A COMPARATIVE STUDY OF LAGRANGIAN RELAXATION SOLUTION AND BENDERS DECOMPOSITION SOLUTION FOR UNIT COMMITMENT PROBLEM

A PROJECT REPORT

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Muthyam Vamshi Krishna

Department of Electrical Engineering

Indian Institute of Technology Madras

Chennai

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Certificate

This is to certify that the thesis entitled,**"A comparative study of Lagrangian Relaxation solution and Benders Decomposition solution for Unit Commitment Problem"** is a bonafide record of the project work done by **Mr. Muthyam Vamshi Krishna** (EE11B115) in the Department of Electrical Engineering, Indian Institute of Technology Madras, as partial requirement for the award of degree of **"BACHELOR OF TECHNOLOGY"** and **"MASTER OF TECHNOLOGY"** in Electrical Engineering by the Indian Institute of Technology Madras. Mr. Muthyam Vamshi Krishna has fulfilled all the requirements of the regulations laid by the institution for the award of said degrees.

Dr. B. Kalyan Kumar
Associate Professor
Department of Electrical Engineering
IIT Madras
Chennai-600036

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EE11B115

Abstract

The Unit Commitment is a complex decision making problem because of multiple constraints which may not be violated while finding the Optimal Commitment Schedule. There are many methods to solve this problem, but each method have merits and demerits.

This report studies Lagrangian Relaxation method and Generalized Benders Decomposition method. A simplest Unit Commitment problem, with objective function being fuel cost alone, with demand-supply balance and generator limit constraints is considered. The above mentioned methods are implemented on this Unit Commitment model problem and then compared. Detailed simulation results are presented

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Chapter 1

Introduction

Power systems form the largest man made complex system. It basically consists of generating sources, transmission network and distribution centers. Economic operation of this system is a challenging task. The primary concern of electric power system operation is to guarantee adequate optimal generation to meet load demand satisfying the numerous constraints enforced from different directions.

All the electrical energy generated in a power station must be consumed immediately as it cannot be stored. So the electrical energy generated in a power station must be regulated according to the demand. The demand of electrical energy or load will also vary with the time and a power station must be capable of meeting the maximum load at any time. So, the active power generated in a power system is controlled in three time based control loops:

A. Unit Commitment
B. Economic Dispatch
C. Automatic Generation Control.

Unit Commitment and **Economic Dispatch** loops are to schedule the generating sources in economic manner to meet the forecasted load demand.
Automatic Generation Control continuously monitors the load variations and adjusts the power output of the generators in optimum manner which results in efficient constant frequency operation for the equipments.

A variety of strategies have been developed to make the operation of these three control loops efficient and fast. In the present economic scenario, the growing sophistication of power systems motivates the development of more and more computationally faster methods, suitable for the existing systems. Several methods have been employed for solving the various power scheduling problems. Dynamic Programming method has been widely used for solving Unit Commitment Problem and Economic Dispatch. Stochastic search methods like Genetic Algorithm, Evolutioruuy Programming and Simulated Annealing also have been used.

The present project work deals with Unit Commitment (UC) problem.

1.1 Literature Survey

Reference [1], presents presents Unit Commitment as a large scale short-term optimization problem, in which the major objective is to distribute and schedule generation to minimize the total fuel cost or to maximize the total profit or revenue over a study period, subject to a large number of constraints that must be satisfied.

The long-term fuel scheduling problem for optimizing the purchase cost, distribution, storage and utilization of fuel is considered in [2]. This problem can be designed as a large-scale linear optimization problem with the objective of minimizing the total fuel and hence the total cost.

Reference [3] deals with a heuristic algorithm based on the average full load cost (ALFC) without network constraints solution of unit commitment problem with network constraints using combination of heuristic algorithm and OPF.

Reference [4], presents the Extended Priority List (EPL) method consists of two stages; in the first stage we get any initial unit commitment problem schedule by Priority List (PL) method. At this step, operational constraints are not taken into account. In the second stage unit schedule is changed using the problem specific heuristics to fulfill operational constraints.

Reference [5], is concerned with the long standing problem of optimal unit commitment in an electric power system. It follows the traditional formulation of this problem which gives rise to a large-scale, dynamic, mixed-integer programming problem. It describes a solution methodology based on duality, Lagrangian relaxation, and non-differentiable optimization that has two unique features. First, computational requirements typically grow only linearly with the number of generating units. Second, the duality gap decreases in relative terms as the number of units increases, and as a result the algorithm tends to actually perform better for problems of large size.

With the fast paced changing technologies in the power industry, new power references addressing new technologies are coming to the market. So there is an urgent need to keep track of international experiences and activities taking place in the field of modern unit-commitment problem. [6] gives a bibliographical survey, mathematical formulations, and general backgrounds of research and developments in the field of UC problem for past 35 years based on more than 150 published articles.

A new Lagrangian relaxation algorithm for unit commitment is proposed in [7]. The algorithm proceeds in three phases. In the first phase, the Lagrangian dual of the unit commitment is maximized with standard subgradient techniques. The second phase finds a reserve feasible dual solution, followed by third phase of economic dispatch. A mathematically based, systematic and generally applicable procedure to search for a reserve feasible dual solution is presented. Both spinning and time-limited reserve constraints are treated.

Reference [8], presents a transmission-constrained unit commitment method using a Lagrangian relaxation approach. Based on a DC power flow model, the transmission constraints are formulated as linear constraints. The transmission constraints, as well as the demand and spinning reserve constraints, are relaxed by attaching Lagrange multipliers. A three-phase algorithmic scheme is devised including dual optimization, a feasibility phase and unit decommitment.

1.2 Motivation

From limited literature survey carried out it was observed that there are many methods to solve unit commitment problem. From last decade, it has become quite common to combine two or more solution techniques to tackle the problem. Although a number of techniques have been applied in solving the problem, there is no single technique that has proved to be universally applicable, as each technique has its own shortcomings. The primary motivation of thesis is to study and compare Lagranian Relaxation solution and Generalized Benders Decomposition solution for unit commitment problem.

1.3 Thesis Organisation

The thesis is organised as follows:
Chapter 1 introduces the problem of power system scheduling operation. The motivation of this work through literature survey along with thesis organisation is presented.
Chapter 2 discusses the Unit Commitment problem, its objective and constraints. It also discusses about Priority List technique and Forward Dynamic programming technique, the primary methods of solving Unit commitment.
Chapter 3 discusses the Lagrangian Relaxation Solution for Unit Commitment. It also includes results of this algorithm on considered systems.
Chapter 4 discusses the Benders Decomposition Solution for Unit Commitment. It also includes results of this algorithm on considered systems.
Chapter 5 presents the conclusions of the project.

Chapter 2

Unit Commitment

Unit Commitment, abbreviated as UC, refers to optimal scheduling of electric power generation. UC optimally chooses generating plants taking into account a wide variety of parameters, technological aspects such as minimum operating point, start-up and shut-down operation time and transient behavior as well as economic considerations like start-up costs and operational costs and social elements such as availability of staff and work-schemes. However latter can be neglected sometimes. UC optimization helps to minimize electricity generation costs.

UC problem is not same as economic dispatch problem. Economic dispatch means optimal schedule of generating units for a fixed power demand in a given time period. The UC problem is a somewhat longer term scheduling problem usually covering a time range from 24 hours (1 day) to 168 hours (1 week) ahead, and is handled by the operator in the pre-dispatch stage. In this problem, the operator needs to take decisions on how to commit or de-commit, keep running or shut down, its available units over the week, or over the next day. The input to the operator is the demand forecast for the next week or next day, as the case may be, aggregated for the whole system. UC decides the set of plants from which dispatching can be chosen.

The difference between UC and economic dispatch occurs in time. In dispatching and allocating decisions, there is practically no time to rapidly start a power plant because the inertia of most plants will not allow this. UC therefore prepares a set of plants and stipulates in which time period they have to be on-line and ready for dispatching. This planning activity is essential due to the fact that the system load varies over a day or even over a week and hence it is not economical to keep all the units on-line for the entire duration. A proper schedule for starting up, or shutting down the units can save costs significantly. UC problems are much more complex to solve, compared to the economic dispatch problem discussed earlier, due to the presence of binary decision variables, that is unit status (on or off). Depending upon the need of the system and computational facilities, the utilities choose to use UC models that suit their requirements.

2.1 General UC Problem Model

In the following sub sections the unit commitment problem, its objectives and optimization problem are discussed.

2.1.1 Objective function

The main objective of the UC problem is to minimize the cost of production of electric power. However, due to the longer time-scale of the problem, the total system cost will be affected by the start-up and shut down decisions of generating units. Hence, three important factors have to be considered for minimizing the cost of electric power generation that is

A. Fuel cost
B. Start up cost
C. Shut down cost.

These three cost are explained below:

Fuel Cost:
There have been two different approaches to represent fuel costs in UC models. The first and the most common approach has been to use cost characteristic derived from the heat-rate characteristics which is represented by a second order polynomial function, and can be written as following,

$$F(P) = aP^2 + bP + c \tag{2.1}$$

a, b and c are constants.
The other approach has been to represent the generator cost as a constant, which is derived from the generator's average full load cost.

Start Up Cost:
This component appears in the UC objective function to take into account the cost incurred during a generator start-up operation. This is often modeled as a function of the time for which the unit was off-line.

$$ST = \alpha + \beta[1 - e^{\frac{-T^{off}}{\tau}}] \tag{2.2}$$

where α is a fixed cost associated with the unit start-up, β is the cost involved in a cold start-up, T^{off} is the time for which the unit has been off and τ is a time-constant representing the cooling speed of the unit. Another approach is to have a constant star t up cost which is included in the objective function whenever a unit is turned on.

Shut Down Cost:
Usually this component of cost is not considered in UC models since it is not very significant compared to other costs. However, a constant cost representation can be used, and is included when the unit undergoes a shut down.
The composite objective function for the UC problem can be constructed using the above as basing on fuel, star up and shut down costs as following:

$$F(P_i^t, U_i^t, V_i^t, W_i^t) = \sum_{t=1}^{T}[\sum_{i=1}^{N}(F_i(P_i^t))U_i^t + ST_i^t V_i^t + SD_i^t W_i^t] \tag{2.3}$$

V, W and U are integer decision variables denoting the status of the unit at hour t.
U denotes the unit status (1 = running, 0 = off)
V denotes the unit start-up state (1 =start-up, 0 = no start-up)
W denotes the unit shut down state (1 = shut down, 0 = no shut down)
T and N denotes number of time levels and generator units respectively

2.1.2 Constraints in Unit Commitment

Unit commitment problem must satisfy the following constraints:

A. Demand-Supply Balance
This constraint ensures that the operator has scheduled enough capacity for a particular hour so that the demand at that hour is met. A typical demand-supply balance constraint is given as follows.

$$\sum_{i=1}^{N} P_i U_i = P_{load} \tag{2.4}$$

B. Generation Limit
This constraint describes the allowable range of generation available for scheduling, as defined by the maximum and minimum limits of the unit.

$$U_i P_i^{min} \leq P_i \leq U_i P_i^{max} \tag{2.5}$$

C. Must-Run Units
Some units such as large coal-based units or nuclear units cannot be start started or shut down on a day-to-day basis following the daily load variations because they involve very high start-up costs and other technical constraints. Such units need to be assigned a must-run status.

$$U_i^t = 1 \ \text{ for all } \ t \tag{2.6}$$

D. Minimum Up and Down Time Constraints on Thermal Units
These constraints ensure the minimum number of hours a unit must be on, before it can be shut down (minimum up-time) or the minimum number of hours a unit must be off-line before it can be brought on-line again (minimum down-time). These constraints are usually applicable to large thermal units.

E. Ramp Rate Constraint on Thermal Units
This constraint limits the inter-hour generation change in a unit, and is particularly applicable to coal-based thermal units. There are several models of the ramp constraint, a typical formulation is shown below.

$$P_i^t \leq RUP_i * P_i^{t-1} \tag{2.7}$$

$$P_i^t \geq RDN_i * P_i^{t-1} \tag{2.8}$$

RUP and RDN are the ramp-up and ramp-down constants of a unit. This constraint links the generation variable of the previous hour to that of the present hour, and hence introduces a dynamic characteristic in the UC models.

F. Spinning Reserves
The spinning reserve in the system is a reserve available to the operator from among its spinning units, i.e. from the generators already running. Therefore, this reserve is available almost instantaneously to the operator in case of need. The operator has a very important responsibility of maintaining adequate spinning reserves in the system, not only on a total-MW basis, but the operator also needs to take care of the location aspect of this reserve, taking into account transmission capacities available in the system.
The operator generally uses his experience or certain rules for determining this reserve to be maintained in the system. The reserve component comprise a base component, a fraction of the load requirement and a fraction of the high operating limit of the largest on-line unit.

G. Crew Constraints
These constraints pertain to the number of units that can be started at the same time in a particular plant.

H. Transmission Constraints
Most UC models neglect the power transmission limits of lines, limits on bus voltages and limits on reactive power generation. Inclusion of these transmission constraints in UC programs helps to represent the transmission losses more accurately and also ensures that the actual dispatch does not deviate much from the UC solution obtained in the pre-dispatch stage. DC load flow representations are used to represent the transmission line capacity limits though the ideal way would be to include an ac load flow model within the UC making the computations extremely complex.

2.1.3 Considered UC Problem

UC problem have different components in objective function and several constraints making it difficult to solve. The things presented above are only common ones. Generally there are other constraints which are specific to season and region.

In this work the simplest UC problem, with objective function being fuel cost alone, with demand-supply balance and generator limit constraints. These assumptions helps in understand Lagrangian Relaxation and Bender Decomposition algorithms easily which are to be discussed in chapter 3 and chapter 4 respectively.

2.2 Methods to solve UC

The importance of UC problem lead to evolution of different techniques. Priority List Approach and Dynamic Programming techniques are most important and basic ones, these techniques are being used widely.

2.2.1 Priority List Approach

This method is considered to be one of the simplest method of unit commitment scheduling. This method consists of creating a priority list of all the generating units based on their Average Full Load Cost (AFLC) value. Unit with the least value of AFLC is assigned the top most priority and the rest according to the increasing value of AFLC.

This method is primarily based on the principle that unit with the least value of AFLC should be loaded to the maximum level and the unit with the lhighest value should be lightly loaded as this may lead to economical unit commitment solution.
The value of AFLC of i^{th} generator is calculated as follows :

$$AFLC_i = \frac{c_i + b_i P_i^{max} + a_i P_i^{max2}}{P_i^{max}} \tag{2.9}$$

Following steps are followed for having unit commitment through Priority List Method
1)According to the AFLC value, arrange each generator in increasing order of their AFLC values. Generator with least value is given the highest priority.
2) Now according to priority order, set up upper and lower bound table using combinations of plants starting from only highest priority plant to all plants
3) For each load level activate number of plants able to meet demand using upper and lower bound table
4) Dispatch activated power plants and calculate costs.

The advantages of using this method for solving UC problem are algorithm is simple and can be easily executed.
The above algorithm gives optimal solution for UC problem if:
1)Unit input-output characteristics are linear between zero output and full load.
2) There are no other restrictions.

2.2.2 Forward Dynamic Programming

Introduction:
In the dynamic-programming approach, we assume that a state consists of an array of units with specified units operating and the rest off-line. A feasible state is one in which the committed units can supply the required load and that meets the minimum amount of capacity each period. Here assume start-up cost of a unit is independent of the time it has been off-line (i.e., it is a fixed amount).

Method
In dynamic programming, we use the following recursive function to compute the minimum cost in hour t with combination I,

$$F_{cost}(t, I) = \min_{\{L\}} \left[P_{cost}(t, I) + S_{cost}(t-1, L; t, I) + F_{cost}(t-1, L) \right] \tag{2.10}$$

where,
$F_{cost}(t, I)$ = least total cost to arrive at state (t, I)

$P_{cost}(t, I)$ = production cost for state (t, I)
$S_{cost}(t-1, L; t, I)$ = transition cost from state $(t-1, L)$to state (t, I)
$\{L\}$ = set of all feasible states in the interval $t-1$

The following steps are followed in dynamic programming approach to solve UC problem:
Step 1:
Start with $t = 1$ and find $F_{cost}(t, I) = P_{cost}(t, I)$ for all possible feasible states (combinations) in period t.
Step 2:
Increment t by one time period, find all feasible state set $\{L\}$in time period $t-1$.
Step 3:
Find $F_{cost}(t, I)$ for all possible feasible states (combinations) in period t using the above recursive formula.
Step 4:
Save the low cost strategies for all feasible state set $\{L\}$in time period $t-1$.
Step 5:
Check whether t has reached the limit i.e. whether t is equal to last time period. If not, go to step 2 and repeat procedure. If yes, proceed to step 6.
Step 6:
Trace the optimal schedule using the saved low cost strategies for each time period.

The dynamic programming algorithm, explained above, moves forward in time starting from the initial hour back to the final hour. Conversely, one could set up the algorithm to run backward in time from the final hour to the initial hour. The forward approach has distinct advantages in solving generator unit commitment. For example, if the start-up cost of a unit is a function of the time it has been off-line then a forward dynamic-program approach is more suitable since the previous history of the unit can be computed at each stage. There are other practical reasons for going forward. The initial conditions are easily specified and the computations can go forward in time as long as required.

The dynamic-programming method of solution of the unit commitment problem has many disadvantages for large power systems with many generating units. This is because of the necessity of forcing the dynamic-programming solution to search over a small number of commitment states to reduce the number of combinations that must be tested in each time period.

Chapter 3

Lagrange Relaxation Solution

Lagrangian Relaxation is a much more recent approach than dynamic programming method to solve UC problem. In this method the separatablity property of generating units are used and the solution is approached by relaxing some constraints. The dual problem theory is used in this method. Dual problem theory is briefly explained below.

3.1 Dual problem Theory

One of the way to solve an optimization problem is to use a technique that solves for the Lagrange variables directly and then solves for the problem variables themselves. This formulation is known as a **dual solution** and in it the Lagrange multipliers are called **dual variables**. Consider a usual optimization problem as below which is formally called **primal problem**.

$$\begin{array}{rl} \text{Minimize:} & f(X) \\ \text{Subject to:} & w(X) = 0 \end{array}$$

where X is a vector variable

and its Lagrangian function is:

$$L(X, \lambda) = f(X) + \lambda w(X)$$

We define a dual function, $q(\lambda)$, as:

$$q(\lambda) = \min_{X} \quad L(X, \lambda) \tag{3.1}$$

Then the dual problem is to find

$$q^*(\lambda) = \max_{\lambda \geq 0} \quad q(\lambda) \tag{3.2}$$

The solution, in the case of the dual problem involves two separate optimization problems. The first requires us to take an initial set of value for X and then find the value of λ. which maximizes $q(\lambda)$. We then take this value of λ and, holding it constant, we find value of X which minimize $L(X, \lambda)$. This process is repeated or iterated until the solution is found.

In case if the objective function is convex then both primal and dual problem will converge to the same solution.
Since the dual problem requires that we find $q^*(\lambda)$ by solving (3.2) and since there is no explicit function in λ a different strategy is adopted. A way is found to adjust λ so as to move $q(\lambda)$ from its initial value to one which is larger. The simplest way to do this is to use a gradient adjustment so that

$$\lambda^1 = \lambda^0 + [\frac{d}{d\lambda}q(\lambda)]\alpha \tag{3.3}$$

where α is a constant. A more useful way to apply the gradient technique is to let λ be adjusted upwards at one rate and downward at a much slower rate; for example:

$$\alpha = 0.5 \text{ when } \frac{d}{d\lambda}q(\lambda) \text{ is positive} \tag{3.4}$$

and

$$\alpha = 0.1 \text{ when } \frac{d}{d\lambda}q(\lambda) \text{ is negative} \tag{3.5}$$

The closeness to the final solution in the dual optimization method is measured by noting the relative size of the gap between the primal function and the dual function. The primal optimization problem can be solved directly and the optimal value will be called J^* and it is defined as:

$$J^* = \text{min } L(X, \lambda) \tag{3.6}$$

This value will be compared to the optimum value of the dual function, $q*$. The difference between them is called the duality gap. A good measure of the closeness to the optimal solution is the relative duality gap, defined as:

$$\frac{J^* - q^*}{q^*}$$

For a convex problem with continuous variables, the duality gap will become zero at the final solution. In case dual optimization method is applied to UC the duality gap will never become zero as the problem is non convex and non continuous.

3.2 Lagrange Relaxation Theory

We start by revisiting the variable U_i^t, where:

U_i^t = 0 if unit i is off-line during period t

U_i^t = 1 if unit i is on-line during period t

Now consider the constraints and the objective function of the unit commitment problem as given below:
1. Demand-Supply balance constraint:

$$P_{load}^t = \sum_{i=1}^{N} P_i^t U_i^t \quad \text{for} \quad t = 1 \dots T \tag{3.7}$$

2. Generator Unit limits:

$$U_i^t P_i^{min} \leq P_i^t \leq U_i^t P_i^{max} \quad \text{for } i = 1 \ldots N \text{ and } t = 1 \ldots T \tag{3.8}$$

3. The objective function is:

$$\sum_{t=1}^{T} \sum_{i=1}^{N} (F_i(P_i^t))U_i^t = F(P_i^t, U_i^t) \tag{3.9}$$

The Lagrange function can be defined as:

$$L(P, U, \lambda) = F(P_i^t, U_i^t) + \sum_{t=1}^{T} \lambda^t (P_{load}^t - \sum_{i=1}^{N} P_i^t U_i^t) \tag{3.10}$$

The unit commitment problem requires that the Lagrange function above is minimized, subject to the local unit constraints 2 , which can be applied to each unit separately. Note:

1. The cost function, $F(P_i^t, U_i^t)$, together with constraints 2 is separable over units. That is, what is done with one unit does not affect the cost of running another unit, as far as the cost function and the unit limits (constraint 2) are concerned.

2. Constraints 1 are coupling constraints across the units so that what we do to one unit affects what will happen on other units if the coupling constraints are to be met.

The Lagrange relaxation procedure solves the unit commitment problem by relaxing or temporarily ignoring the coupling constraints and solving the problem as if they did not exist. This is done through the dual optimization procedure as explained in the dual problem theory . The dual procedure attempts to reach the constrained optimum by maximizing the Lagrangian with respect to the Lagrange multipliers, while minimizing with respect to the other variables in the problem; that is:

$$q^*(\lambda) = \max_{\lambda^t} \quad q(\lambda) \tag{3.11}$$

where

$$q(\lambda) = \min_{P_i^t, U_i^t} \quad L(P, U, \lambda) \tag{3.12}$$

This is done in two basic steps:

Step 1:

Find a value for each λ^t which moves $q(\lambda)$ towards a large value.

Step 2:

Assuming that the λ^t in step 1 are now fixed, find the minimum of L by adjusting the values of P^t and U^t.

The adjustment of the λ^t values will be dealt later in this chapter; assume that a value has been chosen for all the λ and that they are now to be treated as fixed numbers. We shall minimize the Lagrangian as follows. First, we rewrite the Lagrangian as:

$$L = \sum_{t=1}^{T}\sum_{i=1}^{N}(F_i(P_i^t))U_i^t + \sum_{t=1}^{T}\lambda^t(P_{load}^t - \sum_{i=1}^{N}P_i^tU_i^t) \tag{3.13}$$

This is now rewritten as:

$$L = \sum_{t=1}^{T}\sum_{i=1}^{N}(F_i(P_i^t))U_i^t + \sum_{t=1}^{T}\lambda^t P_{load}^t - \sum_{t=1}^{T}\sum_{i=1}^{N}\lambda^t P_i^tU_i^t \tag{3.14}$$

The second term above is constant and can be dropped (since the λ^t are fixed). Finally, we write the Lagrange function as,

$$L = \sum_{i=1}^{N}[\sum_{t=1}^{T}\{F_i(P_i^t)U_i^t - \lambda^t P_i^tU_i^t\}] \tag{3.15}$$

Here, we have achieved our goal of separating the units from one another. The term inside the outer brackets; that is:

$$\sum_{t=1}^{T}\{F_i(P_i^t)U_i^t - \lambda^t P_i^tU_i^t\} \tag{3.16}$$

can be solved separately for each generating unit, without regard for what is happening on the other generating units. The minimum of the Lagrangian is found by solving for the minimum for each generating unit over all time periods; that is,

$$q(\lambda) = \sum_{i=1}^{N} \ min \ \sum_{t=1}^{T} \ \{F_i(P_i^t)U_i^t - \lambda^t P_i^tU_i^t\} \tag{3.17}$$

Subject to

$$U_i^tP_i^{min} \le P_i^t \le U_i^tP_i^{max} \quad \text{for } \ t = 1 \ldots T \tag{3.18}$$

This is easily solved as a dynamic programming problem in one variable. This can be visualized in the figure 3.1 taken from [9], which shows the only two possible states for unit i (i.e., U_i^t = 0 or 1):

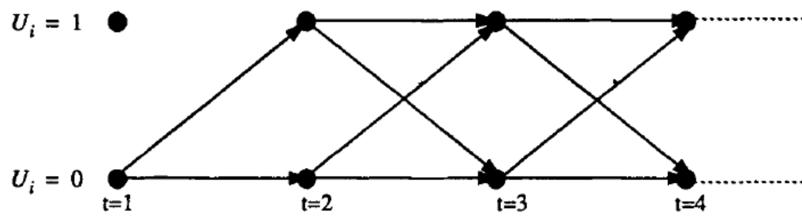
At the U_i^t = 0 state, the value of the function to be minimized is trivial (i.e., it equals zero); at the state where U_i^t = 1, the function to be minimized is :

$$\min \ [F_i(P_i^t) - \lambda^t P_i^t] \tag{3.19}$$

The minimum of this function is found by taking the first derivative:

$$\frac{d}{dP_i^t}[F_i(P_i^t) - \lambda^t P_i^t] = \frac{d}{dP_i^t}F_i(P_i^t) - \lambda^t = 0 \tag{3.20}$$

Figure 3.1: Two states dynamic program



The solution to this equation is

$$\frac{d}{dP_i^t}F_i(P_i^{opt}) = \lambda^t \tag{3.21}$$

There are three cases to be concerned with, depending on the relation of P_i^{opt} and the unit limits:

1. If $P_i^{opt} \leq P_i^{min}$, then:

$$\min\ [F_i(P_i^t) - \lambda^t P_i^t] = F_i(P_i^{min}) - \lambda^t P_i^{min} \tag{3.22}$$

2. If $P_i^{min} \leq P_i^{opt} \leq P_i^{max}$, then:

$$\min\ [F_i(P_i^t) - \lambda^t P_i^t] = F_i(P_i^{opt}) - \lambda^t P_i^{opt} \tag{3.23}$$

3. If $P_i^{opt} \geq P_i^{max}$, then:

$$\min\ [F_i(P_i^t) - \lambda^t P_i^t] = F_i(P_i^{max}) - \lambda^t P_i^{max} \tag{3.24}$$

The solution of the two-state dynamic program for each unit proceeds in the normal manner as was done for the forward dynamic-programming solution of the unit commitment problem itself. Now we seek to minimize $[F_i(P_i^t) - \lambda^t P_i^t]$ at each stage and that when U_i^t=0 this value goes to zero, then the only way to get a value lower is to have

$$[F_i(P_i^t) - \lambda^t P_i^t] < 0$$

Suppose if UC problem had start up cost and shut down cost in objective function with additional constraints (discussed in chapter 2), then the dynamic program should take into account all the start-up costs for each unit, as well as the minimum up and down time for the generator.

3.2.1 Adjusting λ

So far, we have shown how to schedule generating units with fixed values of λ^t for each time period. As shown in the dual problem theory, the adjustment of λ^t must be done carefully so as to maximize $q(\lambda)$. Note that unlike in the section 3.1, the λ here is a vector of values, each of which must be adjusted. Much research in recent years has been aimed at ways to speed the search for the correct values of λ for each hour. The technique of adjusting λ for each hour that is used for the unit commitment problem :

$$\lambda^t = \lambda^t + [\frac{d}{d\lambda}q(\lambda)]\alpha \tag{3.25}$$

where

$$\alpha = 0.01 \text{ when } \tfrac{d}{d\lambda}q(\lambda) \text{ is positive}$$

and

$$\alpha = 0.002 \text{ when } \tfrac{d}{d\lambda}q(\lambda) \text{ is negative}$$

The overall Lagrange relaxation unit commitment algorithm is shown in section 3.3
The relative duality gap or $(J^* - q^*)/q^*$ is used as a measure of the closeness to the solution. So, with this relative duality gap as termination condition in our algorithm. The following facts are to be noted.

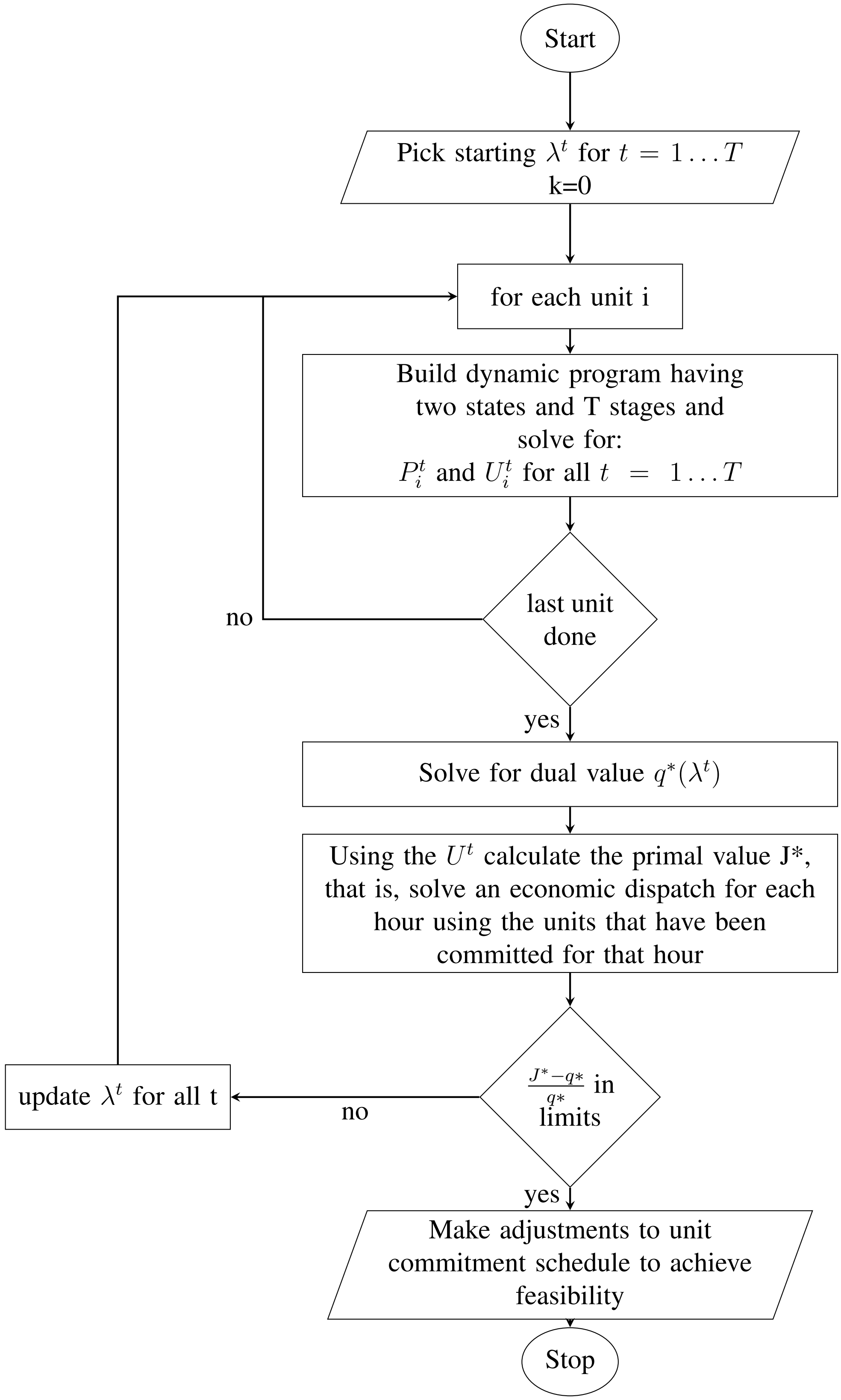
1. For large, real-sized, power-system unit commitment calculations, the duality gap does become quite small as the dual optimization proceeds, and its size can be used as a stopping criterion. The larger the problem (larger number of generating units), the smaller the gap.

2. The convergence is unstable at the end, meaning that some units are being switched in and out, and the process never comes to a definite end.

3. There is no guarantee that when the dual solution is stopped, it will be at a feasible solution.

The duality gap is large at the beginning and becomes progressively smaller as the iterations progress. The solution reaches a commitment schedule when at least enough generation is committed so that an economic dispatch can be run, and further iterations only result in switching marginal units on and off. Finally, the loading constraints are not met by the dual solution when the iterations are stopped.

3.3 Flowchart



3.4 Results

The application of Lagrangian Relaxation algorithm to UC problem is implemented on two systems:

A. 3 generating units with 4 block periods
B. 10 generating units with 24 block periods

3.4.1 Three generating units with four block periods

The cost characteristics of the three generators along with the generation limits are given in Table 3.1. The four block periods load information is given in Table 3.2. Lagrangian relaxation method of UC is applied to this system. The peak load occurs in time during 12 pm - 6 pm with 1100 MW.

The results are obtained starting from an initial condition where all the λ values are set to zero. An economic dispatch is run for each hour, provided there is sufficient generation committed that hour. If there is not enough generation committed, the total cost for that hour is set arbitrarily to 10,000. Once each hour has enough generation committed, the primal value J^* simply represents the total generation cost summed over all hours as calculated by the economic dispatch.

The results are obtained after five iterations. The results are shown in Table 3.3, in which

Table 3.1: Cost characteristics of the three generators

unit	a (Rs/MW^2hr)	b $(Rs/MWhr)$	c (Rs/hr)	Pmin (MW)	Pmax (MW)
1	0.002	10	500	100	600
2	0.0025	8	300	100	400
3	0.005	6	100	50	200

Table 3.2: 4 Load levels system

No	Time Period	Load (MW)
1	12 am - 6 am	170
2	6 am - 12 pm	520
3	12 pm - 6 pm	1100
4	6 pm - 12 am	330

each row consists results for a particular load levels such as units committed, power generation in committed units and cost of production for total time period of that load level.
When load is peak with 1100 MW all the units' status are 1 which indicate all units are committed. When load is low with 170 MW only generator 3 is committed. For other two load levels, generator 2 and generator 3 are committed.

Figure 3.2: Load curve1

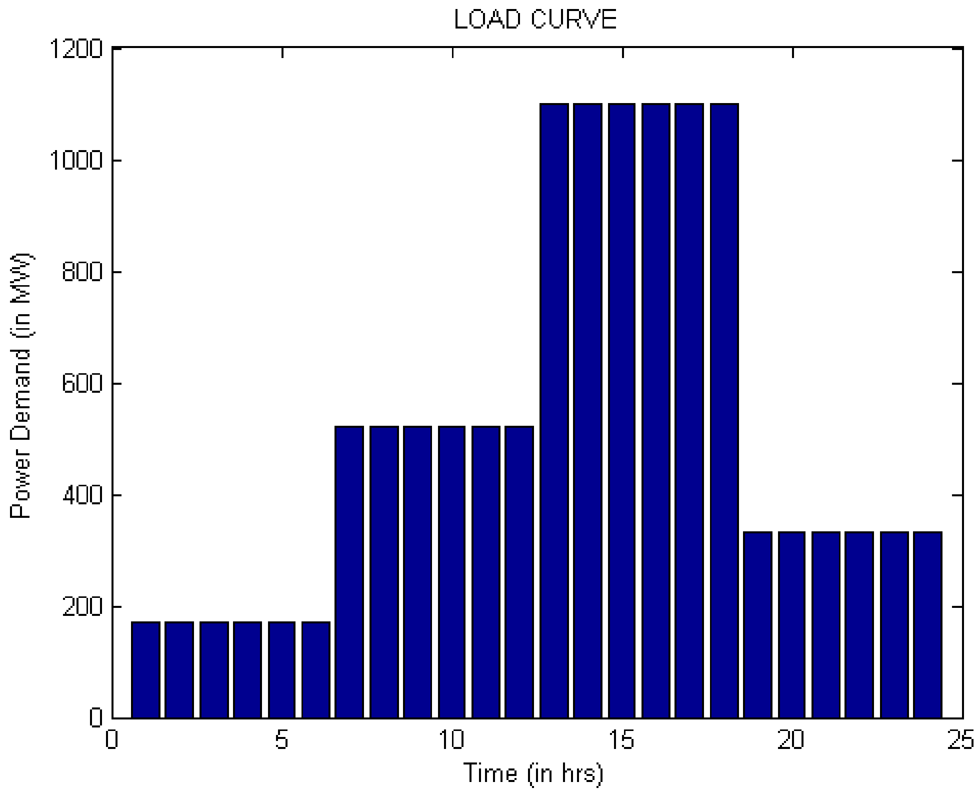


Table 3.3: Results obtained for 3 units with 4 block periods using Lagrangian Relaxation Technique

Load level	u_1	u_2	u_3	P_1 (MW)	P_2 (MW)	P_3 (MW)	Cost (Rs)
1	0	0	1	0	0	170	7587
2	0	1	1	0	320	200	27696
3	1	1	1	500	400	200	68400
4	0	1	1	0	130	200	17293.5
Total Cost (Rs)							120976.5

3.4.2 Ten generating units with twenty four block periods

The actual system in real life is not simple as previous system since it has many generator units and many load levels. So, now we consider slightly bigger system with 10 units and 24 load levels system. this system is solved using the Lagrange relaxation technique.
Generator unit parameters are shown in Table 3.4. Hourly load levels are shown in Table 3.5.
The load curve of 24 load levels system can be seen in figure 3.3. The load varies from 700 MW to 1400 MW throughout the day.
Results obtained from lagrangian relaxation are shown in table 3.6 and 3.7. Table 3.6 shows committed units for each hour.

Table 3.4: 10 Generator units system

Unit	a (Rs/MW^2hr)	b $(Rs/MWhr)$	c (Rs/hr)	Pmin (MW)	Pmax (MW)
1	0.00480	16.19	1000	150	455
2	0.00310	17.26	970	150	455
3	0.00200	16.60	700	20	130
4	0.00211	16.50	680	20	130
5	0.00398	19.70	450	25	162
6	0.00712	22.26	370	20	80
7	0.00790	23.74	480	25	85
8	0.00413	21.92	660	10	55
9	0.00222	24.27	665	10	55
10	0.00473	20.79	670	10	55

Table 3.5: 24 Load level system

No	Time Period	Load (MW)	No	Time Period	Load (MW)
1	12 am - 1 am	700	13	12 pm - 1 pm	1350
2	1 am - 2 am	800	14	1 pm - 2 pm	1300
3	2 am - 3 am	850	15	2 pm - 3 pm	1200
4	3 am - 4 am	950	16	3 pm - 4 pm	1050
5	4 am - 5 am	1000	17	4 pm - 5 pm	1000
6	5 am - 6 am	1100	18	5 pm - 6 pm	1100
7	6 am - 7 am	1150	19	6 pm - 7 pm	1200
8	7 am - 8 am	1200	20	7 pm - 8 pm	1400
9	8 am - 9 am	1250	21	8 pm - 9 pm	1300
10	9 am - 10 am	1300	22	9 pm - 10 pm	1100
11	10 am-11am	1350	23	10 pm-11 pm	900
12	11 am-12 pm	1400	24	11 pm-12 am	850

Figure 3.3: Load curve2

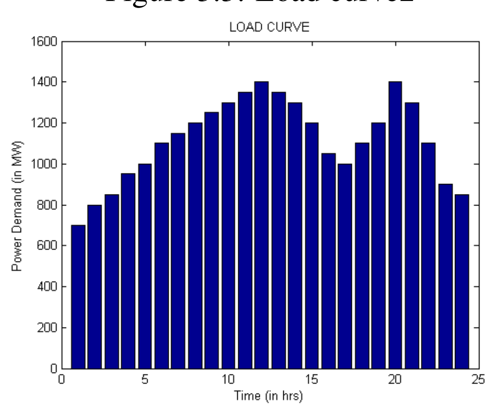


Table 3.6: Unit status obtained for 10 units and 24 load level system

Load level	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}
1	1	1	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0
3	1	1	0	0	0	0	0	0	0	0
4	1	1	1	1	0	0	0	0	0	0
5	1	1	1	1	1	0	0	0	0	0
6	1	1	1	1	0	0	0	0	0	0
7	1	1	1	1	0	0	0	0	0	0
8	1	1	1	1	1	0	0	0	0	0
9	1	1	1	1	1	0	0	0	0	0
10	1	1	1	1	1	0	0	0	0	0
11	1	1	1	1	1	1	0	0	0	0
12	1	1	1	1	1	1	0	0	0	0
13	1	1	1	1	1	1	0	0	0	0
14	1	1	1	1	1	0	0	0	0	0
15	1	1	1	1	1	0	0	0	0	0
16	1	1	1	1	0	0	0	0	0	0
17	1	1	1	1	1	0	0	0	0	0
18	1	1	1	1	0	0	0	0	0	0
19	1	1	1	1	1	0	0	0	0	0
20	1	1	1	1	1	0	0	0	0	0
21	1	1	1	1	1	0	0	0	0	0
22	1	1	1	0	0	0	0	0	0	0
23	1	1	0	0	0	0	0	0	0	0
24	1	1	0	0	0	0	0	0	0	0

Table 3.7 shows power generation in committed units and cost of production for each hour. When load is at peak with 1400 MW generating units from 1 to 6 are committed. When load is low with 700 MW generator 1 and generator 2 are committed. Units 7 to 10 are not committed for any load level, this indicates that they have high fuel cost.

It is to be noted that solution obtained for second load level is infeasible. So no power production and cost is shown corresponding that load level. This is the result of relaxation of demand-supply condition.

Table 3.7: Power Generation obtained for 10 units and 24 load level system

P_1 (MW)	P_2 (MW)	P_3 (MW)	P_4 (MW)	P_5 (MW)	P_6 (MW)	P_7 (MW)	P_8 (MW)	P_9 (MW)	P_{10} (MW)	Cost (Rs)
342.40	357.60	0	0	0	0	0	0	0	0	14644.79
-	-	-	-	-	-	-	-	-	-	-
401.27	448.73	0	0	0	0	0	0	0	0	17608.74
338.48	351.52	130	130	0	0	0	0	0	0	20202.67
348.29	366.71	130	130	25	0	0	0	0	0	21634.82
397.34	442.66	130	130	0	0	0	0	0	0	23160.96
435	455	130	130	0	0	0	0	0	0	24168.47
419.74	455	130	130	65.26	0	0	0	0	0	25611.37
442.40	455	130	130	92.60	0	0	0	0	0	26627.78
455	455	130	130	130	0	0	0	0	0	27655.97
455	455	130	130	160	20	0	0	0	0	29099.64
455	455	130	130	162	68	0	0	0	0	30240.16
455	455	130	130	160	20	0	0	0	0	29099.64
455	455	130	130	130	0	0	0	0	0	27655.97
419.74	455	130	130	65.26	0	0	0	0	0	25611.37
377.72	412.28	130	130	0	0	0	0	0	0	22165.45
348.29	366.71	130	130	25	0	0	0	0	0	21634.82
397.34	442.66	130	130	0	0	0	0	0	0	23160.96
419.74	455	130	130	65.26	0	0	0	0	0	25611.37
455	455	130	130	162	68	0	0	0	0	30240.16
455	455	130	130	130	0	0	0	0	0	27655.97
397.34	442.66	130	130	0	0	0	0	0	0	23160.96
445	455	0	0	0	0	0	0	0	0	18620.15
401.27	448.73	0	0	0	0	0	0	0	0	17608.74
Total Cost (Rs)										552880.94

3.5 Summary

This chapter discussed the Lagrangian Relaxation technique along with dual problem theory. It also discussed a method to solve UC problem using Lagrangian Relaxation technique. The results are presented when Lagrangian Relaxation is implemented on 3 generating units with 4 block periods and 10 generating units with 24 block periods

Chapter 4

Generalized Bender Decomposition Solution

UC problem is nonlinear optimization problems involving unit status which are discrete variables in addition to the continuous variables, that is power generation in the plants. So, it falls in special class of optimization problems called as Mixed Integer Nonlinear Programming (MINLP) problems. Few optimization techniques that deal with MINLP problems are given below:

A. Generalized Benders Decomposition (GBD) algorithm
B. Branch and Bound (BB) algorithm
C. Outer approximation (OA) algorithm

This chapter discusses the GBD algorithm and presents a way to solve UC problem using the algorithm.

4.1 GBD theory

Geoffrion(1972) [10] generalized the approach proposed by Benders(1962) [11], solve to the following class of optimization problems:

$$
\begin{aligned}
\min_{x,y} \quad & f(x,y) \\
\text{s.t.} \quad & h(x,y) = 0 \\
& g(x,y) \leq 0 \\
& x \in X \subseteq \Re^n \\
& y \in Y = \{0,1\}^q
\end{aligned}
\tag{4.1}
$$

under the following conditions:
C1: X is a nonempty, convex set and the functions

$$
\begin{aligned}
f &: \Re^n \text{x} \Re^q \rightarrow \Re \\
g &: \Re^n \text{x} \Re^q \rightarrow \Re^p
\end{aligned}
$$

are convex for each fixed $y \in Y = \{0,1\}^q$ while the functions $h : \Re^n x \Re^l \rightarrow \Re^m$ are linear for each fixed $y \in Y = \{0,1\}^q$.
C2: X is bounded and closed and $h(x,y), g(x,y)$ are continuous on x for each fixed $y \in Y$.
C3: For each fixed $y \in Y \cap V$, where

$$V = \{y : h(x,y) = 0, g(x,y) \leq 0, \text{for some } x \in X\},$$

One of the following two conditions holds :
(i) The resulting problem (4.1) has a finite solution and has an optimal multiplier vector for the equalities and inequalities.
(ii) The resulting problem (4.1) is unbounded, that is, its objective function value goes to $-\infty$.

4.1.1 Basic Idea

The basic idea in GBD is the generation at each iteration, of an upper bound and a lower bound on the sought solution of the MINLP model. The upper bound results from the primal problem, while the lower results from the master problem. The primal problem corresponds to problem (4.1) with fixed y- variable that is, it is in the x-axis only, and its solution provides information about the upper bound and the Lagrange multipliers associated with the equality and inequality constraints. The master problem is decreased via non linear duality theory, makes use of the Lagrange multipliers obtained in the primal problems and its solution provides information about the lower bound, as well as the next set of fixed y variables to be used subsequently in the primal problem. As the iterations proceed , it is above that the sequence of updated upper bounds is non increasing, the sequence of lower bounds is non-decreasing, and that the sequences converge in a finite number of iterations.

4.1.2 Theoretical Development

This section presents the theoretical development of the GBD. The primal problem is analyzed first for the feasible and infeasible cases. Subsequently the theoretical analysis for the derivation of the master problem is presented.

Primal Problem

The primal problem results from fixing the y variables to a particular 0-1 combinations, which is denoted as y^k where k stands for the iteration counter .The formulation of the primal problem $P(y^k)$, at iteration k is

$$P(y^k) = \qquad \begin{array}{ll} \min\limits_{x} & f(x,y^k) \\ \text{s.t.} & h(x,y^k) = 0 \\ & g(x,y^k) \leq 0 \\ & x \in X \subseteq \Re^n \end{array} \tag{4.2}$$

Note that due to conditions C1 and C3(i), the solution of the primal problem $P(y^k)$ is its global solution.Now there are two possible cases. They are:

A. Feasible Primal
B. Infeasible Primal,

Each case is separately described.
A. Feasible Primal:
If the primal problem at iteration k is feasible, then its solution provides information on x^k,$f(x^k, y^k)$, which is the upper bound, and the optimal multiplier vectors λ^k, μ^k, for the equality and inequality constraints . Subsequently, using this information we can formulate the Langrage function as

$$L(x, y, \lambda^k, \mu^k) = f(x, y) + \lambda^{k^T} h(x, y) + \mu^{k^T} g(x, y)$$

B. Infeasible Primal:
If the primal is detected by the NLP solver to be infeasible, then consider its constraints.

$$\begin{aligned} h(x, y^k) &= 0 \\ g(x, y^k) &\leq 0 \\ x \in X &\subseteq \Re^n \end{aligned}$$

where the set X, for instance, consists of lower and upper bounds on the x variables. The feasibility minimization (FP) problem is formulated, which is actually l_1 minimization problem given below:

$$\begin{aligned} \min_{x} \quad & \sum_{i=1}^{p} \alpha_i \\ \text{s.t.} \quad & h(x, y^k) = 0 \\ & g_i(x, y^k) \leq \alpha_i \;\; i = 1, \ldots, p \\ & \alpha_i \geq 0 \;\; i = 1, \ldots, p \end{aligned} \tag{4.3}$$

The solution of the feasibility problem (FP) provides information on the Lagrange multipliers for the equality and inequality constraints which are denoted as $\bar{\lambda}^k, \bar{\mu}^k$, respectively. Then, the Langrange function resulting from an infeasible primal problem at iteration k can be defined as

$$\bar{L}(x, y, \bar{\lambda}^k, \bar{\mu}^k) = (\bar{\lambda}^k)^T h(x, y) + (\bar{\mu}^k)^T g(x, y)$$

It should be noted that two different types of Lagrange function are defined depending on whether the primal problem is feasible or infeasible. Also, the upper bound is obtained only from the feasible primal problem.

Master Problem

The derivation of the master problem in the GBD makes use of nonlinear duality theory are characterized by the following three key ideas:

A. Projection of problem (4.1) onto the y space
B. Dual representation of V
C. Dual representation of the projection of problems (4.1) on the y-space

The theoretical analysis involved in these three key ideas is presented below:
A. Projection of problem (4.1) onto the y space
Problem (4.1) can be written as

$$\begin{aligned} &\min_{y}\ \inf_{x}\ f(x,y) \\ \text{s.t.}\ \ & h(x,y)=0 \\ & g(x,y)\le 0 \\ & x\in X\subseteq \Re^n \\ & y\in Y=\{0,1\}^q \end{aligned} \tag{4.4}$$

where the min operator has been written separately for y and x. Note that it is minimum with respect to x since for given y the inner problems may be unbounded. Let us define $v(y)$ as

$$v(y)= \qquad \begin{aligned} &\inf_{x} && f(x,y) \\ &\text{s.t.} && h(x,y)=0 \\ & && g(x,y)\le 0 \\ & && x\in X\subseteq \Re^n \end{aligned} \tag{4.5}$$

Note that $v(y)$ is parametric in the y variables and therefore, from its definition corresponds to the optimal value of problem (4.1) for fixed y, that is, the primal problem $P(y^k)$ for $y=y^k$. Now recalling the set V in C3, which is:

$$V=\{y:h(x,y)=0,g(x,y)\le 0,\text{for some} x\in X\}, \tag{4.6}$$

Problem (4.4) can be written as

$$\begin{aligned} &\min_{y} && v(y) \\ &\text{s.t.} && y\in Y\cap V \end{aligned} \tag{4.7}$$

where $v(y)$ and V are denoted by (4.5) and (4.6), respectively. Problem (4.7) is the projection of the problem (4.1) onto the y space . Note also that in (4.6) $y\in Y\cap V$ since the projection needs to satisfy the feasibility considerations. Having defined the projection problem of (4.1) onto the y space we can now state the theoretical result of [10].

Projection Theorem:
(i) If (x^*,y^*) is optimal in (4.1) then y^* is optimal in (4.7).
(ii) If (4.1) is feasible or has unbounded solution, then the same is true for (4.7) and vice versa.

Note that the difficulty in (4.7) is due to the fact that $v(y)$ and V are known only implicitly via (4.5) and (4.6).

B. Dual Representation of V
The dual representation of V will be invoked in terms of the intersection of a collection of regions contain it, and it is described in the following theorem of [10].

Theorem: Assuming conditions C1 and C2 a point $y\in Y$ belongs also to the set V if and only if it satisfies the system:

$$0 \geq \inf \bar{L}(x, y, \bar{\lambda}, \bar{\mu}), \forall \bar{\lambda}, \bar{\mu} \in \Lambda, \text{where } \Lambda = \{\bar{\lambda} \in \Re^m, \bar{\mu}^p : \bar{\mu} \geq 0, \sum_{i=1}^{p} \bar{\mu}_i = 1\} \tag{4.8}$$

Note that (4.8) is an indefinite system because it has to be satisfied for all $\bar{\lambda}, \bar{\mu} \in \Lambda$ The dual representation of the set V corresponds to the set of the constraints that have to be incorporated for the case of infeasible primal problem.

C. Dual Representation of $v(y)$

The dual representation of $v(y)$ will be in terms of the pointwise infimum of a collection of function that support it , and it is described in the following theorem due to [10].

Theorem:

$$v(y) = \begin{array}{ll} \inf\limits_{x} & f(x, y) \\ \text{s.t.} & h(x, y) = 0 \\ & g(x, y) \leq 0 \\ & x \in X \subseteq \Re^n \end{array} \tag{4.9}$$

$$= \begin{array}{l} \sup\limits_{\lambda, \mu \geq 0} \inf\limits_{x \in X} \quad L(x, y, \lambda, \mu) \\ \qquad \forall y \in Y \cap V. \end{array} \tag{4.10}$$

$$\text{where } L(x, y, \lambda, \mu) = f(x, y) + \lambda^T h(x, y) + \mu^T g(x, y)$$

The equality of $v(y)$ and its dual is due to having the strong theorem satisfied because of conditions C1,C2 and C3. Substituting (4.10) for $v(y)$ and (4.8) for $y \in Y \cap V$ into the problem (4.7), which is equivalent to (4.4) , lead to

$$\begin{array}{lll} \min\limits_{y \in Y} & \sup\limits_{\lambda, \mu \geq 0} \inf\limits_{x \in X} & L(x, y, \lambda, \mu) \\ \text{s.t.} & 0 \geq \inf\limits_{x \in X} & \bar{L}(x, y, \bar{\lambda}, \bar{\mu}) \end{array} \tag{4.11}$$

Using the definition of supremum as the lowest upper bound and introducing a scalar μ_B gives

$$\begin{array}{lll} \min\limits_{y \in Y, \mu_B} & \mu_B & \\ \text{s.t.} & \mu_B \geq \inf\limits_{x \in X} \quad L(x, y, \lambda, \mu), & \forall \lambda, \mu \geq 0 \\ & 0 \geq \inf\limits_{x \in X} \quad \bar{L}(x, y, \bar{\lambda}, \bar{\mu}), & \forall (\bar{\lambda}, \bar{\mu}) \in \Lambda \end{array} \tag{4.12}$$

$$\text{where } L(x, y, \lambda, \mu) = f(x, y) + \lambda^T h(x, y) + \mu^T g(x, y)$$

$$\bar{L}(x, y, \bar{\lambda}, \bar{\mu}) = \bar{\lambda}^T h(x, y) + \bar{\mu}^T g(x, y)$$

which is called the master problem. If we assume that the optimum solution of $v(y)$ in (4.5) is bounded for all $y \in Y \cap V$, then we can replace infimum with a minimum. subsequently, the master problem will be as follows:

$$\begin{aligned} \min_{y \in Y, \mu_B} \quad & \mu_B \\ \text{s.t.} \quad & \mu_B \geq \min_{x \in X} \quad L(x, y, \lambda, \mu), \quad \forall \lambda, \mu \geq 0 \\ & 0 \geq \min_{x \in X} \quad \bar{L}(x, y, \bar{\lambda}, \bar{\mu}), \quad \forall (\bar{\lambda}, \bar{\mu}) \in \Lambda \end{aligned} \tag{4.13}$$

where $L(x, y, \lambda, \mu)$ and $\bar{L}(x, y, \bar{\lambda}, \bar{\mu})$ are defined as before.
Note that the master problem (4.13) is equivalent to (4.1). It involves however, an infinite number of constraints, and hence it would need to consider a relaxation of the master by dropping a number of constraints, which will represent a lower bound on the original problem. Note also that the master problem features an outer optimization problem with respect to $y \in Y$and inner optimization problems with respect to x which are in fact parametric in y . It is this outer inner nature that makes the solution of even a relaxed master problem difficult.

Geometric Interpretation of the Mater Problem
The inner minimization problems

$$\begin{aligned} & \min_{x \in X} \ L(x, y, \lambda, \mu), \quad \forall \lambda, \forall \mu \geq 0, \\ & \min_{x \in X} \ \bar{L}(x, y, \bar{\lambda}, \bar{\mu}), \quad \forall (\bar{\lambda}, \bar{\mu}) \in \Lambda \end{aligned}$$

are functions of y and can be interpreted as support function of $v(y)$. $\xi(y)$ is support function of $v(y)$ at point y_o if and only if $\xi(y_o) = v(y_o)$ and $\xi(y_o) \leq (y_o) \forall y \neq y_o$. If the support functions are linear in y, then the master problem approximates $v(y)$ by tangent hyperplanes and it can be concluded that $v(y)$ is convex in y. Note that $v(y)$ can be convex in y even though problem (4.1) in the joint $x - y$ space.
Now define the aforementioned minimization problems in terms of the support function that is

$$\xi(y; \lambda, \mu) = \min_{x \in X} \ L(x, y, \lambda, \mu), \quad \forall \lambda, \forall \mu \geq 0, \tag{4.14}$$

$$\bar{\xi}(y; \bar{\lambda}, \bar{\mu}) = \min_{x \in X} \ \bar{L}(x, y, \bar{\lambda}, \bar{\mu}), \quad \forall (\bar{\lambda}, \bar{\mu}) \in \Lambda \tag{4.15}$$

4.1.3 Algorithmic Development

Having discussed the primal and master problem for the GBD. The primal problem being a (linear or) nonlinear programming NLP problem that can be solved. The master problem, however, consists of outer and inner optimization problems and approaches towards attaining its solution are discussed in the following.

Method to solve the Master Problem

The master problem has as constraints the two inner optimization problems, which need to be consider for all λ and all $\mu \geq 0$ in case of feasible primal and all $(\lambda, \mu) \in \Lambda$ in case of infeasible primal. This implies that the master problem has a very large number of constraints.

The most natural approach for solving the master problem is relaxation. The basic idea in the relaxation approach consists of the following:

(i) ignore all but a few of the constraints that correspond to the inner optimization problems, that is consider the inner optimization problems for specific or fixed multipliers (λ^1, μ^1) or ($\bar{\lambda}^1$, $\bar{\mu}^1$).
(ii) Solve the relaxed master problem and check whether the resulting solution satisfies all of the ignored constraints. If not, then generate and add to the relaxed master problem one or more of the violated constraints and solve the new relaxed master problem again
(iii) Continue until a relaxed master problem satisfies all of the ignored constraints, which implies that an optimal solution at the master problem has been obtained or until a termination criterion indicates that a solution of acceptable accuracy has been found.

General Algorithmic Statement of GBD

Assuming that the problem (4.1) has a finite optimal value, [10] stated the following general algorithm for GBD:
Step 1:
Let an initial point $y^1 \in Y \cap V$, that is by fixing $y = y^1$, it is a feasible primal) Solve the resulting primal problem $P(y^1)$ and optimal primal solution and optimal multipliers; vectors λ^1, μ^1. Assume that, somehow the support function $\xi(y; \lambda^1, \mu^1)$ is obtained for the obtained multipliersλ^1, μ^1. Set the counters $k = 1$ for feasible and $l = 1$ for infeasible and the current upperbound $UBD = v(y^1)$. Select the convergence tolerance $\epsilon \geq 0$.
Step 2:
Solve the relaxed master problem:

$$\begin{aligned} \min_{y \in Y, \mu_B} \quad & \mu_B \\ \text{s.t.} \quad & \mu_B \geq \xi(y; \lambda^k, \mu^k), \quad k = 1, \ldots, K \\ & 0 \geq \bar{\xi}(y; \bar{\lambda}^l, \bar{\mu}^l), \qquad l = 1, \ldots, \Lambda \end{aligned} \tag{4.16}$$

Let $(\hat{y}, \hat{\mu}_B)$ be an optimal solution of the above relaxed master problem. $\hat{\mu}_B$is a lower bound on problem on problem (4.1); that is, the current lower bound is $LBD = \hat{\mu}_B$. If $UBD - LBD \leq \epsilon$, then terminate.
Step 3:
Solve the primal problem for $y = \hat{y}$, that is the problem $P(\hat{y})$. Then we distinguish two cases; feasible and infeasible primal:
Step 3a - Feasible Primal $P(\hat{y})$ The primal has $v(y)$ finite with an optimal solution $\hat{x}$ and optimal multiplier vector $\hat{\lambda}^1, \hat{\mu}^1$. Update the upper bound UBD =min$\{UBD, v(y)\}$. If $UBD - LBD \leq \epsilon$, then terminate. Otherwise, set $k = k + 1$, $\lambda^k = \hat{\lambda}$, and $\mu^k = \hat{\mu}$. Return to step 2 and determine the support function $\xi(y; \lambda^{k+1}, \mu^{k+1})$.
Step 3b - Infeasible Primal $P(\hat{y})$
The primal does not have a feasible solution for $y = \hat{y}$. solve a feasibility problem, that is the l_1 minimization, to determine the multiplier vector $\hat{\bar{\lambda}}, \hat{\bar{\mu}}$ of the feasibility problem.

Set $l = l + 1, \bar{\lambda}^l = \bar{\bar{\lambda}}$, and $\bar{\mu}^l = \bar{\bar{\mu}}$. Return to step 2 and determine the support function $\bar{\xi}(y; \bar{\lambda}^{l+1}, \bar{\mu}^{l+1})$.
Note that a feasible initial primal is needed in step 1. However, this does not resist the GBD since it is possible to start with an infeasible primal problem. In this case, after detecting that the primal is infeasible, step3b is applied, in which a support function $\bar{\xi}$ is employed.
The relaxed master problem in step 2 at first iteration will have as a constraint one support function that corresponds to feasible primal and will be of the form.

$$\begin{aligned} \min_{y \in Y, \mu_B} \quad & \mu_B \\ \text{s.t.} \quad & \mu_B \geq \xi(y; \lambda^1, \mu^1) \end{aligned} \tag{4.17}$$

In the second iteration, if the primal is feasible and λ^2, μ^2 are its optimal multiplier vectors, Then the relaxed master problem will feature two constraints and will be of the form.

$$\begin{aligned} \min_{y \in Y, \mu_B} \quad & \mu_B \\ \text{s.t.} \quad & \mu_B \geq \xi(y; \lambda^1, \mu^1) \\ & \mu_B \geq \xi(y; \lambda^2, \mu^2) \end{aligned} \tag{4.18}$$

Note that in this case, the relaxed master problem (4.18) will have a solution that is greater or equal to the solution of (4.17). This is due to having the additional constraint. Therefore, it is evident the sequence of lower bounds that is created from the solution of the relaxed master problems is nondecreasing. A similar argument holds true in the case of having infeasible primal in the second iteration.

Since the upper bounds are produced by fixing the y variables to different 0-1 combinations, there is no reason for the upper bounds to satisfy any monotonicity property. If we consider however the updated upper bounds, that is $UBD = \min_k v(y^k)$, then the sequence for the updated upper bounds is monotonically nonincreasing since by their definition we always keep the best (least) upper bound.

The termination criterion for GBD is based on the difference between the updated upper bound and the current lower bound. If this difference is less than or equal to a prespecified tolerance ϵ then we terminate. Note though that if we introduce in the relaxed master integer cuts that exclude the previously found 0-1 combinations, then the termination criterion can be met by having found an infeasible master problem (i.e., there is no 0-1 combination that makes it feasible)

Finite Convergence of GBD

For formulation (4.1),Geoffrion (1972) proved finite convergence of the GBD algorithm stated, which is as follows.
Theorem (Finite Convergence) If C1, C2, C3 hold and Y is a discrete set , then the GBD algorithms terminates in a finite number of iterations for any given $\epsilon < 0$ and even for $\epsilon = 0$.
Now that in this case exact convergence can be obtained in a finite number of iterations.

4.2 GBD solution for UC Problem

Now recall the UC problem considered in section 3.2, that is,

$$\min_{P_i^t, U_i^t} \sum_{i=1}^{N} \sum_{t=1}^{T} F_i(P_i^t)U_i^t$$

Subject to

$$U_i^t P_i^{min} \leq P_i^t \leq U_i^t P_i^{max} \quad \text{for } i = 1 \ldots N \text{ and } t = 1 \ldots T$$

$$P_{load}^t = \sum_{i=1}^{N} P_i^t U_i^t \quad \text{for } i = 1 \ldots N \text{ and } t = 1 \ldots T$$

rewriting above problem,using separability property with respect load level,as

$$\sum_{t=1}^{T} \min_{P_i^t, U_i^t} \sum_{i=1}^{N} F_i(P_i^t)U_i^t$$

Subject to

$$U_i^t P_i^{min} \leq P_i^t \leq U_i^t P_i^{max} \quad \text{for } i = 1 \ldots N \text{ and } t = 1 \ldots T$$

$$P_{load}^t = \sum_{i=1}^{N} P_i^t U_i^t \quad \text{for } i = 1 \ldots N \text{ and } t = 1 \ldots T$$

Now, it can be solved for each time period separately. The algorithm explained is for each time period.
Clearly, considered UC problem satisfies conditions C1, C2 and C3 mentioned earlier in this chapter. Now proceeding further to primal problem of UC problem. The primal problem results by knowing the unit status of all generators either by guess or as a solution of reduced master problem. Now there are two possibilities as seen earlier,
A. Feasible primal
B. Infeasible primal
Check whether load power requirement is within bounds of all committed units. If is within bounds, then it is feasible primal otherwise it is infeasible primal. Now each case discussed separately.

4.2.1 Feasible Primal

If primal is feasible, the power generation in all committed units can be known by solving the economic dispatch problem. The Lagrangian multiplier corresponding to equality constraint is to be noted. The objective function, that is, total fuel cost of all generator units, is checked for UBD. If total cost is less than previous UBD, then the value is updated, otherwise ignored. The simplified primal Lagrangian function is given as:

$$L(P_i^t, U_i^t, \lambda^t) = \sum_{i=1}^{N}(F_i(P_i^t))U_i^t + \lambda^t(P_{load}^t - \sum_{i=1}^{N} P_i^t U_i^t) \tag{4.19}$$

4.2.2 Infeasible Primal

If primal is infeasible, feasiblity problem (FP) is formulated as:

$$\begin{aligned}
\min_{P} \quad & \sum_{i=1}^{p}(\alpha_i + \bar{\alpha}_i) \\
\text{s.t.} \quad & P_{load}^t - \sum_{i=1}^{p} P_i^t = 0 \\
& P_i^{min} - P_i^t \leq \alpha_i, \;\; i = 1, \dots, p \\
& P_i^t - P_i^{max} \leq \bar{\alpha}_i, \;\; i = 1, \dots, p \\
& \alpha_i, \bar{\alpha}_i \geq 0 \;\; i = 1, \dots, p
\end{aligned} \tag{4.20}$$

where p denotes number of units committed in that period. Now, Lagrangian function is formed using Lagrangian multipliers from FP.

$$\bar{L}(P_i^t, U_i^t, \lambda^l, \mu^l) = \lambda^l(P_{load}^t - \sum_{i=1}^{N} P_i^t U_i^t) + \sum_{i=1}^{N} \mu_i^m (P_i^{min} - P_i^t) U_i^t + \sum_{i=1}^{N} \mu_i^M (P_i^t - P_i^{max}) U_i^t \tag{4.21}$$

4.2.3 Support functions

The support functions are introduced in GBD theory, but their calculation is not mentioned because their calculation is problem specific. This section procedure for calculating support functions for both feasible and infeasible cases is discussed.

Feasible Lagrangian Support function

Recalling the support function corresponding to feasible Lagrangian function, which is

$$\xi(y; \lambda^k, \mu^k) = \min_{x \in X} \;\; L(x, y, \lambda^k, \mu^k) \tag{4.22}$$

In UC problem, using simplified Lagrangian function it is:

$$\xi(U_i^t; \lambda^t, \mu^t) = \min_{P_i^t \in X} \;\; \sum_{i=1}^{N} (F_i(P_i^t)) U_i^t + \lambda^t (P_{load}^t - \sum_{i=1}^{N} P_i^t U_i^t) \tag{4.23}$$

Rearranging the terms, leads to

$$\xi(U_i^t; \lambda^t, \mu^t) = \min_{P_i^t \in X} \;\; \sum_{i=1}^{N} \{(F_i(P_i^t)) - \lambda^t P_i^t\} U_i^t + \lambda^t P_{load}^t \tag{4.24}$$

The last term is constant in the expression, and taking summation out finally gives

$$\xi(U_i^t; \lambda^t, \mu^t) = \lambda^t P_{load}^t + \sum_{\substack{i=1 \\ P_i^t \in X}}^{N} \min \;\; \{(F_i(P_i^t)) - \lambda^t P_i^t\} U_i^t \tag{4.25}$$

Now the function to be minimized is :

$$\min\ [F_i(P_i^t) - \lambda^t P_i^t] \tag{4.26}$$

Similiar function is minimized in Lagrangian Relaxation algorithm, the minimum of this function is found by taking the first derivative,

$$\frac{d}{dP_i^t}[F_i(P_i^t) - \lambda^t P_i^t] = \frac{d}{dP_i^t}F_i(P_i^t) - \lambda^t = 0 \tag{4.27}$$

The solution to this equation is

$$\frac{d}{dP_i^t}F_i(P_i^{opt}) = \lambda^t \tag{4.28}$$

There are three cases to be concerned with, depending on the relation of P_i^{opt} and the unit limits:
1. If $P_i^{opt} \leq P_i^{min}$, then:

$$\min\ [F_i(P_i^t) - \lambda^t P_i^t] = F_i(P_i^{min}) - \lambda^t P_i^{min} \tag{4.29}$$

2. If $P_i^{min} \leq P_i^{opt} \leq P_i^{max}$, then:

$$\min\ [F_i(P_i^t) - \lambda^t P_i^t] = F_i(P_i^{opt}) - \lambda^t P_i^{opt} \tag{4.30}$$

3. If $P_i^{opt} \geq P_i^{max}$, then:

$$\min\ [F_i(P_i^t) - \lambda^t P_i^t] = F_i(P_i^{max}) - \lambda^t P_i^{max} \tag{4.31}$$

Finally, the support function is obtained as function of U_i^t.

Infeasible Lagrangian Support function

Recalling the support function corresponding to feasible Lagrangian function, which is

$$\bar{\xi}(y; \bar{\lambda}^l, \bar{\mu}^l) = \min_{x \in X}\ \bar{L}(x, y, \bar{\lambda}^l, \bar{\mu}^l) \tag{4.32}$$

In UC problem, using Lagrangian function it is:

$$\bar{\xi}(U_i^t; \bar{\lambda}^l, \bar{\mu}^l) = \min_{P_i^t \in X}\ \lambda^l(P_{load}^t - \sum_{i=1}^{N} P_i^t U_i^t) + \sum_{i=1}^{N} \mu_i^m (P_i^{min} - P_i^t)U_i^t + \sum_{i=1}^{N} \mu_i^M (P_i^t - P_i^{max})U_i^t \tag{4.33}$$

Rearranging the terms, leads to

$$\bar{\xi}(U_i^t; \bar{\lambda}^l, \bar{\mu}^l) = \min_{P_i^t \in X}\ \lambda^l P_{load}^t + \sum_{i=1}^{N} \{(-\lambda^l - \mu_i^m + \mu_i^M)P_i^t + \mu_i^m P_i^{min} - \mu_i^M P_i^{max}\} U_i^t \tag{4.34}$$

The first term is constant in the expression, and taking summation out finally gives

$$\bar{\xi}(U_i^t;\bar{\lambda}^l,\bar{\mu}^l) = \lambda^l P_{load}^t + \sum_{i=1}^{N} \min_{P_i^t \in X} \ \{(-\lambda^l - \mu_i^m + \mu_i^M)P_i^t + \mu_i^m P_i^{min} - \mu_i^M P_i^{max}\}U_i^t \quad (4.35)$$

In the above equation, $\mu_i^m P_i^{min} - \mu_i^M P_i^{max}$ is constant with respect to P_i^t. So, minimization of $(-\lambda^l - \mu_i^m + \mu_i^M)P_i^t$ with respect to P_i^t is to be found. There are two cases to be concerned with, depending on the sign of $(-\lambda^l - \mu_i^m + \mu_i^M)$:
a) If $(-\lambda^l - \mu_i^m + \mu_i^M)$ is positive, then $P_i^t = P_i^{min}$
b) If $(-\lambda^l - \mu_i^m + \mu_i^M)$ is negative , then $P_i^t = P_i^{max}$

By substititing these value the values,the support function as function of U_i^t is obtained.

4.2.4 Master Problem

Having the support functions, one can easily formulate the master problem.

$$\begin{aligned} &\min_{U_i^t,\mu_B} \quad \mu_B \\ &\text{s.t.} \quad \mu_B \geq \xi(U_i^t;\lambda^t,\mu^t), \quad t = 1,\dots,T \\ &\quad\ 0 \geq \bar{\xi}(U_i^t;\bar{\lambda}^l,\bar{\mu}^l), \qquad l = 1,\dots,\Lambda \end{aligned} \quad (4.36)$$

(4.36) is a Mixed Integer Linear Program (MILP) in this case. Integer variable is binary, this make problem even simplier. Here $LBD = \mu_B$ is obtained. Following the algorithm specified in section 4.1.3 the solution can be obtained for UC problem.

4.3 Results

The application of GBD algorithm to UC problem is implemented on 3 generating units with 4 block periods.
The cost characteristics of the three generators along with the generation limits are given in Table 3.1. The four block periods load information is given in Table 3.2.

The results are obtained starting from an initial condition where all the generating units' status are set to 1 for each time period. This initial condition lead to feasible primal for second, third and fourth load levels at first iteration. It leads to infeasible primal for first load level.

The results are shown in Table 4.1, in which each row consists results for a particular load levels such as units committed, power generation in committed units and cost of production for total time period of that load level.
When load is peak with 1100 MW all the units' status are 1 which is same as initial condition. When load is low with 170 MW only generator 3 is committed. For other two load levels, generator 2 and generator 3 are committed.

Table 4.1: Results obtained for 3 units with 4 block periods using GBD Technique

Load level	u_1	u_2	u_3	P_1 (MW)	P_2 (MW)	P_3 (MW)	Cost (Rs)
1	0	0	1	0	0	170	7587
2	0	1	1	0	320	200	27696
3	1	1	1	500	400	200	68400
4	0	1	1	0	130	200	17293.5
Total Cost (Rs)							120976.5

4.4 Comparative study

Both the algorithms gave same set of results for 3 generating units with 4 block periods.But Lagrangian Relaxation gave infeasible result when implemented on 10 generating units with 24 block periods. This is due to the relaxation of demand supply balance constraint. Even in GBD, relaxation of constraints is used but here relaxed constraint is not a sysstem constraint. In GBD, the algorithm always converges to feasible solution if it exists.

Lagrangian Relaxation algorithm can be extended to solve UC problem with constant startup costs and shut down cost in objective function, with additional constraints like ramp rates, minimum up time, minimum down time. In this case, a sophisticated two state dynamic problem has to be soved to get the result. The main advantage of GBD algorithm is that it can be extended to all the above additional constraints and many more given satisfying conditions C1,C2 and C3. In this case, one should put a great effort to find support functions.

It may appear that GBD is better technique to solve for considered UC problem, but GBD require a lot more computations to solve the problem than Lagrangian Relaxation technique.

4.5 Summary

This chapter discussed the GBD theory and used that for solve UC problem. The results are presented when GBD is implemented on 3 generating units with 4 block periods. At the end, a comparative study is done between Lagrangian Relaxation solution and GBD solution for UC problem.

Chapter 5

Conclusions

A study is carried out on Lagrangian Relaxation solution and Generalized Benders Decomposition Solution for UC problem. A simplest UC problem model, with objective function being fuel cost alone, with demand-supply balance and generator limit constraints is considered. The Lagrangian Relaxation and GBD techniues are implemented on UC problem.

The simulation results are obtained using Lagrangian Relaxation technique on 3 generating units with 4 block periods and 10 generating units with 24 block periods. The simulation results are obtained using GBD technique on 3 generating units with 4 block periods.The results show that for small system Lagrangian Relaxation technique gives same solution as that from GBD. But for large system Lagrangian Relaxation may converge even giving infeasible solution.

A comparative study between Lagrangian Relaxation solution and GBD solution shows that if one wants a optimal solution even on expense of excessive computations and time then GBD is good option. Otherwise if someone wants easy way to approach towards solution on expense of sub-optimality then Lagrangian Relaxation technique is better option.

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