Capacity of Energy Harvesting Communication Systems

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THESIS CERTIFICATE

This is to certify that the thesis titled **Capacity of Energy Harvesting Communication Systems**, submitted by **Jainam Doshi**, to the Indian Institute of Technology, Madras, for the award of the degree of **Dual Degree : B.Tech + M.Tech**, is a bona fide record of the research work done by him under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

Limitations on the energy that can be stored in compact batteries have severely constrained the capabilities of wireless networks that operate using battery powered nodes. Energy harvesting has the potential to solve this challenging problem. We first consider an energy harvesting channel with fading, where only the transmitter harvests energy from natural sources. We bound the optimal long term throughput by a constant for a class of energy arrival distributions. The proposed method also gives a constant approximation to the capacity of energy harvesting channel with fading.

Next, we consider the a more general case where both the transmitter and the receiver employ energy harvesting to power themselves. In this case we show that finding an approximation to the optimal long term throughput is far more difficult in the general case. We then identify several specific cases where and bound the optimal long term throughput. We also propose policies which are proved to give optimal/near optimal long term throughput.

Towards the end, we look at a Simultaneous Wireless Information and Power Transfer system where two multi antenna stations perform separate Power Transfer (PT) and Information Transfer (IT) to a multi-antenna mobile that dynamically assigns each antenna for either PT or IT. The antenna partitioning results in a tradeoff between the MIMO IT channel capacity and the PT efficiency. The optimal partitioning for maximizing the IT rate under a PT constraint is a NP-hard integer program. We prove this problem to be one that optimizes a sub-modular function over a matroid constraint. This structure allows the application of two well-known greedy algorithms that yield solutions with guaranteed performances.

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CHAPTER 1

Introduction

Limitations on the energy that can be stored in compact batteries have severely constrained the life-time of wireless networks that operate using battery powered nodes, the problem being that once these nodes are out of energy, they need to be replaced. While it is difficult in some cases to replace nodes time and again, there are several applications like space communication, etc. where replacing nodes is not feasible. These constraints have motivated several approaches to increase the life time and reliability of such networks.

The ability to harvest energy from natural sources like solar, wind, vibration and thermoelectric effects has the potential to solve this problem. Unlike the conventional battery powered nodes that die once their battery drains out, an energy harvesting node can harvest energy from the environment and become available for transmission later. Thus, energy harvesting has the potential to provide a maintenance free operation. Such system are also environment friendly, and hence receiving an increasing attention from academia as well as industry.

Finding optimal power/energy transmission policies to maximize the long-term throughput in an energy harvesting (EH) communication system is a challenging problem and has remained open in full generality. The finite horizon problem to maximize average data throughput of a single transmitter, starting with a fixed amount of energy, sending data over a fading channel has been studied in Fu *et al.* (2006). These results can be extended to the case of an energy harvesting transmitter with some modifications. While one can theoretically obtain the optimal power allocation strategy for this case, it is numerically very expensive. Under the assumption of discrete power consumption strategy, there are results obtained in Vaze and Jagannathan (2014). For the case of infinite horizon, where the objective is to maximize long term throughput, Goldsmith and Varaiya (1997) is a classic paper giving an energy allocation strategy for a single transmitter operating over a fading channel with an average power constraint. These results can be extended to the case of energy harvesting transmitter when the battery capacity is assumed to be infinite, as shown in Khairnar and Mehta (2011). However, for the more practical case of a finite battery capacity to store the harvested energy, these policies need to be revamped. For the finite battery case, structural results are known for the optimal solution in Sinha and Chaporkar (2012). However, explicit solutions are only known for a sub-class of problems, for example, binary transmission power Michelusi *et al.* (2012), discrete transmission power Vaze and Jagannathan (2014), etc. Recently, some progress has been reported in approximating the per slot throughput (or long term throughput) by a universal constant in Dong *et al.* (2014), for an AWGN channel. In this work, we take up this problem of energy harvesting communication system and extend the existing work to more general cases.

In chapter 1, we consider the case of discrete rate adaptation, as compared to the conventional continuous rate adaptation where th transmitter can only choose from a predetermined set of modulation and coding schemes, each with it's own fixed rate. We assume that the Bit Error Rate (BER) has to be lesser than a given threshold for a successful transmission. For this case, we observe that the rate function is a step function instead of the normal concave function, which begets water-filling strategies. We find the optimal energy allocation policy for this case, where the objective is to maximize average throughput over a finite horizon. We observe that unlike the case of continuous rate adaptation, this case is more difficult and characterizing the optimal policy is much harder. Also, we observe that the assumption of discrete energy consumption, as assumed in Vaze and Jagannathan (2014) will be highly sub-optimal for this case.

In chapter 2, we approximate the per-slot throughput of the EH system with fading by a universal constant for a class of energy arrival distributions. The fading channel problem is more challenging than the AWGN case, since the energy/power transmitted per-slot depends on the realization of the channel unlike the AWGN problem. Thus, finding an upper bound on the long term throughput is hard. We take recourse in Cauchy-Schwarz inequality for this purpose, and then surprisingly using a channel independent power transmission policy we show that the upper and lower bound on the per-slot throughput differ at most by a constant. Using the techniques of Dong *et al.* (2014), we also show that our universal bound also provides an approximation of the Shannon capacity of the energy harvesting channel with fading upto a constant.

In chapter 3 we consider a more relevant or practical scenario, when energy harvesting is employed at both the transmitter and the receiver. The EH setting at the receiver is simpler than at the transmitter, since the only decision the receiver has to make is whether to stay *on* or not. In the *on* state, the receiver consumes a fixed amount of energy, while in the *off* state, no energy is consumed. We refer to this model as Binary Energy Consumption model. We begin with the case of average power constraint at the receiver, and propose an optimal policy for that case.

Next, we extend that result to the case of energy harvesting receiver, but with an infinite battery capacity to store the harvested energy. For the more practical case of finite battery capacity, there is an inherent lack of information about the receiver energy levels at the transmitter and vice-versa. One can show that the optimal policy at both the transmitter and the receiver is of threshold type, but the thresholds depend on both the energy states in a non-trivial way. Because of the common fading channel state that is revealed to both the transmitter and receiver for each slot, both the transmitter and the receiver have some partial statistical information about others' energy state which is important for finding the optimal policy.

We show that, in general, it is difficult to bound the gap between the upper and lower bound when EH is employed at both the transmitter and the receiver for the case of when the transmitter and receiver do not have access to each others' energy levels. Subsequently, we identify several interesting cases and prove results for them. In specific, we identify a special case of unit battery capacity at both the transmitter and the receiver, and where the transmitter operates with binary transmission power, for which we propose a strategy that achieves at least half of the upper bound on the per-slot throughput, giving a ratio bound.

In chapter 4, we take a recourse by considering a centralized scheduler i.e. both the transmitter and the receiver have complete knowledge about each others' energy levels. Even for this problem, the maximum achievable throughput, and the capacity, is difficult to find in general. Therefore, we restrict ourselves to discrete energy consumption at the transmitter. Under this assumption, we are able to explicitly find optimal energy allocation schemes. We also prove some structural properties of the optimal policy in terms of the channel fade state and the energy arrival parameters which may be useful in designing simple heuristics for this case.

Having looked at the case of a single transmitter-single receiver system, we next move on to a MIMO system in chapter 5. The arguably most desirable new feature for mobile devices is wireless power transfer, which eliminates the need of recharging using cables and avoids interruptions of mobile services due to dead batteries. With rapid advancements in microwave technologies, microwave power transfer has emerged to be a promising solution for wirelessly powering mobiles due to its long transfer ranges (up to hundreds of meters) and support of mobility Huang and Zhou (2014). In contrast, non-radiative technologies for wireless power transfer e.g., inductive coupling and resonant coupling, suffer from extremely short ranges (less than a meter). Using microwaves as carriers, wireless power transfer and information transfer can be seamlessly integrated, which has resulted in the emergence of an active research area called simultaneous wireless information and power transfer (SWIPT) Zhang and Ho (2013); Huang and Zhou (2014). The research on SWIPT, however, requires thorough revamping of classical theories for wireless communications and networking to achieve not only high information transfer rates but also high power transfer efficiencies.

Power Transfer (PT) and Information Transfer (IT) concern two different aspects of data bearing microwaves, namely their information content and absolute power, respectively. As a result, PT can tolerate much less propagation loss and support much shorter transmission distances than IT. Furthermore, depending on the channel and energy states, a mobile may choose to operate in either the IT, PT, or SWIPT modes. Consideration of such factors in realizing SWIPT calls for the design of new algorithms/protocols for MIMO transmissions Zhang and Ho (2013), multiple access Ju and Zhang (2014), resource allocation Huang and Larsson (2013); Ng *et al.* (2013), mobile transceivers Liu *et al.* (2013) and network architectures Huang and Lau (2014).

A simple design of a SWIPT enabled mobile receiver is to combine a conventional information receiver and an RF energy harvester. The form factor of this design can be reduced by sharing antennas between the receiver and harvester where the output of each antenna is split for data processing and energy harvesting Zhou *et al.* (2013). However, the addition of a power splitter with an adjustable splitting ratio for each antenna increases the receiver complexity. A simpler SWIPT-receiver design that allows antenna sharing but requires no splitting, is to partition the set of antennas into two sets, one dedicated for IT and other for PT. This design builds on the classic antenna selection technique for MIMO communications Heath *et al.* (2001) and requires a small number of RF chains, leading to a high-efficient mobile design. The problem of optimal antenna assignment/partitioning for the special case of SWIPT with a single-input-multiple-output IT channel has been explicitly solved in Liu *et al.* (2013) for a simplified objective function. However, the problem for the general case with a MIMO IT channel is much more challenging to solve that depends on the eigenmodes of the channel matrix. To be specific, the problem is an NP-hard integer program.

Our contribution here is to connect the general SWIPT antenna-partitioning problem to the rich field of efficient sub-optimal integer-programming algorithms with guaranteed performance. This important connection is established by analyzing the structure of the antenna-partitioning problem. Specifically, the problem is shown to be equivalent to maximizing a sub-modular function with a matroid constraint. For a sub-modular function, the incremental gain of adding a new element diminishes with increasing set size. The proven structure allows two well-known greedy algorithms to be applied for solving the antenna-partitioning problem. Moreover, the resultant solutions are shown to be equal to the optimal ones with the scaling factors of $(1-1/e)$ and $1/2$. Simulation results reveal that the performance of the said antenna-partitioning algorithms substantially outperform the derived worst-case bounds.

CHAPTER 2

Discrete Rate Adaptation in Finite Horizon

In this chapter, we consider the finite horizon problem for a single transmitter[1] operating over a fading channel. We also consider a slotted time system, where each time slot spans over T seconds. The time slots are indexed by k. At the start of each time slot, a channel realization, denoted by h_k ($h_k \geq 0$), independent of the previous channel realizations, is revealed to the transmitter. The channel state is assumed to be constant throughout the time slot. In our subsequent discussions, we shall deal with the case where h_k has identical distribution denoted by φ for every time slot k. However, the results obtained in the above case can be easily generalized to cases where the distribution of h_k is non-identical (but independent). We shall now define certain notations, which we will be using henceforth. Let the energy available at the source and the energy spent by the source, at time slot k, be denoted by U_k and F_k respectively. Therefore, we can see that the available energy at the source evolves according to the following relation,

$$U_{k+1} = U_k - F_k \tag{2.1}$$

Let the rate achieved at time slot k be denoted by R_k.

Unlike related literature, we focus on the practically relevant case of discrete rate adaptation. At each time slot, the source can choose only from a pre-determined set of modulation and coding schemes, each with it's own fixed rate. For simplicity, we shall consider the situation where the transmission rate is determined solely by the constellation used. Let $\mathscr{M} = \{m_1, m_2, \ldots, m_M\}$ be the set of constellation sizes available to the source for transmission with $m_1 = 1$ corresponding to no transmission and

[1]We use the terms transmitter and source interchangeable.

$m_1 < m_2 <, \ldots, < m_M < \infty$. The corresponding transmission rates are therefore given by $\log_2(m_1) = 0, \log_2(m_2), \ldots, \log_2(m_M)$. The choice of $\mathscr{M}$ depends on the hardware complexity of the system. We impose no further constraints on $\mathscr{M}$.

It has been shown in Goldsmith (2005) that when a node transmits using a constellation of size m, with power P and channel gain h, the Bit Error Rate (BER) is given by the equation,

$$BER = c_1 \exp\left(\frac{-c_2 h P}{N_0 \Omega (m^{c_3} - c_4)}\right), \tag{2.2}$$

where c_1, c_2, c_3 and c_4 are modulation specific constants. For example, for an M-ary Quadrature Amplitude Modulation (M-QAM), $c_1 = 2, c_2 = 1.5, c_3 = 1$ and $c_4 = 1$. In our case, duration of each time slot is T secs, therefore substituting $P = \frac{F_k}{T}$ in (2.2), the BER for time slot k is given by

$$BER_k = c_1 \exp\left(\frac{c_2 h F_k}{T N_0 \Omega (m^{c_3} - c_4)}\right) \tag{2.3}$$

Let the minimum BER required for successful transmission be denoted by P_b. Equating the above BER formula to P_b, we see that the source transmitting with a constellation of size m_j needs to spend a minimum energy $F_{\min}(h, m_j)$ given by

$$F_{\min}(h, m_j) = \frac{d_j}{h_k} \tag{2.4}$$

where $d_1 = 0$ and

$$d_j = \frac{\log\left(\frac{c_1}{P_b}\right)}{c_2} T N_0 \Omega (m_j^{c_3} - c_4) \text{ for } 2 \leq j \leq M \tag{2.5}$$

Note that $d_1 = 0$ implies that when the channel is in a deep fade, the node sets its rate and energy to zero as it cannot transmit reliably.

One can intuitively see that for a given constellation size m_j and channel state h, an optimal strategy would use only $F_{\min}(h, m_j)$ energy, as using energy greater than $F_{\min}(h, m_j)$

will not increase the throughput in the current time slot but instead reduce the energy available for future transmission consequently giving a lesser throughput in future. We will prove this formally in the next section.

We consider the finite horizon problem, where the expected throughput over n slots is $P = \mathbb{E}\left[\sum_{k=1}^{n} R_k\right]$. We refer to this as expected sum throughput or simply, payoff.[2] We are interested in finding the optimal energy utilization F_i that maximizes the expected total throughput summed over n time slots with finite modulation rates available to the transmitter and a per slot BER constraint. We pose this problem in the language of optimization as follows -

$$\begin{aligned}
&\text{Maximize } \mathbb{E}\left[\sum_{k=1}^{n} R_k\right] \\
&\text{subject to} \\
&\sum_{k=1}^{n} F_k \leq E_0; \\
&F_k \geq 0 \;\forall\; k; \\
&BER_k \leq P_b \;\forall\; k
\end{aligned} \tag{2.6}$$

where E_0 is the energy available to the transmitter at $t = 0$ (initial energy).

2.1 Optimal Policy for non Energy Harvesting Node

In this section we derive the optimal energy utilization policy for the case of a non energy harvesting node (non EH node) i.e. The system starts off with a certain amount of energy, say E_0, and does not obtain energy during the entire transmission period of n time slots. At any time slot k, depending on the channel fade state and current battery energy level, the source has to choose one of the available modulation rates and a corresponding transmission energy to maximize the expected sum throughput

[2]We use these terms interchangeably.

constrained to the fact that BER for any slot cannot exceed P_b.

It is important to note here that the payoff function in this case for a particular channel fade state h will be a discontinuous curve with finite jumps whenever an energy enough to transmit at a higher constellation size ensuring BER constraint is spent. Thus instead of the concave payoff function assumed in most of the related work that begets the directional water-filling solution, we have a non concave payoff functions with finite discontinuities. However, the payoff function is non-decreasing in the energy spent. Also, discretizing the battery energy level and restricting to discrete energy consumption will be highly sub-optimal due to discrete rate adaptation and per slot BER constraint. To see this, consider a simple case of $n = 2$ time slots. At time slot 1, let the channel gain be h_1. Given h_1, say the source chooses a constellation of size m_j. Thus, it needs to spend $F_{\min}(h_1, m_j)$ of energy to ensure BER constraint. However, due to discrete energy consumption,if the transmitter spends an integer amount of energy less than $F_{\min}(h_1, m_j)$ the BER constraint will be violated. On the other hand, if it spends an integral amount of energy greater than $F_{\min}(h_1, m_j)$, it spends more than required energy without any increase in throughput. This is unlike the conventional payoff function where an increase in energy spent implies an increase in throughput.

However, we see that the dynamic programming algorithm can be used to find an optimal policy for this problem. As usual in dynamic programming we introduce the value function, $J_k(x, h)$, which provides the measure of the desirability of the transmitter having an energy of a units given that the current channel fade state is h at time k. Also, let $G(x, h)$ denote the set of constellation sizes which can be used to transmit with energy lesser than or equal to x, satisfying the BER constraint, given that the channel fade state is h. Mathematically,

$$G(x, h) = \{m \in \mathscr{M} \mid F_{\min}(h, m) \leq x\} \tag{2.7}$$

Note that the number of elements in $G(x, h)$ will be finite for any choice of x and h as it is upper bounded by $\mathscr{M}$. The functions $J_k(a,h)$ for each stage k are related by the dynamic programming recursion as follows:

$$\text{Base case : } J_n(x,h) = \max_{m \in G(x,h)} \log_2 m, \tag{2.8}$$

$$\text{For } k < n : \; J_k(x,h) =$$
$$\max_{m \in G(x,h)} \left[\log_2 m + \overline{J}_{k+1}\left(x - F_{\min}\left(h,m\right)\right)\right] \tag{2.9}$$

where $\overline{J}_k(x) = \mathbb{E}\left[J_k\left(x,h\right)\right]$, the expectation being taken over the channel fade state distribution φ. The first term in the right hand side of the equation (2.9) represents the rate obtained in the present time slot by consuming $F_{\min}\left(h,m\right)$ units of energy. The available energy in the next stage is then $x - F_{\min}\left(h,m\right)$ and the second term represents the maximum payoff that can be obtained in future given $x - F_{\min}\left(h,m\right)$ units of energy. Now, the task left is to obtain $\overline{J}_k(x)$ using the available information.

2.1.1 Average throughput - $\overline{J}_k(x)$

The usual method to obtain the functions $\overline{J}_k(x)$ $\forall$ k, is to obtain the pay-off vs. energy function for a particular channel fade state and then take expectation over channel fade state distribution, but this computation is two-fold. In this subsection we shall give an intuitive method to compute $\overline{J}_k(x)$ for $k = n$, which accomplishes the same task in a much simpler way. This method can be easily adapted to compute $\overline{J}_k(x)$ for $k < n$ and therefore it is left to the reader.

The method is as follows. Divide the real line for channel fade state in the following manner. In the last slot, the source will transmit with the maximum constellation size possible that satisfies the BER constraint as saving energy for future is of no use.

Define $H_j(x)$ as

$$H_j(x) = \left\{ h \in \mathbb{R} \mid \max_{m \in G(x,h)} = m_j \right\}. \tag{2.10}$$

For every energy level x, we can see that $H_j(x)$ will be a semi-closed semi-open interval on the real line. Now integrating φ (channel distribution) over this interval, we obtain $\mathbb{P}(H_j(x))$. Therefore, the function $\overline{J}_n(x)$ is given by

$$\overline{J}_n(x) = \sum_{j=1}^{M} \mathbb{P}(H_j(x)) \log_2 m_j. \tag{2.11}$$

Note that we can come up with a closed form function $\overline{J}_n(x)$ by following the above stated method. Subsequently, $J_{n-1}(x,h)$ can be obtained by using the recursion in (2.9) and we can obtain $\overline{J}_{n-1}(x)$ by integrating $J_{n-1}(x,h)$ over the channel fade state distribution φ. Following this recursively, one can obtain $\overline{J}_k(x)$ $\forall$ $k < n$. We refer to these functions as *optimal reward functions*.

An important observation in the above method is that the optimal reward functions can be pre-computed since they solely depend on the probability distribution of the channel fade state φ and the number of time slots available for transmission. Also, the maximization in equation (2.9) is over a finite number of choices for m and hence easy to compute.

We now state, and give a proof outline, of an intuitive result which we will use to derive the optimal energy utilization policy.

Theorem 2.1.1. The optimal reward functions $\overline{J}_k(x)$ are non decreasing for every k. Essentially, $\forall k$, we have $\overline{J}_k(a^+) \geq \overline{J}_k(a^-)$ whenever $a^+ \geq a^-$.

Proof: Consider energy states a^+ and a^- where $a^+ \geq a^-$. Let $T = \overline{J}_k(a^-)$ be the optimal expected sum throughput if the source has a^- units of energy at time slot k. Also let $F_i^*(a^-), k \leq i \leq n$ be the optimal energy utilization policy for the same.

Let $F_i(a^+)$ be defined as follows:

$$F_i(a^+) = F_i^*(a^-) + (a^+ - a^-) \text{ for } i = k \tag{2.12}$$

$$F_i(a^+) = F_i^*(a^-) \text{ for } k+1 \leq i \leq n \tag{2.13}$$

The energy utilization policy $F_i(a^+)$ uses an additional $(a^+ - a^-)$ units of energy in the k^{th} time slot as compared to $F_i^*(a^-)$ and exactly follows $F_i^*(a^-)$ in the subsequent time slots. As the source starts with an extra $(a^+ - a^-)$ energy and $F_i^*(a^-)$ is feasible, we see that $F_i(a^+)$ is also feasible.

Also, the payoff functions are non-decreasing in energy spent for every channel fade state h. As a result, we see that $F_i(k,a^+)$ achieves at least as much throughput as $F_i^*(k,a^-)$ in the k^{th} and exactly same throughput for subsequent time slots. Thus, we have a feasible energy utilization policy $F_i(k,a^+)$ which gives expected sum throughput of at least T. Hence the optimal sum throughput if the source starts with a^+ units of energy is lower bounded by T. So,

$$\overline{J}_k(a^+) \geq T \tag{2.14}$$

$$\overline{J}_k(a^+) \geq \overline{J}_k(a^-) \tag{2.15}$$

which is what we wanted to prove.

□

In fact, one can show by similar arguments that the optimal reward functions $\overline{J}_k(x)$ are strictly increasing in the available energy.

Theorem 2.1.2. Let the source have an energy of a units at time slot k and h_k be the channel state realization revealed to the transmitter before transmission. The optimal transmission policy transmits with a constellation of size m^* and an energy of $F_{\min}(h_k, m^*)$ where m^* is given by

$$m^* = \underset{m \in G(a,h_k)}{\arg\max} \left[\log_2 m + \overline{J}_{k+1}(a - F_{\min}(h_k, m))\right] \tag{2.16}$$

Proof: We start by observing that in one of the the optimal policies, the source, transmitting with a constellation of size m_j spends an energy of $F_{\min}(h_k, m_j)$. We prove this in the following way. Let the source have a units of energy in the k^{th} time slot and the optimal policy transmits with a constellation of size m_j using an energy of $F(h_k, m_j) \neq F_{\min}(h_k, m_j)$ units. To ensure that the BER does not exceed P_b, we have

$$F(h_k, m_j) > F_{\min}(h_k, m_j) \tag{2.17}$$

The available energy with the source at $(k+1)^{th}$ time slot is then $a - F(h_k, m_j)$. Thus the expected sum throughput for this policy is

$$T_1 = \log_2 m_j + \bar{J}_{k+1}(a - F(h_k, m_j)) \tag{2.18}$$

Next, consider a policy that transmits with a constellation of size m_j using $F_{\min}(h_k, m_j)$ units of energy. By definition of $F_{\min}(h_k, m_j)$, the BER will not exceed P_b. In this case, the available energy with the source at $(k+1)^{th}$ time slot is $a - F_{\min}(h_k, m_j)$. The expected sum payoff for this policy is thus given by

$$T_2 = \log_2 m_j + \bar{J}_{k+1}(a - F_{\min}(h_k, m_j) \tag{2.19}$$

From (2.17), we have $a - F_{\min}(h_k, m_j) > a - F(h_k, m_j)$. As $\bar{J}_{k+1}(x)$ is a non decreasing function,

$$\bar{J}_{k+1}(a - F_{\min}(h_k, m_j)) \geq \bar{J}_{k+1}(a - F(h_k, m_j)) \tag{2.20}$$

From (2.18),(2.19) and (2.20),

$$T_2 \geq T_1 \tag{2.21}$$

Hence, we have a feasible policy giving at least as much payoff as the optimal policy proving that it is optimal.

We now prove the theorem using the above observation. Let $c_{k-1}, k = 1, 2, ..., n$ be the carried over energy from time slot $k-1$ to time slot k. Hence we have $U_k = c_{k-1}$ with

$c_0 = E_0$ and $c_k = U_k - F_k$.

Then the optimization problem can be posed in the dynamic programming format, by writing the payoff at time slot $k = 1, 2, ..., n$ as

$$J_k(c_{k-1}, h_k) = \max_{F_{\min}(h_k,m): m \in G(c_{k-1},h_k)} \left[\log_2 m + \overline{J}_{k+1}(c_k) \right] \tag{2.22}$$

where $\overline{J}_{k+1}(x) = \mathbb{E}\left[J_{k+1}(x, h_{k+1})\right]$ and $c_k = c_{k-1} - F_{\min}(h_k, m)$.

In order to obtain the optimal solution F_k, when the source has a units of energy (i.e. $U_k = c_{k-1} = a$), we compare the arguments inside the maximization of (2.22). As there are only a finite number of elements in $G(a, h_k)$, we can compute the function value for all the choices of m and find maximum by brute force. In particular, we get

$$F_k = F_{\min}(h_k, m^*) \tag{2.23}$$

where

$$m^* = \arg\max_{m \in G(a,h_k)} \left[\log_2 m + \overline{J}_{k+1}(a - F_{\min}(h_k, m)) \right] \tag{2.24}$$

which completes the proof.

□

2.2 Optimal Policy for Energy Harvesting node

In this section, we consider a single node that harvests energy from the environment. At each time slot, the node harvests an energy E_k, which follows a distribution $f_k(\cdot)$. Without loss of generality, let the source start with zero energy. We assume that the energy harvested at time slot k is available for utilization from the k^{th} time slot itself. We consider a finite battery capacity of B, as is the case for most of the practical scenarios. For ease of notation, we have henceforth assumed $f_k(.)$ to be i.i.d across the time slots and the distribution is denoted by $f(\cdot)$. However, our solution can be easily extended to the case where $f_k(\cdot)$s aren't identical for different time slots (but independent).

As one may guess, the optimal energy utilization policy for this case is motivated from the solution derived in the previous section for a non-EH node. We define the functions $J_k(x,h)$ recursively as follows -

$$\bar{J}_n(x,h) = \int \max_{m \in G(\min(x+p,B),h)} \left[\log_2 m\right] \; f(p)dp$$

$$\text{and for } k < n \tag{2.25}$$

$$J_k(x,h) = \int \left(\max_{m \in G(\min(x+p,B),h)} [\log_2 m + \bar{J}_{k+1}(\min(x+p,B) - F_{\min}(h,m))] \; f(p) \right) dp \tag{2.26}$$

where $\bar{J}_k(a) = \mathbb{E}\left[J_k(a,h)\right]$, the expectation being taken over the channel fade state distribution φ as before. It should be noted that the functions $\bar{J}_k(x)$ depend only on the distributions of the channel fade state, the distribution of energy arrivals and the number of time slots available for transmission and therefore can be pre-computed. After obtaining these functions, the optimal energy utilization policy is same as the one in previous section.

If the source has a units of energy at time slot k after accounting the energy harvested in the present time slot, follow the same energy utilization policy as in theorem 2.

CHAPTER 3

Continuous Rate Adaptation under Fading for an Energy Harvesting Transmitter

As we move towards hand-held devices that use wireless transmitters, there is an exceeding need to prolong the lifetime of their batteries without having to manually recharge them on a regular basis. One natural solution to such a problem is to utilize the environment, i.e., have a renewable energy source recharge the battery. This will enable the system to be self-sustaining. List of renewable sources include solar energy, wind energy, geothermal energy and ocean energy. Such a node/transmitter which recharges itself from ambient energy is called an Energy Harvesting (EH) node.

While an EH node has access to potentially unlimited energy over its lifetime, it needs to grapple with uncertainty in the amount of energy it can harvest at any time and the times at which this energy is available. This uncertainty depends on the EH source, and is abstracted in the form of energy profile, which models the energy harvested as a stochastic process. The operation of an EH node is fundamentally governed by the *Energy Neutrality Constraint*, which mandates that, at any point of time, the total amount of energy utilized must be less than or equal to the sum of initial energy and the total amount of energy harvested thus far.

In the limiting case, when the battery size is taken to be infinite, the capacity of such an energy harvesting communication system has been characterized in literature. It is shown that in this asymptotic case, the capacity of the energy harvesting system is equal to the capacity of the classical fading channel with an average power constraint equal to the average energy harvesting rate.

In this discussion we try to obtain similar results for the more realistic case of a finite battery. We ask for the capacity of an energy harvesting communication system with a fading channel and finite battery size. Despite significant efforts in this field, we currently en lack an understanding about the capacity of such a channel. The typical water filling solution with an average power constraint equal to average energy arrival rate does give an upper bound on the capacity but it is not clear whether the same is achievable. Moreover, that strategy is difficult to analyze.

In this discussion we take an alternate path to approximate the capacity of such a communication system with bounded guarantee on the approximation gap. We see that the bounded gap does depend on the energy harvesting profile and can get bad for some EH profiles. However, we show that for most of the EH profiles, we can bound this gap by a constant which is independent of the key system parameter, the battery size. The discussion henceforth is organized as follows. We describe the system model in the first section. In the second section, we bound the maximum achievable throughput of such a communication system. In the third section, we come up with an energy utilization strategy for Bernoulli energy arrivals and compute the gap between throughput of this achievable strategy and the upper bound on maximum achievable throughput. In the fourth section, we show how the strategy described in the third section can be extended to the more realistic uniform energy arrival profiles and the approximation gap for long term average throughput can be bounded. In the fifth section, we show how the proposed strategy can be extended to most of the energy arrival processes. We also indicate that for some energy arrival profiles, this method can fail to provide a bounded approximation gap to the capacity.

3.1 System Model

We consider slotted time, and a single node that harvests energy from the environment. Let E_t be the amount of energy harvested at time step t which is stored in a battery of size B_{max}. In case the harvested energy exceeds the available space in the battery at time t, the battery is charged to the maximum capacity and the remaining energy is discarded. We consider a fading channel and h_t denotes the channel fade state at time t. The channel fade state h_t is an exponentially distributed random variable with a mean of unity. We denote the energy available in the battery at time t by B_t, and the energy utilized for transmission at time t by X_t. This implies that for all t, we need to satisfy $X_t \leq B_t$. The energy harvested at each time step E_t is a discrete time ergodic and stationary random process dictated by the energy harvesting mechanism. The rate of transmission is assumed to be governed by the classical fading channel rate function given by

$$r(t) = \frac{1}{2}\log(1 + h_t X_t). \tag{3.1}$$

The information theoretic capacity of this energy harvesting system is defined in the usual way as the largest rate at which the transmitter can reliably communicate with the receiver under the system constraints.

3.2 An Upper Bound on maximum achievable throughput

In this section, we upper bound the maximum achievable throughput of the described communication system by the following theorem.

Theorem 3.2.1. Let E_t be an i.i.d. energy harvesting process at the transmitter and h_t be an i.i.d. channel fade state. Let h_t be exponentially distributed with a mean of unity. The maximum achievable average throughput T_{max} of any energy allocation policy for such a communication system is upper bounded by

$$T_{max} \leq \frac{1}{2}\log\left(1 + \sqrt{2}\sqrt{\mathbb{E}[E_t^2]}\right). \tag{3.2}$$

Proof:

Let $X_t = P_h(t)$ be any energy allocation strategy that depends on the channel state h, and satisfies energy neutrality constraint. Then the maximum long term average throughput, T_{max}, is given by

$$T_{max} = \max_{P_h(t)} \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} \frac{1}{2}\log(1+h_t P_h(t)) \tag{3.3}$$

By ergodicity, we have that the average throughput is given by,

$$T_{max} = \max_{P_h(t)} \mathbb{E}\left[\frac{1}{2}\log(1+h_t P_h(t))\right]. \tag{3.4}$$

By Jensen's inequality, we have that for any energy allocation policy $P_h(t)$,

$$\mathbb{E}\left[\log\left(1+h_t P_h(t)\right)\right] \leq \log\Big(1+\mathbb{E}\left[h_t P_h(t)\right]\Big). \tag{3.5}$$

Let $P_h^*(t)$ be the energy allocation strategy that achieves the maximum average throughput. From (3.4) and (3.5), we have

$$T_{max} \leq \frac{1}{2}\log\left(1+\mathbb{E}[h_t P_h^*(t)]\right). \tag{3.6}$$

Now the task essentially is to upper bound $\mathbb{E}[h_t P_h^*(t)]$.

As $P_h^*(t)$ is a channel dependent energy allocation strategy, clearly h_t and $P_h^*(t)$ are dependent random variables. Applying Cauchy-Schwarz inequality, we have

$$\left(\mathbb{E}[h_t P_h^*(t)]\right)^2 \leq \mathbb{E}[h_t^2]\ \mathbb{E}[P_h^{*2}(t)] \tag{3.7}$$

As all the random variables involved are positive valued, we have

$$\mathbb{E}[h_t P_h^*(t)]) \leq \sqrt{\mathbb{E}[h_t^2]}\sqrt{\mathbb{E}[P_h^*(t)^2]}. \tag{3.8}$$

Let $P_{h_t}(t), h_t,$ and E_t be the energy allocated for transmission at time t by any energy allocation strategy, channel fade state at time t, and the energy harvested at time t respectively. By energy neutrality constraint, we have,

$$
\begin{aligned}
\sum_{i=1}^{N} P_{h_i}(i) &\leq \sum_{i=1}^{N} E_i \;\forall N, \\
\left(\sum_{i=1}^{N} P_{h_i}(i)\right)^2 &\leq \left(\sum_{i=1}^{N} E_i\right)^2 \;\forall N, \\
\frac{1}{N}\left(\sum_{i=1}^{N} P_{h_i}(i)\right)^2 &\leq \frac{1}{N}\left(\sum_{i=1}^{N} E_i\right)^2 \;\forall N, \\
\lim_{N\to\infty} \frac{1}{N}\left(\sum_{i=1}^{N} P_{h_i}(i)\right)^2 &\leq \lim_{N\to\infty} \frac{1}{N}\left(\sum_{i=1}^{N} E_i\right)^2, \\
\mathbb{E}\left[P_h^2(t)\right] &\leq \mathbb{E}\left[E_t^2\right].
\end{aligned} \tag{3.9}
$$

where the last step follows as h_i and E_i are assumed to be i.i.d. and hence the cross terms are independent. This is true for every feasible energy allocation strategy and hence,

$$\mathbb{E}\left[P_h^{*2}(t)\right] \leq \mathbb{E}\left[E_t^2\right]. \tag{3.10}$$

Thus, from (5.9), (5.10), and (3.10), we have

$$T_{max} \leq \frac{1}{2}\log\left(1+\sqrt{\mathbb{E}[h_t^2]}\sqrt{\mathbb{E}[E_t^2]}\right). \tag{3.11}$$

As h_t is an exponentially distributed random variable with a mean of unity, we have that $\mathbb{E}[h_t^2] = 2$ and (3.11) can be expressed as

$$T_{max} \leq \frac{1}{2}\log\left(1+\sqrt{2}\sqrt{\mathbb{E}[E_t^2]}\right). \tag{3.12}$$

□

We denote this upper bound on achievable throughput by T_{ub}. We next propose an energy allocation strategy, and compare the average throughput obtained by it with T_{ub}.

3.3 Bernoulli Energy Arrival

In this section, we focus on a simple Bernoulli energy arrival process, where E_t, the energy arrival at time t is i.i.d. Bernoulli random variable. Thus, we have

$$E_t = \begin{cases} E & \text{w.p. } p, \\ 0 & \text{w.p. } 1-p. \end{cases} \tag{3.13}$$

We first look at the case when the battery size B_{max} is less than the size of the energy arrival packets E.

3.3.1 $B_{max} \leq E$ case

Note that when $B_{max} \leq E$, because the energy must be stored in the battery before it can be used, the extra energy is wasted, and the system $B_{max} \leq E$ is equivalent to the system where packet size is exactly of size B_{max}. The energy arrival E_t is thus modified to

$$E_t = \begin{cases} B_{max} & \text{w.p. } p, \\ 0 & \text{w.p. } 1-p. \end{cases} \tag{3.14}$$

In this case, according to our system definition, every time a non-zero energy packet arrives, the battery is charged to full level, and the left over energy is wasted. Since the system is reset to the initial state of full battery level each time a non-zero energy packet arrives, each epoch (time interval between two adjacent energy arrivals) is independent and statistically identical to every other epoch. Motivated by this observation, we propose to use an energy utilization strategy that depends only on the number of time slots since the last time the battery was recharged, and independent of the channel fade state h_t as proposed in Dong *et al.* (2014) i.e. $P_h(t) = P(j)$ where $j = t - \max\left\{t' : E_{t'} = E, \forall t' \leq t\right\}$. Note that j is the number of channel uses since the last time battery was recharged. Based on the system description, any such energy allocation policy should satisfy the following constraints,

$$\sum_{j=0}^{\infty} P(j) \leq B_{max}, \tag{3.15}$$

$$P(j) \geq 0 \; \forall \; j. \tag{3.16}$$

We propose the following energy allocation policy which is similar to Dong *et al.* (2014). However that was proposed for an AWGN channel as compared to the fading channel assumed in this paper.

$$P(j) = p(1-p)^j B_{max}, \;\text{ for }\; j = 0, 1, 2, \ldots \,. \tag{3.17}$$

Note that,

$$\sum_{j=0}^{\infty} P(j) = \sum_{j=0}^{\infty} p(1-p)^j B_{max} = B_{max}.$$

Also,

$$P(j) = p(1-p)^j B_{max} \geq 0 \text{ for } p \in [0,1] \text{ and } B_{max} \geq 0. \tag{3.18}$$

Thus, the proposed energy utilization policy does satisfy the system constraints (3.15) and (3.16).

Note that our energy utilization policy does not depend on the channel fade state h, and hence seems to be highly sub-optimal. It allocates a fraction p of the available energy for transmission in every time slot irrespective of the channel fade state. The motivation behind this strategy is that the inter-arrival time for energy packets is a geometric random variable with parameter p. We know that geometric random variable is memory less and has mean $1/p$. Therefore at each time step, the expected number of time steps to the next energy arrival is $1/p$. Furthermore, as the rate function for each channel fade state h is a concave function, results from Yang and Ulukus (2012),Tutuncuoglu and Yener (2012) tell us that in order to achieve a higher throughput, we would want to allocate the energy as uniformly as possible between the energy arrivals, i.e. if the current energy level in the battery is B_t and we know that the next energy arrival would be after m time slots, we would allocate B_t/m energy to each time slot (ignoring the channel fade state). Here, we do not know when will the next packet of energy arrive. Instead, we use the expected time to the next energy arrival as basis. Since geometric random variable is memory-less, at each time step, the expected number of time slots to next energy arrival is $1/p$ and hence we use a fraction p of the available energy. We call this policy as Constant Fraction Policy (CFP).

Lemma 3.3.1. The average throughput obtained by Constant Fraction Policy is given by,

$$T_{lb} = \sum_{j=0}^{\infty} p(1-p)^j \int_0^{\infty} \frac{1}{2} \log(1 + hp(1-p)^j B_{max}) e^{-h} dh. \tag{3.19}$$

Proof: Recall that the system resets to the full energy state every time a non-zero energy packet arrives. Hence, we can apply renewal reward theorem to find the expected throughput. Without loss of generality, let the first non-zero energy arrival occur at $t = 0$. Let T_1 be the time at which the next non-zero energy arrival occurs. By renewal reward theorem, the time average throughput is given by,

$$
\begin{aligned}
T_{lb} &= \frac{\mathbb{E}\left[\sum_{j=0}^{T_1-1} \frac{1}{2}\log(1+hP(j))\right]}{\mathbb{E}[T_1]}, \\
&\overset{(a)}{=} p\sum_{i=1}^{\infty} \mathbb{P}(T_1 = i) \sum_{j=0}^{i-1} \int_0^{\infty} \frac{1}{2}\log(1+hP(j))e^{-h}dh, \\
&\overset{(b)}{=} p\sum_{i=1}^{\infty} (1-p)^{i-1} p \sum_{j=0}^{i-1} \int_0^{\infty} \frac{1}{2}\log(1+hP(j))e^{-h}dh, \\
&\overset{(c)}{=} p\sum_{j=0}^{\infty} \left(\sum_{i=j+1}^{\infty} (1-p)^{i-1} p\right) \int_0^{\infty} \frac{1}{2}\log(1+hP(j))e^{-h}dh
\end{aligned}
$$

$$
\begin{aligned}
&\overset{(d)}{=} \sum_{j=0}^{\infty} p(1-p)^j \int_0^{\infty} \frac{1}{2}\log(1+hP(j))e^{-h}dh, \\
&\overset{(e)}{=} \sum_{j=0}^{\infty} p(1-p)^j \int_0^{\infty} \frac{1}{2}\log(1+hp(1-p)^j B_{max})e^{-h}dh. \qquad (3.20)
\end{aligned}
$$

(a) follows because T_1 is a geometric random variable with parameter p, and hence $\mathbb{E}[T_1] = 1/p$, (b) follows since for a geometric random variable T_1, $\mathbb{P}(T_1 = i) = (1-p)^{i-1}p$, (c) is obtained by interchanging the order of summations, (d) follows since $\sum_{i=j+1}^{\infty}(1-p)^{i-1}p = (1-p)^j$, and (e) follows from the definition of $P(j)$.

□

From Theorem 1, we know that the maximum achievable average throughput is upper bounded by,

$$
T_{ub} \overset{(a)}{=} \frac{1}{2}\log\left(1+\sqrt{2p}B_{max}\right),
$$

where (a) follows from theorem 1 by substituting $\mathbb{E}[E_t^2] = pB_{max}^2$ for Bernoulli energy arrivals.

Next, we bound the gap between upper bound on maximum achievable throughput T_{ub} and the throughput achieved by Constant Fraction Policy T_{lb}.

Lemma 3.3.2. For a given p, the approximation gap between the upper bound on maximum achievable strategy T_{ub} and the throughput achieved by the Constant Fraction Policy T_{lb} is upper bounded by,

$$\frac{1}{2}\log\left(1+\sqrt{2p}\,k\right), \tag{3.21}$$

where k satisfies

$$\frac{1}{2}\log\left(1+\sqrt{2p}\,k\right) = 0.54 - \frac{1}{4}\log(p) + \frac{1}{2\ln 2}\frac{1}{\sqrt{2p}\,k} + \frac{1-p}{2p}\log\left(\frac{1}{1-p}\right). \tag{3.22}$$

However the approximation gap depends on the value of p and is unbounded for $p = 0$.

Proof: For $B_{max} < k$,

$$\begin{aligned} T_{ub} - T_{lb} = & \frac{1}{2}\log\left(1+\sqrt{2p}B_{max}\right) - \sum_{j=0}^{\infty} p(1-p)^j \int_0^{\infty} \frac{1}{2}\log(1+hp(1-p)^j B_{max})e^{-h}dh, \\ \stackrel{(a)}{\leq} & \frac{1}{2}\log\left(1+\sqrt{2p}B_{max}\right), \\ \stackrel{(b)}{\leq} & \frac{1}{2}\log\left(1+\sqrt{2p}\,k\right), \end{aligned}$$

where (a) follows because $\sum_{j=0}^{\infty} p(1-p)^j \int_0^{\infty} \frac{1}{2}\log(1+hp(1-p)^j B_{max})e^{-h}dh \geq 0$ as the integrand is always positive, and (b) follows since $B_{max} < k$.

For $B_{max} > k$,

$$
\begin{aligned}
& T_{ub} - T_{lb} && (3.23)\\
= \; & \frac{1}{2}\log\left(1+\sqrt{2p}B_{max}\right) - \sum_{j=0}^{\infty} p(1-p)^j \int_0^{\infty} \frac{1}{2}\log(1+hp(1-p)^j B_{max})e^{-h}dh, \\
\overset{(a)}{\leq} \; & \frac{1}{2}\log\left(1+\sqrt{2p}B_{max}\right) - \sum_{j=0}^{\infty} p(1-p)^j \int_0^{\infty} \frac{1}{2}\log(hp(1-p)^j B_{max})e^{-h}dh, \\
= \; & \frac{1}{2}\log\left(1+\sqrt{2p}B_{max}\right) \\
& - \sum_{j=0}^{\infty} p(1-p)^j \int_0^{\infty} \frac{1}{2}\left[\log(h)+\log(p)+j\log(1-p)+\log(B_{max})\right] e^{-h}dh, \\
\overset{(b)}{=} \; & \frac{1}{2}\log\left(\sqrt{2}\right) + \frac{1}{2}\log\left(\sqrt{p}\right) + \frac{1}{2}\log\left(B_{max}\right) + \log\left(1+\frac{1}{\sqrt{2p}B_{max}}\right) \\
& - \int_0^{\infty} \frac{1}{2}\log(h)e^{-h}dh + \int_0^{\infty} \frac{1}{2}\log(p)e^{-h}dh + \frac{1-p}{p}\int_0^{\infty} \frac{1}{2}\log(1-p)e^{-h}dh \\
& + \int_0^{\infty} \frac{1}{2}\log(B_{max})e^{-h}dh, \\
\overset{(c)}{=} \; & 0.25 + \frac{1}{4}\log(p) + \frac{1}{2}\log\left(B_{max}\right) + \log\left(1+\frac{1}{\sqrt{2p}B_{max}}\right) + 0.29 - \frac{1}{2}\log(p) \\
& - \frac{1-p}{2p}\log(1-p) - \frac{1}{2}\log(B_{max}), \\
\overset{(d)}{\leq} \; & 0.54 - \frac{1}{4}\log(p) + \frac{1}{2\ln 2}\frac{1}{\sqrt{2p}B_{max}} + \frac{1-p}{2p}\log\left(\frac{1}{1-p}\right), \\
\overset{(e)}{\leq} \; & \frac{1}{2}\log\left(1+\sqrt{2p}\,k\right). && (3.24)
\end{aligned}
$$

where (a) follows from the fact that removing 1 from the second log term results in an upper bound; (b) follows because $\sum_{j=0}^{\infty} p(1-p)^j = 1$ and $\sum_{j=0}^{\infty} jp(1-p)^j = \frac{1-p}{p}$; (c) follows because $\int_0^{\infty} e^{-h}dh = 1$ and $\int_0^{\infty} \log(h)e^{-h} = -0.29$; (d) uses the identity $\ln(1+x) \leq x$, and finally (e) follows from (3.22)

$$g(p) = \frac{1-p}{2p}\log\left(\frac{1}{1-p}\right),$$

is a monotonically decreasing and continuous function for $p \in (0,1)$ and is upper bounded by $\lim_{p \to 0} g(p) = 0.72$.

$\square$

Thus, we see that for a given value of p ($p \neq 0$), the gap between maximum achievable throughput and the throughput obtained by Constant Fraction Policy can be bounded for all values of B_{max}. However this gap becomes large for small p and is unbounded as $p \to 0$. However this is only a peculiar case and we can indeed bound this gap for most of the realistic energy arrival profiles as we show later.

This can be done because for a given p, the gap can be upper bounded by (3.24) for large B_{max} and can be upper bounded by the first term, ignoring the second term (which is positive) of (3.23) for small values of B_{max}. Hence, a universal upper bound, irrespective of B_{max} can be obtained by equating these two terms. However this gap becomes large for small p and is unbounded as $p \to 0$. However this is only a peculiar case and we can indeed bound this gap for most of the realistic energy arrival profiles as we show later.

We now consider the special case of $p = 0.5$ and show that the gap between the upper bound on maximum achievable throughput, T_{ub} and the throughput obtained by Constant Fraction Policy, T_{lb} is upper bounded by 1.41 bits. The reason behind behind considering this special case will become clear in the subsequent discussion where we use this result to bound the gap between the T_{ub} and T_{lb} for other energy arrival profiles.

Lemma 3.3.3. For Bernoulli energy arrivals with $p = 0.5$, the approximation gap between the upper bound on achievable average throughput T_{ub} and the throughput achieved by the proposed strategy is bounded by 1.41 bits.

Proof: Fixing $p = 0.5$,

For $B_{max} \leq 6.05$,

$$\begin{aligned}
T_{ub} - T_{lb} = & \frac{1}{2}\log\left(1+\sqrt{2p}B_{max}\right) - \sum_{j=0}^{\infty} p(1-p)^j \int_0^{\infty} \frac{1}{2}\log(1+hp(1-p)^j B_{max})e^{-h}dh, \\
\overset{(a)}{\leq} & \frac{1}{2}\log\left(1+\sqrt{2p}B_{max}\right), \\
\overset{(b)}{\leq} & \frac{1}{2}\log\left(1+B_{max}\right), \\
\overset{(c)}{\leq} & 1.41.
\end{aligned}$$

where (a) uses the fact that the $\sum_{j=0}^{\infty} p(1-p)^j \int_0^{\infty} \frac{1}{2}\log(1+hp(1-p)^j B_{max})e^{-h}dh \geq 0$ as the integrand is always positive; (b) follows by substituting $p = 0.5$ and (c) is true because $B_{max} \leq 6.05$.

For $B_{max} \geq 6.05$

$$\begin{aligned}
T_{ub} - T_{lb} = \; & \frac{1}{2}\log\left(1+\sqrt{2p}B_{max}\right) - \sum_{j=0}^{\infty} p(1-p)^j \int_0^{\infty} \frac{1}{2}\log(1+hp(1-p)^jB_{max})e^{-h}dh, \\
= \; & \frac{1}{2}\log\left(1+B_{max}\right) - \sum_{j=0}^{\infty} (0.5)^{j+1} \int_0^{\infty} \frac{1}{2}\log(1+h(0.5)^{j+1}B_{max})e^{-h}dh, \\
\overset{(a)}{\leq} \; & \frac{1}{2}\log\left(1+B_{max}\right) - \sum_{j=0}^{\infty} (0.5)^{j+1} \int_0^{\infty} \frac{1}{2}\log(h(0.5)^{j+1}B_{max})e^{-h}dh, \\
= \; & \frac{1}{2}\log(B_{max}) + \frac{1}{2}\log\left(1+\frac{1}{B_{max}}\right) \\
& - \sum_{j=0}^{\infty} (0.5)^{j+1} \int_0^{\infty} \frac{1}{2}\left[\log(h) + (j+1)\log(0.5) + \log\left(B_{max}\right)\right] e^{-h}dh, \\
\overset{(b)}{=} \; & \frac{1}{2}\log(B_{max}) + \frac{1}{2}\log\left(1+\frac{1}{B_{max}}\right) - \int_0^{\infty} \frac{1}{2}\log(h)e^{-h}dh - 2\int_0^{\infty} \frac{1}{2}\log(0.5)e^{-h}dh \\
& - \int_0^{\infty} \frac{1}{2}\log(B_{max})e^{-h}dh, \\
\overset{(c)}{=} \; & \frac{1}{2}\log(B_{max}) + \frac{1}{2}\log\left(1+\frac{1}{B_{max}}\right) - \int_0^{\infty} \frac{1}{2}\log(h)e^{-h}dh - \log(0.5) - \frac{1}{2}\log(B_{max}), \\
\overset{(d)}{\leq} \; & \frac{1}{2ln(2)B_{max}} + 1.29, \\
\overset{(e)}{\leq} \; & 1.41.
\end{aligned}$$

where (a) follows from the fact that removing the 1 inside the second log results in an upper bound, (b) follows because $\sum_{j=0}^{\infty}(0.5)^{j+1} = 1$ and $\sum_{j=0}^{\infty}(0.5)^{j+1}(j+1) = 2$; (c) follows from the fact that e^{-h} is a probability density function and thus $\int_0^{\infty} e^{-h}dh = 1$; (d) follows from the inequality $\ln(1+x) \leq x$ and also $\int_0^{\infty} \frac{1}{2}\log(h)e^{-h}dh = -0.29$, and (e) is true since $B_{max} \geq 6.05$.

□

We now consider the case when the battery size B_{max} is greater than the energy arrival packet size E.

3.3.2 $B_{max} > E$ case

When $B_{max} > E$, the epochs are non-identical as the residual energy from the last epoch may be different. However, we propose an energy allocation strategy which ensures that the throughput obtained in every epoch is identical and statistically independent from every other epoch. We achieve this by the considering the battery size to be E. Whenever an energy packet arrives, the amount of energy in the battery is the sum of the residual energy and the arrival energy E. However, our strategy assumes that the battery size is only E and the remaining energy is wasted. Clearly this is an energy feasible strategy. Thus, we are essentially assuming that battery size is E which is less than B_{max}. Similar to the last section, we propose an energy allocation strategy $P_h(t) = g'(j)$ that depends only on the number of channel uses since the last energy arrival.

$$g'(j) = p(1-p)^j E \text{ for } j = 0, 1, 2, \ldots, \tag{3.25}$$

where

$$j = t - \max\left\{t' : E_{t'} = E, \forall t' \leq t\right\}. \tag{3.26}$$

Note that this strategy satisfies energy neutrality as $\sum_{j=0}^{\infty} g'(j) = E$. Indeed this energy allocation policy is quite conservative and clearly wastes energy. This is because every time a non-zero energy arrives, this strategy ignores the residual energy, and starts as if the battery energy level is reset to E.

Note that the upper bound on maximum achievable throughput for this case is given by,

$$T_{ub} \overset{(a)}{=} \frac{1}{2} \log\left(1 + \sqrt{2pE}\right) \tag{3.27}$$

where (a) follows from theorem 1 by substituting $\mathbb{E}\left[E_t^2\right] = pE^2$ for Bernoulli energy arrivals of size E.

Also, the throughput achieved by Constant Fraction Policy, which we denote by T_{lb}

can be evaluated as follows. Recall that the system resets to the full energy state every time a non-zero energy packet arrives. Moreover, the throughput obtained between consecutive epochs is independent and statistically identical across time. Hence, we can apply renewal reward theorem to find the expected throughput. Without loss of generality, let the first non-zero energy arrival occur at $t = 0$. Let T_1 be the time at which the next non-zero energy arrival occurs. By renewal reward theorem, the time average throughput is given by,

$$\begin{aligned}
T_{lb} &= \frac{\mathbb{E}\left[\sum_{j=0}^{T_1-1} \frac{1}{2}\log(1+hP(j))\right]}{\mathbb{E}[T_1]}, \\
&\overset{(a)}{=} p\sum_{i=1}^{\infty}\mathbb{P}(T_1 = i)\sum_{j=0}^{i-1}\int_0^{\infty}\frac{1}{2}\log(1+hP(j))e^{-h}dh, \\
&\overset{(b)}{=} p\sum_{i=1}^{\infty}(1-p)^{i-1}p\sum_{j=0}^{i-1}\int_0^{\infty}\frac{1}{2}\log(1+hP(j))e^{-h}dh, \\
&\overset{(c)}{=} p\sum_{j=0}^{\infty}\left(\sum_{i=j+1}^{\infty}(1-p)^{i-1}p\right)\int_0^{\infty}\frac{1}{2}\log(1+hP(j))e^{-h}dh \\
&\overset{(d)}{=} \sum_{j=0}^{\infty}p(1-p)^j\int_0^{\infty}\frac{1}{2}\log(1+hP(j))e^{-h}dh, \\
&\overset{(e)}{=} \sum_{j=0}^{\infty}p(1-p)^j\int_0^{\infty}\frac{1}{2}\log(1+hp(1-p)^jE)e^{-h}dh. \qquad (3.28)
\end{aligned}$$

(a) follows because T_1 is a geometric random variable with parameter p, and hence $\mathbb{E}[T_1] = 1/p$, (b) follows since for a geometric random variable T_1, $\mathbb{P}(T_1 = i) = (1-p)^{i-1}p$, (c) is obtained by interchanging the order of summations, (d) follows since $\sum_{i=j+1}^{\infty}(1-p)^{i-1}p = (1-p)^j$, and (e) follows from the definition of $P(j)$.

Following the exact steps as Lemma 3 one can obtain that for $p = 0.5$,

$$\begin{aligned} T_{ub}-T_{lb} =&\ \frac{1}{2}\log\left(1+\sqrt{2p}E\right)-\sum_{j=0}^{\infty}p(1-p)^j\int_0^{\infty}\frac{1}{2}\log(1+hp(1-p)^jE)e^{-h}dh,\\ =&\ \frac{1}{2}\log(1+E)-\sum_{j=0}^{\infty}(0.5)^{j+1}\int_0^{\infty}\frac{1}{2}\log(1+h(0.5)^{j+1}E)e^{-h}dh,\\ \leq&\ 1.41. \end{aligned}$$

for all $E < B_{max}$.

Thus for a Bernoulli energy arrival profile with $p = 0.5$, we have a near optimal energy allocation strategy which is guaranteed to be within 1.41 bits of the upper bound on the maximum achievable average throughput T_{ub} i.e. the throughput T_{lb} obtained by Constant Fraction Policy will satisfy,

$$\begin{aligned} &T_{lb} \geq T_{ub}-1.41,\\ \therefore\ &T_{lb} \overset{(a)}{\geq} \frac{1}{2}\log(1+E)-1.41,\ \text{for } p=0.5. \end{aligned} \tag{3.29}$$

for $\forall E < B_{max}$ where (a) follows by substituting $\mathbb{E}\left[E_t^2\right] = \frac{E^2}{2}$ for Bernoulli energy arrival with $p = 0.5$ and packet size of E in Theorem 1. An equivalent way of saying the above stated result is,

Lemma 3.3.4. For Bernoulli energy arrivals with $p = 0.5$ and packet size $E < B_{max}$, the gap between the upper bound on achievable throughput $T_{ub} = \frac{1}{2}\log(1+E)$ and throughput T_{lb} achieved by Constant Fraction Policy is upper bounded by 1.41.

$$\frac{1}{2}\log(1+E)-T_{lb} \leq 1.41,\ \forall\, E < B_{max} \text{ and } p=0.5. \tag{3.30}$$

3.4 Uniform Energy Arrival

We now consider the case of uniform energy arrival process i.e. E_t is uniformly distributed between 0 and B_{max}. We fix a threshold of $B_{max}/2$. It should be noted that the probability p of having an energy arrival with packet size at least $B_{max}/2$ i.e. $\mathbb{P}(E_t \geq B_{max}/2)$ is 0.5. We assume that there is no energy arrival when the arrival energy packet size is smaller than $B_{max}/2$ and we assume that the packet size is $B_{max}/2$ when we receive an energy packet of size at least $B_{max}/2$. Clearly, this is an energy feasible strategy. We have thus converted the given uniform energy arrival process into a Bernoulli process with $p = 0.5$ and $E = B_{max}/2$. However, it may seem highly sub-optimal and wasteful of the available energy. We next apply the near optimal strategy discussed in the previous section. From (3.29), we know that the throughput achieved by this strategy, denoted by T will satisfy

$$T \geq \frac{1}{2}\log\left(1+\frac{B_{max}}{2}\right) - 1.41 \tag{3.31}$$

Also, from (3.12), the maximum achievable throughput of this channel is upper bounded by,

$$\begin{aligned} T_{max} &\leq \frac{1}{2}\log\left(1+\sqrt{2}\sqrt{\mathbb{E}[E_t^2]}\right) \\ \therefore\ T_{max} &\leq \frac{1}{2}\log\left(1+\sqrt{2}\frac{B_{max}}{\sqrt{3}}\right) \quad \text{since } \mathbb{E}[E_t^2] = \frac{B_{max}^2}{3} \text{ for uniform energy arrivals} \end{aligned} \tag{3.32}$$

We now bound the difference between the maximum achievable throughput for uniform arrivals and the maximum achievable throughput for Bernoulli energy arrivals of size

$B_{max}/2$ and parameter $p = 0.5$

$$\begin{aligned}
& \frac{1}{2}\log\left(1+\sqrt{2}\frac{B_{max}}{\sqrt{3}}\right) - \frac{1}{2}\log\left(1+\frac{B_{max}}{2}\right) \\
= \; & \frac{1}{2}\log\left(\frac{1+\sqrt{2}\frac{B_{max}}{\sqrt{3}}}{1+\frac{B_{max}}{2}}\right) \\
\overset{(a)}{\leq} \; & \frac{1}{2}\log\left(\frac{\sqrt{2}\frac{B_{max}}{\sqrt{3}}}{\frac{B_{max}}{2}}\right) \\
= \; & \frac{1}{2}\log\left(\frac{2\sqrt{2}}{\sqrt{3}}\right) \\
= \; & 0.35
\end{aligned}$$

where (a) is true because the numerator is greater than the denominator.

We thus have the following,

$$\frac{1}{2}\log\left(1+\sqrt{2}\frac{B_{max}}{\sqrt{3}}\right) - \log\left(1+\frac{B_{max}}{2}\right) \leq 0.35 \;\forall\; B_{max} \tag{3.33}$$

Rearranging (5.19), we get the following,

$$\frac{1}{2}\log\left(1+\frac{B_{max}}{2}\right) \geq \frac{1}{2}\log\left(1+\sqrt{2}\frac{B_{max}}{\sqrt{3}}\right) - 0.35 \;\forall\; B_{max} \tag{3.34}$$

From (3.31) and (5.20), we have the following,

$$T \geq \frac{1}{2}\log\left(1+\sqrt{2}\frac{B_{max}}{\sqrt{3}}\right) - 1.76 \;\forall\; B_{max} \tag{3.35}$$

We thus have an energy allocation policy which is guaranteed to be within 1.76 bits of the upper bound on the maximum achievable throughput derived in section III. Hence we have the following result.

For uniform energy arrival between 0 and B_{max}, the throughput achieved by the pro-

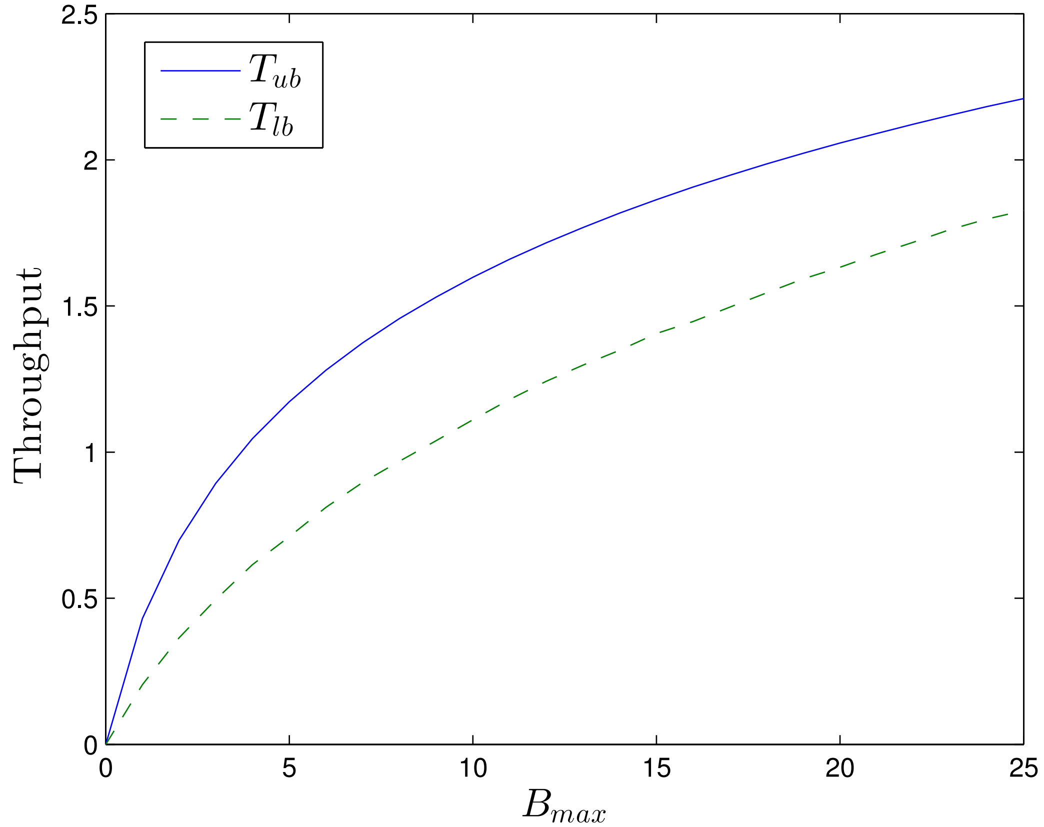

Figure 3.1: Performance of CFP for uniform energy arrivals as compared to the upper bound on maximum achievable long term throughput for different values of B_{max}. The solid curve represents the upper bound T_{ub} and the dashed curve corresponds to per slot throughput achieved by CFP T_{lb}.

posed strategy will satisfy the following

$$\frac{1}{2}\log\left(1+\sqrt{\mathbb{E}[h_t^2]}\sqrt{\mathbb{E}[E_t^2]}\right)-1.76 \leq T \leq \frac{1}{2}\log\left(1+\sqrt{\mathbb{E}[h_t^2]}\sqrt{\mathbb{E}[E_t^2]}\right)\ \forall B_{max} \tag{3.36}$$

where $\mathbb{E}[h_t^2]=2$ for exponentially distributed channel fade state with mean 1 and $\mathbb{E}[E_t^2]=\frac{B_{max}^2}{3}$ for uniform energy arrivals between 0 and B_{max}.

3.5 Generalization to other Energy Profiles

The idea that we discussed in the last section was specific to i.i.d. uniform energy arrival process. In this section, we present a simple way to apply Constant Fraction Policy to other energy arrival processes.

Let E_t, the energy arrival profile, be denoted by a random variable X with cumulative distribution function $F_X(\cdot)$. We assume X to be a continuous random variable. Let γ be the energy value such that $F_X(\gamma) = 0.5$. We will assume that $\gamma \leq B_{max}$. The case $\gamma > B_{max}$ needs some modifications but can be worked out similarly. Thus, we have $\gamma = F_X^{-1}(0.5)$. It should be noted that the probability p of having an energy arrival of at least γ i.e. $\mathbb{P}(E_t \geq \gamma)$ is 0.5.

We now propose to use Constant Fraction Policy as if the energy arrival process were i.i.d. Bernoulli with packet size γ and $p = 0.5$. The throughput achieved is denoted as before by T_{lb}. By this, we mean that we assume that there is no energy arrival when the arrival energy packet size is smaller than γ, and we assume that the energy packet size is equal to γ each time we receive an energy packet of size at least γ. Clearly, this is an energy feasible strategy for general i.i.d. energy arrival process but may seem highly sub optimal and wasteful of energy. However, we have the following theorem for the throughput achieved T_{lb}.

Theorem 3.5.1. The throughput achieved by the proposed policy T_{lb} satisfies the following,

$$T_{lb} \geq T_{ub} - 1.67 - \frac{1}{4}\log\left(\frac{\mathbb{E}[X^2]}{\left(F_X^{-1}(0.5)\right)^2}\right), \tag{3.37}$$

where T_{ub} is as given in Theorem 1 i.e.

$$T_{ub} = \frac{1}{2}\log\left(1 + \sqrt{\mathbb{E}[h_t^2]}\sqrt{\mathbb{E}[X^2]}\right). \tag{3.38}$$

Proof: The proposed strategy views any i.i.d. energy arrival process as Bernoulli with packet size $\gamma \leq B_{max}$ and $p = 0.5$ and uses the Constant Fraction Policy. By Lemma 4, we have the following,

$$T_{lb} \geq \frac{1}{2}\log(1+\gamma) - 1.41 \tag{3.39}$$

We next bound the difference between maximum achievable throughput T_{ub} and the first term on the right hand side of (3.39) as follows,

$$\begin{aligned}
T_{ub} - \frac{1}{2}\log(1+\gamma) &\overset{(a)}{=} \frac{1}{2}\log\left(1+\sqrt{\mathbb{E}[h_t^2]}\sqrt{\mathbb{E}[X^2]}\right) - \frac{1}{2}\log(1+\gamma),\\
&\overset{(b)}{=} \frac{1}{2}\log\left(\frac{1+\sqrt{2}\sqrt{\mathbb{E}[X^2]}}{1+\gamma}\right),\\
&\overset{(c)}{\leq} \frac{1}{2}\log\left(\frac{\sqrt{2}\sqrt{\mathbb{E}[X^2]}}{F_X^{-1}(0.5)}\right),\\
&= 0.25 + \frac{1}{4}\log\left(\frac{\mathbb{E}[X^2]}{\left(F_X^{-1}(0.5)\right)^2}\right).
\end{aligned}$$

where (a) follows from Theorem 1, (b) follows as $h_t \sim \exp(1)$, and (c) follows because the numerator is greater than the denominator.

Hence, we have the following inequality,

$$\frac{1}{2}\log\left(1+\sqrt{2}\sqrt{\mathbb{E}[X^2]}\right) - \frac{1}{2}\log(1+\gamma) \leq 0.25 + \frac{1}{4}\log\left(\frac{\mathbb{E}[X^2]}{\left(F_X^{-1}(0.5)\right)^2}\right). \tag{3.40}$$

Rearranging the terms, one obtains that

$$\frac{1}{2}\log(1+\gamma) \geq \frac{1}{2}\log\left(1+\sqrt{2}\sqrt{\mathbb{E}[X^2]}\right) - 0.25 - \frac{1}{4}\log\left(\frac{\mathbb{E}[X^2]}{\left(F_X^{-1}(0.5)\right)^2}\right). \tag{3.41}$$

From (3.39) and (3.41), we have,

$$T_{lb} \geq \frac{1}{2}\log\left(1+\sqrt{2}\sqrt{\mathbb{E}[X^2]}\right) - 1.67 - \frac{1}{4}\log\left(\frac{\mathbb{E}[X^2]}{\left(F_X^{-1}(0.5)\right)^2}\right),$$

$$T_{lb} \geq T_{ub} - 1.67 - \frac{1}{4}\log\left(\frac{\mathbb{E}[X^2]}{\left(F_X^{-1}(0.5)\right)^2}\right). \tag{3.42}$$

which proves the theorem.

□

We therefore have a strategy which is guaranteed to be within bounded gap from the upper bound on maximum achievable throughput obtained in section III.

$$\frac{1}{2}\log\left(1+\sqrt{2}\sqrt{\mathbb{E}[X^2]}\right) - 1.67 - \frac{1}{4}\log\left(\frac{\mathbb{E}[X^2]}{\left(F_X^{-1}(0.5)\right)^2}\right) \leq T_{lb} \leq \frac{1}{2}\left(1+\sqrt{2}\sqrt{\mathbb{E}[X^2]}\right) \tag{3.43}$$

Lemma 3.5.1. For symmetrically distributed energy arrival profiles i.e. E_t is symmetrically distributed around the mean, with mean less than B_{max}, we have the following result for the throughput achieved by Constant Fraction Policy (after converting it into Bernoulli with packet size $\gamma = \mathbb{E}[X]$ and $p = 0.5$).

$$T_{lb} \geq T_{ub} - 1.67 - \frac{1}{4}\log\left(\frac{\mathbb{E}[X^2]}{\left(\mathbb{E}[X]\right)^2}\right) \tag{3.44}$$

Proof: Note that for symmetric distributions, we have the following

$$F_X^{-1}(0.5) = \mathbb{E}\left[X\right].$$

The lemma then follows from Theorem 2.

□

Remark 3.5.1. The proposed strategy provides a bounded approximation gap to the upper bound on achievable throughput T_{ub}, for energy profiles with finite second moment. However, it is possible to engineer energy arrival profiles with finite mean but unbounded second moment. For such cases, we know that the maximum achievable throughput is bounded but the proposed strategy will fail to provide a bounded gap from T_{ub}. However, such profiles may not be common in practice.

Thus, we can bound the gap between the maximum achievable throughput and the throughput achieved by our proposed strategy for most of the energy arrival profiles with the gap being a function of $\frac{\mathbb{E}[X^2]}{\left(\mathbb{E}[X]\right)^2}$. However, as mentioned earlier, it is possible to come up with counter examples for which the proposed method will fail to give a bounded gap. From (3.44), it is trivial to see that energy arrivals with an infinite second moment will fail to give a bounded gap even though the mean is bounded above by B_{max}. One can easily come up with such en energy arrival profile. However, such profiles may not be common in practice.

3.6 Optimal Policy for Discrete Energy Arrival process and Discrete Energy Consumption

In this section, we restrict ourselves to discrete energy arrivals as well as discrete energy utilization at the transmitter i.e., we assume that instead of the transmitter having the choice to select from a continuum of energy, it can only select a discrete amount of energy to transmit in each time slot. Keeping in mind the discrete transmission policy, we allow only a discrete energy arrival process in this section.

Like before, we consider slotted time. The node is assumed to have a finite battery of size B_{max} to store the harvested energy. Let B_k be the battery energy level, E_k be the amount of energy harvested, and F_k be the energy allocated for transmission at time k.

For each time step, a realization of the channel $h_k \geq 0$ is revealed to the transmitter and the payoff function is assumed as before, $r_k = \log(1 + h_k F_k)$.

As mentioned before, we assume a discrete energy arrival process E_k, where $E_k = i$ with probability p_i for $i = 1, 2, \ldots, B_{max}$.

For such a system, it has been shown that the optimal energy allocation policy (for maximizing the average long term throughput) is stationary, and monotonic in the battery energy level as well as the channel fade state. Owing to discrete energy consumption, this result can be transformed to the fact that the optimal energy utilization for each battery level will be a staircase like function with respect to the channel fade state h_k. For example, if the battery energy level is B_k, there will be thresholds $h_{B_k,i}$, for $i \in \{1, 2, \ldots, B_{max}\}$, with $h_{B_k,0} = 0$ and $h_{B_k,B_{max}+1} = \infty$, so that $F_k^*(B_k, h_k) = i$ if $h_k \in (h_{B_k,i}, h_{B_k,i+1})$.

This in turn can be transformed into

$$\mathbb{P}(\text{using i units of energy when battery level is } B_k) = \int_{h_{B_k,i}}^{h_{B_k,i+1}} \varphi(x)dx, \qquad (3.45)$$

where $\varphi(\cdot)$ is the probability density function of the channel fade state h_k.

Using this knowledge, the evolution of the battery energy level can be modeled as a finite state Markov chain with the transition probabilities in terms of the thresholds $h_{B_k,i}$ for $k \in 1, 2, \ldots, B_{max}$ and $i \in 1, 2, \ldots B_{max}$, and the energy arrival process probabilities p_i. In turn, the average throughput can be obtained by solving the steady state probabilities of the Markov chain. Consequently, the optimal thresholds can be found by optimizing the average throughput.

While it seems easy to solve for this case, even solving it for small cases is non-trivial owing to the non-convex nature of the optimization problem at hand.

CHAPTER 4

Transmitter-Receiver Energy Harvesting link with Distributed Control

Till now, we assumed that the payoff obtained is only a function of the transmit power and the channel fade state. We assumed that the receiver is always *on*. It should be noted that the receiver too has to spend energy to remain *on*, receive the information and decode it. While the *receiver always on* assumption may be true if the receiver has an access to unlimited un-interrupted power source, in certain cases this assumption may not be justified. In cases like space communication, or sending data to remote locations, the receiver too has energy constraints ans should spend energy judiciously.

Unlike the transmitter, the energy consumption at receiver is much simpler. We assume the following energy consumption model at receiver.

- "On" state : If the receiver is in on state, we assume that the throughput achieved is given by the rate transmission function assumed earlier. Consequently, the receiver spends a fixed amount of energy to stay on and decode data.
- "Off" state : If the receiver is in off state, it cannot receive and decode the data sent by the source and the throughput obtained is zero irrespective of the energy spent by the transmitter. In this state, the receiver does not spend any energy.

We refer to this energy consumption model at receiver as Binary receiver Energy Consumption model for obvious reasons. Based on this model, when transmitter spends $P_h(t)$ amount of energy and the channel fade state is h_t, the throughput obtained is given by,

$$r(t) = \mathbf{1}_R(h_t, t) \log\left(1 + h_t P_h(t)\right), \tag{4.1}$$

where $\mathbf{1}_R(h_t,t)$ is the indicator random variable corresponding to the receiver being on or off i.e.

$$\mathbf{1}_R(h_t,t) = \begin{cases} 1 & \text{if the receiver is on,} \\ 0 & \text{if the receiver is off.} \end{cases} \tag{4.2}$$

Unlike the traditional battery powered receiver, an energy harvesting receiver has the potential to solve the limitations on the lifetime of the communication link. Moreover, an energy harvesting receiver would be sustainable and environment friendly. As before, the energy harvesting mechanism is abstracted in the form of an energy profile, which models the energy harvested as a stochastic process. The receiver has a finite battery of size $\tilde{B}_{max}$ (the quantities at the receiver end are denoted by tilde) to store the harvested energy. Like the transmitter, in the case when the harvested energy exceeds the available space in the battery, the battery is charged to the maximum capacity and the remaining energy is discarded.

4.1 Upper Bound on maximum achievable throughput with receiver energy harvesting

In this section, we derive an upper bound on the maximum achievable throughput by any energy allocation strategy at the transmitter and the receiver. We assume that the transmitter does not have information about the receiver energy level, and vice versa, for receiver, but they do have the knowledge of the energy arrival distribution at each other end. Also, both of them see the common channel h to decide $P_h(t)$ and $\mathbf{1}_R(h,t)$, through which they can possibly have an estimate about the energy level at the other end. Hence, the energy allocation strategy at transmitter and receiver has an interesting dependence on each other through the common channel information.

The maximum achievable throughput by any energy allocation policy is then given by,

$$\tilde{T}_{max} = \max_{\mathbf{1}_R(h,t),P_h(t)} \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} \mathbf{1}_R(h_t,t)\log(1+h_tP_h(t)). \tag{4.3}$$

Note that energy allocation strategy now refers to a distributed policy at transmitter and receiver with the knowledge of common channel information and the energy arrival distribution at both the ends.

By ergodicity,

$$\tilde{T}_{max} = \max_{\mathbf{1}_R(h,t);P_h(t)} \mathbb{E}\left[\mathbf{1}_R(h_t,t)\log(1+h_tP_h(t))\right]. \tag{4.4}$$

Clearly, the indicator random variable of the receiver $\mathbf{1}_R(h_t,t)$ is dependent on the channel fade state h_t. Applying Cauchy-Schwarz on the indicator random variable and the log term, we get

$$\mathbb{E}\left[\mathbf{1}_R(h_t,t)\log(1+h_tP_h(t))\right] \le \mathbb{E}\left[\left(\mathbf{1}_R(h,t)\right)^2\right]\mathbb{E}\left[\log^2\left(1+h_tP_h(t)\right)\right], \tag{4.5}$$

for any receiver policy $\mathbf{1}_R(h,t)$ and transmitter policy $P_h(t)$.

Thus, for any $\mathbf{1}_R(h_t,t)$ and $P_h(t)$, we have,

$$\tilde{T} \le \mathbb{E}\left[\left(\mathbf{1}_R(h_t,t)\right)^2\right]\mathbb{E}\left[\log^2\left(1+h_tP_h(t)\right)\right]. \tag{4.6}$$

Let $\mathbf{1}_R^*(h_t,t)$ and $P_h^*(t)$ be the optimal energy allocation strategy at the receiver and the transmitter respectively. Then,

$$\tilde{T}_{max} \le \mathbb{E}\left[\left(\mathbf{1}_R^*(h_t,t)\right)^2\right]\mathbb{E}\left[\log^2\left(1+h_tP_h^*(t)\right)\right]. \tag{4.7}$$

Note that the second moment of the indicator random variable at receiver, $\mathbf{1}_R^*(h_t,t)$, cannot exceed the second moment of the receiver energy arrival $\tilde{E}_t$. Also, the second term can be upper bounded by using Jensen's inequality and then Cauchy-Schawrz by following the exactly same steps as in Theorem 1 . We thus get,

$$\tilde{T}_{max} \leq 2\sqrt{\mathbb{E}\left[\left(\tilde{E}_t\right)^2\right]}\log\left(1+\sqrt{\mathbb{E}\left[(h_t)^2\right]}\sqrt{\mathbb{E}\left[(E_t)^2\right]}\right). \tag{4.8}$$

As we have assumed an exponentially distributed channel with a mean of unity, $\mathbb{E}\left[h_t^2\right] = 2$. Thus,

$$\tilde{T}_{max} \leq 2\sqrt{\mathbb{E}\left[\left(\tilde{E}_t\right)^2\right]}\log\left(1+\sqrt{2}\sqrt{\mathbb{E}\left[(E_t)^2\right]}\right). \tag{4.9}$$

The term on the right hand side of (4.9) serves as an upper bound on the achievable throughput by any energy allocation strategy. We denote it by $\tilde{T}_{ub}$.

Next we consider a simple case of receiver energy harvesting being Bernoulli. The receiver receives one unit of energy with probability q and does not receive any energy with probability $1-q$. We then have $\mathbb{E}\left[\tilde{E}_t^2\right] = q$. Equation (4.9) then becomes

$$\tilde{T}_{ub} = 2\sqrt{q}\log\left(1+\sqrt{2}\sqrt{\mathbb{E}\left[(E_t)^2\right]}\right). \tag{4.10}$$

Let us consider a simple energy allocation policy at the receiver,

$$\mathbf{1}_R(h_t,t) = \begin{cases} 1 & \text{if the receiver has energy,} \\ 0 & \text{otherwise.} \end{cases} \tag{4.11}$$

Using the Constant Fraction Policy at the transmitter, the throughput achieved is given by,

$$\tilde{T}_{lb} = q\sum_{j=0}^{\infty} p(1-p)^j \int_0^{\infty} \frac{1}{2}\log(1+hp(1-p)^j B_{max})e^{-h}dh. \tag{4.12}$$

which follows from Lemma 1.

Note that $\tilde{T}_{ub} = 2\sqrt{q}\,T_{ub}$ and $\tilde{T}_{lb} = qT_{lb}$ where T_{ub}, and T_{lb} are the upper bound on maximum achievable throughput, and the throughput achieved by Constant Fraction Policy respectively without receiver energy harvesting as defined before. From Theorem 1, $T_{ub} - T_{lb}$ is bounded but because of the $\sqrt{q}$ dependence in $\tilde{T}_{ub}$ and q dependence in $\tilde{T}_{lb}$, the difference $\tilde{T}_{ub} - \tilde{T}_{lb}$ is unbounded. Also, $\frac{\tilde{T}_{ub}}{\tilde{T}_{lb}} = \frac{2}{\sqrt{q}} \frac{T_{ub}}{T_{lb}}$ goes unbounded for $q \to 0$. This is because for $\mathbb{E}[E_t] \to \infty$, $T_{ub} \to \infty$ and as $T_{ub} - T_{lb}$ is bounded, we also have $T_{lb} \to \infty$. Thus, $\frac{\tilde{T}_{ub}}{\tilde{T}_{lb}} = \frac{2}{\sqrt{q}} \frac{T_{ub}}{T_{lb}} \to \frac{2}{\sqrt{q}}$ which goes unbounded for $q \to 0$. For other energy arrivals at receiver, it becomes even more complicated. It is not clear how to use the common channel information to get a higher throughput.

While the most general case with receiver energy harvesting is difficult to solve, we look at certain specific cases providing interesting results.

4.2 Finite Horizon problem with Discrete Energy arrival process

We first consider the case of Bernoulli energy arrivals and binary energy consumption model. The Bernoulli parameter at the transmitter is p and that at receiver is q. Also we assume that the transmitter and receiver has complete knowledge of the channel fade state and the battery energy level at the other end. Owing to binary energy consumption, we denote by r_k the reward obtained if both the transmitter and receiver spend one unit of energy at time step k with the channel fade state being h_k.

Theorem 4.2.1. Under Bernoulli energy arrivals and binary energy consumption model, the optimum transmission policy is given by the following threshold rule.

$$\mathbf{1}_T(h_t,t) = 1_R(h_t,t) = \begin{cases} 1 & \text{if } r_k + \gamma_{k+1}^{m-1,n-1} > \gamma_{k+1}^{m,n}, \\ 0 & \text{otherwise} \end{cases} \tag{4.13}$$

where m and n is the energy available at transmitter and receiver respectively at time k.

The thresholds are given by

$$\gamma_n^{0,0} = pq\mathbb{E}[r_n],$$
$$\gamma_n^{i,0} = q\mathbb{E}[r_n] \text{ for } i > 0,$$
$$\gamma_n^{0,j} = p\mathbb{E}[r_n] \text{ for } j > 0,$$
$$\gamma_n^{i,j} = \mathbb{E}[r_n] \text{ for } i, j > 0.$$

For $k < n$,

$$\gamma_k^{m,n} = (1-p)(1-q)\mathbb{E}\left[\max\{r_k + \gamma_{k+1}^{m-1,n-1}, \gamma_{k+1}^{m,n}\}\right] + p(1-q)\mathbb{E}\left[\max\{r_k + \gamma_{k+1}^{m,n-1}, \gamma_{k+1}^{m+1,n}\}\right] + (1-p)q\mathbb{E}\left[\max\{r_k + \gamma_{k+1}^{m-1,n}, \gamma_{k+1}^{m,n+1}\}\right] + pq\mathbb{E}\left[\max\{r_k + \gamma_{k+1}^{m,n}, \gamma_{k+1}^{m+1,n+1}\}\right],$$

where $\gamma_k^{m,n} = -\infty$ if either $m < 0$ or $n < 0$.

Proof: The proof of this theorem is on the same lines as (Vaze and Jagannathan, 2014). It is a result obtained by modeling the given problem as a Dynamic program.

□

Note that the thresholds depend only on the distribution of the channel fade state and the number of slots available for transmission, and can be computed before hand.

We next consider a more general scenario, where both the energy arrival and transmitted energy can take any discrete value between 0 and B_{max}. Here too, we can explicitly characterize the optimal finite horizon throughput maximizing policy. Assume i units of energy arrive during each slot with probability $p_i,\ i = 1, 2, \ldots, B_{max}$, and that this is i.i.d. across time.

Theorem 4.2.2. Suppose m and n units of energy are available at transmitter and receiver respectively at time k. Then the optimal policy is to transmit $F_k^* = q$ units of energy, where $q = \underset{j\in(0,1,\ldots,m)}{\text{argmax}} \left[\log(1+jh_k) + \gamma_{k+1}^{m-j,n-\mathbf{1}(j>0)}\right]$.

Proof: The proof of this theorem is same as the last one and is left to the reader.

□

As before, the thresholds depend only on the distribution of energy arrivals and channel gains and can be pre-computed. Note that the number of thresholds goes as B_{max}^2 for each time step k.

This solution can be extended to the case of multiple nodes. We assume that at each time slot, an i.i.d. channel realization is revealed to each of the transmitter-receiver pair, and all the nodes harvest energy from environment governed by their respective energy arrival profiles. We also assume that at most one of the nodes is allowed to transmit in a particular time slot. For such a system, the optimal energy consumption policy and scheduling of the communication links for finite horizon can again be characterized using pre-computable thresholds. However, in this case the number of thresholds for each time step goes as $\left(B_{max}^2\right)^n$ where n is the number of transmitter-receiver pairs.

The number of thresholds grows very rapidly as the number of communication link increases. To avoid this problem, we propose a sub-optimal policy for the case of multiple nodes. The sub-optimal policy takes decoupled decisions at each node, considering that it were the only node. In case more than one node is to remain on through the decoupled decisions, choose the one with the highest channel fade state and transmit using that communication link (shutting off the other ones). For this sub-optimal policy, the number of thresholds to be computed grows linearly in the number of communication links and simulations show that it performs very well in practice. However, no theoretic bounds on the performance could be obtained.

4.3 Infinite Horizon Case

Having looked at the finite horizon case, we next move on to the infinite horizon case, where the objective is to maximize the long term throughput. Recall that the rate function taking into account the receiver is given by

$$r(t) = \mathbf{1}_R(h_t, t) \log\left(1 + h_t P_h(t)\right). \tag{4.14}$$

We start by looking at the case where the receiver is battery powered, imposing an average power constraint at the receiver end. Later, we will look at the case where the receiver also harvests energy from the environment. Before going into the case of receiver, let us recall the optimal power allocation policy for the case of fading channel and an average power constraint at the transmitter. The following is a well known result.

Theorem 4.3.1. Let the probability density function of the channel fade state h be given by $f(h)$ and let there be an average power constraint at the transmitter upper bounding the maximum allowable average power transmit to $\overline{P}$. Moreover, assume that the rate obtained by consuming P_h amount of power when the fade state is h is given by

$$r(h, P_h) = \log(1 + hP_h). \tag{4.15}$$

Then the optimal power allocation policy is given by

$$P(h) = \begin{cases} \frac{1}{\gamma} - \frac{1}{h} & \text{if } h \geq \gamma, \\ 0 & \text{otherwise,} \end{cases} \tag{4.16}$$

where γ can be obtained by solving

$$\overline{P} = \int_{\gamma}^{\infty} P(h) f(h) dh. \tag{4.17}$$

Note that the maximum average throughput is thus given by

$$\begin{aligned}\tilde{T}_{ub} &= \int_{\gamma}^{\infty} \log\left(1+h\left(\frac{1}{\gamma}-\frac{1}{h}\right)\right) f(h)dh \\ &= \int_{\gamma}^{\infty} \log\left(\frac{h}{\gamma}\right) f(h)dh.\end{aligned}$$

Next, we start looking at the case where there is a similar constraint at the receiver end i.e. the average power allowed to be consumed at the receiver is given by $\overline{P_R}$. From here on, we denote the average transmit power by $\overline{P_T}$. It should be easy to see that if $\overline{P_R} \geq 1$, the receiver can always remain on and the problem converts to the one where only average transmit power is the constraint.

Note that $\tilde{T}_{ub}$ (the maximum achievable throughput without the receiver constraint) is trivially an upper bound on the maximum achievable throughput for this case as well.

Theorem 4.3.2. If $\overline{P_R} \geq \mathbb{P}[h \geq \gamma]$, then the optimal power allocation policy is given by

Transmitter

$$P(h) = \begin{cases} \frac{1}{\gamma} - \frac{1}{h} & \text{if } h \geq \gamma, \\ 0 & \text{otherwise,} \end{cases} \tag{4.18}$$

Receiver

$$\mathbf{1}_R(h) = \begin{cases} 1 & \text{if } h \geq \gamma, \\ 0 & \text{otherwise.} \end{cases} \tag{4.19}$$

Proof: The given power allocation policy consumes an average power of $\mathbb{P}[h \geq \gamma]$ at the receiver, which is less than $\overline{P_R}$ hence it is a feasible policy. Also it achieves an average throughput of $\tilde{T}_{ub}$ proving it is optimal.

□

We now move on the case where $\overline{P_R}$ is such that $\overline{P_R} < \mathbb{P}[h \geq \gamma]$.

Theorem 4.3.3. For an average power constraint at both the ends, the optimal energy policy will not allow (or allow only for a finite number of times) instances where $P_h(t) > 0$ but $\mathbf{1}_R(t) = 0$.

Proof: This can be easily proved by contradiction. The complete proof is left upon the reader but we provide a brief outline. If the event mentioned above happens recurrently, then the given policy can be improvised by shifting the power used for transmission when $\mathbf{1}_R(t) = 0$ to instances where $\mathbf{1}_R(t) = 1$ and then arguing that the rate function is monotonically increasing in transmit power. □

Theorem 4.3.4. Let $\overline{P_R} < \mathbb{P}[h \geq \gamma]$ and let γ_0 be such that $\mathbb{P}[h \geq \gamma_0] = \overline{P_R}$. Then the optimal power allocation strategy at the receiver is

$$\mathbf{1}_R(h) = \begin{cases} 1 & \text{if } h \geq \gamma_0, \\ 0 & \text{otherwise} \end{cases} \tag{4.20}$$

Proof: The proof of this theorem also follows easily by contradiction and is left to the reader. □

It is important to note now that for $h < \gamma_0$, the receiver will be off. At such instances no matter what transmit power is consumed, the throughput achieved will be zero. In other words, with the receiver constraint, all h such that $h < \gamma_0$ are equivalent to $h = 0$. Hence, the channel distribution seen by the transmitter is now a mixed random variable with a non-zero mass of $\int\limits_0^{\gamma_0} f(h)dh$ at $h = 0$ and distributed according to $f(h)$ for $h > \gamma_0$. We denote this modified distribution as $\tilde{f}(h)$. This observation, with the previous two results leads to the following major theorem.

Theorem 4.3.5. Let the average transmit power be given by $\overline{P_T}$ and let the channel fade state h be distributed according to the density $f(h)$. Let the average power allowed to be used at the receiver be $\overline{P_R}$. Let γ, γ_0 and $\tilde{f}(h)$ be as defined before.

- If $\overline{P_R} \geq \mathbb{P}[h \geq \gamma]$, then the optimal power allocation policy is given by

Transmitter

$$P(h) = \begin{cases} \frac{1}{\gamma} - \frac{1}{h} & \text{if } h \geq \gamma, \\ 0 & \text{otherwise} \end{cases} \tag{4.21}$$

Receiver

$$\mathbf{1}_R(h) = \begin{cases} 1 & \text{if } h \geq \gamma, \\ 0 & \text{otherwise} \end{cases} \tag{4.22}$$

- If $\overline{P_R} < \mathbb{P}[h \geq \gamma]$, then the optimal power allocation policy is given by

  Transmitter

  $$P(h) = \begin{cases} \frac{1}{\tilde{\gamma}} - \frac{1}{h} & \text{if } h \geq \tilde{\gamma}, \\ 0 & \text{otherwise} \end{cases} \tag{4.23}$$

  Receiver

  $$\mathbf{1}_R(h) = \begin{cases} 1 & \text{if } h \geq \gamma_0, \\ 0 & \text{otherwise,} \end{cases} \tag{4.24}$$

  where $\tilde{\gamma}$ is obtained by solving

  $$\overline{P_T} = \int_{\tilde{\gamma}}^{\infty} \left(\frac{1}{\tilde{\gamma}} - \frac{1}{h} \right) \tilde{f}(h) dh. \tag{4.25}$$

Proof: The proof follows from the previous theorems and the fact that water-filling is optimal power allocation strategy for any channel state distribution as long as the rate function is $r(h, P_h) = \log\left(1 + hP(h)\right)$.

□

Having looked at the case of average power constraint, we next move on to the case where both the transmitter and the receiver harvest energy from environment. As before, the energy arrivals are modeled as discrete stochastic processes. The harvested energy is stored in a battery with a capacity of B_{max} at the transmitter and $\tilde{B}_{max}$ at the receiver. The operation now is fundamentally governed by *Energy Neutrality Constraint*.

We denote the energy arrival at transmitter by E_T with an average $\mathbb{E}[E_T]$. Similarly, the energy arrival at the receiver is given by E_R with an average $\mathbb{E}[E_R]$. Note that the throughput obtained by the policy mentioned in Theorem 7 with $\overline{P_T} = \mathbb{E}[E_T]$ and $\overline{P_R} =$

$\mathbb{E}[E_R]$ is trivially an upper bound to the achievable throughput for the case of energy harvesting. We denote this upper bound by $\tilde{T}_{ub}$.

We now consider the case where $B_{max} = \tilde{B}_{max} = \infty$. Let B_k and $\tilde{B}_k$ denote the battery levels at transmitter and receiver at time k. The following theorem then describes the maximum achievable throughput.

Theorem 4.3.6. For every $\varepsilon > 0$, there exists power allocation policy which achieves a throughput of $\tilde{T}_{ub} - \varepsilon$.

Proof: This theorem has a constructive proof. In other words, we construct a policy which achieves a throughput of $\tilde{T}_{ub} - \varepsilon$. Let γ be obtained by the following equation.

$$\mathbb{E}[E_T] - \delta_1 = \int_{\gamma}^{\infty} P(h) f(h) dh. \tag{4.26}$$

We start with the case when $\mathbb{E}[E_R] > \mathbb{P}[h \geq \gamma]$. Also define γ_0 such that $\mathbb{E}[E_R] - \delta_2 = \mathbb{P}[h \geq \gamma_0]$. Note that $\gamma_0 \leq \gamma$ for suitably chosen δ_2.

Next, consider the following policy.

Transmitter

$$P(h) = \begin{cases} \frac{1}{\gamma} - \frac{1}{h} & \text{if } h \geq \gamma \text{ and there is sufficient energy,} \\ 0 & \text{otherwise} \end{cases} \tag{4.27}$$

Receiver

$$\mathbf{1}_R(h) = \begin{cases} 1 & \text{if } h \geq \gamma_0 \text{ and there is sufficient energy,} \\ 0 & \text{otherwise,} \end{cases} \tag{4.28}$$

The idea of this policy is that the throughput obtained by the energy allocation policy in Theorem 7 is continuous in $\overline{P_T}$ and $\overline{P_R}$. By using an average power of $\mathbb{E}[E_T] - \delta_1$ and $\mathbb{E}[E_R] - \delta_2$, the available energy at the transmitter and the receiver increases with time. This effectively reduces the randomness in energy harvested as shown in Khairnar and

Mehta (2011). Also, by making δ_1 and δ_2 small enough, a throughput arbitrarily close to $\tilde{T}_{ub}$ can be achieved which completes the proof for $\mathbb{E}[E_R] > \mathbb{P}[h \geq \gamma]$.

Next consider the case of $\mathbb{E}[E_R] < \mathbb{P}[h \geq \gamma]$. For this case let γ_0 be such that $\mathbb{P}[h \geq \gamma_0] = \mathbb{E}[E_R] - \delta_2$. Also let $\tilde{f}(h)$ be as defined before and $\tilde{\gamma}$ be obtained by

$$\overline{P_T} - \delta_1 = \int_{\tilde{\gamma}}^{\infty} \left(\frac{1}{\tilde{\gamma}} - \frac{1}{h}\right) \tilde{f}(h) dh. \tag{4.29}$$

Next, consider the following energy consumption policy

Transmitter

$$P(h) = \begin{cases} \frac{1}{\tilde{\gamma}} - \frac{1}{h} & \text{if } h \geq \tilde{\gamma}, \\ 0 & \text{otherwise} \end{cases} \tag{4.30}$$

Receiver

$$\mathbf{1}_R(h) = \begin{cases} 1 & \text{if } h \geq \gamma_0, \\ 0 & \text{otherwise,} \end{cases} \tag{4.31}$$

By using an average power of $\mathbb{E}[E_T] - \delta_1$ and $\mathbb{E}[E_R] - \delta_2$, the available energy at the transmitter and the receiver increases with time. This effectively reduces the randomness in energy harvested as shown in Khairnar and Mehta (2011). Also, by making δ_1 and δ_2 small enough, a throughput arbitrarily close to $\tilde{T}_{ub}$ can be achieved which completes the proof.

□

Next, we look at the case when $B_{max} = \infty$ and $\tilde{B}_{max} = 1$. Here we have a Bernoulli energy arrival process at receiver with parameter $q = \mathbb{E}[E_R]$.

Theorem 4.3.7. For the above mentioned case, the following policy

Transmitter

$$P(h) = \begin{cases} \frac{1}{\gamma} - \frac{1}{h} & \text{if } h \geq \gamma \text{ and there is sufficient energy,} \\ 0 & \text{otherwise} \end{cases} \tag{4.32}$$

Receiver

$$\mathbf{1}_R(h) = \begin{cases} 1 & \text{if } h \geq \gamma_0 \text{ and there is sufficient energy,} \\ 0 & \text{otherwise,} \end{cases} \tag{4.33}$$

can achieve a throughput arbitrarily close to half the maximum achievable throughput. Here γ_0 is given by

$$\mathbb{P}[h \geq \gamma_0] = q. \tag{4.34}$$

Proof: Note that the time until which you have to wait for an energy arrival at receiver is geometric with parameter q. Let this be denoted by X_1 The waiting time for the event $\mathbf{1}_{h \geq \gamma_0}$ is again geometric with parameter q. Let this be denoted by X_2. Owing to the memory-less property of geometric random variable, the system refreshes whenever X_1 or X_2 fires. Also it should be noted that the system behaves identically to the case where only transmitter has energy constraint (receiver is always on) as long as $X_1 \leq X_2$ which happens with probability $\frac{1}{2}$ proving the theorem.

□

4.4 A special case with $B_{max} = \tilde{B}_{max} = 1$

In this section, we consider a special case of $B_{max} = \tilde{B}_{max} = 1$ and binary energy transmission policy at transmitter. Also, the energy arrivals at the transmitter and the receiver are assumed to be Bernoulli. For this case, we first obtain an upper bound on the maximum achievable throughput by any energy allocation strategy at the transmitter and the receiver. Next, we propose an energy utilization policy for the transmitter and the receiver for which we can universally bound the ratio of the throughput achieved by the proposed strategy and the upper bound on maximum achievable throughput.

For this system, the throughput obtained when the channel fade state is h_t, is given by,

$$r(t) = \mathbf{1}_T(h_t,t)\mathbf{1}_R(h_t,t)\log(1+h_t), \tag{4.35}$$

where $\mathbf{1}_T(h_t,t)$, and $\mathbf{1}_R(h_t,t)$, are the indicator random variables corresponding to the energy usage at transmitter and receiver respectively.

$$\mathbf{1}_T(h_t,t) = \begin{cases} 1 & \text{if the transmitter spends one unit of energy,} \\ 0 & \text{otherwise.} \end{cases} \tag{4.36}$$

The maximum achievable throughput is given by,

$$\tilde{T}_{max} = \max_{\mathbf{1}_T(h_t,t),\mathbf{1}_R(h_t,t)} \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} \mathbf{1}_T(h_i,i)\mathbf{1}_R(h_i,i)\log(1+h_i). \tag{4.37}$$

By egotist,

$$\tilde{T}_{max} = \max_{\mathbf{1}_T(h_t,t),\mathbf{1}_R(h_t,t)} \mathbb{E}\left[\mathbf{1}_T(h_t,t)\mathbf{1}_R(h_t,t)\log(1+h_t)\right]. \tag{4.38}$$

Next, we obtain an upper bound on the maximum achievable throughput for this case.

Lemma 4.4.1. Let the energy arrival process at transmitter and receiver be Bernoulli with parameters p and q respectively. Also, $B_{max} = \tilde{B}_{max} = 1$. Then the maximum achievable throughput by any distributed energy allocation policy with discrete energy usage at transmitter is upper bounded by,

$$\tilde{T}_{ub} = \min\{p,q\}\int_{\gamma^*}^{\infty} \log(1+h)\, f(h)dh, \tag{4.39}$$

where $f(h) = e^{-h}$ is the probability density function of the channel fade state and $\gamma^* =$

$-\ln(\min\{p,q\})$.

Proof: Let us assume that $p > q$. Next, assume that the transmitter always has energy to transmit i.e. at each time t, $\mathbf{1}_T(h_t,t) = 1$. Clearly, this will result in an upper bound on the maximum achievable throughput. The throughput obtained is then given by,

$$r(t) = \mathbf{1}_R(h_t,t)\log(1+h_t). \tag{4.40}$$

For such a system, for $\tilde{B}_{max} = 1$, the optimal transmission policy has been shown to be of threshold type in Michelusi *et al.* (2012), Sinha and Chaporkar (2012) i.e.

$$\mathbf{1}_R(h_t,t) = \begin{cases} 1 & \text{if } h_t > \gamma \text{ and } \tilde{B}_t = 1, \\ 0 & \text{otherwise.} \end{cases} \tag{4.41}$$

Next, we argue that the optimal threshold γ^* will satisfy $\mathbb{P}(h > \gamma^*) = q$. This is true because of the following reasons:

- If the threshold is greater than γ^*, the energy arrival rate at the receiver is greater than the energy usage at the receiver, and essentially there is a wastage of energy owing to unit battery size resulting in sub-optimality.
- If the threshold is less than γ^*, the receiver remains on for not so good channel fade state while it could have obtained a better throughput by remaining on for a higher channel gain.

Note that $\mathbb{P}(h > \gamma^*) = q$ implies $\gamma^* = -\ln q$ as h is exponentially distributed with mean 1.

Thus the maximum achievable throughput of such a communication system is upper bounded by,

$$\tilde{T}_{ub} = q\int_{\gamma^*}^{\infty} \log(1+h)f(h)dh. \tag{4.42}$$

This proves the Lemma for $p > q$. The other case, $q > p$, can be proved by interchanging the role of the transmitter and the receiver.

□

Next, we propose the following energy allocation policy. We call it *Common Threshold Policy*.

- Transmitter Policy : The transmitter simulates an i.i.d. Bernoulli random variable for each slot with parameter q. The corresponding epochs are then geometrically distributed with parameter q. We call this random variable X_1. The transmitter waits till X_1 happens and thereafter transmits whenever $h_t > \gamma^*$ if $B_t = 1$ and saves energy for future use if $h_t < \gamma^*$.
- Receiver Policy : The receiver remains on whenever $h_t > \gamma^*$ if the receiver battery level, $\tilde{B}_t = 1$, and saves energy for future use if $h_t < \gamma^*$, where $\gamma^* = -\ln(\min\{p,q\})$.

Theorem 4.4.1. The throughput achieved by Common Threshold Policy, $\tilde{T}_{lb}$, satisfies,

$$\tilde{T}_{lb} \geq \frac{1}{2}\,\tilde{T}_{ub}. \tag{4.43}$$

Proof:

Consider the case of $p > q$. Let the transmitter simulate an i.i.d. Bernoulli random variable for energy arrivals at each time slot with parameter q. The corresponding inter arrival time for energy packets is now a geometric random variable, X_1, with parameter q. Note that the actual energy arrival at transmitter is Bernoulli with parameter $p > q$ and the actual energy packet arrives before X_1. The channel fade state is assumed to be i.i.d. exponential and hence $\mathbf{1}_{h_t > \gamma^*}$ is Bernoulli with parameter q as $\mathbb{P}(h_t > \gamma^*) = q$. The corresponding epochs are geometric random variable with parameter q. We call this random variable X_2. Finally, the energy arrivals at receiver are i.i.d. Bernoulli with parameter q. Hence, the corresponding inter arrival time are energy packets is a geometric random variable with parameter q. We call this random variable X_3. Note that X_1, X_2, and X_3 are i.i.d. random variables.

We now compare the throughput obtained by Common Threshold Policy $\tilde{T}_{lb}$ with the upper bound on maximum achievable throughput $\tilde{T}_{ub}$. Note that the system is reset to the initial state each time the event $\mathbf{1}_{h_t > \gamma^*}$ occurs owing to the memory-less property of geometric random variable. By this we mean the following. Let the system start at $t=0$ and let $t_0 = \min\{t : h_t > \gamma^*\}$. Then the time taken after $t = t_0$ for the next energy arrival at transmitter, for the channel fade state to be better than γ^*, and the next energy arrival at receiver are again i.i.d. geometric random variables with parameter q. Thus, the system is reset to the initial state of $t=0$ and each epoch corresponding to the event $h_t > \gamma^*$ is independent and statistically independent to every other epoch. Hence, it suffices to compare $\tilde{T}_{ub}$ with $\tilde{T}_{lb}$ for one particular epoch.

Without loss of generality, let us compare $\tilde{T}_{ub}$ with $\tilde{T}_{lb}$ in the first epoch. Let the system start at $t=0$. The time taken for the first energy arrival at transmitter, the first time channel fade state is greater than γ^*, and the first energy arrival at receiver are indeed X_1, X_2, and X_3.

Lemma 4.4.2. The policy used for upper bounding the maximum achievable throughput and Common Threshold Policy will perform identically if $X_1 \leq X_2$.

Proof: Note that the receiver is following the same energy allocation policy under Common Threshold Policy as the policy used for upper bounding the maximum achievable throughput. If $X_1 \leq X_2$ i.e. the transmitter gets an energy packet before $t = t_0$, the transmitter holds on the energy under Common Threshold Policy and uses it at $t = t_0$ when $h_{t_0} > \gamma^*$. Thus, both the policies obtain the same throughput of $\frac{1}{2}\log\left(1 + h_{t_0}\right)$.

□

Next we lower bound $\mathbb{P}(X_1 \leq X_2)$.

Note that X_1 and X_2 are i.i.d. random variables. By symmetry, we have,

$$\mathbb{P}(X_1 > X_2) = \mathbb{P}(X_1 < X_2). \tag{4.44}$$

Also, we have,

$$\mathbb{P}(X_1 > X_2) + \mathbb{P}(X_1 = X_2) + \mathbb{P}(X_1 < X_2) = 1. \tag{4.45}$$

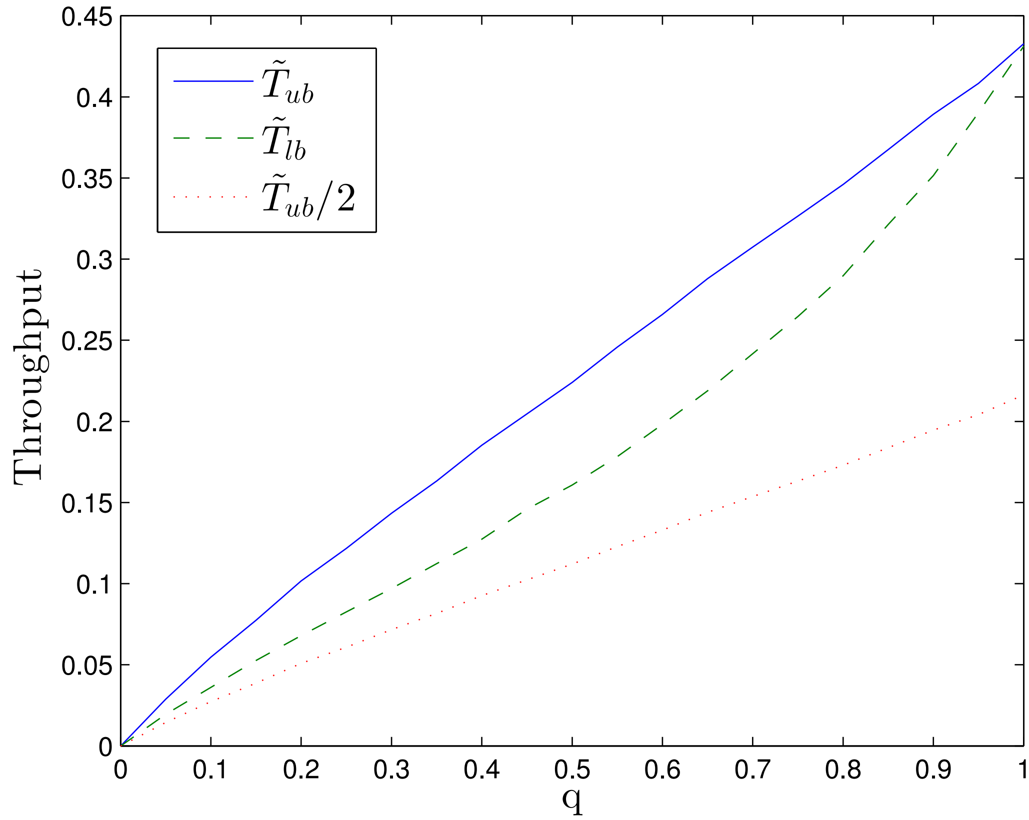

Figure 4.1: Performance CTP as compared to to the upper bound on maximum achievable long term throughput for different values of q. The solid curve represents the upper bound $\tilde{T}_{ub}$ and the dashed curve corresponds to the per slot throughput obtained by CTP $\tilde{T}_{lb}$. The dotted curve is $\frac{1}{2}\tilde{T}_{ub}$.

From (4.44) and (4.45),

$$\mathbb{P}(X_1 \leq X_2) = \mathbb{P}(X_1 < X_2) + \mathbb{P}(X_1 = X_2) \geq 0.5. \tag{4.46}$$

We now get the desired multiplicative bound of $\frac{1}{2}$ from Lemma 7 and (4.46).

This proves the Theorem for $p > q$. The other case can be proved by interchanging the role of the transmitter and the receiver.

□

CHAPTER 5

Transmitter-Receiver Energy Harvesting link with Centralized Control

5.1 System Model

In this chapter, we consider a single transmitter-receiver pair, where both the transmitter and receiver harvest energy from the environment. Let E_t be the amount of energy harvested at the transmitter at time t and $\tilde{E}_t$ be the amount of energy harvested at the receiver at time t. The battery capacity to store the harvested energy are denoted by B_{max} and $\tilde{B}_{max}$ at the transmitter and the receiver respectively. At each time t, the fading channel between the transmitter and the receiver is h_t, where h_t is i.i.d. distributed across the time slots. Let B_t be the battery energy level at the transmitter at time t and let $P_h(t) \leq B_t$ be the energy consumed for transmission at time t. The energy state at the transmitter thus evolves as follows,

$$B_{t+1} = \min\{B_t - P_h(t-1)\mathbf{1}_{P_h(t-1)\leq B_t}, B_{max}\} \tag{5.1}$$

Compared to the transmitter, the receiver decision structure is simpler; it only has to decide whether to stay *on* or *off* in any given slot. When the receiver is *on*, it consumes a fixed amount of energy. In the *off* state, it cannot receive any data and the throughput obtained is zero. In this state, the receiver does not consume any energy. Let $\mathbf{1}_R(h_t,t) = 1$ if receiver is *on* at time t, otherwise 0. Then the rate obtained at time t is

$$\tilde{r}(t) = \mathbf{1}_R(h_t,t)\log\left(1+h_tP_h(t)\right), \tag{5.2}$$

and long-term throughput is given by

$$\tilde{T} = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} \tilde{r}(t). \tag{5.3}$$

Our objective is to find optimal $P_h(t)$ and $\mathbf{1}_R(h_t,t)$, given the energy neutrality constraint $P_h(t) \leq B_t$, and $\mathbf{1}_R(h_t,t) \leq \tilde{B}_{max}$, that maximizes $\tilde{T}$, i.e.

$$\tilde{T}^\star = \max_{P_h(t)\leq B_t, \mathbf{1}_R(h_t,t)\leq \tilde{B}_{max}} \tilde{T}. \tag{5.4}$$

It has been shown that with receiver energy harvesting, the problem of approximating $\tilde{T}^\star$ is very hard and finding the difference of the lower bound and upper bound is challenging for the case when transmitter and receiver are separated and do not have access to each others' energy availability information.

In this chapter, we take a recourse by considering the case where we have a centralized system and the transmitter and receiver have complete access to each others' energy state. We also assume a discrete energy arrival process at the transmitter and the receiver, and impose a discrete energy consumption constraint at the transmitter. With these assumptions, we shall see that an explicit characterization of the optimal energy allocation strategy becomes possible. We also derive some structural properties of the optimal policy which may be of vital importance in understanding the kind of algorithms and approximations needed in real-life scenarios.

5.2 Unit Battery Capacity at Transmitter and Receiver

5.2.1 Binary Channel

In this section, we start with a simple case where $B_{max} = \tilde{B}_{max} = 1$. With a battery capacity of 1, we can assume a Bernoulli energy arrival at the transmitter and receiver without loss of generality. Let the Bernoulli parameters at the transmitter and the receiver be p and q respectively. We assume $p, q > 0$. With the discrete energy consumption constraint, the rate function is given by

$$r(t) = \mathbf{1}_R(t)\mathbf{1}_T(t)\log\left(1 + h_t\right), \tag{5.5}$$

where

$$\mathbf{1}_T(t) = \begin{cases} 1 & \text{if } P_h(t) = 1, \\ 0 & \text{if } P_h(t) = 0. \end{cases} \tag{5.6}$$

To begin with, we assume that the channel fade state has only two states $h_1 < h_2$ i.e. the channel fade state is h_1 with probability $p(h_1)$ and h_2 with probability with $p(h_2) = 1 - p(h_1)$. We assume $p(h_1), p(h_2) > 0$. We model the evolution of such a system as a Markov decision process. With two energy states at transmitter, two energy states at receiver and two channel fade states, we have a total of 8 states.

- $00h_1$
- $00h_2$
- $01h_1$
- $01h_2$
- $10h_1$
- $10h_2$
- $11h_1$
- $11h_2$

In our notation, the first entry denotes the energy level at the transmitter, the second entry denotes the energy level at the receiver, and the third entry denotes the channel fade state. For instance, a state “$10h_1$ corresponds to $B_t = 1$, $\tilde{B}_t = 0$ and channel fade state h_1. Since we are looking at a centralized controller, it is easy to see that in states where $B_t = 0$ or $\tilde{B}_t = 0$, irrespective of the channel fade state, the optimal energy utilization strategy would be $\mathbf{1}_T(t) = 0$ and $\mathbf{1}_R(t) = 0$. This is because $r(t) > 0$ only if $\mathbf{1}_T(t) = \mathbf{1}_R(t) = 1$ which is not possible with $B_t = 0$ or $\tilde{B}_t = 0$. With this observation, we can combine $00h_1$ and $00h_2$ into one state, which we denote now by 00. Similarly $01h_1$ and $01h_2$ can be combined into 01, and $10h_1$ and $10h_2$ can be combined into 10. This leaves us with the following state space, and we number them in the following way for ease of notation.

- $00 \to 1$
- $01 \to 2$
- $10 \to 3$
- $11h_1 \to 4$
- $11h_2 \to 5$

Owing to a centralized controller, note that the only possible action for the optimal policy in the states $1, 2$, and 3 is $\mathbf{1}_R(t) = \mathbf{1}_T(t) = 0$. In the states $11h_1$ and $11h_2$, there are two possible decisions, or two possible *actions* according to the MDP literature. These are

- $1 \to \mathbf{1}_T(t) = \mathbf{1}_R(t) = 0.$
- $2 \to \mathbf{1}_T(t) = \mathbf{1}_R(t) = 1.$

Note that the other two actions, namely $\mathbf{1}_T(t) = 0$ and $\mathbf{1}_T(t) = 1$, and $\mathbf{1}_T(t) = 1$ and $\mathbf{1}_T(t) = 0$ are clearly sub-optimal and hence we eliminate them. Let $r(s,a)$ be the reward obtained by taking action a in state s. We then have

- $r(4,1) = 0$
- $r(4,2) = \log(1 + h_1)$
- $r(5,1) = 0$
- $r(5,2) = \log(1 + h_2)$

Observe that the actions taken in states 4 and 5, will have corresponding transition probabilities. For example, under section 2 in state 5, we have $p_{51} = (1-p)(1-q)$, $p_{52} = (1-p)q$, $p_{53} = p(1-q)$, $p_{54} = pqp(h_1)$ and $p_{55} = pqp(h_2)$.

For this simple case, our objective is to find optimal actions in the states 4 and 5 to maximize the average long-term throughput. It should be noted that the actions at states 4 and 5 maybe random in nature i.e. one may chose to decide to transmit in state 5 with some probability and not transmit with the remaining probability. This leaves us with a continuum of feasible policies at states 4 and 5 any one of which can potentially be optimal.

We now go on to find the optimal policy. We denote the unconditional steady state probability of being in state s and taking action a by $x(s,a)$. With only one possible actions in states 1,2 and 3, we are left to find $x(4,1)$,$x(4,2)$,$x(5,1)$ and $x(5,2)$ that maximize the long-term throughput.

Therefore, our objective now is to find the optimal state action frequencies. These can be obtained as follows. Let $x(s,a)$ be the unconditional probability of being in state s and taking action a, and let $r(s,a)$ be the reward obtained by taking action a in state s. Then the solution to the following linear program will give the optimal state action frequencies.

$$\text{Maximize} \sum_{s\in S}\sum_{a\in A_s} x(s,a)r(s,a)$$

subject to

$$\sum_{a\in A_j} x(j,a) - \sum_{s\in S}\sum_{a\in A_s} p(j|s,a)x(s,a) = 0, \ \forall j \in S$$

$$\sum_{s\in S}\sum_{a\in A_s} x(s,a) = 1$$

where S is the state space and A_s is the set of all possible actions in state s.

Note that once we have these optimal unconditional probabilities, they can be transformed into an energy utilization policy in the following way.

- $\mathbb{P}\left[\text{Taking action 1 given state 4}\right] = \frac{x(4,1)}{x(4,1)+x(4,2)}$
- $\mathbb{P}\left[\text{Taking action 2 given state 4}\right] = \frac{x(4,2)}{x(4,1)+x(4,2)}$
- $\mathbb{P}\left[\text{Taking action 1 given state 5}\right] = \frac{x(5,1)}{x(5,1)+x(5,2)}$
- $\mathbb{P}\left[\text{Taking action 2 given state 5}\right] = \frac{x(5,2)}{x(5,1)+x(5,2)}$

We now prove some structural properties of the optimal energy utilization policy. The following is a well known result in MDP literature.

Theorem 5.2.1. There exists an optimal energy allocation policy with $x(4,1)x(4,2) = 0$ and $x(5,1)x(5,2) = 0$

Proof:

To prove this theorem, we need the following lemma which is a well known result in MDP literature.

Lemma 5.2.1. For an MDP that is either unichain or communicating, there exists a stationary deterministic policy that achieves the optimal average reward.

Note that our MDP is unichain with $p, q, p(h_1), p(h_2) > 0$. There are 4 deterministic policies possible for the MDP at hand.

(P1) Do not transmit in states 4 and 5.

(P2) Transmit in state 4 but not in 5.

(P3) Transmit in state 5 but not in 4.

(P4) Transmit in state 4 as well as 5.

Note that either $x(4,1)$ or $x(4,2)$ is zero for all the four policies. Similarly, either $x(5,1)$ or $x(5,2)$ is zero for all the four policies. This is because in all the four policies we have a deterministic action to be taken in states 4 and 5. For instance in the policy which transmits in state 5 but does not transmit in state 4, we have $x(5,1) = 0$ and $x(5,2) > 0$, and $x(4,1) > 0$ and $x(4,2) = 0$. From lemma 1, we know that one of the above 4 policies is optimal which completes the proof.

□

Knowing that one of the stationary deterministic policy is optimal, we restrict ourselves only to those from now on.

Observe that the long-term throughput achieved by policy P1 is zero as it never chooses to transmit. On the other hand, throughput obtained by P2,P3 and P4 is greater than zero. This is because under each of these policies, all the 5 states ($00, 01, 10, 11h_1$ and $11h_2$) are recurrent and hence there is a non-zero steady state probability of being in each of these states. This in turn ensures a non-zero long-term throughput obtained in at least one of the two states ($00h_1$ and $00h_2$) under P1, P2 or P3.

Theorem 5.2.2. Energy consumption policy P2 cannot be optimal for any choice of the parameters $p, q, h_1, h_2, p(h_1)$ and $p(h_2)$

Proof:

We prove this theorem by contradiction. Let us assume there exists $p, q, h_1, h_2, p(h_1)$ and $p(h_2)$ such that P2 is optimal. Since the energy arrival process is assumed to be independent of the channel fade state, we can also look at the system evolution as a 4 state Markov chain based on the energy states at the transmitter and the receiver i.e. the sates are $00, 01, 10$ and 11. Once the system is in state 11, it looks at the channel fade state h_t. Under policy P2, if $h_t = h_1$, it chooses to transmit. This event occurs with probability $p(h_1)$. With probability $p(h_2) = 1 - p(h_1)$, it chooses not to transmit and remain in the state 11. Under P1, the throughput obtained is given by

$$T(P1) = \pi_4 \; p(h_1) \log(1 + h_1), \tag{5.7}$$

where π_4 is the steady state probability of being in state 11 under P1.

Assume $p(h_2) \geq p(h_1)$. We now propose the following energy allocation policy $\tilde{P}$. In the state 11, do not transmit if the channel fade state is h_1. If the channel fade state is h_2, generate a Bernoulli random variable X with success probability $\frac{p(h_1)}{p(h_2)}$, independently of the channel fade state and transmit if $X = 1$. Note that under $\tilde{P}$, the overall probability of transmission once the system is in 11 state is $\mathbb{P}[h_t = h_2 \text{ and } X = 1] = \mathbb{P}[h_2]\mathbb{P}[X = 1] = p(h_2)\frac{p(h_1)}{p(h_2)} = p(h_1)$. This is same as the transmit probability in state 11 under policy P1. This ensures that the steady state probability of being in state 11 under $\tilde{P}$ is same as the steady state probability of being in state 11 under P2.

Next observe that the long-term throughput obtained by $\tilde{P}$ is given by

$$T(\tilde{P}) = \pi_4 \; p(h_2)\frac{p(h_1)}{p(h_2)} \log(1+h_2) = \pi_4 \; p(h_1) \log(1+h_2). \tag{5.8}$$

Since P2 is optimal $T(P2) \geq T(\tilde{P})$ which results in

$$\pi_4 \; p(h_1) \log(1+h_1) \geq \pi_4 \; p(h_1) \log(1+h_2)$$
$$\log(1+h_1) \geq \log(1+h_2)$$
$$h_1 \geq h_2$$

This is a contradiction as we started with $h_1 < h_2$.

Now consider the case when $p(h_2) < p(h_1)$. We now propose the following energy allocation policy $\tilde{P}$. In the state 11, transmit if the channel fade state is h_2. If the channel fade state is h_2, generate a Bernoulli random variable X with success probability $\frac{p(h_1)-p(h_2)}{p(h_1)}$, independently of the channel fade state and transmit if $X = 1$. Note that under $\tilde{P}$, the overall probability of transmission once the system is in 11 state is $\mathbb{P}[h_t = h_1 \text{ and } X = 1] + p(h_2) = \mathbb{P}[h_1]\mathbb{P}[X = 1] + p(h_2) = p(h_1)\frac{p(h_1)-p(h_2)}{p(h_1)} + p(h_2) = p(h_1)$. This is same as the transmit probability in state 11 under policy P1. This ensures that the steady state probability of being in state 11 under $\tilde{P}$ is same as the steady state probability of being in state 11 under P2.

Next observe that the long-term throughput obtained by $\tilde{P}$ is given by

$$
\begin{aligned}
T(\tilde{P}) &= \pi_4 \left[p(h_1)\frac{p(h_1)-p(h_2)}{p(h_1)}\log(1+h_1)+p(h_2)\log(1+h_2)\right] \\
&= \pi_4\ p(h_1)\log(1+h_1)+p(h_2)\big(\log(1+h_2)-\log(1+h_1)\big).
\end{aligned}
$$

Since P2 is optimal $T(P2) \geq T(\tilde{P})$ which results in

$$
\begin{aligned}
&\pi_4\ p(h_1)\log(1+h_1) \geq \pi_4\ p(h_1)\log(1+h_1)+p(h_2)\big(\log(1+h_2)-\log(1+h_1)\big) \\
&p(h_2)\big(\log(1+h_2)-\log(1+h_1)\big) \leq 0 \\
&h_1 \geq h_2
\end{aligned}
$$

This is a contradiction as we started with $h_1 < h_2$.

□

Remark 5.2.1. From theorem 1, we know that at least one energy consumption policy among P1,P2,P3 and P4 is optimal. The throughput obtained by P1 is zero while theorem 2 proves that P2 cannot possibly be optimal. Thus either P3 or P4 will be throughput optimal. In turn, if we can evaluate system performance, comparing P3 and P4 would result in an optimal energy utilization policy.

Theorem 5.2.3. If the optimal policy for some $p, q, h_1, h_2, p(h_1)$ and $p(h_2)$ is P4, then for every $h_1' > h_1$ with the same distribution for channel fade state and the energy arrival process, the optimal policy P4. In essence, if we were to fix all the system parameters except h_1, $P_T(h_1)$ is monotonically increasing in h_1.

Proof: We prove this by contradiction. Assume that there exists h_1 and h_1' that violate the theorem. Observe that by choosing to transmit at both h_1 and h_2 (P4), we get a corresponding $x^*(4,2), x^*(5,2) > 0$. Let $x(4,1), x(5,2) > 0$ be the state action frequencies under policy P3 (transmit at h_2 but not at h_1). As P4 is optimal, we have

$$x^*(4,2)\log(1+h_1)+x^*(5,2)\log(1+h_2) \geq x(5,2)\log(1+h_2) \tag{5.9}$$

Now, with $h_1' > h_1$ and the same distribution for channel fade state, we have that P3 is optimal. Note that the policy P3 will give the same state action frequencies as before i.e. $x(4,1), x(5,2) > 0$ because all the transition probabilities of the MDP are unchanged. In the same way, policy P4 will give the same state action frequencies as before i.e. $x^*(4,2), x^*(5,2) > 0$. Assuming the contrary, we have that P3 is optimal for the case of h_1'. Hence we have

$$x(5,2)\log(1+h_2) \geq x^*(4,2)\log\left(1+h_1'\right)+x^*(5,2)\log(1+h_2) \tag{5.10}$$

From (5.9) and (5.10), we have

$$x^*(4,2)\log(1+h_1)+x^*(5,2)\log(1+h_2) \geq x^*(4,2)\log\left(1+h_1'\right)+x^*(5,2)\log(1+h_2) \tag{5.11}$$

which implies $h_1 \geq h_1'$ which is a contradiction.

$\square$

Theorem 5.2.4. Let $p, q, h_2, p(h_1)$ and $p(h_2)$ be fixed. As we vary h_1 from 0 to h_2, we have the following.

1. There exists a unique $0 < h_0 < h_2$, such that P3 is optimal for $h_1 \leq h_0$ and P4 is optimal for $h_1 > h_0$.

2. Let X_T and X_R be geometric random variables with parameters p and q respectively. Then h_0 can be determined by the following equation

$$\frac{\log(1+h_2)}{\mathbb{E}[Z]+\frac{1}{p(h_2)}} = \frac{p(h_1)\log(1+h_0)+p(h_2)\log(1+h_2)}{\mathbb{E}[Z]}, \tag{5.12}$$

where $Z = \max\{X_T, X_R\}$.

Proof: To prove this theorem, we look at the average throughput obtained by P3 and P4 as we vary h_1 from 0 to h_2.

Let us first fix the policy P3. Recall that under P3, we choose to transmit in 11 state only if the channel fade state is h_2. Thus, the throughput obtained is given by

$$T(P3) = \pi_4 \left(p(h_2) \log (1 + h_2) \right). \tag{5.13}$$

Note that the steady state probability π_4 does not depend on h_1. It varies only with p, q and the transmit probability in the state 11 (which is $p(h_2)$ under P3). Thus we see that $T(P3)$ does not vary with h_1. It remains the same as we vary h_1 from 0 to h_2. In essence, the plot of $T(P3)$ vs. h_1 is a horizontal line.

Next, let us fix the policy P4. Recall that under P4, we choose to transmit in the 11 state for both h_1 and h_2. The throughput obtained by P4 is thus given by

$$T(P4) = \pi_4' \left(p(h_1) \log (1 + h_1) + p(h_2) \log (1 + h_2) \right) \tag{5.14}$$

where π_4' is the steady state probability of being in state 11 under energy consumption policy P4. Observe that as before, π_4' is a function of p, q and transmit probability in the state 11 (which is 1 under P4). It does not vary with h_1. Therefore, we see that the throughput obtained is a monotonically increasing function of h_1.

Lemma 5.2.2. At $h_1 = 0$, $T(P3) > T(P4)$.

Proof: For $h_1 = 0$, from (5.13) and (5.14), we have

$$T(P3) = \pi_4 \left(p(h_2) \log (1 + h_2) \right). \tag{5.15}$$

and

$$T(P4) = \pi_4' \left(p(h_2)\log\left(1+h_2\right)\right). \tag{5.16}$$

Next, from Blackwell'stheorem, we know that $\pi_4 = \frac{1}{T_{11}}$ and $\pi_4' = \frac{1}{T'_{11}}$. Let the time taken for an energy arrival at the transmitter be denoted by X_T. Note that X_T is geometric with parameter p. Similarly, let the time taken for an energy arrival at the receiver be denoted by X_R, which is a geometrically distributed random variable with parameter q. The time taken to reach state 11 from 00 is now given by $Z = \max\{X_T, X_R\}$. Note that the minimum value Z can take is 1 and hence $\mathbb{E}[Z] > 1$. By linearity of expectations, we now have

$$T_{11} = p(h_1) \times 1 + p(h_2) \times \mathbb{E}[Z]. \tag{5.17}$$

Similarly,

$$T_{11}' = \mathbb{E}[Z] \tag{5.18}$$

As $\mathbb{E}[Z] > 1$, we have $T_{11} < T_{11}'$ resulting in $\pi_4 > \pi_4'$. This, along with (5.15) and (5.16) proves the lemma.

□

Lemma 5.2.3. At $h_1 = h_2$, $T(P3) < T(P4)$.

Proof: For $h_1 = h_2$, we have only a single fade state possible. P3 then translates into a random policy which chooses to transmit in the state 11 with probability $p(h_2)$. Note that the evolution of the energy states is a stochastic process. At every instant when the state 00 is achieved, the system refreshes. We consider these instances as epoch.

Let T denote the time between epochs under P3. We then have $\mathbb{E}[T] = \mathbb{E}[Z] + \frac{1}{p(h_2)}$. The second term $\frac{1}{p(h_2)}$ comes because under P3, once in the state 11, we have a random policy with waiting time being a geometric random variable with parameter $p(h_2)$.

Next, let T' denote the time between epochs under P4. We then have $\mathbb{E}[T'] = \mathbb{E}[Z] + 1$. Under P4, we always transmit in the 11 state.

From renewal reward theory, we have

$$
\begin{aligned}
T(P3) =& \frac{\mathbb{E}[R]}{\mathbb{E}[T]} \\
=& \frac{\log(1+h_2)}{\mathbb{E}[Z] + \frac{1}{p(h_2)}}
\end{aligned}
\tag{5.19}
$$

Similarly,

$$
\begin{aligned}
T(P4) =& \frac{\mathbb{E}[R]}{\mathbb{E}[T]} \\
=& \frac{\log(1+h_2)}{\mathbb{E}[Z] + 1}
\end{aligned}
\tag{5.20}
$$

As we assumed $p(h_1) > 0$, we have $p(h_2) < 1$ resulting in $T(P3) < T(P4)$, completing the proof.

□

From lemma 3 and lemma 4, we have $T(P3) > T(P4)$ for $h_1 = 0$ and $T(P3) < T(P4)$ for $h_1 = h_2$. Moreover, we know that $T(P3)$ is constant with h_1 while $T(P4)$ is a continuous function in h_1. By intermediate value theorem, there exists a $0 < h_0 < h_1$ such that at $h_1 = h_0$, $T(P3) = T(P4)$. Also, since $T(P4)$ is a strictly increasing function in h_1, we have that for $h_1 \geq h_0$, $T(P3) \geq T(P4)$, and for $h_1 > h_0$, $T(P4) > T(P3)$. The first part of the theorem then follows as we have already established that one of $P3$ or $P4$ is optimal.

Also, observe that we have $T(P3) = T(P4)$ at $h_1 = h_0$. We then have the second part of the theorem from (5.19) and (5.20).

□

We next prove the monotonicity of the optimal energy consumption policy with respect to the energy arrival parameters p and q. To prove this, we need a the following result from probability.

Lemma 5.2.4. Let X be a geometric random variable with parameter p and Y be a geometric random variable with parameter q. Let $Z = \max\{X,Y\}$. Also, let X' and Y' be geometric random variables with parameters p' and q' respectively such that $p' > p$ and $q' > q$. Again, let $Z' = \max\{X',Y'\}$. Then we have

$$\mathbb{E}[Z'] > \mathbb{E}[Z]. \tag{5.21}$$

Proof: The proof of this lemma is left to the reader.

□

Theorem 5.2.5. Monotonicity in energy arrival parameters: If the optimal policy for system parameters $p,q,h_1,h_2,p(h_1)$ and $p(h_2)$ is P4. The optimal policy for every $p',q',h_1,h_2,p(h_1)$ and $p(h_2)$ such that $p' > p$ and $q' > q$ is P4.

Proof:

From renewal reward theory, we know that the throughput obtained by P3 and P4 for system parameters $p,q,h_1,h_2,p(h_1)$ and $p(h_2)$ is given by

$$T(P3) = \frac{\log(1+h_2)}{\mathbb{E}[Z] + \frac{1}{p(h_2)}}$$
$$T(P4) = \frac{p(h_1)\log(1+h_1) + p(h_2)\log(1+h_2)}{\mathbb{E}[Z]+1},$$

where $Z = \max\{X_T, X_R\}$ and X_T, X_R are geometric random variables with parameters p and q respectively.

As P4 is optimal, we have

$$\frac{p(h_1)\log(1+h_1) + p(h_2)\log(1+h_2)}{\mathbb{E}[Z]+1} \geq \frac{\log(1+h_2)}{\mathbb{E}[Z] + \frac{1}{p(h_2)}}. \tag{5.22}$$

Next, for the system parameters $p', q', h_1, h_2, p(h_1)$ and $p(h_2)$, the throughput obtained by P3 and P4 is given by

$$T'(P3) = \frac{\log(1+h_2)}{\mathbb{E}[Z] - \delta + \frac{1}{p(h_2)}}$$
$$T'(P4) = \frac{p(h_1)\log(1+h_1) + p(h_2)\log(1+h_2)}{\mathbb{E}[Z] - \delta + 1},$$

where $\delta = \mathbb{E}[Z'] - \mathbb{E}[Z]$. Observe that $\mathbb{E}[Z'] \geq 1$ (the minimum value Z' can take is 1) and we have $\delta \leq \mathbb{E}[Z] - 1$. From lemma 4, we know that $\delta > 0$. Next, we have

$$T'(P4) - T(P4) = \big(p(h_1)\log(1+h_1) + p(h_2)\log(1+h_2)\big) \times \frac{\delta}{\big(\mathbb{E}[Z]+1\big)\big(\mathbb{E}[Z] - \delta + 1\big)}. \tag{5.23}$$

Similarly,

$$T'(P3) - T(P3) = \log(1+h_2) \times \frac{\delta}{\Big(\mathbb{E}[Z] + \frac{1}{p(h_2)}\Big)\Big(\mathbb{E}[Z] - \delta + \frac{1}{p(h_2)}\Big)}. \tag{5.24}$$

Using simple algebra, one can show that

$$T'(P4) - T(P4) \geq T'(P3) - T(P3) \tag{5.25}$$

As we started with $T(P4) \geq T(P3)$, we have the required result.

□

5.2.2 n-Fade state channel

Having looked at the case of binary channel, we now move on the case of discrete channel fade state where $h_t = h_i$ with probability $p(h_i)$ for $i = 1, 2, \ldots, n$. Without loss of generality, we assume $0 < h_1 < h_2 < \cdots < h_n$ and $p(h_i) > 0 \forall i$. Our objective, as before, is to find the optimal action in the state 11. Recall that we look only at the the class of stationary deterministic policies as we are assured of optimality through at least one of them.

Theorem 5.2.6. Monotonicity in channel fade state: The optimal energy consumption policy is of the form $\mathbf{1}_T(h_i) = \mathbf{1}_R(h_i) = 1$ for $i \geq m$ and $\mathbf{1}_T(h_i) = \mathbf{1}_R(h_i) = 0$ for $i < m$ for some $m \in \{1,2,\ldots,n\}$.

Proof: The proof of this theorem is on the same lines as that of theorem 2 and is hence left upon the reader.

□

Remark 5.2.2. There are a total of 2^n deterministic stationary policies possible with a $n-$ fade state system. Theorem 6 proves that one needs to only check n policies of the form specified above to obtain optimality. In turn, if it were possible to evaluate system performance, comparing these n policies will return the optimal energy consumption policy.

Theorem 5.2.7. For a $n-$ fade state channel, if the optimal policy for energy parameters is p and q is $\mathbf{1}_T(h_i) = \mathbf{1}_R(h_i) = 1$ for $i \geq m$ and $\mathbf{1}_T(h_i) = \mathbf{1}_R(h_i) = 0$ for $i < m$, then the optimal policy for energy parameters p' and q' such that $p' > p$ and $q' > q$, the optimal policy will be $\mathbf{1}_T(h_i) = \mathbf{1}_R(h_i) = 1$ for $i \geq m'$ and $\mathbf{1}_T(h_i) = \mathbf{1}_R(h_i) = 0$ for $i < m'$ such that $m' \leq m$.

Proof: The proof of this theorem is on the same lines as theorem 5 and is hence left on the reader to complete.

□

CHAPTER 6

Simultaneous Wireless Information and Power Transfer system

In this chapter, we move on to the case of a Multiple Input Multiple Output system. We consider a practical SWIPT system where two multi-antenna stations perform separate PT and IT to a multi-antenna mobile that dynamically assigns each antenna for either PT or IT. The antenna partitioning results in a tradeoff between the MIMO IT channel capacity and the PT efficiency. The optimal partitioning for maximizing the IT rate under a PT constraint is a NP-hard integer program. We propose solving it via efficient greedy algorithms with guaranteed performance. To this end, we prove that the antenna partitioning problem one that optimizes a sub modular function over a matroid constraint. This structure allows the application of two well known greedy algorithms that yield solutions no smaller than the optimal one scaled by factors (1 - 1/e) and 1/2, respectively.

Consider the system where a mobile is receiving information/data from the basestation and power from a power beacon. Let N_t, N_r, and N_p, denote the numbers of antennas at the base station, at the mobile, and at the power beacon, respectively. The MIMO channel from the base station to the mobile is represented by the complex $N_r \times N_t$ matrix $\mathbf{H}$. Power is beamed from the power beacon to the mobile and the beamforming vector is denoted as $\mathbf{f}$. Let $\mathbf{H}'$ denote the MIMO channel from the power beacon to the mobile. The effective MISO channel after beamforming is defined as $\mathbf{g} = \mathbf{H}'\mathbf{f}$.

Let $s_n \in \{0, 1\}$ indicates whether the n-th receiver antenna of the mobile is assigned for information transfer ($s_n = 1$) or power transfer ($s_n = 0$). For ease of notation, the indicator variables are grouped into a vector $\mathbf{s} = [s_1, s_2, \cdots, s_{N_r}]^\dagger$. Let $\mathbf{S}$ be a $N_r \times$

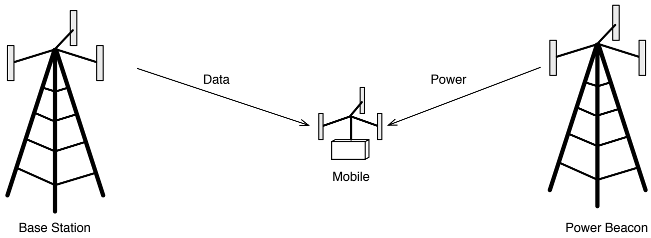

Figure 6.1: A system supporting simultaneous wireless information and power transfer

N_r diagonal matrix with $\mathbf{S}_{ii} = 1$ if $s_i = 1$ and $\mathbf{S}_{ii} = 0$ if $s_i = 0$. We first consider the CSIR case, where the transmitter does not have the knowledge of the channel state $\mathbf{H}$. The case of CSIT is discussed in Section 6.4. With CSIR, without loss of generality, assuming equal power allocation over all transmit antennas, the mutual information can be written as

$$\mathsf{MI} = \log\det\left(\mathbf{I} + \frac{P}{N_t}\mathbf{S}\mathbf{H}\mathbf{H}^{\dagger}\mathbf{S}^{\dagger}\right). \tag{6.1}$$

Note that the MIMO channel matrix $\mathbf{SH}$ used in (6.1) selects the rows from $\mathbf{H}$ corresponding to receiver antennas assigned for information transfer.

Given the antenna assignment specified by $\mathbf{s}$, maximum-ratio combining is applied at the mobile to maximize the received power that is written as

$$P_r = \sum_{n=1}^{N_r}(1 - s_n)|g_n|^2 \tag{6.2}$$

where g_n is the n-th element of $\mathbf{g}$. The power P_r is required to exceed the threshold $p_c > 0$ that represents fixed circuit-power consumption.

6.1 Problem Formulation

Under this model, the problem is to partition the set of receiver antennas of the mobile into two parts, the set of antennas dedicated for information transfer that maximizes the rate of information transfer or mutual information from the base station to the mobile, while the remaining antennas satisfy the circuit power constraint. Formally, we are interested in solving the following optimization problem, where the objective function is the mutual information.

$$
(\mathbf{P_1}) \qquad \begin{aligned} \max_{\{\mathbf{s}\}} \quad & \log\det\left(\mathbf{I}+\frac{P}{N_t}\mathbf{S}\mathbf{H}\mathbf{H}^{\dagger}\mathbf{S}^{\dagger}\right) \\ \text{s.t.} \quad & s_n \in \{0,1\}, \quad n=1,2,\cdots,N_r \\ & \sum_{n=1}^{N_r}(1-s_n)|g_n|^2 \geq p_c. \end{aligned}
$$

This is an integer programming problem which is typically *NP*-hard to solve. Next, we show that there is a lot of structure to the problem that makes it simpler to solve or approximate, compared to a typical integer programming problem. In the next two sections, we describe how to use that structure to find a provably approximate optimal solution.

Remark 6.1.1. If we relax the problem $(\mathbf{P_1})$ to allow $s_n \in [0,1]$, it corresponds to allowing dynamic power splitting of the received signal at all antennas for the purposes of information transfer and power transfer. However, it is worth noting that dynamic power splitting is challenging to incorporate in practice. Moreover, with $s_n \in [0,1]$, problem $(\mathbf{P_1})$ is a concave program since $\log\det(.)$ is a concave function, and the circuit power constraint is linear. Hence the relaxed problem can be solved efficiently. The dynamic power splitting problem for the case of single antenna at the base station and the mobile (SISO) has been solved in Liu *et al.* (2013). In addition, for the single antenna at the base station and multiple antennas at mobile (SIMO) case, solution to a simplified objective function for the antenna selection problem with binary constraint $s_n \in \{0,1\}$ has been found in Liu *et al.* (2013).

6.2 $1-1/e$-Approximate Solution

In this section, rather than finding an optimal solution to problem $\mathbf{P_1}$, we work towards finding algorithms that have efficient approximation ratio, that is defined as follows.

Definition 6.2.1. An algorithm $\mathscr{A}$ is defined to have an approximation ratio of $\mathsf{a} \leq 1$, if for any instance (input values) of the problem, the ratio of the objective function *OBJ* evaluated at the output **a** of the algorithm is at least a times the optimal value OPT, i.e., $\mathsf{a} \leq \min_{input} \frac{OBJ(\mathbf{a})}{\mathsf{OPT}}$.

Remark 6.2.1. All the derived results in this paper on the approximation ratios are worst case, and hence do not depend on the distribution of channel matrix **H**.

To proceed further, we need some preliminaries.

Definition 6.2.2. Let f be a set function defined over all subset of U, $f : 2^U \to \mathbb{R}^+$. Then f is called *monotone* if

$$f(S \cup \{a\}) - f(S) \geq 0,$$

for all $a \in U, S \subseteq U, a \notin S$. In addition, f is called a *sub-modular* function if it satisfies

$$f(S \cup \{a\}) - f(S) \geq f(T \cup \{a\}) - f(T),$$

for all elements $a \in U, a \notin T$ and all pairs of subsets $S \subseteq T \subseteq U$.

Essentially, for a sub-modular function the incremental gain from adding an extra element in the set decreases with the size of the set. Our interest in sub-modular function is because of the sub-modularity of the mutual information expression.

Lemma 6.2.1. The mutual information expression (6.1) is sub-modular in the number of mobile receiver antennas.

Proof: See Theorem 4 Vaze and Ganapathy (2012). □

We now describe an algorithm to obtain a solution to problem $\mathbf{P}_1$ that has a provable approximation ratio.

Definition 6.2.3. Multi-Linear Extension: Let S be a set of cardinality n, and consider a function that assigns value to each subset of S as $f : 2^S \to \mathbb{R}^+$. For each subset $T \subseteq S$, let $\mathbf{x}_T = [x_1 \ldots x_n]$ be the n-length vector, where $x_i = 1$ if the i^{th} element of S is contained in T and $x_i = 0$ otherwise. Thus, we can think of f as a function assigning value to each vertex of $\{0,1\}^n$ hypercube. The multi-linear extension F extends f to whole of $[0,1]^n$ such that for $\mathbf{x} \in [0,1]^n$

$$F(\mathbf{x}) = \mathsf{E}\{f(\hat{\mathbf{x}})\} = \sum_{R \subseteq S} f(R) \prod_{i \in R} x_i \prod_{j \notin R} (1 - x_j),$$

where the sum is over all subsets R of S, and $\hat{\mathbf{x}}$ denotes the random vector whose j^{th} coordinate is independently 1 with probability x_j and 0 otherwise.

Note that S is our case is the set of receiver antennas $[1 : N_r]$ with cardinality $n = N_r$.

Definition 6.2.4. Matroid: Consider a set S of n elements. An independence family $\mathscr{I} \subseteq 2^S$ of subsets of S is called a matroid $\mathscr{M}(S, \mathscr{I})$ if i) $X \subseteq Y$, and $Y \in \mathscr{I}$ then $X \in \mathscr{I}$ and ii) $X \in \mathscr{I}$, and $Y \in \mathscr{I}$ with $|X| \leq |Y|$, then $\exists \{e\} \in Y \backslash X$ such that $X \cup \{e\} \in \mathscr{I}$.

Consider the following optimization problem $\mathbf{P_2}$:

$$\max_{\mathbf{x} \in C(\mathscr{M})} f(\mathbf{x}),$$

where f is a monotone sub-modular function, and $C(\mathscr{M}) =$ convex hull $\{\mathbf{1}_I : I \in \mathscr{I}\}$, where $\mathbf{1}_I$ is the indicator vector of length $|S|$, where $\mathbf{1}_I(j) = 1$ if $j \in I$. Thus, $\mathbf{P_2}$ is to maximize a sub-modular function over the convex hull of a matroid.

Using the following Lemma 6.2.2, we show that problem $\mathbf{P}_1$ is of the form $\mathbf{P}_2$.

Lemma 6.2.2. The linear circuit power constraint of $\mathbf{P_1}$ is a matroid.

Proof: The linear circuit power constraint of $\mathbf{P_1}$ is equivalent to $-\sum_{n=1}^{N_r}(1-s_n)|g_n|^2 \leq -p_c$. Thus, if $-\sum_{n=1}^{N_r}(1-s_n)|g_n|^2 \leq -p_c$ is true for a set of receiver antennas S, then clearly it is true for a subset $T \subseteq S$, thus satisfying axiom i) of matroid definition. Axiom ii) can also be verified immediately.

□

Definition 6.2.5. Weight of an independent set: Consider a set S of n elements and a matroid $\mathscr{M} = (S, \mathscr{I})$ of subsets of S. Let w_j for $j = 1, 2, \ldots, n$ be the weights of the elements of S. Let $T \in \mathscr{M}$ and let $\mathbf{x}_T = [x_1 \ldots x_n]$ be the n-length vector, where $x_i = 1$ if the i^{th} element of S is contained in T and $x_i = 0$ otherwise. The weight of the subset T is then given by

$$w(T) = \sum_{i=1}^{n} w_i x_i \tag{6.3}$$

Given a set S, a matroid $\mathscr{M} = (S, \mathscr{I})$, and the weights of the elements $w_j \ \forall \ j$, the maximum weight independent set is given by $\underset{I \in \mathscr{I}}{\operatorname{argmax}} \ w(I)$.

Consider the following greedy algorithm (GREEDY) for finding the maximum weight independent set problem Schrijver (2003). GREEDY

1. Rearrange the elements of S and obtain $S = \{e_1, e_2 \ldots, e_n\}$ such that $w_{e_1} \geq w_{e_2} \geq \cdots \geq w_{e_n}$.

2. Initialize $X \leftarrow \emptyset$.

3. For $i = 1$ to n, do
   if $(X + e_i \in \mathscr{M})$ and $w(X + e_i) \geq w(X)$ then $X \leftarrow X + e_i$.

4. Output X.

Lemma 6.2.3. The greedy algorithm outputs the optimal solution to the maximum weight independent set problem Schrijver (2003).

From hereon, we concentrate our attention on solving $\mathbf{P}_2$ and obtaining theoretical bound on the approximation ratio.

To solve $\mathbf{P_2}$: a continuous greedy algorithm has been proposed in Vondrák (2008), that is described as follows.

The Continuous Greedy Algorithm:

Given matroid $\mathscr{M} = (S, \mathscr{I})$, and a function f.

1. Let $\delta = \frac{1}{n^2}$, where $|S| = n$. Start with $t = 0$ and $\mathbf{x}(0) = (x_t(1) \dots x_t(n)) = \mathbf{0}$.

2. Let R_t be a vector of size n, where $R_t(j) = 1$ independently with probability $x_t(j)$. For each $j \in S$, estimate $\omega_j(t) = \mathsf{E}\{f_{R(t)}(j)\}$ say by taking the average of n^5 samples, where

$$f_{R(t)}(j) = f(R(t) \cup \{j\}) - f(R(t)).$$

3. Let $I(t)$ be a maximum weight independent set in $\mathscr{M}$ computed by GREEDY, according to weights $\omega_j(t)$. Let $\mathbf{x}(t+\delta) = \mathbf{x}(t) + \delta.\mathbf{1}_{I(t)}$.

4. Increment $t := t + \delta$ if $t < 1$, go to Step 2. Otherwise, return $x(1)$.

Lemma 6.2.4. Vondrák (2008) The fractional solution $\mathbf{x}$ of the optimization problem $\mathbf{P_2}$ found by the continuous greedy algorithm satisfies

$$F(\mathbf{x}) = \mathsf{E}\{f(\hat{\mathbf{x}})\} \geq \big(1 - 1/e\big)\,\mathsf{OPT}$$

with high probability, where OPT is the optimal value of $\mathbf{P_2}$.

Note: All high probability results in this paper mean the error falls exponential in n.

The solution $\mathbf{x}$ output by the continuous greedy algorithm may be fractional. Hence an additional rounding method called the pipage rounding (described below) is used to obtain an integer solution from $\mathbf{x}$.

Pipage Rounding:

We now discuss the pipage rounding technique, developed by Ageev and Sviridenko Ageev and Sviridenko (2004), and adapted for the matroid polytope by Calinescu et al. Calinescu *et al.* (2007), to convert the fractional solution obtained by the continuous greedy algorithm to an integral solution.

Lemma 6.2.5. Given a matroid $\mathscr{M} = (S, \mathscr{I})$, and a monotone sub modular function

$f : 2^S \to \mathbb{R}_+$, and a fractional solution $x \in \mathscr{C}(\mathscr{M})$, there exists a polynomial time randomized algorithm, which returns an independent set $\mathscr{X} \in \mathscr{I}$ of value $f(\mathscr{X}) \geq (1 - o(1))F(x)$, where F is the multi-linear extension, and the $o(1)$ term can be made polynomially small in $n = |S|$.

Given a $y \in [0,1]^n$, we say that i is fractional in y if $0 < y_i < 1$, and for $y \in \mathscr{C}(\mathscr{M})$, define $y(A) = \sum_{i \in A} y_i$. Then, a set $A \subseteq S$ is defined to be *tight* if $y(A) = r_{\mathscr{M}}(A)$, where $r_{\mathscr{M}}(A) = \max\{|I| : I \subseteq A \text{ and } I \in \mathscr{I}\}$ is the rank function of the matroid.

The pipage rounding algorithm is described below.

Input : the fractional solution y.

1. Find A, the minimal tight set containing at least 2 fractional variables i, j.
2. Let $y_{ij}(\varepsilon)$ be the vector obtained by adding ε to y_i, subtracting ε from y_j and leaving the other values unchanged. Define $\varepsilon^+_{ij}(y) = \max lbrace \varepsilon \geq 0 \mid y_{ij}(\varepsilon) \in \mathscr{C}(\mathscr{M})\}$ and $\varepsilon^-_{ij}(y) = \min\{\varepsilon \leq 0 \mid y_{ij}(\varepsilon) \in \mathscr{C}(\mathscr{M}) rbrace$.
3. If $F(y_{ij}(\varepsilon^+_{ij})) > F(y_{ij}(\varepsilon^-_{ij}))$, then $y \leftarrow y_{ij}(\varepsilon^+_{ij})$ else $y \leftarrow y_{ij}(\varepsilon^-_{ij})$.
4. If y is fractional, go to step 1. Otherwise, return y.

Lemma 6.2.6. Ageev and Sviridenko (2004) The integer solution $\mathbf{x}_{int}$ for problem $\mathbf{P_2}$ obtained by applying pipage rounding to the fractional output $\mathbf{x}$ of the continuous greedy algorithm satisfies $F(\mathbf{x}_{int}) \geq \left(1 - 1/e\right) \mathsf{OPT}$ with high probability.

Finally, we describe the main result of this section as follows.

Theorem 6.2.1. The pipage rounded solution $\mathbf{x}_{int}$ of the fractional solution $\mathbf{x}$ found by the continuous greedy algorithm for the antenna partitioning problem $\mathbf{P_1}$ satisfies

$$f(\mathbf{x}_{int}) \geq \left(1 - 1/e\right) \mathsf{OPT}$$

with high probability, where OPT is the optimal value of the mutual information in $\mathbf{P_1}$.

Proof: From Lemma 6.2.1 and 6.2.2, problem $\mathbf{P}_1$ is a special case of problem $\mathbf{P}_2$, and the result follows from Theorem 6.2.1.

□

Thus, exploiting the sub-modularity of the mutual information expression and matroidal circuit power constraint, the pipage rounding + greedy continuous algorithm gives us a guaranteed worst case approximation ratio of $\left(1 - 1/e\right)$. Even though the guarantees obtained by the pipage rounding + greedy continuous algorithm are good, the time complexity of running both the algorithms is significant. The continuous greedy algorithm starts with $t = 0$ and increments in the direction of the maximum weight independent set with a size of $\frac{1}{n^2}$ $(n = N_r)$. In each increment, the weights of elements are found by taking average of n^5 independent samples. Thereafter, finding the maximum weight independent set has a time complexity of $O(n \log n)$. This results in an overall time complexity of $O(n^2(n^5(n) + n\log n)) = O(n^8)$. Subsequently, the pipage rounding algorithm takes n^2 iterations to convert the fractional solution given by the continuos greedy algorithm into an integral solution. Thus, the overall time complexity of the $1 - 1/e$ algorithm is $O(n^8)$.

Next, we propose a simpler greedy algorithm and show that it achieves a $1/2$ approximation to $\mathbf{P_1}$.

6.3 $1/2$-Approximate Solution

Greedy Algorithm (GA): Start with set $s_0 = \mathbf{0}$. At step i, $s_i = s_{i-1} + [0 \ldots 0 \underbrace{1}_{i^\star} 0 \ldots 0]$, where

$$i^\star = \underset{i \in \{1,2,\ldots,N_r\}}{\operatorname{argmax}} \ \log\det\left(\mathbf{I} + \frac{P}{N_t}\mathbf{S}_i\mathbf{H}\mathbf{H}^\dagger\mathbf{S}_i^\dagger\right),$$

where $s_i = s_{i-1} + [0 \ldots 0 \underbrace{1} 0 \ldots 0]$ and $\mathbf{S}_i$ is the diagonal matrix corresponding to s_i as mentioned before. If $s_{i^\star}$ satisfies $\sum_{j=1}^{n} (1 - s_{i^\star})|g_n|^2 \geq p_c$, repeat for $i = i+1$, else, output

s_{i-1}.

The following result is known for using greedy algorithms for maximizing monotone sub-modular functions with matroid constraint.

Theorem 6.3.1. Nemhauser *et al.* (1978) For a non-negative, monotone sub-modular function f and a matroid constraint, let subset S be obtained by selecting elements one at a time, each time choosing an element that provides the largest marginal increase in the function value that is feasible with respect to the matroid constraint. Let $S^\star$ be a set that maximizes the value of f over all subsets of the matroid. Then $f(S) \geq \frac{1}{2} f(S^\star)$.

Theorem 6.3.2. The objective function of problem $\mathbf{P_1}$ evaluated at the greedy algorithm GA's output $\geq \mathsf{OPT}/2$, where OPT is the value of the optimal solution.

Proof: From Lemma 6.2.1, we know that the objective function of $\mathbf{P_1}$ is monotone and sub-modular. Moreover, the linear circuit power constraint is a matroid from Lemma 6.2.2. Thus, using Theorem 6.3.1, we have that the greedy algorithm gives a $1/2$ approximation to $\mathbf{P_1}$.

□

If S has n elements, the $1/2$ approximate algorithm will terminate in a maximum of n iterations, as a new element is included in every iteration. In each iteration, the element with the highest marginal gain can be obtained by finding the maximum of n entries. This results in a running time complexity of $O(n^2)$.

Thus, even though, the continuous greedy algorithm gives a better bound on performance than the greedy algorithm, it has a running time complexity of $O(n^8)$ as compared to $O(n^2)$ for the greedy algorithm. Hence, both the algorithms have their own significance and which algorithm to be used depends on the problem at hand.

In this section, we considered the CSIR case, where only the receiver has CSI and uses it judiciously to obtain an efficient solution to the antenna splitting problem. Next, we discuss the more general scenario where the base station also has CSI, typically referred to as the CSIT.

6.4 CSIT

In this section, we assume that the base station has CSIT for the channel $\mathbf{H}$ towards the mobile. As before, let $s_n \in \{0,1\}$ indicates whether the n-th receiver antennas at the mobile is assigned for information transfer ($s_n = 1$) or power transfer ($s_n = 0$). With CSIT, the power allocation (input covariance matrix) at different antennas of the base station depends on receiver antenna assignment vector $\mathbf{s}$, and the capacity is given by

$$\begin{aligned} C &= \max_{\mathbf{Q},\mathrm{tr}(\mathbf{Q})\le P} \log\det\left(\mathbf{I}+\mathbf{SHQH}^\dagger\mathbf{S}^\dagger\right), \\ &= \max_{p_i,\sum_{i=1}^{N_r} p_i\le P} \sum_{i=1}^{N_r} \log(1+s_i\lambda_i(\mathbf{s})p_i), \end{aligned} \tag{6.4}$$

where $\lambda_i(\mathbf{s})$ are the eigen-values of the submatrix $\mathbf{H}(\mathbf{s})$ of $\mathbf{H}$ which is obtained by keeping all rows of $\mathbf{H}$ for which $s_i = 1$, and without loss of generality we have assumed that $N_\mathrm{t} \ge N_r$. The power allocation p_i at the basestation depends on the receiver antennas allotted for data transfer, i.e., p_i depend on s_i, and the optimal power allocation is given by waterfilling.

Compared to (6.1), the expression inside the max in (6.4) is simple (sum of N_r parallel channels), however, together with the max, the overall capacity expression is more complicated. Moreover, note that the choice of s_i and p_i depend on each other. So solving the CSIT case, is far more non-trivial than the CSIR case.

The power received from the power beacon is same as in the CSIR case, given by

$$P_r = \sum_{n=1}^{N_r}(1-s_n)|g_n|^2, \tag{6.5}$$

where P_r is required to exceed the threshold $p_c > 0$ that represents fixed circuit-power consumption.

So the integer program $\mathbf{P}_3$ is

$$(\mathbf{P3}) \qquad \begin{aligned} \max_{\{\mathbf{s}\}} \quad & \max_{p_i, \sum_{i=1}^{N_r} p_i \leq P} \sum_{i=1}^{N_r} \log(1 + s_i \lambda_i(\mathbf{s}) p_i) \\ \text{s.t.} \quad & s_n \in \{0,1\}, \quad n = 1, 2, \cdots, N_r \\ & \sum_{n=1}^{N_r} (1 - s_n)|g_n|^2 \geq p_c. \end{aligned}$$

Even though the capacity expression (6.4) involves a maximization, it is still a sub-modular function as shown in Thekumparampil *et al.* (2014). We summarize the result of Thekumparampil *et al.* (2014) as follows.

Theorem 6.4.1. For a set S, and fixed $s_i, i \in S$, the rate

$$\mathbf{R}(S) = \max_{p_i, \sum_{i \in S} p_i \leq P} \sum_{i \in S} \log(1 + s_i \lambda_i(\mathbf{s}) p_i),$$

obtained with a set S of parallel Gaussian channels using the optimal waterfilling algorithm is a sub-modular function over the set of channels S.

It is easy to show that (6.4) is a monotone function, since adding more receiver antennas cannot decrease the rate. Thus, we have that similar to the CSIR case, $\mathbf{P}_3$ is a special case of problem $\mathbf{P}_2$, and we get results on solving $\mathbf{P}_3$ as described below.

Theorem 6.4.2. The pipage rounded solution $\mathbf{x}_{int}$ of the fractional solution $\mathbf{x}$ found by the continuous greedy algorithm on $\mathbf{P_3}$ satisfies

$$f(\mathbf{x}_{int}) \geq \left(1 - 1/e\right) \mathsf{OPT}$$

with high probability, where OPT is the optimal value of the mutual information in $\mathbf{P_3}$.

Theorem 6.4.3. The objective function of problem $\mathbf{P_3}$ evaluated at the greedy algorithm GA's output $\geq \mathsf{OPT}/2$ for solving $\mathbf{P}_3$, where OPT is the value of the optimal solution of $\mathbf{P}_3$.

6.5 Simulation Results

In this section, we illustrate the numerical performance of the two approximation algorithms presented in this paper to maximize the mutual information while satisfying the circuit power constraint. For the CSIR case, in Fig. 6.2, we plot the throughput (mutual information) as a function of receiver antennas N_r for fixed $N_t = 5$ and circuit power constraint of $0.2N_r$ with total transmit power $P = 5W = 6.9dB$ under a Rayleigh fading assumption on the channel matrix $\mathbf{H}$. We scale the circuit power constraint with N_r since larger the number of receiver antennas more is the power required for their operation. As can be seen from Fig. 6.2, both the continuous greedy and the greedy algorithm perform better than their worst case bound of $1 - 1/e$ and $1/2$, respectively. The continuous greedy algorithm performs better than the greedy algorithm, however, its running complexity is also much higher and typically of the order N_r^8, while greedy algorithm runs in N_r^2 time.

In Fig. 6.3, we plot the throughput as a function of receiver antennas N_r for the CSIT case, with identical parameters used for Fig. 6.2. Once again we see that the continuous greedy and the greedy algorithm perform better than their worst case bound of $1 - 1/e$ and $1/2$. An important observation one can make from Fig. 6.2 and Fig. 6.3, is that the greedy algorithm outperforms the continuous greedy + pipage rounding algorithm, even though, the worst-case performance guarantees of the continuous greedy + pipage rounding algorithm is better than the greedy algorithm.

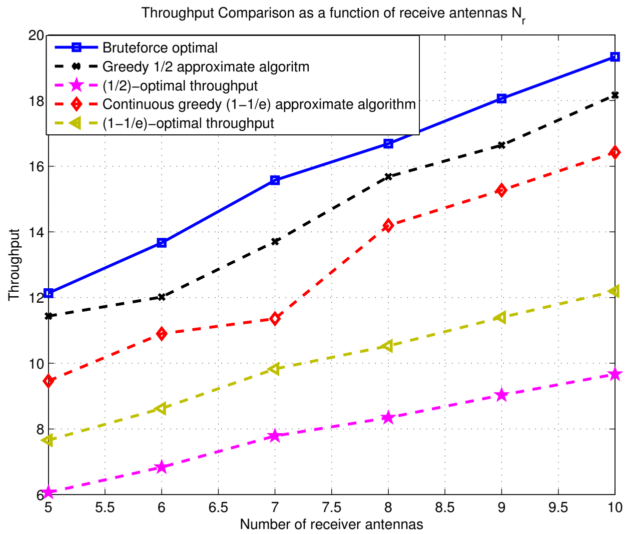

Figure 6.2: Mutual information comparison with different algorithms for CSIR

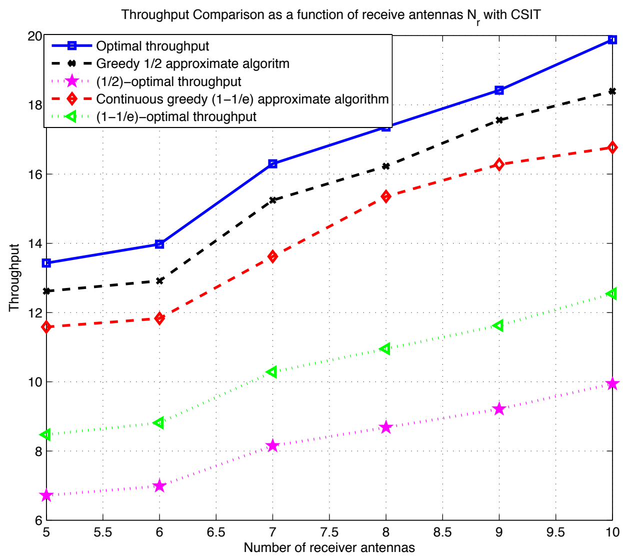

Figure 6.3: Mutual information comparison with different algorithms for CSIT

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CHAPTER 7

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