Performance of Large MIMO (Multiple Input Multiple Output) Systems

A Project Report

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THESIS CERTIFICATE

This is to certify that the thesis titled **Performance of Large MIMO (Multiple Input Multiple Output) Systems**, submitted by **N Avinash**, to the Indian Institute of Technology, Madras, for the award of the degree of **Master of Technology**, is a bonafide record of the research work done by him under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

KEYWORDS: Long term evolution ; Link Reliability;Lattice Reduction; LLL algorithm; Seysen's algorithm; EBLR ; LAS

Multiple-input multiple-output (MIMO) technology is maturing and is being incorporated into emerging wireless broadband standards like long-term evolution (LTE). When number of antennas will be increased, it provides the better performance in terms of data rate or link reliability and decrease the probability of outage. Not only advantages, we have face few difficulties like correlation also occur, when we increase number of antennas at both transmitter and receiver. In this thesis, we are using some lattice reduction algorithms like LLL (Lenstra, Lenstra and Lovasz), Seysen's and EBLR (Element- Based Lattice Reduction) techniques to increase the orthogonality . These Lattice reduction techniques are used to increase the orthogonality between the basis of the channel matrices and also get the shortest basis. One more algorithm is used for better detection at the receiver is LAS (Likelihood Ascent Search) iteration algorithm. In this following thesis, we are going to detail discussion about these algorithms.

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ρ	Signal to Noise Ratio at any receive antenna
N_T	Number of transmitted antennas
N_R	Number of receiving antennas
T	Uni modular matrix i.e. det(T)= ± 1
λ_{ij}	Number of non- zero diagonal elements in a matrix A
$\tilde{H}$	New transformation matrix of H
α	Scaling factor that restrict total transmit to predefine limit
k	The number of complex data symbols sent in one STBC matrix
p	Number of time slots
$d^{(0)}$	Initial solution vector by using MMSE/ZF filters
$C^{(k)}$	ML cost function after kth iteration
$\mathcal{F}\left(l_p^{(k)}\right)$	The ML cost difference between current and neighborhood solution vectors
Y_c	The receive space-time signal matrix

CHAPTER 1

Introduction

Current wireless standards have adopted multiple-input and multiple-output techniques to achieve the benefits of transmit diversity and high data rates. But this work limited to up to some extent like two or four antennas. If we can use large number of antennas like Large- MIMO systems with tens or hundreds of antennas have great potential for generating multi- giga bit rate transmission at high spectral efficiency.There are advantages in using multiple antennas, namely they proved that the capacity of a multiple input multiple output system increases linearly with the minimum number of receive and transmit antennas. Actually in MIMO system, we can use some detection methods like ZF (Zero Forcing), MMSE (Minimum Mean Square Error), ZF- SIC (ZF- Successive Interference Cancellation), MMSE- SIC and ML (Maximum Likelihood) equalizers. Above all detection methods, ML detection method gives optimal solution. But the complexity of the method is increased with increasing number of antennas. In this discuss about large- MIMO systems. From the above all other techniques will be poorer performance. Based on those results we are going to introduce new detection methods like lattice reduction (LR) techniques have been applied improve the performance of detectors for MIMO systems but complexity is high. Now the main goal of this thesis is how we can get better performance at low complexity?. we proposed multistage likelihood ascent search (M- LAS) detector will produce better performance at low complexity. In lattice reduction techniques, the MIMO detection problem translates to closest lattice point search problem in lattice theory. Lattice reduction methods have proved themselves to be powerful tools in solving the closest lattice point problems. There is no unique definition for lattice reduction, and therefore, there exist many different methods for lattice reduction. Among the lattice reduction methods, the LLL methods due to Lenstra, Lenstra, and Lovasz, is the most practical one, due to its efficiency in finding near orthogonal vectors with short norms. Generally, in most of the recent works , the complexity of using the LLL algorithm is ignored. This can be justified in a case that, the channel variations are slow enough, to make it possible to use the result of the LLL reduction for quite a large number of received signals. Seysen's algorithm is also one of the best lattice reduction method compare with the LLL algorithm. This algorithm differs from the LLL algorithm and its variants in that it considers all vectors in the lattice simultaneously, and perform operations on those vectors which will reduce the lattice according to some measure. One more reduction technique, Element based lattice reduction (ELR) algorithms that reduce the diagonal elements of the noise co-variance matrix of the linear detectors and enhance the asymptotic performance of linear detectors. From the above, we just introduced about lattice reduction techniques. Here we are going to introduce one more algorithm is maximum likelihood ascent search (LAS) algorithm. We proposed this algorithm by employing a low- complexity multistage multi-symbol update based strategy.

1.1 THESIS OUTLINE

The rest of the work organized as follows.

In chapter 2, the system model for MIMO detection scenario is explained, different algorithms for MIMO detection are explored along with their advantages and disadvantages and provide the simulation results. In chapter 3, Introducing Large- MIMO systems, followed by briefly studied about need of lattice reduction techniques then Detail explanation about lattice reduction techniques and Simulation results, compare those results. In Chapter 4, Here we used system model model is some what different compare with previous methods. So firstly, we give the system mode, followed by detail discussion about likelihood search algorithm (LAS) for one- symbol update and M-symbols update and provide Simulation results. In chapter 5, the reader can find the conclusion and future work.

CHAPTER 2

MIMO Detection System Model

In this chapter, we discuss some of the important concepts regarding MIMO systems (Telatar (1999)) as well as discuss about some linear detection techniques and non- linear detection techniques. But, in this chapter we have maximum 4 antennas at both transmitter and receiver side. In MIMO system is that the transmitted signals from distinct antennas must be decorrelated, and hence, the antenna elements must be sufficiently separated. It has been shown in the literature that the spacing between antenna elements must exceed half the wavelength of the transmitted signals. In this work, we consider a multiple input multiple output (MIMO) system with M transmit, and $N \geq M$ receive antennas. If we consider $x^c = [x_1^c, x_2^c, ..., x_M^c]^T$, $y^c = [y_1^c, y_2^c, ..., y_M^c]^T$, $w^c = [w_1^c, w_2^c, ..., w_M^c]^T$ and the $N \times M$ matrix H_c respectively as the transmitted signal, the received signal, the noise vector and the channel matrix, it will lead to the popular base-band model

$$y^c = H^c x^c + w^c \tag{2.1}$$

The channel is assumed to be Rayleigh, and the noise is Gaussian, i.e., the elements of H , namely $h_{i,j}^c$, are independent and identically distributed (i.i.d), with zero mean and unit variance complex Gaussian distribution. The complex input signal x_c is composed of components, c_i^c , chosen from a $M^2 - QAM$ constellation with energy $\frac{\rho}{M}$,in which ρ can be interpreted as the signal-to-noise ratio (SNR) observed at any receive

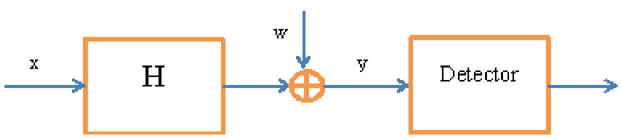

Figure 2.1: Block Diagram of System Model

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antenna. We can convert the whole system to its real counterpart using the following transformations defined for vectors and matrices,

$$x^c = x_I + j\,x_Q \quad y^c = y_I + j\,y_Q \tag{2.2}$$

$$w^c = w_I + j\,w_Q \quad H^c = H_I + j\,H_Q \tag{2.3}$$

Further, we define

$$x = [x_I^T\ x_Q^T]^T,\ y = [y_I^T\ y_Q^T]^T \tag{2.4}$$

$$w = [w_I^T\ w_Q^T]^T,\ H = [H_I^T\ H_Q^T]^T \tag{2.5}$$

Using the aforementioned transformations, the resulting real model is given by

$$y = Hx + w \tag{2.6}$$

From the above equation, y denotes the input at the receiver. Now we have to decode the x by using some detection techniques. We already told that, Maximum Likelihood (ML) produce optical performance. Following we will discuss about all detection techniques used in MIMO systems (Larsson (2009)).

2.1 Detection Methods

2.1.1 Maximum Likelihood (ML) Receiver

Our intuition correctly suggests that an optimal detector should return $\hat{x} = x$, the symbol vector whose (posterior) probability of having been sent, given the observed signal vector y, is the largest (Mow (1994)):

$$\hat{x} \triangleq argmax_{x \epsilon c^c} \; P(x \, was \, sent \, | \, y \, is \, observed)$$

$$\hat{x} \triangleq argmax_{x \epsilon c^c} \frac{P(y \, is \, observed \, | \, x \, was \, sent) P(x \, was \, sent)}{P(y \, is \, observed)} \tag{2.7}$$

Equation (2.7) is known as the Maximum A Posterior Probability (MAP) decision rule. If all symbol vectors are equiprobable, i.e.,that P(s was sent) is constant, the optimal MAP detection rule can be written as:

$$\hat{x} \triangleq argmax_{x \epsilon c^c} \, P(y \, is \, observed \, | \, x \, was \, sent) \tag{2.8}$$

A detector that always returns an optimal solution satisfying (2.8) is called a Maximum Likelihood (ML) detector. If we further assume that the additive noise $\mathbf{n}$ is white and Gaussian, then we can express the ML detection problem of Fig. 2.1 as the minimization of the squared Euclidean distance metric to a target vector y over an M-dimensional finite discrete search set:

$$\hat{x} = argmin_{x \epsilon c^c} ||y - Hx||^2 \tag{2.9}$$

2.1.2 ZF (Zero Forcing) Equalizer

To solve for x, we need to find the matrix Q which satisfies $QH = I$. The zero forcing linear detector for this constraint is given by,

$$Q = (H^H H)^{-1} H^H \tag{2.10}$$

This matrix known as the pseudo matrix for a general $M \times N$ matrix. Note that the off-diagonal terms in the matrix $H^H H$ are not zero. Because the off diagonal terms are not zero, the zero forcing equalizer tries to null out the interfering terms when performing the equalization, i.e when solving for the interference from is tried to be nullified and vice verse. While doing so, there can be amplification of noise. Hence Zero Forcing equalizer is not the best possible equalizer to do the job. However, it is simple and reasonably easy to implement.

2.1.3 MMSE (Minimum Mean Square Error) Equalizer

MMSE Aims at minimizing the variance of the difference between the transmitted data and the signal at the equalizer output. The MMSE approach tries to find a coefficient Q which minimizes the criterion, $E\left\{[Qy-x]\,[Qy-x]^H\right\}$, solving this equation

$$Q = \left[H^H H + N_0 I\right]^{-1} H^H \tag{2.11}$$

By using the above equation we can multiply receiver input with this equation, then the receiver estimate of the transmitted symbols.

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ . \\ \hat{x}_n \end{bmatrix} = \left[H^H H + N_0 I\right]^{-1} H^H \begin{bmatrix} y_1 \\ y_2 \\ . \\ y_n \end{bmatrix} \tag{2.12}$$

When comparing to the equation in ZF equalizer, MMSE will not let infinite noise as ZF does when the channel spectral null. When N_0 is zero, it will be same as ZF equalizer. When N_0 is not equal to zero, residual ISI (Inter Symbol Interference) and noise will be observed at the output of the MMSE equalizer.

2.1.4 ZF- SIC (Successive Interference Cancellation)

To solve for x, we need to find the matrix Q which satisfies $QH = I$. The zero forcing linear detector for this constraint is given by, $Q = (H^H H)^{-1} H^H$ To do the successive interference cancellation (SIC), the receiver needs to perform the following steps: Using ZF equalization, the receiver can obtain an estimate of the transmitted symbols. Suppose we can take 2 symbols and Take one of the estimated symbols and subtract the effect from the receiver vector $y-1$ and y_2 i.e

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_{1,2}\hat{x}_2 \\ y_2 - h_{2,2}\hat{x}_2 \end{bmatrix} = \begin{bmatrix} h_{1,1}x_1 + n_1 \\ h_{2,1}x_1 + n_2 \end{bmatrix} \tag{2.13}$$

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} \\ h_{2,1} \end{bmatrix} x_1 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \tag{2.14}$$

The above equation can be written as

$$\mathbf{r} = \mathbf{h}x_1 + \mathbf{n} \tag{2.15}$$

The above equation is same as equation obtained for receive diversity case. Optimal way of combining the information from multiple copies of the received symbols in receive diversity case is to apply Maximal Ratio Combining(MRC). The equalized symbol is,

$$\hat{x}_1 = \frac{\mathbf{h}^H \mathbf{r}}{\mathbf{h}^H \mathbf{h}} \tag{2.16}$$

This forms the simple explanation for Zero Forcing Equalizer with Successive Interference Cancellation (ZF-SIC) approach. MMSE- SIC also we can follow the same procedure instead of ZF equalization we can do MMSE equalization in the first step.

2.1.5 Lattice Reduction Aided Detector for 2×2 MIMO System

This algorithm (Yao and Wornell (2002)) works only for 2×2 MIMO case. If you increase the number of antennas, it will not work. We are going to discuss the lattice reduction methods for Large- MIMO systems detail in the next chapter. But her we are giving brief material about lattice reduction (LR) techniques. Lattice reduction means to change basis of channel matrix to get better performance. But every time when we change the basis, we did not achieve optimal performance but sometimes we will get optimal performance. In particular, changing lattice basis to be more orthogonal and shorter. If channel matrix have more correlated columns, then by using this procedure significant improvements occurred.

For any lattice $\mathcal{L}$ there are many possible bases. Indeed, if $H = [h_1, h_2, ..., h_M]$ is a matrix whose columns are basis vectors for the lattice.If H is a basis, so is $H' = HP$ for any matrix P such that both P and P^{-1} have integer entries. Specifically, a points represented by x in the basis H is represented by $z = P^{-1}x$ in the basis H', i.e., $s = Hx = (HP)(P^{-1}x) = H'z$.

The basic idea behind using lattice reduction in conjunction with traditional low-complexity detectors is to operate in a chosen lattice basis that is optimized for those detectors, as shown in fig

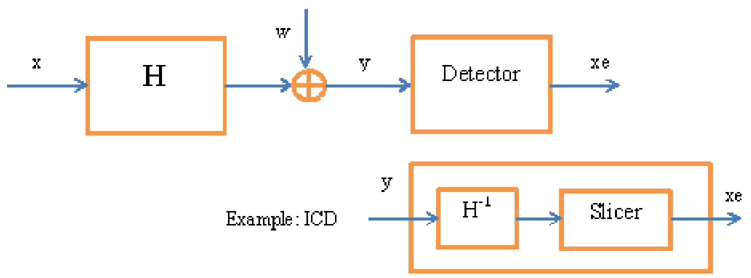

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Figure 2.2: Traditional Detector

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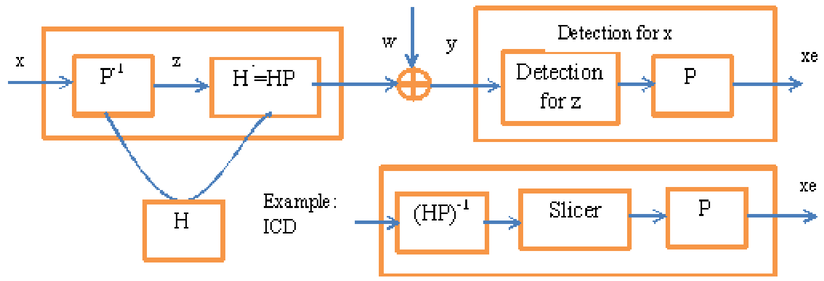

Figure 2.3: Using lattice reduction in conjuction with traditional detectors

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In the traditional system, the detector compensates for the original channel H to produce $\hat{x}$. In the new system, we perform a basis change via a matrix P, specifically

$$y = Hx + w = (HP)(P^{-1}x) + w = H'z + w \tag{2.17}$$

Reduction algorithm:

Given an original set of basis vectors (h_l, h_2) for a lattice with $||h_1|| \leq ||h_2||$, we develop an iterative algorithm to progressively reduce their correlation and converge to the desired basis vectors (u, v). One intuitive way to reduce the correlation between two lattice basis vectors is to subtract integer copies of one vector out of the other. Since the rounding errors for real and imaginary parts are each not more than $\frac{1}{2}$.

Step 1: Reduce the correlation between the basis Check the correlation, $Re\{< h_1 \;\; h_2 >\} \leq \frac{1}{2}||h_1||^2$ and $Im\{< h_1 \;\; h_2 >\} \leq \frac{1}{2}||h_1||^2$,stop. Otherwise replace h_2 with $h_2^1 = (h_2 - nh_1)$ where n is replaced by $n = \left\lceil \frac{< h_1 \;\; h_2 >}{||h_1||^2} \right\rfloor$. Here $\lceil \;\; \rfloor$ it represents the rounding operation.

Step 2: Check h_2^1 is shorter than h_1,then swap go to step (1)

After change the basis we apply ZF or MMSE equalization employed, then $(H')^{-1}y$ is quantized to get $\hat{z}$. After that we can multiply $\hat{z}$ with P then we can get $\hat{x} = P\hat{z}$. We presented an iterative lattice reduction algorithm for optimal decoding and studied its complexity. We showed that the number of iterations needed is typically low and it is increasingly unlikely to need more. We also showed that, relative to optimal MLD, LR techniques is sub-optimal by no more than 3dB in terms of SNR for any Gaussian channel, and allows us to achieve the same diversity on the Rayleigh fading channel, assuming sufficiently large constellations are used.

2.2 Simulation Results

In the section performance and complexity of the lattice reduction methods are studied and compare those reduction algorithm methods with linear detection methods also. We consider the MIMO channel with $N_T = N_R$, transmit and receive antennas. Channel is assumed to be Rayleigh fading channel. Here we consider less number of antennas. The Zero Forcing equalizer is not the best possible way to equalize the received symbol. The zero forcing equalizer helps us to achieve the data rate gain, but NOT take advantage of diversity gain.In this case, channels are correlated so it might not able to solve unknown transmitted symbols at receiver. It is claimed that there can be receiver structures which enables us to have both diversity gain and data rate gain. If we consider MMSE equalization, we get better performance in terms of 3dB improvement. Compared to ZF equalization alone case, addition of successive interference cancellation (SIC) results in around 2.2dB of improvement for BER of 10^{-3}. In the same way, MMSE- SIC gives better results compare with above all methods. The results for 2x2 MIMO with Maximum Likelihood (ML) equalization helped us to achieve a performance closely matching the 1 transmit 2 receive antenna Maximal Ratio Combining (MRC). But if we can increase the order of constellation, then computing of ML equalization is very complex. So normally we can't use this equalization. See fig (2.4), that figure shows comparison of all detection techniques discussed in 2×2 MIMO system detection techniques.

Here we consider one of lattice reduction technique, it will be worked only for 2×2 MIMO system. The incremental complexity inherent in the use of lattice reduction is

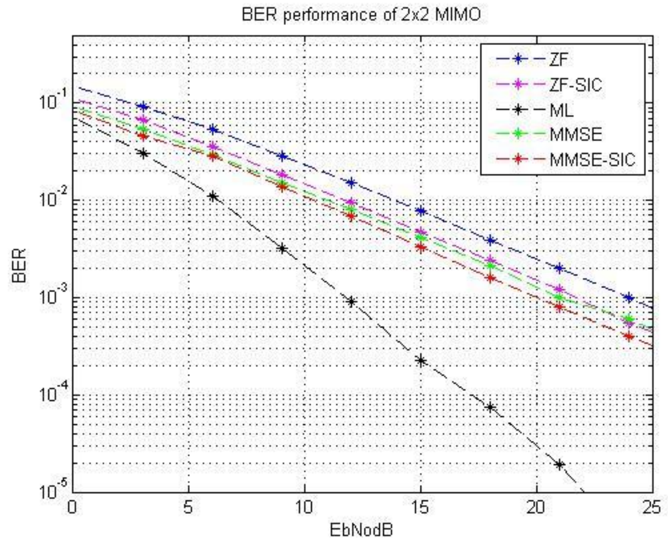

Figure 2.4: Comparision of BER Vs SNR plots for different detection techniques

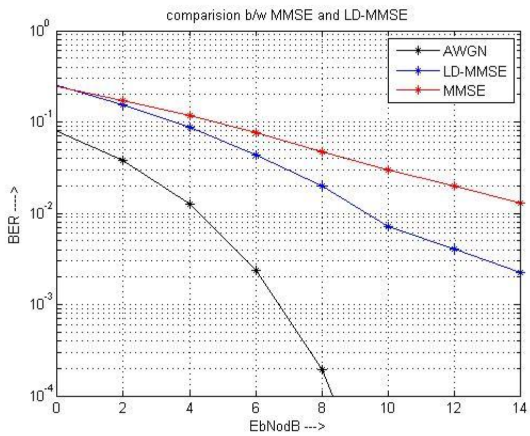

Figure 2.5: Comparision between MMSE-LD and LR-MMSE

determined by the number of iterations required to reduce the basis. In the 2 x 2 case, lattice reduction improves the diversity achieved by ICD and BLAST detection to that of MLD (see in fig 2.5).

CHAPTER 3

Large- MIMO Systems

Large Multiple-Input Multiple-Output (MIMO) systems (Rusek *et al.* (2013)) with tens or hundreds of antennas have shown great potential for next generation of wireless communications to support high spectral efficiencies. The more number of antennas equipped with transmitter/receiver, and the more degrees of freedom that the propagation channel can provide, the better performance in terms of data rate or link reliability. Such large-number of antennas can be employed in large/medium sized communication terminals like set top boxes, laptops and TVs for spectral efficient wireless delivery of high data rate applications. The gains in multiuser systems are even more impressive, because such systems offer the possibility to transmit simultaneously to several users as well the flexibility to select what users to schedule for reception at any given point in time.

The price to pay for MIMO is increased complexity of the hardware and energy consumption of the signal processing at both ends. when number of antennas increased,the complexity of signal detection is also increased.Based on the complexity at the transmitter, some advanced coding schemes are used to transmit the information simultaneously more than one transmitter with inter symbol interference to be controlled manner. Large number of terminals can always be accommodated by combining very large MIMO technology with conventional time and frequency- division multiplexing via orthogonal frequency division multiplexing (OFDM). When we are using large-MIMO, each antenna allotted to extremely low power because we know tha power of each antenna is inversely proportional to number of transmit antennas.Very- large MIMO system made extremely robust because any failure occur in any one of the antenna that won't affect much on system performance.

When number of antennas increased, the area of the aperture is also increased. If the aperture of the array grows, the resolution of the array also increased. Increasing the number of antenna elements implies that antenna separation decreased.It results spatial correlation increased between the antennas, that results more inter symbol interference

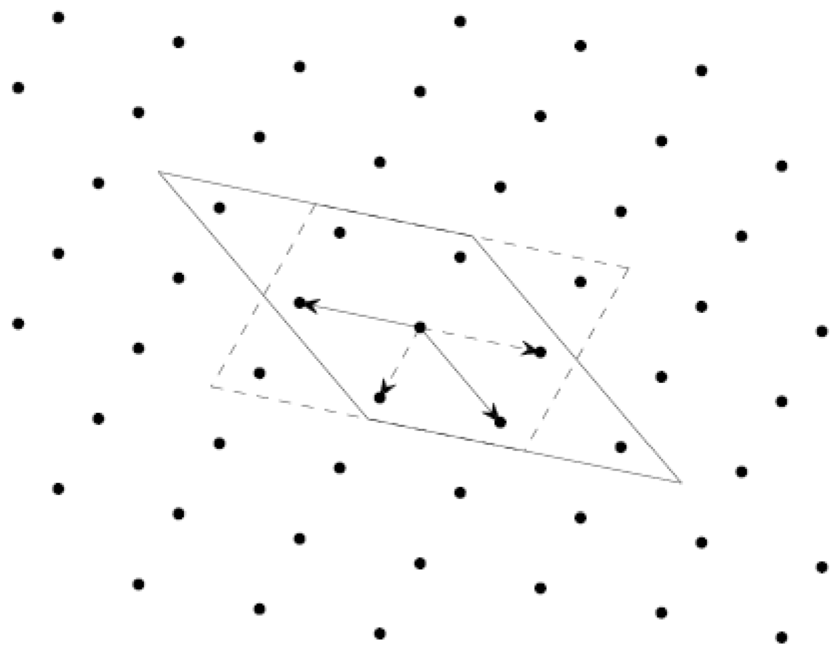

Figure 3.1: Reduction Effect, Solid line: Original basis, Dashed Line: Reduced basis

(ISI) causes degrade the performance of the system. If we consider line of sight (LOS) propagation,we can get the disappointing performance can occur so we always consider independent and identical distributed (iid) Rayleigh fading channel for the propagation. If the channel is not asymptotically orthogonal i.e. coupling and correlation is present. So by using some lattice reduction methods we can decrease the correlation between the antennas and get better performance. Now following topics in this chapter discuss about the lattice reduction algorithms and how performance affect regarding those algorithms.

3.0.1 Lattice Reduction Algorithms

Any lattice L may be described by many different lattice bases. LetB_1, B_2..... be distinct set of vectors, all of which from the bases of lattice L. We can imagine that there exists some ordering or ranking of the bases B_i, and thus one or more of the B_i might be considered good lattice bases of L. Mainly lattice reduction theory deals with identifying good lattice bases for a particular lattice.

In the next following sections, we will discuss about the various reduction techniques like Lenstra,Lenstra,Lovasz (LLL) algorithm, Seysen's algorithm and Element Based Lattice Reduction (ELR) algorithms.

3.1 The LLL Algorithm

In Wubben *et al.* (2004) LLL- reduction-algorithm is discussed. It was originally meant to find "short" vectors in lattices, i.e. to determine a so called reduced Basis for a given lattice.The following pages we will describe the LLL-Algorithm and derive all its steps. We will then determine the relation between lattice reduction and the problem of factoring polynomials, and the relation between lattice reduction and finding integer relations.

Before proceeding further, we will define some expressions and recall the Gram-Schmidt orthogonalization process since it is crucial in the algorithm.

Lattices, Gram- Schmidt and some properties:

Definition 3.1:

subset L of the real vector space R^n is called a lattice if there exist a basis $b_1, b_2...., b_n$ of R^n such that

$$L = \left\{ \sum_{i=1}^{n} r_i b_i \mid r_i \epsilon \mathcal{Z} \ for \, i \epsilon \{1, 2, ..., n\} \right\} \tag{3.1}$$

We call $b_1, b_2...., b_n$ a basis for L and n is the rank of L. Moreover we define $d(L) :=\mid det(b_1, b_2...., b_n) \mid$ to be the determinant of the lattice.

Gram-Schmidt orthogonalization process:

Let $b_1, b_2...., b_n$ be some independent vectors in R^n. We define inductively

$$b_1^* = b_1 \tag{3.2}$$

$$b_i^* = b_i - \sum_{j=1}^{i-1} \mu_{i,j} b_j^* \ for \ 1 \leq i \leq n \tag{3.3}$$

$$\mu_{i,j} = \frac{(b_i, b_j^*)}{|b_j^*|^2} \ for \ 1 \leq i \leq n \tag{3.4}$$

This process produces vectors $b_1^*, b_2^*...., b_n^*$ that form an orthogonal basis of R^n.

Definition 3.2

We call a basis $b_1, b_2...., b_n$ of a lattice L if

$$|\ \mu_{i,j}\ | \leq \frac{1}{2}\ for\ 1 \leq i \leq n \tag{3.5}$$

and

$$|\ b_i^* + \mu_{i,i-1} b_{i-1}^*\ |^2 \geq \frac{3}{4}\ |\ b_{i-1}^*\ |^2\ \ for\ 1 \leq i \leq n \tag{3.6}$$

The second condition can be rewritten as $|\ b_i^*\ | \geq \left(\frac{3}{4} - \mu_{i,i-1}^2\right)\ |\ b_{i-1}^*\ |^2$, which is known as the Lovasz's condition. Note that the constant $\frac{3}{4}$ in the definition is arbitrary chosen. Indeed, we could take any other constant between $\frac{1}{4}$ and 1 . In the original LLL algorithm, to check if a basis is LLL reduced, only adjacent columns are checked against each other. One can argue that this condition can be strengthened, to take into account the earlier columns too. This leads to a non polynomial algorithm both in theory and practice. Obviously this was one of the reasons that the authors of LLL chose the relaxed condition.

The following we give the full LLL- algorithm:

Table 3.1: LLL Algorithm details

Step 1: Initialization

Set $k = 2,\ k_{max} = 1,\ b_1^* = b_1,\ B_1 =< b_1, b_1 >, H = I_n$

Step 2: Incremental Gram- Schmidt

If $k < k_{max}$ go to Step 3

else $k < k_{max},\ b_k^* = b_k$

for $j = 1, ..., k-1$

$\mu_{i,j} = \frac{\left(b_i, b_j^*\right)}{B_j},\ b_k^* = b_k^* - \mu_{k,j} b_j^*\ B_k =< b_k^*, b_k^* >$

Step 3: Test for LLL Condition Run $RED\,(k, k-1)$

If $B_k < (0.75 - \mu_{k,k-1}^2) B_{k-1}$ Run $SWAP(k)\ k = max\,(k-1, 2)$ Go to **Step 3**

else for $l = k-2, k-3, ..., 1$ Run $RED\,(k, l)\ k = k+1$

Step 4: Test for termination If $k \leq m$ go to **Step 2**

Terminate the program and Output b_i 's and the transformation matrix H

Table 3.2: $RED(k, l)$ sub- algorithm

If $\mid \mu_{k,l} \mid \leq 0.5$ exit the sub algorithm

else $q = \lfloor mu_{k,l} \rceil$; $b_k = b_k - qb_l,\ H_k = H_k - qH_l,\ \ \mu_{k,l} = \mu_{k,l} - q$

for $i = 1, 2, ..., i - 1$ $\mu_{k,i} = \mu_{k,i} - q\mu_{l,i}$

Terminate the sub-algorithm

Table 3.3: $SWAP\,(k)$ sub-algorithm

Swap vectors b_k and b_{k-1}

If $k > 2$ for $j = 1, 2, ..., k - 2$, exchange $\mu_{k,j}$ and $\mu_{k-1,j}$ $\mu = \mu_{k,k-1}, B = B_k + \mu^2 B_{k-1}, mu_{k,k-1} = \frac{\mu B_{k-1}}{B},\ b = b^*_{k-1},\ b^*_{k-1} = b^*_k + \mu b$

$b^*_k = -\mu_{k,k-1} b^*_k + \left(\frac{B_k}{B}\right) b,\ B_k = \frac{B_{k-1} B_k}{B}, B_{k-1} = B$

for $i = k + 1, k + 2, ..., k_{max}$ $t = \mu_{i,k},\ \mu_{i,k} = \mu_{i,k-1} - \mu t, mu_{i,k-1} = t + \mu_{k,k-1} \mu_{i,k}$

Terminate the sub-program

At high SNR the performance is dominated by the minimum distance in the decision region. The decision region of ZF decoder is a fundamental parallelogram centered at the transmitted code-word. This decision region can be specified by the Gram-Schmidt orthogonal vectors. The size reduction of a vector which is done by sub-algorithm $RED(k; l)$ in Table 3.3, does not affect the size reduction of the other vectors. By studying the LLL algorithm it is not hard to see that, the size reduction does not affect the Gram-Schmidt orthogonal vectors.

As the decision region of the ZF decoder is determined by Gram-Schmidt orthogonal vectors b_i, and the size reduction does not affect these vectors, therefore a version of the LLL algorithm can be used in the MIMO detection applications.

3.2 Seysen's Algorithm

Seethaler *et al.* (2007) proposed a new method for performing lattice basis reduction. This algorithm differs from the LLL algorithm and its variants in that it considers all vectors in the lattice simultaneously , and performs on those vectors which will reduce the lattice according to some measure. This measure is called seysen's measure. We define a lattice $L(H)$ whose basis vectors are $h_1, h_2, ..., h_n$. The dual of this lattice $L(H)$ is $L(H)^* = (L(H)^{-1})^T$, then whose basis vectors are $h_1^*, h_2^*, ..., h_n^*$. Here h_i is i^{th} column of the channel matrix. Before proceeding further each bases in the lattice follow these properties:

$$(h_i, h_j^*) = 1, for\ i = j$$

$$(h_i, h_j^*) = 0, otherwise \tag{3.7}$$

Here (a, b) indicates the dot product between two vectors a and b. Now we are going to define two matrices A and A^*. Let a_{ij} is the element in the matrix A similarly a_{ij}^* is the element in the matrix A^*.

$$A = [a_{ij}] = [(h_i, h_j)]$$

$$A^* = \left[a_{ij}^*\right] = \left[(h_i^*, h_j^*)\right] \quad for\ 1 \leq i, j \leq 2N_T \tag{3.8}$$

After change the basis in the lattice, the actual channel H is transformed in $\tilde{H}$ in the lattice $L(H)$. It can be written as

$$\tilde{H} = HT \tag{3.9}$$

Here T is called uni modular matrix whose determinant is always ± 1. Now the main aim in this algorithm is after finding matrix A we will get non zero diagonal elements λ_{ij}. Based on the diagonal elements we follow the below algorithm to find out the unimodular matrix T. Once if you find the matrix T, we can get the transformation channel matrix $\tilde{H}$ from the above equation. After finding new channel matrix

Table 3.4 Seysen's algorithm details

Step 1: $T = eye(2N_T), S(A) = \sum_{i=1}^{2N_T} a_{ii}a_{ii}^* = \sum_{i=1}^{2N_T} \| h_i \|^2 \| h_i^* \|^2$

While $\lambda_{ij} \neq 0$ for all (i, j)

Select (i, j) for $i \neq j$ that minimize Δ

Calculate $\lambda_{ij} = round \left\{ \frac{1}{2} \left(\frac{a_{i,j}^*}{a_{j,j}^*} - \frac{a_{i,j}}{a_{j,j}} \right) \right\}$

$\Delta(i, j, \lambda) = S \left(\left(T_{ij}^{\lambda_{ij}} \right)^T A T_{ij}^{\lambda_{ij}} \right) - S(A)$

Update

$A' = \left(\left(T_{ij}^{\lambda_{ij}} \right)^T A T_{ij}^{\lambda_{ij}} \right) A^{*'} = \left(\left(T_{ij}^{\lambda_{ij}} \right)^{-1} A \left(T_{ij}^{\lambda_{ij}-1} \right)^T \right)$

Calculate only for the chosen (i, j)

$\lambda_{ij} = round \left\{ \frac{1}{2} \left(\frac{a_{i,j}^*}{a_{j,j}^*} - \frac{a_{i,j}}{a_{j,j}} \right) \right\}$

$T = T \times T_{ij}^{\lambda_{ij}}$ End

Return Transformation matrix T

After finding new transformation matrix $\tilde{H}$ instead of using normal precoding method, we will change small modifications in the way of pumping data into the transmit antennas. The block diagram of the LRA precoding is shown : fig (3.1) Linear MMSE precoding is a well-known linear precoding technique An *et al.* (2009) that pre-filters the transmitted symbols using the pseudo-inverse of the channel matrix. The precoded vector is defined as x =Ws , where W is defined as following

$$W = \alpha H^{-1}, \ where \ \alpha = \sqrt{\frac{N_T}{trace(H^T.H)^{-1}}} \tag{3.10}$$

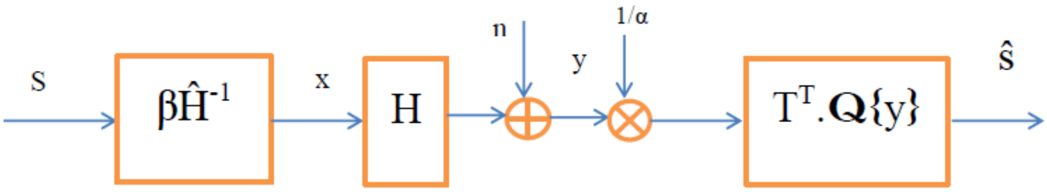

Figure 3.2: LRA Precoding

the scaling factor α is used to restrict the total transmit power to the predefined limit N_T. At the receiver side, the symbols are recovered by dividing the received vector by α. In case of Large-MIMO systems, inter-user interference can be therefore totally canceled by using this approach. Nonetheless, if the channel is ill-conditioned, serious noise amplification arises. The application of LR before the linear MMSE precoding leads to tremendous reduction in α, which consequently reduces the noise amplification. Since that SA makes more orthogonal lattice basis compared to LLL algorithm, a better performance can be obtained.

3.3 Element Based Lattice Reduction

Compared to the existing LLL methods and SAs, the proposed ELR algorithm (Zhou and Ma (2013)) has a different designing goal: to minimize the diagonal elements in noise co-variance matrix. First, we show the relationship between the diagonal elements in noise co-variance matrix with symbol-wise asymptotic pairwise error probability (PEP) and develop two LRs to minimize the asymptotic PEP, which are called "shortest longest vector (SLV) reduction" and a stronger version, "shortest longest basis (SLB) reduction",âĂİ respectively.

The pairwise error probability (PEP) of the ZF-LD (Linear Detection) is if we transmit the i^{th} symbol is s_i, the error is detected as $\hat{s}_i \neq s_i$ given channel matrix H is

$$P\left(s_i \to \hat{s}_i \mid H\right) = Q\left(\sqrt{\frac{\mid e_{s_i} \mid^2}{2\sigma_w^2 C_{i,i}}}\right) \tag{3.11}$$

Here $e_{s_i} = s_i - \hat{s}_i$, $Q(x) = (2\pi)^{-\frac{1}{2}}\int_x^{\infty} exp(-t^2/2)dt$, $C = (H^H H)^{-1}$ is scaled covariance matrix after equalization, and $C_{i,i}$ denotes the i^{th} diagonal element of C.

After Lattice reduction algorithms applied, the channel matrix transformed in to $\tilde{H} = HT$. The noise covariance matrix after the lattice reduction methods is $\tilde{C} = \left(\tilde{H}^H\tilde{H}\right)^{-1} = T^{-1}C\left(T^{-1}\right)^H$. The PEP of LR-ZF is if we transmit the i^{th} symbol is z_i, the error is detected as $\hat{z}_i \neq z_i$ given channel matrix $\tilde{H}$ is

$$P\left(z_i \rightarrow \hat{z}_i \mid \tilde{H}\right) = Q\left(\sqrt{\frac{\mid e_{z_i} \mid^2}{2\sigma_w^2\tilde{C}_{i,i}}}\right) \tag{3.12}$$

From the above equation, we understand that PEP is depends upon the noise covariance matrix. So here we update the unimodular matrix T as well as noise covariance matrix $\tilde{C}$.Mathematically, we formulate the optimization problem as finding a unimodular matrix T by minimizing the largest diagonal element of $\tilde{C}$ as

$$\begin{aligned} &min \;\; max_i^1\left(\tilde{C}_{i,i}\right) \\ &s.t.\tilde{C} = T^{-1}C\left(T^{-1}\right)^H, \\ &T\,\epsilon_N\left(\mathbb{Z}[j]\right) \end{aligned} \tag{3.13}$$

the above optimization is equivalent to minimizing the longest basis vector in the dual basis. Here, we refer to the optimization as the "dual shortest longest vector (D-SLV) reduction".But the optimal solution is not unique i.e. there exists two or more bases of lattice L that satisfy the reduction. To improve the PEP performance by minimizing the second largest diagonal element of $\tilde{C}$ with respect to T as

$$\begin{aligned} &min \;\; max_i^2\left(\tilde{C}_{i,i}\right) \\ &s.t.\tilde{C} = T^{-1}C\left(T^{-1}\right)^H \qquad (3.14) \\ &max_i\left(\tilde{C}_{i,i}\right) = \tilde{C}^{(0)} \\ &T\,\epsilon\,GL_N\left(\mathbb{Z}[j]\right) \qquad (3.15) \end{aligned}$$

After solving optimization problem , we can further optimize the third largest diagonal element, the fourth one, and so on. This procedure continues until all the diagonal elements of $\tilde{C}$ are minimized. In this paper, we call this process the "dual shortest longest basis (D-SLB) reduction".

Table 3.5: The Element- Based Lattice Algorithm

Input: H, Output: $\tilde{H}, T$

(1) $\tilde{C} = (H^H H)^{-1}, \quad T^{'} = I_N$

(2) DO

(3) $\lambda_{i,j} = -\left\lfloor \frac{\tilde{C}_{i,k}}{\tilde{C}_{i,i}} \right\rceil, \quad \forall i \neq k$

(4a) For the D-ELR-SLV: If the largest element of $\tilde{C}$ is irreducible, go to 11

(4b) For the D-ELR-SLB: If all $\lambda_{i,k} = 0, \quad \forall i \neq k$, go to 11

(5) Find the largest reducible $\tilde{C}_{k,k}$

(6) Choose $i = arg\, max_{\tilde{i}=1,\tilde{i}\neq k}^{N} \triangle_{\tilde{i},k}$

(7) $t_k^{'} = t_k^{'} + \lambda_{i,k}\, t_i^{'}$

(8) $\tilde{c}_k^{'} = \tilde{c}_k^{'} + \lambda_{i,k}\, \tilde{c}_i^{'}$

(9) $\tilde{c}^{(k)} = \tilde{c}^{(k)} + \lambda_{i,k}\tilde{c}^{(i)}$

(10) While (true)

(11) $T = (T^{'-1})^H, \quad \tilde{H} = HT$

Here given the initial matrix $\tilde{C} = (H^H H)^{-1}$, for each iteration, the algorithm selects a reducible $\tilde{C}_{k,k}$ and i such that $\triangle_{i,k} > 0$. Then we can update the $\tilde{C}$. For next iteration, the algorithm just picks up another reducible $\tilde{C}_{n,n}$ from the update $\tilde{C}$. This procedure continues until the termination condition satisfied.

D-ELR-SLV algorithm requires, at most, $\mathcal{O}(N)$ iterations requires to find the largest $\tilde{C}_{k,k}$ and $i = arg\, max_{\tilde{i}=1,\tilde{i}\neq k}^{N} \triangle_{\tilde{i},k}$. While, the D-ELB-SLB algorithm requires $\mathcal{O}(N^2)$ iterations to find the index pair (i, k) at the worst case. But finally, D-ELB-SLB algorithm yields better error performance than the D-ELB-SLV algorithm at the cost of higher complexity.

3.4 Simulation Results

In the section performance and complexity of the lattice reduction methods are studied and compare those reduction algorithm methods with linear detection methods also. We consider the MIMO channel with $N_T = N_R$, transmit and receive antennas. Channel is assumed to be Rayleigh fading channel. Here i work out on high order QAM constellation is used to investigate error performance of the proposed methods. In this simulations, the following proposed algorithms are used LLL algorithm, Seysen's algorithm and Element Based Reduction techniques.

We can see fig (3.2 3.3), we can increase the number of antennas from ten to hundreds. Her fig (3.2) displays the error performance of the LR- aided detection with 4QAM, SNR at 20dB. If we can see the figure, ZF- LD gives worst performance compare with the other with other techniques. If we consider, MMSE and LLL- aided MMSE- LD, upto $N_T \leq 20$, LLL- aided MMSE gives good performance compare with MMSE but after the MMSE gives good performance. SA- aided MMSE- LD exhibit better performance compare with MMSE and LLL- aided MMSE. The proposed D-ELR-SLB-aided MMSE-LD show significant improvement over the other existing LR-aided MMSE-LD for large MIMO systems. If we can see in fig (3.3), demonstrates the error performance of the LR- aided detectors with 64QAM, SNR at 30dB.

We can see fig (3.4), If number of antennas 64×64 then the SNRs varies, we can see the the how performance varies for different LR- aided detectors. Here we can get good performance from D-ELR-SLB aided MMSE- LD, but the complexity of this method is very high. When we are going to increase more number of antennas, the complexity also increases with respect to that.

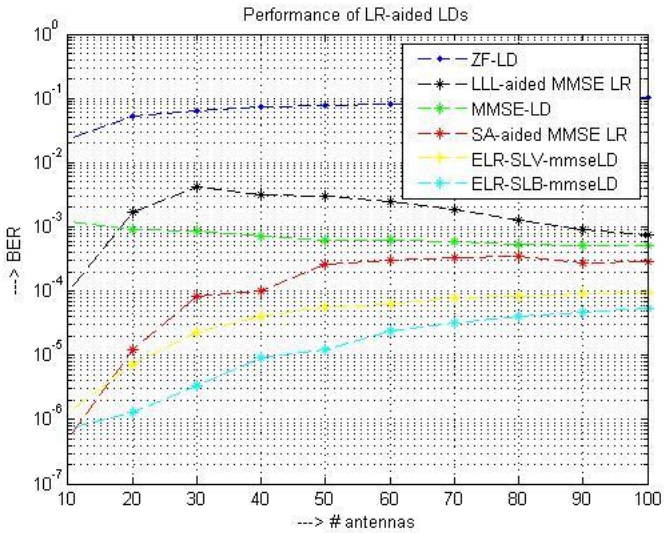

Figure 3.3: Performance of LR-aided LD for MIMO systems with 4QAM, SNR = 20dB

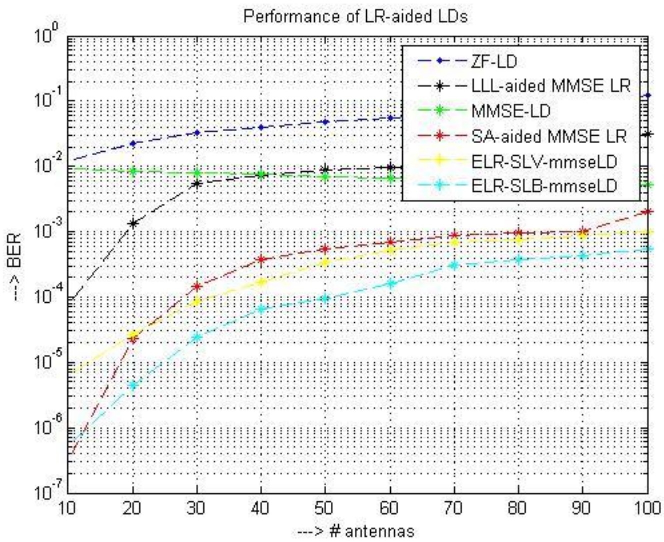

Figure 3.4: Performance of LR-aided LD for MIMO systems with 64QAM, SNR = 30dB

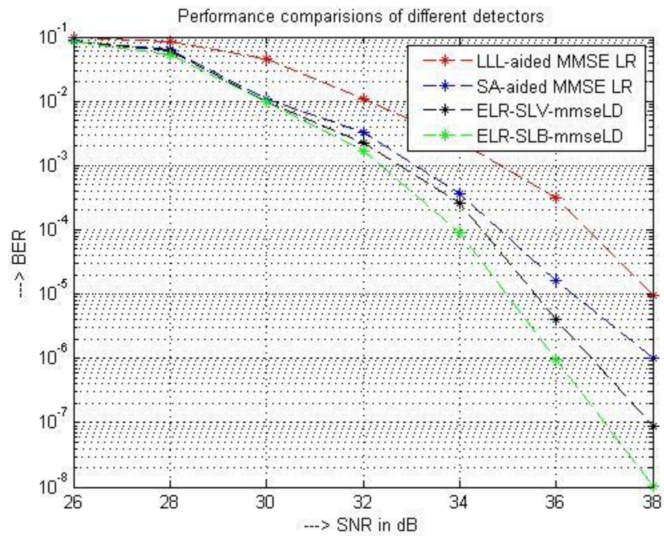

Figure 3.5: Performance comparison with different detection methods, $N_T = N_R = 64$, and 256QAM

CHAPTER 4

High- Rate Space Time Coded Large- MIMO Systems

In the previous chapter, we learn many lattice reduction algorithms when number of antennas increases. But if we observe those algorithms, they give good performance but complexity is very high. So, In this chapter we are going to discuss about low complexity detection algorithm (Mohammed *et al.* (2008)) for detection of high data rate, non- orthogonal. space-time block coded (STBC), large- MIMO systems that achieve high spectral efficiency. Here we present a training based iterative detection/ channel estimation scheme for such large STBC MIMO systems. That algorithm is Multistage- Likelihood Ascent Search (M- LAS) algorithm.

This chapter is organized as follows. We present the STBC MIMO system model considered. Later discuss about the detection algorithm proposed.

4.1 System Model

Consider a STBC MIMO system with multiple transmit and multiple receive antennas. An (n, p, k) STBC is represented by a matrix $X_c \epsilon \mathbb{C}^{N_t \times p}$, where as N_t denotes the number of transmit antennas and p number of time slots, respectively, and k denotes the number of complex data symbols sent in one STBC matrix. The (i, j)th entry in $X_c \epsilon \mathbb{C}^{N_t \times p}$ represents the complex number transmitted from the ith transmit antenna in the jth time slot. The rate of an STBC is given by $r \triangleq \frac{k}{p}$. Let $H_c \epsilon \mathbb{C}^{N_r \times N_t}$ denotes the channel gain matrix, where the (i, j)th entry in H_c is the complex channel gain from the jth transmit antenna to the ith receive antenna. The receive space- time signal matrix $Y_c \epsilon \mathbb{C}^{N_r \times p}$ can be written as

$$Y_c = H_c X_c + N_c \tag{4.1}$$

where $Y_c \epsilon \mathbb{C}^{N_r \times p}$ is the noise matrix at the receiver and its entries are modeled as i.i.d $\mathcal{CN}\left(0, \sigma^2 = (N_t E_s)/(\gamma)\right)$, where E_s be the average energy of the transmitted symbols, γ is the average received SNR per receive antenna. In a linear dispersion (LD) STBC, where X_c can be written in the form

$$X_c = \sum_{i=1}^{k} x_c^{(i)} A_c^{(i)} \tag{4.2}$$

From above two equations, applying the vec(.) operation then we have

$$vec\left(Y_c\right) = \sum_{i=1}^{k} x_c^{(i)} vec\left(H_c A_c^{(i)}\right) + vec\left(N_c\right) \tag{4.3}$$

If U, V, W, D are matrices such that $D = UWV$, then $vec(D) = \left(V^T \otimes U\right) vec(W)$, $\otimes$ denotes tensor product of matrices. So we can be written as

$$vec\left(Y_c\right) = \sum_{i=1}^{k} x_c^{(i)} \left(I \otimes H_c\right) vec\left(A_c^{(i)}\right) + vec\left(N_c\right) \tag{4.4}$$

$$y_c = vec\left(Y_c\right) \epsilon \mathbb{C}^{N_r p \times 1}, \;\; \hat{H}_c \triangleq I \otimes H_c \epsilon \mathbb{C}^{N_r p \times N_t p}$$

$$a_c^{(i)} \triangleq vec\left(A_c^{(i)}\right) \epsilon \mathbb{C}^{N_t p \times 1}, \; n_c \triangleq vec\left(N_c\right) \epsilon \mathbb{C}^{N_t p \times 1}, \;\; x_c \epsilon \mathbb{C}^{k \times 1}$$

From above these definitions, we can write as

$$y_c \triangleq \sum_{i=1}^{k} x_c^{(i)} \left(\hat{H}_c a_c^{(i)}\right) + n_c = \tilde{H}_c x_c + n_c \tag{4.5}$$

we can decomposed into real and imaginary parts as

$$\begin{aligned} y_c = y_I + j y_Q, \quad x_c = x_I + j x_Q \\ n_c = n_I + j n_Q, \quad \tilde{H}_c = \tilde{H}_I + j \tilde{H}_Q \end{aligned} \tag{4.6}$$

Further, we define $x \,\epsilon\, \mathbb{R}^{2k \times 1}$, $y \,\epsilon\, \mathbb{R}^{2N_r p \times 1}$, $\mathrm{H} \,\epsilon\, \mathbb{R}^{2N_r p \times 2k}$, and $n \epsilon \mathbb{R}^{2N_r p \times 1}$ as

$$\begin{aligned} x = \left[x_I^T \; x_Q^T\right]^T, \;\; y = \left[y_I^T \; y_Q^T\right]^T \\ H = \begin{pmatrix} \tilde{H}_I & -\tilde{H}_Q \\ \tilde{H}_Q & \tilde{H}_I \end{pmatrix}, \;\; n = \left[n_I^T \; n_Q^T\right]^T \end{aligned} \tag{4.7}$$

Hence we work with the real-valued system can be written as

$$y = Hx + n \tag{4.8}$$

The ML solution is given by

$$\mathtt{d}_{ML} = argmin_{\mathtt{d}\epsilon\mathbb{S}} \parallel y - Hd \parallel^2$$

$$\mathtt{d}_{ML} = argmin_{\mathtt{d}\epsilon\mathbb{S}}\ d^T H^T H d - 2y^T H d \tag{4.9}$$

whose complexity exponential in k.

When $\delta = e^{\sqrt{5}j}$ and $t = e^j$, the STBC in achieves full transmit diversity (under ML decoding) as well as information-losslessness. When $\delta = t = 1$, the code ceases to be of full-diversity (FD), but continues to be information-lossless (ILL). High spectral efficiency with large n can be achieved using this code construction.

4.2 LAS Algorithm

The M-LAS algorithm starts with an initial solution $d^{(0)}$, given by $d^{(0)} = By$, where B is the initial solution filter, which can be a matched filter (MF) or zero-forcing (ZF) filter or MMSE filter. The index m in $d^{(m)}$ denotes the iteration number in a substage of a given search stage. The ML cost function after the kth iteration in a given search stage is

$$C^{(k)} = d^{(k)^T} H^T H d^{(k)} - 2y^T H d^{(k)} \tag{4.10}$$

4.2.1 One- Symbol Update

In this algorithm (Vishnu Vardhan *et al.* (2008)), we can update each symbol every time. According to that updating vector, we can find the cost function. Later, we can find the cost difference between previous vector and update vector. If that difference is less than zero, then we can update that symbol according to that. Otherwise keep it previous symbol in that vector as it is. We can write the total algorithm with equations as follows:

The update rule can be written as

$$d^{(k+1)} = d^{(k)} + \lambda_p^{(k)} e_p \tag{4.11}$$

Here e_p denotes the unit vector with its pth entry is only one, and all other entries as zero. $\lambda_p^{(k)}$ can take only certain integer values. For example, in case of 4-PAM or 16-QAM , take $\lambda_p^{(k)}$ can take value only from $\{-6, -4, -2, 0, 2, 4, 6\}$.

We can write the cost difference as

$$\triangle C_p^{k+1} \triangleq C^{(k+1)} - C^{(k)}$$

$$\triangle C_p^{k+1} = \lambda_p^{(k)^2} (G)_{p,p} - 2\lambda_p^{(k)} z_p^{(k)} \tag{4.12}$$

Here $G \triangleq H^T H$, $\mathbf{z}^{(k)} = \mathbf{H}^T \left(y - Hd^{(k)}\right)$, where $\mathbf{z}_p$ is the pth entry of the $\mathbf{z}$ and $(G)_{p,p}$ is the (p,p)th entry of the G matrix.

The ML cost difference can be rewritten as

$$\mathcal{F}\left(l_p^{(k)}\right) \triangleq \triangle C_p^{k+1} = l_p^{(k)^2} a_p - 2 l_p^{(k)} \left| z_p^{(k)} \right| \tag{4.13}$$

Here $a_p = (G)_{p,p}$, $l_p^{(k)} = \left| \lambda_p^{(k)} \right|$

However, for the case of one-symbol update, we could obtain a closed-form expression for the optimum $l_p^{(k)}$ that minimizes $\mathcal{F}\left(l_p^{(k)}\right)$, which is given by

$$l_{p,opt}^{(k)} = 2 \left\lfloor \frac{\left| z_p^{(k)} \right|}{2 a_p} \right\rceil \tag{4.14}$$

The new value of the symbol would be given by

$$\bar{d}_p^{(k+1)} = d_p^{(k)} + l_p^{(k)} sgn\left(z_p^{(k)}\right) \tag{4.15}$$

If the values in $\bar{d}_p^{(k+1)}$, be greater than $(\mathcal{M}-1)$ then the adjusted value of $l_p^{(k)}$ is $(\mathcal{M}-1)$.

In the same way, If the update symbol value is less than $-(\mathcal{M}-1)$, then the value is adjusted value of $l_p^{(k)}$ is $-(\mathcal{M}-1)$.

Finally if $\mathcal{F}\left(l_p^{(k)}\right) < 0$, the update for the $(k+1)$th iteration is

$$d^{(k+1)} = d^{(k)} + l_{p,opt}^{(k)} sgn\left(z_s^{(k)}\right) e_{(s)} \tag{4.16}$$

$$\mathbf{z}^{(k+1)} = \mathbf{z}^{(k)} - l_{p,opt}^{(k)} sgn\left(z_s^{(k)}\right) e_{(s)} \tag{4.17}$$

If $\mathcal{F}\left(l_p^{(k)}\right) \geq 0$, then the one-symbol update search terminates. The data vector at this point is referred to as "one-symbol update local minima".

4.2.2 Multistage LAS (Likelihood Ascent Search) Algorithm

The proposed M-LAS algorithm consists of a sequence of likelihood-ascent search stages, where the likelihood increases monotonically with every search stage. Each search stage consists of several substages.In the first substage, the algorithm updates one symbol per iteration such that the likelihood monotonically increases from one iteration to the next until a local minima is reached. Upon reaching this local minima, the algorithm initiates the second substage. In the second substage, a two-symbol update is tried to further increase the likelihood. If the algorithm succeeds in increasing the likelihood by two-symbol update, it starts the next search stage. If the algorithm does not succeed in the second substage, it goes to the third substage where a three-symbol update is tried to further increase the likelihood. The following steps we can write whole M- LAS algorithm with equations. The following block diagram clearly shows how the algorithm executes.

The update rule can be written as $d^{(k+1)} = d^{(k)} + \sum_{j=1}^{K} \lambda_{i,j}^{(k)} e_{i,j}$. Here also we can take specific integers $\lambda_p^{(k)}$. For example, for 16- QAM,$\mathbb{A}_{i,j} = \{-3,-1,1,3\}$ we can take the $\lambda_{i,j}$ values as $\{-6,-4,-2,0,2,4,6\}$.

we can write the cost difference function as

$$\triangle C_u^{k+1}\left(\lambda_{i_1}^{(k)}, \lambda_{i_2}^{(k)}, ..., \lambda_{i_K}^{(k)}\right) \triangleq C^{k+1} - C^k$$

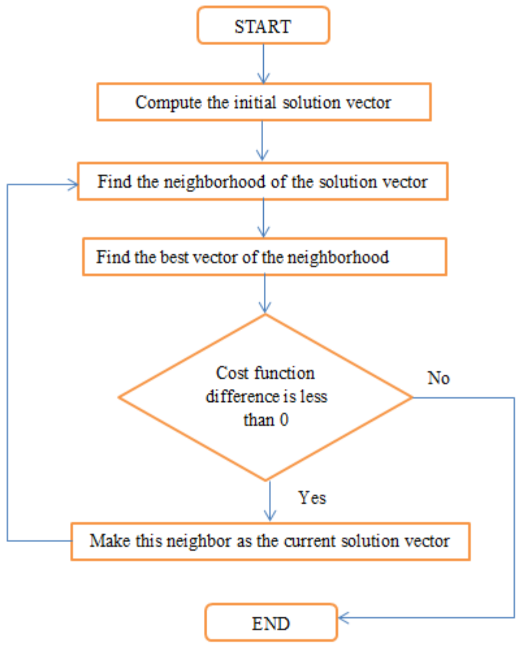

Figure 4.1: LAS Algorithm

$$C^{k+1} - C^{k} = \sum_{j=1}^{K} \lambda_{i_j}^{(k)^2} (G)_{i_j,i_j} + 2\sum_{q=1}^{K} \sum_{p=q+1}^{K} \lambda_{i_p}^{(k)} \lambda_{i_q}^{(k)} (G)_{i_p i_q} - \sum_{j=1}^{K} \lambda_{i_j}^{(k)} z_{i_j}^{(k)} \quad (4.18)$$

Approximate methods can be adopted to solve this problem using lesser complexity. One method based on zero-forcing is as follows. The cost difference function in can be rewritten as

$$\triangle C_u^{k+1}\left(\lambda_{i_1}^{(k)}, \lambda_{i_2}^{(k)}, ..., \lambda_{i_K}^{(k)}\right) = \Lambda_u^{(k)^T} F_u \Lambda_u^{(k)} - 2\Lambda_u^{(k)^T} \mathbf{z}_u^{(k)} \quad (4.19)$$

Here $\Lambda_u^{(k)} \triangleq \left[\lambda_{i_1}^{(k)}, \lambda_{i_2}^{(k)}, ..., \lambda_{i_K}^{(k)}\right]^T$, $\mathbf{z}_u^{(k)} \triangleq \left[z_{i_1}^{(k)}, z_{i_2}^{(k)}, ..., z_{i_K}^{(k)}\right]^T$, where $(F_u)_{p,q} = (G)_{i_p,i_p}$,and $p, q \in \{1, 2, ..., K\}$

The final update rule for the $z^{(k)}$ and $d^{(k)}$ vectors are given by

$$\mathbf{z}^{(k+1)} = \mathbf{z}^{(k)} - \sum_{j=1}^{K} \hat{\lambda}_{\hat{i}_j}^{(k)} g_{\hat{i}_j} \quad (4.20)$$

$$\mathbf{d}^{(k+1)} = \mathbf{d}^{(k)} + \sum_{j=1}^{K} \hat{\lambda}_{\hat{i}_j}^{(k)} e_{\hat{i}_j} \quad (4.21)$$

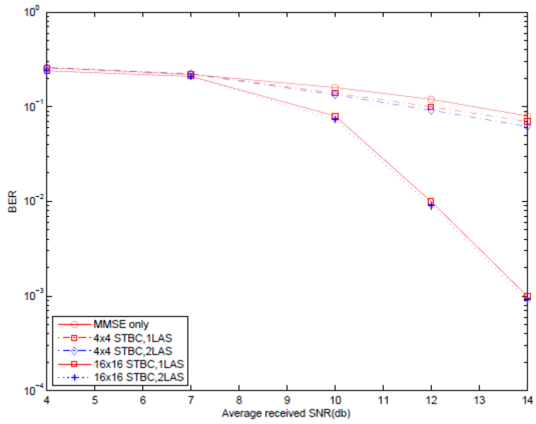

Figure 4.2: Uncoded BER of the proposed 1-LAS, 2-LAS detector for ILL only STBCs

4.3 Simulation Results

It can be seen that MMSE-only performance does not improve with increasing STBC size (i.e., increasing $N_t = N_r$). However, it is interesting to see that, when the proposed search using LAS is performed following the MMSE operation, the performance improves for increasing $N_t = N_r$. If we can see the results, we understand that 2-LAS performs better than 1-LAS. In ILL-only STBCs can be taken advantage of without incurring much performance loss compared to FD-ILL STBCs.

The complexity of the proposed LAS algorithm comprises of three components, namely, 1) computation of the initial vector $d^{(0)}$, 2) computation of $H^T H$, and 3) the search operation.Two good properties of the STBCs from CDA are useful in achieving low orders of complexity for the computation of $d^{(0)}$ and $H^T H$. They are: 1) the weight matrices $A_c^{(i)}$'s are permutation type, and 2) the matrix $N_t^2 \times N_t^2$ formed with $N_t^2 \times 1$ sized vectors $a_c^{(i)}$ as columns is a scaled unitary matrix. These properties allow the computation of MMSE/ZF initial solution in $\mathcal{O}\left(N_t^3 N_r\right)$ complexity, i.e., in $\mathcal{O}\left(N_t N_r\right)$ per-symbol complexity since there are N_t^2 symbols in one STBC matrix. Likewise, the computation of $H^T H$ can be done in $\mathcal{O}\left(N_t^3\right)$ per-symbol complexity. The average per-symbol complexities of the 1-LAS and 2-LAS search operations are $\mathcal{O}\left(N_t^2\right)$ and $\mathcal{O}\left(N_t^2 log N_r\right)$, respectively, which can be explained as follows. The average search complexity is the complexity of one search stage times the mean number of search stages till the algorithm terminates.

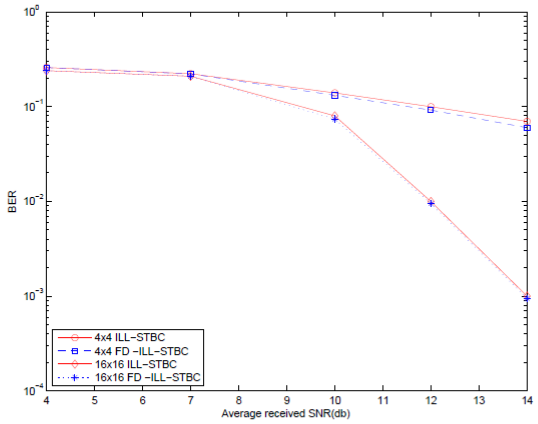

Figure 4.3: Uncoded BER comparision between FD-ILL and ILL- only

CHAPTER 5

Conclusion and future work

5.1 Conclusion

In this work different methods used in MIMO detection scenario were studied. Most of the efficient decoding techniques require a preprocessing stage that involves lattice reduction. In a Rayleigh fading channel that the channel realizations are correlated in time, it was shown that it is possible to take advantage of this temporal correlation, and reduce the complexity of the lattice reduction, and therefore the preprocessing stage. This helps the decoding systems to be practical to be used in today's communication devices that have a constraint on energy and processing power.

The lattice reduction algorithms in a Rayleigh fading channel MIMO system, without any loss in error performance. This makes the proposed algorithm to be quite practical and appealing to be used in any MIMO scenario that needs lattice reduction.Each lattice reduction algorithm has its own identity. But final conclusion of each algorithm is common i.e. to get the orthogonal and shortest basis. When we are using LLL algorithm we need not worry about complexity much but it gives good performance. From Seysen's algorithm , we are getting lattice reduction basis for that matrix as well as dual. Element reduction techniques are following different approach but we are getting better performance but complexity is more compare with the other techniques.

Further, we discussed about the high-data space- time coded large- MIMO systems. Here we concentrate not only on the performance but also decrease in the complexity. According to the results, we are getting satisfied response from this algorithm. But this is not only breaking point of the research. We have many more methods to get better performance compare with the this algorithm.

5.2 Future Work

We are discussed very few algorithms in this thesis. They are many more algorithm are used to get the good performance like reactive tabu search, layered tabu search algorithms etc. Now a days to transmit data through the antennas we are using many types of advanced codes like LDPC and TURBO codes. In this thesis, we discussed only STBC codes. If work on further we can use different codes and different algorithms to improve system performance.

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