Stochastic Game Theoretical Modelling of Trade Credit

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CERTIFICATE

This is to certify that the report titled **Stochastic Game Theoretical Modelling of Trade Credit**, submitted by **Talluri Shakthi Krishna Sravanth**, to the Indian Institute of Technology, Madras, for the award of the degree of **Bachelor of Technology**, is a bona fide record of the work done by him under our supervision. The contents of this report, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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1 ABSTRACT

In this report, we make mathematical model of a trade credit practice between a retailer and a supplier including all the costs incurred. We shall discuss the strategies of cooperate and defect among the retailer and supplier and try to find out the optimum ordering quantity and optimum ordering time period for the retailer to gain maximum profit and the value of interest rate for the supplier to maximize his profit. We finally conclude by showing the working of the model for a case of input values for the variables.

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2 INTRODUCTION

When suppliers deliver goods to retailers, they often do not require to be paid immediately. Instead, suppliers offer credit terms that allow the retailers to delay the payment. This practice is called *trade credit.* Trade credit transactions normally involve short-term (e.g. thirty to sixty days) delayed payment of purchases of intermediate goods or services. Through delayed payment, trade credit suppliers are effectively funding their clients with short-term debt.

The concept of trade credit is popular because the supplier has better information about retailers' creditworthiness which helps him to discriminate prices between different retailers leading to minimizing his risk of loss. It also helps the supplier to control the retailer with better enforcement options. Commonly used credit terms lower a retailer's inventory carrying charges for a limited period of time. If the buyer does not pay in a timely fashion, these terms imply a schedule of escalating finance charge rates. Credit-constrained retailers that have productive investment opportunities benefit from receiving credit, because they are able to increase their purchase of inputs. At the same time, the possibility of obtaining higher sales gives suppliers with easier access to capital markets an incentive to offer trade credit to their customers.

The importance of trade credit can also be seen from the proportion of investment that is financed through it. Peterson and Rajan (1997, p. 661) state, "Trade credit is the single most important source of short-term external finance for firms in the United States." Similar observations have been made about firms in Europe as well. A study by Beck, Demirgüç-Kunt and Maksimovic (2008) shows, using a survey that covers 48 countries, that on average 19.7% of all investment financed through external sources was done using trade credit; in fact, the authors found that in most countries trade credit is the second most important source of external finance, preceded only by bank credit. Trade credit represents more than 30% of all external finance in developed economies such as France and the UK.

However, trade credit has three main differences with respect to other types of corporate debt. First, suppliers lend 'in kind'; they seldom lend cash. Second, in contrast with bonds or loans, trade credit is frequently not subject to specific, formal contracts between the lender and the borrower. Finally, trade credit is issued by non-financial firms.

Almost all of the models assume that retailers can freely implement their optimal operational decisions that are based on their production information, and there exist no budget constraint on purchasing decisions. In real applications, especially for start-up and growing firms, a firm is often short of capital, and as a result, the purchasing decisions have to be restrained by the availability of capital. Indeed, cash flow is a major reason for the bankruptcy of small- and medium- sized firms. However, only a few operational decision models consider budgetary constraints. By assuming the availability of market hedges, Birge (2000) adapts option pricing theory for incorporating risk into planning models by adjusting capacity and resource levels. Other models include those of Hadley and Whitin (1963), Sherbrooke (1968) and Rosenblatt (1981). However, these models assume that the budget in each period is fixed and will not be affected by inventory decisions. An exception is Rosenblatt and Rothblum (1990), who treat the capacity as a decision variable for studying multi-item inventory systems under a single resource capacity constraint. However, their model does not incorporate financial considerations.

In this model, we first look at the implications of decision taken by supplier and retailer at the end of every period whether to cooperate or defect each other and then proceed to formulating the profit gained if they cooperate with each other. We then look at maximizing this profit and finding the decision variables namely order quantity, re-order time period, credit rate of interest.

3 TRADE CREDIT MODEL

Assumptions made:

1) The demand distribution remains same throughout, for all the periods where the trade credit is considered.
2) Zero lead time for delivery of products by the supplier.
3) A constant rate of interest is maintained for the trade credit.

As the demand distribution and credit terms are assumed to be constant for every period, it can be said that the amount of on-hand inventory required to maximize the retailer's profit for a period also remains the same. Hence, a base stock policy would result in maximum profit for the retailer.

So, the retailer has to decide the base stock level (order quantity) and the re-order period. The supplier has to decide on the rate of interest to maximize his profit.

3.1 Effects Of Decision of Cooperate or Defect Taken By Either Party

Each period, the supplier and retailer can either cooperate with each other or one of them defects.

<u>Supplier Cooperates – Retailer Cooperates:</u>

The supplier cooperates by providing the retailer with the required quantity of products. The retailer cooperates by making payments on time. Both of them gain profit and try to maximize their profits by changing the variables in their control.

<u>Supplier Cooperates - Retailer Defects:</u>

The supplier cooperates by providing the retailer with the required quantity of products. The retailer defects by not paying the supplier. In this case, even though the retailer gets benefitted for that period, the supplier would defect from the next period which isn't profitable for the retailer in the long run.

<u>Supplier Defects - Retailer Cooperates/Defects:</u>

In case the supplier doesn't provide required product to the retailer, neither the supplier nor the retailer will gain profit.

Hence, to obtain maximum profit in the long run, it would be beneficial to both the supplier and retailer to cooperate with each other.

3.2 MATHEMATICAL MODEL

We consider the case that both the retailer and supplier cooperate with each other.

The variables used in the model are defined below:

c – Cost price of item for retailer

p – Selling price of item for retailer

a – Interest rate offered by supplier per period per item

C_o - Ordering cost

i - Inventory holding cost per item per period

h_n – Optimum ordering quantity if ordering is done by retailer for n periods

D_n - Demand for the period n

$f(D)$ - Probability density function for demand

We need to find out 1) Optimum order quantity for the retailer, 2) Re-order period, 3) Interest rate by supplier

3.2.1 Finding the optimum order quantity

Cost incurred by retailer = $C_o + C_i + C_p$, where C_i is the inventory holding cost and C_p is the payment that is to be made to the supplier

Calculation of C_i and C_p:

The retailer orders h_n for n periods. Let us assume that this quantity is consumed in *j* periods, owing to the non-deterministic demand.

Period	Starting inventory	Ending inventory	Inventory cost
1	h_n	$h_n - D_1$	$\frac{(2h_n - D_1) * i}{2}$
2	$h_n - D_1$	$h_n - D_1 - D_2$	$\frac{(4h_n - 2D_1 - D_2) * i}{2}$
k	$h_n - (D_1 + D_2 + .. + D_{k-1})$	$h_n - (D_1 + D_2 + .. + D_{k-1} + D_k)$	$\frac{[(2k)h_n - (2k-1)D_1 - (2k-3)D_2 - \cdots - 3D_{k-1} - D_k] * i}{2}$
j	$h_n - (D_1 + D_2 + .. + D_{j-1})$	0	$\frac{\left[(2j-1)h_n - (2j-2)D_1 - (2j-4)D_2 - \cdots - 2D_{j-1}\right] * i}{2}$

Hence,

Inventory cost for j periods $C_{ij} = \frac{[(2j-1)h_n - (2j-2)D_1 - (2j-4)D_2 - \cdots - 2D_{j-1}] * i}{2}$

Payment to supplier for j periods $C_{pj} = c(1+a)D_1 + c(1+a)^2 D_2 + \cdots + c(1+a)^j D_j$

$(D_j = h_n - (D_1 + D_2 + .. + D_{j-1}))$

Retailer expenditure for the h_n products = C_o+

(expenditure if all products are sold in 1 period)*probability(all products are sold in 1 period)+ (expenditure if all products are sold in 2 periods)*probability(all products are sold in 2 periods)+…

$$= C_o + \int_{D_1=h_n}^{\infty} \left(\frac{ih_n}{2} + c(1+a)h_n\right) f(D_1) dD_1 +$$

$$\int_{D_1=0}^{h_n} \int_{D_2=h_n-D_1}^{\infty} \begin{bmatrix} \frac{i(3h_n - 2D_1)}{2} + c(1+a)D_1 \\ + c(1+a)^2 (h_n - D_1) \end{bmatrix} f(D_2) dD_2 f(D_1) dD_1 + \cdots +$$

$$\int_{D_1=0}^{h_n} \int_{D_2=0}^{h_n-D_1} \int_{D_3=0}^{h_n-(D_1+D_2)} \ldots \int_{D_{j-1}=0}^{h_n-(D_1+\cdots+D_{j-2})} \int_{D_j=D_1+\cdots+D_{j-2}}^{\infty} (C_{ij} + C_{pj}) dD_j \, dD_{j-1} \ldots dD_1]$$

The series goes on up to infinity but can be stopped at any point after which the probability if happening is very less. For the sake of convenience, we consider the series up to $(n+1)$ periods, if the ordering is done for n periods.

Calculation of retailer's profit:

Profit obtained by retailer is given by

$$\pi_{r_n} = ph_n - \Big[C_o + \int_{D_1=h_n}^{\infty}\left(\frac{ih_n}{2} + c(1+a)h_n\right) f(D_1)dD_1 + \int_{D_1=0}^{h_n}\int_{D_2=h_n-D_1}^{\infty}\begin{bmatrix}\frac{i(3h_n-2D_1)}{2} + c(1+a)D_1 \\ +c(1+a)^2(h_n-D_1)\end{bmatrix} f(D_2)dD_2 f(D_1)dD_1 + \cdots + \int_{D_1=0}^{h_n}\int_{D_2=0}^{h_n-D_1}\int_{D_3=0}^{h_n-(D_1+D_2)} \ldots \int_{D_n=0}^{h_n-(D_1+\cdots+D_{n-1})}\int_{D_{n+1}=h_n-(D_1+\cdots+D_n)}^{\infty}\left(C_{i(n+1)} + C_{p(n+1)}\right) dD_{n+1}\, dD_n \ldots dD_1\Big]$$

To obtain the optimum inventory level (order quantity), the above equation should be differentiated with h_n and equated to zero.

3.2.2 Finding the constraint on interest rate

The supplier will get profits as long as the retailer doesn't defect. The retailer will not defect voluntarily as it would affect his profits too. Hence, the retailer would defect in a case only when he is bankrupt. Hence, the supplier's credit terms (decision of interest rate) should be in such a way that the retailer doesn't go bankrupt in the worst of cases. Going by the assumption that order for n periods will be consumed in at most $n+1$ periods, the worst case for the retailer would be when all the products are sold in the $(n+1)^{th}$ period i.e. $D_1 = D_2 = \cdots = D_n = 0, D_{n+1} = h_n$. The profit for retailer in that case will be $ph_n - [C_o + c(1+a)^{n+1}h_n]$. Hence, the constraint on the interest rate is:

$$ph_n - [C_o + c(1+a)^{n+1}h_n] > 0$$

3.2.3 Finding optimum re-order period

The retailer has to decide on whether to order for 1 period or 2 periods or so on. Firstly, the values of $h_1, h_2, \ldots$ have to be found out using the profit equation and interest rate constraint. Substituting that, the profit values $\pi_{r_1}, \pi_{r_2} \ldots$ have to be found. The optimum re-order period is found by:

$$\max(\pi_{r_1}, \frac{\pi_{r_2}}{2}, \frac{\pi_{r_3}}{3}, \dots)$$

If $\frac{\pi_{r_i}}{i}$ is maximum, the retailer has to order h_i every i periods to obtain maximum profits.

4 RESULTS

To check the working of the model, the following input is considered and optimum ordering policy for 1-period, 2-period and 3-period orderings is compared:

$c = 10, p = 15, C_o = 200, i = 1.$

The demand distribution for any period is assumed to be uniform between 750 and 1250 and zero elsewhere.

$$f(D) = \begin{cases} \frac{1}{500}, 750 \leq D < -1250 \\ 0, otherwise \end{cases}$$

4.1 1-PERIOD ORDERING

If the retailer orders every period, the profit per period is given by:

$$\pi_{r1} = ph_1 - \{C_o + \int_{D_1=0}^{h_1} \left[\frac{i(3h_1 - 2D_1)}{2} + c(1+a)D_1 + c(1+a)^2(h_1 - D_1)\right] f(D_1)dD_1 + \int_{D_1=h_1}^{\infty} \left[\frac{ih_1}{2} + c(1+a)h_1\right] f(D_1)dD_1\}$$

Differentiating it withh_1,

$$p - \int_{D_1=0}^{h_1} \left[\frac{3i}{2} + c(1+a)^2\right] f(D_1)dD_1 - \int_{D_1=h_1}^{\infty} \left[\frac{i}{2} + c(1+a)\right] f(D_1)dD_1 = 0$$

The constraint for interest rate is

$$ph_1 - \{C_o + \frac{3ih_1}{2} + \mathrm{c}(1+\mathrm{a})^2 h_1\} > 0$$

Making the above inequality into an equation and solving both equations yields $h_1 = 1278.024, x = 15.51\%$

4.2 2-PERIOD ORDERING

If the retailer orders once every 2 periods, the profit is given by:

$$\pi_{r2} = ph_2 - \{C_o + \int_{D_1=h_2}^{\infty} \left(\frac{ih_2}{2} + c(1+a)h_2\right) f(D_1)dD_1 +$$

$$\int_{D_1=0}^{h_2} \int_{D_2=h_2-D_1}^{\infty} \begin{bmatrix} \frac{i(3h_2-2D_1)}{2} + c(1+a)D_1 \\ +c(1+a)^2(h_2-D_1) \end{bmatrix} f(D_2)dD_2 f(D_1)dD_1$$

$$+ \int_{D_1=0}^{h_2} \int_{D_2=0}^{h_2-D_1} [\, i(5h_2 - 4D_1 - 2D_2)/2 + c(1+a)\, D_1 + c(1+a)^2 D_2 + c(1+a)^3 (h_2 - D_1 - D_2)] f(D_2)dD_2 f(D_1)dD_1\}$$

Differentiating it withh_2,

$$p + \left[\frac{\mathrm{i}h_2}{2} + \mathrm{c}(1+\mathrm{a})h_2\right] \mathrm{f}(h_2) - \int_{D_1=h_2}^{\infty} \left[\frac{i}{2} + c(1+a)\right] f(D_1)dD_1$$

$$- \int_{D_2=0}^{\infty} \left[\frac{ih_2}{2} + c(1+a)h_2\right] f(h_2) f(D_2)dD_2$$

$$- \int_{D_1=0}^{h_2} \int_{D_2=h_2-D_1}^{\infty} \left[\frac{3i}{2} + c(1+a)^2\right] f(D_2)dD_2\, f(D_1)dD_1$$

$$- \int_{D_1=0}^{h_2} \int_{D_2=0}^{h_2-D_1} \left[\frac{5i}{2} + c(1+a)^3\right] f(D_2)dD_2\, f(D_1)dD_1 = 0$$

The constraint for interest rate is

$$ph_2 - \{C_o + \frac{5ih_2}{2} + c(1+a)^3 h_2\} > 0$$

Making the above inequality into an equation and solving both equations yields $h_2 = 2273.614, x = 7.468\%$

4.3 3-Period Ordering

If the retailer orders once every 3 periods, the profit is given by:

$$\pi_{r3} = ph_3 - \{C_o + \int_{D_1=h_3}^{\infty} \left(\frac{ih_3}{2} + c(1+a)h_3\right) f(D_1)dD_1 +$$

$$\int_{D_1=0}^{h_3}\int_{D_2=h_3-D_1}^{\infty}\left[\begin{matrix}\frac{i(3h_3-2D_1)}{2}+c(1+a)D_1\\ +c(1+a)^2(h_3-D_1)\end{matrix}\right]f(D_2)dD_2f(D_1)dD_1$$

$$+\int_{D_1=0}^{h_3}\int_{D_2=0}^{h_3-D_1}\int_{D_3=h_3-(D1+D2)}^{\infty}[i(5h_3-4D_1-2D_2)/2+c(1+a)\,D_1+c(1+a)^2D_2$$
$$+c(1+a)^3(h_3-D_1-D_2)f(D_3)dD_3f(D_2)dD_2f(D_1)dD_1]\}$$
$$+\int_{D_1=0}^{h_3}\int_{D_2=0}^{h_3-D_1}\int_{D_3=0}^{h_3-(D1+D2)}[i(7h_3-6D_1-4D_2-2D_3)/2+c(1+a)\,D_1$$
$$+c(1+a)^2D_2+c(1+a)^3D_3$$
$$+c(1+a)^4(h_3-D_1-D_2-D_3)f(D_3)dD_3f(D_2)dD_2f(D_1)dD_1]\}$$

Differentiating it withh_3,

$$p+\left[\frac{\mathrm{i}h_3}{2}+\mathrm{c}(1+\mathrm{a})h_3\right]\mathrm{f}(h_3)-\int_{D_1=h_3}^{\infty}\left[\frac{i}{2}+c(1+a)\right]f(D_1)dD_1$$
$$-\int_{D_2=0}^{\infty}\left[\frac{ih_3}{2}+c(1+a)h_3\right]f(h_3)f(D_2)dD_2$$
$$-\int_{D_1=0}^{h_3}\int_{D_2=h_3-D_1}^{\infty}\left[\frac{3i}{2}+c(1+a)^2\right]f(D_2)dD_2\,f(D_1)dD_1$$
$$+\int_{D_1=0}^{h_3}\left[\begin{matrix}\frac{i(3h_3-2D_1)}{2}+c(1+a)D_1\\ +c(1+a)^2(h_3-D_1)\end{matrix}\right]f(h_3-D_1)f(\,D_1)dD_1$$
$$-\int_{D_1=0}^{h_3}\int_{D_2=0}^{h_3-D_1}\int_{D_3=h_3-(D_1+D_2)}^{\infty}\left[\frac{5i}{2}+c(1+a)^3\right]f(D_3)dD_3f(D_2)dD_2\,f(D_1)dD_1$$
$$-\int_{D_1=0}^{h_3}\int_{D_2=0}^{h_3-D_1}\int_{D_3=0}^{h_3-(D_1+D_2)}\left[\frac{7i}{2}+c(1+a)^4\right]f(D_3)dD_3f(D_2)dD_2\,f(D_1)dD_1=0$$

The constraint for interest rate is

$$ph_3-\{C_o+\frac{7ih_3}{2}+c(1+a)^4h_3\}>0$$

Making the above inequality into an equation and solving both equations yields $h_3 = 2774.723, x = 3.649\%$

4.4 Comparison of Results

	1 period ordering	2 period ordering	3 period ordering
Optimal Ordering quantity	1278.024	2273.614	2774.7223
Total profit	2792.8	5732	7689.9
Maximum interest rate (per period rate)	15.51%	7.468%	3.649%
Profit per period	2792.8	2866	2563.3

Hence, for the values considered, 2-period ordering is better than 1-period or 3-period ordering.

5 CONCLUSION

In this report, we have developed a model for trade credit considering the various costs involved for the retailer and found the optimum strategies of retailer and supplier to maximize their long-term profits. We then took an example and looked at the variation of profit based on variation in the re-order time period of the retailer.

The model developed, however, has some shortcomings as it assumes zero lead time, which is highly improbable in real life situations. This model can be used as a starting point to develop complex models by relaxing the assumptions considered. Also, various other trade credit terms exist in usage, such as no interest for the first period or varying interest rate as payment gets delayed etc. Models for different trade credit terms can be made in a similar procedure.

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