HOLOMORPHIC EMBEDDED LOAD FLOW METHOD

A Dual Degree Project Report

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PROJECT CERTIFICATE

This is to certify that the report titled **HOLOMORPHIC EMBEDDED LOAD FLOW METHOD**, submitted by **Varsha Vattikonda**, to the Indian Institute of Technology Madras, Chennai for the award of the degree of **Dual Degree**, is a bonafide record of the research work done by her under our supervision. The contents of this report, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

With the growing power demand there is an inescapable need for the expansion of Power Systems and operating them at their limits. A prior power flow analysis before resorting to any kind of alteration in the power system would be the best way to plan these expansions. Hence the importance of power flow analysis devoid of convergence issues is growing exponentially. Traditional iterative methods are extremely initial-estimate dependent and do no guarantee convergence to the required solution. Holomorphic embedding Load Flow Method is strictly independent of the initial-estimate. While the theory behind the method is deeply rooted in the theory of complex numbers the implementation of the method in a high language software like MATLAB is quite straight forward. Experience has proven that it is competitive with respective established iterative methods, but its important characteristics are that it yields the correct solution when it exists and, unequivocally signals voltage collapse when it does not.

TABLE OF CONTENTS

Chapter **Page**

LIST OF TABLES

Table **Page**

LIST OF FIGURES

CHAPTER I: Introduction

Power System :

Power system is a network of electrical components used for the supply, transfer and usage of power. It comprises of generators, transmission lines and distribution system that supplies power to the residential and industrial areas.

Buses are of 3 types namely PQ, PV and Slack. PQ buses are the buses at which the power active and reactive is known. One of the objectives of the load flow is to find the bus voltage magnitude |V| (Magnitude) and δ (Phase) at these buses. PV buses are at which the power generation is controlled through a prime mover while the terminal voltage is controlled through the generator excitation. Keeping the input power constant through turbine-governor control and keeping the bus voltage constant using automatic voltage regulator, P and |V| are constant for these buses. This is why such buses are also referred to as PV buses. A Slack bus (or swing bus), defined as a Vδ bus, is used to balance the active power P and reactive power Q in a system while performing load flow studies. It is used to provide for system losses by generating or absorbing active and/or reactive power to and from the system.

Bus Type	**P**	**Q**	**|V|**	**δ**
Slack	Unknown	unknown	Known(reference)	Known(reference)
Load	Known	Known	unknown	Unknown
Generator	Known	unknown	Known	Unknown

Table 1-1. Types of Buses

Load Flow Study:

Load flow or Power flow study is the analysis of all the AC parameters, such as voltages and its phases, real and reactive power flows in each transmission line in the Power-System considered. It focuses on evaluating and comparing them against their respective limits. They are important for planning future expansion of power systems, control and determining the best way of operating the existing system. The successful operation of power systems depends upon knowing the effects of adding interconnections, adding new loads, connecting new generators or connecting new transmission line before being installed. These analyses also help in determining the unit-commitment, economic dispatch etc. It is especially valuable for a multi-load bus system to know if the demand can be met and also the best way of meeting this demand with minimal transmission line losses.

At each bus there are two known and two unknown values and two equations which relate the active and reactive powers at each bus to the voltages, phases and transmission line parameters.

As Power systems' power balance equations are non-linear system of equations, the solution for these equations can be found using iterative methods like Gauss-Siedel, Newton Raphson, Fast-Decoupled Load Flow etc. There are several different iterative methods to solve the power flow problems. The iterative methods give the accurate solution as they approach towards the convergence of the solution by correcting the calculated value which minimizes the mismatch in the known values. The memory and time complexities increase enormously as the problem size increases (increase in number of buses).

CHAPTER II: BACK GROUND AND LITERATURE REVIEW

Power Balance Equations:

Consider a power system of N buses. These buses are numbered from 1 to N. The Nodal equation of all the buses in a power system in a matrix form can be written as

$$\vec{I} = Y_{bus}\vec{V} \quad (2.1)$$

Where

$\vec{I}$	*Column vector of currents injected at all the buses*
Y_{bus}	*Bus admittance matrix*
$\vec{V}$	*Column vector of Nodal voltages*

$$I_i = \overline{V_i}y_{ii} + (\overline{V_i} - \overline{V_1})y_{i1} + \cdots (\overline{V_i} - \overline{V_p})y_{ip} + \cdots (\overline{V_i} - \overline{V_N})y_{iN}$$

$$(2.2)$$

$$I_i = V_1Y_{i1} + (V_2)Y_{i2} + \cdots (V_p)Y_{ip} + \cdots (V_N)Y_{iN}$$

In short

$$Y_{ij} = -\frac{1}{r_{ij} + jx_{ij}} \quad \forall i \neq j \quad where\ \ 1 \leq i, j \leq N$$

$$(2.3)$$

$$Y_{ii} = \sum_{j=1}^{N} \frac{1}{r_{ij} + jx_{ij}} \quad \forall i = 1\ to\ N$$

Where

$\overline{I_i}$	$Current\ injected\ at\ Bus\ i$
$\overline{V_i}$	$Voltage\ at\ Bus\ i$
$y_{ij} = r_{ij} + jx_{ij}$	$Summed\ impedance\ of\ all\ transmission\ lines\ connecting\ directly\ the\ buses\ i\ and\ j$

To summarize the admittance matrix of a power system can be formed by the following steps

- The diagonal element Y_{ii} is the sum of the admittances connected to bus i.
- The off-diagonal element Y_{ij} is the negative of the summed admittance between the buses i and j.

For a 2 bus matrix it would look like Eq.(1) would look like

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{2.4}$$

Now for power equations

$$\overline{S_i} = P_i + jQ_i = \overline{V_i I_i}^* = \overline{V_i}\left[\sum_{k=1}^{N} \overline{V_k} Y_{ik}\right]^* \tag{2.5}$$

This gives the Power equations as

$$P_i = \left|\overline{V_i}\right| \sum_{k=1}^{N} \left|\overline{V_k}\right| [G_{ik}\cos(\theta_i - \theta_k) + B_{ik}\sin(\theta_i - \theta_k)] \qquad \forall\ 1 \le i \le N$$

(2.6)

$$Q_i = \left|\overline{V_i}\right| \sum_{k=1}^{N} \left|\overline{V_k}\right| [G_{ik}\sin(\theta_i - \theta_k) - B_{ik}\cos(\theta_i - \theta_k)] \qquad \forall\ 1 \le i \le N$$

where

$$\overline{V_i} = |\overline{V_i}|e^{j\theta_i} \qquad and \qquad Y_{ik} = G_{ik} + jB_{ik}$$

Gauss-Seidel Method:

The Gauss–Seidel method is an iterative method used to solve a system of equations (linear or non-linear).

Consider a system of equations $f_i(x_1, x_2, x_3, \ldots . x_n) = 0 \; for \; 1 \leq i \leq N$

From the above system of equations let $x_i = g_i(x_1, x_2, x_3, \ldots . x_n)$

1. Let the initial guess be $(x_1^0, x_2^0 \ldots x_n^0)$
2. Substituting the initial values in the equation we calculate new x_1 from the equation $x_1^1 = g_1(x_1^0, x_2^0 \ldots x_n^0)$
3. Next we calculate the new update of x_2 from $x_2^1 = g_2(x_1^1, x_2^0 \ldots x_n^0)$
4. Progressively new updates are calculated for all the values from the equation $x_i^1 = g_i(x_1^1, x_2^1 \ldots x_{i-1}^1, x_i^0, x_{i+1}^0 \ldots x_n^0)$
5. Steps 2, 3, 4 are iterated until the difference between the new updated values and the old values is below tolerance.

Newton Raphson Method:

It is an iterative method to find successive better solutions for a system of equation $x : \; f(x) = 0$s.The method for a single variable in general can be represented by $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ where ***n*** represents the number of iterations. This solution is iterated until the solution reaches required precision. Convergence of NR is referred to as quadratic convergence and so NR is the most widely used iterative method to arrive at the solution. Let x_r be the actual solution

$$0 = f(x_r) = f(x_n + \delta)$$

$$= f(x_n) + f'(x_n)(x_r - x_n) + \frac{1}{2!}f''(x_n)(x_r - x_n)^2 \qquad (Approxmn.)$$

$$f(x_n) = f'(x_n)(x_n - x_{n+1}) \quad (\,From\ Newton\ Raphson) \tag{2.7}$$

From the above two equations we have

$$\Rightarrow \quad 0 = f'(x_n)(x_r - x_{n+1}) + f''(x_n)(x_r - x_n)^2$$

$$\Rightarrow \quad (x_r - x_{n+1}) = -f''(x_n)(x_r - x_n)^2 /\ f'(x_n) \quad (Quadratic\ convergence)$$

Newton Raphson For Load Flow:

Let us consider a power system of **N** buses where the *Bus 1* represents *Slack* bus and the buses numbered from *2 to p+1* are *PQ* buses and the rest are *PV* buses.

In each iteration we form a new Jacobian matrix which solves for the correction in the unknowns using the mismatches in the known quantities.

$$\begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \begin{pmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \vdots \\ \Delta\delta_N \\ \frac{\Delta V_2}{V_2} \\ \frac{\Delta V_3}{V_3} \\ \vdots \\ \frac{\Delta V_{p+1}}{V_{p+1}} \end{pmatrix} = \begin{pmatrix} \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_N \\ \Delta Q_2 \\ \vdots \\ \Delta Q_{p+1} \end{pmatrix}$$

Jacobian Matrix — Correction vector — Mismatch of Powers from known values of P and Q

$$J_{11} = \begin{pmatrix} \partial P_2/\partial\delta_2 & \cdot & \cdot & \partial P_2/\partial\delta_N \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \partial P_n/\partial\delta_2 & \cdot & \cdot & \partial P_N/\partial\delta_N \end{pmatrix}$$

$$J_{21} = \begin{pmatrix} \partial Q_2/\partial\delta_N & \cdot & \cdot & \partial Q_2/\partial\delta_N \\ . & \cdot & \cdot & . \\ . & \cdot & \cdot & . \\ \partial Q_{p+1}/\partial\delta_N & \cdot & \cdot & \partial Q_{p+1}/\partial\delta_N \end{pmatrix}$$

$$J_{12} = \begin{pmatrix} V_2\,\partial P_2/\partial V_2 & \cdot & \cdot & V_p\partial P_2/\partial V_{p+1} \\ . & \cdot & \cdot & . \\ . & \cdot & \cdot & . \\ V_2\,\partial P_N/\partial V_2 & \cdot & \cdot & V_p\partial P_N/\partial V_{p+1} \end{pmatrix} \quad (2.8)$$

$$J_{22} = \begin{pmatrix} V_2\,\partial Q_2/\partial V_2 & \cdot & \cdot & V_{p+1}\partial Q_2/\partial V_{p+1} \\ . & \cdot & \cdot & . \\ . & \cdot & \cdot & . \\ V_2\,\partial Q_{p+1}/\partial V_2 & \cdot & \cdot & V_{p+1}\partial Q_{p+1}/\partial V_{p+1} \end{pmatrix}$$

Steps for Newton-Raphson :

1. Form the Bus admittance matrix (***Y_{ij}***)
2. Assume initial set of Voltage magnitudes for all the PQ buses and Voltage phases for all the buses except slack. Slack is taken as the reference and is set to $1\angle 0^0$
3. Calculate the active and reactive powers using the power equations respectively.
4. Form the Jacobian matrix by partially differentiating the power equations respectively.
5. Find the power mismatches with the known values of powers of PQ and PV buses.
6. Choose the tolerance value.
7. Evaluate the corrections in the assumed values.
8. Repeat the steps 4, 5 ,7 and 8 until the mismatch is greater than the chosen tolerance.

Problems With Newton Raphson :

- Involves re-evaluation and inversion of the Jacobian matrix in each and every iteration. High computational costs which are of time complexity O(N^2).

- Might converge to any possible solution nearer to the initial guess.
- If Jacobian matrix is not invertible anywhere in the path from initial estimate to the best estimate this method will not lead to the solution.

Small Systems vs Large System:

For a small system the selection is mainly based on the time per iteration and total convergence time. It is expected that the number of iterations for this system is less and so is the total convergence time. In this case the method with less time per iteration is expected to give better convergence time than high average iteration time method. The average iteration time of Gauss-Seidel method is less compared to the Newton-Raphson method owing to the complexity of calculations in each iteration.

It is expected that the number of iterations for a large system are large and so is the total convergence time. In this case the method with less number of iterations gives better convergence time than one with large number of iterations. As Newton-Raphson converges faster (quadratic convergence) than Gauss and Gauss-Seidel method it is preferred for larger systems.

Necessity of Holomorphic Embedded method:

The Power Equations are a nonlinear system of equations with multiple solutions. Traditional Methods do not guarantee that it will converge to a solution even though the solution actually exists. If a test case does not converge, there is no way to know if a solution does not exist or if the iterative method is to blame for the non-convergence. Also, there is no control to which solution the method converges.

For example for a 2 bus system, there exist two solutions for a specified value of power injection. One solution is desired for practical purposes. Depending on the initial estimate, NR method may converge to that solution or the other solution or may not converge at all. These problems are inherent in all iterative methods applied to nonlinear problems and occur irrespective of the problem size.

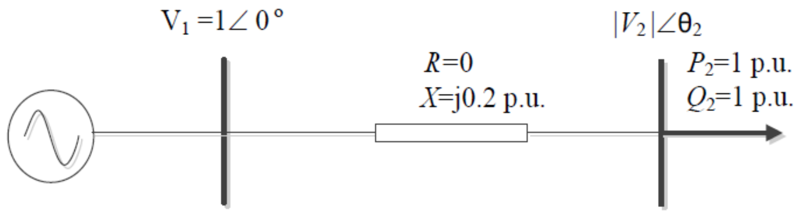

Figure 2.1- 2 bus system (Ref[8])

Varying the initial guess from *0 pu* to *1pu* and the phase from -180^0 to +180^0 the solution to which Newton-Raphson converges oscillates between the practical voltage and the other solution.

CHAPTER III: SOLVING POWER FLOW PROBLEM USING HOLOMORPHIC EMBEDDING

Introduction to holomorphic functions:

A holomorphic function is a complex-valued function of one or more complex variables that is complex differentiable in a neighborhood of every point in its domain. The existence of a complex derivative in a neighborhood is a very strong condition, for it implies that any holomorphic function is actually infinitely differentiable and equal to its own Taylor series. The term analytic function is often used interchangeably with "holomorphic function", although the word "analytic" is also used in a broader sense to describe any function (real, complex, or of more general type) that can be written as a convergent power series in a neighborhood of each point in its domain. All holomorphic functions are complex analytic functions.

Analytic functions can be entire or non-entire. A function is entire if its Taylors series' value matches with the actual function in the whole complex plane. In short their convergence radius is infinity. For non-entire functions the function value matches with the Taylors series expansion within a finite radius. Analytic continuation (explained in the later part of the chapter) comes handy to extend the radius of convergence.

Holomorphic Embedding :

Holomorphic embedding is the technique of embedding(submerging) the non-analytic problem(solving for a function which is non-analytic) within a large problem of complex variables. As this large problem is analytic and all the properties of holomorphism come in hand it is easier to arrive at the solution of the larger problem where the required power-flow solution is sub-solution of the complex-analytic problem's solution.

So for the power flow solution the power balance equations of slack, PV and PQ buses are embedded into a bigger complex problem whose sub-solution is the required power-flow solution. In formulating this complex problem the embedding of Slack, PQ bus and PV bus power balance equations are to be considered separately.

Holomorphic embedded Power Balance Equations:

We consider the system with **N** buses and this algorithm holds good for an **N** value as high as 3000. Let ***p*** be the set of PQ and ***q*** be the set of PV buses.

PQ BUS Power Balance Equation Embedding :

Let the system have N buses named from 1 to N.

Let ***i*** be a PQ bus whose power injection S_i is known. The power equation at any PQ bus ***i*** is given by

$$\overline{V_i}^* \sum_{k=1}^{N} Y_{ik}\overline{V_k} = S_i^* \qquad for\ all \quad i \in p \tag{3.1}$$

Where

$$Y_{ik} - \quad (i,k) element\ of\ the\ bus\ admittance\ matrix\ of\ the\ system$$
$$S_i^* \ - \quad Complex\ power\ injection\ at\ the\ i\ (PQ) bus$$
$$\overline{V_i} \ - \quad Complex\ voltage\ at\ bus\ i$$

The shunt elements (of the transmission lines) in the bus admittance values in the Eq.(9) are moved to the RHS and the complex variable ***s*** is embedded in the equation to arrive at the following equation

$$\sum_{k=1}^{N} Y_{ik\,series}\overline{V_k(s)} = \frac{sS_i^{\,*}}{\overline{V_i^*(s^*)}} - sY_{i\,shunt}\overline{V_i(s)} \qquad for\ all \quad i \epsilon p$$

(3.2)

Where

$Y_{ik\,series}$ $-$ $series\ branch\ part\ of\ Y_{ik}$

$Y_{ik\,shunt}$ $-$ $shunt\ admittance\ components\ of\ transmission\ line, transformer\ tap\ model\ etc$

s $-$ $Any\ complex\ variable$

For ***s=1*** the original power balance equation is retrieved from the Eq.(10). At ***s=0*** RHS evaluates to zero and on the LHS the row entries of the admittance matrix add-up to zero (from Eq.(3)) as the shunt admittances are excluded(present on RHS side) if voltages at all the buses are the same. We start by assuming that per unit voltages at all the buses are ***1∠0^0***.

With the parameter ***s*** as an embedded complex variable, ***V(s)*** is used to emphasize that the voltage has become a holomorphic function of the complex parameter ***s***. The complex conjugate of the voltage, ***V**** that is present in the Power equation is replaced by, ***V*(s*)*** and not ***V*(s).*** The presence of ***s**** rather than ***s*** retains the property of holomorphism and therefore analyticity.

In the field of complex analysis, the Cauchy–Riemann equations consist of a system of two partial differential equations which, together with certain continuity and differentiability criteria, form a necessary and sufficient condition for a complex function to be complex differentiable that is holomorphic. The equations for a function $f(z = x + iy) = u(x,y) + iv(x,y) \quad u(x,y), v(x,y), x, y \in R$ are

$$a)\quad \frac{\partial \boldsymbol{u}}{\partial x} = \frac{\partial \boldsymbol{v}}{\partial y}$$

(3.3)

$$b)\quad \frac{\partial \boldsymbol{u}}{\partial y} = -\frac{\partial \boldsymbol{v}}{\partial x}$$

Where

Wirtinger's derivatives are partial differential operators of the first order which behave in a very similar manner to the ordinary derivatives with respect to one real variable, when applied to holomorphic functions, anti-holomorphic functions or simply differentiable functions on complex domains. These operators permit the construction of a differential calculus for such functions that is entirely analogous to the ordinary differential calculus for functions of real variables. Let $\boldsymbol{z = x + iy}$

$$a)\quad \frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)$$

(3.4)

$$b)\quad \frac{\partial}{\partial z^*} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)$$

An equivalent condition from Wirtinger's derivative and Cauchy Riemann requires the function ***V(s)*** should be independent of ***s**** in order to be holomorphic. Substituting ***V*(s*)*** instead of ***V*(s)*** helps in making the power balance equation independent of ***s**** . The Maclaurin series expansion of the ***V*(s)*** (if it were to exist) and ***V*(s*)*** truncated at *n* terms are as below

$$V^*(s) = V^*[0] + V^*[1]s^* + \ldots\ldots\ldots + V^*[n](s^*)^n$$

(3.5)

$$V^*(s^*) = V^*[0] + V^*[1]s + \dots\dots\dots + V^*[n](s)^n$$

.

The function ***V*(s)*** is not ***s**** independent and hence substituting ***V*(s*)*** is essential so as to make it holomorphic. And also from the Cauchy Riemann equations (Eq.(11)) if ***V(s)*** is holomorphic ***V*(s*)*** is holomorphic and **V*(s)** is not holomorphic. Also embedding ***V*(s*)*** instead of ***V*(s)*** does not effect the actual power flow solution i.e the solution at ***s=1***.

Assuming we can use Maclaurin series to represent ***V(s)*** and its conjugate the holomorphic embedded model of PQ bus is given by

$$\sum_{k=1}^{N} Y_{ik\,series}(V_k[0] + V_k[1]s + V_k[2]s^2 + \cdots) = \frac{sS_i^*}{V_i^*[0] + V_i^*[1]s + V_i^*[2](s)^2 + \cdots} - sY_{i\,shunt}(V_i[0] + V_i[1]s + V_i[2]s^2 + \cdots)$$

(3.6)

Let W(s) be the inverse power series of V(s) i.e

$$W(s) = \frac{1}{V(s)} = W[0] + W[1]s + W[2]s^2 + \cdots$$

(3.7)

Hence the final PQ bus model turns out to be

$$\sum_{k=1}^{N} Y_{ik\ trans}(V_k[0] + V_k[1] + V_k[2]s^2 + \cdots) = sS_i^*(W_i^*[0] + W_i^*[1]s + W_i^*[2]s^2 + \cdots) - sY_{i\ shunt}(V_i[0] + V_i[1]s + V_i[2]s^2 + \cdots)$$

(3.8)

Now the power flow problem is reduced to finding the coefficients of the Maclaurin series by equating the coefficients of ***s, s², s³….***

Eq.(16) is a complex-analytic problem in which the power flow problem is the submerged problem. Solving for the coefficients is evaluating the power series expansion of complex variable embedded Voltage function. The value of power series at s=1 is the power flow solution. Thus the power flow solution is embedded in the larger holomorphic problem. Hence this method is called the Holomorphic Embedded load Flow Method.

By generalizing the coefficients of s^n on LHS and RHS we have

$$\sum_{k=1}^{N} Y_{ik\ trans}(V_k[n]) = S_i^* W_i^*[n-1] - Y_{i\ shunt} V_i[n-1]$$

(3.9)

Where

$V_k[n]$ — $Coefficient\ of\ s^n\ in\ the\ Maclaurin\ series\ of\ Voltage\ of\ kth\ bus$

$W_i\ [n-1]$ — $Coefficient\ of\ s^{n-1}\ in\ the\ Maclaurin\ series\ of\ inverse\ of\ Voltage\ of\ kth\ bus$

$$W(s) \times V(s) = 1$$

$$(W[0] + W[1]s + W[2]s^2 + \cdots)(V[0] + V[1]s + V[2]s^2 + \cdots) = 1$$

(3.10)

$$W[0] = {}^{1}/_{V[0]}$$

$$W[n] = -\frac{\sum_{k=0}^{n-1} W[k]V[n-k]}{V[0]} \ , \ n \geq 1$$

We assume V[0] of all the buses to be 1∟0° . So W[0] of all the PQ buses is 1∟0° from Eq.(18). From these V[1] can be evaluated from Eq.(17) and then W[1] and so on. Hence by using Eq.(17) and Eq.(18) all the coefficients of the power series can be evaluated.

Slack Bus Power balance Equations Embedding :

$$\delta_{ni} = \begin{cases} 1 & if \quad n = i \\ 0 & if \quad n \neq i \end{cases}$$

$$V_i[n] = \delta_{n0} + \delta_{n1}(\left|V_i^{sp}\right| - 1) \tag{3.11}$$

Hence for a slack bus $V_i[0]$ is one ($since \quad \delta_{n0} = 1$) and $V_i[1]$ is $\left|V_i^{sp}\right| - 1$ ($since \quad \delta_{n1} = 1$) and coefficients of higher powers of s are zero ($since \quad \delta_{n0} = 0$ and $\delta_{n1} = 0$). Thus V_i finally adds up to $\left|V_i^{sp}\right|$.

PV Bus Power Balance Equations Embedding:

Let ***q*** represent the set of PV buses. The power equations of a PV bus are

$$P_i = Re\left(V_i \sum_{k=1}^{n} V_k^* Y_{ik}^*\right) \qquad for\ all\ i \in q$$

(3.12)

$$|V_i| = V_i^{sp} \qquad for\ all\ i \in q$$

Where P_i $-$ *Power injection at bus i*

V_i^{sp} $-$ *Specified voltage at the PV bus i*

$$S_i + S_i^* = 2P_i$$

$$V_i^* \sum_{k=1}^{N} Y_{ik} V_k + V_i \sum_{k=1}^{N} V_k^* Y_{ik}^* = 2P_i$$

(3.13)

$$V_i^* \sum_{k=1}^{N} Y_{ik\,series} V_k + V_i^* Y_{i\,shunt} V_i + V_i \sum_{k=1}^{N} V_k^* Y_{ik\,series}^* + V_i^* Y_{i\,shunt}^* V_i = 2P_i$$

The Holomorphic embedded power equation turns out to be

$$V_i^*(s^*) \sum_{k=1}^{N} Y_{ik\,series} V_k(s) + V_i(s) \sum_{k=1}^{N} V_k^*(s^*) Y_{ik\,series}^* = 2sP_i - 2sRe(Y_{i\,shunt})|V_i^{sp}|^2$$

(3.14)

And also for a PV bus we have

$$V_i(s) \times V_i^*(s^*) = 1 + s\left(|V_i^{sp}|^2 - 1\right)$$

(3.15)

For solution at ***s=0*** we have

$$\left(V_i[0]\sum_{k=1}^{N} V_k^*[0]Y_{ik\,series}^*\right) + V_i^*[0]\sum_{k=1}^{N} Y_{ik\,series}V_k[0] = 0 \;\; for\ all\ \boldsymbol{i} \in \boldsymbol{q}$$

$$V_i[0] \times V_i^*[0] = 1 \;\; for\ all\ \boldsymbol{i} \in \boldsymbol{q}$$

(3.16)

$$\sum_{k=1}^{N} Y_{ik\,series}V_k[0] = 0 \quad for\ all\ \boldsymbol{i} \in \boldsymbol{q}$$

For a solution at ***s=0*** all these equations of PQ , PV and slack are to be simultaneously satisfied and hence 1∟$\mathbf{0}^{o}$ would satisfy all these equations as the diagonal element in Y_{series} matrix is the negative of the sum of the off-diagonal matrix elements and hence $\sum_{k=1}^{N} Y_{ik\,trans}V_k[0] = 0$ is satisfied and thus the obvious solution is **1**∟$\mathbf{0}^{o}$ for all the buses.

We need a recurrence relation for PV buses similar to PQ buses such that given we know the coefficients of the power series of the voltage till $V_i[n-1]$ we can arrive at the value of $V_i[n]$.Since we have the value of $V_i[0]$ we have a starting value to start off the recurrence relation with. Now let

$$Y_{ik\,series} = G_{ik} + jB_{ik}$$

$$V_i[n] = V_{i\,re}[n] + jV_{i\,im}[n]$$

On the LHS of PV bus we have two terms, one conjugate of the other. The first term is

$$V_i^*(s^*)\sum_{k=1}^{N} Y_{ik\ series}V_k(s)$$

$$= (V_i^*[0] + V_i^*[1]s \ldots V_i^*[n]s^n)\left(\sum_{k=1}^{N} Y_{ik\ series}(V_k[0] + V_k[1]s + \cdots V_k[n]s^n)\right)$$

(3.17)

$$= ((V_{i\,re}[0] - jV_{i\,im}[0]) + (V_{i\,re}[1] - jV_{i\,im}[1])s + \cdots (V_{i\,re}[n] - jV_{i\,im}[n])s^n\ ) \times \left(\sum_{k=1}^{N}(G_{ik+}\,jB_{ik})((V_{k\,re}[0] + jV_{k\,im}[0]) + (V_{k\,re}[1] + jV_{k\,im}[1])s + \cdots (V_{k\,re}[n] + jV_{k\,im}[n])s^n)\right)$$

Coefficient of s^n from this term is

$$\sum_{x+y=n,0\le x,y\le n}\big((V_{i\,re}[x] - jV_{i\,im}[x])\big) \times \left(\sum_{k=1}^{N}(G_{ik} + jB_{ik})(V_{k\,re}[y] + V_{k\,im}[y])\right)$$

$$= \sum_{x+y=n,0\le x,y\le n}\big((V_{i\,re}[x] - jV_{i\,im}[x])\big)$$

$$\times \left(\sum_{k=1}^{N}\left([G_{ik} \quad -B_{ik}]\begin{bmatrix}V_{k\,re}[y]\\ V_{k\,im}[y]\end{bmatrix} \quad + j[B_{ik} \quad G_{ik}]\begin{bmatrix}V_{k\,re}[y]\\ V_{k\,im}[y]\end{bmatrix}\right)\right)$$

$$= \sum_{x+y=n}\left\{\sum_{k=1}^{N}[G_{ik}V_{i\,re}[x] + B_{ik}V_{i\,im}[x] \quad -B_{ik}V_{i\,re}[x] + G_{ik}V_{i\,im}[x]]\begin{bmatrix}V_{k\,re}[y]\\ V_{k\,im}\,[y]\end{bmatrix}\right\}$$

$$+ \ \ j\sum_{x+y=n}\left\{\sum_{k=1}^{N}[B_{ik}V_{i\,re}[x] - G_{ik}V_{i\,im}[x] \quad G_{ik}V_{i\,re}[x] + B_{ik}V_{i\,im}[x]]\begin{bmatrix}V_{k\,re}[y]\\ V_{k\,im}\,[y]\end{bmatrix}\right\}$$

(3.18)

And since there exists a conjugate of the first term, on adding both we have

$$2\sum_{x+y=n}\left\{\sum_{k=1}^{N}[G_{ik}V_{i\,re}[x]+B_{ik}V_{i\,im}[x] \quad -B_{ik}V_{i\,re}[x]+G_{ik}V_{i\,im}[x]]\begin{bmatrix}V_{k\,re}[y]\\V_{k\,im}[y]\end{bmatrix}\right\}$$

The objective now is to express the calculation of power series coefficients of an order **n** using a recursive linear relation. Hence, the unknown quantities $V_{i\,re}[n]$ and $V_{i\,im}[n]$ are retained on the LHS and the coefficients up to degree n-1 are supposed to be moved to the RHS. The unknown coefficients occur only when $x = 0\ and\ y = n\ \boldsymbol{or}\ x = n\ and\ y = 0$.

For easier representation, the coefficients of the elements $V_{k\,re}[n]$ and $V_{k\,im}[n]$ are denoted by γ_{ik} and β_{ik} respectively where *i* is the index of the PV bus whose real power is to be balanced.

$$\gamma_{ik} = 2(G_{ik}V_{i\,re}[0] + B_{ik}V_{i\,im}[0]),\ \ k \neq i$$

$$\beta_{ik} = 2(-B_{ik}V_{i\,re}[0] + G_{ik}V_{i\,im}[0])\ ,k \neq i$$

$$\gamma_{ii} = 2\left(G_{ii}V_{i\,re}[0] + B_{ii}V_{i\,im}[0] + \sum_{k=1}^{N}(G_{ik}V_{k\,re}[0] - B_{ik}V_{k\,im}[0])\right)$$

(3.19)

$$\beta_{ii} = 2\left(-B_{ii}V_{i\,re}[0] + G_{ii}V_{i\,im}[0] + \sum_{k=1}^{N}(B_{ik}V_{k\,re}[0] + G_{ik}V_{k\,im}[0])\right)$$

In RHS power value comes into picture for ***s*** raised to 1. Hence the multiplication factor δ_{n1} so that power injection comes into picture only when $n = 1$.Hence the other terms that include lower powers of ***s*** are transferred onto the RHS so that the unknowns remain on the LHS

$$\zeta_i[n] = 2\delta_{n1}P_i - \sum_{x=1}^{n-1}\left\{\sum_{k=1}^{N}[G_{ik}V_{i\,re}[x] + B_{ik}V_{i\,im}[x] \quad -B_{ik}V_{i\,re}[x] + G_{ik}V_{i\,im}[x]]\begin{bmatrix}V_{k\,re}[n-x]\\V_{k\,im}[n-x]\end{bmatrix}\right\}$$

(3.20)

The other known value for a PV bus is the magnitude of voltage.

$$V_i(s) \times V_i^*(s^*) = 1 + s(\left|V_i^{sp}\right|^2 - 1)$$

(3.21)

$$(V_i[0] + V_i[1]s + V_i[2]s^2 + \cdots) \times (V_i^*[0] + V_i^*[1]s + V_i^*[2]s^2 + \cdots) = 1 + s(\left|V_i^{sp}\right|^2 - 1)$$

Equating the coefficients on both the sides we have

$$V_i[0] \times V_i^*[0] = 1$$

(3.22)

$$V_i[0]V_i^*[1] + V_i[1]V_i^*[0] = |V_i^{sp}|^2 - 1 \;\Rightarrow V_i[1] + V_i^*[1] = 2V_{i\,re}[1] = |V_i^{sp}|^2 - 1$$

$$V_i[0]V_i^*[2] + V_i[1]V_i^*[1] + V_i[2]V_i^*[0] = 0 \;\Rightarrow\; 2V_{i\,re}[2] = -V_i[1]V_i^*[1]$$

Generalizing this we have

$$V_{i\,re}[n] = \delta_{n0} + \delta_{n1}\frac{V_i^{sp\,2} - 1}{2} - 0.5\left[\sum_{k=1}^{n-1} V_i[k]V_i^*[n-k]\right]$$

(3.23)

Where δ_{n0} vanishes for every n except when $n = 0$ and δ_{n1} which vanishes for every n except when $n = 1$

Bus type of Bus i	Original Equation	Holomorphic Embedded Equation
PQ	$\sum_{k=1}^{N} Y_{ik}V_k = \frac{S_i^*}{V_i^*}$	$\sum_{k=1}^{N} Y_{ik\,series}V_k(s) = \frac{sS_i^*}{V_i^*(s^*)} - sY_{i\,shunt}V_i(s)$
PV	$V_i^* \sum_{k=1}^{N} Y_{ik}V_k + V_i \sum_{k=1}^{N} V_k^* Y_{ik}^* = 2P_i$ $\lvert V_i\rvert = V_i^{sp}$	$V_i^*(s^*) \sum_{k=1}^{N} Y_{ik\,series}V_k(s) + V_i(s) \sum_{k=1}^{N} V_k^*(s^*)Y_{ik\,series}^* = 2sP_i - 2sRe(Y_{i\,shunt})\lvert V_i^{sp}\rvert^2$ $V_i(s) \times V_i^*(s^*) = 1 + s(\lvert V_i^{sp}\rvert^2 - 1)$
Slack	$V_i = V_i^{sp}$	$V_i = 1 + s(\lvert V_i^{sp}\rvert - 1)$

Table 3-1. Original power equations and corresponding holomorphic embedded equations

Thus for a 3bus system with ***Bus 1*** as ***Slack*** and ***Bus 2*** as ***PV*** and ***Bus 3*** as ***PQ*** the linear set of equations represented as a matrix would be

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \gamma_{21} & \beta_{21} & \gamma_{22} & \beta_{22} & \gamma_{23} & \beta_{23} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ G_{31} & -B_{31} & G_{32} & -B_{32} & G_{33} & -B_{33} \\ B_{31} & G_{31} & B_{32} & G_{32} & B_{33} & G_{33} \end{bmatrix} \times \begin{bmatrix} V_{1\,re}[n] \\ V_{1\,im}[n] \\ V_{2\,re}[n] \\ V_{2\,im}[n] \\ V_{3\,re}[n] \\ V_{3\,im}[n] \end{bmatrix}$$

$$= \begin{bmatrix} \delta_{n0} + \delta_{n1}(\lvert V_{slack}^{sp}\rvert - 1) \\ 0 \\ \zeta_2[n] \\ \delta_{n0} + \delta_{n1}\frac{V_i^{sp^2} - 1}{2} - 0.5[\sum_{k=1}^{n-1} V_i[k]V_i^*[n-k]] \\ Re(S_3^* W_3^*[n-1] - Y_{3shunt}V_3[n-1]) \\ Im(S_3^* W_3^*[n-1] - Y_{3shunt}V_3[n-1]) \end{bmatrix} \qquad (3.24)$$

Analytic Continuation:

Calculation of the coefficients of the power series can be carried on progressively till infinity**.** The solution may not always converge for ***s=1*** as the radius of convergence can be less than 1. Analytic continuation comes in hand in these scenarios.

In complex analysis, analytic continuation is a technique to extend the domain of a given analytic function. Analytic continuation is defining further values of a function, in a new region where an infinite series representation of this function becomes divergent.

For example consider a series given $f_1(s)$ given by

$$f_1(s) = 1 + s + s^2 + s^3 + \cdots \tag{3.25}$$

This function converges for values with $|s| < 1$. For the same domain $f_1(s)$ can be represented as

$$f_2(s) = \frac{1}{1-s} \tag{3.26}$$

The function $f_2(s)$ represents $f_1(s)$ accurately when $|s|$<1. In addition, $f_2(s)$is valid over a larger domain in the complex plane. Thus, $f_2(s)$ is an analytic continuation of the original series $f_1(s)$. It is valid for all values of s except for $s = 1$ and hence is called the maximal analytic continuation of $f_1(s)$.

There are different approaches for finding the analytic continuation function of another function. The problem of interest is to find that analytic continuation that extends the analytic function f_1 to the largest domain possible in the complex plane, i.e. the maximal analytic continuation. Stahl's Pade convergence theory shows that "*For an analytic function f with finite singularities, the close to diagonal sequence of diagonal Padé approximants converge to the original function in the extremal domain".* In other words, Pade approximants can be used to evaluate the maximal analytic continuation.

Pade Approximates:

A Pade approximant is the best approximation of a function by a rational function of given order – under this technique, the approximant's power series agrees with the power series of the function it is approximating. The technique was developed around 1890 by Henri Pade, but goes back to Georg Frobenius who introduced the idea and investigated the features of rational approximations of power series. The Pade approximant often gives better approximation of the function than truncating its Taylor series, and it may still work where the Taylor series does not converge. For these reasons Pade approximants are used extensively in computer calculations.

Pade Approximate of Truncated Taylor Series:

Consider the power series representation of an analytic function

$$f(s) = c_0 + c_1 s + c_2 s^2 + \cdots \infty = \sum_{n=0}^{\infty} c_n s^n \tag{3.27}$$

Where

$$c_n \ \ is\ the\ coefficient\ of\ s^n\ in\ the\ power\ series\ expansion$$

If L is the degree of the numerator and M is the degree of the denominator conventionally it is represented as

$$[L/M]\ Pade \ \ = \frac{a_0 + a_1 s + a_2 s^2 + \cdots a_L s^L}{b_0 + b_1 s + b_2 s^2 + \cdots b_M s^M} \tag{3.28}$$

$$f(s) = a_0 + a_1 s + a_2 s^2 + \cdots a_{L+M} s^{L+M} + O(s^{L+M+1})$$

$$(b_0 + b_1 s + b_2 s^2 + \cdots b_M s^M)(c_0 + c_1 s + c_2 s^2 + \cdots c_{L+M} s^{L+M})$$
$$= \ a_0 + a_1 s + a_2 s^2 + \cdots a_L s^L$$

Equating the coefficients of s^{L+1} to s^{L+M} on LHS to 0 we have

$$b_M c_{L-M+1} + b_{M-1} c_{L-M+2} + \cdots b_0 c_{L+1} = 0$$

$$b_M c_{L-M+2} + b_{M-1} c_{L-M+2} + \cdots b_0 c_{L+2} = 0$$

.

. (3.29)

.

.

$$b_M c_L + b_{M-1} c_{L+1} + \cdots b_0 c_{L+M} = 0$$

This is a system of M linear equations which can be represented in the form of a matrix and solved using MATLAB.

Equating the coefficients of s^L to s^0 on LHS and RHS we have

$$c_0 = a_0$$

$$b_0 c_1 + b_1 c_0 = a_1$$

.

.

. (3.30)

.

$$\sum_{k=0}^{L} c_k b_{L-k} = a_L$$

The coefficients of the *c* series are known; the *b* series coefficients are evaluated from Eq.(37). Thus from Eq.(38) the numerator polynomial coefficients can be evaluated.

As to the question of the number of terms needed in the power series in order to attain a given level of precision, practice has shown that typically anywhere from 10 to 40 terms suffice to reach 5-digit precision in large networks, and about 60 terms will exhaust the limits of the computer arithmetic in double precision.

In particular, the diagonal Pade approximants converge quicker than the truncated power series. Stahl's results, after the seminal works of Nuttall [11], reveal that Pade approximants are really a means for maximal analytic continuation. This method very strongly guarantees if the Pade approximants do not converge at s = 1 then it is guaranteed that there is no solution (that is, the system is beyond voltage collapse).

To recapitulate, the **Holomorphic Embedding Load Flow Method** boils down to these steps:

1. Choose a suitable complex embedding using a complex parameter ***s***. It needs to be holomorphic (uses $V^*(s^*)$, not $V^*(s)$). At ***s = 0***, this embedding should be such that the system becomes linear and trivial to solve (the no-load, no generation case). This unambiguously selects the reference solution at ***s = 0***.
2. Calculate the power series of ***V(s)*** corresponding to the reference solution, by means of a sequence of linear systems that yield the coefficients progressively. The matrix in those systems remains always constant and the RHS can always be calculated from the results of the previous system (Eq.(32)).
3. Compute the solution at s = 1 as the analytical continuation of the power series obtained in step 2, by using Pade Approximants. These are guaranteed to yield maximal analytical continuation, therefore the solution is obtained when it exists, or a divergence is obtained when it does not exist.
4. Steps 2 and 3 are typically performed in an interleaved fashion, that is, after a new coefficient of the series is obtained, a new Pade approximant can be computed. The procedure stops when the desired accuracy is obtained, or when oscillation or divergence is detected. Section IV below will illustrate the whole procedure step by step.

CHAPTER IV: RESULTS

The comparison of power mismatches for IEEE systems [Ref.13] of both the traditional method of Newton-Raphson and the novel method of Holomorphic Embedding.

Sl.No	No.of Buses	NR(MAX MISMATCH)	HELM(MAX MISMATCH)
1	**4**	$\mathbf{1.065 \times 10^{-9}}$	$\mathbf{0.075 \times 10^{-8}}$
2	**14**	$\mathbf{5.9779 \times 10^{-9}}$	$\mathbf{3.045 \times 10^{-9}}$
3	**30**	$\mathbf{3.541 \times 10^{-9}}$	$\mathbf{4.415 \times 10^{-9}}$
4	**57**	$\mathbf{4.781 \times 10^{-9}}$	$\mathbf{9.971 \times 10^{-9}}$
5	**162**	$\mathbf{5.781 \times 10^{-9}}$	$\mathbf{1.456 \times 10^{-9}}$

Table 4-1.Power Mismatches (in pu)

Where

NR(MAX MISMATCH) Maximum of the absolute values of the mismatches (in pu) in Newton Raphson

HELM(MAX MISMATCH) Maximum of the absolute values of the mismatches (in pu) in Holomorphic Embedded Load Flow Method

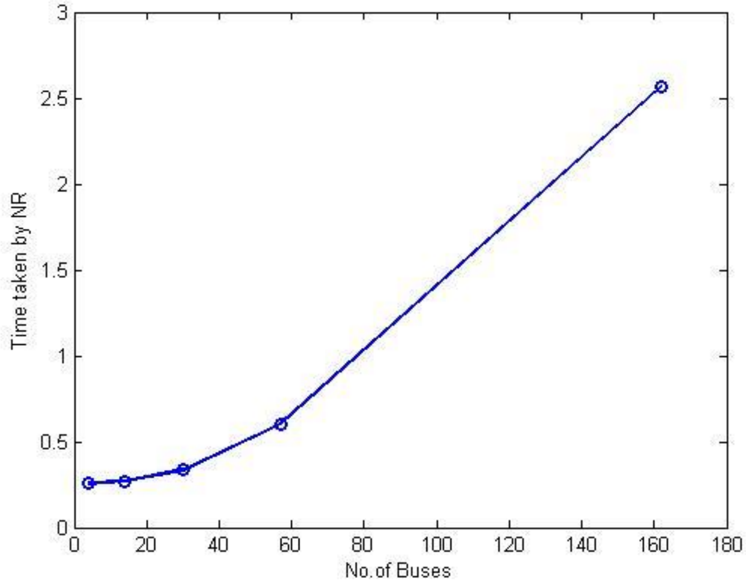

Figure 4-1. Time Taken by NR vs Total No.of Buses

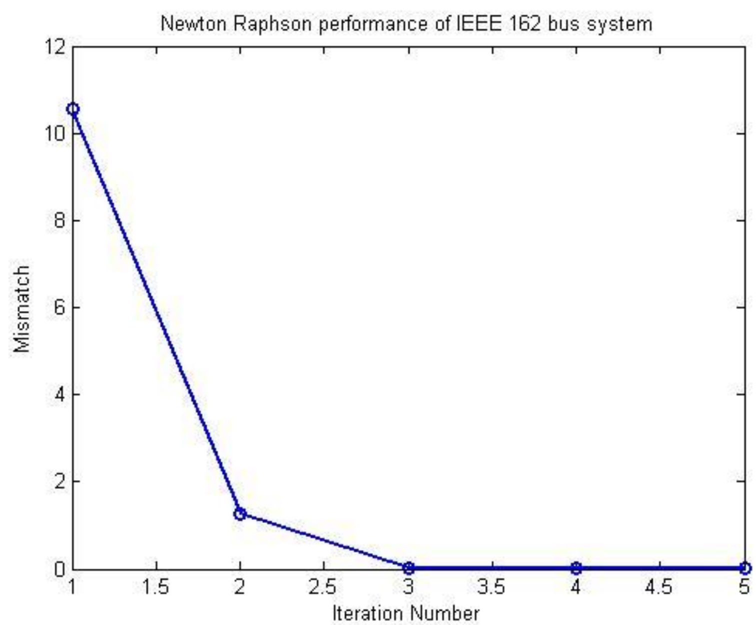

Figure 4-2. Mismatch vs Iteration number for IEEE 162 bus power system

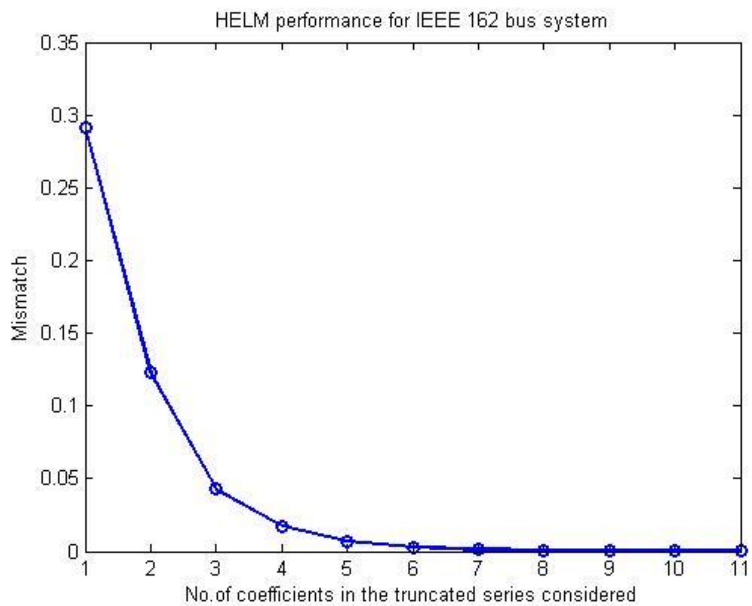

Figure 4-3. Mismatch vs Iteration number for HELM for IEEE 162 bus system

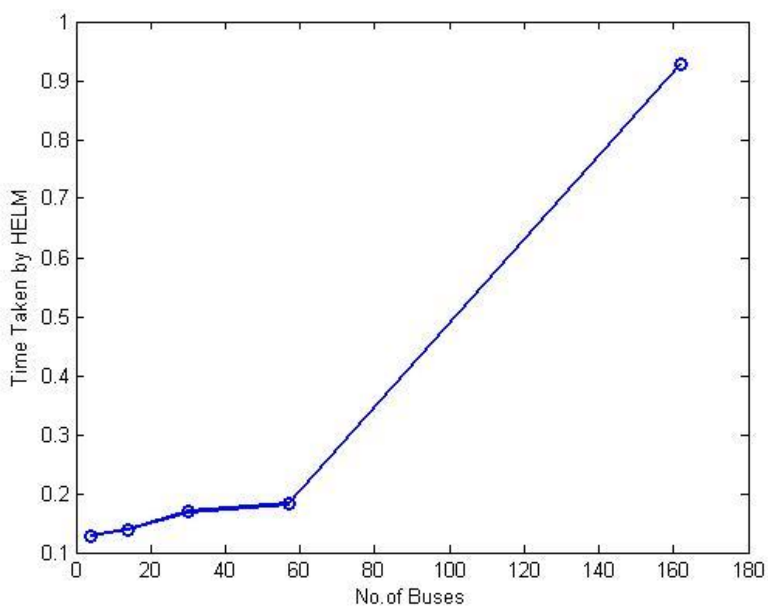

Figure 4-4.time Taken by HELM vs Total No.of Buses

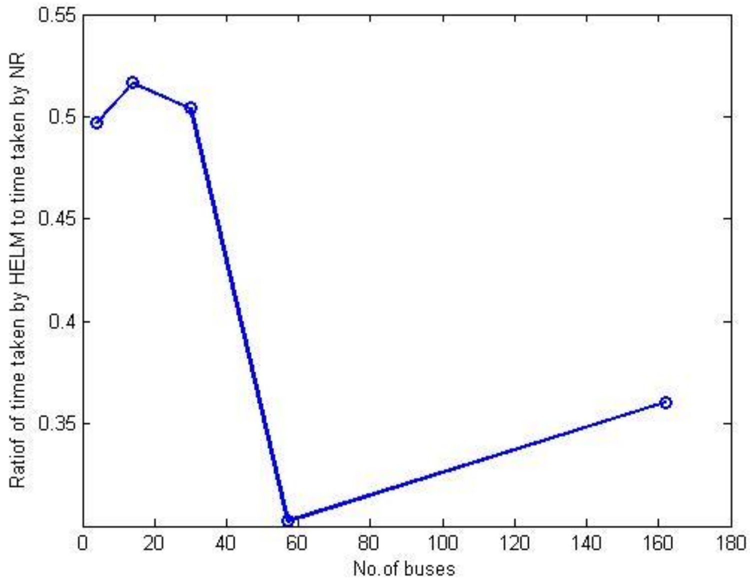

Figure 4-5. Ratio of times taken by HELM and NR for different test cases of IEEE power systems

Time taken by both HELM and NR to arrive at the solution increases with increase in size of the problem. Increase in size implies higher computational costs owing to the increase in the size of matrices to be dealt with.

Ratio of times taken by HELM and NR are observed to be less than unity for any value of total number of buses. Thus HELM is a faster and more reliable algorithm compared to Newton Raphson.

CHAPTER V: CONCLUSION

The NR method requires updating and re-factorizing of the Jacobian matrix in each iteration. Thus in the NR method, calculation of a new voltage estimate involves calculating the mismatch vector, updating the Jacobian matrix, LU factorization of the Jacobian matrix, forward and backward substitution. For the HE method, the modified bus admittance matrix needs to be factorized only once. Calculation of mismatches is a necessary step in NR method whereas the HE method, like Gauss-Seidel type the difference in the voltage solution with one additional term in the power series can be used to determine convergence. The NR method does not involve the analytic continuation process.

In brief the report discusses the need for a power flow solution and then gives the details of the vastly used traditional methods. It points out the problems with the traditional methods and brings to light the need for an initial estimate independent method. It then jumps to theory of complex numbers and how holomorphism and analyticity help in arriving at the solution. The Power balance non-analytics are converted to holomorphic functions. It also highlights the comparison between the performances of the traditional method of Newton Raphson and the new Holomorphic Embedded Load Flow Method. The method was tested for IEEE test cases with buses of 14, 30, 57, and 162.

REFERENCES

[1]. Antonio Trias, 2012, The Holomorphic Embedded Load Flow Method (IEEE paper)

[2]. https://en.wikipedia.org/wiki/Power-flow_study

[3]. https://en.wikipedia.org/wiki/Slack_bus

[4]. http://nptel.ac.in/courses/Webcourse-contents/IIT-KANPUR/power-system/chapter_4/4_3a.html

[5]. https://en.wikipedia.org/wiki/Newton%27s_method

[6]. http://nptel.ac.in/courses/Webcourse-contents/IIT-KANPUR/power-system/chapter_4/4_10.html

[7]. http://scholarworks.uno.edu/cgi/viewcontent.cgi?article=2220&context=td

[8]. https://repository.asu.edu/attachments/137256/content/SUBRAMANIAN_asu_0010N_14139.pdf

[9]. http://nptel.ac.in/courses/122104019/numerical-analysis/Rathish-kumar/ratish-1/f3node7.html

[10]. "The convergence of Pade approximants to functions with branchpoints," Journal of Approximation Theory, vol. 91, no. 2, pp. 139–204,1997.

[11]. G. Baker and P. Graves-Morris, Pade approximants, ser. Encyclopedia of mathematics and its applications. Cambridge University Press, 1996.

[12]. J. Nuttall, "On convergence of Pad´e approximants to functions with branch points," in Pad´e and Rational Approximation, E. B. Saff and R. S. Varga, Eds. Academic Press, New York, 1977, pp. 101–109.

[13]. https://www.ee.washington.edu/research/pstca/