Design of Power System Stabilizer

A Project Report

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ABSTRACT

The study of small-signal instability in a power system is presented. The formulation of a reduced model for multi-machine system is done in order to achieve robustness and avoid computational issues while performing numerical integration of machines' non-linear ordinary differential equations. An efficient algorithm for numerical integration of ODE's is also discussed. Small-signal analysis of the system is also performed in order to identify the modes that lead the system to small-signal instability. The procedure for the design of Power System Stabilizer (PSS) is detailed. The participation factor method is used to select the PSS location for any given mode. The tuning of PSS, thus designed, is done in order to obtain an appropriate PSS gain that can provide adequate damping to the troublesome modes.

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CHAPTER I: Introduction

Power systems in practice are subjected to various disturbances frequently. These disturbances occur because of various causes like faults at buses, change in mechanical torque and failure of transmission lines, etc. These disturbances, even of short duration, deviates the system behavior from steady state or worse can make the system unstable. So, exciters are used for regulating generator terminal voltages. Exciters mostly achieve this by acting as a feedback to the system taking terminal voltage of the generator as input. In practical purposes, the fast acting exciters are mostly used for achieving better results. So, these exciters generally result in low frequency modes that are not adequately damped. This gives rise to small-signal instability in the system. The formulation of system equations in order to study the small-signal instability in a multi-machine system is discussed in Chapter-2.

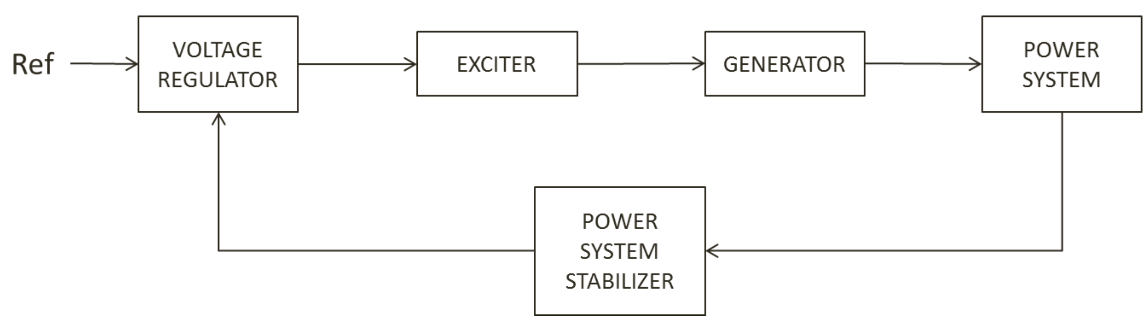

Figure 1: Block Diagram of an Excitation System

Power system Stabilizers (PSS) are used to provide adequate damping for the modes that lead the system to small-signal instability. PSS is a feedback controller which takes speed or power signal as input and outputs a control signal V_s as an input to the exciter. The block diagram of an Excitation system at a generator is shown in Figure 1. The design of PSS is critical as the PSS doesn't just have to provide sufficient damping for troublesome modes, instead it also should not destabilize other modes. So, a detailed procedure for design of PSS is discussed in Chapter 3. The concepts discussed in Chapter 2 and 3 are applied to a Single-Machine Infinite Bus (SMIB) system and a 10-bus, 4-machine system in Chapter-4.

CHAPTER II: Multi-Machine System Behavior

Model 1.1, a field circuit with one equivalent damper on q-axis, is assumed for the representation of the Synchronous Machine in this chapter. This model is governed by non-linear ordinary differential equations given in equation (1):

$$\begin{gathered}
\frac{d\delta}{dt} = w_B(S_m - S_{m0}) \\
\frac{dS_m}{dt} = \frac{1}{2H}[T_m - T_e - D(S_m - S_{m0})] \\
\frac{dE'_q}{dt} = \frac{1}{T'_{d0}}\left[E_{fd} - E'_q + (x_d - x'_d)I_d\right] \\
\frac{dE'_d}{dt} = \frac{1}{T'_{q0}}\left[-E'_d - \left(x_q - x'_q\right)I_q\right] \\
\frac{dE_{fd}}{dt} = \frac{1}{T_E}\left[-E_{fd} + K_E\left(V_{ref} + V_s - V_t\right)\right] \\
E_{fdmin} \leq E_{fd} \leq E_{fdmax}
\end{gathered} \qquad (1)$$

where δ is load angle of the synchronous generator, S_m is operating slip of the generator, $E_{fd}$is the exciter's output voltage, $E'_q$and E'_d are components of the rotor's internal voltage in q-axis and d-axis respectively. The variables T'_{d0} and T'_{q0} are open-circuit transient time-constant of d-axis and q-axis respectively whereas S_{m0} is initial operating slip of the generator. The variables T_E and K_E are exciter time constant and exciter gain respectively whereas the variable V_s is PSS output given to the exciter. The mechanical torque T_m and the reference voltage V_{ref} are considered as inputs to the system given in equation (1) whereas the electrical torque T_e and terminal voltage V_t are given by:

$$\begin{gathered}
T_e = E'_q I_q + E'_d I_d + \left(x'_d - x'_q\right) I_d I_q \\
V_t = \sqrt{V_q^2 + V_d^2}
\end{gathered} \qquad (2)$$

The stator variables I_q, I_d, V_q and V_d are calculated using network equations. In a single-machine system, the calculation of these variables is relatively easy because of simplicity of the network. But the complexity and bulkiness of multi-machine system brings in computational issues while trying to study the small-signal instability. Hence, a reduced model is considered and its formulation is discussed in this chapter.

The study of small-signal instability, in other words, is studying the variation of the state variables $\delta, S_m, E'_q, E'_d, E_{fd}$ over duration of time when the system is under small-signal instability. Hence, the numerical integration of the ODE's in equation (1) by fourth-order Runge-Kutta Method is detailed in this chapter. So, the usage of the network equations of the reduced model in the numerical integration of the machine equations will help us study the system behavior and in turn its instability.

2.1 Reduced model

While studying the small-signal instability in a multi-machine system, observation of the generators' output is crucial. So, the reduced model must include all the generators in addition to components which reflect the system's transmission line network. So, a reduced model where all buses except generator buses are eliminated should be considered. In other words, we need to reduce the admittance matrix Y. For the system of n buses and m generators, the ith row and jth column element Y_{ij} of the $n \, x \, n$ admittance matrix Y is given as follows:

$$\boldsymbol{Y_{ij}} = \begin{Bmatrix} -\boldsymbol{y_{ij}} & \boldsymbol{for\ i \neq j} \\ \boldsymbol{y_i} + \sum_{\substack{k=1 \\ k \neq i}}^{n} \boldsymbol{y_{ik}} & \boldsymbol{for\ i = j} \end{Bmatrix} \quad (3)$$

where $y_{ip} = g_{ip} + jb_{ip}$ is the summed admittance of all the power lines going from bus i to bus p and $y_i = jb_{ii}$ is the summed admittance to ground at bus i. The admittance to ground y_i will include transmission lines' shunt susceptance and load admittance as the loads are considered to be of constant impedance. The load admittance y_{Li} at a particular bus i can be calculated as follows:

$$\boldsymbol{y_{Li}} = \frac{\boldsymbol{P_{Li}^0 - jQ_{Li}^0}}{|\underline{\boldsymbol{V}}_i^{\boldsymbol{0}}|^2} \tag{4}$$

where P_{Li}^0, Q_{Li}^0, $\underline{V_i}^0$ are initial values of real load power, reactive load power and voltage at the bus i respectively. The admittance matrix Y, so obtained, is sparse and satisfies the equation:

$$[\boldsymbol{Y}]\begin{bmatrix}\underline{\overrightarrow{\boldsymbol{V_l}}}\\ \underline{\overrightarrow{\boldsymbol{V_g}}}\end{bmatrix} = \begin{bmatrix}\overrightarrow{\boldsymbol{O}}\\ \underline{\overrightarrow{\boldsymbol{I_g}}}\end{bmatrix} \tag{5}$$

where $\underline{\overrightarrow{V_g}}, \underline{\overrightarrow{I_g}}$ are column vectors of phasors of generators' terminal voltages and generators' currents respectively and are of order m, m respectively whereas $\underline{\overrightarrow{V_l}}$ is column vector of order $(n - m)$ and its elements are phasors of voltages at buses other than generator buses . In order to obtain a reduced model, we need to understand influence of generator terminal voltages on the generator currents. In other words, we have to eliminate $\underline{\overrightarrow{V_l}}$. So, rewriting Y as:

$$[\boldsymbol{Y}] = \begin{bmatrix}\boldsymbol{U} & \boldsymbol{M}\\ \boldsymbol{M^T} & \boldsymbol{L}\end{bmatrix} \tag{6}$$

where U, M, L are matrices of order $(n - m)\ x\ (n - m), (n - m)\ x\ m$ and $m\ x\ m$ respectively. So, the admittance matrix Y_{red} satisfying the equation $[Y_{red}]\underline{\overrightarrow{V_g}} = \underline{\overrightarrow{I_g}}$ is as follows:

$$[\boldsymbol{Y_{red}}] = \boldsymbol{L} - \boldsymbol{M^T U^{-1} M} \tag{7}$$

where Y_{red} is a matrix of order $m \times m$, m being the number of generators in the system.

The network equation given by $[Y_{red}]\overrightarrow{\underline{V_g}} = \overrightarrow{\underline{I_g}}$ doesn't give information about q-axis and d-axis components of stator phase voltages and stator currents. But these components are important as they appear in ordinary differential equations of the generator given in equation (1). The terminal voltage $\underline{V}_{gi}$ and generator current $\underline{I}_{gi}$ at any given generator i can be expressed in terms of the stator components as follows:

$$\begin{aligned} \underline{\boldsymbol{V}}_{\boldsymbol{gi}} &= (\boldsymbol{V_{qi}} + \boldsymbol{jV_{di}})\boldsymbol{e^{j\delta_i}} \\ \underline{\boldsymbol{I}}_{\boldsymbol{gi}} &= (\boldsymbol{I_{qi}} + \boldsymbol{jI_{di}})\boldsymbol{e^{j\delta_i}} \end{aligned} \tag{8}$$

Substituting the above equation in the network equation $[Y_{red}]\overrightarrow{\underline{V_g}} = \overrightarrow{\underline{I_g}}$, we obtain:

$$(I_{qk} + jI_{dk})e^{j\delta_k} = \sum_{i=1}^{m} Y_{ki}(V_{qi} + jV_{di})e^{j\delta_i}$$

or,

$$(\boldsymbol{I_{qk}} + \boldsymbol{jI_{dk}}) = \sum_{i=1}^{m}(\boldsymbol{G_{ki}} + \boldsymbol{jB_{ki}})(\boldsymbol{V_{qi}} + \boldsymbol{jV_{di}})\boldsymbol{e^{j(\delta_i - \delta_k)}} \tag{9}$$

where $Y_{ki} = G_{ki} + jB_{ki}$ is k^{th} row and i^{th} column element of the reduced admittance matrix Y_{red}. Simplifying the above equation by dividing it into real and imaginary parts, we obtain a relation between voltage components and current components of the generator as follows:

$$\begin{aligned} \boldsymbol{I_{qk}} &= \sum_{i=1}^{m}(\boldsymbol{M_{ki}V_{qi}} - \boldsymbol{N_{ki}V_{di}}) \\ \boldsymbol{I_{dk}} &= \sum_{i=1}^{m}(\boldsymbol{N_{ki}V_{qi}} + \boldsymbol{M_{ki}V_{di}}) \end{aligned} \tag{10}$$

where

$$\begin{aligned} \boldsymbol{M_{ki}} &= \boldsymbol{G_{ki}}\,\mathbf{cos}(\boldsymbol{\delta_i} - \boldsymbol{\delta_k}) - \boldsymbol{B_{ki}}\,\mathbf{sin}(\boldsymbol{\delta_i} - \boldsymbol{\delta_k}) \\ \boldsymbol{N_{ki}} &= \boldsymbol{G_{ki}}\,\mathbf{sin}(\boldsymbol{\delta_i} - \boldsymbol{\delta_k}) + \boldsymbol{B_{ki}}\,\mathbf{cos}(\boldsymbol{\delta_i} - \boldsymbol{\delta_k}) \end{aligned}$$

Writing equation (10) in matrix form, we obtain:

$$\begin{bmatrix} \overrightarrow{\boldsymbol{I_q}} \\ \overrightarrow{\boldsymbol{I_d}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{M} & -\boldsymbol{N} \\ \boldsymbol{N} & \boldsymbol{M} \end{bmatrix} \begin{bmatrix} \overrightarrow{\boldsymbol{V_q}} \\ \overrightarrow{\boldsymbol{V_d}} \end{bmatrix} \tag{11}$$

where $\overrightarrow{I_q}, \overrightarrow{I_d}, \overrightarrow{V_q}, \overrightarrow{V_d}$ are column vectors of order m and their kth element is represented by kth generator's stator variables $I_{qk}, I_{dk}, V_{qk}, V_{dk}$ respectively. M_{ki}, N_{ki} are elements of the $m \ x \ m$ matrices M, N . Rewriting the above equation as follows:

$$\begin{bmatrix} \overrightarrow{\boldsymbol{I_q}} \\ \overrightarrow{\boldsymbol{I_d}} \end{bmatrix} = [\boldsymbol{D_1}] \begin{bmatrix} \overrightarrow{\boldsymbol{V_q}} \\ \overrightarrow{\boldsymbol{V_d}} \end{bmatrix} \tag{12}$$

where D_1 is a matrix of order $2m \ x \ 2m$, m being the number of generators in the system.

The network equations thus obtained doesn't have significance unless one of vectors on either side is known. So, we need to express one of these non-state vectors in terms of the state variables δ, E'_q, E'_d. Considering the synchronous machine to be of model 1.1, we can write stator equations for any kth generator as follows:

$$\begin{aligned} \boldsymbol{E'_{qk} + x'_{dk} I_{dk} - r_{ak} I_{qk} = V_{qk}} \\ \boldsymbol{E'_{dk} - x'_{qk} I_{qk} - r_{ak} I_{dk} = V_{dk}} \end{aligned} \tag{13}$$

where x'_{dk}, x'_{qk}, r_{ak} are d-axis transient reactance, q-axis transient reactance and armature resistance of generator k respectively. Writing equation (13) in matrix form, we obtain:

$$\begin{bmatrix} \overrightarrow{V_q} \\ \overrightarrow{V_d} \end{bmatrix} = \begin{bmatrix} \overrightarrow{E'_q} \\ \overrightarrow{E'_d} \end{bmatrix} + \begin{bmatrix} -R_a & X'_d \\ -X'_q & -R_a \end{bmatrix} \begin{bmatrix} \overrightarrow{I_q} \\ \overrightarrow{I_d} \end{bmatrix}$$

where X'_d, X'_q, R_a are diagonal matrices of order m and its diagonal elements are x'_{dk}, x'_{qk}, r_{ak} respectively whereas $\overrightarrow{E'_q}, \overrightarrow{E'_d}$ are column vectors of order m and their kth element is represented by E'_{qk}, E'_{dk} respectively . Rewriting the above equation as follows:

$$\begin{bmatrix} \overrightarrow{\boldsymbol{V_q}} \\ \overrightarrow{\boldsymbol{V_d}} \end{bmatrix} = \begin{bmatrix} \overrightarrow{\boldsymbol{E'_q}} \\ \overrightarrow{\boldsymbol{E'_d}} \end{bmatrix} + [\boldsymbol{B_1}] \begin{bmatrix} \overrightarrow{\boldsymbol{I_q}} \\ \overrightarrow{\boldsymbol{I_d}} \end{bmatrix} \tag{14}$$

where B_1 is a matrix of order $2m\ x\ 2m$. Substituting equation (12) in the above equation, we obtain:

$$\begin{bmatrix} \overrightarrow{V_q} \\ \overrightarrow{V_d} \end{bmatrix} = \begin{bmatrix} \overrightarrow{E'_q} \\ \overrightarrow{E'_d} \end{bmatrix} + [B_1][D_1] \begin{bmatrix} \overrightarrow{V_q} \\ \overrightarrow{V_d} \end{bmatrix}$$

or,

$$\begin{bmatrix} \overrightarrow{\boldsymbol{V_q}} \\ \overrightarrow{\boldsymbol{V_d}} \end{bmatrix} = [\boldsymbol{I} - \boldsymbol{B_1 D_1}]^{-1} \begin{bmatrix} \overrightarrow{\boldsymbol{E'_q}} \\ \overrightarrow{\boldsymbol{E'_d}} \end{bmatrix} \tag{15}$$

If the states variables δ, E'_q, E'_d at any given time are known, then the network equation (12) and (15) will helpful in evaluating the unknown d-axis and q-axis components of stator phase voltages and stator currents of the generators at that time.

2.2 Runge-Kutta Method

The numerical integration of non-linear ordinary differential equations is critical due to its non-linearity. So, Runge-Kutta method is used for numerical integration of ordinary differential equations in equation (1) and the variation of state variables $\delta, S_m, E'_q, E'_d, E_{fd}$ under various disturbances is studied. Fourth-order Runge-Kutta Method is an explicit single-step algorithm that uses intermediate points in the interval $(t-1, t)$ to calculate state at time t. Rewriting equation (1) as follows:

$$\frac{\boldsymbol{d\vec{X}}}{\boldsymbol{dt}} = \boldsymbol{f}(\vec{\boldsymbol{X}}, \vec{\boldsymbol{U}}) \tag{16}$$

where,

$$\overrightarrow{X} = \begin{bmatrix} \vec{\delta} & \overrightarrow{S_m} & \overrightarrow{E'_q} & \overrightarrow{E'_d} & \overrightarrow{E_{fd}} \end{bmatrix}^t$$
$$\vec{U} = [\overrightarrow{T_m} \quad \overrightarrow{(V_{ref} + V_s)}\ ]^t$$

The value of state $\vec{X}(t)$at time t is given by this method as follows:

$$\vec{\boldsymbol{X}}(\boldsymbol{t}) = \vec{\boldsymbol{X}}(\boldsymbol{t}-\mathbf{1}) + \frac{\boldsymbol{h}}{\mathbf{6}}(\overrightarrow{\boldsymbol{K_1}} + \mathbf{2}\overrightarrow{\boldsymbol{K_2}} + \mathbf{2}\overrightarrow{\boldsymbol{K_3}} + \overrightarrow{\boldsymbol{K_4}}) \tag{17}$$

where

$$\overrightarrow{K_1} = f(\vec{X}(t-1), \vec{U}(t-1))$$

$$\overrightarrow{K_2} = f(\vec{X}(t-1) + \frac{h}{2}\overrightarrow{K_1}, \vec{U}(t-1) + \frac{h}{2})$$

$$\overrightarrow{K_3} = f(\vec{X}(t-1) + \frac{h}{2}\overrightarrow{K_2}, \vec{U}(t-1) + \frac{h}{2})$$

$$\overrightarrow{K_4} = f(\vec{X}(t-1) + h\overrightarrow{K_3}, \vec{U}(t-1))$$

Implementation of an algorithm of above kind at a given time T will give us state variables of the system at time T. In order to evaluate the system behavior at further times (t>T), the system variables need to be evaluated using the equations (12), (15) and (17) at every instant. It's also necessary to include the exciter's limiter in the algorithm as the value of the state variable E_{fd} depends on the limiter used. The Runge-Kutta Method can also be used to verify the effectiveness of PSS designed. In that case, the PSS output V_s and derivatives of V_s are also considered as state variables and Runge-Kutta method is implemented with equations (17) and PSS equations in differential form. So, the Runge-Kutta method will be helpful in understanding the system behavior.

CHAPTER III: Design of Power System Stabilizer (PSS)

The function of Power System Stabilizers (PSS) is to provide sufficient damping for the modes which lead to small-signal instability. These modes normally are local modes (0.8-1.8 Hz) for single-machine systems whereas for multi-machine systems, small-signal instability also occurs due to inter-area modes (0.2-0.7 Hz) in addition to the local modes. These modes have to be identified and its effect on each generator has to be analyzed in order to choose an appropriate PSS location in multi-machine system. Usage of participation factors to determine a PSS location is discussed in this chapter. The design of PSS involves choosing appropriate parameters for its components: Washout block, Compensator and Limiter. The conditions involved in choosing these parameters are discussed in this chapter. Also, the root locus of the system including PSS is performed in order to select an appropriate gain for PSS. Hence, the PSS will be tuned such that the troublesome mode is damped.

3.1 PSS Location

Eigenvalues of the system can used to determine the modes which lead the system to instability. So, the system equations (1) have to be written in state-space form given by equation (18) in order to determine eigenvalues of the system.

$$\dot{\vec{\boldsymbol{X}}} = [\boldsymbol{A}]\vec{\boldsymbol{X}} + [\boldsymbol{B}]\vec{\boldsymbol{U}} \tag{18}$$

where A and B are matrices of order $5m \, x \, 5m$ whereas the state X and input U are column vectors of order $5m$ and are defined by:

$$\vec{X} = \left[\Delta\delta_1, \Delta\delta_2, \ldots \Delta\delta_m, \Delta S_{m1}, \ldots, \Delta E'_{q1}, \ldots . \Delta E'_{qm}, \Delta E'_{d1}, .. \Delta E'_{dm}, \Delta E_{fd1}, \ldots \Delta E_{fdm}\right]^T$$

$$\vec{U} = \left[\Delta T_{m1}, \Delta T_{m2}, \ldots \ldots, \Delta V_{ref1} + \Delta V_{s1}, \ldots \ldots, \Delta V_{refm} + \Delta V_{sm}\right]^T$$

As the generator's differential equations (1) are non-linear, they have to be linearized in order to represent system in state-space form. The linearized form of differential equations for the k^th generator is as follows:

$$\Delta\boldsymbol{\delta_k} = \boldsymbol{w_B}\Delta\boldsymbol{S_{mk}}$$

$$\Delta\boldsymbol{S_{mk}} = \frac{\boldsymbol{1}}{\boldsymbol{2H_k}}[\Delta\boldsymbol{T_{mk}} - \Delta\boldsymbol{T_{ek}} - \boldsymbol{D}\Delta\boldsymbol{S_{mk}}]$$

$$\Delta\boldsymbol{E'_{qk}} = \frac{\boldsymbol{1}}{\boldsymbol{T'_{d0k}}}\left[\Delta\boldsymbol{E_{fdk}} - \Delta\boldsymbol{E'_{qk}} + (\boldsymbol{x_{dk}} - \boldsymbol{x'_{dk}})\Delta\boldsymbol{I_{dk}}\right] \qquad \textbf{(19)}$$

$$\Delta\boldsymbol{E'_{dk}} = \frac{\boldsymbol{1}}{\boldsymbol{T'_{q0k}}}\left[-\Delta\boldsymbol{E'_{dk}} - \left(\boldsymbol{x_{qk}} - \boldsymbol{x'_{qk}}\right)\Delta\boldsymbol{I_{qk}}\right]$$

$$\Delta\boldsymbol{E_{fdk}} = \frac{\boldsymbol{1}}{\boldsymbol{T_{Ek}}}\left[-\Delta\boldsymbol{E_{fdk}} + \boldsymbol{K_{Ek}}\left(\Delta\boldsymbol{V_{refk}} - \Delta\boldsymbol{V_{tk}}\right)\right]$$

where the variables T_{Ek} and K_{Ek} are exciter time constant and exciter gain of k^th generator respectively whereas the variables ΔV_{refk} and ΔT_{mk} are change in reference voltage and change in the mechanical torque of the k^th generator respectively. The obtained linearized differential equations have two non-state variables $(\Delta T_{ek}, \Delta V_{tk})$ that are linked to network properties. So, the dependence of these variables on state variables needs to be determined. The change in electrical torque ΔT_{ek} of k^th generator can be computed from equation (2) as follows:

$$\Delta\boldsymbol{T_{ek}} = \Delta\boldsymbol{E'_{qk}}\boldsymbol{I_{qk}} + \Delta\boldsymbol{E'_{dk}}\boldsymbol{I_{dk}} + \left((\boldsymbol{x'_{dk}} - \boldsymbol{x'_{qk}})\boldsymbol{I_{dk}} + \boldsymbol{E'_{qk}}\right)\Delta\boldsymbol{I_{qk}} + \left((\boldsymbol{x'_{dk}} - \boldsymbol{x'_{qk}})\boldsymbol{I_{qk}} + \boldsymbol{E'_{dk}}\right)\Delta\boldsymbol{I_{dk}} \qquad \textbf{(20)}$$

Writing equation (20) in matrix form, we obtain:

$$\overrightarrow{\Delta\boldsymbol{T_e}} = [\boldsymbol{R_7}]\overrightarrow{\Delta\boldsymbol{E'_q}} + [\boldsymbol{R_8}]\overrightarrow{\Delta\boldsymbol{E'_d}} + [\boldsymbol{R_9}]\overrightarrow{\Delta\boldsymbol{I_q}} + [\boldsymbol{R_{10}}]\overrightarrow{\Delta\boldsymbol{I_d}} \qquad \textbf{(21)}$$

where R_7, R_8, R_9, R_{10} are diagonal matrices of order m and I_{qk}, I_{dk} are k[th] diagonal elements of R_7, R_8 respectively whereas k[th] diagonal element of matrices R_9, R_{10} is as follows:

$$R_{9k} = \left(x'_{dk} - x'_{qk}\right)I_{dk} + E'_{qk}$$
$$R_{10k} = \left(x'_{dk} - x'_{qk}\right)I_{qk} + E'_{dk}$$

The change in terminal voltage ΔV_{tk} at k[th] generator can be computed from equation (2) as follows:

$$\Delta \boldsymbol{V_{tk}} = \frac{\boldsymbol{V_{qk}}}{\boldsymbol{V_{tk}}} \Delta \boldsymbol{V_{qk}} + \frac{\boldsymbol{V_{dk}}}{\boldsymbol{V_{tk}}} \Delta \boldsymbol{V_{dk}} \quad (22)$$

Writing equation (22) in matrix form, we obtain:

$$\overrightarrow{\Delta \boldsymbol{V_t}} = [\boldsymbol{R_5}]\overrightarrow{\Delta \boldsymbol{V_q}} + [\boldsymbol{R_6}]\overrightarrow{\Delta \boldsymbol{V_d}} \quad (23)$$

where R_5, R_6 are diagonal matrices of order m and the k[th] diagonal element of the matrices R_5, R_6 is as follows:

$$R_{5k} = \frac{V_{qk}}{V_{tk}}; \quad R_{6k} = \frac{V_{dk}}{V_{tk}}$$

The change in the stator components $\overrightarrow{\Delta I_q}, \overrightarrow{\Delta I_d}, \overrightarrow{\Delta V_q}$ and $\overrightarrow{\Delta V_d}$ needs to be calculated using network equation (12). By linearizing the network equation (12), we obtain:

$$\begin{bmatrix} \overrightarrow{\Delta I_q} \\ \overrightarrow{\Delta I_d} \end{bmatrix} = [\Delta D_1] \begin{bmatrix} \overrightarrow{V_q} \\ \overrightarrow{V_d} \end{bmatrix} + [D_1] \begin{bmatrix} \overrightarrow{\Delta V_q} \\ \overrightarrow{\Delta V_d} \end{bmatrix}$$

or,

$$\begin{bmatrix} \overrightarrow{\Delta \boldsymbol{I_q}} \\ \overrightarrow{\Delta \boldsymbol{I_d}} \end{bmatrix} = \begin{bmatrix} \Delta \boldsymbol{M} & -\Delta \boldsymbol{N} \\ \Delta \boldsymbol{N} & \Delta \boldsymbol{M} \end{bmatrix} \begin{bmatrix} \overrightarrow{\boldsymbol{V_q}} \\ \overrightarrow{\boldsymbol{V_d}} \end{bmatrix} + [\boldsymbol{D_1}] \begin{bmatrix} \overrightarrow{\Delta \boldsymbol{V_q}} \\ \overrightarrow{\Delta \boldsymbol{V_d}} \end{bmatrix} \quad (24)$$

The k[th] row and i[th] column element ΔM_{ki}, ΔN_{ki} of $m \times m$ matrices ΔM and ΔN can be written as follows:

$$\Delta M_{ki} = (-G_{ki}\sin(\delta_i - \delta_k) - B_{ki}\cos(\delta_i - \delta_k))(\Delta\delta_i - \Delta\delta_k)$$
$$= -N_{ki}(\Delta\delta_i - \Delta\delta_k)$$
$$\Delta\, N_{ki} = (G_{ki}\cos(\delta_i - \delta_k) - B_{ki}\sin(\delta_i - \delta_k))\,(\Delta\delta_i - \Delta\delta_k)$$
$$= M_{ki}(\Delta\delta_i - \Delta\delta_k)$$

We can rewrite the above equation in matrix form as follows:

$$[\Delta \boldsymbol{M}] = \textstyle\sum_{\boldsymbol{k=1}}^{\boldsymbol{m}}[\boldsymbol{N}_{\boldsymbol{k}}']\Delta\boldsymbol{\delta}_{\boldsymbol{k}}$$
$$[\Delta \boldsymbol{N}] = \textstyle\sum_{\boldsymbol{k=1}}^{\boldsymbol{m}}[\boldsymbol{M}_{\boldsymbol{k}}']\Delta\boldsymbol{\delta}_{\boldsymbol{k}} \quad (25)$$

where N_k', M_k' are matrices of order $m\ x\ m$ for every k and the ith row and jth column element N_{kij}', M_{kij}' of the matrices N_k', M_k' are given by:

$$N_{kij}' = \begin{Bmatrix} N_{ij} & for\ \ i = k\ and\ j > k \\ -N_{ij} & for\ i = k\ and\ j < k \\ 0 & otherwise \end{Bmatrix}$$
$$M_{kij}' = \begin{Bmatrix} -M_{ij} & for\ \ i = k\ and\ j > k \\ M_{ij} & for\ i = k\ and\ j < k \\ 0 & otherwise \end{Bmatrix}$$

Using the above equations in finding ΔD_1, we obtain:

$$[\Delta D_1] = \sum_{k=1}^{m}\begin{bmatrix} N_k' & -M_k' \\ M_k' & N_k' \end{bmatrix}\Delta\delta_k$$

or,

$$[\Delta \boldsymbol{D_1}] = \textstyle\sum_{\boldsymbol{k=1}}^{\boldsymbol{m}}[\boldsymbol{D}_{\boldsymbol{k}}']\Delta\boldsymbol{\delta}_{\boldsymbol{k}} \quad (26)$$

where D_k' is a matrix of order $2m\ x\ 2m$ for every k .Substituting the above relation in equation (24), we obtain:

$$\begin{bmatrix} \overrightarrow{\Delta \boldsymbol{I_q}} \\ \overrightarrow{\Delta \boldsymbol{I_d}} \end{bmatrix} = (\textstyle\sum_{\boldsymbol{k=1}}^{\boldsymbol{m}}[\boldsymbol{D}_{\boldsymbol{k}}']\Delta\boldsymbol{\delta}_{\boldsymbol{k}})\begin{bmatrix} \overrightarrow{\boldsymbol{V_q}} \\ \overrightarrow{\boldsymbol{V_d}} \end{bmatrix} + [\boldsymbol{D_1}]\begin{bmatrix} \overrightarrow{\Delta \boldsymbol{V_q}} \\ \overrightarrow{\Delta \boldsymbol{V_d}} \end{bmatrix} \quad (27)$$

Defining a column matrix D_{2k} of order $2m$ for every k as follows:

$$[\boldsymbol{D_{2k}}] = [\boldsymbol{D'_k}]\begin{bmatrix}\overrightarrow{\boldsymbol{V_q}}\\ \overrightarrow{\boldsymbol{V_d}}\end{bmatrix} \tag{28}$$

So, equation (27) can be rewritten as:

$$\begin{bmatrix}\overrightarrow{\Delta \boldsymbol{I_q}}\\ \overrightarrow{\Delta \boldsymbol{I_d}}\end{bmatrix} = [\boldsymbol{D_2}]\overrightarrow{\Delta \boldsymbol{\delta}} + [\boldsymbol{D_1}]\begin{bmatrix}\overrightarrow{\Delta \boldsymbol{V_q}}\\ \overrightarrow{\Delta \boldsymbol{V_d}}\end{bmatrix} \tag{29}$$

where D_2 is a $2m \, x \, m$ matrix formed by the column matrices D_{2k} $\forall k = 1,2,..,m$. A linearized form of network equations has been obtained in equation (29) but a relation between non-state variables and state variables needs to be determined in linearized form. Hence, equation (14) needs to be linearized:

$$\begin{bmatrix}\overrightarrow{\Delta \boldsymbol{V_q}}\\ \overrightarrow{\Delta \boldsymbol{V_d}}\end{bmatrix} = \begin{bmatrix}\overrightarrow{\Delta \boldsymbol{E'_q}}\\ \overrightarrow{\Delta \boldsymbol{E'_d}}\end{bmatrix} + [\boldsymbol{B_1}]\begin{bmatrix}\overrightarrow{\Delta \boldsymbol{I_q}}\\ \overrightarrow{\Delta \boldsymbol{I_d}}\end{bmatrix} \tag{30}$$

Substituting (29) in above equation, we obtain:

$$\begin{bmatrix}\overrightarrow{\Delta V_q}\\ \overrightarrow{\Delta V_d}\end{bmatrix} = [I - B_1 D_1]^{-1}\begin{bmatrix}\overrightarrow{\Delta E'_q}\\ \overrightarrow{\Delta E'_d}\end{bmatrix} + [I - B_1 D_1]^{-1}[B_1][D_2]\overrightarrow{\Delta \delta}$$

or,

$$\begin{bmatrix}\overrightarrow{\Delta \boldsymbol{V_q}}\\ \overrightarrow{\Delta \boldsymbol{V_d}}\end{bmatrix} = [\boldsymbol{R_1}]\begin{bmatrix}\overrightarrow{\Delta \boldsymbol{E'_q}}\\ \overrightarrow{\Delta \boldsymbol{E'_d}}\end{bmatrix} + [\boldsymbol{R_2}]\overrightarrow{\Delta \boldsymbol{\delta}} \tag{31}$$

where R_1, R_2 are matrices of order $2m \, x \, 2m$, $2m \, x \, m$ respectively. Rearranging the matrices R_1, R_2 in following manner:

$$[\boldsymbol{R_1}] = \begin{bmatrix}\boldsymbol{x_2} & \boldsymbol{y_2}\\ \boldsymbol{z_2} & \boldsymbol{w_2}\end{bmatrix};\ [\boldsymbol{R_2}] = \begin{bmatrix}\boldsymbol{p_2}\\ \boldsymbol{q_2}\end{bmatrix}$$

where $x_2, y_2, z_2, w_2, p_2, q_2$ are matrices of order $m \, x \, m$. So, equation (31) can be rewritten as follows:

$$\overrightarrow{\Delta \boldsymbol{V_q}} = [\boldsymbol{x_2}]\overrightarrow{\Delta \boldsymbol{E'_q}} + [\boldsymbol{y_2}]\overrightarrow{\Delta \boldsymbol{E'_d}} + [\boldsymbol{p_2}]\overrightarrow{\Delta \boldsymbol{\delta}}$$
$$\overrightarrow{\Delta \boldsymbol{V_d}} = [\boldsymbol{z_2}]\overrightarrow{\Delta \boldsymbol{E'_q}} + [\boldsymbol{w_2}]\overrightarrow{\Delta \boldsymbol{E'_d}} + [\boldsymbol{q_2}]\overrightarrow{\Delta \boldsymbol{\delta}} \quad \textbf{(32)}$$

So, a relation between the non-state variables $\Delta Vq, \Delta Vd$ and state variables $\Delta\delta, \Delta E'_q, \Delta E'_d$ is obtained.

In order to obtain a similar relation between the non-state variables $\Delta Iq, \Delta Id$ and the state variables, we need to substitute equation (31) in (29):

$$\begin{bmatrix} \overrightarrow{\Delta I_q} \\ \overrightarrow{\Delta I_d} \end{bmatrix} = [D_1][R_1]\begin{bmatrix} \overrightarrow{\Delta E'_q} \\ \overrightarrow{\Delta E'_d} \end{bmatrix} + ([D_2] + [D_1][R_2])\overrightarrow{\Delta\delta}$$

Or,

$$\begin{bmatrix} \overrightarrow{\Delta \boldsymbol{I_q}} \\ \overrightarrow{\Delta \boldsymbol{I_d}} \end{bmatrix} = [\boldsymbol{R_3}]\begin{bmatrix} \overrightarrow{\Delta \boldsymbol{E'_q}} \\ \overrightarrow{\Delta \boldsymbol{E'_d}} \end{bmatrix} + [\boldsymbol{R_4}]\overrightarrow{\Delta \boldsymbol{\delta}} \quad \textbf{(33)}$$

where R_3, R_4 are matrices of order $2m\ x\ 2m$, $2m\ x\ m$ respectively. Rearranging the matrices R_3, R_4 in following manner

$$[R_3] = \begin{bmatrix} x_1 & y_1 \\ z_1 & w_1 \end{bmatrix};\ [R_4] = \begin{bmatrix} p_1 \\ q_1 \end{bmatrix}$$

where $x_1, y_1, z_1, w_1, p_1, q_1$ are matrices of order $m\ x\ m$ .So, equation (33) can be rewritten as follows:

$$\overrightarrow{\Delta \boldsymbol{I_q}} = [\boldsymbol{x_1}]\overrightarrow{\Delta \boldsymbol{E'_q}} + [\boldsymbol{y_1}]\overrightarrow{\Delta \boldsymbol{E'_d}} + [\boldsymbol{p_1}]\overrightarrow{\Delta \boldsymbol{\delta}}$$
$$\overrightarrow{\Delta \boldsymbol{I_d}} = [\boldsymbol{z_1}]\overrightarrow{\Delta \boldsymbol{E'_q}} + [\boldsymbol{w_1}]\overrightarrow{\Delta \boldsymbol{E'_d}} + [\boldsymbol{q_1}]\overrightarrow{\Delta \boldsymbol{\delta}} \quad \textbf{(34)}$$

So, equations (32) and (34) provide the relation between non-state variables and state variables of the system.

The linearized differential equations (19) refer only to the states of single generator. So, the linearized differential equations have to be formulated in matrix form for multi-machine systems. Rewriting equation (19) in matrix form and substituting the equations (21), (23), (32), (34) in it, we obtain the system in the state-space form :

$$\dot{\vec{\boldsymbol{X}}} = [\boldsymbol{A}]\vec{\boldsymbol{X}} + [\boldsymbol{B}]\vec{\boldsymbol{U}} \tag{35}$$

where state vector X and input vector U are column vectors of order $5m$ and given by

$$\vec{X} = \left[\Delta\delta_1, \Delta\delta_2, \ldots \Delta\delta_m, \Delta S_{m1}, \ldots, \Delta E'_{q1}, \ldots . \Delta E'_{qm}, \Delta E'_{d1}, \ldots \Delta E'_{dm}, \Delta E_{fd1}, \ldots \Delta E_{fdm}\right]^T$$
$$\vec{U} = \left[\Delta T_{m1}, \Delta T_{m2}, \ldots \ldots, \Delta V_{ref1}, \ldots \ldots, \Delta V_{refm}\right]^T$$

and A and B are $5m \; x \; 5m$ matrices given by:

$$[\boldsymbol{A}] = \begin{bmatrix} \bar{\boldsymbol{O}} & \boldsymbol{a}_{12} & \bar{\boldsymbol{O}} & \bar{\boldsymbol{O}} & \bar{\boldsymbol{O}} \\ \boldsymbol{a}_{21} & \boldsymbol{a}_{22} & \boldsymbol{a}_{23} & \boldsymbol{a}_{24} & \bar{\boldsymbol{O}} \\ \boldsymbol{a}_{31} & \bar{\boldsymbol{O}} & \boldsymbol{a}_{33} & \boldsymbol{a}_{34} & \boldsymbol{a}_{35} \\ \boldsymbol{a}_{41} & \bar{\boldsymbol{O}} & \boldsymbol{a}_{43} & \boldsymbol{a}_{45} & \bar{\boldsymbol{O}} \\ \boldsymbol{a}_{51} & \bar{\boldsymbol{O}} & \boldsymbol{a}_{53} & \boldsymbol{a}_{54} & \boldsymbol{a}_{55} \end{bmatrix}; \quad [\boldsymbol{B}] = \begin{bmatrix} \bar{\boldsymbol{O}} & \bar{\boldsymbol{O}} \\ \boldsymbol{b}_{21} & \bar{\boldsymbol{O}} \\ \bar{\boldsymbol{O}} & \bar{\boldsymbol{O}} \\ \bar{\boldsymbol{O}} & \bar{\boldsymbol{O}} \\ \bar{\boldsymbol{O}} & \boldsymbol{b}_{52} \end{bmatrix}$$

The matrix elements a_{ij}, b_{ij} of A and B are of order $m \; x \; m$ and given by:

$$[\boldsymbol{a}_{21}] = -[\boldsymbol{M}][\boldsymbol{R}_9][\boldsymbol{p}_1] - [\boldsymbol{M}][\boldsymbol{R}_{10}][\boldsymbol{q}_1] \qquad [\boldsymbol{a}_{31}] = [\boldsymbol{R}_{13}][\boldsymbol{q}_1]$$
$$[\boldsymbol{a}_{23}] = -[\boldsymbol{M}][\boldsymbol{R}_7] - [\boldsymbol{M}][\boldsymbol{R}_9][\boldsymbol{x}_1] - [\boldsymbol{M}][\boldsymbol{R}_{10}][\boldsymbol{z}_1] \qquad [\boldsymbol{a}_{33}] = [\boldsymbol{R}_{11}] + [\boldsymbol{R}_{13}][\boldsymbol{z}_1]$$
$$[\boldsymbol{a}_{24}] = -[\boldsymbol{M}][\boldsymbol{R}_8] - [\boldsymbol{M}][\boldsymbol{R}_9][\boldsymbol{y}_1] - [\boldsymbol{M}][\boldsymbol{R}_{10}][\boldsymbol{w}_1] \qquad [\boldsymbol{a}_{34}] = [\boldsymbol{R}_{13}][\boldsymbol{w}_1]; \quad [\boldsymbol{b}_{21}] = [\boldsymbol{M}]$$
$$[\boldsymbol{a}_{51}] = [\boldsymbol{R}_{17}][\boldsymbol{R}_5][\boldsymbol{p}_2] + [\boldsymbol{R}_{17}][\boldsymbol{R}_6][\boldsymbol{q}_2] \qquad [\boldsymbol{a}_{35}] = -[\boldsymbol{R}_{11}]; \quad [\boldsymbol{b}_{52}] = -[\boldsymbol{R}_{17}]$$
$$[\boldsymbol{a}_{53}] = [\boldsymbol{R}_{17}][\boldsymbol{R}_5][\boldsymbol{x}_2] + [\boldsymbol{R}_{17}][\boldsymbol{R}_6][\boldsymbol{z}_2] \qquad [\boldsymbol{a}_{41}] = [\boldsymbol{R}_{15}][\boldsymbol{p}_1]; \quad [\boldsymbol{a}_{43}] = [\boldsymbol{R}_{15}][\boldsymbol{x}_1]$$
$$[\boldsymbol{a}_{54}] = [\boldsymbol{R}_{17}][\boldsymbol{R}_5][\boldsymbol{y}_2] + [\boldsymbol{R}_{17}][\boldsymbol{R}_6][\boldsymbol{w}_2] \qquad [\boldsymbol{a}_{44}] = [\boldsymbol{R}_{14}] + [\boldsymbol{R}_{15}][\boldsymbol{y}_1]$$

where $a_{12}, a_{55}, R_{11}, R_{13}, R_{14}, R_{15}, R_{17}$ are diagonal matrices of order m and their kth diagonal element is as follows:

$$a_{12k} = w_B \qquad R_{11k} = -\frac{1}{T'_{d0k}} \qquad M_k = \frac{1}{2H_k} \qquad R_{14k} = -\frac{1}{T'_{q0k}}$$
$$a_{55k} = -\frac{1}{T_{Ek}} \qquad R_{13k} = \frac{x_{dk} - x'_{dk}}{T'_{d0k}} \qquad R_{15k} = \frac{x_{qk} - x'_{qk}}{T'_{q0k}} \qquad R_{17k} = -\frac{K_{Ek}}{T_{Ek}}$$

So, the matrix A thus obtained is of order $5m\,x\,5m$ where m is number of generators in the system. As rotor angles are measured relative to reference generator, we need to eliminate the state corresponding to the rotor angle of the reference generator. Generally, the generator numbered as 1 is taken as reference generator. So, the matrix A is modified by entering $-w_B$ in the column for ΔS_{m1} in rows corresponding to $\Delta\delta_i, i$=2, 3…..m. So the modified matrix A_{fin} is of order$(5m-1)\;x\;(5m-1)$ and the system can be represented as:

$$\dot{\vec{\boldsymbol{X}}} = [\boldsymbol{A_{fin}}]\vec{\boldsymbol{X}} + [\boldsymbol{B}]\vec{\boldsymbol{U}} \tag{36}$$

where state vector X and input vector U are column vectors of order $(5m-1)$ and $5m$ respectively and they are given by

$$X = \left[\Delta\delta_{21}, \Delta\delta_{31}, \ldots \Delta\delta_{m1}, \Delta S_{m1}, \ldots, \Delta E'_{q1}, \ldots . \Delta E'_{qm}, \Delta E'_{d1}, .. \Delta E'_{dm}, \Delta E_{fd1}, \ldots \Delta E_{fdm}\right]^T$$
$$U = \left[\Delta T_{m1}, \Delta T_{m2}, \ldots .., \Delta V_{ref1}, \ldots .., \Delta V_{refm}\right]^T$$

Hence, the eigenvalues of the system can be obtained using A_{fin} in equation (36). So, the modes which lead to instability can be identified and an appropriate location for the PSS has to be chosen so that these modes are efficiently damped. For this purpose, we use participation factors method. The participation factor P_{ih} of the h^{th} state variable in the i^{th} mode is given by:

$$\boldsymbol{P_{ih}} = |\boldsymbol{v_{ih}}||\boldsymbol{w_{hi}}| \tag{37}$$

where v_{ih} and w_{hi} are h^{th} entries of i^{th} right and left eigenvectors respectively. Participation factor is good parameter for selection of PSS location because the right eigenvector term v_{ih} reflects the activity of h^{th} state variable when i^{th} mode is excited whereas left eigenvector term w_{hi} weighs the contribution of this activity to the i^{th} mode.

So, by evaluating the participation factors for any particular mode, the PSS location for damping of the mode can be chosen at a generator that participates significantly in that mode.

3.2 Design of PSS components

The main components comprising PSS are compensator and washout block. Washout block is used to eliminate the steady state bias in the output of PSS. Washout block is a high pass filter.

$$\boldsymbol{W(s) = \frac{sT_w}{1+sT_w}} \qquad (38)$$

It is designed such that it passes the frequencies that are of interest. So, T_w is chosen depending on the modes that are of interest. If only the local modes are of interest, the time constant T_w can be chosen in the range of 1 to 2. However, if inter area modes are also to be damped, then T_w must be chosen in the range of 10 to 20.

PSS with speed input is chosen for design here and the procedure mainly involves design of a compensator that can provide adequate damping for the troublesome mode. So, a lead-lag compensator of one or two stages is used for this:

$$\boldsymbol{T(s) = \left(\frac{1+sT_1}{1+sT_2}\right)^p ; p = 1 \ or \ 2} \qquad (39)$$

For evaluating the time constants in compensator, we need to study how these compensator parameters affect the compensated phase of $P(s) = GEP(s) * T(s)$ where $GEP(s)$ is the transfer function determining the relation between ΔT_e and ΔV_s of the generator. So, $GEP(s)$can be written as follows:

$$\boldsymbol{GEP(s) = \frac{\Delta T_e}{\Delta V_s} : for \ \overrightarrow{\Delta w} = 0} \qquad (40)$$

In order to obtain ΔT_e in terms of state variables, equation (34) is substituted in (21):

$$\overrightarrow{\Delta T_e} = ([R_9][p_1] + [R_{10}][q_1])\overrightarrow{\Delta\delta} + ([R_7] + [R_9][x_1] + [R_{10}][z_1])\overrightarrow{\Delta E'_q} + ([R_8] + [R_9][y_1] + [R_{10}][w_1])\overrightarrow{\Delta E'_d}$$

or,

$$\overrightarrow{\Delta \boldsymbol{T_e}} = [\boldsymbol{K_1}]\overrightarrow{\Delta\boldsymbol{\delta}} + [\boldsymbol{K_2}]\overrightarrow{\Delta \boldsymbol{E'_q}} + [\boldsymbol{K_3}]\overrightarrow{\Delta \boldsymbol{E'_d}} \quad (41)$$

where K_1, K_2, K_3 are matrices of order $m\ x\ m$. By eliminating the state corresponding to the rotor angle of the reference generator (i.e. generator 1) in K_1, we obtain:

$$\overrightarrow{\Delta \boldsymbol{T_e}} = [\boldsymbol{K_4}]\overrightarrow{\Delta\boldsymbol{\delta_m}} + [\boldsymbol{K_2}]\overrightarrow{\Delta \boldsymbol{E'_q}} + [\boldsymbol{K_3}]\overrightarrow{\Delta \boldsymbol{E'_d}} \quad (42)$$

where K_4 is a matrix of order $m\ x\ (m-1)$ and $\overrightarrow{\Delta\delta_m}$ is a column matrix of order m and is given by

$$\overrightarrow{\Delta\delta_m} = [\Delta\delta_{21}, \dots, \Delta\delta_{m1}]^T$$

Rewriting equation (42) as follows:

$$\overrightarrow{\Delta \boldsymbol{T_e}} = [\boldsymbol{C}]\overrightarrow{\boldsymbol{X}} \quad (43)$$

where C is a matrix of order $m\ x\ (5m-1)$ and the state vector $\vec{X}$ is column vector of order $(5m-1)$. They are given by:

$$\overrightarrow{X} = \left[\Delta\delta_{21}, \Delta\delta_{31}, \dots \Delta\delta_{m1}, \Delta S_{m1}, \dots, \Delta E'_{q1}, \dots . \Delta E'_{qm}, \Delta E'_{d1}, \dots \Delta E'_{dm}, \Delta E_{fd1}, \dots \Delta E_{fdm}\right]^T$$

$$[C] = [K_4\ \bar{O}\ K_2\ K_1\ \ \bar{O}]$$

So, $GEP_k(s)$ for a generator k in a multi-machine system can be evaluated by representing it in state-space form, i.e. considering ΔV_{sk} as input and ΔT_{ek} as output. Using equations (36) and (43), we can write:

$$\dot{\overrightarrow{\boldsymbol{X}}} = [\boldsymbol{a_1}]\overrightarrow{\boldsymbol{X}} + [\boldsymbol{b_1}]\Delta \boldsymbol{V_{sk}}$$
$$\Delta \boldsymbol{T_{ek}} = [\boldsymbol{c_1}]\overrightarrow{\boldsymbol{X}} + [\boldsymbol{d_1}]\Delta \boldsymbol{V_{sk}} \quad (44)$$

where $a_1 = A_{fin}$ is a square matrix of order $(5m-1)$, b_1 is $(4m+k-1)^{\text{th}}$ column of matrix B. c_1 is k$^{\text{th}}$ row of matrix C and $d_1 = 0$. So, the transfer function $GEP_k(s)$ can be written as:

$$\boldsymbol{GEP_k}(\boldsymbol{s}) = \frac{\Delta \boldsymbol{T_{ek}}}{\Delta \boldsymbol{V_{sk}}} = \boldsymbol{c_1}(\boldsymbol{sI} - \boldsymbol{a_1})^{-1}\boldsymbol{b_1} + \boldsymbol{d_1} \tag{45}$$

The transfer function $GEP_k(s)$, evaluated here, has to be compensated such that the compensated phase lag of $P(s) = GEP_k(s) * T(s)$ satisfies the below criteria:

- It should pass through $90°$ at frequency around 3.5 Hz (22 rad/sec)
- It should be below $45°$ at local mode frequency, preferably at $20°$
- It is preferable to be lagging at inter-area modes

So, the compensator parameters T_1, T_2, p are chosen such that above conditions are met. The PSS designed here without gain ($PSS_k(s) = W(s) * T(s)$) is used in next section to plot the root locus of the overall system in order to find the gain of PSS.

3.3 Tuning of PSS

The PSS designed has to be tuned so that all the critical modes are damped. So, a gain which provides maximum damping torque for the troublesome mode but doesn't destabilize other modes must be chosen. This is obtained by analyzing the root locus of the system including the designed PSS. In order to plot root locus, we need to find out the open loop transfer function $O(s)$ of the system:

$$\boldsymbol{O}(\boldsymbol{s}) = \frac{\Delta w_k}{\Delta V_{sk}} \boldsymbol{PSS_k}(\boldsymbol{s}) \tag{46}$$

So, $O(s)$ for a multi-machine system can be evaluated by representing it in state-space form, i.e. considering ΔV_{sk} as input and Δw_k as output. Using equation (36), we can write:

$$\begin{aligned}\dot{\vec{\boldsymbol{X}}} &= [\boldsymbol{a_2}]\vec{\boldsymbol{X}} + [\boldsymbol{b_2}]\Delta \boldsymbol{V_{sk}} \\ \Delta \boldsymbol{w_k} &= [\boldsymbol{c_2}]\boldsymbol{X} + [\boldsymbol{d_2}]\Delta \boldsymbol{V_{sk}}\end{aligned} \tag{47}$$

where $a_2 = a_1$ is a square matrix of order $(5m - 1)$, $b_2 = b_1$ is a column vector of order $(5m - 1)$, $d_2 = 0$ and c_2 is a row matrix of order $(5m - 1)$ and its ith element c_{2i} is given by:

$$c_{2i} = \begin{Bmatrix} w_B & for\ i = m - 1 + k \\ 0 & otherwise \end{Bmatrix}$$

So, $O(s)$ can be written as:

$$\boldsymbol{O}(\boldsymbol{s}) = (\boldsymbol{c_2}(\boldsymbol{sI} - \boldsymbol{a_2})^{-1}\boldsymbol{b_2} + \boldsymbol{d_2})\boldsymbol{PSS_k}(\boldsymbol{s}) \tag{48}$$

This open loop transfer $O(s)$ evaluated here is used in plotting the root locus of system and the gains for which the real parts of all the closed loop poles are in negative X-axis are noted. Out of these, the gain for which the troublesome mode is most damped should be chosen. This gain is incorporated into the designed PSS and is verified by studying the system behavior using Runge-Kutta method as discussed in Chapter 2.

CHAPTER IV: RESULTS

4.1 SMIB System

The SMIB system shown in Figure 2 is chosen for small-signal analysis. Initially, the system's small signal instability is studied and then the design of PSS for the same is discussed.

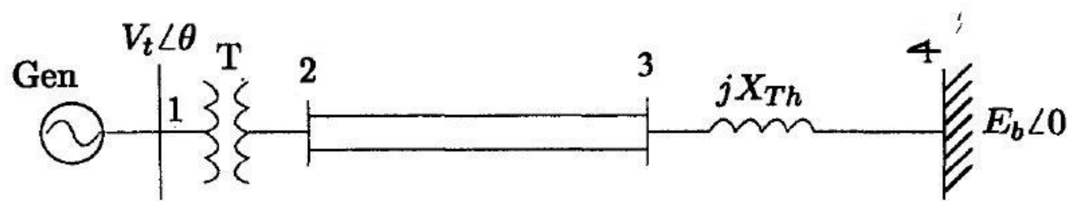

Figure 2: A SMIB System

The system and operating data are given in Tables 1 to 4.

Table 1:Transmission Line Data of the SMIB System

From Bus Number	To Bus Number	Series Resistance (R_s)	Series Reactance (X_s)	Shunt Susceptance(B)
2	3	0.08593	0.8125	0.1184
2	3	0.08593	0.8125	0.1184

Table 2: Machine Data of the SMIB System

Variable	Value
R_a	0.00327
X_d	1.7572
X_d'	0.4245
T_{d0}'	6.66
X_q	1.5845
X_q'	1.04
T_{q0}'	0.44
H	3.542
D	0

Table 3: Operating Data of the SMIB System

Variable	Value
E_b	0.00327
P_t	1.7572
Q_t	0.4245
V_t	6.66
Θ	21.65°
X_{Th}	0.13636
X_t	0.1364
R_t	0

Table 4: Excitation System Data of the SMIB System

Variable	Value
K_E	400
T_E	0.025
E_{fdmin}	-6.0
E_{fdmax}	6.0

The numerical integration of the system's differential equations (1) using Fourth-order Runge-Kutta Method and h was chosen as 0.01. The initial conditions of the state variables are calculated from the operating data given in Table 3. Subsequently, the voltage fault at the generator was applied at 1 sec and cleared in 4 cycles. The variation of state variables was recorded up to time duration of 10sec.The results obtained are given in Figure 3.

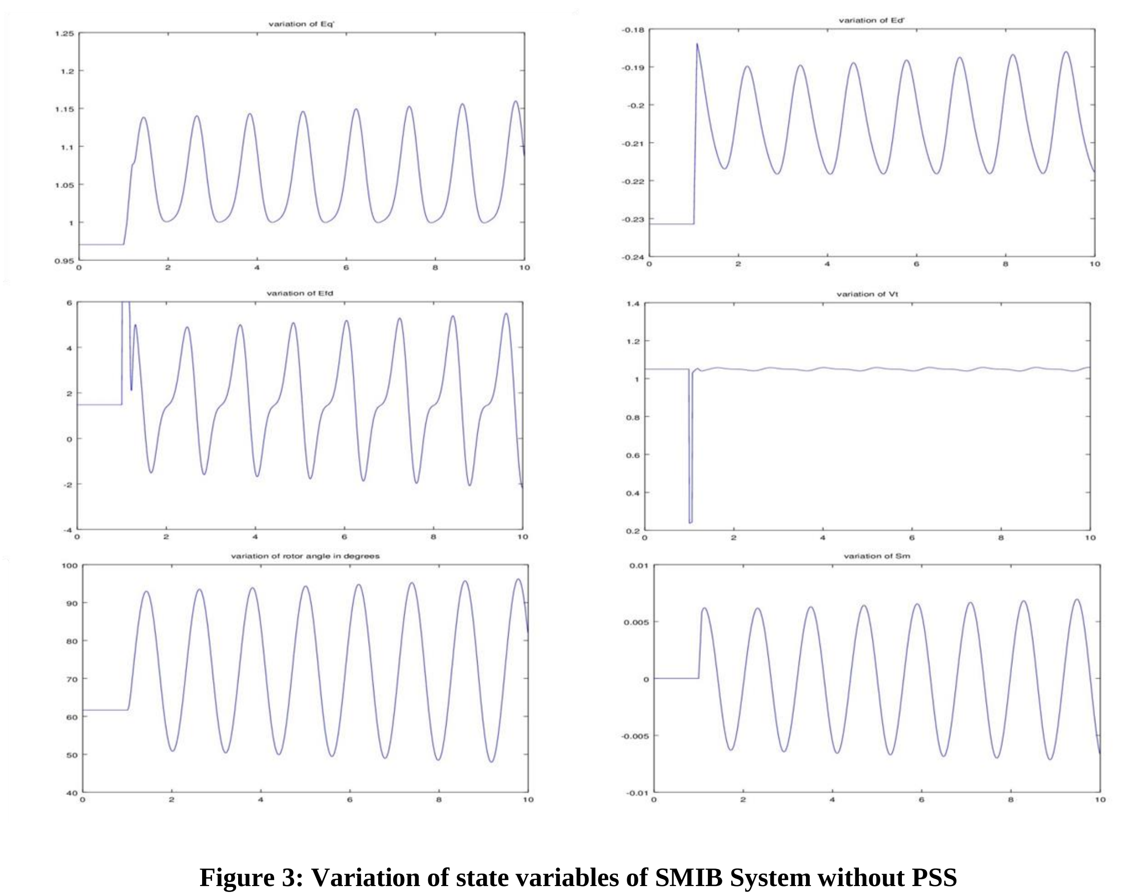

Figure 3: Variation of state variables of SMIB System without PSS

From the results above, we can see that the fault at 1 sec has moved the system behavior from steady state. Even though the fault is cleared in a small duration, we can see that small signal oscillations still persist. Hence, a power system stabilizer (PSS) needs to be designed in order to damp the mode of oscillations.

A state-space model was formulated for the system in the form of equation (36) and the eigenvalues of the system were obtained as shown in Table 5:

Table 5: Eigenvalues of the SMIB System

Eigenvalues
-2.70483
-0.06263+6.2329**i**
-0.06263-6.2329**i**
-20.2352+28.333**i**
-20.2352-28.333**i**

We observe that eigenvalue at 6.23 rad/sec has less damping and hence it is the cause for small-signal instability. So, the mode needs to be damped by a PSS. Let us design a PSS which damps the mode 6.23 rad/sec. As discussed earlier, the compensator for PSS should be designed such that

1. The compensated phase lag of $P(s) = GEP(s) * T(s)$ should pass through $90°$ at frequency around 3.5 Hz (22 rad/sec).
2. The compensated phase lag of $P(s)$ should be below $45°$ at local mode frequency, preferably at $20°$.
3. It is preferable for the compensated phase to be lagging at inter-area modes.

So, $GEP(s)$ is calculated using equation (45) at the generator and its Bode plot is obtained as shown in Figure 4.

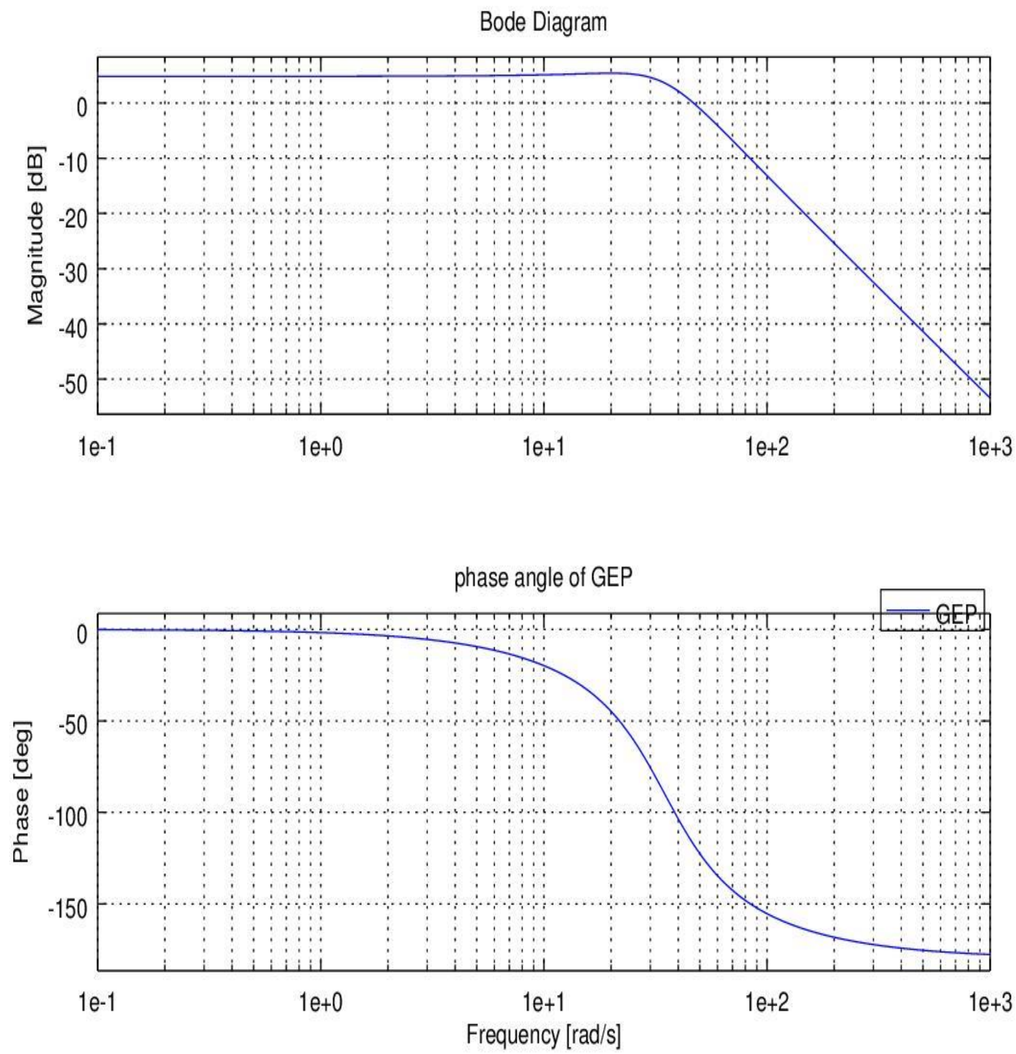

Figure 4: Bode plot of GEP(s) of SMIB System

We observe that phase angle of $GEP(s)$ at the cut-off frequency 3.5 Hz (22 rad/sec) is $-50.576°$ whereas phase is $-13.457°$ at local mode frequencies (around 7 rad/sec). As the $GEP(s)$ already satisfies the compensator design criteria by itself, a compensator is not needed for the system. Also as the local modes are of interest, the time constant T_w is chosen as 2. Hence, the PSS designed without the gain is $PSS(s) = \frac{2s}{1+2s}$. The root locus of the system is plotted by incorporating the designed PSS into open loop transfer function. The root locus is shown in Figure 5.

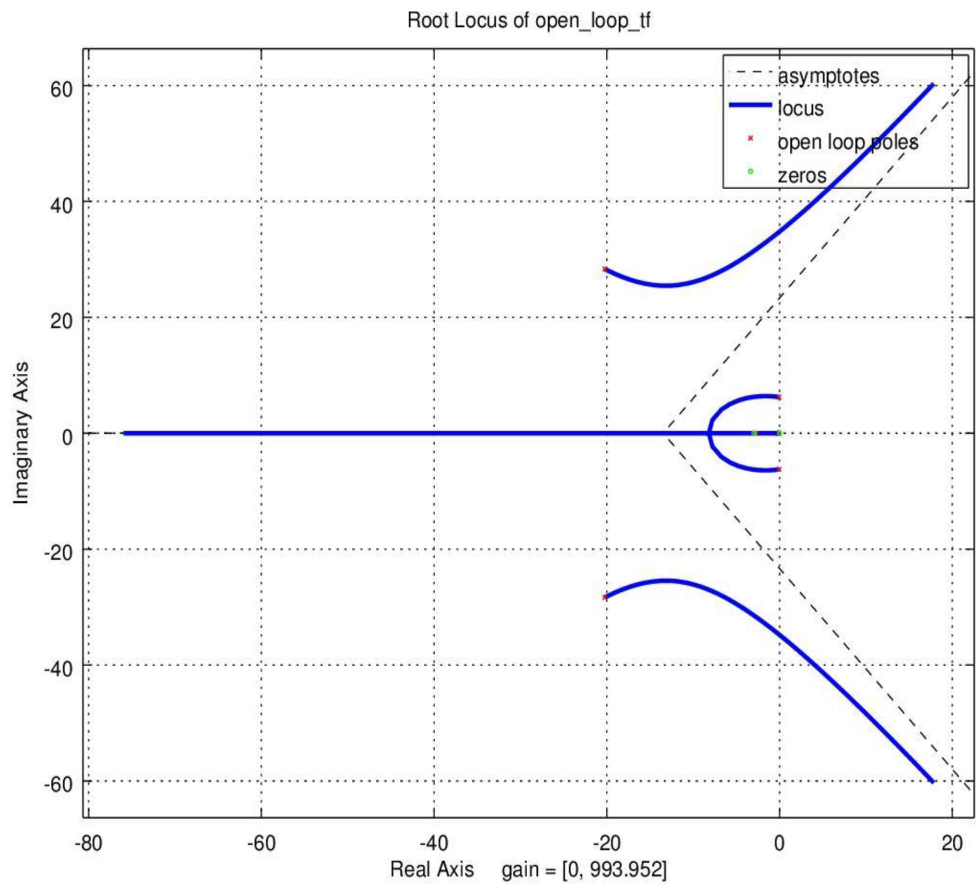

Figure 5: Root Locus of the SMIB System including the PSS designed

From the root locus data, we infer that at PSS gain K_{PSS} =24, both modes (6.233, 28.333 rad/sec) of the system are significantly damped. Hence, the PSS designed with the gain is $PSS(s) = \frac{48s}{1+2s}$. The numerical integration of the system's differential equations (1) and PSS equations in differential form is performed to verify the effectiveness of PSS designed. Subsequently, the voltage fault at the generator was applied at 1 sec and cleared in 4 cycles. The variation of state variables was recorded up to time duration of 10sec.The results obtained are given in Figure 6.

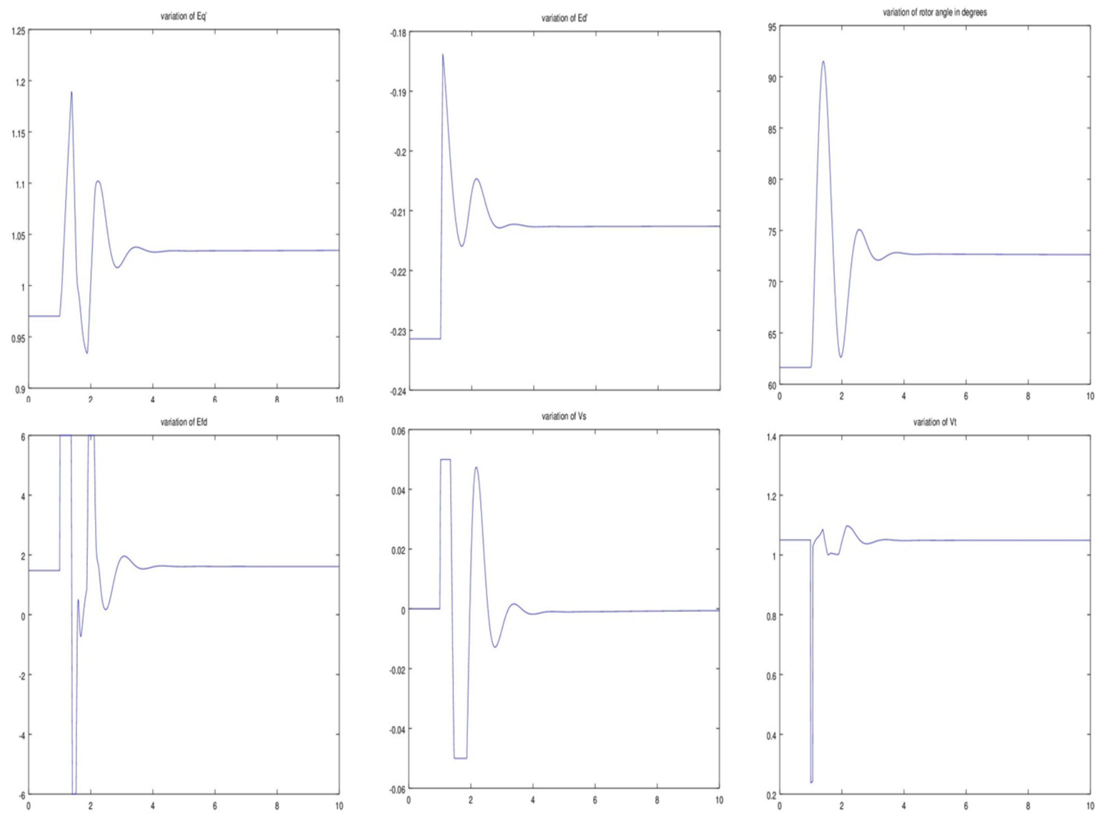

Figure 6: Variation of state variables of SMIB System with the PSS designed

We see that the system doesn't have small signal oscillations after the designed PSS is incorporated. So, the effectiveness of the PSS designed is verified.

4.2 Multi-Machine System

A 4-generator, 10-bus system shown in Figure 7 is chosen for small-signal analysis. Initially, the system's small signal instability is studied and then the design of PSS for the same is discussed.

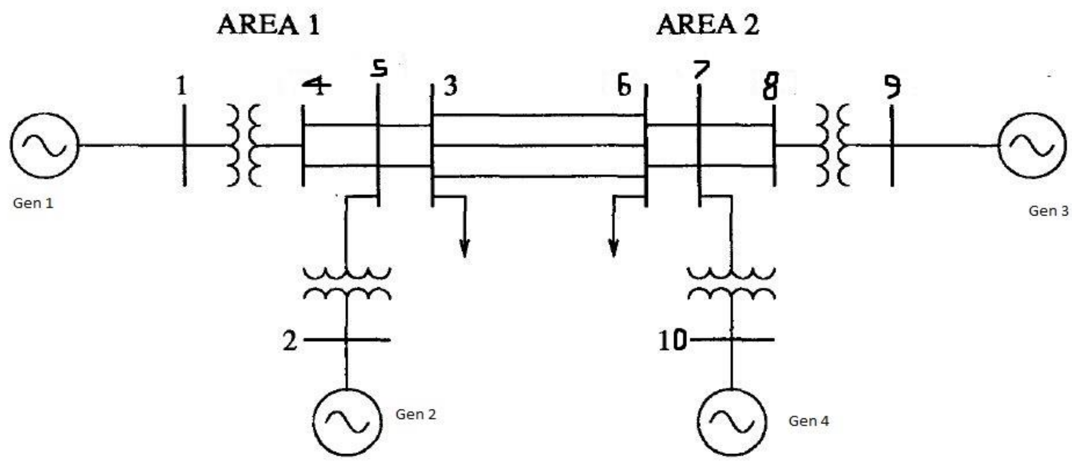

Figure 7: Two-Area System

The system and operating data are given in Tables 6 to 9.

Table 6: Transmission line data on 100 MVA base of the multi-machine system

From Bus Number	To Bus Number	Series Resistance (R_s)	Series Reactance (X_s)	Shunt Susceptance(B)
1	4	0.001	0.012	0
2	5	0.001	0.012	0
3	6	0.022	0.22	0.33
3	6	0.022	0.22	0.33
3	6	0.022	0.22	0.33
3	5	0.002	0.02	0.03
3	5	0.002	0.02	0.03
9	8	0.001	0.012	0
10	7	0.001	0.012	0
6	7	0.002	0.02	0.03
6	7	0.002	0.02	0.03
4	5	0.005	0.05	0.075
4	5	0.005	0.05	0.075
8	7	0.005	0.05	0.075
8	7	0.005	0.05	0.075

Table 7: Initial Load Flow Data of the multi-machine system

Bus No.	Voltage (mag)	Angle (deg)	Real Power Gen	Reactive Power Gen	Real Power Load	Reactive Power Load	Shunt Susceptance
1	1.03	8.2154	7	1.3386	0	0	0
2	1.01	-1.504	7	1.592	0	0	0
9	1.03	0	7.2172	1.4466	0	0	0
10	1.01	-10.2051	7	1.8083	0	0	0
4	1.0108	3.6615	0	0	0	0	0
5	0.9875	-6.2433	0	0	0	0	0
8	1.0095	-4.6977	0	0	0	0	0
7	0.985	-14.9443	0	0	0	0	0
3	0.9761	-14.4194	0	0	11.59	2.12	3
6	0.9716	-23.2922	0	0	15.75	2.89	4

Table 8: Machine Data of the multi-machine system

Variable	Machine at Bus 1	Machine at Bus 2	Machine at Bus 9	Machine at Bus 10
X_l	0.022	0.022	0.022	0.022
R_a	0.00028	0.00028	0.00028	0.00028
X_d	0.2	0.2	0.2	0.2
X_d'	0.033	0.033	0.033	0.033
T_{d0}'	8	8	8	8
X_q	0.19	0.19	0.19	0.19
X_q'	0.061	0.061	0.061	0.061
T_{q0}'	0.4	0.4	0.4	0.4
H	54	54	63	63
D	0	0	0	0

Table 9: Excitation System Data of multi-machine system

Variable	Bus 1	Bus 2	Bus 9	Bus 10
K_E	200	200	200	200
T_E	0.02	0.02	0.02	0.02

The numerical integration of the system's differential equations was done after reducing the network equations to equations (12) and (15) and h was chosen as 0.01. The initial conditions of the state variables are calculated from the load flow data given in Table 2. Subsequently, the voltage fault at generator 1 was applied at 1 sec and cleared in 4 cycles. The variation of state variables was recorded up to time duration of 10sec.The results obtained are given in Figure 8.

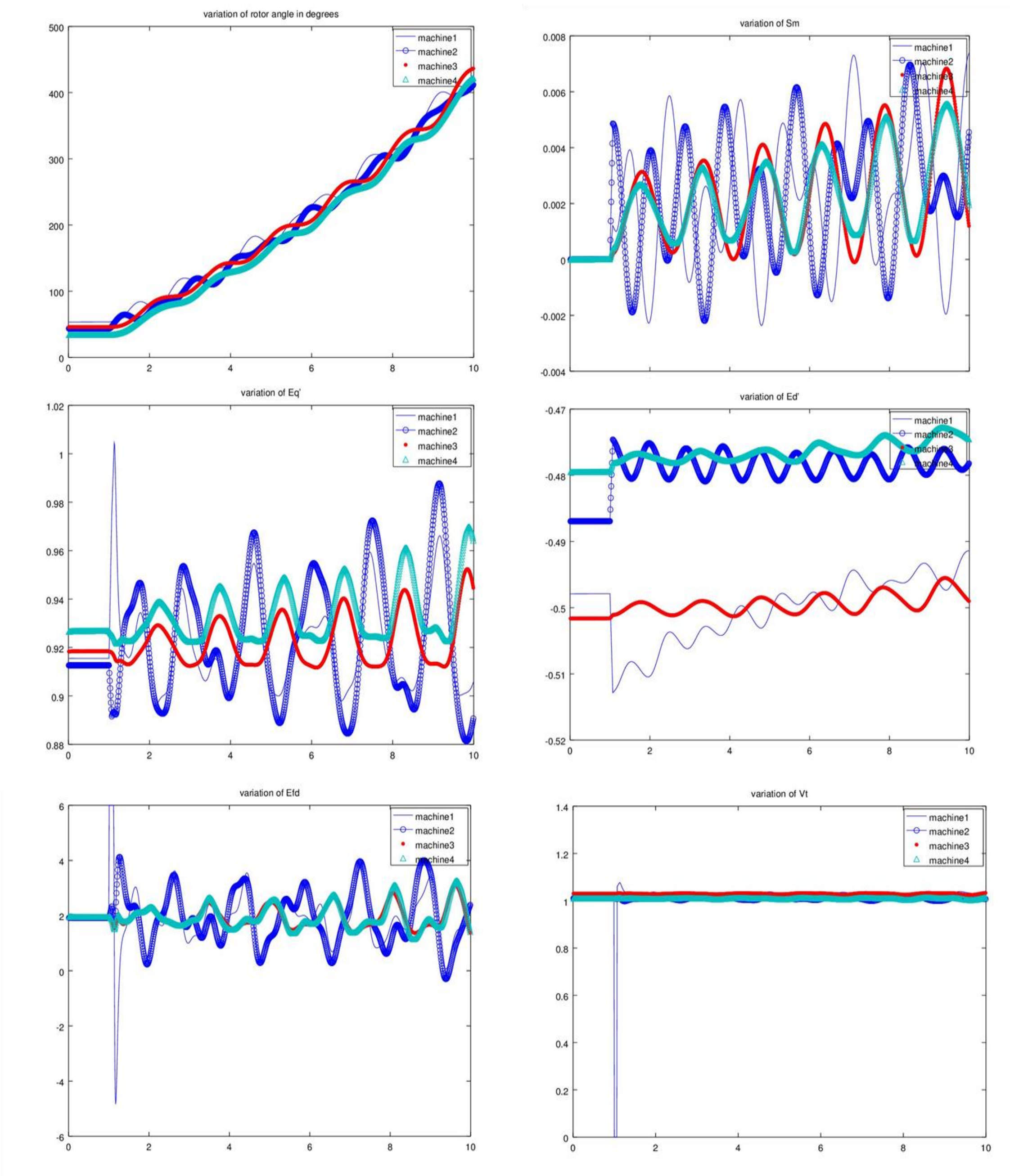

Figure 8: Variation of state variables of the multi-machine system without PSS

From the results above, we can see that the fault at 1 sec has moved the system behavior from steady state. Even though the fault is cleared in a small duration, we can see that small signal oscillations still persist. Hence, a power system stabilizer (PSS) needs to be designed in order to damp the modes of oscillations. We can also observe from the results that each generator is affected differently by each mode. So, we also need to study the effect of an unstable mode on each generator by which an appropriate PSS location can be chosen which in turn will be followed by designing of a PSS at that location.

A state-space model was formulated for the system in the form of equation (36) and the eigenvalues of the system were obtained as shown in Table 10:

Table 10: Eigenvalues of the multi-machine system

Eigenvalues
-40.023
-39.527
-11.556
-11.148
-4.570
-4.477
-4.242
-4.109
-9.111e-016
-0.00428-4.456**i**
-0.00428+4.456**i**
-0.734-6.71**i**
-0.734+6.71**i**
-0.755-7.316**i**
-0.755+7.316**i**
-25.0383-11.959**i**
-25.0383+11.959**i**
-24.506-20.629***i***
-24.506+20.629***i***

We observe that eigenvalues at 4.45 ,$6.71, 7.31$ rad/sec has less damping and hence they are the cause for small-signal instability. So, these modes are needed to be damped by the PSS. Hence, the participation factors of state variables ΔS_m of all four generators in all the three modes (4.45 ,$6.71, 7.31$ rad/sec) were calculated and obtained as shown in Table 6:

Table 11: Participation Factors of state variable ΔS_m in a multi-machine system

	Mode 1 (4.456 rad/sec)	**Mode 2 (6.71 rad/sec)**	**Mode 3 (7.31 rad/sec)**
Generator 1	0.1375	0.00341	0.1763
Generator 2	0.0852	0.00048	0. 228
Generator 3	0.131	0.188	0.000282
Generator 4	0.0849	0.239	0.0033

We can see that for the swing mode of 6.71 rad/sec, only generators 3 and 4 participate pre-dominantly whereas in the case of swing mode of 7.31 rad/sec, generators 1 and 2 participate pre-dominantly. But in the case of swing mode of 4.456 rad/sec, all the generators participate significantly. This implies that the first swing mode (4.45 rad/sec) is an inter-area mode. So, for each mode, location of PSS should be at either of the generators that participate in the mode significantly.

CHAPTER V: CONCLUSION

A reduced model is formulated and Fourth-order Runge-Kutta method is used for study of small-signal instability in a SMIB and a 10-bus, 4-machine system. A linearized model in state-space form is formulated in order to identify the modes that lead the system to small-signal instability for two systems. The lead compensator and washout block in PSS for the SMIB system are designed to provide adequate damping to the local mode 6.23 rad/sec. The effectiveness of the PSS designed for SMIB system is verified by using Runge-Kutta method. The PSS location for damping each of the troublesome modes in the 10-bus, 4-machine system is determined using participation factors.

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