Fairness Based Resource Allocation in OFDMA Downlink using Imperfect CSIT

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THESIS CERTIFICATE

This is to certify that the thesis titled **Fairness Based Resource Allocation in OFDMA Downlink using Imperfect CSIT**, submitted by **Maria George**, to the Indian Institute of Technology, Madras, in partial fulfillment of the requirements for the award of the degree of **Master of Technology**, is a bona fide record of the project work done by her under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

KEYWORDS: Resource Allocation; OFDMA; Imperfect CSIT; Fairness; Outage

In today's wireless scenario, OFDMA is one of the promising technologies which can meet the current data requirement and mitigate the spectrum scarcity. This project focuses on the efficient management of resources - power, rate and subcarriers on the OFDMA downlink. In a practical setting, it is likely that the channel knowledge at the transmitter is imperfect either due to channel estimation errors or feedback delays. We consider a novel approach of considering three factors together for OFDMA resource allocation - imperfect CSIT, outage, and fairness.

We decouple the resource allocation scheme into two steps - rate, power allocation step and subcarrier allocation step. To reduce complexity, equal power allocation is done and optimal rate to be allocated on each link to maximise throughput is determined. Then in the second step, certain heuristic strategies are proposed for subcarrier allocation which will incorporate fairness. This thesis considers methods to ensure both short-term and long-term fairness and also inclusion of fairness among users with different rate requirements. The effectiveness of the proposed scheme is demonstrated through simulations. It is found that in terms of throughput, the proposed scheme bridges the gap between perfect and imperfect CSIT. Jain's fairness index is used to measure fairness and the strategies are found to have high Jain's index. At the same time, there is minimal degradation in throughput from the performance of throughput maximising schemes. Thus the proposed resource allocation scheme achieves a good balance between throughput and fairness at low complexity under conditions of imperfect CSIT.

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ABBREVIATIONS

CSIT	Channel State Information at Transmitter
AWGN	Additive White Gaussian Noise
EWMA	Exponentially Weighted Moving Average
BS	Base Station
MS	Mobile Station
BER	Bit Error Rate
OFDM	Orthogonal Frequency Division Multiplexing
OFDMA	Orthogonal Frequency Division Multiple Access
OPA	Optimal Power Allocation

NOTATION

$\mathbf{K}$	Number of users
$\mathbf{N}$	Nmber of subcarriers
$\mathbf{h}_{k,n}$	Actual channel gain for the kth user on the nth subcarrier
$\hat{\mathbf{h}}_{k,n}$	Estimated channel gain for the kth user on the nth subcarrier
$\mathbf{e}_{k,n}$	Estimation error for the kth user on the nth subcarrier
$\sigma_{\mathbf{e}}^{\mathbf{2}}$	Estimation error variance
$\mathbf{C}$	Actual channel capacity
$\hat{\mathbf{C}}$	Estimated channel capacity
$\mathbf{r}$	Rate allocated
$\mathbf{p}$	Power allocated
$\mathbf{G}$	Goodput
$\mathbf{P_{out}}$	Outage probability
$\hat{\mathbf{a}}$	Subcarrier allocation indicator
$\mathbf{W}$	Weighting factor
$\mathbf{R}_k$	Total successfully transmitted rate by the kth user
$\overline{\mathbf{R}}_k$	Average successfully transmitted rate by the kth user
$\hat{\mathbf{R}}_k$	Used for EWMA update
$\mathbf{r}'_k$	Used for dynamic update
$\triangle\mathbf{C}$	Rate back off
$\mathbf{\Psi}$	Set of subcarriers to be allocated
$\mathbf{J}$	Jains' Fairness Index

CHAPTER 1

INTRODUCTION

The number of users of wireless communications services and their demands are ever increasing, thus posing an important problem of efficient management of communications resources. Different modulation and multiple access schemes have been proposed in the past to deal with the requirements. Currently what is now looked upon as a promising technology of the future is Orthogonal Frequency Division Multiplexing (OFDM) and OFDMA. OFDM is popular especially in broadband wireless communication systems primarily due to its resistance to multipath fading, and its ability to deliver high data rates with reasonable computational complexity. OFDM divides a broadband channel into multiple parallel narrowband subchannels, wherein each subchannel carries low data rate stream, which sums up to a high data rate transmission [16]. The concept of OFDM can be extended to multiple users also, known as OFDMA. The focus of this project is on the resource allocation in the downlink transmission channel for OFDMA. We especially focus on the more practical case of doing resource allocation - i.e., power, rate, and subcarrier allocation, with imperfect CSIT. In such a case, since the estimated channel capacity is likely to have errors, and there is a probability that the rate allocated results in outage and hence that has to be considered. Also when there are multiple users in the system, it is essential to maintain fairness among the users [9]. This adds complexity to the problem.

This chapter gives an overview about OFDM and OFDMA and is concluded with the organisation of thesis.

1.1 Overview

OFDM is a digital communication scheme which is becoming increasingly popular because of its high spectral efficiency and robust performance in wireless links which are heavily impaired by inteference or multipath. It combines a large number of low data rate carriers to construct a composite high data rate communication system. Since the carriers are orthogonal, they can be closely-spaced, with spectral overlap, without inter-carrier interference. The low data rate of each carrier implies long symbol duration, which greatly diminishes inter-symbol interference due to multipath. Although the idea of OFDM originated back in 1966, it has been widely utilized only in the last two decades. It is now being used in several communication systems such as IEEE 802.11a/g wireless local area networks, IEEE 802.16-2004/802.16e-2005 wireless metropolitan area networks, 3GPP-LTE, ADSL, and power line communications [16].

OFDMA allows multiple users to transmit simultaneously on the different subcarriers per OFDM symbol. Since the probability that all users experience a deep fade in a particular subcarrier is typically quite low, intelligent subcarrier allocation mechanisms can be used to assure that subcarriers are assigned to the users who see good channels on them. [16]. Hence there is an increased scope for OFDMA in the current scenario.

1.2 Thesis Organization

This thesis is organized as follows: Chapter 2 explains the problem statement. It states the objective, the assumptions made in the thesis and also includes the description of the system model. Chapter 3 gives a consolidated overview about the current work related to this area of OFDMA resource allocation. In Chapter 4, the different resource allocation schemes are proposed. Chapter 5 contains the simulation results which validate the performance of the proposed schemes in terms of throughput (goodput) and fairness. Finally, Chapter 6 and Chapter 7 give the conclusions of this project and the future work and potential extensions that can be explored.

CHAPTER 2

PROBLEM STATEMENT

2.1 Objective

A multiuser Orthogonal Frequency Division Multiplexing (OFDM) system is one of the emerging systems in cellular and broadband wireless communication. There is a need to identify the techniques which will obtain the maximum throughput from an OFDM-based system. For this, transmit power and bandwidth are two vital resources which have to be effectively allocated. An ideal system has to be designed by incorporating fairness of resource allocation among users, which makes the problem more complex. Though there are many papers in the literature which address this issue of resource allocation, many of them assume perfect channel knowledge at transmitter. However in a practical scenario, due to channel estimation errors or feedback delays, the resource allocation has to be done with imperfect channel knowledge and in such scenarios, the rate of transmission also becomes an important parameter to be allocated.

Thus the aim of this project is to propose a resource allocation algorithm for the multiuser OFDM system which achieves

- An effective trade off between throughput and fairness.
- Robustness to channel estimation errors
- Low computational complexity for the optimal resource allocation technique.

2.2 Assumptions in the Thesis

The following assumptions are made in this thesis:

- We assume the more realistic scenario of imperfect channel knowledge at the transmitter. The error in the estimate of channel gain is modeled as a complex Gaussian [2], [10], [15].

- A single cell scenario is considered and the interference from other cells is modelled as additive white Gaussian noise which increases the noise variance of the signal model. [16]
- The fading is slow enough for the channel to be considered constant during the transmission of one OFDM symbol.
- The number of subcarriers is more than the number of users.
- One subcarrier is assigned to one user only. In literature, it is proved that such an exclusive subcarrier assignment is the optimal solution while considering throughput maximisation [10]. We consider the same for the case with fairness also.
- We use continuous Shannon channel capacity formula as the user throughput measure. In practical systems, discrete data rates are adopted due to different modulation and coding schemes. The continuous Shannon capacity formula, however, simplifies the analysis of adaptive resource allocation and provides an upper bound on the achievable throughput [8].
- It is assumed that the users have full buffer queues. i.e., the user will always have some data to transmit when we assign resources to the user. Although the amount of user data is limited in practice, there is always a subset of users who require an opportunity to communicate. Hence, the resource allocation algorithms presented in this thesis can be applied to those active users. [8]

2.3 System Model

In this section, the system model that has been used in this thesis is described in detail.

We consider the downlink of a single cell OFDMA base station (BS). There is a total power constraint on the BS which is P. There are K users in the system sharing N subcarriers (Figure 2.1) [16].

The channel is assumed to be frequency selective, Rayleigh-faded and the channel gain of the k^{th} user on the n^{th} subcarrier is denoted as $h_{k,n}$. However the base station doesn't have perfect channel knowledge. At the transmitter, the channel is estimated as $\hat{h}_{k,n}$. The channel estimation error has been characterised as an additive Gaussian

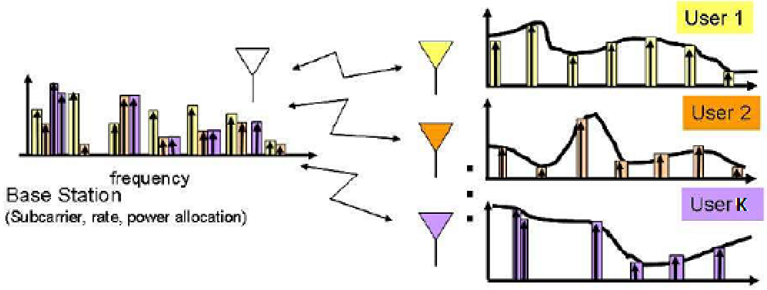


Figure 2.1: System Model

noise, independent of the channel itself [2]. Hence the relation between the estimate and the actual channel gain can be stated as:

$$\hat{h}_{k,n} = h_{k,n} + e_{k,n} \tag{2.1}$$

where, $e_{k,n}$ is a circularly-symmetric, complex Gaussian random variable with zero mean and variance σ_e^2, i.e.,

$$e_{k,n} = \mathcal{CN}(0, \sigma_e^2)$$

The value of σ_e^2, the error variance, depends on the duration of feedback delay, quantization errors and accuracy of the channel estimate. If these effects are larger, the value of the variance also becomes larger.

CHAPTER 3

REVIEW OF RELATED WORK

In this chapter, some of the recent publications in the area of downlink resource allocation for OFDMA systems are reviewed. There has been a considerable work in this area covering different aspects of resource allocation. The idea of using channel information at the transmitter to improve the performance of communication systems have been around since 1968 [16]. Much of the earlier work has focused on resource allocation based on perfect channel knowledge at the transmitter [14], [7], [6], [17], [13]. However imperfect CSIT seems to be a more valid assumption and is recognised in recent publications [15], [2], [10], [12].

The most common approach in the literature for solving resource allocation problem is by considering it as an optimisation problem. In general, there are two main classes of resource allocation schemes depending on the objective function [16]:

* **Margin adaptive** - The optimization problem is posed as minimization of transmit power subject to Quality of Service (QoS) constraints for each user which may be a combination of data rate, bit error rate, delay etc.

* **Rate adaptive** - The optimization problem is posed as maximisation of data rate subject to power constraint and other QoS constraints.

We consider Rate adaptive resource allocation scheme in this thesis.

The papers can be classified based on the optimization metrics that they have employed. Various metrics have been used in literature like the capacity lower bound [12], ergodic sum utility [2], goodput (average successfully transmitted rate) [10], channel capacity, throughput, outage probability [13], SINR, BER [7] etc. One major effect of imperfect channel knowledge is that the transmission may result in outage. Hence when we consider imperfect CSIT, maximising goodput appears to be a good metric as it incorporates the impact of the imperfect channel knowledge on the system performance. Also, rate allocation becomes important in imperfect CSIT cases, since it is important to minimize outage [4].

Though different metrics are used in the different papers, one common aspect that is found in all the papers which are based on imperfect CSIT assumption is that the channel uncertainty model used mostly is circular symmetric complex Gaussian [12], [10], [2], [15] and hence the same assumption is taken in this thesis too.

Different power allocation schemes have been suggested in literature. In case of perfect CSIT and no fairness constraint, the optimal power allocation method is water-filling [11] which results in maximising the capacity and hence, the throughput of the system. In the case of imperfect CSIT, modified waterfilling strategies are suggested as in [12]. When fairness constraints are incorporated, methods like Multilevel Water-filling are suggested [2] wherein each user is allotted a different water level. Iterative procedures are used to find this water level such that both fairness constraints and power constraints are satisfied.

In general, there are two kinds of OFDMA subcarrier allocation algorithms :

1. Throughput-oriented [10]
2. Fairness-oriented [9], [2].

Throughput-oriented allocation means that the allocation scheme will aim to maximise the sum rate of all users, so that if one user experiences a good channel, he will be given more resources and a user with a poor channel may be neglected. On the other hand, in fairness-oriented approaches [9], subcarrier allocation has been done to meet some measure of fairness.

There are different notions of fairness [7]:

- In terms of bandwidth, wherein all users are assigned equal number of subcarriers.
- In terms of power, wherein all users are assigned same power.
- In terms of data rate, wherein all users get equal data rate.
- Proportional fairness, wherein all users are assigned rates according to some weighting factor etc.

Some references like [7] suggest asymptotically-fair subcarrier allocation schemes. They achieve fairness by first grouping the users and using fairness oriented approach for subcarrier allocation for groups and then throughput oriented approach for subcarrier allocation to users inside a group. The grouping size determines the degree of fairness. However this has been proposed with perfect CSIT assumption [7]. Among the papers that consider fairness along with imperfect CSIT, [15], [2], define an Ergodic Sum Utility function that includes ergodic data rate and the long term fairness requirement.

Complexity reduction is also an important aspect in resource allocation schemes. Different papers have used multiple methods to address it. Ways to reduce complexity that have been identified are:

1. Using suboptimal algorithms [12]
2. Reformulating the optimisation problem as a dual problem [15]
3. Separating subcarrier allocation and power allocation [3], [14]
4. Sub-channelization - instead of per subcarrier allocation, a subchannel which is a group of sub-carriers is considered [2]
5. Using closed form approximations of quantities involved in the computations [10].

3.1 Comparison of Related work

A brief comparison of the most relevant papers read during literature survey is tabulated below:

Table 3.1: Literature Survey Analysis

Ref	Authors	Imperfect CSIT	Outage consideration	Fairness consideration
[15]	I. Wong et. al.	Yes	No	Yes
[7]	H. Rasouli	No	No	Yes
[2]	L. Vandendorpe et. al.	Yes	No	Yes
[9]	Zukang Shen et. al.	No	No	Yes
[10]	S. Stefanatos et. al.	Yes	Yes	No
[5]	Mehrdad. et. al.	No	No	Yes

From Table 3.1, it can be observed that there hasn't been a resource allocation scheme which included all the three factors together - Imperfect CSIT, Outage and Fairness considerations. Thus the literature survey leads to the following question - **How to allocate resources in a multiuser OFDM downlink with imperfect CSIT such that**

the successfully transmitted rate is maximised and fairness is maintained among the users? The key aim of this project is to find an answer to this question. To the best of our knowledge, the approaches presented in this thesis are the first to consider all the three requirements.

CHAPTER 4

PROPOSED RESOURCE ALLOCATION SCHEMES

4.1 Problem Formulation

The objective is to allocate the resources - rate, power and subcarrier in the downlink of an OFDMA system. Assume there are K users and N subcarriers with the received signal at each user terminal being corrupted by additive white Gaussian noise (AWGN). Assume $N \geq K$. Without loss of generality the AWGN sample variance is normalized to unity. The signal gain is varied to achieve the desired SNR. Let the channel gain experienced by user k on subcarrier n be denoted as $h_{k,n}$ and power allotted on that link be $p_{k,n}$. Then the bandwidth normalized capacity is given by [11]

$$C_{k,n} = log_2(1 + |h_{k,n}|^2 p_{k,n})$$

If the rate allotted on a particular link exceeds the capacity, then outage occurs. Especially since resource allocation has to be done based on imperfect CSIT, there is a finite probability that the rate allocated may result in outage. It is expressed as $P_{out} \triangleq Pr\{r_{k,n} > C_{k,n}\}$. Hence even though many authors consider the ergodic capacity as a metric to be maximised through resource allocation [15], [2], a more appropriate measure is the average successfully transmitted rate over a subcarrier, referred to as goodput and defined as [10]

$$G_{k,n} \triangleq r_{k,n}\overline{P}_{out}$$

where $\overline{P}_{out} \triangleq 1 - P_{out}$ is the probability of successful transmission.

Let $\hat{a}_{k,n}$ denote the subcarrier assignment indicator, i.e., $\hat{a}_{k,n}$ =1 indicates that subcarrier n is allocated to user k and $\hat{a}_{k,n}$ = 0 otherwise. It is also assumed that the maximum power the base station can allocate is P. The aim of our work is to maximise the goodput subject to a fairness constraint that the average successful rate $\overline{R_k}$ is proportional to the set of weighting factors W_k. The weighting factor is particularly useful in multiservice networks where the users demand different services like data service, voice service etc [5]. Such a fairness constraint will take into account the different rate requirements of different users. For example, consider a 4 user system where user 1 and user 2 need the same rate whereas user 3 requires twice the rate of user 1 and user 4 requires thrice the rate of user 1. The weighting factors chosen such that this is incorporated are $W_1 = 1, W_2 = 1, W_3 = 2, W_4 = 3$.

The problem can thus be mathematically formulated as : [10], [5] :

Objective function: Maximise sum goodput

$$\max_{\{r_{k,n}\},\{p_{k,n}\},\{\hat{a}_{k,n}\}} \sum_{k=1}^{K}\sum_{n=1}^{N} \hat{a}_{k,n} G_{k,n} \tag{4.1}$$

$$\begin{aligned}
\sum_{k=1}^{K}\sum_{n=1}^{N} \hat{a}_{k,n} p_{k,n} &\leq P && (C_1) \text{ Power Constraint}\\
\sum_{k=1}^{K} \hat{a}_{k,n} &\leq 1 \ \forall\, n && (C_2) \text{ Subcarrier Allocation Constraints}\\
\hat{a}_{k,n} &\geq 0 \ \forall\, k,n && (C_3)\\
p_{k,n} &\geq 0 \ \forall\, k,n && (C_4) \text{ Non negativity of power}\\
r_{k,n} &\geq 0 \ \forall\, k,n && (C_5) \text{ Non negativity of rate}\\
\frac{\overline{R}_1}{W_1} = \frac{\overline{R}_2}{W_2} = &\ \ldots \ = \frac{\overline{R}_K}{W_K} && (C_6) \text{ Fairness Constraint}
\end{aligned}$$

The aim is to achieve a good balance between throughput and fairness. The optimal method will be to solve this resource allocation problem jointly. However to reduce complexity, it is done in 2 steps [9] :

Step 1: Rate and Power Allocation - Find the rates and power that can be allotted for every possible subcarrier - user combination assuming subcarrier allocation is done to maximise goodput without considering fairness, i.e., only constraints C_1 to C_5 are considered.

Step 2: Subcarrier Allocation - Heuristic approaches for subcarrier allocation such that fairness constraint C_6 is satisfied.

4.2 Rate and Power Allocation (Step 1)

As described above, the aim is to find the rate and power to be allocated to maximise goodput. So the optimisation problem considered will be the same as earlier but with constraints C_1 to C_5. This is similar to the approach in [10]. For reducing complexity of the algorithm, optimization of power allocation is not done. Instead, we choose equal power allocation for all subcarriers.

$$p_{k,n} = \hat{a}_{k,n}\frac{P}{N}$$

This approach may not reach the same performance as 'Optimal Power Allocation' (OPA) but it is shown that OPA is advantageous only under operation in very low signal to noise ratios [10]. The fact that the difference between OPA and equal power allocation is small is justified through simulations and depicted in Figure 2 of [10].

At the transmitter, we have only imperfect channel knowledge ($\hat{h}_{k,n}$), so the capacity of the channel as estimated at the transmitter end will be

$$\hat{C}_{k,n} = log_2(1 + |\hat{h}_{k,n}|^2 p_{k,n})$$

where $\hat{h}_{k,n}$ is the estimate of the channel gain. Since there is error in estimation, if we allocate a rate suggested by capacity given by above equation, there is a finite probability of outage. So to reduce the outage probability and thereby increase average successful transmission rate, the rate allocated should be less than the nominal capacity. In other words, a rate back off is required. Mathematically, the rate allocated in each link should be

$$r_{k,n} = \hat{C}_{k,n} + \Delta C_{k,n}$$

where $\Delta C_{k,n}$ is a negative quantity. If the absolute value of ΔC is larger, then the transmission rate will be less and if the value is lower, the outage probability is higher and so the probability of successful transmission will be lower. Thus an optimal value is required. As mentioned earlier, this optimisation problem has been considered in [10] and under equal power allocation scheme, the optimal solution proposed there is:

$$\Delta C_{k,n} = \frac{2\sigma_{k,n}^2 - \sqrt{\sigma_{k,n}^2(2\hat{C}_{k,n}^2 - \hat{C}_{k,n}\sqrt{2\pi\sigma_{k,n}^2} + 4\sigma_{k,n}^2)}}{\hat{C}_{k,n}}$$

where

$$\sigma_{k,n}^2 = a_{k,n}2ln2/(a_{k,n} + b_{k,n})^2 \text{ , } a_{k,n} \triangleq |\hat{h}_{k,n}|^2/\sigma_e^2 \text{ , } b_{k,n} \triangleq 1/p_{k,n}\sigma_e^2,$$

$\hat{h}_{k,n}$ is the estimate of the actual channel gain $h_{k,n}$ and the variance of error made in this estimation is σ_e^2 as described in the system model.

The above mentioned equation for $\Delta C_{k,n}$ is used to find $r_{k,n}$ only when $C_{k,n} \geqslant \sigma_{k,n}\sqrt{\pi/2}$ holds. Otherwise, no solution exists and that particular subcarrier-user combination is not considered as a valid transmission link. The detailed derivation of the rate back off is given in Appendix A.

4.3 Subcarrier Allocation (Step 2)

It is proposed that fairness can be achieved through proper subcarrier allocation [5]. In general, fairness means that the resources are shared equitably and in our context, it means that we meet the proportional rate constraint given by C_6.

There are two notions of fairness in literature :-

1. Short-term fairness
2. Long-term fairness

Short-term fairness of a scheme refers to its ability to allocate the resources fairly to all the users in a short time scale whereas long-term fairness means that we ensure fairness by considering the resources allocated over a longer time scale. This can be clearly understood from figure 4.1 which shows how long-term and short-term fairness schemes perform in a two user system where both users require same data rate. It shows that even if the successfully transmitted rate for two users are different in earlier slots, the short-term based scheme allocates equal rates to them whereas long-term based scheme allocates different rates so that the average rates are same for the two users.

In our context, short-term fairness will mean that the metric used in subcarrier allocation will have no memory of previous allocations and is completely dependent on the current allocation. Let such a variable for the k^{th} user be denoted as r'_k. For the realization of long-term fairness, we use the concept of Exponentially Weighted Moving Average (EWMA). It means that after each channel realization, the metric, say, $\hat{R}_k$ will be updated as

$$\hat{R}_k(t) = \lambda \hat{R}_k(t-1) + (1-\lambda) R_k(t-1), \tag{4.2}$$

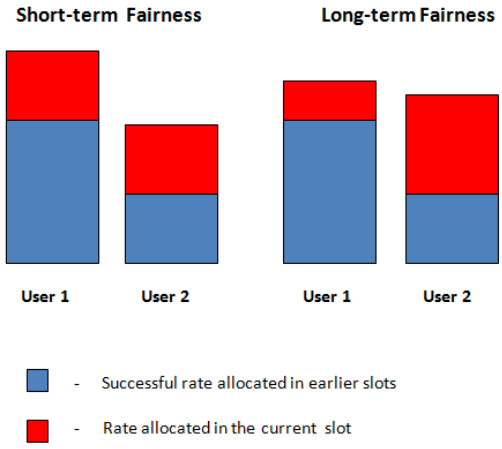


Figure 4.1: Short-term and Long-term Fairness

where $R_k(t-1)$ is the successful transmitted rate to user k in the previous channel realization and λ is forgetting factor, a constant which will determine how much of the earlier allocation, the algorithm should consider to decide the present resource allocation. Typically $\lambda \approx 0.8 - 0.95$. In thesis, the value used is $\lambda = 0.9$.

4.4 Resource Allocation Strategies

Based on the above mentioned constraints, five strategies have been proposed. All strategies use the rate back-off calculated after step 1 of the resource allocation policy as described in the section 4.2. Let this rate be denoted as $r_{k,n}$ i.e., the rate after back-off for user k on subcarrier n. Also let G be the KxN matrix denoting goodput as defined in eqn (A.4) A brief overview of the strategies is as below:

4.4.1 Strategy 1

The motivation behind Strategy 1 is [9] which suggests a subcarrier allocation scheme which will guarantee proportional fairness. However that scheme is based on the assumption of perfect CSIT and hence the algorithm is based on channel gain. Since in our case, we have imperfect channel knowledge, goodput seems to be a more appropriate metric for comparison and hence the algorithm is modified to incorporate that. The various steps of the algorithm are:

Figure 4.2: Flowchart representing the proposed resource allocation schemes

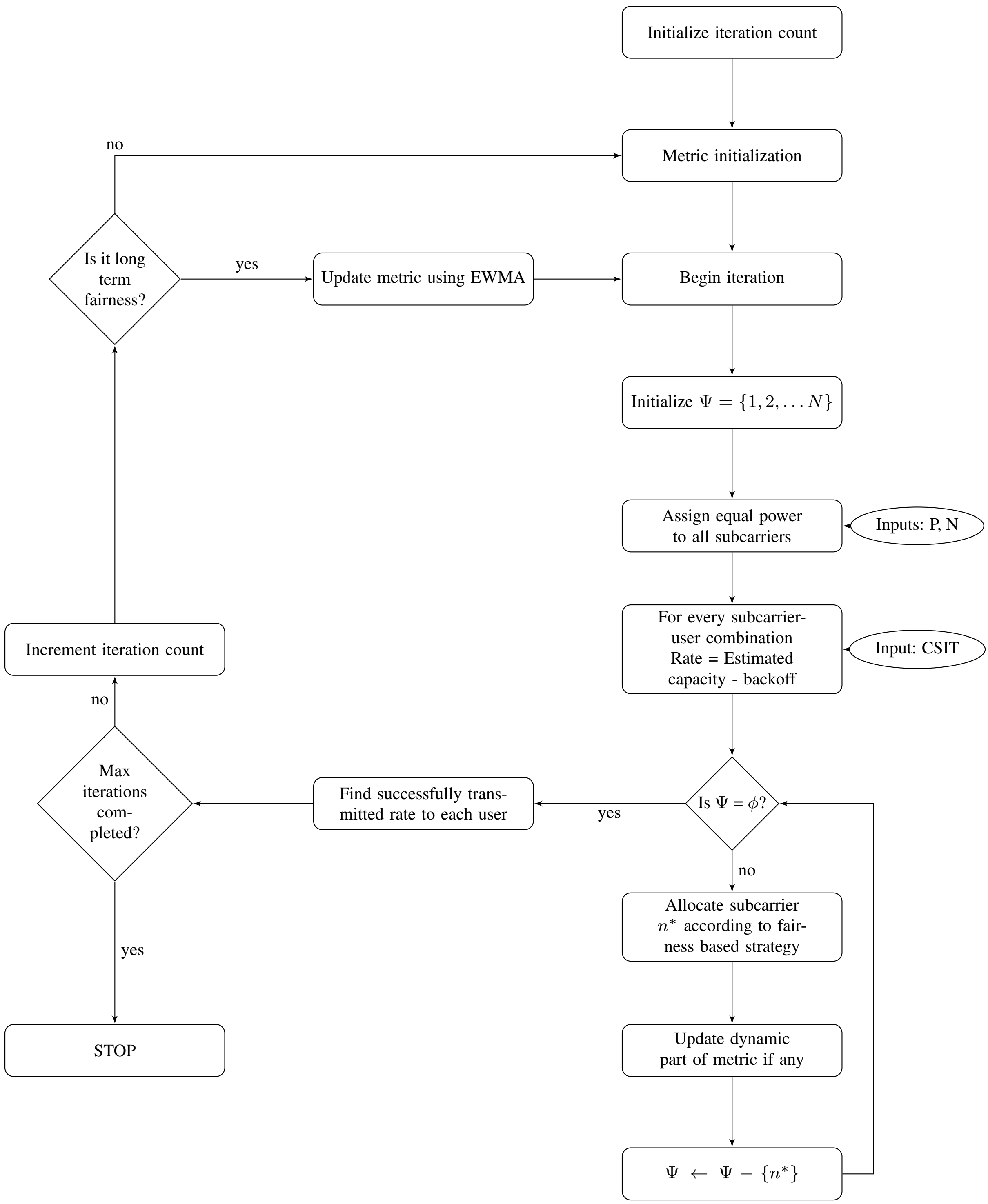


Table 4.1: Subcarrier Allocation Strategies

Strategy	Short or long term	Metric deciding User Priority	Dynamic updating of R_k
1	Short term	$\frac{r'_k}{W_k}$	Yes
2	Long term	$\frac{G_{k,n}}{(\hat{R}_k/W_k)}$	No
3	Long term	$\frac{\hat{R}_k + r'_k}{W_k}$	Yes
4	Long term	$\frac{\hat{R}_k}{W_k}$	No
5	Long term	$\frac{G_{k,n}}{((\hat{R}_k + r'_k)/W_k)}$	Yes

Step 1: Initialization

- For k=1 to K , set $r'_k = 0$
- Let Ψ be the set of all subcarriers to be allotted, so initialise $\Psi = \{1, 2, \ldots, N\}$
- Let S_k be the set of subcarriers allotted to user k. So, set all $S_k = \phi$

Step 2: Subcarrier Assignment - This is done in two steps:

(a) Each user is assigned the subcarrier on which the goodput is maximum.

- For k = 1 to K , find $n^* \in \Psi$ such that $G_{k,n^*} \geqslant G_{k,i}\ \forall\, i \in \Psi$ and update $r'_k = r_{k,n^*}$. Also remove the subcarrier n^* from the set Ψ and add to set S_k

(b) Assign subcarriers to users based on r'/W

Find $k^* = \underset{k}{\text{argmin}}\ \ r'_k/W_k$ and then $n^* = \underset{n\in\Psi}{\text{argmax}} G_{k^*,n}$ and update the following:

- Allocate to user k^*, the subcarrier n^* i.e., $S_{k^*} \leftarrow S_{k^*} \cup \{n^*\}$
- Update the rate $r'_{k^*} = r'_{k^*} + r_{k^*,n^*}$
- Remove the selected subcarrier n^* from the set Ψ.

Repeat Step 2b until the set Ψ is empty.

Step 3: Re-initialisation Once all the subcarriers are allotted, r'_k is again set to zero for every user and all the steps are repeated for the next channel realization.

4.4.2 Strategy 2

A drawback of Strategy 1 is that it is based on the notion of short-term fairness whereas in a scenario where resource allocation is done based on imperfect CSIT, it is better to employ long-term fairness since at the time of allocation, there is no guarantee that the rate allocated will not result in outage. This is taken care of in Strategy 2. Though there are different ways of incorporating long-term fairness, we adopt the method of Exponentially Weighted Moving Average (EWMA) as suggested in reference papers like [9]. In Strategy 2, we propose a procedure which combines both goodput and $\hat{R}_k/W_k$ ratio into a single step by adopting $\dfrac{G_{k,n}}{\hat{R}_k/W_k}$ as the metric to decide priority of users. The steps are:

Step 1:Initialisation

- $\hat{R}_k$ is initialised to a small value for every user. This is only for the first channel realization.
- λ is the forgetting factor and is assigned a suitable value, $\lambda\epsilon[0.8, 0.95]$. If the value is closer to one, more emphasis is given to earlier resource allocations.
- Let S_k be the set of subcarriers allotted to user k. So, set all $S_k = \phi$

Step 2: Subcarrier Assignment

For n=1 to N

- Find $k^* = \underset{k}{\text{argmax}} \dfrac{G_{k,n}}{(\hat{R}_k/W_k)}$
- Allocate to user k^*, the subcarrier n i.e., $S_{k^*} = S_{k^*} \cup \{n\}$

Suppose the $\dfrac{G}{\hat{R}/W}$ matrix (K x N matrix) is like :

$$\begin{bmatrix} 3 & 1 & 2 & 2 & 1 \\ 1 & 2 & 3 & 3 & 2 \\ 2 & 3 & 1 & 1 & 3 \end{bmatrix}$$

where, $3 > 2 > 1$ and it denotes the order of the corresponding $\frac{G}{\hat{R}/W}$. Then the subcarrier allocation matrix (K x N matrix) obtained through Strategy 2 is:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Step 3: Updates

- $\hat{R}_k$ is constant throughout the slot (channel realization) under consideration. At the end of the slot, after all subcarriers are allotted, find the successful transmitted bits for each user, say, R_k. Transmission on a link is successful if there is no outage i.e., $r_{k,n} \leq C_{k,n}$
- $\hat{R}_k$ is updated for the next slot with this successful transmitted rate based on EWMA as given in eqn (4.2)

4.4.3 Strategy 3

Since long-term fairness appears to be a right metric for imperfect CSIT case, a modification to Strategy 1 is being proposed to update the rate based on EWMA. This is the basis of Strategy 3. The steps are:

Step 1: Initialisation

- For the first channel realization, $\hat{R}_k$ is initialized to zero.
- Set $r'_k = 0$, to denote the rate that has been allocated to user k in the current channel realization (it is not the successfully transmitted bits but rather the rate allocated). This is for dynamically updating the rate. The reason for using a separate variable for the dynamic update is that it is not known beforehand whether the transmission on that link will be successful or not, since the transmitter has only imperfect channel knowledge.
- Let Ψ be the set of all subcarriers to be allotted, so initially $\Psi = \{1, 2, \ldots, N\}$

- Let S_k be the set of subcarriers alloted to user k. So set all $S_k = \phi$

Step 2: Subcarrier Assignment

- For every user k from 1 to K
  - ∗ Find $n^* \epsilon \Psi$ such that $G_{k,n^*} \geqslant G_{k,i} \; \forall \; i \; \epsilon \; \Psi$
  - ∗ Update $r'_k = r_{k,n^*}$
  - ∗ $\Psi \leftarrow \Psi$ - $\{n^*\}$
  - ∗ $S_k \leftarrow S_k \cup \{n^*\}$
- While $\Psi \neq \phi$
  - ∗ Pick the user $k^* = \underset{k}{\text{argmin}}(\hat{R}_k + r'_k)/W_k$
  - ∗ Allocate the subcarrier on which the goodput is best, i.e., $S_{k^*} = S_{k^*} \cup \{n^*\}$ where $n^* = \underset{n \in \Psi}{\text{argmax}} G_{k^*,n}$.
  - ∗ Update the dynamic rate variable as $r'_{k^*} = r'_{k^*} + r_{k^*,n^*}$
  - ∗ $\Psi \leftarrow \Psi$ -$\{n^*\}$

Step 3: Updates

- Once all subcarriers are allocated, find the successfully transmitted bits for each user, say, R_k.

$$R_k = \sum_{n \in S_k} r_{k,n} I_{k,n}$$

  where,

$$I_{k,n} = \begin{cases} 1 & r_{k,n} \leq C_{k,n} \\ 0 & \text{Otherwise} \end{cases}$$

- R_k is updated for the next iteration based on EWMA with this successful transmitted rate as in eqn (4.2).
- Repeat from Step 1 for subsequent channel realizations

4.4.4 Strategy 4

Strategy 4 results from the search for a method which uses the similar metric as Strategy 1 and 3, namely $\hat{R}_k/W_k$ for deciding priority of users, but incorporates long-term fairness without using dynamic updates. This requirement is met through Strategy 4 and it involves the following steps:

Step 1: Initialise $\hat{R}_k$ to be zero for all users.

Step 2: Arrange the users in increasing order of $\hat{R}_k/W_k$. Let the order be $u_1, u_2, u_3 \ldots u_K$

Step 3: Set $\Psi = \{1, 2, \ldots, N\}$ and $S_k = 0$ for $k = 1, 2 \ldots, K$

Step 4: The users are selected in the order $u_1, u_2, u_3 \ldots u_K$. This sequence is repeated until all subcarriers are allocated. For each user :

- Find $n^* = \underset{n \in \Psi}{\text{argmax}} G_{k,n}$.
- $S_k \leftarrow S_k \cup \{n^*\}$
- $\Psi \leftarrow \Psi$ -$\{n^*\}$

Step 5: Once all subcarriers are allocated, find the successfully transmitted bits for each user, say, R_k. $\hat{R}_k$ is updated for the next iteration based on EWMA with this successful transmitted rate, as in eqn (4.2).

Step 6: Repeat from Step 2 for subsequent channel realizations.

4.4.5 Strategy 5

Strategy 5 includes the dynamic updating of the rate in Strategy 2. The importance of dynamic updating is that it helps to satisfy the fairness constraint more closely, since the priority of users may change with dynamic updating. The following steps are proposed for this strategy:

Step 1:Initialisation

- Initialise $\hat{R}_k$ to a small value for every user.
- Set r'_k to zero for $k = 1, 2, \ldots K$

- Let Ψ be the set of all subcarriers to be allotted, so initialise $\Psi = \{1, 2, \ldots, N\}$

Step 2: Subcarrier Assignment

- Find $(k^*, n^*) = \underset{(k,n), n \in \Psi}{\operatorname{argmax}} \frac{G_{k,n}}{(\hat{R}_k + r'_k)/W_k}$
- Allocate subcarrier n^* to user k^* and update $r'_{k^*} = r'_{k^*} + r_{k^*,n^*}$
- Eliminate n^* from the set Ψ and iterate until all subcarriers are allocated. i.e., until set Ψ is empty.

Step 3: Updates

- Once all the subcarriers are allocated, find the successfully transmitted bits for each user, say, R_k.
- $\hat{R}_k$ is updated for the next iteration based on EWMA with this successful transmitted rate according to eqn (4.2).
- Re-initialise r' to 0 and $\Psi = \{1, 2, \ldots, N\}$. Repeat the procedure from Step 2 for subsequent channel realizations.

4.5 Analysis of Proposed Algorithm

The key features of the proposed algorithms are:

* To reduce complexity, the resource allocation problem is decoupled into two - The Rate and power allocation step, and The Subcarrier allocation step. Also flat power allocation to all subcarriers further reduces complexity and gives good performance which is close to the optimal power allocation.
* To account for imperfect CSIT, rate back off is used.
* To account for outage, we use goodput as the optimization metric rather than capacity
* Long term fairness is achieved through EWMA
* Use of dynamic updating provides flexibility.

4.6 Summary

In this chapter, the resource allocation schemes that have been proposed in this thesis for the OFDMA downlink are discussed. It is done with imperfect CSIT with outage considerations but maintaining proportional fairness among users. In a nutshell, the scheme is:

1. Allocate equal power to all subcarriers.
2. Allocate a rate that is less than the estimated nominal capacity by a back off value found using optimization techniques
3. Allocate subcarriers according to any of the five strategies proposed to maintain fairness.

CHAPTER 5

SIMULATION RESULTS

In this chapter, the performance of the proposed resource allocation schemes is evaluated using MATLAB simulations. The performance metrics used are the system's total successful throughput (normalized by the number of subcarriers and bandwidth of the system) and fairness. To measure fairness, Jain's fairness index [1] is used which is defined as :

$$J = \frac{\left(\sum_{k=1}^{K} z_k\right)^2}{K\sum_{k=1}^{K} z_k^2}, \text{where } z_k = \frac{\overline{R}_k}{W_k},$$

$\overline{R}_k$ is the total successful rate received by user k. The system is absolutely fair when $J = 1$ and absolutely unfair when $J = \frac{1}{K}$

A single cell scenario with 1 BS, K users (MS) and N=32 subcarriers is considered. The wireless channel is modeled as a six tap frequency-selective Rayleigh faded channel. Jakes' model is used to generate the Rayleigh coefficients for the channel. The channel estimate is assumed to be imperfect and the error variance of estimate is taken as $\sigma_e^2 = 0.1$. **The value of forgetting factor λ that is used is 0.9.** The results are averaged over 10,000 channel realizations.

We will consider two categories of users [9]:

1. **Homogeneous users:** Users which are at the same distance from the base station. This scenario is depicted in Figure The average SNR is considered to be the same for all the users. In the simulations, the average SNR is taken to be 10 dB. The users may or may not have the same rate requirements. This is incorporated using weighting factors in the fairness constraint.

2. **Heterogeneous users:** Users which are at different distances from the base station. This scenario is depicted in Figure .The users will experience large scale

fading which is distance dependent and the path loss exponent is 2 [9]. Regarding rate requirement, the users maydiffer, however, in the simulation, we consider that all the users are having same rate requirement (unity weighting factor).

Simulations are performed both for homogeneous users (users having same average SNR) and heterogeneous users(users having different average SNR). The average SNR is taken to be 10 dB in the case of homogeneous users.

Six resource allocation schemes are compared (Schemes A to F) as described in Table 5.1. This is done for each of the strategies proposed- If the scheme considers fairness, the subcarriers are allotted according to the strategy under consideration and if the scheme doesn't consider fairness, then subcarriers are allotted to maximise throughput.

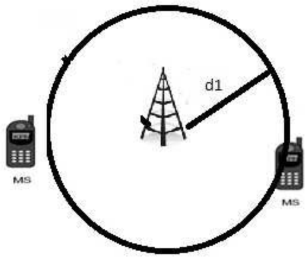
Table 5.1: Resource Allocation Schemes Compared

Scheme	Description		
	CSIT	Rate allocated	Fairness
A	Perfect	$r_{k,n} = C_{k,n}$	No
B	Perfect	$r_{k,n} = C_{k,n}$	Yes
C	Imperfect	$r_{k,n} = \hat{C}_{k,n}$	No
D	Imperfect	$r_{k,n} = \hat{C}_{k,n}$	Yes
E	Imperfect	$r_{k,n} = \hat{C}_{k,n} + \Delta C_{k,n}$	No
F	Imperfect	$r_{k,n} = \hat{C}_{k,n} + \Delta C_{k,n}$	Yes

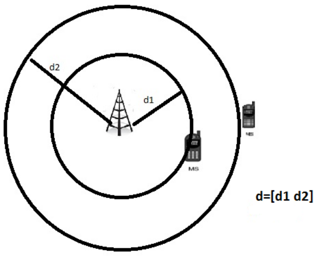
5.1 Performance Analysis of different Strategies

This section contains simulation results obtained using different strategies. The performance is obtained for 4 cases:

- Case 1: Homogeneous users experiencing uncorrelated fading - This is the case of high Doppler.
- Case 2: Homogeneous users experiencing correlated fading - This is the case of low Doppler.
- Case 3: Homogeneous users with different rate requirement
- Case 4: Heterogeneous users



a)Homogeneous Users



b)Heterogeneous Users

Simulation of uncorrelated fading : In case 1, to simulate uncorrelated fading, frequency domain channel coefficients are generated as independent Rayleigh fading samples. This can be explained mathematically as follows:
Let the channel response in time domain be represented as $h[n]$. $H[k] = DFT(h[n])$ Output at the receiver (in frequency domain) is given by the relation $Y[k] = H[k] \times X[k]$. For the simulation of uncorrelated fading, we have generated N independent complex Gaussian values of H for every user.

Simulation of correlated fading : In case 2, to simulate correlated fading, sets of samples (each sample corresponds to one channel realization) are taken from the Jakes' model which are separated by $5T_c$, where T_c is the coherence time. Coherence time is calculated as

$$T_c = \frac{9}{16\pi f_d}$$

where, f_d is the Doppler frequency and it is taken as 30 Hz in the simulations. Each set contains 10 samples which are within the coherence time and are thus correlated. The channel is modeled as a six tap frequency-selective Rayleigh faded channel. So in the simulations, during each channel realization, the Jakes' model is invoked six times to generate the channel coefficients in each of the taps (i.e., $h[n]$ is obtained from the Jakes' model) and then 32 point FFT is taken to get the channel gains in frequency domain (i.e., $H[k]$).

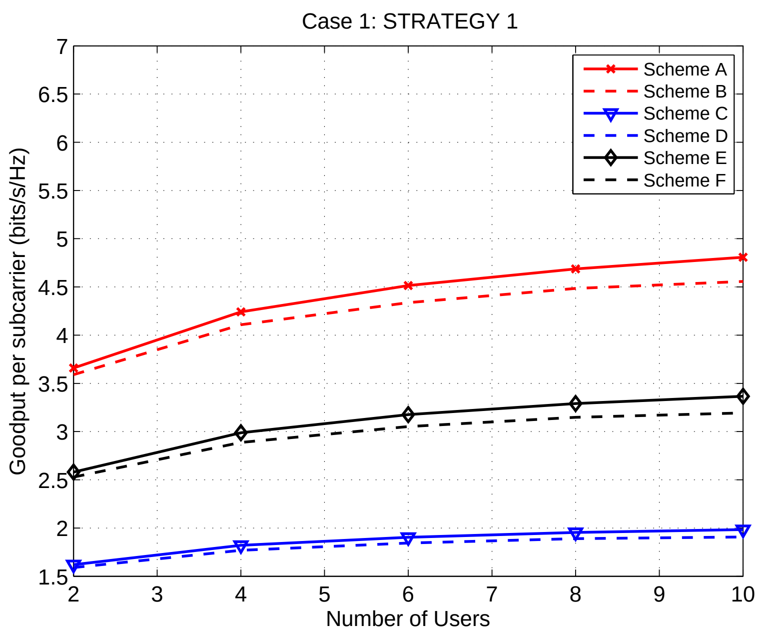


Figure 5.1: Uncorrelated Fading - Strategy 1

The observations that can be made from these simulation results (Figures 5.1 - 5.10) are as follows:

- As the number of users are increasing, the goodput is also showing an increasing trend. This is a manifestation of multiuser diversity in the system.

- Comparing schemes A,C,E in all the simulated cases, it is seen that average successful rate (goodput) follows the relation

$$\text{Goodput in A} > \text{Goodput in E} > \text{Goodput in C}.$$

  This is as expected since when we do resource allocation based on perfect CSIT (scheme A), there is no outage and hence goodput is maximum. In scheme E, since we are doing a rate back off, the outage has decreased compared to scheme C and hence the goodput is higher.

- When fairness is introduced also, similar trend is seen

$$\text{Goodput in B} > \text{Goodput in F} > \text{Goodput in D}$$

  It thus implies that the strategies proposed are robust to channel estimation errors.

- When fairness is introduced, the goodput is marginally reduced.

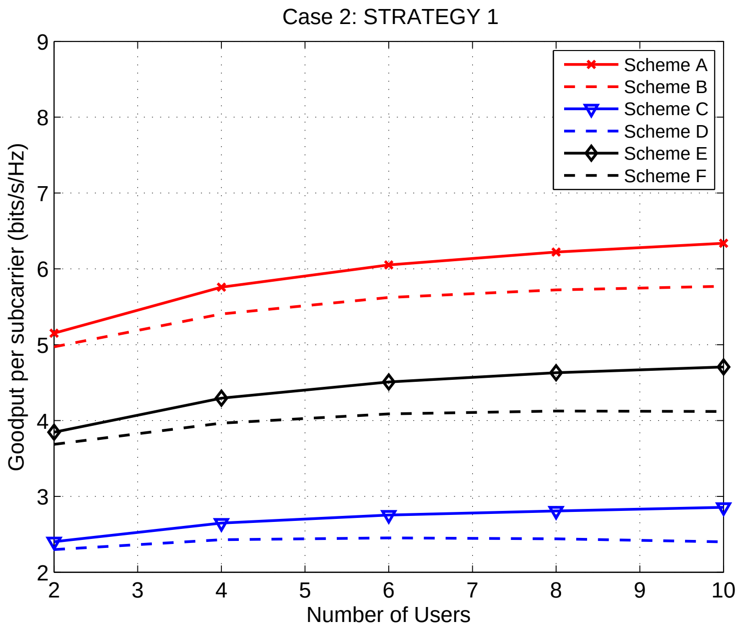


Figure 5.2: Correlated Fading - Strategy 1

Goodput in B < Goodput in A

Goodput in D < Goodput in C

Goodput in F < Goodput in E

This is justified because any throughput maximising scheme neglects poor channels whereas fairness based schemes allocate resources to them also thus reducing overall goodput. However the gap is not too much reflecting the fact that the strategies maintain an effective balance between throughput and fairness.

- The gap between fairness-based and throughput-based allocation schemes is more in the case of correlated fading than in uncorrelated fading.

(Goodput in A- Goodput in B) Case 2 > (Goodput in A- Goodput in B) Case 1

(Goodput in C- Goodput in D) Case 2 > (Goodput in C- Goodput in D) Case 1

(Goodput in E- Goodput in F) Case 2 > (Goodput in E- Goodput in F) Case 1

This is because, in the case of correlated fading, if the channel becomes bad, it remains so for a longer time but to ensure fairness, we need to use that channel also. In such a situation, it is likely that the average goodput becomes lower. Whereas in the case of uncorrelated fading since consecutive channel realisations are independent, it is unlikely that one channel remains bad for an extended time, so average goodput is not much affected even if it is a fairness-based scheme.

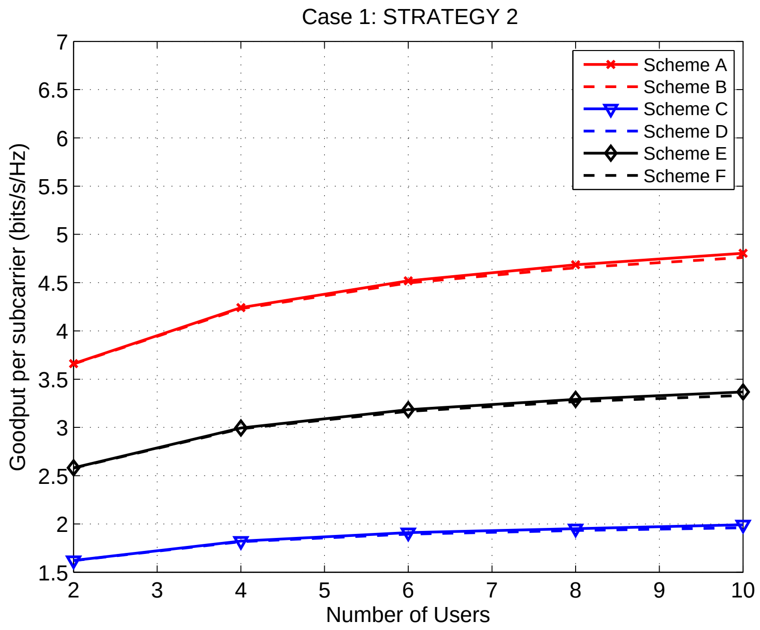


Figure 5.3: Uncorrelated Fading - Strategy 2

Table 5.2: Case 3: Strategy 1

$W = [W_1 W_2]$	Scheme A	Scheme B	Scheme C	Scheme D	Scheme E	Scheme F
[1 1]	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
[2 1]	0.9029	0.9998	0.9029	1.0000	0.9027	0.9999
[3 1]	0.7978	0.9994	0.7986	0.9999	0.7975	0.9998
[4 1]	0.7267	0.9986	0.7280	1.0000	0.7273	0.9996

Case 3 is represents a scenario wherein even though two users are having similar channel conditions, the rate requirement is different (incorporated as different weighting factors). A throughput oriented scheme in such a scenario may possibly end up allocating resources in such a way that the differences in rate requirements are not taken care of, hence actually being unfair. This is seen from the results tabulated in Tables 5.2 - 5.6. It is seen that the fairness-oriented schemes, Scheme B, D, and F attain higher fairness indices compared to Schemes A, C, and E respectively. This asserts the fact that the proposed strategies are effective in improving fairness in the system.

From the Tables 5.7 - 5.11, it can be seen that as the distance of user 2 from the base station increases with respect to user 1, there is more heterogeneity in the system. In other words, user 2 is more likely to have poorer channel conditions than user 1. In such a case, throughput-oriented approaches allocate more resources to user 1 thus decreasing fairness. This is shown by lower value of the Jain's fairness index for schemes

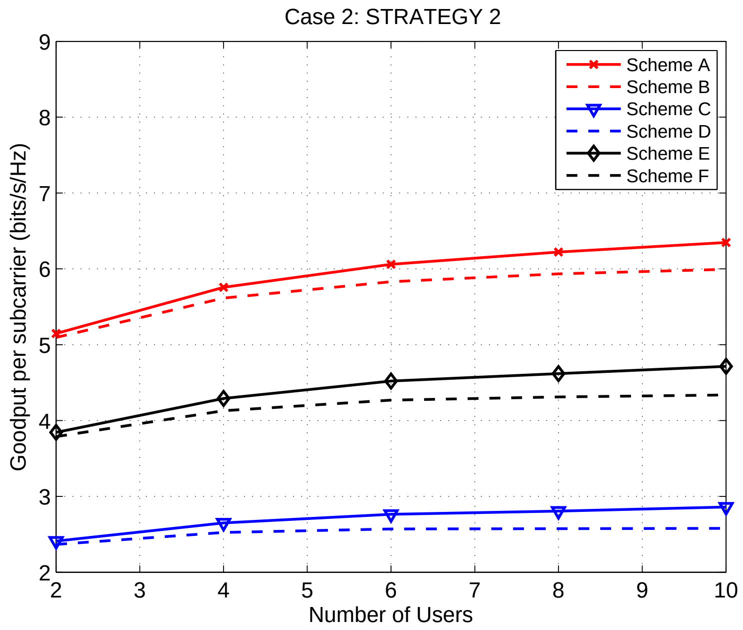


Figure 5.4: Correlated Fading - Strategy 2

Table 5.3: Case 3: Strategy 2

$W = [W_1 W_2]$	Scheme A	Scheme B	Scheme C	Scheme D	Scheme E	Scheme F
[1 1]	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
[2 1]	0.9000	0.9933	0.8994	0.9934	0.9007	0.9926
[3 1]	0.7945	0.9814	0.7938	0.9814	0.7941	0.9793
[4 1]	0.7351	0.9720	0.7358	0.9723	0.7351	0.9688

A, C, and E. On the other hand, fairness-oriented approaches allocate some resources to user 2 also and try to meet the fairness constraint. Hence Schemes B, D, and F achieve a higher fairness index. This is observed with all the strategies proposed. Hence it can be concluded that the strategies proposed are effective in ensuring fairness.

Fairness Measure using Gini Fairness Index

To measure fairness, there are different fairness indices that are mentioned in literature. Jain's fairness index that has been discussed above is one of the most popular fairness indices. Another fairness index that can be used is Gini fairness index [5]. It is defined as :

$$I = \frac{1}{2K^2\overline{z}} \sum_{x=1}^{K} \sum_{y=1}^{K} |z_x - z_y|$$

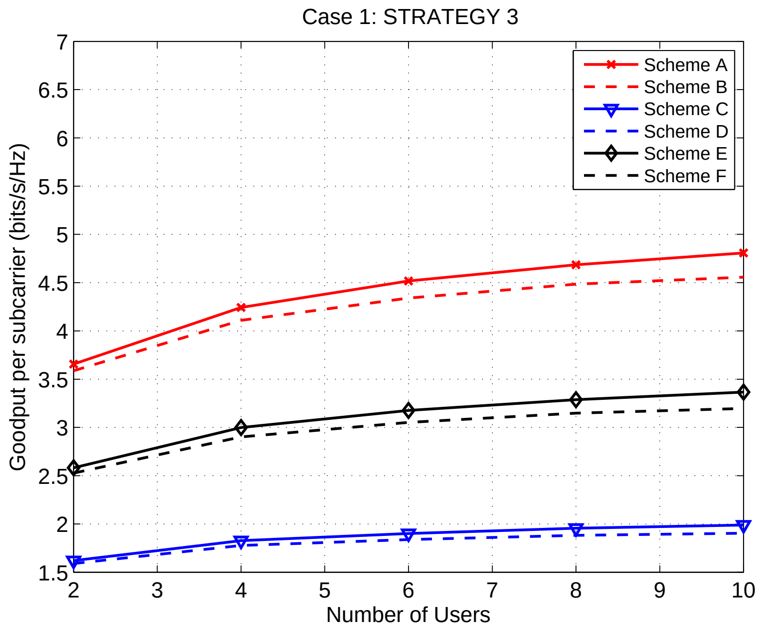


Figure 5.5: Uncorrelated Fading - Strategy 3

Table 5.4: Case 3: Strategy 3

$W = [W_1 W_2]$	Scheme A	Scheme B	Scheme C	Scheme D	Scheme E	Scheme F
[1 1]	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
[2 1]	0.9028	1.0000	0.9028	1.0000	0.9030	1.0000
[3 1]	0.7969	0.9998	0.7978	1.0000	0.7967	0.9999
[4 1]	0.7360	0.9997	0.7364	1.0000	0.7358	0.9998

where, $\overline{z} = \frac{\sum_{k=1}^{K} z_k}{K}$ and $z_k = \overline{R}_k / W_k$. The Gini fairness index varies between 0 and 1. A rate allocation is perfectly fair if $I = 0$. The higher value of I indicates higher unfairness among the users.

The performance of the strategies have been found using Gini Fairness index metric also. The same trend i.e., the Schemes B, D, and F are better in terms of fairness compared to Schemes A, C, and E is seen. This is justified by a lower value of Gini Fairness Index. The results for Strategy 3 performance in Case 4 is tabulated in Table 5.12.

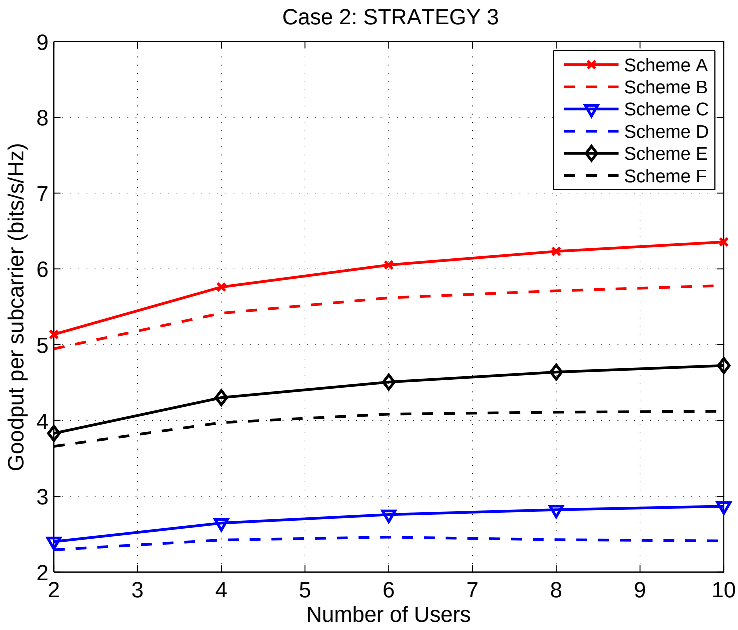


Figure 5.6: Correlated Fading - Strategy 3

Table 5.5: Case 3: Strategy 4

$W = [W_1 W_2]$	Scheme A	Scheme B	Scheme C	Scheme D	Scheme E	Scheme F
[1 1]	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
[2 1]	0.8996	0.9040	0.9005	0.9041	0.8996	0.9041
[3 1]	0.8004	0.8045	0.8007	0.8034	0.8009	0.8048
[4 1]	0.7385	0.7392	0.7386	0.7390	0.7384	0.7394

5.2 Comparison of Strategies

All the simulations for comparison have been done for Scheme F and correlated fading. However the observations can be extended to other fairness based schemes discussed also. On comparing the different strategies, it is found that

- From Figure 5.11, in terms of goodput, for homogeneous users:
  Strategy 2 $>$ Strategy 5 $>$ Strategy 4 $>$ Strategy 1 $>$ Strategy 3

- From Figure 5.12, in terms of fairness, for users with same data rate requirements (i.e., same weighting factors):
  Strategy 3 $>$ Strategy 1 $>$ Strategy 4 $>$ Strategy 5 $>$ Strategy 2

- From Figure 5.13, in terms of fairness, for users with different data rate requirements (i.e., different weighting factors):
  Strategy 3 $>$ Strategy 1 $>$ Strategy 2 $>$ Strategy 5 $>$ Strategy 4

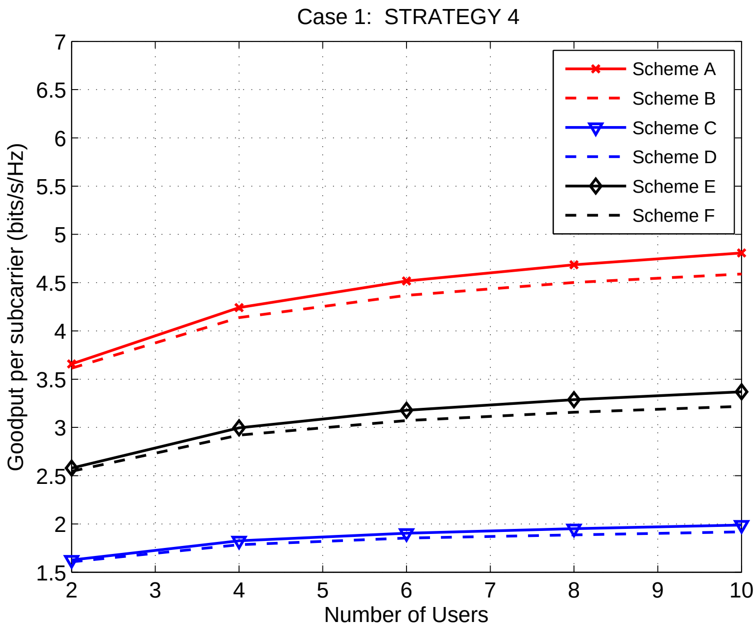


Figure 5.7: Uncorrelated Fading - Strategy 4

Table 5.6: Case 3: Strategy 5

$W = [W_1 W_2]$	Scheme A	Scheme B	Scheme C	Scheme D	Scheme E	Scheme F
[1 1]	1.0000	1.0000	0.9999	1.0000	1.0000	1.0000
[2 1]	0.8980	0.9872	0.8981	0.9861	0.8984	0.9855
[3 1]	0.8017	0.9704	0.8033	0.9689	0.8023	0.9666
[4 1]	0.7383	0.9525	0.7378	0.9496	0.7382	0.9464

- All strategies maintain a balance between goodput and fairness.

- For the case in which the different users require the same data rate, Strategy 4 may be better since it performs well in terms of both goodput and fairness. However, in the case in which different users require different weighting factors, Strategy 4 may not perform well in terms of fairness because in that strategy, since the ordering of users is fixed beforehand, the resources allocated may not reach the demand. In such cases, Strategy 3 can be considered as the best strategy in terms of fairness.

- From Figure 5.14, it is found that on increasing error variance, the gap between fairness indices of strategies based on short-term and long-term fairness increases and the one based on long-term fairness has higher Jain's index.

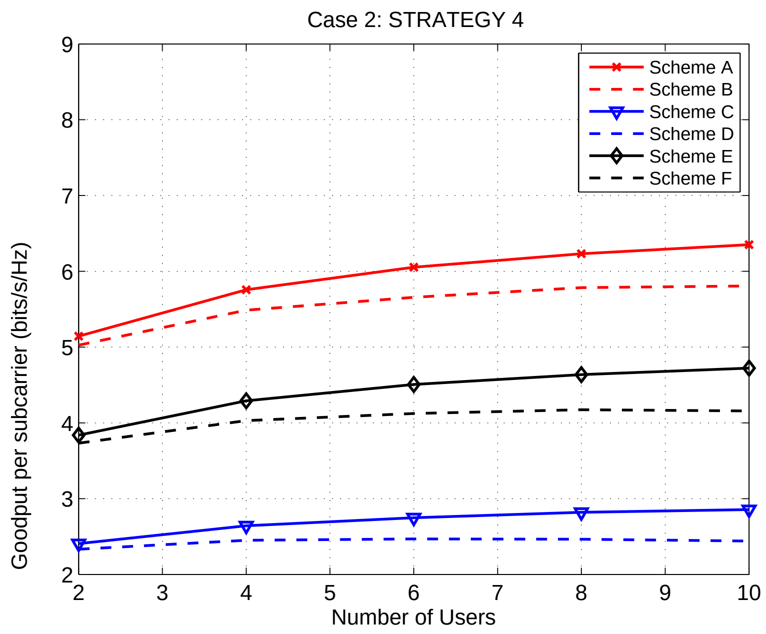


Figure 5.8: Correlated Fading - Strategy 4

Table 5.7: Case 4: Strategy 1

d	Scheme A	Scheme B	Scheme C	Scheme D	Scheme E	Scheme F
[1 1]	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
[1 2]	0.6769	0.9999	0.6722	1.0000	0.7071	0.9923
[1 3]	0.5644	0.9995	0.5620	1.0000	0.5789	0.9887
[1 4]	0.5295	0.9990	0.5282	1.0000	0.5359	0.9886

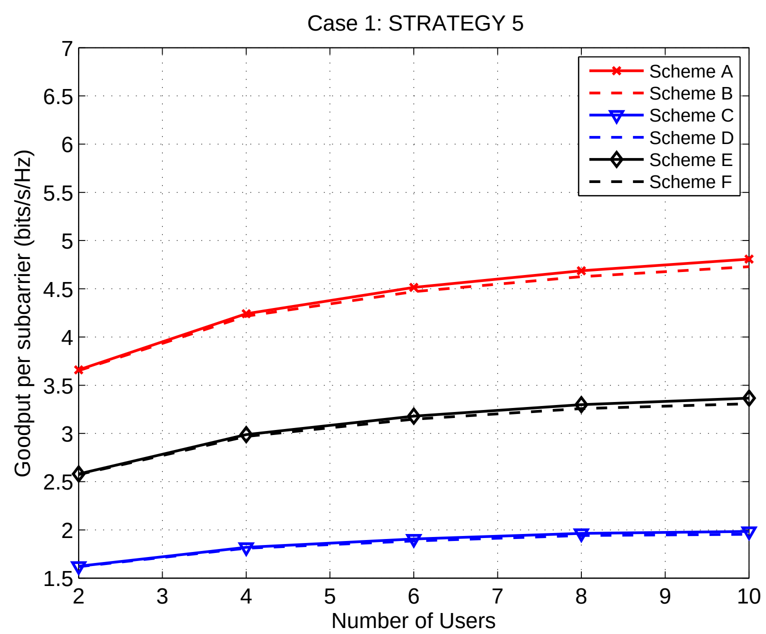


Figure 5.9: Uncorrelated Fading - Strategy 5

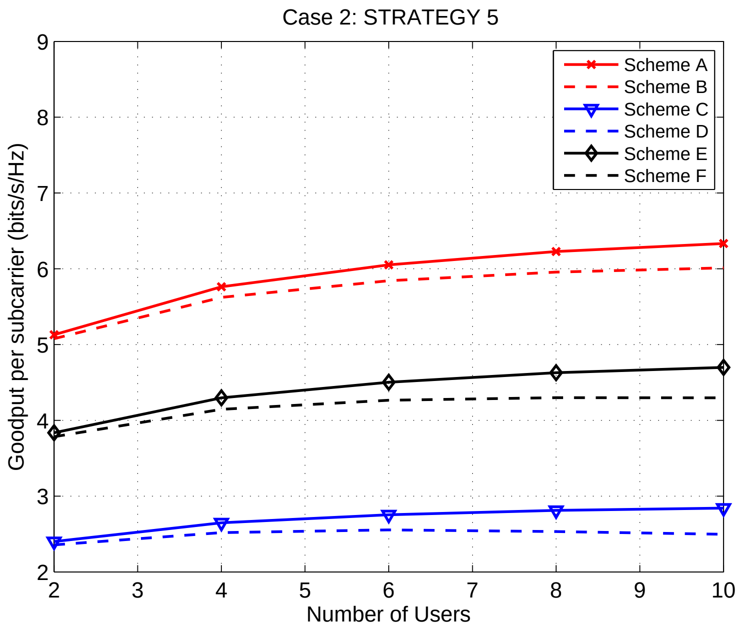


Figure 5.10: Correlated Fading - Strategy 5

Table 5.8: Case 4: Strategy 2

d	Scheme A	Scheme B	Scheme C	Scheme D	Scheme E	Scheme F
[1 1]	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
[1 2]	0.6768	0.9493	0.6728	0.9489	0.7072	0.9528
[1 3]	0.5669	0.8678	0.5651	0.8661	0.5827	0.8691
[1 4]	0.5275	0.7880	0.5268	0.7861	0.5335	0.7817

Table 5.9: Case 4: Strategy 3

d	Scheme A	Scheme B	Scheme C	Scheme D	Scheme E	Scheme F
[1 1]	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
[1 2]	0.6781	1.0000	0.6733	1.0000	0.7088	0.9977
[1 3]	0.5672	0.9999	0.5650	1.0000	0.5830	0.9968
[1 4]	0.5268	0.9997	0.5258	1.0000	0.5326	0.9966

Table 5.10: Case 4: Strategy 4

d	Scheme A	Scheme B	Scheme C	Scheme D	Scheme E	Scheme F
[1 1]	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
[1 2]	0.6764	0.9555	0.6722	0.9538	0.7066	0.9816
[1 3]	0.5625	0.8790	0.5609	0.8775	0.5769	0.9261
[1 4]	0.5278	0.8055	0.5264	0.8022	0.5340	0.8509

Table 5.11: Case 4: Strategy 5

d	Scheme A	Scheme B	Scheme C	Scheme D	Scheme E	Scheme F
[1 1]	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
[1 2]	0.6764	0.9428	0.6737	0.9398	0.7070	0.9590
[1 3]	0.5620	0.8522	0.5596	0.8455	0.5764	0.8723
[1 4]	0.5269	0.7740	0.5255	0.7663	0.5328	0.7847

Table 5.12: Case 4: Strategy 3 - Gini Fairness Index

d	Scheme A	Scheme B	Scheme C	Scheme D	Scheme E	Scheme F
[1 1]	0.0028	0.0000	0.0033	0.0002	0.0038	0.0005
[1 2]	0.3438	0.0029	0.3464	0.0000	0.3196	0.0238
[1 3]	0.4403	0.0055	0.4422	0.0004	0.4272	0.0301
[1 4]	0.4709	0.0080	0.4718	0.0023	0.4646	0.0291

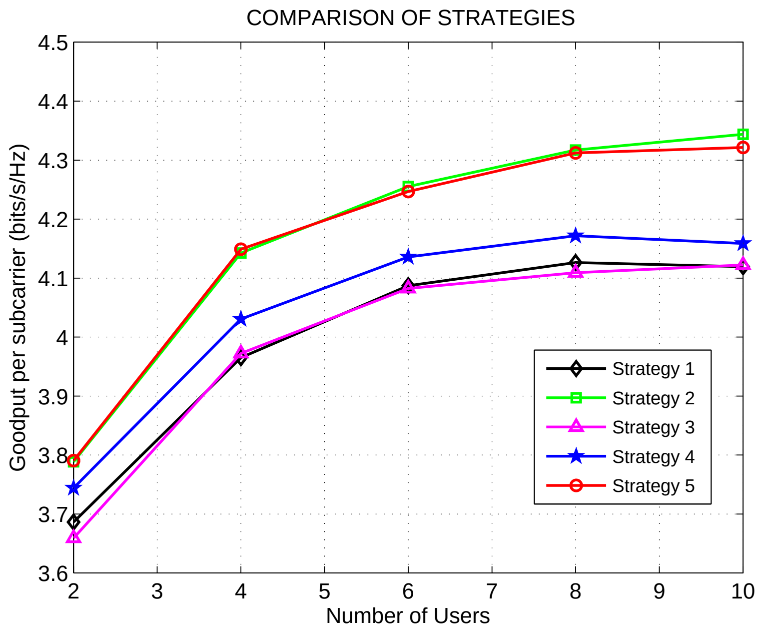


Figure 5.11: Comparison of different Strategies

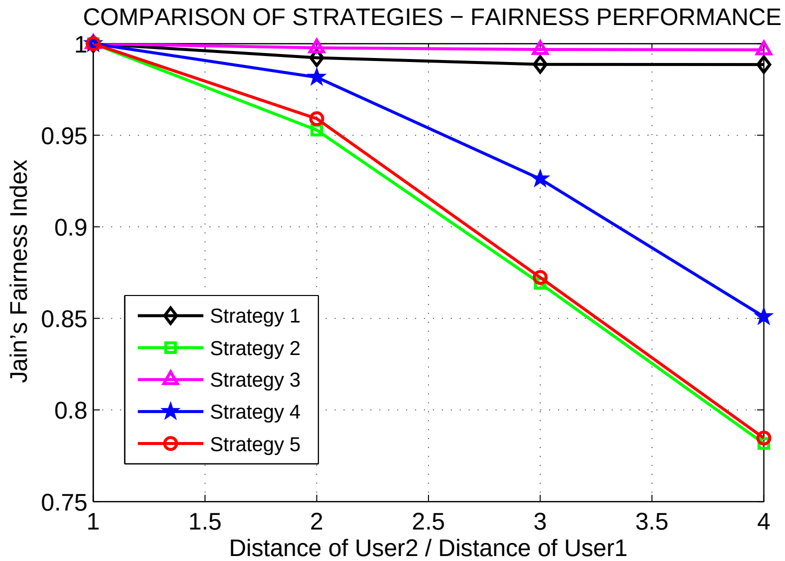


Figure 5.12: Case 4: Comparison of Fairness achieved by different Strategies

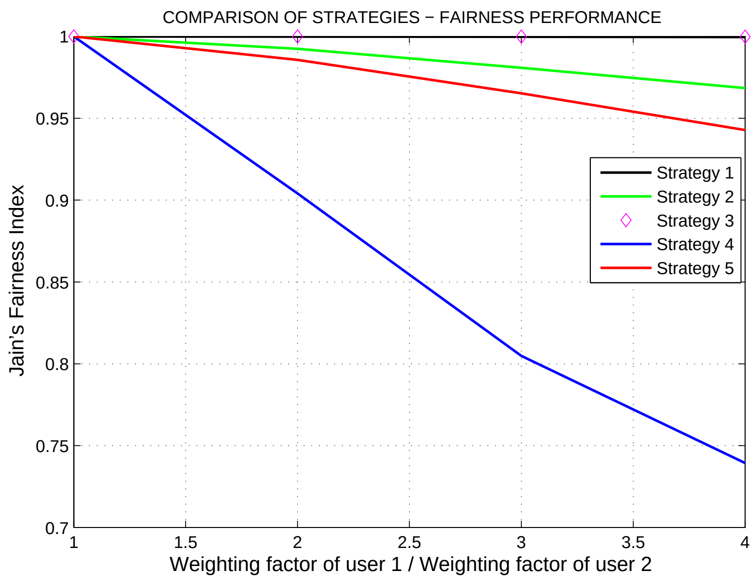


Figure 5.13: Case 3: Comparison of Fairness achieved by different Strategies

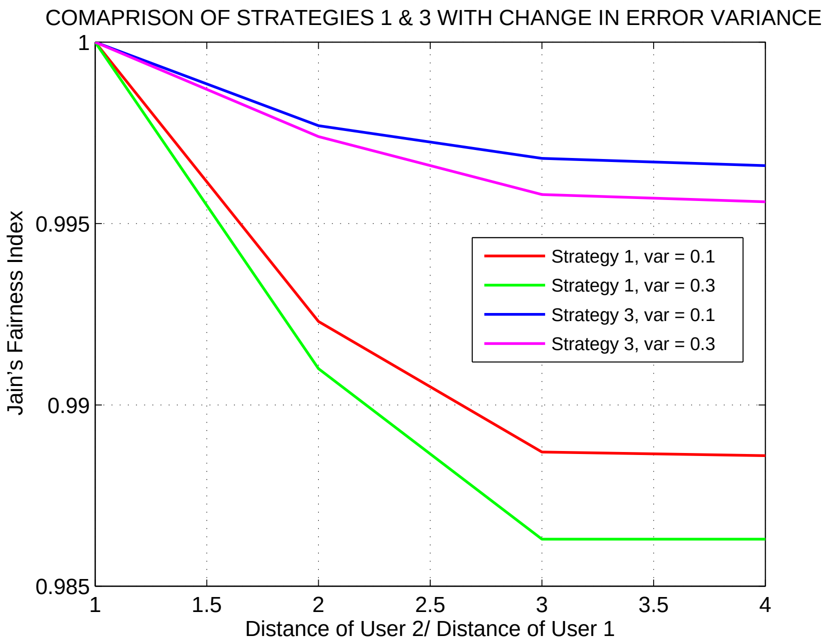


Figure 5.14: Comparison of Short-term fairness and Long-term fairness

CHAPTER 6

CONCLUSION

In the current scenario, where there is an increasing demand for higher data rate on one hand and an acute spectrum scarcity on the other hand, efficient resource allocation techniques are the need of the hour. In particular, OFDMA-based systems are attaining higher popularity and are considered as future broadband communication technology. In this project, we focused on the allocation of resources - power, rate and subcarriers, in an OFDMA downlink. The main contribution of the thesis is in the novel resource allocation schemes proposed which assume imperfect CSIT and consider both outage and fairness for allocating resources.

The initial phase of the project was a review of the related work. It was understood that though there had been a lot of research in this area, each resource allocation problem was unique because of the assumptions that are considered, the constraints that are incorporated and the objective function that is utilized. This determined how realistic and close to a practical system, the solution obtained is. In terms of assumptions, some authors assumed perfect CSIT while some assumed imperfect CSIT. Similarly some considered single cell, while others considered multi-cell scenario. In terms of constraints also, there were constraints like power constraint, fairness constraint, rate constraint etc. The most common approach in literature was to formulate the resource allocation problem as an optimisation problem. However, different authors considered different objective functions like ergodic capacity, goodput, BER, capacity lower bound etc. By doing a comparison of the related work, it was found that there wasa gap -there was no paper that combined imperfect CSIT, outage and fairness considerations together. So the aim of this project was to come up with novel resource allocation schemes considering all the factors.

The system considered was a single-cell scenario with one BS and K users sharing N subcarriers in a frequency-selective, Rayleigh-faded channel. The transmitter had channel knowledge but with an estimation error that was modeled as a complex Gaussian random variable. The resource allocation policy that was proposed, decouples the problem into two steps:

Step 1: Power and rate allocation

Step 2: Subcarrier allocation step

In the first step, we assign equal power to all subcarriers. This was suggested since it reduced complexity with minimal deviation from the optimal solution. For rate allocation, it was proposed to allocate a rate less than the estimated capacity to account for outage. The amount of rate back off required was found as a solution to the optimisation problem [10].

In the second step, the subcarrier allocation was done so as to ensure the required fairness. For the fairness constraint, five heuristic strategies were proposed. Out of these, one was for ensuring short-term fairness whereas the remaining four were addressed long-term fairness.

In the latter part of the project, the performance of the proposed resource allocation schemes were evaluated through MATLAB simulations. Performance in terms of both goodput and fairness was evaluated. In terms of goodput, since this scheme considers outage, it bridges the gap between goodput performance of perfect CSIT and imperfect CSIT. A key observation of this work is that even with fairness introduced, there isn't a large deviation from the throughput maximising schemes. For fairness criterion evaluation, Jain's fairness index was used. Heterogeneous systems such as users at different distances and users with different rate requirements were considered. It was found that in both the cases, the proposed scheme gave high Jain's index suggesting that the system achieved the desired fairness.

Thus a low-complexity OFDMA downlink resource allocation scheme has been proposed which achieves a good balance between throughput and fairness even in the presence of channel estimation errors at transmitter (imperfect CSIT).

CHAPTER 7

FUTURE WORK

In this chapter, we outline the research possibilities and the directions in which this work can be extended.

One possible extension of the present work is to include relays and study the possibility of transmission in a multihop link. During the course of literature survey, some papers have been referred to which deals with relays. Some of them dealt with perfect CSIT [14], [7], [6], [17] whereas some have introduced imperfect CSIT [12], [4]. However none of the papers, to the best of our knowledge, has addressed Imperfect CSIT, Outage and Fairness considerations in the presence of relays. Since relay based OFDM combination has been proposed for the 4G systems, it will be interesting to extend the resource allocation scheme presented in this thesis for a single-hop system to the case with relays (multi-hop systems).

This work has been based on the assumption of a single cell scenario. It can be modified to study how robust the algorithm is to the interference caused by a multi-cell scenario. Another way of extending the work is to include MIMO principles. In the present scheme, capacity calculations have been done with the assumption that it is a single antenna system both at the transmitter and receiver. With the inclusion of MIMO, the throughput that can be obtained from the system can be increased as well as fading can be combated due to the inherent diversity possibility. So resource allocation schemes that can get the best out of such a system have an increased scope.

The work presented in this thesis has the possibility of being extended and can serve as a baseline to a variety of research investigations which address the emerging scenarios of the future.

APPENDIX A

Derivation of Rate Back Off

The scenario considered is OFDMA downlink transmission in a Rayleigh faded channel. The transmitter knows only an estimate of the channel gain. Consider transmission by kth user on nth subcarrier with the received signal being corrupted by AWGN. The channel uncertainity model is defined as in

$$\hat{h}_{k,n} = h_{k,n} + e_{k,n}$$

where,

$$e_{k,n} = \mathcal{CN}(0, \sigma_e^2)$$

Then bandwidth normalized capacity is given by

$C_{k,n} = log_2(1 + |h_{k,n}|^2 p_{k,n})$

Conditioned on the estimate of the channel gain, $C_{k,n}$ is a random variable and its pdf, parametrized by the allocated power $\hat{p}_{k,n}$ is given as :

$$p\left(C_{k,n}|\hat{h}_{k,n}; \hat{p}_{k,n}\right) = b_{k,n} e^{C_{k,n} - a_{k,n} - b_{k,n}(e^{C_{k,n}-1})} \times I_0\left(2\sqrt{a_{k,n} b_{k,n}(e^{C_{k,n}-1})}\right), \quad \text{(A.1)}$$

where $a_{k,n} \triangleq |\hat{h}_{k,n}|^2/\sigma_e^2$ and $b_{k,n} \triangleq 1/(\hat{p}_{k,n}\sigma_e^2)$.

For simplification, this can also be approximated as a Gaussian, namely,

$$p\left(C_{k,n}|\hat{h}_{k,n}; \hat{p}_{k,n}\right) \approx \mathcal{N}\left(\hat{C}_{k,n}(\hat{p}_{k,n}), \sigma_{k,n}^2(\hat{p}_{k,n})\right), \quad \text{(A.2)}$$

where, $\hat{C}_{k,n} \triangleq log_2(1+|\hat{h}_{k,n}|^2\hat{p}_{k,n})$ is the nominal capacity, $\sigma_{k,n}^2(\hat{p}_{k,n} \triangleq a_{k,n} 2ln2/(a_{k,n}+ b_{k,n})^2$ is the corresponding variance.

Since the transmitter has only imperfect CSIT, there is a nontrivial outage probability $P_{out}(\hat{r}_{k,n}, \hat{p}_{k,n}) \triangleq Pr(\hat{r}_{k,n} > C_{k,n}(h_{k,n}; \hat{p}_{k,n}) | \hat{h}_{k,n})$. Using the above approximation to the pdf of $C_{k,n}$, we get

$$\overline{P}_{out}(\hat{r}_{k,n}, \hat{p}_{k,n}) = 1 - P_{out}(\hat{r}_{k,n}, \hat{p}_{k,n}))$$

$$\overline{P}_{out}(\hat{r}_{k,n}, \hat{p}_{k,n}) \approx Q\left(\frac{\hat{r}_{k,n} - \hat{C}_{k,n}(\hat{p}_{k,n})}{\sigma_{k,n}(\hat{p}_{k,n})}\right) \tag{A.3}$$

where Q(.) is the Gaussian *Q*- function.

The optimisation metric used is *goodput* , defined as

$G_{k,n}(\hat{r}_{k,n}, \hat{p}_{k,n}) \triangleq \hat{r}_{k,n}\overline{P}_{out}(\hat{r}_{k,n}, \hat{p}_{k,n})$

$$G_{k,n}(\hat{r}_{k,n}, \hat{p}_{k,n}) = \hat{r}_{k,n} Q\left(\frac{\hat{r}_{k,n} - \hat{C}_{k,n}(\hat{p}_{k,n})}{\sigma_{k,n}(\hat{p}_{k,n})}\right) \tag{A.4}$$

In this framework, we define the optimisation problem for resource allocation as defined in Section (4.1).

Introducing the non-negative Lagrange multipliers $\mu, \lambda_n, \beta_{k,n}, \gamma_{k,n}, \delta_{k,n}$ for constraints C_1 to C_5 respectively, the following Karush-Kuhn-Tucker(KKT) conditions should be satisfied at the optimal solution ($(.)^*$ denotes optimal values).

$$\hat{a}^*_{k,n} \frac{\partial \phi_{k,n}(\hat{r}^*_{k,n}, \hat{p}_{k,n}, \mu^*)}{\partial \hat{p}_{k,n}} |_{\hat{p}_{k,n}=\hat{p}^*_{k,n}} + \beta^*_{k,n} = 0, \forall\, k, n. \tag{A.5}$$

$$\hat{a}^*_{k,n} \frac{\partial \phi_{k,n}(\hat{r}_{k,n}, \hat{p}^*_{k,n}, \mu^*)}{\partial \hat{r}_{k,n}} |_{\hat{r}_{k,n}=\hat{r}^*_{k,n}} + \gamma^*_{k,n} = 0, \forall\, k, n. \tag{A.6}$$

$$\phi_{k,n}(\hat{r}^*_{k,n}, \hat{p}^*_{k,n}, \mu^*) - \lambda^*_n + \delta^*_{k,n} = 0, \forall\, k, n. \tag{A.7}$$

$$\mu^* \left(P - \sum_{k=1}^{K} \sum_{n=1}^{N} \hat{a}_{k,n} \hat{p}_{k,n}\right) = 0 \tag{A.8}$$

$$\lambda^*_n \left(1 - \sum_{k=1}^{K} \hat{a}_{k,n}\right) = 0, \forall\, n, \tag{A.9}$$

$$\beta^*_{k,n}\hat{p}^*_{k,n} = 0, \gamma^*_{k,n}\hat{r}^*_{k,n} = 0, \delta^*_{k,n}\hat{a}^*_{k,n} = 0, \forall\, k, n, \tag{A.10}$$

$$\mu^*, \lambda^*_n, \beta^*_{k,n}, \gamma^*_{k,n}, \delta^*_{k,n} \geq 0, \forall\, k, n, \tag{A.11}$$

where $\phi_{k,n}(\hat{r}^*_{k,n}, \hat{p}^*_{k,n}, \mu^*) \triangleq G_{k,n}(\hat{r}^*_{k,n}, \hat{p}^*_{k,n}) - \mu^*\hat{p}^*_{k,n}$

For obtaining a low complex algorithm, equal power has been given to all subcarriers. Thus there is no longer a need to find μ^*. To get the optimal rate, eqn (A.6) can be used. For a positive optimal rate, from eqn(A.10), $\gamma^*_{k,n} = 0$. Using this result and eqn (A.4) in (A.6), the optimal rate can be found as solution to :

$$Q\left(\frac{\hat{r}_{k,n} - \hat{C}_{k,n}}{\sigma_{k,n}}\right) - \frac{\hat{r}_{k,n}}{\sigma_{k,n}\sqrt{2\pi}} e^{-\frac{(\hat{r}_{k,n} - \hat{C}_{k,n})^2}{2\sigma^2_{k,n}}} = 0, \tag{A.12}$$

However this equation is highly nonlinear in $\hat{r}^*_{k,n}$, so we substitute $\hat{r}^*_{k,n} = \hat{C}^*_{k,n} + \Delta C_{k,n}$, where $\Delta C_{k,n}$ is a small non-positive constant, representing the optimal rate back off required. Then the equation becomes:

$$Q\left(\frac{\Delta C_{k,n}}{\sigma_{k,n}}\right) - \frac{\hat{C}_{k,n} + \Delta C_{k,n}}{\sigma_{k,n}\sqrt{2\pi}} e^{-\frac{(\Delta C_{k,n})^2}{2\sigma^2_{k,n}}} = 0, \tag{A.13}$$

Using second order Taylor series expansion of left hand side of eqn (A.13), we get

$$\frac{1}{2} - \frac{\Delta C_{k,n}}{\sqrt{2\pi}\sigma_{k,n}} - \frac{\hat{C}_{k,n}}{\sqrt{2\pi}\sigma_{k,n}} - \frac{\Delta C_{k,n}}{\sqrt{2\pi}\sigma_{k,n}} + \frac{\hat{C}_{k,n}(\Delta C_{k,n})^2}{2\sqrt{2\pi}\sigma^3} = 0 \tag{A.14}$$

The solution to this is obtained as:

$$\Delta C_{k,n} = \frac{2\sigma^2_{k,n} \pm \sqrt{\sigma^2_{k,n}(2\hat{C}^2_{k,n} - \hat{C}_{k,n}\sqrt{2\pi\sigma^2_{k,n}} + 4\sigma^2_{k,n})}}{\hat{C}_{k,n}} \tag{A.15}$$

The non-positive solution corresponds to the minus sign and it exists only when $\hat{C}_{k,n} \geq \sigma_{k,n}\sqrt{\pi/2}$. Else, that link is not considered suitable for transmission.

APPENDIX B

Matlab Code Used for Simulation of Strategy 3 - Correlated Fading Case

```
%STRATEGY 3 - HOMOGENEOUS USERS  - CORRELATED FADING

clc;
close all;
clear all;

N=32;
B=1000000;
SNR_dB=[10];
SNR1=10.^(0.1*SNR_dB);
lambda=0.9;
itermax=10000;
diffuservector=[2 4 6 8 10];
flag11=0; % To see if delC is always -ve

tic

for mm=1:length(SNR1)
    SNR=SNR1(mm);

    %for different user number
    for ii=1:length(diffuservector)
        diffuser=diffuservector(ii);
        K=diffuser; %Total no: of users
        ch_error=zeros(K,N);
        ch=zeros(K,N);
        ch_value=zeros(K,N);

        W=ones(K,1); % Weighting factor
        R1=zeros(K,1);  %For EWMA update
        R5=zeros(K,1);
        R6=zeros(K,1);
```

```
capacity_fair1=zeros(1,K);
capacity_fair2=zeros(1,K);
capacity_fair3=zeros(1,K);
capacity_fair4=zeros(1,K);
capacity_fair5=zeros(1,K);
capacity_fair6=zeros(1,K);


goodput1=0; %For successful rate in one slot
goodput2=0;
goodput3=0;
goodput4=0;
goodput5=0;
goodput6=0;


ratesum1=zeros(1,K); % Total successful rate
ratesum2=zeros(1,K);
ratesum3=zeros(1,K);
ratesum4=zeros(1,K);
ratesum5=zeros(1,K);
ratesum6=zeros(1,K);

var_e=0.1; % Estimation error variance

for k=1:K  % 6 Channel taps
    h0(k,:) = Jakes(30,0,itermax);
    h1(k,:) = Jakes(30,0,itermax);
    h2(k,:) = Jakes(30,0,itermax);
    h3(k,:) = Jakes(30,0,itermax);
    h4(k,:) = Jakes(30,0,itermax);
    h5(k,:) = Jakes(30,0,itermax);
end

for iter1=1:itermax
    iter1+(ii-1)*itermax
    toc

    for k=1:K
```

```
    h(k,:)=[h0(k,iter1) h1(k,iter1) h2(k,iter1) h3(k,iter1)
        h4(k,iter1) h5(k,iter1)];
end
ch_value=fft(h',N)';
ch_value= ch_value*sqrt(SNR/2);

ch=abs(ch_value).^2;
p=ones(K,N);
Cactual=log2(1+ch.*p); % Actual capacity
e=zeros(1,N);

R1=lambda*R1+(1-lambda)*capacity_fair1';
R5=lambda*R5+(1-lambda)*capacity_fair5';
R6=lambda*R6+(1-lambda)*capacity_fair6';

capacity_fair1=zeros(1,K);
capacity_fair2=zeros(1,K);
capacity_fair3=zeros(1,K);
capacity_fair4=zeros(1,K);
capacity_fair5=zeros(1,K);
capacity_fair6=zeros(1,K);

for i=1:K
    for n=1:N
        e(n)=(randn(1,1)+sqrt(-1)*randn(1,1))*sqrt(var_e/2);
    end
    ch_error(i,:)=abs(ch_value(i,:)+e).^2;



end
a=ch_error/var_e;
b=1./(p.*var_e);
var_rate=log(2)*(a*2)./((a+b).^2);
C=log2(1+ch_error.*p);

delC=zeros(K,N);
r=zeros(K,N);
Psuc=zeros(K,N);

for k=1:K
```

```
            for n=1:N
                %a(k,n)=ch_error(k,n)/var_e;
                %b(k,n)=1/(p(k,n)*var_e);
                if C(k,n)>=sqrt(var_rate*pi/2)
                    delC(k,n)=(2*var_rate(k,n)-sqrt(var_rate(k,n)*
(2*(C(k,n)^2)-C(k,n)*sqrt(2*pi*var_rate(k,n))+4*var_rate(k,n))))/C(k,n);
                    r(k,n)=C(k,n)+delC(k,n);
                    if delC >0
                        flag11=flag11+1;
                    end
                 Psuc(k,n)=qfunc((r(k,n)-C(k,n))/sqrt(var_rate(k,n)));
                end
            end
        end
        G=r.*Psuc;

        %SUBCARRIER ALLOCATION
        occupy1=zeros(1,N);
        occupy2=zeros(1,N);
        occupy3=zeros(1,N);
        occupy4=zeros(1,N);
        occupy5=zeros(1,N);
        occupy6=zeros(1,N);

        rtemp1=zeros(K,1); %For dynamic update
        rtemp5=zeros(K,1);
        rtemp6=zeros(K,1);

        %SUBCARRIER ALLOCATION BASED ON OUTAGE AND FAIRNESS

        Gtemp=G;
        suballo=zeros(1,N);
        for k=1:K
            flag1=0;
            while(flag1<1)
                [~,index]=max(Gtemp(k,:));
                if(suballo(index)==0)
                    flag1=flag1+1;
                    suballo(1,index)=suballo(1,index)+1;
                    rtemp1(k)=rtemp1(k)+r(k,index);
```

```
                occupy1(1,index)=k;
            end
            Gtemp(k,index)=-1;
        end
    end

    Gtemp=G;
    while (sum(suballo)<N)
        [~,minuser]=min((R1+rtemp1)./W);
        temp=0;
        maxindex=0;
        for j=1:N
            if ((suballo(j)==0)&&(Gtemp(minuser,j)>=temp))
                temp=Gtemp(minuser,j);
                maxindex=j;
            end
        end
        rtemp1(minuser)=rtemp1(minuser)+r(minuser,maxindex);
        suballo(maxindex)=1;
        occupy1(1,maxindex)=minuser;
    end

%SUBCARRIER ALLOCATION BASED ON FAIRNESS WITHOUT OUTAGE
%CONSIDERATION

    Gtemp=C;
    suballo=zeros(1,N);
    for k=1:K
        flag1=0;
        while(flag1<1)
            [~,index]=max(Gtemp(k,:));
            if(suballo(index)==0)
                flag1=flag1+1;
                suballo(1,index)=suballo(1,index)+1;
                rtemp5(k)=rtemp5(k)+C(k,index);
                occupy5(1,index)=k;
            end
            Gtemp(k,index)=-1;
        end
    end
```

```
Gtemp=C;
while (sum(suballo)<N)
    [~,minuser]=min((R5+rtemp5)./W);
    temp=0;
    maxindex=0;
    for j=1:N
        if ((suballo(j)==0)&&(Gtemp(minuser,j)>=temp))
            temp=Gtemp(minuser,j);
            maxindex=j;
        end
    end
    rtemp5(minuser)=rtemp5(minuser)+C(minuser,maxindex);
    suballo(maxindex)=1;
    occupy5(1,maxindex)=minuser;
end

%SUBCARRIER ALLOCATION BASED ON FAIRNESS WITH PERFECT CSIT

Gtemp=Cactual;
suballo=zeros(1,N);
for k=1:K
    flag1=0;
    while(flag1<1)
        [~,index]=max(Gtemp(k,:));
        if(suballo(index)==0)
            flag1=flag1+1;
            suballo(1,index)=suballo(1,index)+1;
            rtemp6(k)=rtemp6(k)+Cactual(k,index);
            occupy6(1,index)=k;
        end
        Gtemp(k,index)=-1;
    end
end

Gtemp=Cactual;
while (sum(suballo)<N)
    [~,minuser]=min((R6+rtemp6)./W);
    temp=0;
    maxindex=0;
```

```
    for j=1:N
        if ((suballo(j)==0)&&(Gtemp(minuser,j)>=temp))
            temp=Gtemp(minuser,j);
            maxindex=j;
        end
    end
    rtemp6(minuser)=rtemp6(minuser)+Cactual(minuser,maxindex);
    suballo(maxindex)=1;
    occupy6(1,maxindex)=minuser;
end

%SUBCARRIER ALLOCATION WITHOUT FAIRNESS

for n=1:N
    %Subcarrier allocation based on maximum goodput - With rate
    %back off
    [~,occupy2(1,n)]=max(G(:,n));
     %Subcarrier allocation based on maximum throughput -
     %Imperfect CSIT
    [~,occupy3(1,n)]=max(C(:,n));
     %Subcarrier allocation based on maximum throughput -
     %Perfect CSIT
    [~,occupy4(1,n)]=max(Cactual(:,n));

end

% GOODPUT CALCULATION

for n=1:N
    user1=occupy1(1,n);
    user2=occupy2(1,n);
    user3=occupy3(1,n);
    user4=occupy4(1,n);
    user5=occupy5(1,n);
    user6=occupy6(1,n);

    if r(user1,n)<=Cactual(user1,n)
        goodput1=goodput1+r(user1,n);
        capacity_fair1(user1)=capacity_fair1(user1)+r(user1,n);
        ratesum1(user1)=ratesum1(user1)+r(user1,n);
```

```
        end

        if r(user2,n)<=Cactual(user2,n)
            goodput2=goodput2+r(user2,n);
            capacity_fair2(user2)=capacity_fair2(user2)+r(user2,n);
            ratesum2(user2)=ratesum2(user2)+r(user2,n);
        end

        if C(user3,n)<=Cactual(user3,n)
            goodput3=goodput3+C(user3,n);
            capacity_fair3(user3)=capacity_fair3(user3)+C(user3,n);
            ratesum3(user3)=ratesum3(user3)+C(user3,n);
        end

        if C(user5,n)<=Cactual(user5,n)
            goodput5=goodput5+C(user5,n);
            capacity_fair5(user5)=capacity_fair5(user5)+C(user5,n);
            ratesum5(user5)=ratesum5(user5)+C(user5,n);
        end

      goodput4=goodput4+Cactual(user4,n);
      capacity_fair4(user4)=capacity_fair4(user4)+Cactual(user4,n);
      ratesum4(user4)=ratesum4(user4)+Cactual(user4,n);

       goodput6=goodput6+Cactual(user6,n);
       capacity_fair6(user6)=capacity_fair6(user6)+Cactual(user6,n);
       ratesum6(user6)=ratesum6(user6)+Cactual(user6,n);

    end

end

%FAIRNESS INDEX CALCULATION

ratesum1=ratesum1/itermax;
ratesum2=ratesum2/itermax;
ratesum3=ratesum3/itermax;
ratesum4=ratesum4/itermax;
ratesum5=ratesum5/itermax;
ratesum6=ratesum6/itermax;
```

```
    %Jain's Index Calculation

    z=ratesum1./W';
    J1=sum(z)^2/(K*sum(z.^2));
    z=ratesum2./W';
    J2=sum(z)^2/(K*sum(z.^2));
    z=ratesum3./W';
    J3=sum(z)^2/(K*sum(z.^2));
    z=ratesum4./W';
    J4=sum(z)^2/(K*sum(z.^2));
    z=ratesum5./W';
    J5=sum(z)^2/(K*sum(z.^2));
    z=ratesum6./W';
    J6=sum(z)^2/(K*sum(z.^2));

    %UPDATING THE VALUE TO THE CORRESPONDING USER

    goodputVec1(ii)=goodput1/itermax/N;
    goodputVec2(ii)=goodput2/itermax/N;
    goodputVec3(ii)=goodput3/itermax/N;
    goodputVec4(ii)=goodput4/itermax/N;
    goodputVec5(ii)=goodput5/itermax/N;
    goodputVec6(ii)=goodput6/itermax/N;


    JainIndex1(ii)=J1;
    JainIndex2(ii)=J2;
    JainIndex3(ii)=J3;
    JainIndex4(ii)=J4;
    JainIndex5(ii)=J5;
    JainIndex6(ii)=J6;

end


%PLOTTING

figure;
hold on;
```

```
    plot(diffuservector,goodputVec4, 'rx-','Linewidth',1.5);
    hold on;
    plot(diffuservector,goodputVec6, 'r--','Linewidth',1.5);
    hold on;
    plot(diffuservector,goodputVec3, 'bv-','Linewidth',1.5);
    hold on;
    plot(diffuservector,goodputVec5, 'b--','Linewidth',1.5);
    hold on;
    plot(diffuservector,goodputVec2, 'kd-','Linewidth',1.5);
    hold on;
    plot(diffuservector,goodputVec1, 'k--','Linewidth',1.5);
   legend('Scheme A','Scheme B','Scheme C','Scheme D','Scheme E', 'Scheme F')
    xlabel('Number of Users','Fontsize',12);
    ylabel('Normalised successful sum rate','Fontsize',12);
    title('Case 2: STRATEGY 3','Fontsize',12);
    set(gca,'Fontsize',12);
    box on;


    figure;
    hold on;
    plot(diffuservector,JainIndex4, 'rx-','Linewidth',1.5);
    hold on;
    plot(diffuservector,JainIndex6, 'r--','Linewidth',1.5);
    hold on;
    plot(diffuservector,JainIndex3, 'bv-','Linewidth',1.5);
    hold on;
    plot(diffuservector,JainIndex5, 'b--','Linewidth',1.5);
    hold on;
    plot(diffuservector,JainIndex2, 'kd-','Linewidth',1.5);
    hold on;
    plot(diffuservector,JainIndex1, 'k--','Linewidth',1.5);
    legend('Scheme A','Scheme B','Scheme C','Scheme D','Scheme E', 'Scheme F')
    xlabel('Number of Users','Fontsize',12);
    ylabel('Jain Fairness Index','Fontsize',12);
    title('Case 2: STRATEGY 3','Fontsize',12);
    set(gca,'Fontsize',12);
    box on;

end
```

APPENDIX C

Jakes' Model

```
function Z=Jakes(fd,t0,Num)
% Z=Jakes( ) - returns fading waveform generated using Jakes model
% fd= maximum Doppler frequency
% Tsample= Time resolution of signal in seconds
% t0 = starting time of oscillators
% Num=Number of samples of fading to be generated
Tc=9/(16*pi*fd);
Tsample=Tc/10;
N0=20; %Number of oscillators
N=4*N0+2; %N is even but not multiple of 4
scale=2;
scale1=1/sqrt(scale*N0); %Scale factor for real part of fading channel
scale2=1/sqrt(scale*(N0+1)); %Scale factor for imag part of fading channel
n=1:N0; %Oscillator index
bt=(0:Tsample:9*Tsample)+t0
ct=repmat(bt,1,Num/10);
at=[];
for count =1:Num/10
    at=[at ones(1,10)*(count-1)*5*Tc];
end
t=ct+at;
fn=fd*cos(2*pi*n/N); %fn   frequencies of oscillators in Jakes model
alp=0;
beta=n*pi/(N0+1);
phi=2*pi*rand(N0,1); %Random phases for each oscillator
ZC1=zeros(1,length(t));
ZS1=zeros(1,length(t));
for k=1:N0
  ZC1=ZC1+2*cos(beta(k))*cos(2*pi*fn(k)*t+phi(k)); %Real part
 ZS1=ZS1+2*sin(beta(k))*cos(2*pi*fn(k)*t+phi(k));  %Imag part
end
ZC=(ZC1+sqrt(2)*cos(alp)*cos(2*pi*fd*t))*scale2;
ZS=(ZS1+sqrt(2)*sin(alp)*cos(2*pi*fd*t))*scale1;
Z=complex(ZC,ZS);
end
```

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