MULTICELL CO-OPERATIVE BEAMFORMING

A THESIS

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THESIS CERTIFICATE

This is to certify that the thesis titled **MULTICELL BEAMFORMING**, submitted by **K Meghana Reddy (EE09B105)**, to the Indian Institute of Technology Madras, Chennai for the award of the degree of **Bachelor and Master of Technology**, is a bona fide record of the research work done by her under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

KEYWORDS: Interference, Pareto Optimal Boundary, Multi-antenna Base Station and Mobile Station

The problem of resource allocation arises in every scenario where the resources are limited. It becomes important to allocate the resources optimally in order to maximize the aggregate output. This is a major problem currently faced by communication networks where frequency band is limited. Conventional mobile networks are designed with a cellular architecture where the entire frequency band is distributed among the cells such that only the non-adjacent cells use the same frequency band. The resulting interference is treated as noise and is minimized by applying the predesigned frequency reuse pattern.

The growing demand for high data rates has put these conventional methods to throughput limits. Hence, there is a dire need to make available the entire frequency band in all the cells. However, this factor one frequency reuse pattern deteriorates the overall network performance by the inter-cell interference; As a result, more sophisticated interference management techniques with multi-cell cooperation become crucial. Over the years many techniques were introduced which have fully cooperative downlink channels where large amounts of data is exchanged between the base stations. But this method turned ineffective due to the large amounts of power required for the exchange of information between the base stations.

The problem dealt with in this work is to come up with an effective method to remove the effect of the inter-cell interference via joint signal processing across various base stations. Unlike the fully cooperative downlink transmission the problem setup we have considered is a decentralized implementation of the multi-cell cooperative downlink beamforming.

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ABBREVIATIONS

BS	Base Station
MS	Mobile Station
IT	Interference Temperature
MISO	Multiple Input Single Output
IC	Interference Channel
LTE	Long Term Evolution
ZF	Zero Forcing
KKT	Karash Kuhn Tucker conditions
DPC	Dirty Paper Coding

NOTATIONS

h_{ij} Channel coefficients from BS_i to MS_j

$h_{i,jk}$ Channel coefficients from BS_i to MS_{jk}

S_k Transmit Covariance Matrix of the BS_k

Γ_{jk} Interference power at MS_k due to BS_j

$Tr(S_k)$ Trace of the matrix S_k

σ_i^2 Noise variance

$(B)^{-1}$ Inverse of the matrix B

$||B||$ Norm of the matrix B

P_k Power constraint at BS_k

CHAPTER 1

INTRODUCTION

Conventional wireless mobile networks are designed with a cellular architecture, where base stations (BSs) from different cells control communications for their associated mobile stations (MSs) independently. The resulting inter-cell interference is treated as additive noise and is minimized by predesigned frequency reuse pattern such that the same frequency is reused only by the non-adjacent cells.

1.1 Conventional Cellular Network

In conventional cellular networks the entire region is divided into cells. A group of cells form a cluster. The entire frequency band is divided equally among all the cells in this cluster. The frequency band allocation is done such that the distance between the cells using the same frequency band is as large as possible.

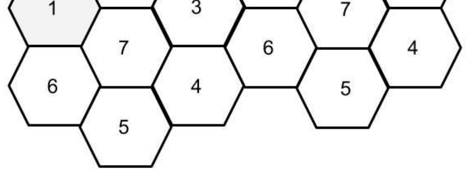

Figure 1.1: Graphical Representation of cellular network with 7-cell clusters

1.2 Cooperative Beamforming Techniques

Due to the rapidly growing demand for high-rate wireless multimedia applications, these conventional cellular networks have been pushed to their throughput limits. Consequently, the constraint on the frequency reuse has been relaxed such that the entire frequency is available in all the cells.

In this work, we study a particular type of multi-BS cooperation for the downlink transmission, given by Zhang and Cui (1), and find a solution to the 2-cell scenario. This thesis mainly focuses on the decentralized implementation of the multi-cell cooperative downlink beamforming assuming only the neighboring channel information at each BS.

1.3 A Summary

We start with the algorithm proposed by Zhang and Cui (1) to characterize the Pareto optimal boundary for a general k-user case with 1MS in each. Based on the problem setup given, we derive a closed form solution to finding the covariance matrix and hence, the global Pareto optimal rates for a 2-cell network. This is followed by the rate region and the Pareto optimal boundary results of the network simulation obtained by applying the closed form solution as well as the decentralized algorithm provided by Zhang and Cui (1).

A comparison of the results obtained by both the methods is also given in order to check the correctness of the implementation. The results obtained by using the Decentralized algorithm slightly differ from that of the closed form solution. Work on improving these results in still in progress. It is important to be able to characterize the Pareto optimal boundary using the algorithm since it helps us extend it easily from a 2-user network to a general k-user network.

The final part of this thesis is characterizing the Pareto optimal boundary for a 2-MS per cell network. It concludes with an algorithm in line with that provided in Zhang and Cui (1) for a 1MS per cell network.

CHAPTER 2

Multicell Beamforming

In this chapter, we deal with the system model given by Zhang and Cui (1) and derive a closed form solution for the 2-cell network, where each cell has a multi antenna base station and a single mobile station with a single antenna.

2.1 System Model

We consider a system consisting of K cells each served by its own base station (BS), one mobile station (MS), each equipped with one receiver antenna. The multi antenna base stations of the corresponding cell serve these mobile stations. All the K mobile stations use the same frequency band. The i^{th} base station is equipped with M_i antennas. Every mobile station, besides the data sent from its own base station also receives the data from the other neighboring base stations. Therefore, the discrete time signal received by each user is given by:

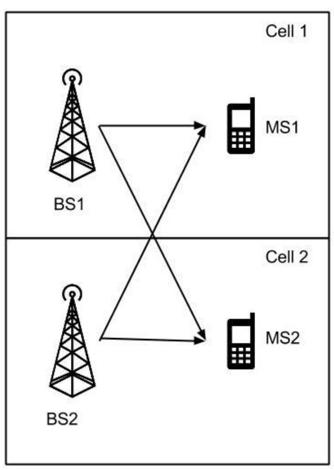

Figure 2.1: Wireless Cellular Network with 2 cells and 1MS each

$$y_k = h_{kk}^H x_k + \sum_{j=1, j\neq k}^{K} h_{jk}^H x_j + z_k \qquad (1.1)$$

where $x_k, h_{kk} \in C^{M_k \times 1}$ and h_{jk}, $x_j \in C^{M_j \times 1}$. We assume that $z_k \sim CN\,(0, \sigma_k^2)$ and that the signals transmitted and the noise at the receivers are independent of each other.

The achievable rate by user k is given by

$$R_k(S_1, S_2, \dots . S_k) = \log(1 + \frac{h_{kk}^H S_k h_{kk}}{\sum_{j\neq k} h_{jk}^H S_j h_{jk} + \sigma_k^2})(1.2)$$

It has been shown in Zhang and Cui (1) that the covariance matrices are rank one and therefore beam forming is optimal. The achievable rate region is given by:

$$\mathcal{R} = \cup_{||w_k||^2 \leq P_k} \{(r_1, r_2, \dots r_k): 0 \leq r_k \leq R_k(w_1, w_2 \dots w_3)\} \qquad (1.3)$$

where w_i is the i^{th} beam forming vector used by the base station i and P_i is the power constraint on the base station i.

The optimal rates achievable on this rate region constitute the Pareto optimal boundary. The rate tuples lying on the Pareto optimal boundary are as defined below:

Definition:

The rate tuple $\{r_1, r_2, \dots r_k\}$is Pareto-optimal if there is no other is no other tuple $r'_1, r'_2, \dots r'_k$ such that $r'_k \geq r_k$ *and* $\exists i: r'_i \geq r_i$

2.2 Interference temperature based optimization

In Zhang and Cui (1), it is proposed that we have a convex optimization problem at each BS and these problems are to be solved at each BS, each time paired with another BS. Solving these problems in all the given K cells results in an achievable rate tuple. The boundary of these achievable rate tuples has been shown to be the Pareto optimal boundary of the rate region. The convex optimization problem for the k^{th} base station is given by

$$max_{S_k} \log\left(1 + \frac{h_{kk}^H S_k h_{kk}}{\sum_{j\neq k} \Gamma_{jk} + \sigma_k^2}\right) \qquad (1.4)$$

$$s.t.\ h_{kj}^H S_k h_{kj} \leq \Gamma_{kj}$$

$$Tr(S_k) \leq P_k, S_k \succcurlyeq 0$$

where Γ_{kj} is the constraint on interference power due to the k-th base station at the j-th user, h_{kj} and h_{kk} define the channel parameters, S_k is the transmit covariance matrix.

The optimal rate is obtained by maximizing the above function for the given constraints.

Given the parameters Γ_k, the problem in the k-th base station can be solved as follows. Let the optimal rate be denoted by $C_k(\Gamma_k)$. The following results have been shown in Zhang and Cui(1):

- For every point the Pareto optimal boundary of rate region of the given network, we will have a set of interference temperature parameters such that the optimal covariance matrix solutions (of all the cells), for this particularΓ satisfy the interference temperature constraints exactly. The rate$C_k(\Gamma_k)$thus obtained is called the Pareto optimal rate of the k^{th} cell for the IT Γ_k.
- We define D_{ij} as follows

$$D_{ij} = \begin{bmatrix} \frac{\partial C_i(\Gamma_i)}{\partial \Gamma_{ij}} & \frac{\partial C_i(\Gamma_i)}{\partial \Gamma_{ji}} \\ \frac{\partial C_j(\Gamma_j)}{\partial \Gamma_{ij}} & \frac{\partial C_j(\Gamma_j)}{\partial \Gamma_{ji}} \end{bmatrix}$$

where $\frac{\partial C_i(\Gamma_i)}{\partial \Gamma_{ij}} = \lambda_{ij} \geq 0$ and

The Lagrangian function for this problem can be written as

$$L(S_k, \lambda_k) = \log\left(1 + \frac{h_{kk}^H S_k h_{kk}}{\sum_{j \neq k} \Gamma_{jk} + \sigma_k^2}\right) - \sum_{j \neq k} \lambda_{kj}\left(h_{kj}^H S_k h_{kj} - \Gamma_{kj}\right) - \lambda_{kk}(Tr(S_k) - P_k) \quad (1.5)$$

where $\lambda_k = [\lambda_{k1}, \lambda_{k2} \ldots \lambda_{kK}] \geq 0$ (component wise inequality). The dual problem is given by:

$$g(\lambda_k) = max_{S_k} L(S_k, \lambda_k) \quad (1.6)$$

$$\frac{\partial C_i(\Gamma_i)}{\partial \Gamma_{ij}} = \frac{-h_{ii}^H S_i^* h_{ii}}{ln2(\sum_{l\neq i} \Gamma_{li} + \sigma_i^2 + h_{ii}^H S_i^* h_{ii})} \leq 0$$

- The necessary condition for the Pareto-optimality on the parameters Γ is $|D_{ij}|$=0

- If the determinant is non-zero, we update the IT constraints such that we drive the value closer to Pareto optimality. The updating rule for Γ is given by

$$\begin{bmatrix} \Gamma'_{ij} \\ \Gamma'_{ji} \end{bmatrix} = \begin{bmatrix} \Gamma_{ij} \\ \Gamma_{ji} \end{bmatrix} + \delta_{ij} . d_{ij}$$

d_{ij} is a 2 by 1 vector such that $D_{ij} d_{ij} > 0$.

2.3 Closed form solution for the global Pareto optimality in a 2-cell case

The 2-cell network problem can be solved analytically and we can write the closed form solution to the rate tuple in terms of the interference temperatures$\{\Gamma_{12}, \Gamma_{21}\}$. It is given in Zhang and Cui (1) that the covariance matrix S is rank one. Hence, the 2-cell network problem can be written as:

$$max_{w_1} \log(1 + \frac{h_{ii}^H w_i w_i^H h_{ii}}{\Gamma_{ji} + \sigma_i^2}) \qquad (1.7)$$

$$s.t. h_{ij}^H w_i w_i^H h_{ij} \leq \Gamma_{ij}$$

$$||w_i||^2 \leq P_i, i = 1,2, j \neq i$$

where w_i is the beamforming vector which needs to be derived to arrive at the optimal rate.

It has been shown in E.Jorsweick and D.Danev (2) that the beamforming vector that achieves the Pareto optimal rate lies in the span of the channel vectors from its corresponding base station i.e.,

$$w_i = \sum_{j=1}^{K} a_{ij} h_{ij} \qquad (1.8)$$

Here a_{ij} is complex. Let us consider the optimal beamforming solution for w_1 for the BS1 subject to the interferences Γ_{12} and Γ_{21}. Upon Gram-Schmidt orthogonalization, the channel vectors h_{11} and h_{12} can be represented as

$$h_{11} = a_{11}u_1$$

$$h_{12} = a_{12}^1 u_1 + a_{12}^2 u_2$$

$$w_1 = b_1 u_1 + b_2 u_2$$

The closed form solution for the optimal beamforming vector can be obtained as shown by Karthikeyan (3). The problem can be written as:

$$max_{b_1,b_2} |b_1|^2 a_{11}^2 \qquad (1.9)$$

$$s.t. |b_1^H a_{12}^1 + b_2^H a_{12}^2|^2 \leq \Gamma_{12}$$

$$|b_1|^2 + |b_2|^2 \leq P_1$$

Let $|b_1| = r_1$, $|b_2| = r_2$, $arg\ (b_1^H a_{12}^1) = \theta_1$ and arg $(b_2^H a_{12}^2) = \theta_2$. Hence, the objective here is to maximize the value of r_1. The above two constraints of the problem can be written as

$$r_1^2|a_{12}^1|^2 + r_2^2|a_{12}^2|^2 + 2r_1r_2|a_{12}^1||a_{12}^2|\cos(\theta_1 - \theta_1) \leq \Gamma_{12} \qquad (1.10)$$

$$r_1^2 + r_2^2 \leq P_1 \qquad (1.11)$$

Solving the above equations we have,

$$0 \leq |\theta_1 - \theta_2| \leq \pi$$

$$-1 \leq \cos(\theta_1 - \theta_2) \leq 1$$

Solving the equations at the boundary points of the values of $\cos(\theta_1 - \theta_2)$, we get

$\cos(\theta_1 - \theta_2) = 1$	$\cos(\theta_1 - \theta_2) = -1$
$r_1^2\|a_{12}^1\|^2 + r_2^2\|a_{12}^2\|^2 + 2r_1r_2\|a_{12}^1\|\|a_{12}^2\| \leq \Gamma_{12}$	$r_1^2\|a_{12}^1\|^2 + r_2^2\|a_{12}^2\|^2 - 2r_1r_2\|a_{12}^1\|\|a_{12}^2\| \leq \Gamma_{12}$

$\|r_1\|a_{12}^1\|+r_2\|a_{12}^2\|\|^2 \le \Gamma_{12}$	$\|r_1\|a_{12}^1\|+r_2\|a_{12}^2\|\|^2 \le \Gamma_{12}$
$-\sqrt{\Gamma_{12}} \le r_1\|a_{12}^1\|+r_2\|a_{12}^2\| \le \sqrt{\Gamma_{12}}$	$-\sqrt{\Gamma_{12}} \le r_1\|a_{12}^1\|-r_2\|a_{12}^2\| \le \sqrt{\Gamma_{12}}$
$\frac{-\sqrt{\Gamma_{12}} - r_2\|a_{12}^2\|}{\|a_{12}^1\|} \le r_1 \le \frac{\sqrt{\Gamma_{12}} - r_2\|a_{12}^2\|}{\|a_{12}^1\|}$	$\frac{-\sqrt{\Gamma_{12}} + r_2\|a_{12}^2\|}{\|a_{12}^1\|} \le r_1 \le \frac{\sqrt{\Gamma_{12}} + r_2\|a_{12}^2\|}{\|a_{12}^1\|}$

Table2.1 Range of r_1 obtained for various values $\cos(\theta_1 - \theta_2)$

We obtain the following equations in terms of r_1 and r_2

(i) $r_1 = \frac{\sqrt{\Gamma_{12}}-r_2|a_{12}^2|}{|a_{12}^1|}$

(ii) $r_1 = \frac{-\sqrt{\Gamma_{12}}-r_2|a_{12}^2|}{|a_{12}^1|}$

(iii) $r_1 = \frac{\sqrt{\Gamma_{12}}+r_2|a_{12}^2|}{|a_{12}^1|}$

(iv)$r_1 = \frac{-\sqrt{\Gamma_{12}}+r_2|a_{12}^2|}{|a_{12}^1|}$

Plotting the above equations (for the condition where $\sqrt{\Gamma_{12}} \le P_1|a_{12}^1|$) and the eqn.1.11 we get,

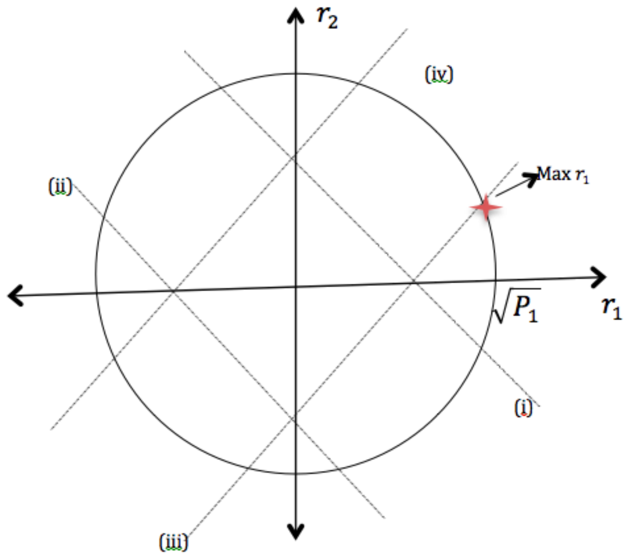

Figure2.2: $r_1 - r_2$ region under interference and power constraints

From the above figure, we see that the maximum value of r_1 is achieved at two points on the outer circle. But only the point marked above achieves the maximum value besides keeping the value of r_2 positive (since it is the magnitude of b_2).

Hence, solving the equations

(i) $r_1 = \frac{\sqrt{\Gamma_{12}}+r_2|a_{12}^2|}{|a_{12}^1|}$

(ii) $r_1^2 + r_2^2 \leq P_1$

gives us the solution to the Pareto optimal beamforming vector for the given Γ_{12} and Γ_{21}.

The solution to the above equations has been given in Karthikeyan(2009)(3) as follows:

$$r_1 = \frac{|a_{12}^1|}{|a_{12}^1|^2 + |a_{12}^2|^2}\sqrt{\Gamma_{12}} + \frac{|a_{12}^2|}{|a_{12}^1|^2 + |a_{12}^2|^2}\sqrt{P_1|a_{12}^1|^2 + P_2|a_{12}^2|^2 - \Gamma_{12}}\sqrt{\Gamma_{12}} \leq P_1|a_{12}^1| \qquad (1.13)$$

When $\sqrt{\Gamma_{12}} \geq P_1|a_{12}^1|$, the maximum value of r_1 that could be achieved lies on the boundary of the circle since the lines intersect outside the circle, which represents the power constraints set for the transmission of data. Hence, we assign P_1 as the maximum rate achievable.

We plot the equation in table 2.1 along with the constraint $\sqrt{\Gamma_{12}} \geq P_1|a_{12}^1|$ to illustrate the above statement.

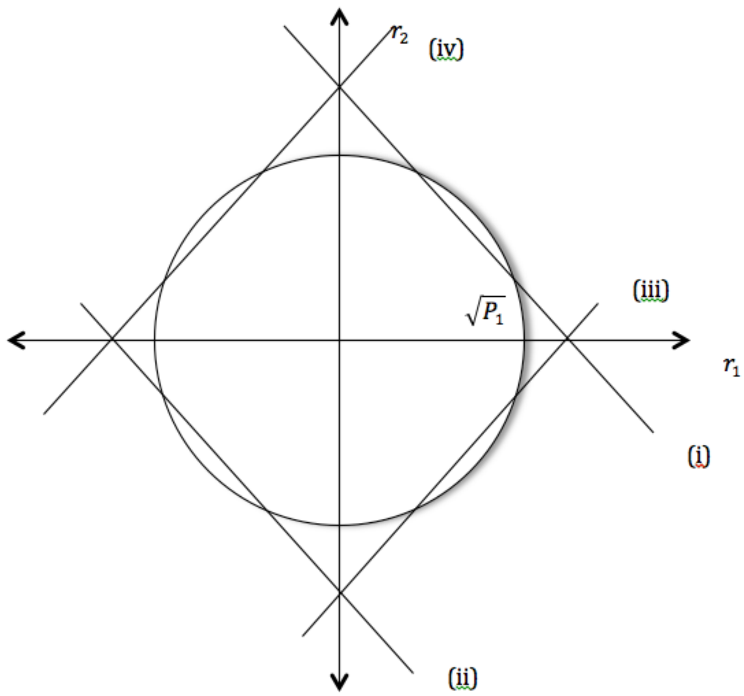

Figure2.3:$r_1 - r_2$ region under interference and power constraints

$r_1 = \sqrt{P_1}\sqrt{\Gamma_{12}} \geq P_1|a_{12}^1|$ (1.14)

Thus, we have the closed for solution to obtain the beamforming vectors for which the rate lies on the Pareto optimal boundary of the rate region. Next, we look at the simulation of a network for these results

CHAPTER 3

Pareto Optimal Boundary

In this chapter, we characterize the Pareto optimal boundary for the 2-cell network by implementing the closed form solution derived in chapter-2 and also the general solution as given in Zhang and Cui (1) via MISO IC interference temperature control.

3.1 Pareto Optimal Boundary for the 2-cell network by implementing the closed form solution

3.1.1 Rate Region

Rate region is the union of all the rates achievable by all the users present in a network given the interference and the power constraints for the given network.

The channel vectors of the MISO Gaussian IC, after Gram-Schmidt orthogonalization, can be represented as shown in the section 2.3. For any given IT constraint the rate is given by the expression

$$C_1(\Gamma) = \log(1 + \frac{r_1^2 a_{11}^2}{\Gamma_{21} + n_1}) \qquad (2.1)$$

where r_1 is derived in section 2.3.

The expression for $C_2(\Gamma)$ can be obtained similarly. To check the correctness of the above expressions we plot the rate region for the 2-user case using these expressions and the parameterization program as given by Lindblom et al (4).

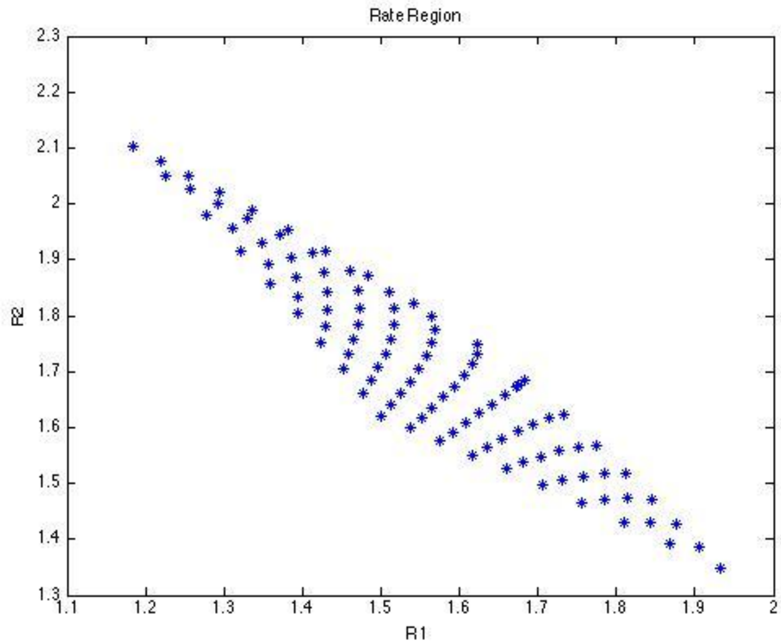

Figure3.1: Rate region obtained by simulating a 2-cell network using the closed form α

$P_1 = 5$	$P_1 = 5$
$h11 = 0.5[1+i, 1-i]^T$	$h21 = [0.35-\sqrt{2}i, 0.35+\sqrt{2}i]^T$
$h12 = [0.35+\sqrt{2}i, 0.35-\sqrt{2}i]^T$	$h22 = 0.5[1-i, 1+i]^T$
$\Gamma_{12} \in (0.1,1)$ in steps of 1	$\Gamma_{21} \in (0.1,1)$ in steps of 1
$\alpha_{12} = 1$	
$\delta_{12} = 0.1$	

Table 3.1: Parameters used in the simulation of rate region

The channel vectors are considered such that the rate regionmatches the rate region shown by Lindblom et al (4).

3.1.2 Pareto Optimal Boundary

From the above figure we see that the rates lie either in the rate region or on the boundary depending on the value of the interference temperature. We use the boundary conditions as shown by Zhang and Cui (1) to arrive at the Pareto optimal boundary. In order to check the optimal conditions we need to obtain the derivative of the rate expressions with respect to the interference temperatures.

$$\frac{\partial C_1}{\partial \Gamma_{12}} = \frac{2r_1|a_{11}|^2}{(|a_{11}|^2 r_1^2 + \Gamma_{21} + \sigma_1^2)} \times \frac{\partial r_1}{\partial \Gamma_{12}}$$

$$\frac{\partial r_1}{\partial \Gamma_{12}} = \frac{1}{(|a_{12}^1|^2 + |a_{12}^2|^2)}\left(\frac{1}{2\sqrt{\Gamma_{12}}} - \frac{|a_{12}^2|}{2|a_{12}^1|\sqrt{P_1|a_{12}^1|^2 + P_2|a_{12}^2|^2 - \Gamma_{12}}}\right)$$

$$\frac{\partial C_1}{\partial \Gamma_{21}} = \frac{-a_{11}^2 r_1^2 (\Gamma_{21} + \sigma_1^2)^3}{|a_{11}|^2 r_1^2 + \Gamma_{21} + \sigma_1^2}$$

The expressions for $\frac{\partial C_2}{\partial \Gamma_{12}}$ and $\frac{\partial C_2}{\partial \Gamma_{21}}$ can be obtained by deriving the corresponding rate expressions.

For each IT value (as considered in section 2.4.1) we derive the rates of both the users and also the value of the matrix D. If $|D| = 0$, then we say the corresponding rates are on the Pareto optimal boundary. In any other case we update the IT values.

Updating rule:

$$\begin{bmatrix} \Gamma'_{12} \\ \Gamma'_{21} \end{bmatrix} = \begin{bmatrix} \Gamma_{12} \\ \Gamma_{21} \end{bmatrix} + \delta_{12}.d_{12} \qquad (2.2)$$

where $d_{12} = sign(ad - bc).(\alpha_{12}d - b, a - \alpha_{12}c)^T$ and $D = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Using the channel vectors and the initial IT values as assumed in the above section we obtain the Pareto optimal boundary of the rate region.

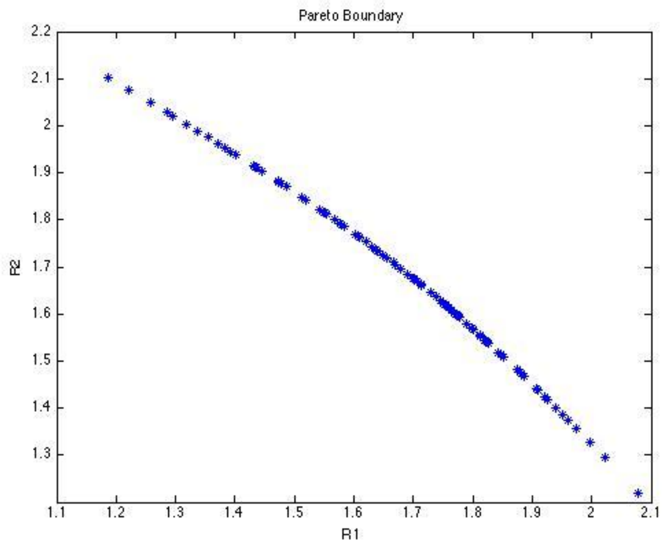

Figure3.2: Pareto Optimal Boundary obtained using the closed form solution

The parameters used here are the same as ones given in the table3.1.

Since, in real situations, it is not possible for the stopping condition i.e., the determinant to exactly approach zero we provide a threshold 0.01.

3.1.3 Zero Forcing Rate

In this section we obtain the rates of the two users by zero forcing the interference through the appropriate selection of the beamforming vectors.

$$y_1 = h_{11}^H x_1 + h_{21}^H x_2 + n_1 \qquad (2.3)$$

$$y_2 = h_{12}^H x_1 + h_{22}^H x_2 + n_2$$

$$x_1 = w_1 d_1, \qquad x_2 = w_2 d_2$$

In order to achieve zero forcing we need, $h_{21}^H w_1 = 0, h_{12}^H w_2 = 0, ||w_1||^2 = 1, ||w_2||^2 = 1$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \sqrt{P'_1} \begin{bmatrix} h_{11}^H \\ h_{12}^H \end{bmatrix} w_1 d_1 + \sqrt{P'_2} \begin{bmatrix} h_{21}^H \\ h_{22}^H \end{bmatrix} w_2 d_2 \qquad (2.4)$$

We require,

$$\begin{bmatrix} h_{11}^H \\ h_{12}^H \end{bmatrix} w_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} h_{21}^H \\ h_{22}^H \end{bmatrix} w_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Hence,

$$w_1 = [\begin{matrix} h_{11}^H \\ h_{12}^H \end{matrix}]^{-1}(1,0) w_2 = [\begin{matrix} h_{21}^H \\ h_{22}^H \end{matrix}]^{-1}(1,0)$$

By using the above beamforming vectors, the rates can be derived using the rate expression where we equate the values of P'_1 and P'_2 to r_1 and r_2 derived above.

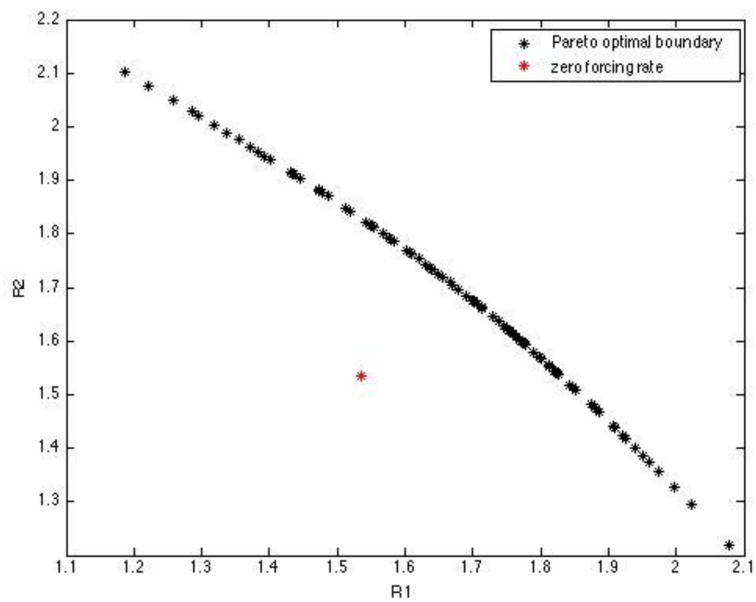

Figure3.3: Pareto Optimal Boundary in comparison with the Zero Forcing Rate

The parameters used here are the same as ones given in the table3.1.

From the above figure, we see that the zero forcing rate is lower than the Pareto optimal rate that could be achieved.

The following figure shows us that the zero forcing rate converges to the Pareto optimal boundary by iteratively updating the IT values.

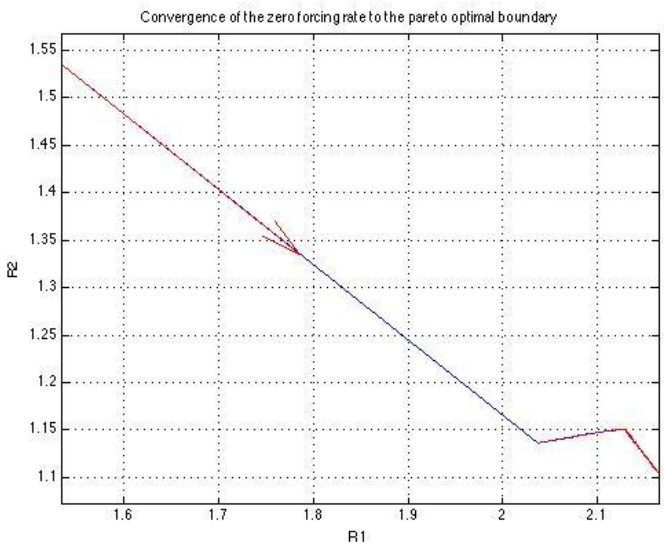

Figure 3.4: Illustrating the convergence of the zero forcing rate to the Pareto boundary

The parameters used here are the same as ones given in the table3.1.

3.1.4 Converging values of Interference Temperature

The convergence to the optimal boundary depends on the initialization values of IT in the network. The IT values cease to converge when initialized to very large values.

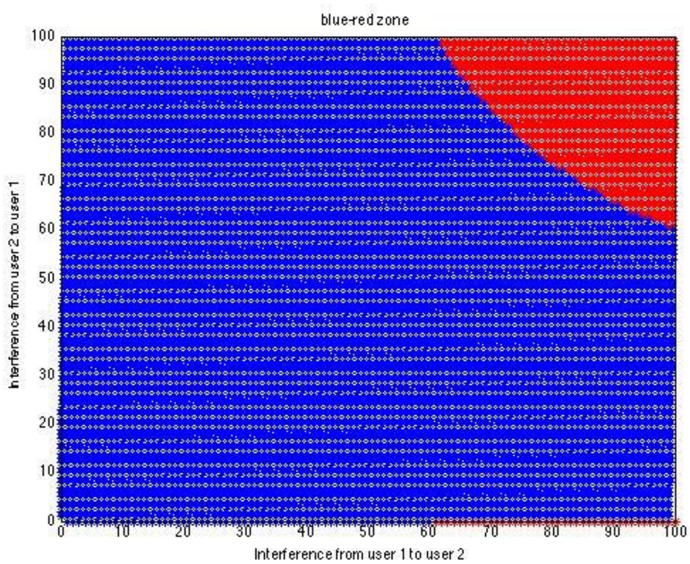

Figure 3.5 Illustrates the IT initialization values that do not converge

The parameters used here are the same as ones given in the table 3.1. except the range of the IT. Here we used

$\Gamma_{12} \in (0.1,10)$ in steps of 1	$\Gamma_{12} \in (0.1,10)$ in steps of 1

Table 3.2 Parameters used in obtaining the Figure 3.5

In the above figure, the IT values that are initialized to the red region do not converge to the Pareto Boundary. This fact may be illustrated in the following figure.

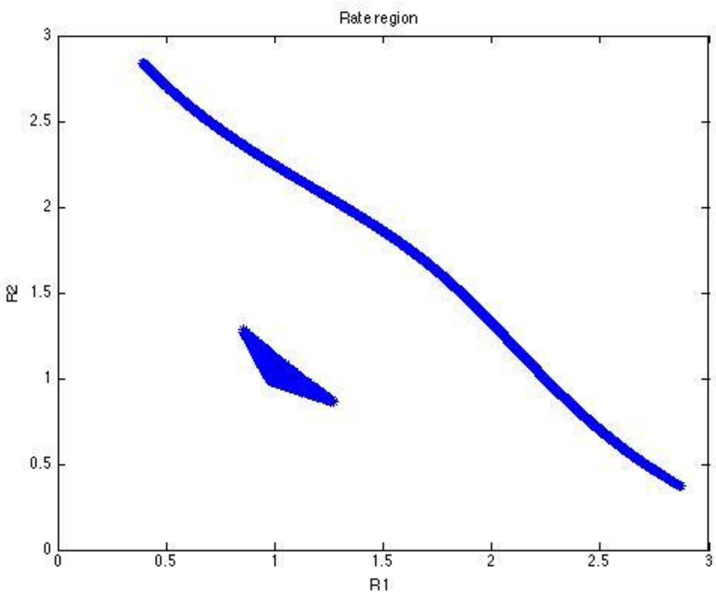

Figure 3.6 Illustrates the non-convergence of certain IT initializations

The parameters used here are the same as ones given in the Table 3.1. except the range of the IT. We use the IT values mention in Table 3.2.

3.2 Simulation of the Pareto Optimal Boundary using the Decentralised Algorithm for Multi-Cell Cooperative Beamforming

3.2.1 Solving the dual of the optimization problem

The convex optimisation problem (1.4) can be solved by the standard Langrange duality method given by S.Boyd (6). Let λ_{kj} , λ_{kk} be the non-negative dual variables for problem (1.4) associated with k^{th} BS's IT constraint for the j^{th} MS and its own transmit power constraint respectively. The Lagrangian function for this problem can be written as:

$$L(S_k, \lambda_k) = \log\left(1 + \frac{h_{kk}^H S_k h_{kk}}{\sum_{j \neq k} \Gamma_{jk} + \sigma_k^2}\right) - \sum_{j \neq k} \lambda_{kj} \left(h_{kj}^H S_k h_{kj} - \Gamma_{kj}\right) - \lambda_{kk} (Tr(S_k) - P_k) \tag{2.5}$$

The dual function of the problem is given by

$$g(\lambda_k) = max_{S_k \geq 0} L(S_k, \lambda_k) \tag{2.6}$$

The dual problem is defined as

$$min_{\lambda_k \geq 0}\, g(\lambda_k)$$

The duality gap between the optimal and the dual problem is zero and hence the problem(1.4) can be solved by the solving the dual.

This can be obtained by solving the maximisation problem which can be written as (by eliminiating the constant terms)

$$max_{S_k} \log\left(1 + \frac{h_{kk}^H S_k h_{kk}}{\sum_{j \neq k} \Gamma_{jk} + \sigma_k^2}\right) - Tr(B_k(\lambda_k) S_k) \tag{2.7}$$

$$S_k \geq 0$$

where $B_k(\lambda_k) = \sum_{j \neq k} \lambda_{kj} h_{kj}^H h_{kj} + \lambda_{kk} I$

In order to have a bounded objective value for problem (2.7) $B_k(\lambda_k)$ has to be full rank.

Proof: If $B_k(\lambda_k)$ is rank deficient, such that we could define $S_k = q_k v_k v_k^H$, satisfying $||v_k|| = 1$ and $B_k(\lambda_k)v_k = 0$. Thereby the objective function of the problem 2.7 reduces to

$$\log\left(1 + \frac{q_k ||h_{kk}^H v_k||^2}{\sum_{j\neq k}\Gamma_{jk} + \sigma_k^2}\right) \qquad (2.8)$$

Due to independence of h_{kk} and v_k, it follows that $||h_{kk}^H v_k|| > 0$ with probablity one such that problem 2.8 goes to infinity by letting $q_k \to \infty$. Hence, we have that $B_k(\lambda_k)$ should be of rank one.

By the definition of $B_k(\lambda_k)$ and Karash-Kuhn-Tucker(KKT) optimality conditions (8), it follows that $B_k(\lambda_k)$ is rank one only when one of the following two conditions is satisfied:

- $\lambda_{kk} > 0$
- $\lambda_{kk} = 0$

Since, $B_k(\lambda_k)$ is rank one, we have that $(B_k(\lambda_k))^{-1}$ exists.

Thus, we can introduce a new variable $S_k^{'}$ such that

$$S_k^{'} = (B_k(\lambda_k))^{\frac{1}{2}} S_k (B_k(\lambda_k))^{1/2}$$

Substituting it in 2.7 yields

$$Max_{S_k} \log\left(1 + \frac{h_{kk}^H (B_k(\lambda_k))^{-\frac{1}{2}} S_k (B_k(\lambda_k))^{-\frac{1}{2}} h_{kk}}{\sum_{j\neq k}\Gamma_{jk} + \sigma_k^2}\right) - Tr(S_k) \qquad (2.9)$$

$S_k^{'}$ can be eigen value decomposed as $S_k^{'} = U_k \varphi_k U_k^H$. Substituting in eqn. 2.8 we get,

$$Max_{S_k} \log\left(1 + \frac{\sum_i \cdot \theta_{ki} ||h_{kk}^H (B_k(\lambda_k))^{-\frac{1}{2}} u_{ki}||^2}{\sum_{j\neq k}\Gamma_{jk} + \sigma_k^2}\right) - \sum \theta_{ki} \qquad (2.10)$$

such that, $||u_{ki}|| = 1, u_{ki} u_{kj} = 0$

For any given U_k, the eqn. 2.9 can be maximised by solving

$$\frac{\partial \log\left(1+\frac{\sum_i \theta_{ki} ||h_{kk}^H (B_k(\lambda_k))^{-\frac{1}{2}} u_{ki}||^2}{\sum_{j\neq k} \Gamma_{jk} + \sigma_k^2}\right) - \sum \theta_{ki}}{\partial \theta_{ki}} = 0 \qquad (2.11)$$

$$\frac{1}{ln2} \frac{1}{1 + \frac{\sum_i \theta_{ki} ||h_{kk}^H (B_k(\lambda_k))^{-\frac{1}{2}} u_{ki}||^2}{\sum_{j\neq k} \Gamma_{jk} + \sigma_k^2}} \frac{||h_{kk}^H (B_k(\lambda_k))^{-\frac{1}{2}} u_{ki}||^2}{\sum_{j\neq k} \Gamma_{jk} + \sigma_k^2} - 1 = 0$$

Only one of the $\theta_k{}'s$ will be non-zero positive. Hence we have,

$$\frac{1}{ln2} \frac{||h_{kk}^H (B_k(\lambda_k))^{-\frac{1}{2}} u_{ki}||^2}{\sum_i . \theta_{ki} ||h_{kk}^H (B_k(\lambda_k))^{-\frac{1}{2}} u_{ki}||^2 + \sum_{j\neq k} \Gamma_{jk} + \sigma_k^2} - 1 = 0$$

$$\theta_{ki} = \left(\frac{1}{ln\,2} - \frac{\sum_{j\neq k} \Gamma_{jk} + \sigma_k^2}{||h_{kk}^H (B_k(\lambda_k))^{-\frac{1}{2}} u_{ki}||^2}\right)^+ \qquad (2.12)$$

For any given $\theta_k > 0$, the eqn. 2.9 is maximised by

$$u_k = \frac{(B_k(\lambda_k))^{-\frac{1}{2}} h_{kk}}{||(B_k(\lambda_k))^{-\frac{1}{2}} h_{kk}||} \qquad (2.13)$$

From the equations 2.12 and 2.13, we get

$$S_k^{'} = \frac{\left(\frac{1}{ln\,2} - \frac{\sum_{j\neq k} \Gamma_{jk} + \sigma_k^2}{||h_{kk}^H (B_k(\lambda_k))^{-\frac{1}{2}}||^2}\right)^+}{||(B_k(\lambda_k))^{-\frac{1}{2}} h_{kk}||^2} (B_k(\lambda_k))^{-\frac{1}{2}} h_{kk} h_{kk}^H (B_k(\lambda_k))^{-\frac{1}{2}}$$

3.2.2 A simulation

We employ this method to simulate the Pareto optimal boundary for a 2-user network. This method can be extended to a general k-user network.

Procedure:

- Step1: Initialize the range for the values of $\lambda_{11}, \lambda_{12}, \lambda_{22}\ and\ \lambda_{21}$
- Step2: Evaluate $S_k^{'}$ using these values of λ and thereby evaluate the values of S_k

- Step3: For these values of S_k, find the value of $g(\lambda_k)$
- Step4: Find the values of $\lambda_{11}, \lambda_{12}$ that minimise $g(\lambda_1)$ and the values of $\lambda_{21}, \lambda_{22}$ that minimise $g(\lambda_2)$
- Step5: Evaluate the determinant D using the values of λ that minimise $g(\lambda_k)$
- Step6: Update the values of Γ as shown in equation 2.2
- Step7: Repeat the procedure from Step1 until the determinant approaches zero.

3.2.3 Rate Region

Using the channel coefficients as used in the section 3.1.1 we obtain the following rate region using the decentralised algorithm.

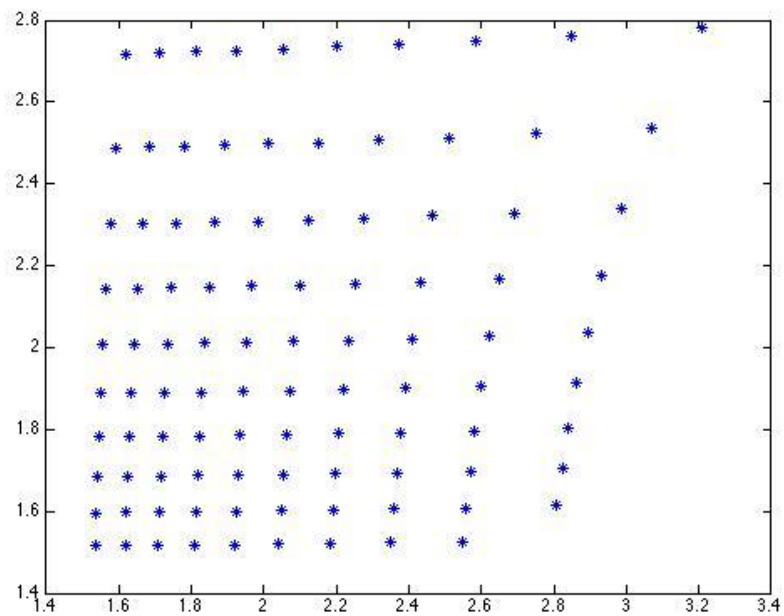

Figure 3.7: Rate region obtained by using the decentralized algorithm

The parameters used here are the same as ones given in the Table 3.1.

$\lambda_{11} \in (0.001, 0.1)$	$\lambda_{21} \in (0.001, 0.1)$
$\lambda_{12} \in (0.001, 0.1)$	$\lambda_{22} \in (0.001, 0.1)$

in steps of 0.001	in steps of 0.001

Table 3.3 Parameters used to obtain the Figure 3.7.

3.2.4 Pareto Optimal Boundary

We implement the decentralised algorithm to obtain the Pareto optimal boundary for the 2-cell network.

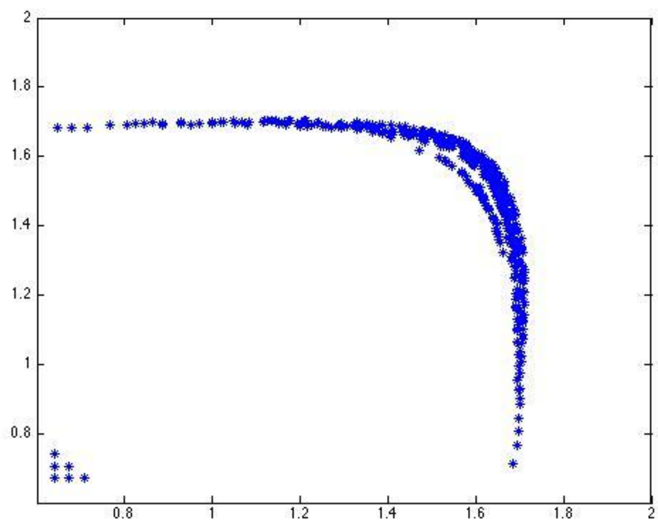

Figure 3.8: Pareto optimal boundary obtained by using the decentralised algorithm

The parameters used here are the same as ones given in the Table 3.1. and Table 3.3.

In the above figure, we notice that the value of the rates does not cross a certain threshold for the given parameters. This was a huge setback in the characterization of the Pareto optimal boundary using the decentralised method.

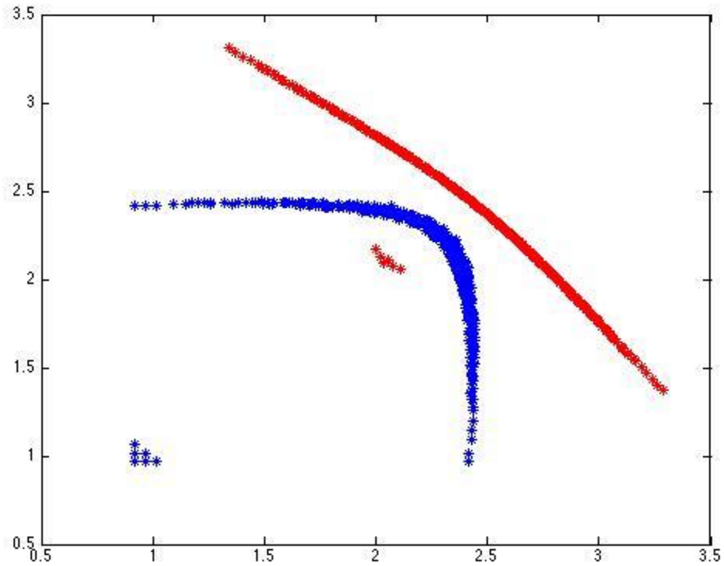

Figure3.9: Comparison between the Pareto boundaries obtain by both the methods

The parameters used here are the same as ones given in the Table 3.1. and Table 3.2.

In the above figure, we see that the 2 Pareto optimal boundaries differ. The implementation of the decentralized algorithm could not characterize the Pareto optimal boundary exactly due to the crudeness of the searching algorithm while finding the solution to the dual problem of the optimization problem (1.7).

CHAPTER 4

Characterization of Pareto optimal boundary for 2 MSs per cell case

In this chapter, we study the problem of characterizing the Pareto optimal boundary for the 2-cell network in which 2 MSs are present in each cell. The problem has been solved only partially. Hence, this chapter leaves some directions for future work.

4.1 System Model

We consider a 2-cell network in which 2 MSs are present in each cell. Thus, we form a Multiple Input Multiple Output (MIMO) Broadcast channel. In this set up each transmitter has to send independent messages to both the receivers. The Broadcast Channel is an additive noise channel and hence we write the discrete time signals received by each MS as:

$$y_{11} = h_{1,11}^{H} x_1 + h_{2,11}^{H} x_2 + \eta_{11} \qquad (4.1)$$

$$y_{12} = h_{1,12}^{H} x_1 + h_{2,12}^{H} x_2 + \eta_{12}$$

$$y_{21} = h_{1,21}^{H} x_1 + h_{2,21}^{H} x_2 + \eta_{21}$$

$$y_{22} = h_{1,22}^{H} x_1 + h_{2,22}^{H} x_2 + \eta_{22}$$

where x_k is the signal transmitted by BS_k intended for MSs in cell k and $h_{i,jk}$ denotes the channel gain between BS_i and MS_{jk} and S_{jk} denotes covariance matrix for MS_{jk}.

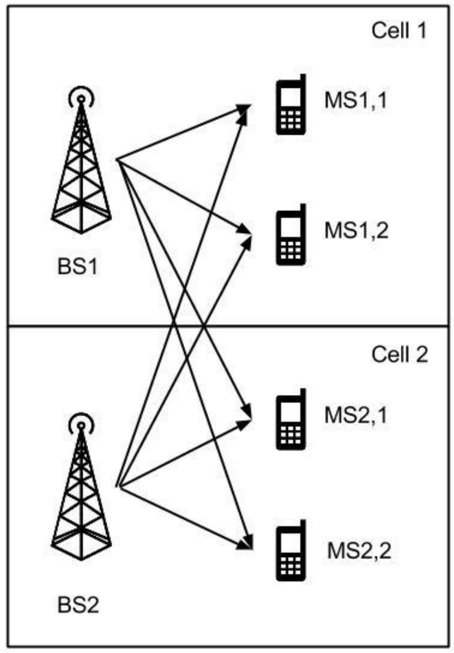

Figure4.1: Cellular Network with 2 cell with 2MSs each

Using the Dirty Paper Coding (DPC) in each cell, to characterize the achievable rates in a MIMO Broadcast channel as shown in Wiengarten et al (6). Let π and σ denote the encoding orders in cell 1 and cell 2 respectively (as mentioned in Vinay H (7)). For convenience we follow the notation:

$$x_1 = x_{1\pi_1} + x_{1\pi_2}$$

$$x_2 = x_{2\sigma_1} + x_{2\sigma_2}$$

$$S_1 = S_{1\pi_1} + S_{1\pi_2}$$

$$S_2 = S_{2\sigma_1} + S_{2\sigma_2}$$

By treating interference from other cell as noise, the achievable rates using DPC in each cell can be written as follows

$$R_{1\pi_1} = log\frac{h_{1,1\pi_1}^H(S_{1\pi_1}+S_{1\pi_2})h_{1,1\pi_1}+h_{2,1\pi_1}^H(S_{2\sigma_1}+S_{2\sigma_2})h_{2,1\pi_1}+N_{1\pi_1}}{h_{1,1\pi_1}^H(S_{1\pi_2})h_{1,1\pi_1}+h_{2,1\pi_1}^H(S_{2\sigma_1}+S_{2\sigma_2})h_{2,1\pi_1}+N_{1\pi_1}} \quad (4.2)$$

$$= log\frac{h_{1,1\pi_1}^H(S_1)h_{1,1\pi_1} + h_{2,1\pi_1}^H(S_2)h_{2,1\pi_1} + N_{1\pi_1}}{h_{1,1\pi_1}^H(S_{1\pi_2})h_{1,1\pi_1} + h_{2,1\pi_1}^H(S_{2\sigma_1} + S_{2\sigma_2})h_{2,1\pi_1} + N_{1\pi_1}}$$

$$R_{1\pi_2} = \log\frac{h_{1,1\pi_2}^H S_{1\pi_2} h_{1,1\pi_2} + h_{2,1\pi_2}^H S_2 h_{2,1\pi_2} + N_{1\pi_2}}{h_{2,1\pi_2}^H S_2 h_{2,1\pi_2} + N_{1\pi_2}}$$

$$R_{2\sigma_1} = log\frac{h_{2,2\sigma_1}^H(S_2)h_{2,2\sigma_1} + h_{1,2\sigma_1}^H(S_1)h_{1,2\sigma_1} + N_{2\sigma_1}}{h_{2,2\sigma_1}^H(S_{2\sigma_2})h_{2,2\sigma_1} + h_{1,2\sigma_1}^H(S_1)h_{1,2\sigma_1} + N_{2\sigma_1}}$$

$$R_{2\sigma_2} = log\frac{h_{2,2\sigma_2}^H(S_{2\sigma_2})h_{2,2\sigma_1} + h_{1,2\sigma_2}^H(S_1)h_{1,2\sigma_2} + N_{2\sigma_2}}{h_{1,2\sigma_2}^H(S_1)h_{1,2\sigma_2} + N_{2\sigma_2}}$$

As defined in section 3.1.1, the rate region for the MIMO broadcast channel can also be defined as:

$$\mathcal{R} = \bigcup\nolimits_{\pi,\sigma}^{.} \bigcup\nolimits_{\mathbb{S}\in S_{\pi\sigma}}^{.} R(\pi,\sigma,\mathbb{S}),$$

where

$$R(\pi,\sigma,\mathbb{S}) = \left\{ (r_{1\pi_1}, r_{1\pi_2}, r_{2\sigma_1}, r_{2\sigma_2}) \middle| \begin{matrix} 0 \le r_{1\pi_1} \le R_{1\pi_1}(\mathbb{S}), \\ 0 \le r_{1\pi_2} \le R_{1\pi_2}(\mathbb{S}), \\ 0 \le r_{2\sigma_1} \le R_{2\sigma_1}(\mathbb{S}), \\ 0 \le r_{2\sigma_2} \le R_{2\sigma_2}(\mathbb{S}) \end{matrix} \right\}$$

$$S_{\pi\sigma} = \{S_{1\pi_1}, S_{1\pi_2}, S_{2\sigma_1}, S_{2\sigma_2} \ge 0, Tr(S_1) \le P_1, Tr(S_2) \le P_2\}$$

where r is the rate tuples in the rate region of the

With the given interference temperatures constraints, we can write the problem at BS1 as

$$\max(C_{1\pi_1}, C_{1,\pi_2}) \quad (4.3)$$

$$s.t.\, h_{1,2\sigma_1}^H S_1 h_{1,2\sigma_1} \le \Gamma_{1,2\sigma_1}$$

$$h_{1,2\sigma_2}^{H} S_1 h_{1,2\sigma_2} \leq \Gamma_{1,2\sigma_2},$$

$$Tr(S_1) \leq P_1$$

where

$$C_{1\pi_1} = log \frac{h_{1,1\pi_1}^{H}(S_1)h_{1,1\pi_1} + \Gamma_{2,1\pi_1} + N_{1\pi_1}}{h_{1,1\pi_1}^{H}\left(S_{1\pi_2}\right)h_{1,1\pi_1} + \Gamma_{2,1\pi_1} + N_{1\pi_1}}$$

$$C_{1\pi_2} = \log \frac{h_{1,1\pi_2}^{H} S_{1\pi_2} h_{1,1\pi_2} + \Gamma_{2,1\pi_1} + N_{1\pi_2}}{\Gamma_{2,1\pi_1} + N_{1\pi_2}}$$

The optimization problem for the MIMO broadcast channel is a multi-objective optimization problem. Solving this gives the global Pareto optimal solutions. Similarly, the problems to the BS2 could be obtained. Now, we try to obtain a solution to these problems in line with that provided in Zhang and Cui (1).

4.2 Characterization of the Pareto Boundary

Lemma 4.1: For an arbitrarily chosen $\Gamma \geq 0$, if the optimal rate values of the IT-problems, C, are on the Pareto boundary of 2MS rate region, then there exists no vector $c > 0$ in the column space of $\boldsymbol{D}$, where

$$D = \begin{pmatrix} \frac{\partial C_{1\pi_1}}{\partial \Gamma_{1,2\sigma_1}} & \frac{\partial C_{1\pi_1}}{\partial \Gamma_{1,2\sigma_2}} & \frac{\partial C_{1\pi_1}}{\partial \Gamma_{2,1\pi_1}} & \frac{\partial C_{1\pi_1}}{\partial \Gamma_{2,1\pi_2}} \\ \frac{\partial C_{1\pi_2}}{\partial \Gamma_{1,2\sigma_1}} & \frac{\partial C_{1\pi_2}}{\partial \Gamma_{1,2\sigma_2}} & \frac{\partial C_{1\pi_2}}{\partial \Gamma_{2,1\pi_1}} & \frac{\partial C_{1\pi_2}}{\partial \Gamma_{2,1\pi_2}} \\ \frac{\partial C_{2\sigma_1}}{\partial \Gamma_{1,2\sigma_1}} & \frac{\partial C_{2\sigma_1}}{\partial \Gamma_{1,2\sigma_2}} & \frac{\partial C_{2\sigma_1}}{\partial \Gamma_{2,1\pi_1}} & \frac{\partial C_{2\sigma_1}}{\partial \Gamma_{2,1\pi_2}} \\ \frac{\partial C_{2\sigma_2}}{\partial \Gamma_{1,2\sigma_1}} & \frac{\partial C_{2\sigma_2}}{\partial \Gamma_{1,2\sigma_2}} & \frac{\partial C_{2\sigma_2}}{\partial \Gamma_{2,1\pi_1}} & \frac{\partial C_{2\sigma_2}}{\partial \Gamma_{2,1\pi_2}} \end{pmatrix} \qquad (4.4)$$

Proof: Proof is given by Vinay H (2013).

Let $\Gamma \geq 0$ be such that $C(\Gamma)$ is Pareto optimal. Then there should not be any improvement on Γ such that $C(\Gamma)$ can be improved

The stopping criterion for Pareto optimality is $|D| = 0$. When it is non zero we have the following updating rule which leads the rate towards the global Pareto optimality.

For some $\delta > 0$, consider $\Gamma^{'}$ as follows

$$\begin{bmatrix} \Gamma^{'}_{1,2\sigma_1} \\ \Gamma^{'}_{1,2\sigma_2} \\ \Gamma^{'}_{2,1\pi_1} \\ \Gamma^{'}_{2,1\pi_2} \end{bmatrix} = \begin{bmatrix} \Gamma_{1,2\sigma_1} \\ \Gamma_{1,2\sigma_2} \\ \Gamma_{2,1\pi_1} \\ \Gamma_{2,1\pi_2} \end{bmatrix} + \delta d \qquad (4.5)$$

Let (S_1^*, S_2^*) be the new optimal solutions found using the IT-problems. Consider

$$r_{1\pi_1} = \ log \frac{h^H_{1,1\pi_1}(S_1^*)h_{1,1\pi_1} + h^H_{2,1\pi_1}(S_2^*)h_{2,1\pi_1} + N_{1\pi_1}}{h^H_{1,1\pi_1}\left(S^*_{1\pi_2}\right)h_{1,1\pi_1} + h^H_{2,1\pi_1}(S_2^*)h_{2,1\pi_1} + N_{1\pi_1}}$$

$$\geq log \frac{h^H_{1,1\pi_1}(S_1^*)h_{1,1\pi_1} + \Gamma^{'}_{2,1\pi_1} + N_{1\pi_1}}{h^H_{1,1\pi_1}\left(S^*_{1\pi_2}\right)h_{1,1\pi_1} + \Gamma^{'}_{2,1\pi_1} + N_{1\pi_1}} = C_{1\pi_1}(\Gamma^{'})$$

$$r_{1\pi_2} = log \frac{h^H_{1,1\pi_1}\left(S^*_{1\pi_2}\right)h_{1,1\pi_1} + h^H_{2,1\pi_1}(S_2^*)h_{2,1\pi_1} + N_{1\pi_1}}{h^H_{2,1\pi_1}(S_2^*)h_{2,1\pi_1} + N_{1\pi_1}}$$

$$\geq log \frac{h^H_{1,1\pi_1}\left(S^*_{1\pi_2}\right)h_{1,1\pi_1} + \Gamma^{'}_{2,1\pi_1} + N_{1\pi_1}}{\Gamma^{'}_{2,1\pi_1} + N_{1\pi_1}} = C_{1\pi_2}(\Gamma^{'})$$

Similarly, we can write $r_{1\sigma_1} \geq C_{2\sigma_1}(\Gamma^{'})$,$r_{1\sigma_2} \geq C_{2\sigma_2}(\Gamma^{'})$.

Therefore for the new IT constraints we have that,

$$\begin{bmatrix} r_{1,2\sigma_1} \\ r_{1,2\sigma_2} \\ r_{2,1\pi_1} \\ r_{2,1\pi_2} \end{bmatrix} \geq \begin{bmatrix} C_{2\sigma_1}(\Gamma^{'}) \\ C_{2\sigma_2}(\Gamma^{'}) \\ C_{1\pi_1}(\Gamma^{'}) \\ C_{1\pi_2}(\Gamma^{'}) \end{bmatrix} \approx \begin{bmatrix} C_{2\sigma_1}(\Gamma) \\ C_{2\sigma_2}(\Gamma) \\ C_{1\pi_1}(\Gamma) \\ C_{1\pi_2}(\Gamma^{'}) \end{bmatrix} + \delta Dd$$

4.3 Updating Rule and the Stopping Criterion

From the arguments given in Zhang and Cui, we have that, for an arbitrarily chosen $\Gamma > 0$, if the optimal rate values of all k's, $C_k(\Gamma_k)'s$ are all Pareto optimal on the boundary of the MISO-IC rate region, then for any pair $(i,j), i \in (1,2,..k), j \in (1,2..k)$ and $i \neq j$, it must hold that $|D_{ij}| = 0$, where D_{ij} is as defined in equation 4.4.

We also have that if $|D_{ij}| \neq 0$, then there exists a d_{ij}, such that, for $\Gamma' = \Gamma + \delta_{ij} D_{ij} d_{ij}$, the rate corresponding to Γ' is higher than that of Γ, where δ_{ij} is a small step size and d_{ij} is such that $D_{ij} d_{ij} > 0$.

In order to formulate the updating rule we fix $D_{ij} d_{ij} = l$, where l is some vector such that $l > 0$ such as $l = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ 1 \end{bmatrix}$.

Summarizing, we have stopping criterion to updating the values of $\Gamma's$ as $|D_{ij}| = 0$ and the updating rule as $\begin{bmatrix} \Gamma'_{1,2\sigma_1} \\ \Gamma'_{1,2\sigma_2} \\ \Gamma'_{2,1\pi_1} \\ \Gamma'_{2,1\pi_2} \end{bmatrix} = \begin{bmatrix} \Gamma_{1,2\sigma_1} \\ \Gamma_{1,2\sigma_2} \\ \Gamma_{2,1\pi_1} \\ \Gamma_{2,1\pi_2} \end{bmatrix} + \delta d_{ij}$, where $d_{ij} = l D_{ij}{}^{-1}$.

CHAPTER 5

Conclusions and Directions to Future Work

5.1 Conclusions

This thesis focuses on the characterization of the Pareto optimal boundary and implementation of the various techniques through network simulation. For better understanding of the problem, we have considered a 2-cell network to demonstrate the Pareto optimal boundary characterization. We have provided a detailed derivation of the closed form solution to the optimization problem.

We initialize the interference temperature values and iteratively arrive at the IT-values that give the rate that is global Pareto optimal. We have observed that not all initializations converge to the Pareto boundary. When we initialize very large interference values, in comparison to the power constraint that we assume, the rate doesn't converge to the boundary. We have also shown that the Pareto optimal rate is better than the zero forcing rate.

In the implementation of the decentralized algorithm, the Pareto optimal boundary for the 2-cell network does not match exactly with that obtained on implementation of the analytical solution provided in the chapter 2. The last topic we dealt with, is the 2MS per cell network where we gave the algorithmic solution to characterizing the Pareto optimal boundary in line with that given for a 1MS network.

5.2 Directions to Future Work

This thesis dealt with only 2-cell network for both simulation and for the closed form analytical solution. Research could be done in the direction of finding solution to a general k-user network. Also, most of the work considers only a single receiving antenna in each cell. It is very important to extend it to a multiple receiving antenna network in order to further address the real life scenario.

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