDM symbol there are 390 data carriers so for one block there will be 3510 data symbols we can transmit.

So final data vector of 3510 data symbols is filled with data of 3 users using heuristic scheduling algorithm. In order to know at the receiver which data symbols they have to decode i am putting the information of each sub channel to which it is allocated in the final data vector to be send. Also CRC32 is calculated for each final data vector and it is padded to the final data vector will be send using USRP.

RECEIVER SIDE:

At the Receiver side the channel taps are decoded using Channel estimation and it is send to transmitted to Base station using socket programming . After receiving Constellation points , they are decoded to binary array and then converted to character equivalent array . The first 39 elements of decoded character array will contain information for users which data symbols they have to decode And the last 4 elements of decoded array will contain CRC32, excluding last 4 elements of the decoded character array we will again compute CRC32 at the receiver side . If the Resultant CRC of Decoded Char array and Transmitted CRC is equal then we can say that the transmit data is correctly received .

For the CDF plot in fig.4.2using Heuristic scheduling algorithm i have generated Random channel and the instantaneous rates are not higher when compare to Heuristic PFS with Channel Feedback . In this case all the users average instantaneous rates of the users are approximately equal. The no of iteration is 10000.
For the CDF plot in fig.4.2 using Heuristic scheduling algorithm with the channel feedback from each user.In this case we can notice that we can achieve instantaneous rates of the users are very high when compared to random channel case.

The CDF plot is computed at the transmitter side. The modulation scheme used for this CDF plots is QAM 64. Using USRP we are sending 1 Mega complex symbols per second. so for QAM 64 rate will become 6 Mbps . It can be calculated That for one sub channel rate is 117 Kbps . In fig.4.2 the big step is due to allocation of all sub channels of one OFDM symbol to only one user. It will vanish if we use more number of users in the downlink.
I have written code for optimal proportional fair scheduling but we have to implement dynamic power allocation for sub carriers in each iteration.

CHAPTER 4

Results

4.1 Matlab simulations of PFS, Heuristic Scheduling Algorithms

For Matlab simulations i have considered OFDM downlink system of with M=50 users and system bandwidth of 5 Mhz of 512 OFDM tones, grouped into 64 sub channels (8 tones for sub channel). Resource allocation is done every OFDM symbol . Simulation is done for 60000 OFDM and the results are calculated over last 2000 OFDM symbols. Channel matrix (e) is generated using uniform random function ranges from 12 to 40. All users are infinitely back-logged and assigned a throughput-based utility functions with parameter c_i = 1 and the fairness parameter $\alpha = 0$ across users.
The rate of user i on sub channel j is calculated as

$$r_{ij} = 0.28B\log\Big(1 + 0.56p_{ij}e_{ij}\Big) \tag{4.1}$$

where B is the sub channel bandwidth. Here 0.56 accounts for the 'SNR gap' due to limited modulation and coding choices and 0.28 accounts for various factors such as hybrid ARQ transmission scheme and the overhead due to guard tones and control symbols, etc.
The fig 4.1 is for Heuristic algorithm . Avg. throughput per user is calculated for the last 2000 OFDM symbols is approximately equal to 45 Kbps.

For the same parameters mentioned above i have simulated optimal Proportional Fair Scheduling . The average Throughput per user in this case is 55 Kbps and the CDF plot for The PFS scheduling is given in figure(4.1).
In the matlab simulations of optimal PFS algorithm for finding optimalλ^* value i have used the method which is given by math works. Optimal λ will change with the range

of channel SNR per unit power i.e e_{ij}. For calculation of optimal λ using golden section search .We have to give the interval of λ over which we have to perform search. I have given the interval from 0 to 100. Because in optimization solution in the paper(Reference paper) the condition is $\lambda \geq 0$.

Comparing to CDF plots of the fig.4.1 We can say that we are able to achieve higher instantaneous rates in optimal PFS than the heuristic Scheduling Algorithm And the average Throughput achievable in the former case is more than the previous case.

In PFS implementation there are two cases in finding the optimal user for each sub channel j. (1) only one user have maximum value in column j of μ matrix. for the sub channel . (2) Multiple users have maximum value in the column j of μ matrix if there is any tie in the selection of the user for a particular sub channel we are storing the feasible powers of the users for that particular sub channel in a matrix and selecting the users for all sub channels such that $\sum_{ij} p_{ij} \leq P$

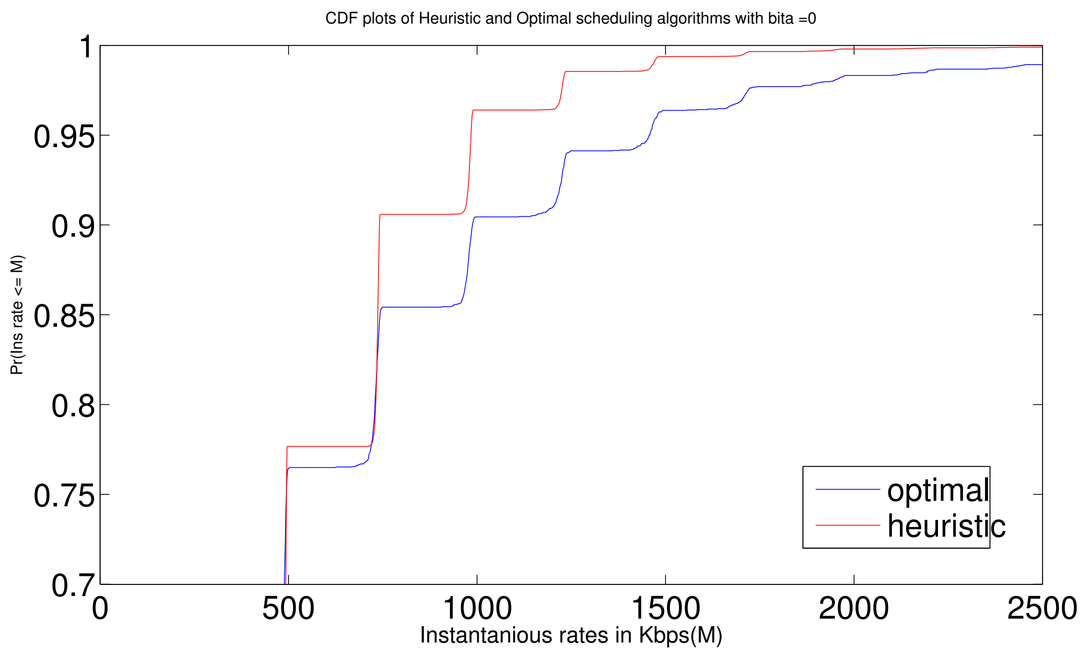

Figure 4.1: CDF plots of instantaneous rates of users using matlab

4.2 CDF plots of instantaneous rates of users using USRP

Plots of CDF of instantaneous rates of users without Channel feedback and with feed back are shown in the figures fig.4.2,fig.4.3 . From the plots we can say that we are achieving higher rates using heuristic scheduling algorithm with channel feedback than without feedback of channel feedback. For the fig.4.2, fig.4.3 the system model we are considering is one basestation and 2 users, 3 users at the receiver side respectually .

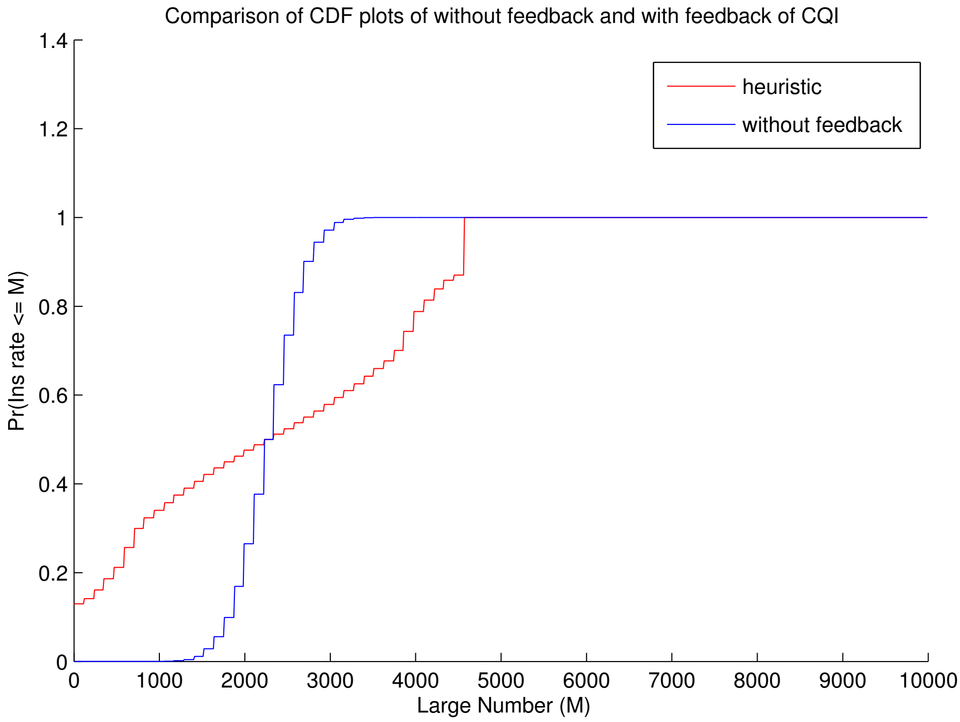

Figure 4.2: CDF plots Instantenious rates of 2users with channel feedback and without feedback

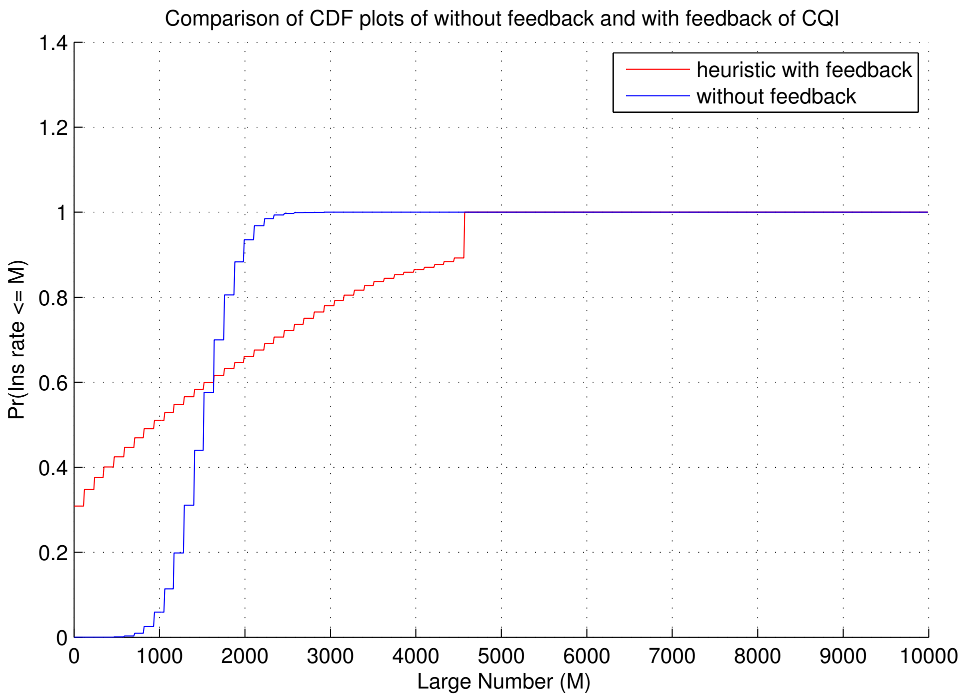

Figure 4.3: CDF plots Instantenious rates of 3users with channel feedback and without feedback

4.3 Variation of Avg. Throughputs of the users for Different values of Alpha

In this section the system model we are considering is the base station and two users one is near user and other user is far from the base station . The plot in the 4.4 shows the variation of throughput of both users with different fairness parameter values. For $\alpha = 0$ the far user's throughput is 400 characters. But as the $\alpha \rightarrow 1$ as we can see from the plot 4.4 far user's throughput is decreasing linearly.

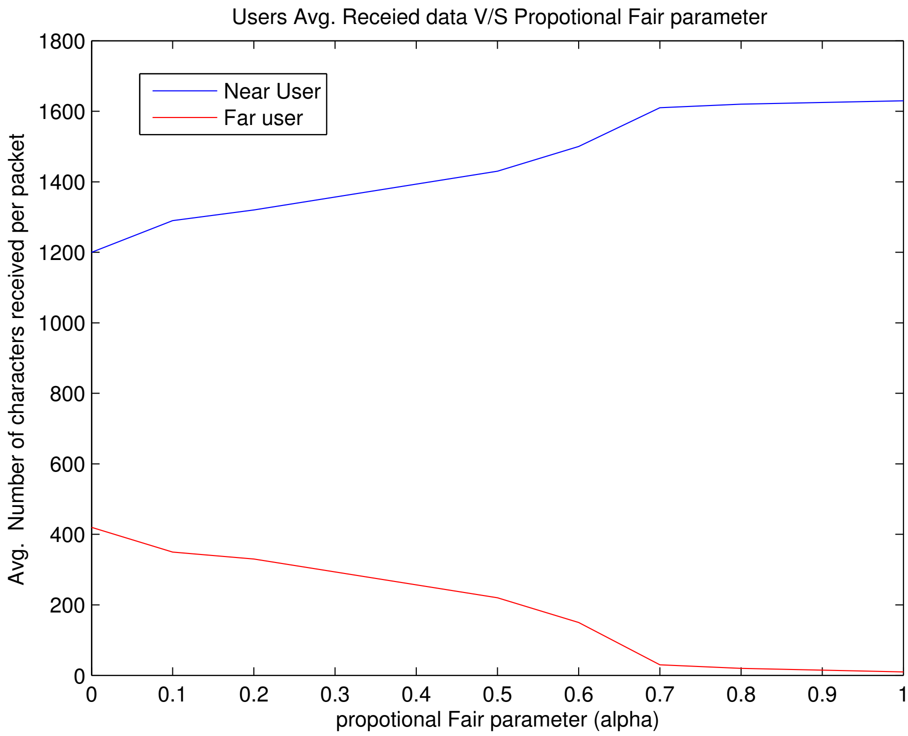

Figure 4.4: Avg. Throughputs of the users for Different values of Alpha

4.4 Calculation of Jain's Fairness Index

Jain's Fairness Index is Used to characterize the fairness of the Scheduling Algorithm. Let $r_1, r_2, ..., r_N$ are throughputs of N users. Fairness is calculated by the equation ((4.2)).

$$\jmath = \frac{(\sum_{i=1}^{N} r_i)^2}{N * (\sum_{i=1}^{N} r_i^2)} \tag{4.2}$$

The result ranges from $\frac{1}{N}$ (worst case) to 1 (best case). It is maximum when all users receive the same allocation. This index is $\frac{k}{N}$ when k users equally share the resource, and the other N-k users receive zero allocation.

Plot 4.5 shows the decrease in the Jain's fairness fairness parameter as$\beta \rightarrow 1$.

Table 4.1: Jain's Fairness Index Vs Fairness Parameter

Fairness Parameter	Jain's Fairness Index
0	0.8118
0.1	0.7527
0.2	0.73529
0.5	0.65028
0.6	0.5990
0.7	0.51862
1	0.506

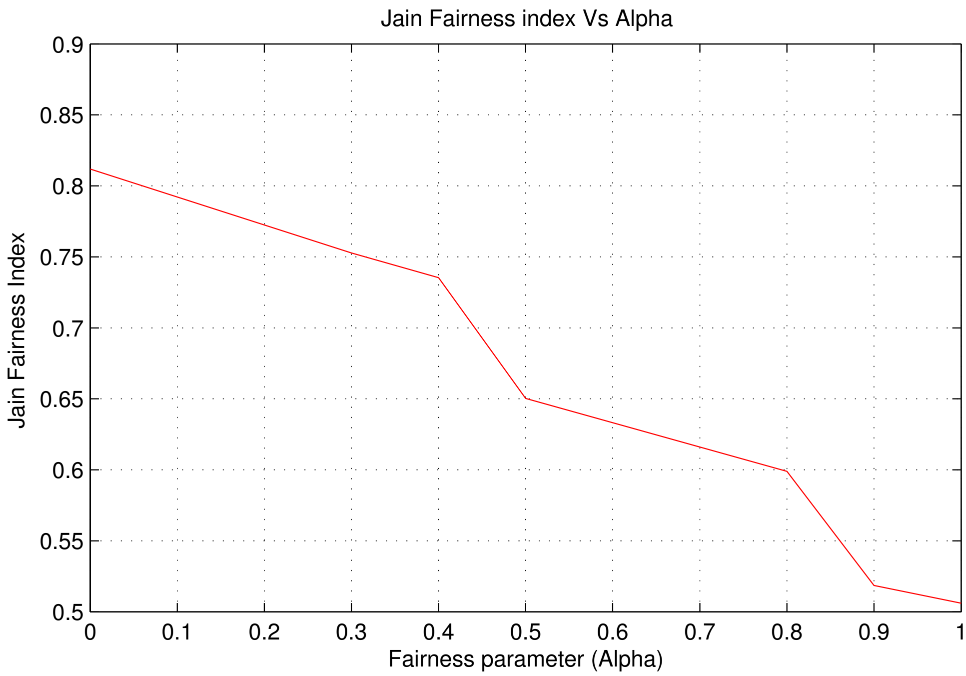

Figure 4.5: Plot of Jain Fairness Index for various Values of Fairness parameter

CHAPTER 5

Conclusion

We have Considered Implementation of Proportional Fair Scheduling on USRP.First We have implemented PFS, Heuristic Scheduling Algorithms using Matlab . From the CDF plots instantaneous rates of the users we can conclude that we are able to achieve higher instantaneous rates using optimal PFS than heuristic scheduling algorithm.

We have implemented the heuristic scheduling algorithm on USRP. Using the CDF plots we conclude that we are achieving the higher instantaneous rates with the channel feedback than without feedback. In with channel feedback case we are just generating random channel and scheduling with the same heuristic algorithm. We did the detection of the errors of the packets using CRC32 checksum.

5.1 Future work

- Implementing Dynamic power allocation to the sub channels of OFDM symbol on USRP.
- Implementation of Proportional Fair Scheduling,other scheduling algorithms using Software Defined Radio.

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