Gaussian Minimum Shift Keying (GMSK) Modulation and Demodulation

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Contents

Abstract

This Report discusses the implementation of Gaussian Minimum Shift Keying (GMSK) Modulation and Demodulation Schemes using Symbol by Symbol detection and Viterbi decoding. GMSK is most prominent standards around the world. Global System for Mobile communication (GSM), Digital European Cordless Telephone (DECT), Cellular Digital Packet Data (CDPD), Digital communications system in the 1800 MHz band (DCS1800 in Europe), Personal communications services in the 1900 MHz band (PCS1900) in U.S., all use GMSK as their modulation format. The reason GMSK is used for GSM is its High spectral efficiency, MSK uses phase variation for modulation so better immune to noise. Non linear amplifiers are used to give better response and consumes less power so low battery usage which is an important parameter in cellular technologies. Gaussian Minimum Shift keying modulation scheme is a derivative of MSK. In GMSK, the side lobe levels of the spectrum introduced in MSK are further reduced by passing the modulating NRZ data waveform through a pre modulation Gaussian pulse-shaping filter. ISI degrade the performance of GMSK. GMSK performance is improved by using optimum filters, soft decision Viterbi decoding.

Index Terms— Bit Error Rate (BER), Gaussian filter, Gaussian Minimum Shift Keying (GMSK), Inter Symbol Interference (ISI), Minimum Shift Keying (MSK), Signal to Noise Ratio (SNR).

1 Introduction

1.1 Motivation

The need for people to communicate with each other anytime, anywhere has greatly expanded the field of wireless communication. Due to user's demands for more and better wireless services, there is a need for higher transmission bandwidth. However, in a wireless communication system, adding bandwidth is not as easy as in a wire-line system. As a result, spectral congestion becomes a serious problem. Spectrally efficient modulation scheme, which maximize the bandwidth efficiency, is a very promising solution. To achieve spectral efficiency, certain modulation constraints are imposed, which lead to a more complex receiver design. wireless communication becomes more popular, there is also a big demand for a more compact receiver. Therefore, there is a need to reduce the complexity of the receiver structures without losing much performance.

Gaussian Minimum Shift Keying(GMSK) is a spectrum and power efficient modulation scheme, used in many wireless communication systems. The GSM cellular, Cellular Digital Packet Data (CDPD) and Mobitex standards are some of the best known uses of GMSK. However, because of the phase modulation and Gaussian filtering, GMSK is not a linear modulation scheme. It is well known that the receiver structure for a linear modulation scheme(e.g. Pulse Amplitude Modulation) is less complex than that for the nonlinear modulation scheme.The goal of this thesis is to attempt to reduce the complexity of GMSK receiver by using a linear approximation of GMSK and coherent detection. The reduction in demodulation cornplexity is achicved by using the Laurent represcntation of GMSK. The Laurent representation decomposes Continuous Phase Modulation (CPM) signals into summations of PAM signals. It has been successfully implemented in [1], [3] to reduce the complexity of coherent CPM receivers.

1.2 Organization of the thesis

Chapter 2 presents a brief tutorial on GMSK, the Laurent representation, AWGN channel. Chapter 3 describes the design of simple coherent GMSK receiver for BT=0.5 based on Laurent representation. It's performance in the AWGN channel will be compared to a corresponding coherent MSK receiver. Chapter 4 describes the design of reduced-complexity coherent GMSK receiver for BT=0.3 based on Laurent representation. It's performance in the AWGN channel will be compared to a corresponding coherent MSK receiver.Chapter 5 describes the design of coherent GMSK receiver in flat fading channel.Chapter 6 provides a summary and conclusions of this thesis along with suggestions for further research.

2 GMSK and its Laurent representation

2.1 GMSK

Gaussian Minimum Shift Keying belongs to the class of CPM schemes CPM is a modulation scheme in which the phase of the carrier is instantaneously varied by the modulating signal while the RF signal envelope is kept constant. Due to the constant envelope, the CPM signals are not sensitive to amplifier nonlinearities, which results in a more compact power spectrum.

The Baseband GMSK signal can be represented as

$$s(t) = \exp(j * \varphi(t, \alpha))$$

. $\varphi(t, \alpha)$ is the Phase of the signal given by

$$\varphi(t, \alpha) = pi * h * \int_{-inf}^{t} (\sum_{i=0}^{i=n} \alpha[i] g(t - iT))$$

The Phase can be written as

$$\varphi(t, \alpha) = pi * h * \sum_{i=0}^{i=n} \alpha[i] q(t - i * T)$$

where

$$q(t) = \int_{-inf}^{t} g(\tau) d\tau$$

$g(t)$is the impulse response of the premodulation filter given by

$$g(t) = B\sqrt{\frac{2pi}{ln(2)}} \exp[\frac{-2pi^2 B^2 t^2}{ln(2)}]$$

.

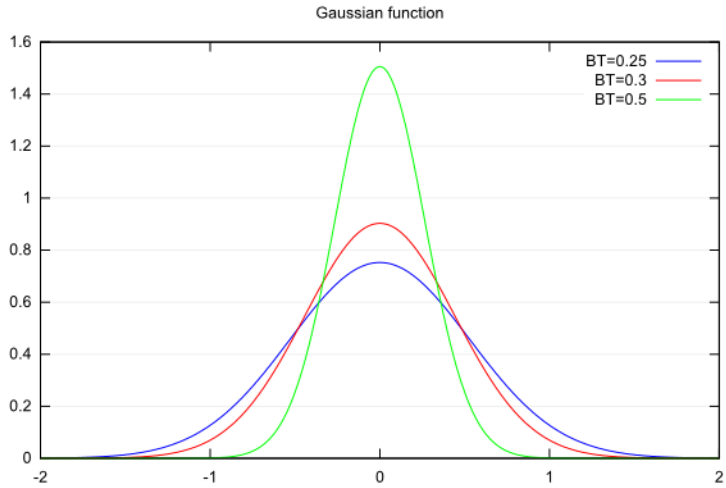
$B$is the $3db$ bandwidth of the filter.By convention, B is normally expressed in terrns of the inverse of T; therefore the 3-dB bandwidth of filter is defined as BT. The frequency pulse g(t) for GMSK is :

$$g(t) = \frac{1}{2T}\{Q(2pi * B\frac{(t - \frac{T}{2})}{\sqrt{ln2}}) - Q(2pi * B\frac{(t + \frac{T}{2})}{\sqrt{ln2}})\}$$

where

$$Q(t) = \int_{t}^{inf} \frac{1}{\sqrt{2pi}} \exp(\frac{-\tau^2}{2}) d\tau$$

For GMSK, BT is used to control the bandwidth efficiency. In general, as BT decreases, the bandwidth efficiency increases. However, as BT decreases, g(t) spreads more in time and causes more ISI. Therefore, for GMSK, there is a trade off between bandwidth efficiency and ISI. In the GSM standard, a BT of 0.3 is chosen. The resulting frequency pulse g(t) is shown in below figure .As mentioned before, since g(t) has infinite duration, it has to be truncated. For BT=0.3, g(t) is truncated to a duration of 3T.



2.2 GMSK Modulation

The generation of Baseband GMSK signal is shown in Fig 2.1.

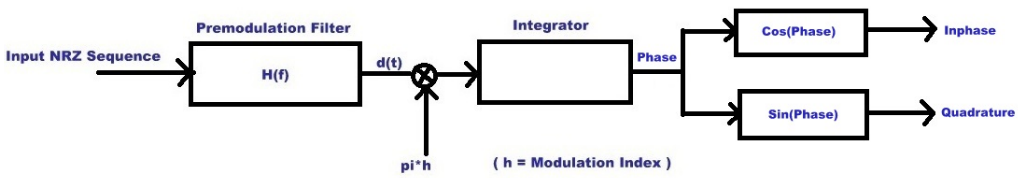


Fig : 2.1

In the Figure 2.1, Input NRZ Sequence taken from the M-ary alphabet -$(m-1)$,-$(m-3)$,......,$(m-3)$,$(m-1)$ where $m = 2^k$, $k = 1, 2, 3....$ In this thesis, we only deal with the binary alphabet +1 or -1. i.e k=1 and $m = 2$. After passing through the pre-modulation filter with frequency response $H(f)$, the output signal $d(t)$ can be expressed as

$$d(t) = \sum_{i=0}^{i=n} \alpha[i] g(t - i * T)$$

$g(t)$ is the gaussian pulse and $H(f)$ is the Gaussian frequency response.

$$g(t) = \frac{1}{2T} \{Q(2pi * B \frac{(t - \frac{T}{2})}{\sqrt{ln2}}) - Q(2pi * B \frac{(t + \frac{T}{2})}{\sqrt{ln2}})\}$$

where

$$Q(t) = \int_{t}^{inf} \frac{1}{\sqrt{2pi}} \exp(\frac{-\tau^2}{2}) d\tau$$

and

$$H(f) = \exp(\frac{-\ln 2}{2B^2} f^2)$$

$\alpha[i]$ is the Input NRZ Sequence .
h is modulation index. For GMSK $h = \frac{1}{2}$.
The resulting signal $d(t)$is passed through a Integrator to get the phase of the signal. Phase of the signal can be written as

$$\varphi(t, \alpha) = pi * h * \int_{t=-inf}^{t=t} \sum_{i=0}^{i=n} \alpha[i] g(t - i * T)$$

The Phase can be written as

$$\varphi(t, \alpha) = pi * h * \sum_{i=0}^{i=n} \alpha[i] q(t - i * T)$$

where

$$q(t) = \int_{-inf}^{t} g(\tau) d\tau$$

.

The In-phase component of the baseband representation is the Cosine of the phase generated and the Quadrature component of the baseband representation is the Sine of the phase generated.

$$I = cos(\varphi(t, \alpha))$$

$$Q = sin(\varphi(t, \alpha))$$

$$s(t) = I + j * Q$$

2.3 Laurent representation of Continous Pulse Modulation(CPM)

In baseband, CPM signals can be represented as $s(t) = \exp(j * \varphi(t, \alpha))$, where

$$\varphi(t, \alpha) = pi * h * \sum_{i=0}^{i=n} \alpha[i]q(t - i * T)$$

. For nT< t < nT + T, we have

$$s(t) = \exp(jpi * h \sum_{i=0}^{i=n-L} \alpha[i]) \prod_{i=n-L+1}^{i=n} \exp[j * h * pi * \alpha[i] * q(t - iT)] \qquad (2.1)$$

Let $J = \exp(j * pi * h)$, then $J^{\alpha[i]} = \cos(pi * h) + j\alpha[i] \sin(pi * h)$.Using this relation, the product terms of (2.1) can be expressed as

$$\exp[j * pi * h\alpha[i]q(t - iT)] = \cos(pi * hq(t - iT)) + j\alpha[i] \sin(pi * hq(t - iT))$$

$$= J^{\alpha[i]} \frac{\sin[pi * hq(t - iT)]}{\sin(pi * h)} + \frac{\sin[pi * h - pi * hq(t - iT)]}{\sin(pi * h)}$$

If we also notice that $1 - q(t) = q(LT) - q(t) = q(LT - t)$ and define

$$C(t) = \frac{\sin[pi * h - pi * hq(t)]}{\sin(pi * h)} : 0 \leq t \leq LT$$

$$C(t) = C(-t) : 0 \leq t \leq LT$$

$$a_0[n - L] = \exp(j * pi * h \sum_{i=0}^{i=n-L} \alpha[i])$$

we have

$$s(t) = \exp(jpi * h \sum_{i=0}^{i=n-L} \alpha[i]) \prod_{i=n-L+1}^{i=n} \exp[j * h * pi * \alpha[i] * q(t - iT)]$$

$$= a_0[n - L] \prod_{i=n-L+1}^{i=n} [J^{\alpha[i]} C(t - iT - LT) + C(t - iT)] \qquad (2.2)$$

Expanding 2.2 ,Laurent shows that there are only 2^{L-1} different pulses and each pulse is obtained by the product of L shifted version of C(t) . For binary GMSK(h=0.5.& J=j), it can be shown that the Laurent representation is given by

$$s(t) = \sum_{i=0}^{2^{L-1}-1} \sum_{n=0}^{n=n} a_i[n] h_i[t - nT]$$

where $h_i[t]$terms are the impulse responses of the Laurent PAM pulses and

$$a_0[n] = a_0[n-1] * j^{\alpha[n]}$$

$$a_i[n] = a_0[n-L] \prod_{i \epsilon I_k} j^{\alpha[n-i]}$$

$I_k$is a non empty subset of the set {0,1,2,....,L-1}.

2.4 GMSK BT=0.3, L=3 Laurent representation

For GMSK with BT=0.3 and L=3, $s(t)$ can be represented by 4 PAM signals

$$s(t) = \sum_{k=0}^{3} \sum_{n} j^n a_{k,n} h_k[t - nT]$$

$$h_0(t) = C(t-3T)C(t-2T)C(t-T) : 0 \leq t \leq 4T$$
$$h_1(t) = C(t-3T)C(t+T)C(t-T) : 0 \leq t \leq 2T$$
$$h_2(t) = C(t-3T)C(t+2T)C(t-2T) : 0 \leq t \leq T$$
$$h_3(t) = C(t-3T)C(t+2T)C(t+T) : 0 \leq t \leq T$$

where

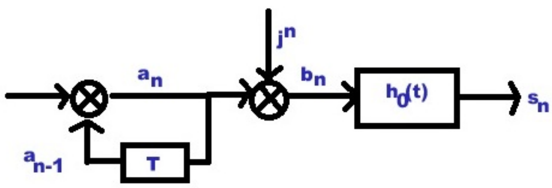
$$C(t) = \sin(\frac{pi}{2}(1 - q(t))) : 0 \leq t \leq 3T$$
$$C(t) = C(-t) : -3T \leq t \leq 0$$

The functions $h_0(t)$,$h_1(t)$,$h_2(t)$,$h_3(t)$ are plotted in fig 2.2 and it is found that 99.63% of the energy is present in $h_0(t)$. Hence the $s(t) = \sum_{k=0}^{3} \sum_n j^n a_{k,n} h_k[t - nT]$can be approximated as

$$s(t) = \sum j^n a_{0,n} h_0[t - nT]$$

The antipodal symbols $a_{0,n}$are related to transmitted antipodal symbols by the encoding rule $a_{0.n} = \alpha_n a_{0,n-1}$

Therefore the Linear approximation of the GMSK Modulator can be represented by the following figure:



2.5 AWGN Channel

$s(t)$ is the transmitted signal.

$r(t)$ is the received signal .

If the transmitted signal is passed through a AWGN Channel of $SNR(db)$, then $r(t) = s(t) + n(t)$:

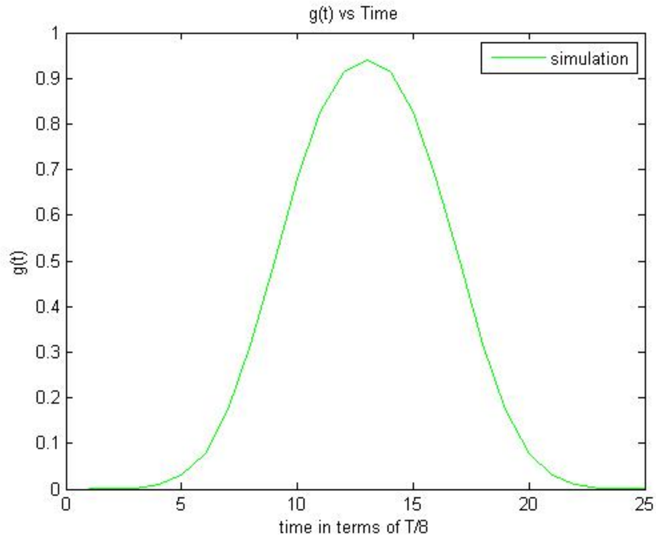
where

$$SNR = 10 \log \frac{E(||s(t)||^2)}{E(||n(t)||^2)}$$

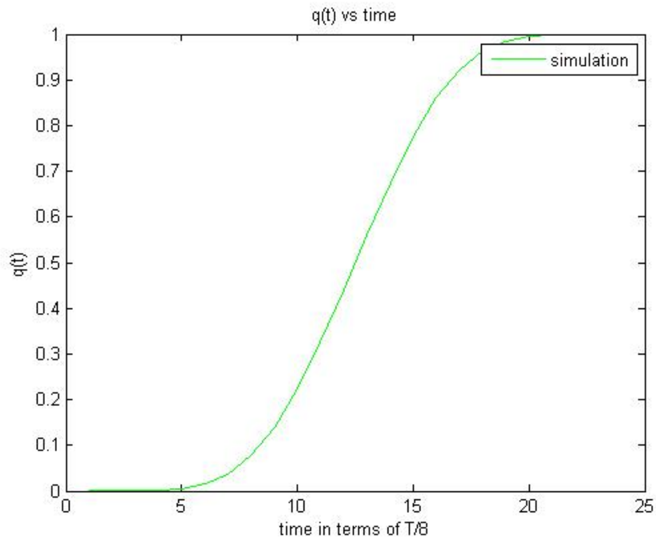
where s(t) is the transmitted signal, n(t) represents complex additive Gaussian noise with a one-sided power spectral density of N.

3 GMSK Demodulator for BT=0.5

For the BT=0.5 , $g(t)$ for 3T is plotted in following figure :



$q(t)$is for 3T is plotted in the following figure :



One can infer that most of the pulse energy in g(t) is located in between samples 9-17 (One Symbol Time period) . So the ISI , Intersymbol Interference is negligible in this case and can be neglected. Thus , one should be able to acheive optimal performance without using Viterbi decoder since there is no ISI.

3.1 Demodulator

Using the Laurent PAM representation of the GMSK , BT = 0.5 ,

we can write

$$s(t) = \sum j^n a_{0,n} h_0[t - nT]$$

The antipodal symbols $a_{0,n}$are related to transmitted antipodal symbols by the encoding rule $a_{0.n} = \alpha_n a_{0,n-1}$ and $b_n = a_{0,n} j^n$

and

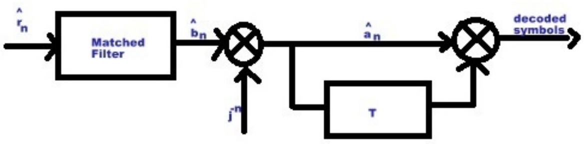
$$r(t) = s(t) + n(t) :$$

where

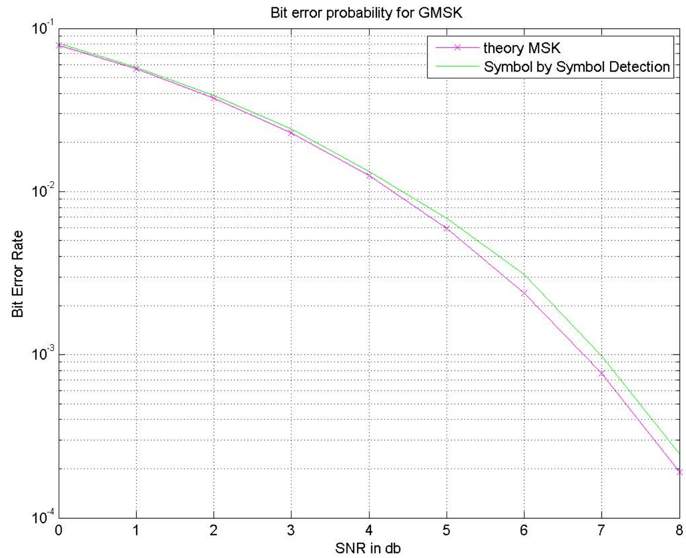
$$SNR(db) = 10 \log \frac{E(||s(t)||^2)}{E(||n(t)||^2)}$$

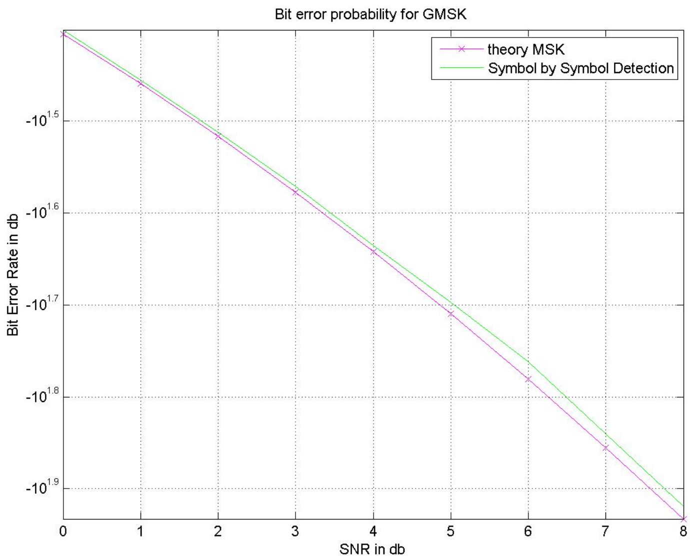
where s(t) is the transmitted signal, n(t) represents complex additive Gaussian noise with a one-sided power spectral density of N.

Optimum Demodulator Figure is :



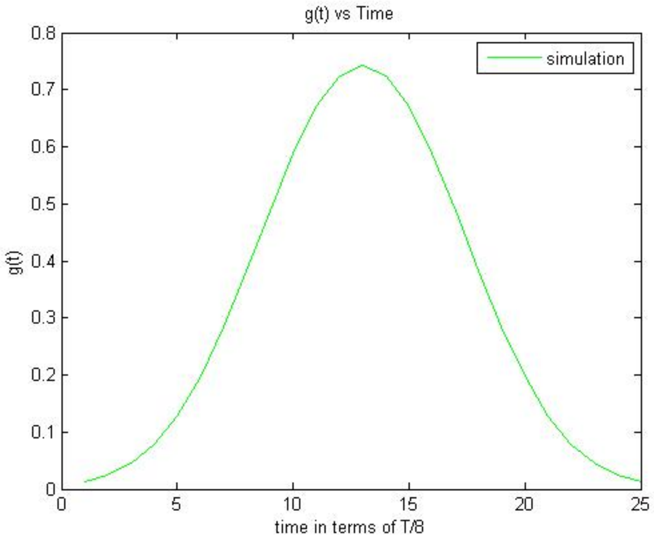
3.2 Plot for BER Vs SNR(db)



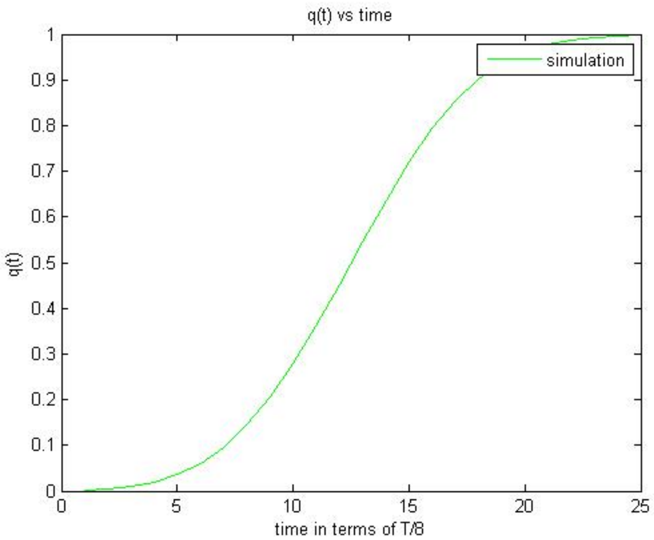


4 GMSK Demodulator for BT=0.3

For the BT=0.3, $g(t)$ for 3T is plotted in following figure :



$q(t)$ for 3T is plotted in the following figure :



If we look at the Gaussian pulse $g(t)$, we can infer that g(t) is spread out in 3 Symbol time periods. Thus one can expect ISI (Inter Symbol Interference) in the modulation.

The Gaussian pre-filter in GMSK modulation for BT =0.3 introduces intersymbol interference (ISI) that spreads over several bit intervals (For BT=0.3, 3 symbols interfere) thus, degrading performance from MSK when coherent symbol-by-symbol detection is used. Maximum likelihood sequence estimation (MLSE) using the Viterbi algorithm is well known to achieve optimal performance in the presence of ISI. The optimal MLSE demodulator for GMSK requires $4 * 2^{L-1}$ (where L is the ISI duration in bit intervals) states on AWGN channels. The presence of severe multipath fading and narrow-band receive filtering further increases the number of states making implementation complexity prohibitively high. In this report, MLSE GMSK demodulator that requires 2^{L-1} states and achieves almost the same BER performance as MSK. we utilize a linear representation of GMSK signals in terms of basic pulse amplitude modulation (PAM) signals, to design MLSE demodulator. However, linearized MLSE GMSK demodulator has several advantages namely :

1) Usage of a standard off-the-shelf Viterbi algorithm (VA), that requires 2^{L+1} additions for updating state metrics and 2^L comparisons to select survivor path, with one sample per bit,

2) Because of its linearity, GMSK demodulator can be readily applied to channels with multipath fading simply by increasing the number of states according to the multipath delay spread.

Initially the symbol by symbol conherent detection was presented in 4.1 and then the MLSE using the Viterbi algorithm was presented in 4.3.

4.1 Symbol by symbol detection(Without Viterbi)

First , the symbol by symbol detection was done to check the degradation in performance due to ISI.

Using the Laurent PAM representation of the GMSK , BT = 0.3 ,
we can write

$$s(t) = \sum j^n a_{0,n} h_0[t - nT]$$

The antipodal symbols $a_{0,n}$are related to transmitted antipodal symbols by the encoding rule $a_{0.n} = \alpha_n a_{0,n-1}$ and $b_n = a_{0,n} j^n$
and

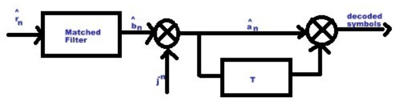
$$r(t) = s(t) + n(t) :$$

where

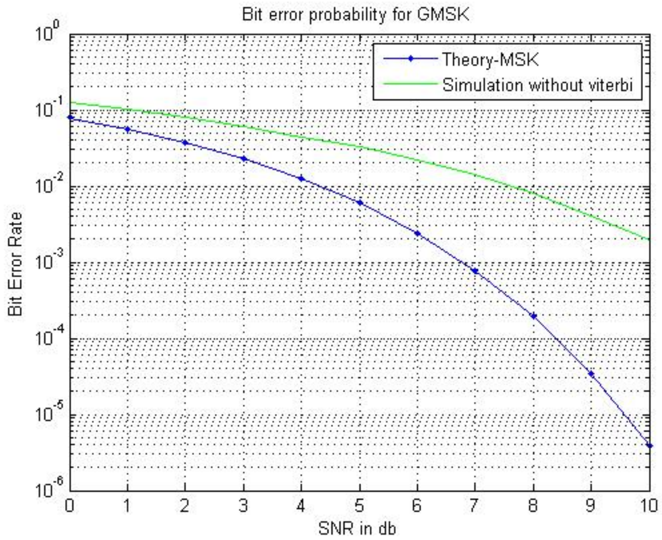
$$SNR = 10 \log \frac{E(||s(t)||^2)}{E(||n(t)||^2)}$$

where s(t) is the transmitted signal, n(t) represents complex additive Gaussian noise with a one-sided power spectral density of N.

Symbol by symbol detection figure is :



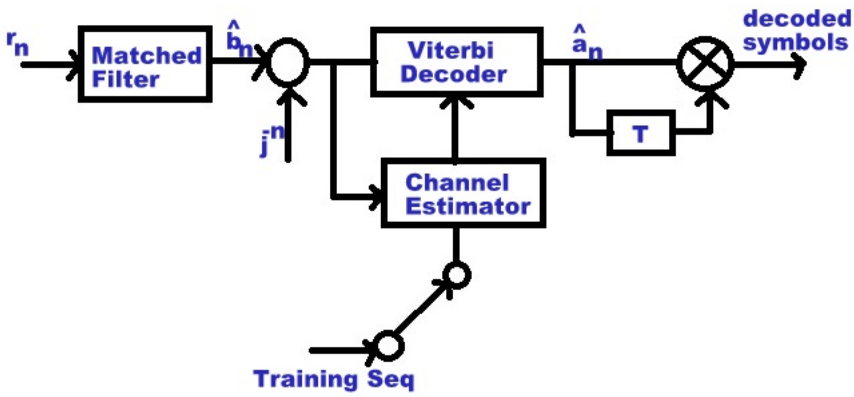
4.2 Plot for BER Vs SNR(db) for BT=0.3 without Viterbi.



4.3 MLSE using Viterbi Decoding for BT=0.3

As we can see, the symbol by symbol detection is not performing well because of ISI(Inter-Symbol Interference) caused because of narrowing down the bandwidth. One solution to improve the performance and remove the ISI(Inter-Symbol Interference) is MLSE(Maximum Likelihood Sequence Estimation) using the Viterbi Algorithm.

The demodulator for BT=0.3 using Viterbi Decoding :



In baseband, CPM signals can be represented as $s(t) = \exp(j * \varphi(t, \alpha))$, where

$$\varphi(t, \alpha) = pi * h * \sum_{i=0}^{i=n} \alpha[i] q(t - i * T)$$

. For nT< t < nT + T, we have

$$s(t) = \exp(jpi * h \sum_{i=0}^{i=n-L} \alpha[i]) \prod_{i=n-L+1}^{i=n} \exp[j * h * pi * \alpha[i] * q(t - iT)]$$

In the above case for BT=0.3 , L = 3 .

For nT< t < nT + T, we have

$$s(t) = \exp(jpi * h \sum_{i=0}^{i=n-3} \alpha[i]) \prod_{i=n-2}^{i=n} \exp[j * h * pi * \alpha[i] * q(t - iT)]$$

$$s(t) = \exp(jpi*h \sum_{i=0}^{i=n-3} \alpha[i])\{\exp[j*h*pi*\alpha[i]*q(t-(n-2)T)]*\exp[j*h*pi*\alpha[i]*q(t-(n-1)T)]*\exp[j*h*pi*\alpha[i$$

implies at t=nT ,

$$s(nT) = \exp(j*pi*h \sum_{i=0}^{i=n-3} \alpha[i])\{\exp[j*h*pi*\alpha[n-2]*q(2T)]*\exp[j*h*pi*\alpha[n-1]*q(T)\}$$

So, the state at S_n at t=nT can be represented in terms of $(\theta_n, \alpha[n-1], \alpha[n-2])$ and the received signal can be written as $\theta_n$being the phase accumulated$\exp(j*pi*h \sum_{i=0}^{i=n-3} \alpha[i])$

$$r(nT) = s(nT) + z(nT)$$

The optimal MLSE demodulator for GMSK requires $4 * 2^{L-1}$ (where L is the ISI duration in bit intervals) states on AWGN channels. $2^{L-1}$for previous two symbols interfering with current symbol and 4 for the θ_n.

θ_n can take values of $pi/2$ or pi or $3 * pi/2$ or 0 . Thus for BT=0.3 we need 16 state viterbi algorithm to optimally estimate the sequence of bits sent through MLSE.

The 16 states of the MLSE demodulator can be reduced to 4 by using Laurent Linear Representation of the transmitted sigmal and demodulating it accordingly as shown in the demodulator diagram.

By using Laurent Linear Representation ,

For GMSK with BT=0.3 and L=3, $s(t)$ can be represented by 4 PAM signals

$$s(t) = \sum_{k=0}^{3} \sum_{n} j^n a_{k,n} h_k[t - nT]$$

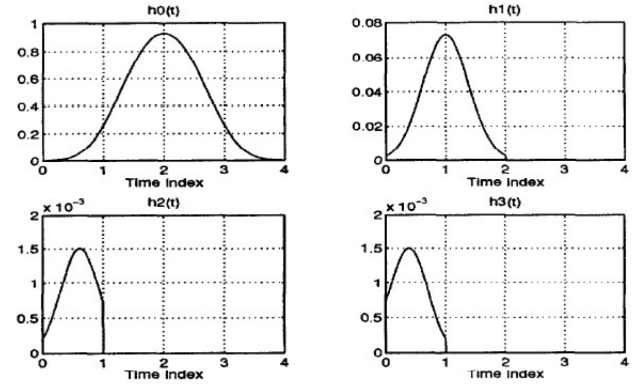
$$h_0(t) = C(t - 3T)C(t - 2T)C(t - T) : 0 \leq t \leq 4T$$

$$h_1(t) = C(t-3T)C(t+T)C(t-T) : 0 \le t \le 2T$$
$$h_2(t) = C(t-3T)C(t+2T)C(t-2T) : 0 \le t \le T$$
$$h_3(t) = C(t-3T)C(t+2T)C(t+T) : 0 \le t \le T$$

where

$$C(t) = \sin(\frac{pi}{2}(1-q(t))) : 0 \le t \le 3T$$
$$C(t) = C(-t) : -3T \le t \le 0$$

The functions $h_0(t)$,$h_1(t)$,$h_2(t)$,$h_3(t)$ are plotted in below figure :



[1]

and it is found that 99.63% of the energy is present in $h_0(t)$. Hence the $s(t) = \sum_{k=0}^{3}\sum_n j^n a_{k,n} h_k[t-nT]$can be approximated as

$$s(t) = \sum j^n a_{0,n} h_0[t-nT]$$

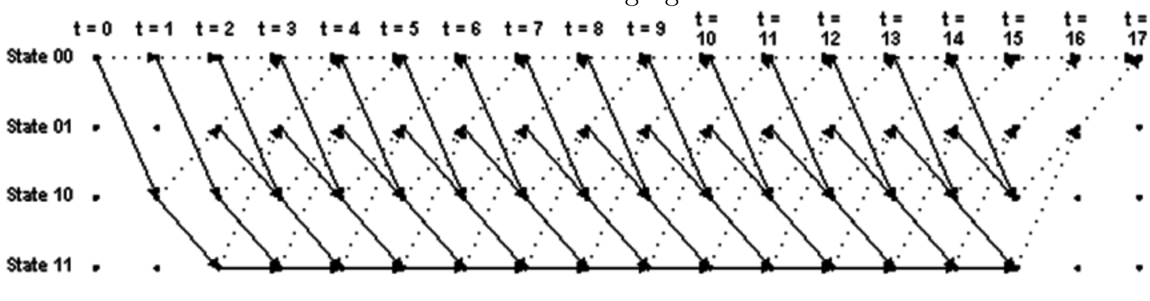
$$s(t) = \sum b_n h_0[t-nT]$$

The antipodal symbols $a_{0,n}$are related to transmitted antipodal symbols by the encoding rule $a_{0.n} = \alpha_n a_{0,n-1}$ where$\alpha_n$is the transmitted symbol .

Thus the received signal is passed through a matched filter $h_0^*[-t]$, to get $b_n$and multiply $b_n$with j^{-n} to get $a_{0.n}$.Once $a_{0.n}$ sequence is received, it is passed thorugh viterbi decoder to find the optimal sequence that was sent.The sequence from the viterbi decoder is differentially decoded to give the transmitted symbols α_n.

Viterbi Decoder Branch Metric calculations :

Viterbi state transitions can be seen in the following figure.



$\cdots\cdots$ represent that current symbol is -1 while $\rightarrow$ represent that current symbol is $+1$

.

and the branch metric calculations $branchmetric(n) = ||a(n) - \hat{a}(n))||^2$ where $a(n)$ is the received signal after multiplying $b_n$with j^{-n}. $\hat{a}(n)$ can be written as $h_0 a(n) + h_1 a(n-1) + h_2 a(n-2)$ since past two symbols are interfering with the current symbol.

$h_0, h_1, h_2$are estimated by using a training sequence of 200 symbols.

4.3.1 Estimating h_0, h_1, h_2:

Estimating h_0, h_1, h_2 is done by sending known symbols. Let us send around 200 symbols which are known at the receiver.

Let the sent symbols be $\alpha(0), \alpha(1), \alpha(2), ...\alpha(198), \alpha(199)$ and differentially encoded symbols are $a(0), a(1), a(2), a(3), ...a(198), a(199)$. and the received symbols be $\hat{a(0)}$,$\hat{a(1)}$,$\hat{a(2)}$.....$\hat{a(198)}$,$a($

Then

$$\hat{a(r)} = h_0 a(r) + h_1 a(r-1) + h_2 a(r-2) \quad r = 2, 3, ..., 199$$

which implies that $Y = XH$ where $Y = \begin{bmatrix} \hat{a(2)} \\ \hat{a(3)} \\ \hat{a(4)} \\ ... \\ \hat{a(199)} \end{bmatrix}$ $X = \begin{bmatrix} a(2) & a(1) & a(0) \\ a(3) & a(2) & a(1) \\ a(4) & a(3) & a(2) \\ ... & ... & ... \\ a(199) & a(198) & a(197) \end{bmatrix}$ $H = \begin{bmatrix} h_0 \\ h_1 \\ h_2 \end{bmatrix}$

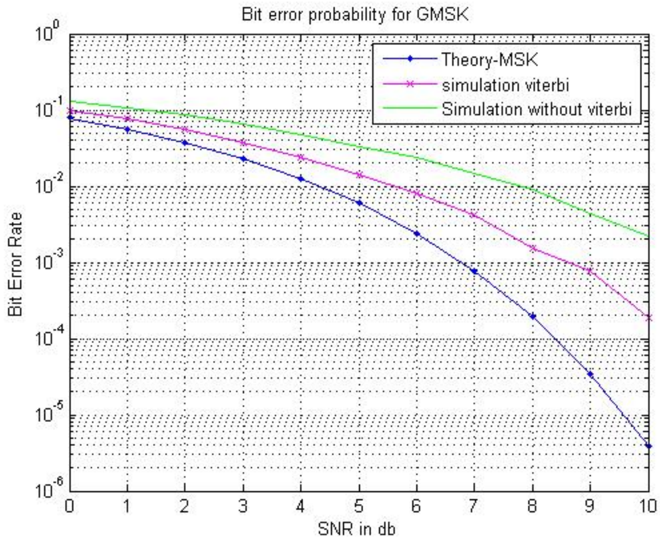
Using LMS Solution , we can find $H = \left(X^T X\right)^{-1} X^T Y$

Once h_0, h_1, h_2 are estimated , we can calculate the branchmetric as followed:

$$branchmetric(n) = ||a(n) - \{h_0 a(n) + h_1 a(n-1) + h_2 a(n-2)\}||^2$$

Then MLSE (Maximum Likelihood Sequence Estimation) can be done using Standard Viterbi Algorithm.

4.4 Plot for BER Vs SNR(db) for BT=0.3 using Viterbi Algorithm.



5 GMSK Demodulation in flat fading

In mobile radio channels, the Rayleigh distribution is commonly used to describe the statistical time varying nature of the received envelope of a flat fading signal, or the envelope of an individual multipath component. It's well known that the envelope of the sum of two quadrature Gaussian noise signals obeys a Rayleigh distribution. The Rayleigh distribution has a probability density function given by:

$$p(r) = (r/\sigma^2)exp(-r^2/2\sigma^2) \quad (0 < r < inf)$$

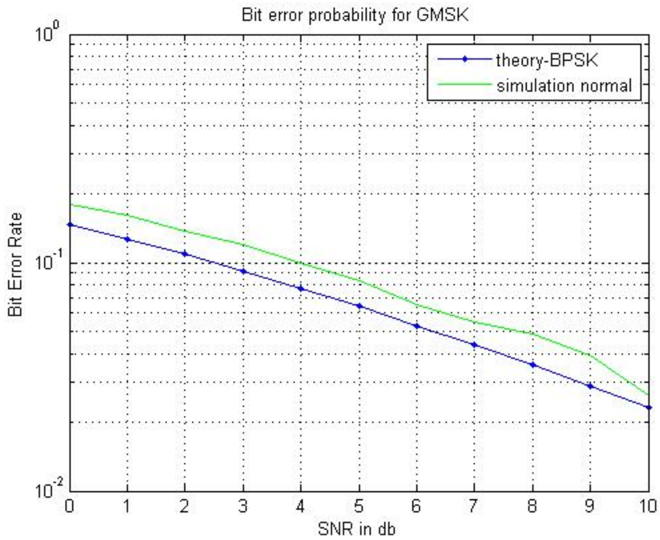
where σ is the rms value of the received voltage signal before envelope detection, and σ^2 is the time-average power of the received signal before envelope detection.

We can write the received symbol in flat fading channel as

$$y(n) = h * x(n) + z(n)$$

$z(n)$ is the AWGN Noise and $||h||$ follows rayleigh distribution.

5.1 Plot for BER Vs SNR(db) for BT=0.5

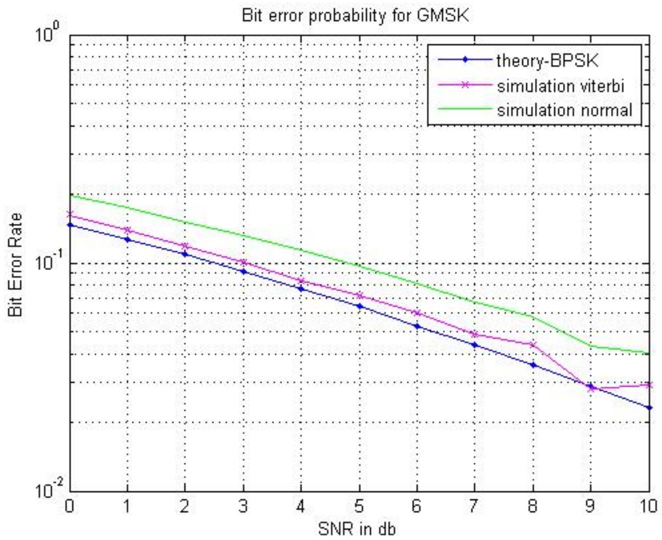


Since for BT=0.5 , there is negligible ISI in GMSK Modulation scheme , the demodulation can be done symbol by symbol and using the metric $Re\{r(n) * \hat{h}\}$ where $\hat{h}$ is the estimated based on training symbols as explained in 4.3.1

$$Theoretical\, BPSK\, BER = (1/2) * (1 - (sqrt(snr/(1 + snr))))$$

There is degradation of 0.107 db performance from BPSK.

5.2 Plot for BER Vs SNR(db) for BT=0.3 using Viterbi



Since for BT=0.3 , there is ISI in GMSK Modulation scheme , the demodulation can be done using MLSE.

$$Theoretical\,BPSK\,BER = (1/2) * (1 - (sqrt(snr/(1 + snr))))$$

There is degradation of 0.176 db performance from BPSK on an average for Symbol by Symbol detection without MLSE. But by using MLSE the degradation in performance is reduced to 0.0485 db.

6 GMSK Demodulation in multipath fading

GMSK demodulation for multipath fading of 3 taps is described as below.

The received signal y(n) can be written in terms of transmitted signal x(n) as

$$y(n) = h_0 x(n) + h_1 x(n-1) + h_2 x(n-2) + z(n) \tag{6.1}$$

where $||h_0||$, $||h_1||$, $||h_2||$ follows Raleigh distribution and $z(n)$ is the AWGN.

For BT=0.5 , since there is negligible ISI , 4 state MLSE can be used to estimate the received symbols.

For BT=0.3 , since there is ISI , we can further write $x(n) = a * s(n) + b * s(n-1) + c * s(n-2)$ where $s(n), s(n-1), s(n-2)$are the current and two previous symbols respectively.

Therefore, one can write $y(n) = h_0[a * s(n) + b * s(n-1) + c * s(n-2)] + h_1[a * s(n-1) + b * s(n-2) + c * s(n-3)] + h_2[a * s(n-2) + b * s(n-3) + c * s(n-4)] + z(n)$

Thus, 16 state MLSE can be used to estimate the received symbols since the received signal has components from past 4 symbols.

7 Conclusion

In this report, the following things are described :

1. Symbol by symbol detector for BT=0.5 is described and simulated .It is shown that there is negligible ISI(Inter Symbol Interference) in this case and the plot for BER Vs SNR(in db) was Plotted. It is found that , it performs equally to MSK.

2. Symbol by symbol detector for BT=0.3 is described and simulated .It is shown that there is ISI(Inter Symbol Interference) in this case. It is shown that past two symbols interfere with the current symbol.The plot for BER Vs SNR(in db) was Plotted. It is found that , it doesn't perform equally to MSK due to ISI.

3. 4 state reduced MLSE GMSK demodulator that achieves optimum BER performance (almost equal to MSK) on AWGN channels is described and simulated. The linear nature of the demodulator allows to use a standard off-the-shelf Viterbi processor.

4. 4-state MLSE GMSK demodulator that acheives optimum BER performance (almost equal to bpsk) in Rayleigh Fading channel for BT=0.3 was simulated and plots were shown.

Nomenclature

GMSK	Gaussian Minimum Shift Keying
MSK	Minimum Shift Keying
PAM	Pulse Amplitude Modulation
DECT	Digital Enhanced Cordless Telecommunications
GSM	Global System for Mobile Communications
CPM	Continous Phase Modulation
CDPD	Cellular Digital Packet Data
AWGN	Additive White Gaussian Channel
SNR	Signal to Noise Ratio
ISI	Inter Symbol Interference
BER	Bit Error Rate

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