Study of Subsynchronous Resonance in Power Systems

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Contents

Abstract

This report presents a study of subsynchronous resonance. The IEEE first benchmark model [1] is used for this purpose. The study includes modeling of synchronous generator, turbine-shaft rotating elements and network elements. The stability of this system is studied by determining the eigenvalues. The system stability is studied for different values of the series capacitor. Three phase fault is modeled in the network and the system is simulated to determine the transient response.

Notations

ω_B	base angular frequency
Ψ_k	flux linkage along the k^{th}axis after transformation and normalization $k = d, q, F, 1D, 1Q, 2Q$
ω_i	angular frequency of the i^{th}mass
v_d, v_q, v_0	transformed and normalized generator voltages
$T_d^{'}, T_d^{''}, T_q^{'}, T_q^{''}$	short circuit time constants
$X_d, X_d^{'}, X_d^{''}, X_q, X_q^{'}, X_q^{''}$	generator reactances
$E_f^{'}$	excitation
$v_a, v_{b,} v_c$	phase voltages at generator terminals
H_i	inertia constant of the i^{th}mass
δ_i	position of the i^{th}mass
$T_e^{'}$	electrical torque
K_{ij}	spring constant
T_i	mechanical torque of the i^{th}turbine section
r_i	ratio of the steady state mechanical torque of the i^{th}turbine section to $T_m^{'}$
$T_m^{'}$	Total mechanical torque
i_d, i_q	currents flowing in the network
v_c	voltage across the capacitor
v_t	voltage of the infinite bus
X_L	sum of reactance of transformer, transmission line and the infinite bus
X_{L1}	sum of reactance of transformer and transmission line
X_2	reactance of infinite bus
X_C	series capacitor reactance

R total resistance

D damping factor in pu torque per pu speed deviation,

D_{HP},D_{IP},D_{LA},D_{LB} self damping coefficients

D_{HI},D_{IA},D_{AB},D_{BG},D_{GE} mutual damping coefficients

P power generated by the generator

V voltage at the generator bus

I current flowing out of generator terminals

ϕ phase angle of voltage at generator

ψ phase angle of current

1 Introduction

Subsynchronous resonance (SSR) is a case where the electric network exchanges significant amount of power with the mechanical system. This phenomenon arises as a result of the interaction between a fixed series capacitor, used for compensating transmission lines and the turbine generator shaft. This results is excessively high oscillatory torque on machine shaft causing fatigue and damage. Since the two shaft failures at Mohave station in Navada in 1970 and 1971, subsynchronous resonance has become topic of interest in utilities where this phenomenon is a problem, and the determination of conditions that excite these SSR oscillations are important to those who design and operate these power systems [2-3].

1.1 Introduction to SSR

The formal definition of SSR is provided by IEEE [4] to be,

Subsynchronous resonance is an electric power system condition where the electric network exchanges energy with a turbine generator at one or more of the natural frequencies of the combined system below the synchronous frequency of the system.

Subsynchronous resonance can exist in a power system wherein the network has natural frequencies that fall below the nominal frequency of the network voltages. Currents flowing in the ac network have two components: one component at the frequency of the driving voltages and another sinusoidal component at a frequency that depends entirely on the elements of the network. Park's transformation makes the $50/60Hz$ component of current appear, as viewed from the rotor, as a dc current in the steady state, but the currents of frequency that depends on the network elements are transformed into currents of frequencies containing the sum and difference of the two frequencies. The difference frequencies are called subsynchronous frequencies. These subsynchronous currents produce shaft torques on the turbine-generator rotor that cause the rotor to oscillate at subsynchronous frequencies. The presence of subsynchronous torques on the rotor causes concern because the turbine-generator shaft itself has natural modes of oscillation. It happens that the shaft oscillatory modes are at subsynchronous frequencies. If the induced subsynchronous torques coincide with one of the shaft natural modes of oscillation, the shaft will oscillate at this natural frequency, with a high amplitude. This is called subsynchronous resonance, which causes shaft fatigue and possible damage or failure[5].

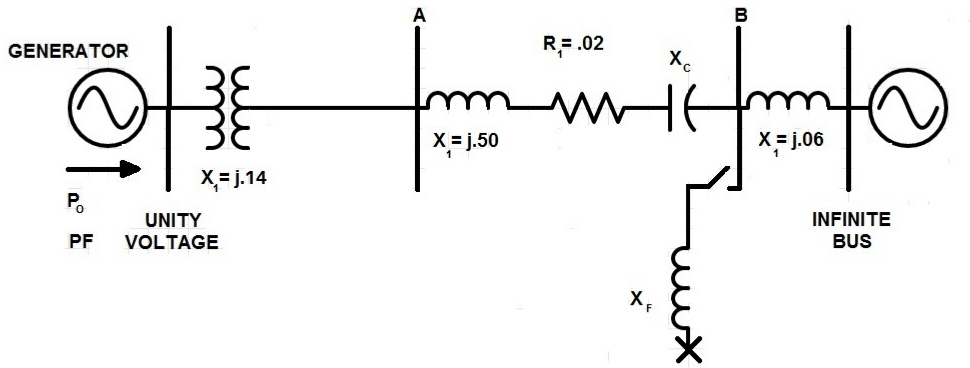
1.2 Report outline

Chapter 2 gives the equations to model synchronous generator, turbine shaft system, and the network. It includes a discussion on Park's transformation to convert the network equations into Park variables. Chapter 2 also presents fault analysis. In Chapter 3, small disturbance stability is evaluated while varying the value of series capacitor reactance. Chapter 4 describes the results of the simulation.

2 Equipment Modeling

The IEEE first benchmark model is used for the study of subsynchronous resonance [1]. Fig. 2.1 shows the single line diagram of this sysytem.

Figure 2.1: network for sub synchronous resonance studies



2.1 Generator modeling

The following generator equations were taken from [6]

$$\frac{d\Psi_d}{dt} = -\omega_2\Psi_q - \omega_B v_d \tag{2.1}$$

$$\frac{d\Psi_q}{dt} = \omega_2\Psi_d - \omega_B v_q \tag{2.2}$$

$$\frac{d\Psi_0}{dt} = -\omega_B v_0 \tag{2.3}$$

$$\frac{d\Psi_F}{dt} = \frac{1}{T'_d}(-\Psi_F + \Psi_d - \frac{X'_d}{X'_d - X_d}E'_f) \tag{2.4}$$

$$\frac{d\Psi_{1D}}{dt} = \frac{1}{T''_d}(-\Psi_{1D} + \Psi_d) \tag{2.5}$$

$$\frac{d\Psi_{1Q}}{dt} = \frac{1}{T'_q}(-\Psi_{1Q} + \Psi_q) \tag{2.6}$$

$$\frac{d\Psi_{2Q}}{dt} = \frac{1}{T''_q}(-\Psi_{2Q} + \Psi_q) \tag{2.7}$$

$$\Psi_0 = X_0 i_0 \tag{2.8}$$

$$i_d = \frac{1}{X''_d}\Psi_d + (\frac{1}{X_d} - \frac{1}{X'_d})\Psi_F + (\frac{1}{X'_d} - \frac{1}{X''_d})\Psi_{1D} \tag{2.9}$$

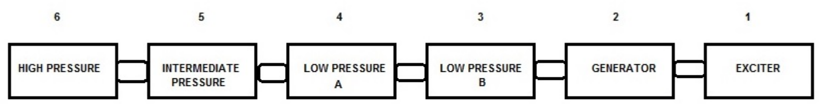
$$i_q = \frac{1}{X''_q}\Psi_q + (\frac{1}{X_q} - \frac{1}{X'_q})\Psi_{1Q} + (\frac{1}{X'_q} - \frac{1}{X''_q})\Psi_{2Q} \tag{2.10}$$

Neglect the zero component terms Ψ_0, v_0

2.2 Modeling rotating masses

The following equations are used to model the rotating masses [6]

Figure 2.2: rotating masses



The masses corresponding to $i = 1, 2, 3, 4, 5, 6$ are
mass 1 : exciter
mass 2 : generator
mass 3 : low pressure B
mass 4 : low pressure A
mass 5 : intermediate pressure
mass 6 : high pressure

$$\frac{d\delta_1}{dt} = \omega_1 - \omega_o \tag{2.11}$$

$$\frac{d\delta_2}{dt} = \omega_2 - \omega_o \tag{2.12}$$

$$\frac{d\delta_3}{dt} = \omega_3 - \omega_o \tag{2.13}$$

$$\frac{d\delta_4}{dt} = \omega_4 - \omega_o \tag{2.14}$$

$$\frac{d\delta_5}{dt} = \omega_5 - \omega_o \tag{2.15}$$

$$\frac{d\delta_6}{dt} = \omega_6 - \omega_o \tag{2.16}$$

$$\frac{d\omega_1}{dt} = \frac{\omega_B}{2H_1}[K_{12}(\delta_2 - \delta_1)] \tag{2.17}$$

$$\frac{d\omega_2}{dt} = \frac{\omega_B}{2H_2}[K_{23}(\delta_3 - \delta_2) - \bar{k}_{12}(\delta_2 - \delta_1) - T_e^{'}] \tag{2.18}$$

$$\frac{d\omega_3}{dt} = \frac{\omega_B}{2H_3}[T_3 + K_{34}(\delta_4 - \delta_3) - K_{23}(\delta_3 - \delta_2)] \tag{2.19}$$

$$\frac{d\omega_4}{dt} = \frac{\omega_B}{2H_4}[T_4 + K_{45}(\delta_5 - \delta_4) - K_{34}(\delta_4 - \delta_3)] \tag{2.20}$$

$$\frac{d\omega_5}{dt} = \frac{\omega_B}{2H_5}[T_5 + K_{56}(\delta_6 - \delta_5) - K_{45}(\delta_5 - \delta_4)] \tag{2.21}$$

$$\frac{d\omega_6}{dt} = \frac{\omega_B}{2H_6}[T_6 - K_{56}(\delta_6 - \delta_5)] \tag{2.22}$$

Use $T_e^{'} = \Psi_d i_q - \Psi_q i_d$ and re-write equation (2.18) as

$$\frac{d\omega_2}{dt} = \frac{\omega_B}{2H_2}[K_{23}(\delta_3 - \delta_2) - K_{12}(\delta_2 - \delta_1) - (\Psi_d i_q - \Psi_q i_d)] \tag{2.23}$$

Including damping in the analysis modifies the equations of the rotating masses to the following [7]

$$\frac{d\omega_1}{dt} = \frac{\omega_B}{2H_1}[K_{12}(\delta_2 - \delta_1) - D_{GE}\frac{\omega_1 - \omega_2}{\omega_o}] \tag{2.24}$$

$$\begin{aligned}\frac{d\omega_2}{dt} &= \frac{\omega_B}{2H_2}[K_{23}(\delta_3 - \delta_2) - K_{12}(\delta_2 - \delta_1) - (\Psi_d(\frac{1}{X_q^{''}}\Psi_q + (\frac{1}{X_q} - \frac{1}{X_q^{'}})\Psi_{1Q} \\ &\quad +(\frac{1}{X_q^{'}} - \frac{1}{X_q^{''}})\Psi_{2Q}) - \Psi_q(\frac{1}{X_d^{''}}\Psi_d + (\frac{1}{X_d} - \frac{1}{X_d^{'}})\Psi_F \\ &\quad +(\frac{1}{X_d^{'}} - \frac{1}{X_d^{''}})\Psi_{1D})) - D_{GE}\frac{\omega_2 - \omega_1}{\omega_o} - D_{BG}\frac{\omega_2 - \omega_3}{\omega_o}]\end{aligned} \tag{2.25}$$

$$\begin{aligned}\frac{d\omega_3}{dt} &= \frac{\omega_B}{2H_3}[T_3 + K_{34}(\delta_4 - \delta_3) - K_{23}(\delta_3 - \delta_2) - \\ &\quad D_{BG}\frac{\omega_3 - \omega_2}{\omega_o} - D_{AB}\frac{\omega_3 - \omega_4}{\omega_o} - D_{LB}\frac{\omega_3 - \omega_o}{\omega_o}]\end{aligned} \tag{2.26}$$

$$\frac{d\omega_4}{dt} = \frac{\omega_B}{2H}[T_4 + K_{45}(\delta_5 - \delta_4) - K_{34}(\delta_4 - \delta_3) - D_{AB}\frac{\omega_4 - \omega_3}{\omega_o} - D_{LA}\frac{\omega_4 - \omega_o}{\omega_o} - D_{IA}\frac{\omega_4 - \omega_5}{\omega_o}] \quad (2.27)$$

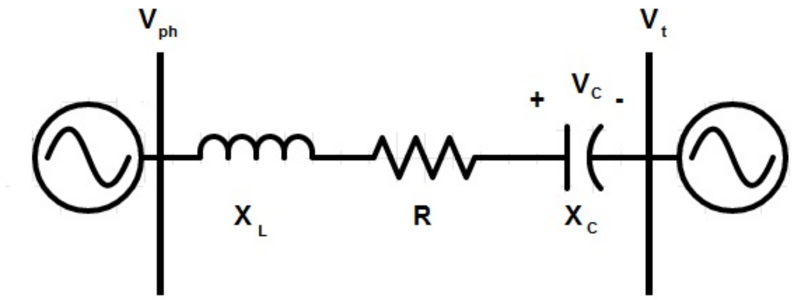
$$\frac{d\omega_5}{dt} = \frac{\omega_B}{2H_5}[T_5 + K_{56}(\delta_6 - \delta_5) - K_{45}(\delta_5 - \delta_4) - D_{IA}\frac{\omega_5 - \omega_4}{\omega_o} - D_{IP}\frac{\omega_5 - \omega_o}{\omega_o} - D_{HI}\frac{\omega_5 - \omega_6}{\omega_o}] \quad (2.28)$$

$$\frac{d\omega_6}{dt} = \frac{\omega_B}{2H_6}[T_6 - K_{56}(\delta_6 - \delta_5) - D_{HI}\frac{\omega_6 - \omega_5}{\omega_o} - D_{HP}\frac{\omega_6 - \omega_o}{\omega_o}] \quad (2.29)$$

2.3 Network equations

The following simplifiedRLC circuit is used to model the network

Figure 2.3: network for modeling the RLC elements



Writing network equations in a,b,c phases, we get two sets of normalized equations

$$-\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} + \frac{1}{\omega_B}X_L\begin{bmatrix} \frac{di_a}{dt} \\ \frac{di_b}{dt} \\ \frac{di_c}{dt} \end{bmatrix} + R\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} v_{ca} \\ v_{cb} \\ v_{cc} \end{bmatrix} + \begin{bmatrix} v_{ta} \\ v_{tb} \\ v_{tc} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (2.30)$$

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \frac{1}{\omega_B X_C}\begin{bmatrix} \frac{dv_{ca}}{dt} \\ \frac{dv_{cb}}{dt} \\ \frac{dv_{cc}}{dt} \end{bmatrix} \quad (2.31)$$

The above set of equations are transformed to

$$-v_d + \frac{\omega_2}{\omega_B} X_L i_q + \frac{X_L}{\omega_B} \frac{di_d}{dt} + Ri_d + v_{cd} + v_{td} = 0 \tag{2.32}$$

$$-v_q - \frac{\omega_2}{\omega_B} X_L i_d + \frac{X_L}{\omega_B} \frac{di_q}{dt} + Ri_q + v_{cq} + v_{tq} = 0 \tag{2.33}$$

$$\frac{dv_{cd}}{dt} = i_d \omega_B X_C - \omega_2 v_{cq} \tag{2.34}$$

$$\frac{dv_{cq}}{dt} = i_q \omega_B X_C + \omega_2 v_{cd} \tag{2.35}$$

Transform the Park parameters $v_{td}$and v_{tq}into Kron parameters.

$$v_{tq} + jv_{td} = (V_{tQ} + jV_{tD})e^{-j\delta_2}$$

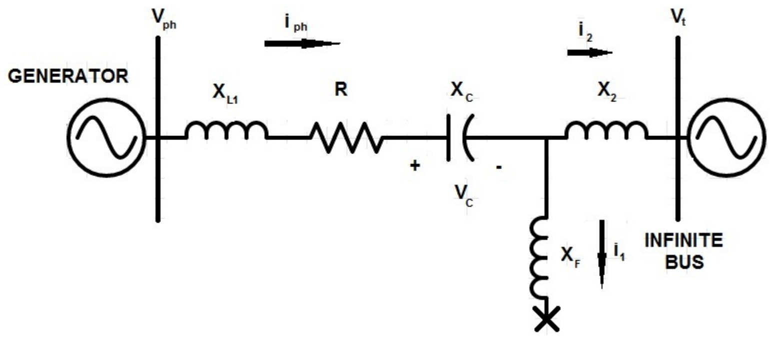
$$v_{td} = V_{tD}\cos(\delta_2) - V_{tQ}\sin(\delta_2) \tag{2.36}$$

$$v_{tq} = V_{tQ}\cos(\delta_2) + V_{tD}\sin(\delta_2) \tag{2.37}$$

2.4 Fault modeling

The following circuit model is used for deriving network equations for simulating transient response during fault [1]
Writing equations in a,b,c phases before the fault is cleared from any of the three phases:

Figure 2.4: network for fault analysis



$$
\begin{aligned}
\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} &= -\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} + \frac{1}{\omega_B} X_{L1} \begin{bmatrix} \frac{di_a}{dt} \\ \frac{di_b}{dt} \\ \frac{di_c}{dt} \end{bmatrix} + R \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \\
& \begin{bmatrix} v_{ca} \\ v_{cb} \\ v_{cc} \end{bmatrix} + \frac{1}{\omega_B} X_2 \begin{bmatrix} \frac{di_{2a}}{dt} \\ \frac{di_{2b}}{dt} \\ \frac{di_{2c}}{dt} \end{bmatrix} + \begin{bmatrix} v_{ta} \\ v_{tb} \\ v_{tc} \end{bmatrix}
\end{aligned} \tag{2.38}
$$

$$
\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = -\frac{1}{\omega_B} X_F \begin{bmatrix} \frac{di_{1a}}{dt} \\ \frac{di_{1b}}{dt} \\ \frac{di_{1c}}{dt} \end{bmatrix} + \frac{1}{\omega_B} X_2 \begin{bmatrix} \frac{di_{2a}}{dt} \\ \frac{di_{2b}}{dt} \\ \frac{di_{2c}}{dt} \end{bmatrix} + \begin{bmatrix} v_{ta} \\ v_{tb} \\ v_{tc} \end{bmatrix} \tag{2.39}
$$

$$
\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \frac{1}{\omega_B X_C} \begin{bmatrix} \frac{dv_{ca}}{dt} \\ \frac{dv_{cb}}{dt} \\ \frac{dv_{cc}}{dt} \end{bmatrix} \tag{2.40}
$$

Using the relation

$$
i_{2abc} = i_{abc} - i_{1abc}
$$

And doing Park's Transformation we get,

$$
\begin{aligned}
0 &= -v_d + \frac{\omega_2}{\omega_B}(X_{L1} + X_2 - \frac{X_2 X_2}{X_F + X_2}) i_q + \frac{1}{\omega_B}(X_{L1} + X_2 - \frac{X_2 X_2}{X_F + X_2}) \frac{di_d}{dt} + \\
& R i_d + v_{cd} + v_{td}(1 - \frac{X_2}{X_F + X_2})
\end{aligned} \tag{2.41}
$$

$$
\begin{aligned}
0 &= -v_q - \frac{\omega_2}{\omega_B}(X_{L1} + X_2 - \frac{X_2 X_2}{X_F + X_2}) i_d + \frac{1}{\omega_B}(X_{L1} + X_2 - \frac{X_2 X_2}{X_F + X_2}) \frac{di_q}{dt} + \\
& R i_q + v_{cq} + v_{tq}(1 - \frac{X_2}{X_F + X_2})
\end{aligned} \tag{2.42}
$$

$$
\frac{dv_{cd}}{dt} = i_d \omega_B X_C - \omega_2 v_{cq} \tag{2.43}
$$

$$
\frac{dv_{cq}}{dt} = i_q \omega_B X_C + \omega_2 v_{cd} \tag{2.44}
$$

Use equation (2.41) to equation (2.44) to model the network during the fault

3 Small disturbance stability analysis

Use

$$P = VI\cos(\phi - \psi) \tag{3.1}$$

$$I = \frac{P}{V\cos(\phi - \psi)}$$

and then ψ can be found using

$$\psi = \phi - cos^{-1}(\frac{P}{VI}) \tag{3.2}$$

V_D, I_D, V_Q, I_Q are found using the following equations

$$V_D = V\sin\phi \tag{3.3}$$

$$V_Q = V\cos\phi \tag{3.4}$$

$$I_D = I\sin\psi \tag{3.5}$$

$$I_Q = I\cos\psi \tag{3.6}$$

Consider equation (2.1) to equation (2.7), the two algebraic equations (2.9) and (2.10) and equation (2.11) to equation (2.16). Equate the derivatives to zero. Use subscript o to denote initial value of the state variables

$$0 = -\omega_{2o}\Psi_{qo} - \omega_B v_{do} \tag{3.7}$$

$$0 = \omega_2\Psi_d - \omega_B v_q \tag{3.8}$$

$$0 = \frac{1}{T_d^{'}}(-\Psi_F + \Psi_d - \frac{X_d^{'}}{X_d^{'} - X_d}E_f^{'}) \tag{3.9}$$

$$0 = \frac{1}{T_d^{''}}(-\Psi_{1D} + \Psi_d) \tag{3.10}$$

$$0 = \frac{1}{T_q^{'}}(-\Psi_{1Q} + \Psi_q) \tag{3.11}$$

$$0 = \frac{1}{T_q''}(-\Psi_{2Q} + \Psi_q) \tag{3.12}$$

$$0 = \omega_1 - \omega_o \tag{3.13}$$

$$0 = \omega_2 - \omega_o \tag{3.14}$$

$$0 = \omega_3 - \omega_o \tag{3.15}$$

$$0 = \omega_4 - \omega_o \tag{3.16}$$

$$0 = \omega_5 - \omega_o \tag{3.17}$$

$$0 = \omega_6 - \omega_o \tag{3.18}$$

and

$$i_d = \frac{1}{X_d''}\Psi_d + (\frac{1}{X_d} - \frac{1}{X_d'})\Psi_F + (\frac{1}{X_d'} - \frac{1}{X_d''})\Psi_{1D}$$

$$i_q = \frac{1}{X_q''}\Psi_q + (\frac{1}{X_q} - \frac{1}{X_q'})\Psi_{1Q} + (\frac{1}{X_q'} - \frac{1}{X_q''})\Psi_{2Q}$$

we get the following relations:

$$\omega_{1o} = \omega_{2o} = \omega_{3o} = \omega_{4o} = \omega_{5o} = \omega_{6o} = \omega_o \tag{3.19}$$

we assume that

$$\omega_o = \omega_B$$

therefore,

$$\Psi_{qo} = -v_{do} \tag{3.20}$$

$$\Psi_{do} = v_{qo} \tag{3.21}$$

$$\Psi_{Fo} = \Psi_{do} - \frac{X_d'}{X_d' - X_d}E_{fo}' \tag{3.22}$$

$$\Psi_{1Do} = \Psi_{do} \tag{3.23}$$

$$\Psi_{1Qo} = \Psi_{qo} \tag{3.24}$$

$$\Psi_{2Qo} = \Psi_{qo} \tag{3.25}$$

$$i_{do} = \frac{1}{X_d''}\Psi_{do} + (\frac{1}{X_d} - \frac{1}{X_d'})(\Psi_{do} - \frac{X_d'}{X_d' - X_d}E_{fo}') + (\frac{1}{X_d'} - \frac{1}{X_d''})\Psi_{do}$$

which gets simplified to

$$i_{do} = \frac{(\Psi_{do} - E_{fo}')}{X_d} = \frac{(v_{qo} - E_{fo}')}{X_d} \tag{3.26}$$

which is equal to

$$i_{qo} = \frac{1}{X_q''}\Psi_{qo} + (\frac{1}{X_q} - \frac{1}{X_q'})\Psi_{qo} + (\frac{1}{X_q'} - \frac{1}{X_q''})\Psi_{qo}$$

which gets simplified to

$$i_{qo} = \frac{\Psi_{qo}}{X_q} = \frac{-v_{do}}{X_q}$$

To get δ_o,

$$v_{qo} + jv_{do} = X_d i_{do} - jX_q i_{qo} + E_{fo}'$$

$$v_{qo} + jv_{do} + jX_q(i_{qo} + ji_{do}) = X_d i_{do} + E_{fo}' - X_q i_{do}$$

$$(V_{Qo} + jV_{Do}) + jX_q(I_{Qo} + jI_{Do}) = (X_d i_{do} + E_{fo}' - X_q i_{do})e^{j\delta_o}$$

$$\delta_o = \angle(V_{Qo} + jV_{Do}) + jX_q(I_{Qo} + jI_{Do}) \tag{3.27}$$

also,

$$v_{qo} = Re\{e^{-j\delta_o}(V_{Qo} + jV_{Do})\} \tag{3.28}$$

$$v_{do} = Im\{e^{-j\delta_o}(V_{Qo} + jV_{Do})\} \tag{3.29}$$

$$i_{qo} = Re\{e^{-j\delta_o}(I_{Qo} + jI_{Do})\} \tag{3.30}$$

$$i_{do} = Im\{e^{-j\delta_o}(I_{Qo} + jI_{Do})\} \tag{3.31}$$

Therefore, $\Psi_{qo}, \Psi_{do}, \Psi_{Fo}, \Psi_{1Do}, \Psi_{1Qo}, \Psi_{2Qo}, E_{fo}'$can be found
Now consider equation (2.26) to equation (2.29) and equate their derivatives to zero

$$0 = \frac{\omega_B}{X_L}(v_d - \frac{\omega_2}{\omega_B} X_L i_q - R i_d - v_{cd} - v_{td}) \tag{3.32}$$

$$0 = \frac{\omega_B}{X_L}(v_q + \frac{\omega_2}{\omega_B} X_L i_d - R i_q - v_{cq} - v_{tq}) \tag{3.33}$$

$$0 = i_d \omega_B X_C - \omega_2 v_{cq} \tag{3.34}$$

$$0 = i_q \omega_B X_C + \omega_2 v_{cd} \tag{3.35}$$

From equation (3.34), we get

$$v_{cqo} = X_C i_{do} \tag{3.36}$$

And from equation (3.35), we get

$$v_{cdo} = -X_C i_{qo} \tag{3.37}$$

We can substitute for v_{cqo} and v_{cdo} and find v_{tdo} and v_{tqo}
V_{tD} and V_{tQ} are found using

$$V_{tD} = Im\{e^{j\delta_2}(v_q + jv_d)\} \tag{3.38}$$

$$V_{tQ} = Re\{e^{j\delta_2}(v_q + jv_d)\} \tag{3.39}$$

To find δ and torque (T) values in steady state, consider equation (2.17) to equation (2.22)

$$0 = \frac{\omega_B}{2H_1}[K_{12}(\delta_2 - \delta_1)] \tag{3.40}$$

$$0 = \frac{\omega_B}{2H_2}[K_{23}(\delta_3 - \delta_2) - K_{12}(\delta_2 - \delta_1) - T_e^{'}] \tag{3.41}$$

$$0 = \frac{\omega_B}{2H_3}[T_3 + K_{34}(\delta_4 - \delta_3) - K_{23}(\delta_3 - \delta_2)] \tag{3.42}$$

$$0 = \frac{\omega_B}{2H_4}[T_4 + K_{45}(\delta_5 - \delta_4) - K_{34}(\delta_4 - \delta_3)] \tag{3.43}$$

$$0 = \frac{\omega_B}{2H_5}[T_5 + K_{56}(\delta_6 - \delta_5) - K_{45}(\delta_5 - \delta_4)] \tag{3.44}$$

$$0 = \frac{\omega_B}{2H_6}[T_6 - K_{56}(\delta_6 - \delta_5)] \tag{3.45}$$

$T_e^{'} = P$; $T_3 = r_1 T_e^{'}$; $T_4 = r_2 T_e^{'}$; $T_5 = r_3 T_e^{'}$; $T_6 = r_4 T_e^{'}$; r_1, r_2, r_3, r_4 values are provided

$$\delta = \delta_{2o} = \delta_o \tag{3.46}$$

$$\delta_{3o} = \frac{T_e^{'}}{K_{23}} + \delta_{2o} \tag{3.47}$$

$$\delta_{4o} = \frac{T_e^{'} - T_3}{K_{34}} + \delta_{3o} \tag{3.48}$$

$$\delta_{5o} = \frac{T_e^{'} - T_3 - T_4}{K_{45}} + \delta_{4o} \tag{3.49}$$

$$\delta_{6o} = \frac{T_e^{'} - T_3 - T_4 - T_5}{K_{56}} + \delta_{5o} \tag{3.50}$$

3.1 Eigenvalues of the system

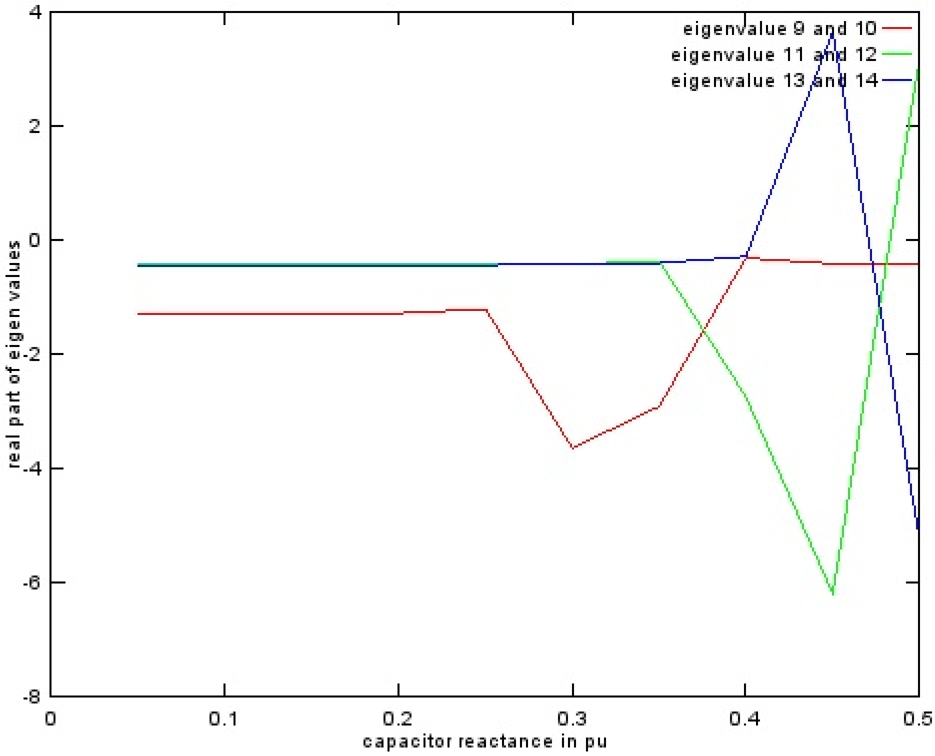
Table 3.1: Computed Eigenvalues for the First Benchmark Model with Damping with $X_C = .35$

Eigenvalue number	Real Part, (s^{-1})	Imaginary Part,(rad/s)
1,2	-4.7121744130	±616.6209083590
3,4	-3.7008370093	±298.1489949776
5,6	-0.7304785937	±202.7832149399
7,8	-1.2872748318	±160.2733094920
9,10	-2.9058597402	±136.7709702242
11,12	-0.3706471041	±127.2403263027
13,14	-0.3999191450	±99.8004681100
15	-32.9865535860	
16	-20.3727594459	
17,18	-0.7519406710	±10.2114921725
19	-3.8483197170	
20	-0.3151077171	

3.2 Variation of eigenvalues with change in capacitance

The series capacitor reactance is varied from 10% to 100% of transmission line reactance, and the real parts of eigenvalues are plotted.

Figure 3.1: graph of eigenvalues with different values of capacitance



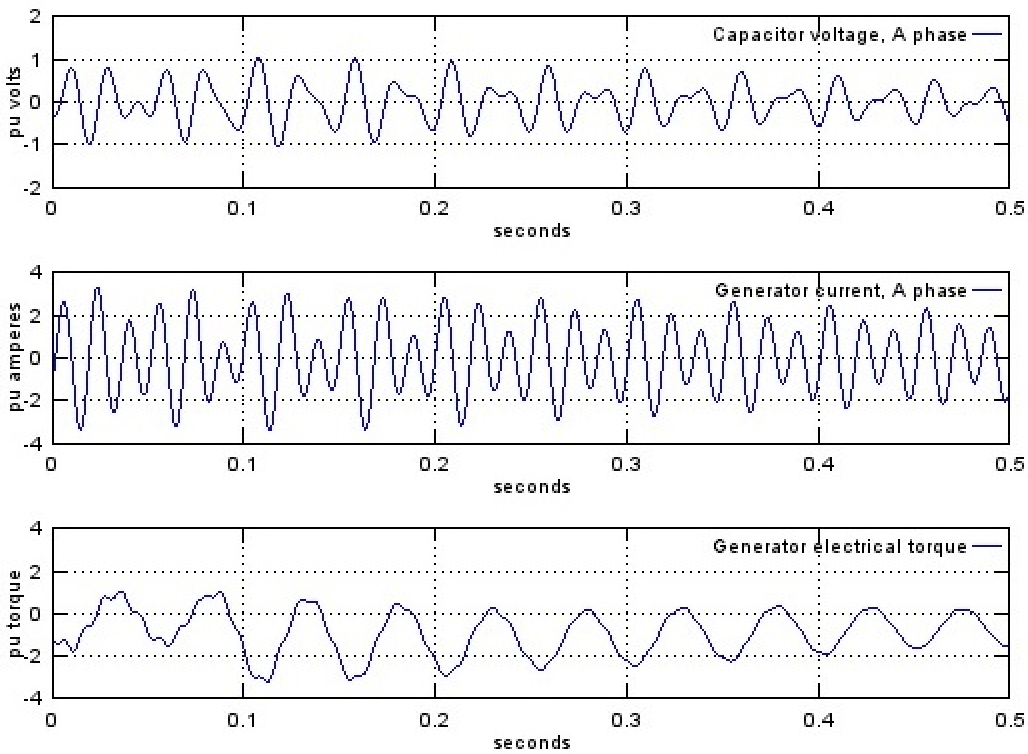
4 System Simulation

Parameter values for simulation taken from [1] are presented
Generator power output P = 0.9 pu
Generator power factor PF = 0.9 pu (lagging)
Fault reactance XF = 0.04 pu
Series capacitor reactance X_C = .350 pu

4.1 Fault simulation

A three phase fault is simulated using the equations described in Section 2.4, and capacitor voltage, generator current and generator electrical torque are plotted.

Figure 4.1: response curves for transient case



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