Optimal Downlink Scheduling under Time-varying Interference

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THESIS CERTIFICATE

This is to certify that the thesis titled **Optimal Downlink Scheduling under Time-varying Interference**, submitted by **R.Ravi Kiran**, to the Indian Institute of Technology, Madras, for the award of the degree of **Bachelor and Master of Technology**, is a bona fide record of the research work done by him under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

KEYWORDS: Throughput Optimal; Max weight ; Downlink Scheduling; Time-varying interference ; Delay

We address the problem of downlink resource allocation in the presence of time-varying interference. We consider a scenario where users served by a base station face interference from neighboring base stations that use the same channels. In particular, we model the interference from the neighboring base stations as ON/OFF renewal processes, that arise due to their idle and busy cycles. The users feedback their downlink SINR values to their base station, but these values are outdated owing to processing and propagation delays. In this setting, we characterize how Layer 2 can optimally exploit the reported SINR values, which could be unreliable due to time-varying interference. In particular, we propose a resource allocation policy that can stably support the largest possible set of traffic rates under the interference scenario considered. Our approach involves estimating the current SINR statistics using tools from renewal theory, and combining it with a Lyapunov stability framework to obtain a throughput optimal policy. We will also briefly study the proportional fair scheduling scheme.

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ABBREVIATIONS

CQI	Channel Quality Information
SINR	Signal to Interference plus Noise Ratio
UE	User Equipment
BS	Base Station
AWGN	Additive White Gaussian Noise
TDMA	Time Division Mulitple Access
OFDMA	Orthogonal Frequency Division Multiple Access
OFDM	Orthogonal Frequency Dvision Multiplexing
i.i.d	Independent and identically distributed
pdf	Probability density function
cdf	Cumulative distribution function
pmf	Probability mass function

NOTATION

K	Number of users in the Network
$\gamma(t)$	Signal to Interference plus Noise Ratio
$C(t)$	Channel capacity
δ	Information Delay
$D_1, D_2, \ldots$	Delay classes
$Q(t)$	Queue Length
$r(t)$	Rate of transmission
Γ	Average fading power gain
N_0	AWGN noise power
$A(t)$	Packet arrival process for queues
$P(t)$	Transmit power in slot t

CHAPTER 1

Introduction

Scheduling of users for downlink transmissions on a cellular network forms a critical component of the functioning of a base station. Such transmissions are often affected by a combination of fading, noise and interference and the objective of this dissertation is to study one such environment that is further complicated by information delay in the network.

We know that the users in a cell suffer from effects of noise (typically Additive White Gaussian (AWGN) in nature), multi-path fading and path loss. In particular, cell-edge users typically suffer from high path loss. This consequently decreases the received signal power and thus the channel capacity.

It is well known that a user equipment (UE) served by a base station does not suffer significant interference from transmissions to other UEs in the same cell. This is because UEs in the same cell are typically served on orthogonal resources like time, frequency and code. For example, in a TDMA[1] system, different UEs are served during different time-slots, whereas in a multi-carrier system such as OFDMA[2], different UEs in a cell are served on different frequency sub-bands. On the other hand, a given UE *does* suffer interference from neighboring base stations that use the same resources to transmit to UEs in the neighboring cells. In addition, this interference is usually time-varying, due to the underlying idle and busy cycles of the interfering base stations on that particular channel. Since this time-varying interference from neighboring base stations can have a detrimental effect on the throughput obtained by the UEs – particularly cell-edge UEs (who in addition to high path loss also experience high interference from said neighbour) – it is important to understand its implications on resource allocation.

While different scheduling schemes try to attain a varied set of objectives, most scheduling policies adopted by base stations fall under the category of "opportunistic

[1]Time Division Multiple Access
[2]Orthogonal Frequency Division Multiple Access

scheduling". The main idea behind opportunistic resource allocation (Viswanath *et al.* (2002)) is to preferentially transmit on channels with favorable conditions. In order to enable opportunistic resource allocation, the UEs have to feedback their instantaneous "channel quality information" (CQI) to the base station from time to time. The CQI is a parameter fixed by the protocol, indicative of the channel strength of a user. It could be the Signal to Interference plus Noise ratio (SINR), the channel capacity or one of several such other properties of a wireless channel.

However, the CQI available at the base station is usually outdated by a few time-slots due to propagation and processing delays, or due to other practical reasons. In a scenario with time-varying interference, delayed CQI implies that it is possible for the channels to have changed drastically between the time that the CQI is measured and resource allocation is performed. For example, consider a UE that reports a relatively strong channel to the base station when an interfering base station is idle. When the user is scheduled however, the interfering base station could turn active on that channel, thereby causing a transmission outage. The opposite of such a scenario also seems likely. Say a cell-edge user reports a weak channel to the base station owing to strong interference from a neighboring base station. This might result in this particular user not being allocated resources at a later point in time when this CQI is received. However, it is quite possible for the interferer to have gone idle in this delay period thereby improving the channel of the user; yet another instance of the delay having a detrimental effect in the opportunistic scheduling framework. This clearly indicates that it is not desirable to directly use the outdated CQI to perform resource allocation.

In this dissertation, we specify how Layer 2 should interpret the delayed CQI in the presence of time-varying interference, in order to perform resource allocation efficiently. Specifically, we exploit the statistics of the time-varying interference to design an optimal resource allocation policy for the scenario considered. We consider a base station serving several UEs. Each UE suffers time-varying interference from one or more neighboring base stations. Any base station can be abstractly viewed as a server serving a set of queues and thereby can be associated with a G/G/1 queue. We know that the Busy-Idle cycles of a G/G/1 queue constitutes a renewal process - in specific, an ON/OFF renewal process. We thus model the interference from each neighboring base station using an ON/OFF renewal process (Ross, 1996, Section 3.4.1).

In this setting, we use tools from renewal theory to compute the conditional distribution of the current channel quality, given the delayed CQI available at the base station. This conditional distribution of the current channel quality allows us to compute the conditional outage probability of each user, for any given rate of transmission. Next, we combine our outage probability expression with a Lyapunov stability framework to come up with a resource allocation policy that provably maximizes the set of all supportable traffic rates for the scenario considered. We first consider a single-carrier system, and later generalize our policy to a multi-carrier system. We will also discuss the $\alpha-$fair scheduling policy. We will discuss existing optimal strategies and also talk about on-going work aimed at developing provably optimal $\alpha-$fair scheduling policies under this framework.

1.1 Related Work

The throughput optimal class of scheduling algorithms has been widely studied over the past two decades. The Lyapunov stability framework for designing throughput optimal scheduling policies was introduced in Tassiulas and Ephremides (1993). The paper proposes the Max weight scheme for dynamic server allocation to a set of parallel queues based on the connectivity of the queues to the server. The work goes on to obtain necessary and sufficient conditions for stabilizing the system of queues and proposes server allocation schemes which maximize the throughput and minimize delay.

Several papers under the domain of Max Weight scheduling have followed this work. The paper, Neely *et al.* (2005), provides centralized and decentralized policies for routing and dynamic power allocation in time-varying channels and proves throughput optimality of the policies. In particular it sheds light on the importance of scheduling based on maximizing the rate-backlog product - a term that has evolved to be the governing principle for max weight scheduling.

Subsequently, Ying and Shakkottai (2011) addressed the issue of throughput optimal scheduling with delayed network state information and time-varying channels.

Similarly, Gopalan *et al.* (2007), Manikandan *et al.* (2009) and Ahmed *et al.* (2013) address scenarios with partial, infrequent and sparse channel information, respectively. An important take back from these papers is the way the rate-backlog product adapts to account for the uncertainties in the CQI. Specifically, we can show that the use of "goodput"-backlog product achieves throughput optimality. Here the term "goodput" refers to the product of the rate and the probability of successful transmission at that rate. Thus, at the very outset, we expect the policy that falls in the throughput optimal regime for our fading-interference environment would hold a similar structure.

Even though the above mentioned papers do capture the effect of CQI uncertainties, the channel in itself is often considered to be independent and identically distributed (i.i.d) across slots. While the assumption assists in the analytical study of the system, it fails to capture the underlying temporal correlations in the channel. In a realistic wireless network however, the communication channels are likely to have temporal correlations, owing to fading and/or interference. This notion of exploiting temporal correlations in channels has been studied, quite recently, in Ouyang *et al.* (2014) where the authors try to exploit these correlations to enhance scheduling under the domain of imperfect CQI and Markovian channels using a Partially Observable Markov Decision Process framework. This dissertation focuses on working with a more generic channel model where the temporal correlations in the channel are brought about through a combination of block fading and renewal process based interference.

The domain of $\alpha-$fair scheduling has also been rigorously studied over the past few years. The most commonly used policy, the "gradient scheduling algorithm" was proved to be asymptotically optimal in Kushner and Whiting (2004) by using the theory of Stochastic Approximation (Borkar (2008)). The work goes on to show that the backbone of the proportional fair scheduling regime is formed by decisions that are based on maximizing the product of the gradient of the utility function and the rate of transmission.

Subsequent works like Huang *et al.* (2009) propose proportional fair policies based on this gradient-rate product structure for specific systems like the OFDM framework in this work. A more elaborate resource allocation policy of deciding the packet transmission schedule, power allocation, modulation and coding scheme to ensure fairness in a packet based system is studied in Agrawal *et al.* (2001). The work studies the efficiency vs fairness tradeoff too.

We can see that the optimal policy has a similar structure to the rate-backlog product characteristic of the max weight policies. An intuitive extension of results seems to suggest that the product of "goodput" and gradient of utility should characterize the scheduling policies working with uncertain CQI. This indeed has been proven to be the case in Pantelidou and Ephremides (2009) where the scheduler works with an estimated channel and probability of outage for such an estimate. Here again the theory of stochastic approximation is invoked to prove convergence. The concavity of the utility function is used to prove the utility maximization of the policy and a Lyapunov structure is exploited to prove uniqueness of the optimal point. A stark difference to be noted in the model used here and that being studied in this dissertation is the lack of temporal correlations in the channel assumed in this paper. However, we expect the uncertainty structure to still hold and thus we intuitively expect a similar scheme to result.

1.2 Thesis Layout

In this work, we will characterize the CQI distribution based on the available delayed information for a slow, block fading model that follows a Rayleigh distribution. We assume that the base stations donot use adaptive power control strategies. The resource allocation under this constraint is restricted to the allocation of slots for transmission to users in a single carrier system and additionally, of distributing sub carriers in a multi-carrier setting. We first devise throughput optimal and α-fair scheduling policies under the given scenario for the case of a single-carrier system and later extend the same to multi-carrier systems. We will also prove the optimality of the throughput optimal scheme and elaborate on the on-going work in the domain of $\alpha-$fair scheduling.

This thesis is structured as follows. In Chapter 2 we will describe the system model and elaborate on the fading and interference structure in the network. In Chapter 3 we will give the mathematically define the two scheduling problems. Chapter 4 is allotted to the Throughput optimal scheme where we describe the optimal policy and use a Lyapunov stability framework to prove optimality of the proposed algorithm. In

Chapter 5 we will propose the algorithm to compute the conditional outage probability under the delayed information constraints assumed. We will highlight on-going work in the domain of $\alpha-$fair scheduling in Chapter 6 and in Chapter 7 we give the concluding remarks with possible future extensions to the problem tackled here.

CHAPTER 2

System Model

Consider a time-slotted downlink system comprising of a base station B_0 serving K UEs. We shall assume here for ease of analysis that all base stations under consideration have synchronized slot boundaries.

The base station maintains separate downlink buffers for each UE. Exogenous arrivals for UE i are characterized by the arrival process $A_i(t)$. We assume that the arrivals are independent and identically distributed (i.i.d) from slot to slot and are independent of the channel realizations. Let $\mathbb{E}[A_i(t)] = \lambda_i$ and $\mathbb{E}[A_i^2(t)] = \widetilde{A}_i < \infty$. We will denote by $Q_i(t)$, the length of the downlink buffer corresponding to user i.

For ease of exposition, we first elaborate our schemes for single-carrier downlink system and later generalize to the case of multi-carrier systems.

2.1 Interference

With advancements in interference mitigation techniques, vast portion of the interference observed by a user is, more often than not, owing to one neighboring base station. Thus, in this work, we shall assume that the UEs experience interference owing to a single neighboring base station B_{Int}, the single strong interferer. Let the transmit power of the interfering base station at time t, be given by $P_{Int}(t)$ and that of base station B_0 be $P_0(t)$. We shall further assume that the base stations transmit with constant power P. This interference model is clearly depicted in Figure 2.1.

The idle and busy cycles of transmission of the interfering base station is assumed to constitute an ON/OFF renewal process (Ross, 1996, Section 3.4.1). Since the transmit power level is a constant, we can see that the interference offered by the base station also constitutes an ON/OFF renewal process. Our renewal model is well motivated, since the idle and busy cycles of fairly general buffering systems (such as a G/G/1 queue, for

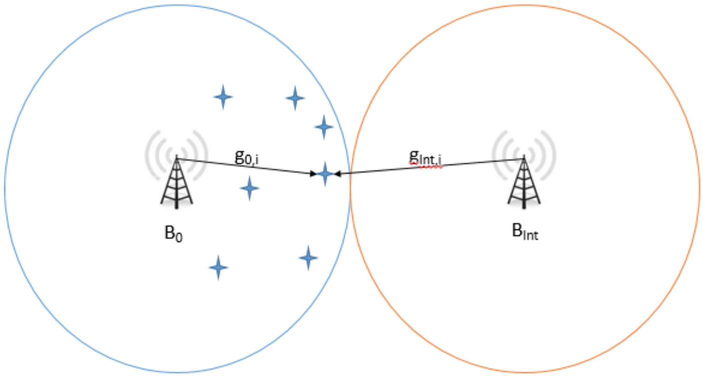


Figure 2.1: Base Station Layout

example) constitute renewal processes. The OFF and ON periods of the interferer are assumed to be distributed according to the random variables Z and Y respectively. We will assume that their joint distribution, given by $\mathbb{P}_{Z,Y}$, is known to B_0. Consequently, the distribution of the renewal period, $X = Y + Z$, is also known to B_0. Let the probability mass function (pmf) of X be given by $\mathbb{P}_X$.

Further, we assume that the traffic at the interfering base station is light tailed. This essentially translates to a light tailed distribution for the renewal period. Thus, we know that $\exists T_0 > 0$ such that $\mathbb{P}_X(t+1) \leq \mathbb{P}_X(t), \forall t \geq T_0$. Further, we can conclude that $\exists s > 0,\ s.t.\ \mathbb{E}[e^{sX}] < \infty$. Thus the Chernoff Inequality is applicable on the renewal period and hence, $\mathbb{P}(X \geq t) \leq \mathbb{E}[e^{sX}]e^{-st}$.

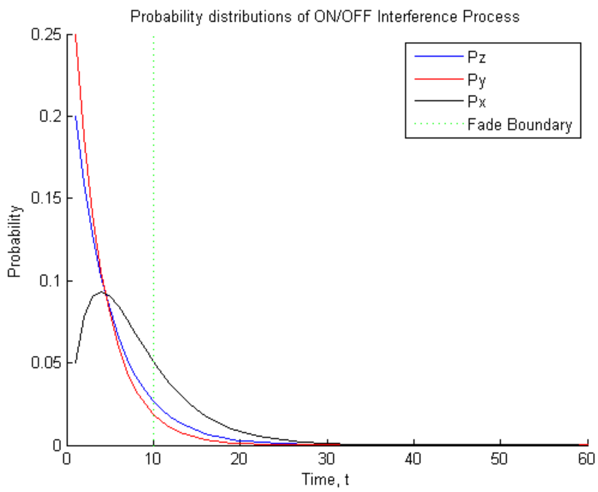


Figure 2.2: On/Off Process Distributions

An illustration of such a distribution is shown in Figure 2.2. The figure shows the distributions that constitute the On/Off process with independent, geometrically distributed, On and Off periods. We can notice that the tail of the distribution of the renewal period following the fade boundary satisfies the Chernoff bound.

Also, the renewal cycles represent the busy and idle cycles of the base station and thus are expected to be fairly large when compared to the slot durations. Thus we shall assume that the probability that of short renewal periods is small and shall characterize this as

$$\mathbb{P}(X \leq \delta_0) \leq \frac{\delta_0}{\bar{X}} = \chi << 1 \tag{2.1}$$

where $\delta_0 << \bar{X}$, represents the number of slots over which cdf of the renewal period is assumed to be upper bounded as shown above, and as will be assumed later, a bound on the information delay in the system as well.

2.2 Fading

We shall consider a frequency-flat block fading channel that offers a constant fading gain to the UEs over a block of size M. We will assume that the fading environment is slow such that $M = \kappa \bar{X}$. Here, κ is such that

$$\forall t \geq M \geq T_0, \mathbb{P}_X(t + M) \leq \alpha \mathbb{P}_X(t),$$

for some $\alpha < 1$. We are justified in assuming such an environment as we know that the renewal period distribution is light tailed.

Although not essential, we assume for ease of computation that all the UEs experience a Rayleigh fading channel from B_0 and B_{Int}. The average fading power gain for the channels is assumed to be known to B_0 and given by $\Gamma_{0,i}$ and $\Gamma_{Int,i}$ for UE i corresponding to B_0 and B_{Int} respectively.

The ON/OFF interference and the consequent fluctuations in Capacity of the channel as a function of time are depicted in Figure 2.3. The dotted lines represent the fade boundaries. This scenario is observed for the ON/OFF interference process characterized by the geometric distributions represented in Figure 2.2. The Channel capacity is

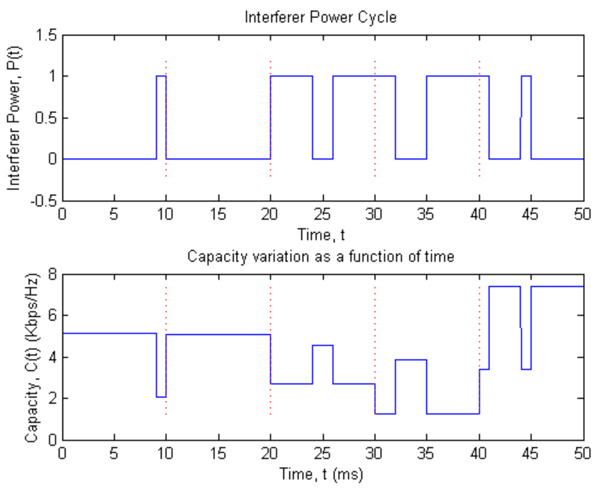


Figure 2.3: Time-varying Interference and Capacity

computed according to the Shannon capacity expression given below. The figure clearly depicts the extent of variation in the Channel capacity over time thereby highlighting the effect the delay can potentially have on scheduling.

2.3 Channel Quality Information

The signal to interference plus noise ratio (SINR) for UE i at time t is given by

$$\gamma_i(t) = \frac{g_{0,i}(t)P_0(t)}{N_0 + g_{Int,i}(t)P_{Int}(t)}, \tag{2.2}$$

We assume that the values of $\mathbb{E}[g_{Int,i}(t)]$ are known to the UE i. N_0 is the average additive white Gaussian noise (AWGN) power for the channel. We will assume that the Shannon Capacity expression, given by

$$C_i(t) = \log_2(1 + \gamma_i(t)) \tag{2.3}$$

defines the capacity of the channel to user i at time t.

We assume that the UEs follow an aperiodic CQI feedback scheme wherein the CQI is reported to B_0 whenever a change in the CQI value is observed in a slot. Owing to delays like propagation and processing, the information is received at the base station with a constant, known delay of δ_i corresponding to the user i. Thus B_0 observes

$\gamma_i(t - \delta_i)$ at slot t. These delays, albeit significant, are relatively small when compared to the average renewal period. Thus we shall assume that $\delta_i \leq \delta_0, \forall i \in \{1, 2, ..., K\}$.

We will assume in this dissertation that the CQI reported by user i is the SINR, $\gamma_i(t)$, observed by the user over slot t. Equivalently, it may also be any one-to-one function of the observed SINR like the Channel capacity (2.3). Since UEs report the CQI as and when a change is observed, we know that the BS B_0 is aware of the SINR values for all slots, albeit with a delay.

We also assume that the delays apply to all information, including ACKs, that is fed back by the UEs. Thus, the success or failure of a transmission attempted for UE i at time t is learnt only at $t + \delta_i$. Consequently, B_0 has access only to outdated information like Queue lengths and average throughput.

Having elaborately outlined the system model, we will now move on to the problem of using these assumed statistics to aid our scheduling problem.

CHAPTER 3

Problem Formulation

This work primarily aims at studying downlink scheduling algorithms under the specified domain of time-varying interference using delayed CQI. In particular, we will try to study two of the most widely used scheduling policies : Throughput Optimal Scheduling and $\alpha-$fair Scheduling. In this chapter, we will give a brief introduction to the two said policies and introduce some important notations used hereafter.

3.1 Throughput Optimal Scheduling

Alternately known as the Max Weight scheduling policy, the throughput optimal class of scheduling policies aim at making scheduling decisions such that the queues in the network are stabilized. We shall now define this notion more formally.

3.1.1 Definitions of Throughput Optimality

Consider the queuing system comprising of K queues. The stability of this system is defined as follows.

Definition 1. *A queuing system is said to be strongly stable, if* $\forall 1 \leq i \leq K$,

$$\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T} E[Q_i(t)] < \infty.$$

Let Π be the class of all possible scheduling algorithms operating under the fading and information delay constraints mentioned above.

Definition 2. *The stability region,* Λ*, of the system is defined as the set of all arrival rate vectors,* $\underline{\lambda}$*, such that there exists a policy* $\pi \in \Pi$ *that stabilizes the queuing system.*

The optimal algorithm under this regime is thus the algorithm that performs best in the stability region.

Definition 3. *A policy $\pi^* \in \Pi$ is said to be throughput optimal if it maintains strong stability in the queuing system for all arrival rates in the interior of the stability region Λ.*

3.1.2 Queuing Dynamics

Let $\mu_i(t)$ be the number of packets of queue i successfully transmitted by B_0 in slot t and is given by:

$$\mu_i(t) = a_i r_i(t) I_i(t), \tag{3.1}$$

where, $r_i(t)$ is the transmit rate assigned for user i. Let a_i denote the fraction of slot t that is allocated to the user i. starting from the We can characterize the dynamics of the queue corresponding to user i as:

$$Q_i(t+1) = (Q_i(t) - \mu_i(t))^+ + A_i(t), \tag{3.2}$$

where, $A_i(t)$ and $\mu_i(t)$ are the arrival and service processes for user i and $(x)^+ = \max\{x, 0\}$. Further, $I_i(t)$ is the indicator random variable that is 1 if the transmission is successful and 0 in the case of outage. The transmission is said to face outage if $C_i(t) < r_i(t)$, where $C_i(t)$ is the capacity of the channel to user i at time t.

Thus during each slot, the base station has to determine the user to transmit to, fraction of the slot to be allocated to that particular user and the optimal rate of transmission using the available delayed CQI and the statistics of the time-varying interference.

In this work, we plan to work on a CQI structure that is influenced by an underlying renewal process (interference). In Chapter IV we develop a provably throughput optimal scheduling policy for single-carrier systems and later extend it to multi-carrier systems.

3.2 $\alpha-$fair Scheduling

The second class of scheduling policy that we aim to study under the specified domain is the $\alpha-$fair scheduling algorithm. This algorithm aims at ensuring fair usage among all users through the objective of maximizing a specific utility function. In this setting, we shall assume that the system is "saturated", i.e, each user has infinite amount of data to be served.

The average downlink throughput served to user i by B_0 at slot t is given by

$$\theta_i(t) = \frac{1}{t}\sum_{i=1}^{t} \mu_i(t) \tag{3.3}$$

Assume that we are given a non-decreasing, concave utility function,

$$U(\bar{\theta}(t)) = \sum_{i=1}^{K} U_i(\theta_i(t)) \tag{3.4}$$

One such class of possible utility functions is given by:

$$U_i(\theta_i(t)) = \begin{cases} \frac{1}{\alpha}(\theta_i(t))^{\alpha}, & \alpha \leq 1, \alpha \neq 0 \\ \log(\theta_i(t)), & \alpha = 0 \end{cases} \tag{3.5}$$

The specific case of $\alpha = 0$ is the criterion that defines the Proportional Fair Scheduler.

Given such a function, the objective of the $\alpha-$fair scheduler is to make scheduling decisions such that the overall utility of the system under steady state is maximized.

Having had a glimpse at the two scheduling schemes, we will study the optimal policies for the defined channels in ensuing chapters.

CHAPTER 4

Throughput Optimal Scheduling

In this chapter we propose a scheduling policy that is provably throughput optimal for the time-varying interference model considered in Chapter 2.

Let $p_i(r)$ be the conditional outage probability for UE i given the delayed SINR history, $\bar{\gamma}_i(t) = \{\gamma_i(\tau) : \tau \leq t - \delta_i\}$. That is,

$$p_i(r) = F_{C_i(t)|\bar{\gamma}_i(t)}(r|\bar{\gamma}) = \mathbb{P}(C_i(t) \leq r|\bar{\gamma}_i(t) = \bar{\gamma}).$$

In Chapter 5, we will describe how this outage probability can be computed by B_0.

It is to be noted that owing to the information delay, the queue length, a critical component of the throughput optimal scheme, is also available only with a delay. That is, B_0 is aware only of $Q_i(t - \delta_i)$ at time t for each user i.

Let us define the goodput for UE i as $G_i(r_i) = r_i(1 - p_i(r_i))$, which is the average successful rate of transmission to UE i.

We will now introduce the Throughput Optimal Scheduling Policy for a single carrier system and study a Lyapunov drift analysis for the queueing system. Subsequently, we will prove that the policy we describe here is in fact throughput optimal under the conditions specified in the model. We will then extend the scheme to adapt to a multi-carrier downlink system.

4.1 Throughput Optimal Policy - Single Carrier System

During every slot t, B_0 observes $\bar{\gamma}_i(t)$ and $Q_i(t - \delta_i)$ of each UE i. This data is used to make throughput optimal scheduling decisions according to the below mentioned algorithm :

Algorithm 1 Throughput Optimal Policy, π^*

for $i = 1$ **to** K **do**

Evaluate $\mathbb{P}_{i,out}(r)$

Compute $r_i^* = \arg\max_{r\geq 0}\{r(1 - \mathbb{P}_{i,out}(r))\}$

end for

Determine $k = \arg\max_{1\leq i\leq K}\{Q_i(t-\delta_i)r_i^*(1 - \mathbb{P}_{i,out}(r_i^*))\}$

Transmit to UE k with rate r_k^* with power P

In each slot, this policy transmits to the UE that maximizes the goodput-backlog product. We first compute the conditional outage probability for every UE using the algorithms described in later sections. We then evaluate the rate that maximizes the goodput for each UE. Finally, we determine the UE with the maximum queue-length goodput product and transmit to that UE at a rate of r_i^* with ties broken at random.

4.2 Lyapunov Drift Analysis

In this section, we prove the throughput optimality of the proposed policy using Lyapunov drift techniques. We use the quadratic Lyapunov function:

$$L(\bar{Q}(t)) = \sum_{i=1}^{K}(Q_i(t))^2.$$

The Lyapunov drift is defined as :

$$\Delta(t) = \mathbb{E}[L(\bar{Q}(t+1)) - L(\bar{Q}(t))|\bar{y}(t-\bar{\delta})]. \tag{4.1}$$

where $\bar{y}(t-\bar{\delta}) = (Q_1(t-\delta_1), Q_2(t-\delta_2), ..., Q_K(t-\delta_K))$. We know the expression for the queue dynamics from (3.2). Also, by our assumption in Section II, the arrival processes for each UE have bounded second moments. Further, we know that the transmit rate for the base station is upper bounded by the capacity offered, i.e, $a_i r_i I_i(t) \leq C_i(t)$. We also know that

$$E[a_i r_i I_i(t)|Q_i(t-\delta)] = a_i r_i(1 - p_i(r_i)) = a_i G_i(r_i).$$

Using these relations we can bound (4.1) as

$$\Delta(t) \leq \sum_{i=1}^{K}\{\mathbb{E}[(C_i(t))^2] + \widetilde{A_i} + (\delta)\mathbb{E}[C_i(t)]^2 + (\delta)\lambda_i^2 + 2Q_i(t-\delta)\lambda_i \quad (4.2)$$

$$-2Q_i(t-\delta_i)\mathbb{E}[\mu_i(t)|\bar{y}(t-\bar{\delta})]\} \quad (4.3)$$

In obtaining the above bound, we have also made use of the fact $A_i(t)$ is independent of the $Q_i(t)$. Under Rayleigh fading conditions, we can trivially assume that the second moment of the capacity of the channels of the users are bounded, i.e, $\mathbb{E}[(C_i(t))^2] < \widetilde{C}_i < \infty$. Thus, we can conclude that the squared mean of capacity is also bounded, i.e, $\mathbb{E}[C_i(t)]^2 \leq \mathbb{E}[C_i(t)^2] < \widetilde{C}_i < \infty$. We also know that $\lambda_i^2 \leq \widetilde{A}_i$. Also, $\mathbb{E}[\mu_i(t)|Q_i(t-\delta_i)] = a_i G_i(r_i)$. Thus, we get,

$$\Delta(t) \leq (\delta+1)\sum_{i=1}^{K}(\widetilde{C}_i + \widetilde{A}_i) + \sum_{i=1}^{K} Q_i(t-\delta_i)(\lambda_i - a_i G_i(r_i)) \quad (4.4)$$

We will now argue that our policy π^* ensures the most negative Lyapunov drift. To see this, consider:

$$\max_{\{a_i\},\{r_i\}} \sum_{i=1}^{K} Q_i(t) G_i(r_i) a_i, \quad (4.5)$$

subject to

$$\begin{aligned} \sum_{i=1}^{K} a_i &\leq 1, && (C_1) \\ a_i &\geq 0,\ 1 \leq i \leq K, && (C_2) \\ r_i &\geq 0,\ 1 \leq i \leq K. && (C_3) \end{aligned}$$

We have assumed that modulation and coding schemes for any desired rate r_i are available. We will now solve the above mentioned Convex Optimization problem by using the Karush-Kuhn-Tucker (KKT) conditions.

4.3 Minimizing Lyapunov Drift

We shall now solve (4.5) thereby arriving at the scheduling policy described earlier.

Introduce non-negative Lagrange multipliers $\eta, \{\alpha_i\}, \{\beta_i\}$ for constraints $(C_1) - (C_3)$ respectively. Thus applying the KKT conditions, we get the following set of conditions that are to be satisfied by the optimal solution (The $(.)^*$ values represent the optimal solutions to the optimization problem) :

$$Q_i(t-\delta_i)G_i(r_i^*) + \alpha_i^* - \eta^* = 0, \ \forall i \tag{4.6}$$

$$Q_i(t-\delta_i)a_i^* \frac{\partial G_i(r_i^*)}{\partial r_i^*} + \beta_i^* = 0, \ \forall i \tag{4.7}$$

$$\eta^*(\sum_{i=1}^{K} a_i^* - 1) = 0 \tag{4.8}$$

$$\alpha_i^* a_i^* = 0, \ \forall i \tag{4.9}$$

$$\beta_i^* r_i^* = 0, \ \forall i \tag{4.10}$$

Proposition 1. *The optimal resource allocation scheme for the problem in (4.5) is that which assigns the channel exclusively to the UE with the largest queue-length goodput product and transmits at the rate that maximizes its goodput.*

Proof. From (4.10) and (4.7), we can see that if $r_i^* > 0$, then $\beta_i^* = 0$ and thus $\frac{\partial G_i(r_i^*)}{\partial r_i^*} = 0$. Thus the optimal rate allotted to UE i is that which maximizes his goodput.

Similarly, from (4.9) and (4.6), if $a_i^* > 0$, then $\alpha_i^* = 0$ and thus $\eta^* = Q_i(t-\delta_i)G_i(r_i^*)$. Also, if $a_i^* = 0$, then $\alpha_i^* \geq 0$. Thus, $\eta^* \geq Q_i(t-\delta_i)G_i(r_i^*)$. This clearly shows that $\eta^* = \max_{1\leq i\leq K} Q_i(t-\delta_i)G_i(r_i^*)$. We can thus see that the optimal allocation is to transmit to the UE with the maximum queue length goodput product. □

Proposition 1 shows that transmissions made at a rate that maximizes goodput to the user with the maximum queue length goodput product, solves the optimization problem described in (4.5). We can see that this is essentially the same as what has been described in our policy π^*. Thus, π^* minimizes the Lyapunov drift.

4.4 Throughput Optimality

Since the proposed policy, π^*, minimizes the Lyapunov drift, we expect it to be throughput optimal under the scenario considered in Section II. The following theorem asserts this.

Theorem 1. *The scheduling policy, π^*, is throughput optimal over the class of all policies constrained to the time-varying interference and CQI mechanism described in Section II.*

Proof. Consider an arrival rate vector $\underline{\lambda} \in \Lambda^0$. Then there exists a policy $\pi \in \Pi$ stabilizes the system for this arrival rate. Then, $\exists \underline{\epsilon} = (\epsilon_1, \epsilon_2, \ldots, \epsilon_K)$ with $\epsilon_i > 0, \forall i \in \{1, 2, \ldots, K\}$ such that

$$\lambda_i \leq b_i \hat{r}_i (1 - P_i(\hat{r}_i)) - \epsilon_i, \forall i \in \{1, 2, \ldots, K\}, \tag{4.11}$$

for some $\underline{b} = (b_1, b_2, \ldots, b_K)$ with $b_i \geq 0, \forall i \in \{1, 2, \ldots, K\}$ and $\sum_{i=1}^{K} b_i = 1$. Here, $\hat{r}_i$ is the rate assigned by π to UE i.

Now consider the convex optimization problem given by the following :

$$\max_{\underline{b}, \underline{r}} \sum_{i=1}^{K} Q_i(t - \delta_i) b_i G_i(r_i) \tag{4.12}$$

subject to

$$\begin{aligned} r_i &\geq 0, \qquad \forall i \in \{1, 2, \ldots, K\}, \\ b_i &\geq 0, \qquad \forall i \in \{1, 2, \ldots, K\}, \\ \sum_{i=1}^{K} b_i &= 1. \end{aligned}$$

We know from Proposition 1 that the optimal assignment for (4.12) is given by $\underline{b}^*$ such that, $b_k^* = 1$, where $k = \arg\max_{1 \leq i \leq K} (Q_i(t - \delta_i) G_i(r_i))$ and $b_i^* = 0, \forall i \neq k$. We can see that π^* does exactly this selection in every slot. Hence, we can conclude that

$$\sum_{i=1}^{K} Q_i(t-\delta_i) b_i G_i(\hat{r}_i) \leq \sum_{i=1}^{K} Q_i(t-\delta_i) a_i G_i(r_i^*), \;\; \forall \underline{b} \tag{4.13}$$

where r_i^* is the rate assignment chosen for UE i under policy π^*.

From (4.11) and (4.13), we can see that

$$\sum_{i=1}^{K} Q_i(t-\delta_i)\lambda_i \leq \sum_{i=1}^{K} Q_i(t-\delta_i)(a_i G_i(r_i^*) - \epsilon_i). \tag{4.14}$$

Substituting (4.14) in (4.3), we get :

$$\Delta(t) \leq \delta \sum_{i=1}^{K} \left(\widetilde{A}_i + (C_i)^2 \right) - 2 \sum_{i=1}^{K} Q_i(t-\delta_i)\epsilon_i.$$

Now, since $\epsilon_i > 0$, the Lyapunov drift becomes negative when the queues are large. This in turn establishes that π^* stabilizes the system for an arrival rate of $\underline{\lambda}$ Thus we have proved that π^* stabilizes the system for any arrival rate, $\underline{\lambda} \in \Lambda$. □

Thus, we have proved that the proposed policy is throughput optimal among the class of policies constrained to the same time-varying interference and CQI mechanism. We will now extend the policy to the scenario of a multi-carrier downlink system.

4.5 Throughput Optimal Policy - Multi-carrier System

We will now extrapolate the policy outlined for the single carrier system to work in a multi-carrier downlink setting. Assume that the base station has M sub carriers to serve the users. We will assume that the fading and interference on each sub carrier is independent of each other. We will also assume the same CQI feedback mechanism as was adopted earlier.

The throughput optimal policy under the above mentioned regime is given by :

Algorithm 2 Multi-carrier Throughput Optimal Policy, π^*_{multi}

for $j = 1$ **to** M **do**

  for $i = 1$ **to** K **do**

    Compute $p_{i,j}(r)$

    Compute $r^*_{i,j} = \arg\max_{r\geq 0}\{r(1 - p_{i,j}(r))\}$

  end for

  Compute $k_j = \arg\max_{1\leq i\leq K}\{Q_i(t-\delta)r^*_{i,j}(1 - p_{i,j}(r^*_{i,j}))\}$

  Transmit to user k_j on sub-carrier j with power P/M at rate $r^*_{i,j}$.

end for

In the above algorithm, $p_{i,j}(r)$ is the outage probability for a transmission of rate r to user i on sub carrier j.

The independence criterion assumed on the fading and interference processes translates to an independence in the channel capacities across sub carriers. Thus, the problem of scheduling on a multi-carrier system essentially simplifies to solving M scheduling problems on single carriers. Hence, the algorithm π^*_{multi} iterates over the sub carriers and assigns users with maximal goodput-backlog product. The proof for throughput optimality of this policy proceeds very similar to what was done for the single carrier case and thus is omitted for brevity.

Hence, in this chapter, we have described the algorithm that maximizes throughput for both the single and multi carrier downlink systems. It may be noted that the computation of outage probability using the delayed information is critical to the execution of the algorithms. In the next chapter, we will outline the procedure to compute the outage probability for the channel.

CHAPTER 5

Estimation of Outage Probability

In this section, we describe how B_0 computes the conditional outage probability for UE i, given the SINRs of the past, by using the statistics of the ON-OFF processes. We will start with a scenario having just one user in the network. For ease of notation, we will drop the suffixes and assume only one user in the system whose SINR at time t is given by $\gamma(t)$ with delay of δ and average fading gain of Γ_1 from B_{Int}.

We will first introduce some notations that characterize the renewal process. Let $N(t)$ be the number of renewals up to and including time t. Let S_n be the time of the n^{th} renewal. Owing to the nature of the renewal process, we know that $P_{Int}(t)$ is dependent on $(P_{Int}(t-1), P_{Int}(t-2), \ldots, P_{Int}(S_{N(t)}))$. Since the process renews at $S_{N(t)}$, $P_{Int}(t)$ is independent of $P_{Int}(t'), \forall t' < S_{N(t)}$.

We will describe the method to compute the outage probability in steps. We will first consider the case of a static network with constant channel gains. In such a network, the interference is predominantly characterized by the renewal process. That being the case, we will show how the outage probability can be computed using the distributions of the ON/OFF process. We will then analyze the scenario of a fast fading channel with Markovian interference. Then we will look at the more generic scenario described in Chapter 2.

5.1 Static Network

In this section we will look at a network under steady state with static channel gains. Since the system is in steady state, it is fair to state that B_0 knows the channel gains g_0 and g_{Int} almost surely (w.p.1). Thus we know the state of the interferer directly from the CQI data. Let $\bar{P}_{Int}(t) = (P_{Int}(t-\delta), P_{Int}(t-\delta-1), \ldots, P_{Int}(S_{N(t-\delta)}))$ and let $\bar{\gamma}(t) = (\gamma(t-\delta), \gamma(t-\delta-1), \ldots, \gamma(S_{N(t-\delta)}))$. Thus, for a static network under steady state, given $\bar{\gamma}(t)$, we can find $\bar{P}_{Int}(t), w.p.1$.

Since we have constant fades, we know that the SINR takes one of 2 values depending on the interferer state. Let these 2 values be

$$\gamma_0 = \frac{g_0 P}{N_0}, \quad \gamma_1 = \frac{g_0 P}{N_0 + g_1 P}$$

The outage probability is given by

$$\mathbb{P}_{out}(r) = \mathbb{P}(C(t) < r | \bar{\gamma}(t)) = \mathbb{P}(\gamma(t) < 2^r - 1 | \bar{\gamma}(t)) \tag{5.1}$$

where $C(t)$ is the Shannon channel capacity at time t. Since only 2 SINRs are possible, only 2 corresponding capacities and thus rates are to be considered in the static network. Let these be $\{C_0, C_1\}$ and $\{r_0, r_1\}$ corresponding to the interferer being "OFF" and "ON" respectively. Thus we have outage only when B_0 transmits at r_0 when $C(t) = C_1$. That is

$$\begin{aligned} \mathbb{P}_{out}(r_1) &= 0, \text{and} \\ \mathbb{P}_{out}(r_0) &= \mathbb{P}(C(t) = C_1 | \bar{\gamma}(t)) \\ &= \mathbb{P}(P_{Int}(t) = P | \bar{P}_{Int}(t)) \end{aligned} \tag{5.2}$$

For convenience, let us assume that $P_t = P_{Int}(t)$ and $T = t - S_{N(t-\delta)}$.

Since the base station can keep track of the interference process subject to the delay, B_0 is privy to the age and state information of the most recent, observed renewal period.

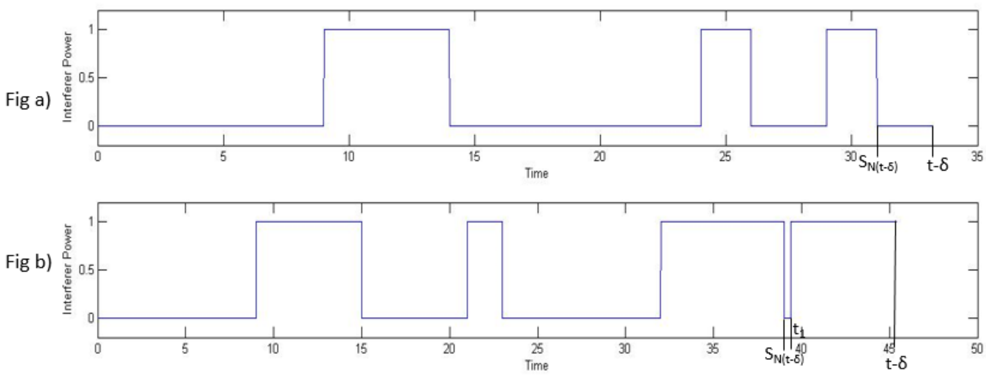


Figure 5.1: Static Network Interference Process

To get a better picture, we can look at Figure (5.1) which shows two scenarios. One where the interferer is OFF at $t - \delta$ and the other where it is ON. Given such a history, it is possible to compute the probabilities of the state of the interferer at t. We shall explain this through the help of the figures we have shown.

In Figure (5.1 a) we can see that the state of the interferer at $t - \delta$ is OFF. Let $z = t - \delta - S_{N(t-\delta)}$, the OFF period which in this case is the age of the process. In this scenario, we can compute the outage probability through the following sequence of steps.

$$\begin{aligned} \mathbb{P}(P_t = 0, \bar{P}_{Int}(t) = \bar{0}) &= \mathbb{P}(Z > T) \\ &+ \sum_{\tau=z}^{T} \mathbb{P}_X(\tau)\mathbb{P}(Z > T - \tau) + o(\chi) \end{aligned} \tag{5.3}$$

where we get the $o(\chi)$ term as we know that the probability of a renewal period being shorter than the delay is bounded by $\chi << 1$ as stated in Chapter 2. Further, we know that $\mathbb{P}(\bar{P}_{Int}(t) = \bar{0}) = \mathbb{P}(Z \geq z)$. Using these, we can compute $\mathbb{P}(P_t = 0|\bar{P}_{Int}(t) = \bar{0})$ trivially using Bayes' theorem. Thus, we can also compute $\mathbb{P}(P_t = P|\bar{P}_{Int}(t) = \bar{0}) = 1 - \mathbb{P}(P_t = 0|\bar{P}_{Int}(t) = \bar{0})$.

Now, we will describe the case shown in Figure (5.1 b). Let $z = t_1 - S_{N(t-\delta)}$ and $y = t - \delta - t_1$. Consequently, $y + z$ represents the age of the renewal period. Proceeding in a similar fashion with the interference history presented in Figure (5.1 b), we get

$$\begin{aligned} \mathbb{P}(P_t = P, \bar{P}_{Int}(t) = \bar{p}) &= \mathbb{P}(Z = z, Y \geq t - z) \\ &+ \sum_{\tau=y+z}^{T} \mathbb{P}_X(\tau)\mathbb{P}(Z \leq T - \tau) + o(\chi) \end{aligned} \tag{5.4}$$

where, $\bar{p}$ represents the interference history corresponding to the figure. Here again, we know that $\mathbb{P}(\bar{P}_{Int}(t) = \bar{p}) = \mathbb{P}(Z = z, Y \geq y)$. Using these, we can once again compute the conditional probabilities.

Hence the outage probability can finally be computed as

$$\mathbb{P}_{out}(r_0) = \mathbb{P}(P_{Int}(t) = P|\bar{P}_{Int}(t)) \tag{5.5}$$

appropriate to the observed history.

Having exploited the properties of renewal processes to compute the outage probability in the static network scenario described in this section, we will now move to another scenario of having memoryless channels and Markovian interference.

5.2 Memoryless Channel and Memoryless Interference

In this section, we will study a fast fading environment wherein the fading gains vary i.i.d from slot to slot with a Rayleigh distribution. As assumed earlier, we will assume that the fading for different users is independent and that the means of the fading gains are known to B_0.

Further, we will also assume that B_0 doesn't store CQI values. That is, B_0 uses the most recent SINR data to make scheduling decisions. Such an estimate works optimally when the interference is characterized either by a Markov chain or by memoryless distributions. In particular, if the ON and OFF distributions are independent of each other and hold geometric distributions, then owing to the memoryless property of the distribution, the interference fluctuation simplifies to that of a Markov chain.

We will however describe the outage probability computation from a generic renewal process standpoint whose distributions are known apriori.

The conditional outage probability for a transmission of rate r, $\mathbb{P}_{out}(r)$ is given by :

$$\begin{aligned}\mathbb{P}_{out}(r) &= \mathbb{P}(C(t) \leq r|\gamma(t-\delta) = \gamma_0) \\ &= F_{\gamma(t)|\gamma(t-\delta)}(2^r - 1|\gamma_0), \qquad (5.6)\end{aligned}$$

where (5.6) follows from (2.3). It is thus sufficient to compute the conditional cdf of $\gamma(t)$ given $\gamma(t-\delta)$. We will introduce some simplified notation first.

$$\begin{aligned}F_\gamma(\gamma|\gamma_0) &= F_{\gamma(t)|\gamma(t-\delta)}(\gamma|\gamma_0), \\ F_P(\gamma|P) &= F_{\gamma(t)|P_{Int}(t)}(\gamma|P), \\ \mathbb{P}(P|P_0) &= \mathbb{P}(P_{Int}(t) = P|P_{Int}(t-\delta) = P_0), \\ \mathbb{P}(P|\gamma) &= \mathbb{P}(P_{Int}(t) = P|\gamma(t) = \gamma).\end{aligned}$$

We will describe the method to compute the outage probability through a sequential process of computation of conditional distributions and exploitation of independence relations.

5.2.1 SINR-Power Dependence Relations

We will first begin by analyzing the underlying conditional independence relations between the SINR ($\gamma(t)$) process and the interferer transmit power ($P_{Int}(t)$) process.

Lemma 1. *Under the channel fading conditions described in this section, $\gamma(t)$ and $\gamma(t-\delta)$ are conditionally independent given $P_{Int}(t)$. Also, the conditional cdf of the SINR given the state of the interferer at $t-\delta$ is given by:*

$$F_P(\gamma|P) = 1 - \left(\frac{\alpha_0}{\gamma\alpha_{Int} + \alpha_0}\right) \exp\left(\frac{-\gamma}{\alpha_0}\right).$$

where $\alpha_0 = \frac{\Gamma_0 P_0(t)}{N_0}$ and $\alpha_{Int} = \frac{\Gamma_{Int} P_{Int}(t)}{N_0}$.

Proof. We use $X \perp Y$ to mean that two random variables X and Y are independent.

Condition on $P_{Int}(t) = P$, for some $P \in \{0, P\}$. Thus from (2.2), the SINR is a deterministic function of the fading coefficients, i.e, $\gamma(t) = \psi(g_0(t), g_{Int}(t))$. Further, from the assumption made on the fading coefficients, we know that

$$(g_0(t), g_{Int}(t)) \quad \perp \quad (g_0(t-\delta), g_{Int}(t-\delta)) \text{ and} \tag{5.7}$$

$$(g_0(t), g_{Int}(t)) \quad \perp \quad P_{Int}(t-\delta). \tag{5.8}$$

Indeed, (5.8) follows from the independence of the fading coefficients and the interference profile. Thus from (5.7) and (5.8),

$$(g_0(t), g_{Int}(t))) \perp \gamma(t-\delta) \implies \phi((g_0(t), g_{Int}(t))) \perp \gamma(t-\delta), \tag{5.9}$$

for any function $\phi(.)$ on $\underline{g}$.

Using (5.9), we can conclude that

$$\text{given } P_{Int}(t) = P,\ \gamma(t) \perp \gamma(t-\delta), \forall P \in \{0, P\}.$$

Hence, we have proved that, $\gamma(t)$ is conditionally independent of $\gamma(t-\delta)$, given $P_{Int}(t)$.

Using this conditional independence, we perform the following computation :

$$\begin{aligned} F_{\gamma P}(\gamma|\gamma_0, P) &= F_P(\gamma|P) \\ &= \mathbb{P}\left(\frac{g_0 P}{N_0 + g_{Int}P_{Int}} \leq \gamma \middle| P \right) \qquad (5.10) \\ &= 1 - \left(\frac{\alpha_0}{\gamma\alpha_{Int} + \alpha_0} \right) \exp\left(\frac{-\gamma}{\alpha_0} \right), \qquad (5.11) \end{aligned}$$

where we have $F_{\gamma P}(\gamma|\gamma_0, P) = F_{\gamma(t)|P_{Int}(t),\gamma(t-\delta)}(\gamma|\gamma_0, P)$. Here, (5.10) comes directly from (2.2) and (5.11) follows from the exponential distribution of the fading power gain. □

Using Lemma (1), we obtain the pdf of the conditional distribution by taking the derivative of the cdf:

$$f(\gamma|P) = \left(\frac{\alpha_0}{\gamma\alpha_{Int} + \alpha_0} \right) \exp\left(\frac{-\gamma}{\alpha_0} \right) \left(\frac{\alpha_{Int}}{\gamma\alpha_{Int} + \alpha_0} + \frac{1}{\alpha_0} \right). \qquad (5.12)$$

Next, invoking the Key Renewal Theorem for the ON-OFF renewal process, (Ross, 1996, Theorem 3.4.4), we obtain $\mathbb{P}(P_{Int}(t) = P) = \rho$ under steady state. Here, $\rho = \frac{\mathbb{E}[Y]}{\mathbb{E}[X]}$, is the duty cycle of the ON/OFF process. Now, using Bayes' Theorem, we get :

$$\mathbb{P}(P_0|\gamma_0) = \frac{f(\gamma_0|P_0)\mathbb{P}(P_0)}{\sum_{P' \in \{0,P\}} f(\gamma_0|P')\mathbb{P}(P')},$$

where $f(\gamma|P)$ is given by (5.12).

Lemma 2. *The interference power, $P_{Int}(t)$ is conditionally independent of $\gamma(t-\delta)$ given $P_{Int}(t-\delta)$.*

Proof. Assume that $P_{Int}(t-\delta) = P$ for some $P \in \{0, P\}$. Thus from (2.2), the SINR is only a function of the fading coefficients, i.e, $\gamma(t-\delta) = \psi(g(t-\delta))$. Further, we know that the interferer transmissions are independent of the fading power gains. Hence, we have

$$P_{Int}(t) \perp g(t-\delta) \implies P_{Int}(t) \perp \phi(g(t-\delta)), \tag{5.13}$$

for any function $\phi(.)$ on g.

Using (5.13), we can conclude that

$$\forall P \in \{0, P\}, \text{ given } P_{Int}(t-\delta) = P, \; P_{Int}(t) \perp \gamma(t-\delta). \tag{5.14}$$

Hence we have proved that $P_{Int}(t)$ is conditionally independent of $\gamma(t-\delta)$, given $P_{Int}(t-\delta)$. □

5.2.2 Conditional Distribution of Transmit Power

We will now compute $\mathbb{P}(P|P_0)$ using the statistics of the ON-OFF process. We will denote by Y and Z, the ON and OFF periods of the renewal process respectively. Let F_Y and F_Z be the cumulative distribution functions of Y and Z respectively. Assume that $P_{i,j} = \mathbb{P}(P_{Int}(t) = iP | P_{Int}(t-\delta) = jP), \; \forall i, j \in \{0, 1\}$. In the next lemma, we will compute the conditional distribution $P_{1,1}$.

Lemma 3. *The conditional probability of the interferer being ON at* t *given that it is on at* $t-\delta$ *is given by:*

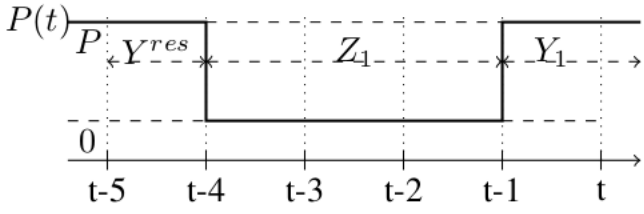
$$P_{1,1} = 1 - F_Y^{res}(\delta) + \mathbb{P}(E_1) + o(\chi^2),$$

where, $\chi = \frac{\delta}{\mathbb{E}[X]}$*.* $F_Y^{res}(.)$ *is the cdf of the residual period of the ON process at* $t-\delta$*, given* $P(t-\delta) = P$ *and has the pmf:*

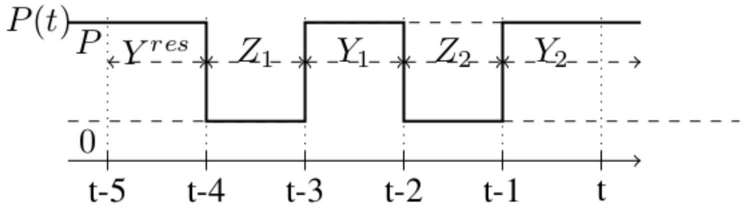
$$p_Y^{res}(i) = \frac{1 - F_Y(i)}{\mathbb{E}[Y]}. \tag{5.15}$$

Further, $\mathbb{P}(E_1) = \sum_{y=0}^{\delta} \sum_{z=0}^{\delta-y} F_Y^c(\delta - z - y) p_Z(z) p_Y^{res}(y)$*, where* $F_Y^c(.)$ *is the complementary cdf of* Y *and* $p_Z(.)$ *is the pmf of* Z*.*

Proof. Consider the event E of having $P(t) = P$, given $P(t-\delta) = P$. Let us assume



(a) One renewal in $(t, t-\delta)$



(b) Two renewals in $(t, t-\delta)$

Figure 5.2: On/Off Process examples in a system with $\delta = 5$

that E_0, E_1, E_2 are events with $P(t) = P$, given $P(t-\delta) = P$ with no renewals, one renewal and at least 2 renewals in $(t-\delta, t)$ respectively. Figure (5.2) shows sample ON-OFF renewal processes with $\delta = 5$. Figures (5.2 a) and (5.2 b) show the cases under consideration with one renewal and two renewal respectively. From Figure (5.2 b), we can see that we necessarily need $Y^{res} < \delta$ and $X_1 = Y_1 + Z_1 < \delta$ when there is at least one renewal. Thus we have

$$\begin{aligned}
\mathbb{P}(E_2) &\leq \mathbb{P}(Y^{res} < \delta, X_1 < \delta | P(t-\delta) = P) \\
&= \mathbb{P}(Y^{res} < \delta)\mathbb{P}(X_1 < \delta | P(t-\delta) = P) \qquad (5.16) \\
&\leq \left(\frac{\delta}{\mathbb{E}[Y]}\right)\left(\frac{\delta}{\mathbb{E}[X]}\right) = o(\chi^2), \qquad (5.17)
\end{aligned}$$

where, (5.16) follows from the fact X_1 and Y^{res} are independent owing to the renewal nature of the process. From the assumption made in Chapter 2 on the nature of the distribution of the renewal period, $\mathbb{P}(X_1 < \delta) \leq \frac{\delta}{\mathbb{E}[X]}$. Further, using the distribution of the residual ON period we get $P(Y^{res} < \delta) \leq \frac{\delta}{\mathbb{E}[Y]}$. Assuming that $\rho \neq 0$, and that $\chi = \frac{\delta}{\mathbb{E}[X]}$, is typically small (as the propagation delays are much smaller than the duration of the average transmit cycle) we can assume that $o(\chi^2)$ terms are negligible. This implies that the probability that we have more than one renewals, i.e, one full renewal period, in the span of $(t-\delta, t)$ and that $P(t) = P$, given that $P(t-\delta) = P$ is a very low probability event.

Let us now consider the case where there is exactly one renewal in $(t-\delta, t)$. Then, from Figure (5.2 a), we can verify that the probability is given by:

$$\mathbb{P}(E_1) = \sum_{y=0}^{\delta} \sum_{z=0}^{\delta-y} F_Y^c(\delta - z - y) p_Z(z) p_Y^{res}(y), \tag{5.18}$$

where, $F_Y^c(.)$ is the complementary cdf of Y, $p_Z(.)$ is the pmf of Z and p_Y^{res} is the pmf of the residual period as given in (5.15).

For the case of no renewals, we can compute the probability as:

$$\begin{aligned}\mathbb{P}(E_0) &= \mathbb{P}(Y^{res} > \delta) \\ &= 1 - F_Y^{res}(\delta).\end{aligned}$$

The events, E_0, E_1 and E_2 are not just disjoint, but also encompass all possible renewal scenarios in the delay interval. Thus by the law of total probability, we have $P_{1,1} = \mathbb{P}(E_0 \cup E_1 \cup E_2) = 1 - F_Y^{res}(\delta) + \mathbb{P}(E_1) + o(\chi^2)$ where $\mathbb{P}(E_1)$ is given by (5.18). □

Having obtained $P_{1,1}$, we can easily obtain $P_{0,1} = 1 - P_{1,1}$. Further, we know that

$$\begin{aligned}\mathbb{P}(P_{Int}(t) = P) &= P_{1,0}\mathbb{P}(P(t-\delta) = 0) + P_{1,1}\mathbb{P}(P(t-\delta) = P) \\ \implies \rho &= (1-\rho)P_{1,0} + \rho P_{1,1},\end{aligned}$$

where ρ, as mentioned earlier, is the duty cycle of the ON/OFF process. Using the above relation, we get

$$P_{1,0} = \left(\frac{\rho}{1-\rho}\right) P_{0,1}.$$

We can thereby compute $P_{0,0} = 1 - P_{1,0}$.

5.2.3 Conditional Outage Probability

We will now piece together, the different conditional distributions we computed to obtain the conditional outage probability. Using the results from Lemma (1) and (2), and using the law of total probability, we can deduce the following:

$$\begin{aligned} F_\gamma(\gamma|\gamma_0) &= \sum_{\underline{P}\in\mathcal{P}} F_{\gamma P}(\gamma|\gamma_0, \underline{P})\mathbb{P}(\underline{P}|\gamma_0) \\ &= \sum_{\underline{P}\in\mathcal{P}} \sum_{\underline{P}_0\in\mathcal{P}} F_P(\gamma|\underline{P})\mathbb{P}(\underline{P}|\underline{P}_0)\mathbb{P}(\underline{P}_0|\gamma_0). \end{aligned}$$

Finally we can use this in (5.6), to get the expression for the conditional outage probability.

5.3 M Block Fading

Having obtained algorithms to compute the conditional outage probability in simpler fading constraints, we will now analyze the M-block fading model described in Chapter 2 and devise optimal and computationally efficient algorithms to compute the outage probability. The M-block fading model comprises of two possible sources of variation in the SINR - fading gain and interference power. In order to compute the outage probability, it is critical to be able to keep track of the renewals in the interference process. However, it is not always feasible to observe the renewals in interference when fading changes are also present. Thus we will devise the algorithm to compute outage by tracking the dynamic changes in reported the CQI values. To facilitate the same, we will build set up a set of probable states that the interference process could currently be in, based on the observed set of CQI values.

To be able to spot renewals, one will have to keep track of the "change points" in the observed CQI data. Let us define a change point as follows:

Definition 4. *A time slot $t \in \mathbb{Z}^+$ is a change point if $\gamma(t) \neq \gamma(t-1)$ or if $\exists k \in \mathbb{Z}^+$, such that, $t = kM$.*

Let $\mathcal{T}$ be the set of all change points. We will now make use of these change points to devise our algorithm to compute outage. Let $\tau(t)$ be the most recent change point. That is,

$$\tau(t) = \arg \min_{\{t' < t : t' \in \mathcal{T}\}} (t - t')$$

Lets define $S(t) = (\omega(t), z(t), y(t), p(t), \psi(t))$ as a probable state of the system, i.e, one possible renewal cycle that could result in the observed set of CQI values. Here $\omega(t)$ represents a probable state of the interferer, i.e, $\omega(t) = 1$ represents the interferer being in the "On" state at time t and 0 otherwise. $z(t)$ and $y(t)$ are the off and on counters of the current renewal cycle of the interferer corresponding to this particular state. Let $p(t)$ be the probability associated with this particular state, i.e, $p(t) = \mathbb{P}(S(t)|\underline{\gamma}(t))$. $\psi(t)$ is the probability of the state at the last change point $\tau(t)$. Let $\mathcal{S}(t) = \{S_1(t), S_2(t), \dots\}$ be the set of all possible states at time t and let $L_S(t)$ be the cardinality of $\mathcal{S}$.

For the block fading model we have assumed, the fading gains within a block are constant and change only at boundaries (kM). Thus within a single fading block, the CQI values are constant unless and until the interference changes. This clearly shows that the transition of the interferer from "ON" to "OFF", which also constitutes a renewal, within the fading block translates to an increase in the CQI value. Similarly the transition from "OFF" to "ON" has the opposite effect on the CQI value. However, at fading boundaries, we are likely to observe a change in CQI value which could be due to a change in the fading gain, a change in interference or a combination of both. Thus the boundaries of the fading blocks present an uncertainty in the renewal structure under consideration and thus has to be accounted for appropriately in the outage computation.

We will describe our algorithm in four parts. For ease of analysis, we will first start with a system which has only one user. For this user, we will describe the algorithm for CQI data observed within a fading block. We will then highlight the steps to be taken for observed CQI values that lie on the fade boundaries. We will then describe how to extend the proposed algorithm to apply to the multiple user scenario. Finally we will mention how to utilize these probable states to compute the conditional outage probability.

5.3.1 Within a Fading Block

Assume that at some time t we have the set $\mathcal{S}(t) = \{S_1(t), S_2(t), \dots, S_{L_{S(t)}}(t)\}$ as the set of probable states of the system having observed CQI information up to t, i.e, $\bar{\gamma}(t+\delta)$. For ease of use, lets define $\underline{\gamma}(t) = \bar{\gamma}(t+\delta)$. We will first consider the evolution of this set of states upon observation of CQI value for time slot $t+1$ when it lies within a fading block, i.e, $kM+2 \leq t+1 \leq (k+1)M$, for some $k \in \mathbb{Z}^+$. The state transitions for observations at such time slots are governed by the following set of rules.

Algorithm 3 State Transitions within a Fading Block

```
if γ(t+1) > γ(t) then
    S_1(t+1) ← (0, 1, 0, 1, t+1, 1)
    𝒮(t+1) ← {S_1(t+1)}
    τ(t+1) ← t+1
else if γ(t+1) = γ(t) then
    for 1 ≤ i ≤ L_S(t) do
        ω_i(t+1) ← ω_i(t)
        z_i(t+1) ← z_i(t) + 𝕀_{ω_i(t)=0}
        y_i(t+1) ← y_i(t) + 𝕀_{ω_i(t)=1}
        p_i(t+1) ← q_i / Σ_{k=1}^{L_S(t+1)} q_k
        ψ_i(t+1) ← ψ_i(t)
    end for
    𝒮(t+1) ← {S_1(t+1), S_2(t+1), ..., S_{L_S(t)}(t+1)}
else
    for 1 ≤ i ≤ L_S(t),  mod (i,2) = 1 do
        ω_i(t+1) ← 1
        z_i(t+1) ← z_i(t)
        y_i(t+1) ← 1
        p_i(t+1) ← p_i(t) / Σ_{k:k is odd, k≤L_S(t)} p_k(t)
        ψ_i(t+1) ← p_i(t+1)
    end for
    𝒮(t+1) ← {S_i(t+1) : i is odd}
    τ(t+1) ← t+1
end if
```

where $q_i = \psi_i(t+1)\Pi_{j=\tau(t)}^{t}\{\mathbb{P}(\omega_i(j+1)|z_i(j), y_i(j))\}$

In essence, for any time slot that is not at the boundary of a fading block, we can update the state space as mentioned above. As was described earlier, when we observe a renewal or equivalently an increase in the SINR, we refresh the state space. When we observe a new value of the SINR that equals the previous value, the possible states are maintained and the probabilities of each state are updated as stated above. In the event of a transition from Off to On, we remove the states that predict the opposite in this block and update the probabilities of each branch appropriately. The reason for maintaining the change point information and the correctness of the update expressions are elucidated in the following lemma.

Lemma 4. *For all time slots within a fading block, i.e,* $\forall kM \leq t < (k+1)M - 1$*, for some* $k \in \mathbb{Z}^+$*, the state probabilities are governed by the following dynamics :*
if $\gamma(t+1) > \gamma(t)$*,*

$$p_1(t+1) = 1 \tag{5.19}$$

if $\gamma(t+1) = \gamma(t)$*,* $\forall 1 \leq i \leq L_S(t)$

$$p_i(t+1) = \frac{q_i}{\sum_{k=1}^{L_S(t+1)} q_k} \tag{5.20}$$

where,

$$q_i = \psi_i(t)\Pi_{j=\tau(t)}^{t}\mathbb{P}(\omega_i(j+1)|z_i(j), y_i(j)) \tag{5.21}$$

if $\gamma(t+1) < \gamma(t)$*,* $\forall 1 \leq i \leq L_S(t)$

$$p_i(t+1) = \frac{p_i(t)}{\sum_{\{k:k \textit{ is odd}, k \leq L_S(t)\}} p_k(t)}, \textit{ if } i \textit{ is odd} \tag{5.22}$$

Proof. For the first case, we know that $\gamma(t+1) > \gamma(t)$ indicates a renewal at the interferer and the state space is refreshed. Thus trivially, we know that $p_1(t+1) = 1$ and is the only possible state in the system.

For the second case, we have $\gamma(t+1) = \gamma(t)$. As defined earlier, $p_i(t+1) = \mathbb{P}(S_i(t+1)|\underline{\gamma}(t+1))$. We can observe that

$$\mathbb{P}(S_i(t+1)|\underline{\gamma}(t+1)) = \mathbb{P}(S_i(t)|\underline{\gamma}(t), \gamma(t+1) = \gamma(t)) \tag{5.23}$$

Thus by induction on t, we can say that

$$\begin{aligned} p_i(t) &= \mathbb{P}(S_i(\tau(t))|\underline{\gamma}(\tau(t)), \{\gamma(j) = \gamma(j-1), j > \tau(t)\}) \\ &= \frac{q_i}{\sum_{k=1}^{L_S(t+1)} q_k} \end{aligned} \tag{5.24}$$

where (5.24) follows by Bayes' theorem, with

$$q_i = \psi_i(t)\Pi_{j=\tau(t)}^{t}\mathbb{P}(\omega_i(j+1) = \omega_i(\tau(t))|z_i(j), y_i(j))$$

Here, $\mathbb{P}(\omega_i(j+1)|z_i(j), y_i(j))$ is computed as was done in the static network case using expressions (5.3) and (5.4).

The final case marks a transition from Off to On state at the interferer. Now, for all even i, we know that $\omega_i(t) = 1$. Thus those states can be removed from the system. For all odd i, we have

$$\begin{aligned} p_i(t+1) &= \mathbb{P}(S_i(t+1)|\underline{\gamma}(t), \gamma(t+1) < \gamma(t)) \\ &= \mathbb{P}(S_i(t+1)|\underline{\gamma}(\tau_i(t)), \{\omega_i(j) = 0, j \geq \tau_i(t)\}) \\ &= \mathbb{P}(S_i(\tau_i(t))|\underline{\gamma}(\tau_i(t)), \{\omega_i(j) = 0, j \geq \tau_i(t)\}) \\ &= \frac{p_i(t)}{\sum_{\{k: k \text{ is odd }, k \leq L_S(t)\}} p_k(t)} \end{aligned} \tag{5.25}$$

This concludes that the set of transition probabilities defined above are indeed the valid state probabilities. □

Thus in this section, we have seen how to keep track of the set of probable states for observations belonging to time slots within the fading block. We will now move to devising the governing dynamics for the fading block boundaries.

5.3.2 At the Fading Boundary

Yet again, assume that at some time t, we have the set of probable states $\mathcal{S}(t)$. However, in this subsection, we will deal with fading block boundaries, i.e, $\forall t = kM$, for some $k \in \mathbb{Z}^+$. At the block boundary, we know that the fading gain changes. Hence it is quite likely that the CQI also change. However, the CQI value, whether different or the same, is still a function of not just the fading gains, but also the interference process. Thus we have to account for a probable transition at the boundary. This implies that from each probable state, we get 2 new probable states - one that maintains its transmission state and one that undergoes a flip. This in turn doubles the number of states under consideration and the following algorithm describes the set of rules that govern this change point.

Algorithm 4 State Transitions at a Fading Boundary

for $1 \leq i \leq L_S(t)$ **do**

$\omega_{2i-1}(t+1) \leftarrow 0$

$\omega_{2i}(t+1) \leftarrow 1$

$z_{2i-1}(t+1) \leftarrow (z_i(t))\mathbb{I}_{\{\omega_i(t)=0\}} + 1$

$z_{2i}(t+1) \leftarrow z_i(t)$

$y_{2i-1}(t+1) \leftarrow 0$

$y_{2i}(t+1) \leftarrow (y_i(t))\mathbb{I}_{\{\omega_i(t)=1\}} + 1$

$p_{2i-1}(t+1) \leftarrow \frac{f(\gamma|0)\mathbb{P}(S_i(t),0|\underline{\gamma}(t))}{\sum_{P' \in \{0,P\}} f(\gamma|P')(\sum_{k=1}^{L_S(t)} \mathbb{P}(S_k(t),P'|\underline{\gamma}(t)))}$

$p_{2i}(t+1) \leftarrow \frac{f(\gamma|P)\mathbb{P}(S_i(t),P|\underline{\gamma}(t))}{\sum_{P' \in \{0,P\}} f(\gamma|P')(\sum_{k=1}^{L_S(t)} \mathbb{P}(S_k(t),P'|\underline{\gamma}(t)))}$

$\psi_{2i-1}(t+1) \leftarrow p_{2i-1}(t+1)$

$\psi_{2i}(t+1) \leftarrow p_{2i}(t+1)$

end for

$\mathcal{S}(t+1) \leftarrow \{S_1(t+1), S_2(t+1), \ldots, S_{2L_S(t)}(t+1)\}$

$\tau(t+1) \leftarrow t+1$

where $f(\gamma|j)$ is computed as was given in (5.12) and $\mathbb{P}(S_i(t), P'|\underline{\gamma}(t)) = \mathbb{P}(S_i(t), P(t+1) = P'|\underline{\gamma}(t)), P' \in \{0, P\}$

In essence, at the boundary of every fading block, there arises an uncertainty with respect to the possibility of a change occurring at the boundary. Thus, we account for this in the form of a doubling of the number of states under consideration as described above. Given this idea of evolution, the dynamics of ω, z, y and ψ are clear from the above mentioned algorithm. The expressions for the state probabilities are derived in the following lemma.

Lemma 5. *The state probabilities at state boundaries, $t = kM$ for some $k \in \mathbb{Z}^+$, are governed by the following expressions :*

$$
\begin{aligned}
p_{2i-1}(t+1) &\leftarrow \frac{f(\gamma|0)\mathbb{P}(S_i(t), 0|\underline{\gamma}(t))}{\sum_{j\in\{0,P\}} f(\gamma|j)(\sum_{k=1}^{L_S(t)} \mathbb{P}(S_k(t), j|\underline{\gamma}(t)))} \\
p_{2i}(t+1) &\leftarrow \frac{f(\gamma|P)\mathbb{P}(S_i(t), P|\underline{\gamma}(t))}{\sum_{j\in\{0,P\}} f(\gamma|j)(\sum_{k=1}^{L_S(t)} \mathbb{P}(S_k(t), j|\underline{\gamma}(t)))}
\end{aligned}
$$

Proof. Starting from the definition, we can observe that

$$
\begin{aligned}
p_{2i-1}(t+1) &= \mathbb{P}(S_{2i-1}(t+1)|\underline{\gamma}(t+1)) \\
&= \mathbb{P}(S_i(t), P(t+1) = 0|\underline{\gamma}(t+1)) \\
&= \frac{f(\gamma|0)\mathbb{P}(S_i(t), 0|\underline{\gamma}(t))}{\sum_{j\in\{0,P\}} f(\gamma|j)(\sum_{k=1}^{L_S(t)} \mathbb{P}(S_k(t), j|\underline{\gamma}(t)))}
\end{aligned}
$$

where, last step follows from Bayes' theorem and the fact that $\gamma(t+1)$ is conditionally independent of $\{\underline{\gamma}(t), S_i(t)\}$ given $P(t+1)$, since we are at a fading boundary, i.e, $t+1 = kM$. Further,

$$
\begin{aligned}
\mathbb{P}(S_i(t), j|\underline{\gamma}(t)) &= \mathbb{P}(S_i(t), P(t+1) = j|\underline{\gamma}(t)), j \in \{0, P\} \\
&= \mathbb{P}(S_i(t)|\underline{\gamma}(t))\mathbb{P}(P(t+1) = j|S_i(t)) \qquad (5.26) \\
&= p_i(t)\mathbb{P}(j|z_i(t), y_i(t)) \qquad (5.27)
\end{aligned}
$$

and again $\mathbb{P}(j|z_i(t), y_i(t))$ is calculated as in (5.3) and (5.4). Here, (5.26) follows from the fact that $P(t+1)$ is conditionally independent of $\underline{\gamma}(t)$ given $S_i(t)$.

Following a similar procedure, we can obtain the expression for $p_{2i}(t+1)$ as well.

□

There are essentially 4 steps that the algorithm does with regard to the states. This can be pictorially seen in Figures (5.3), (5.4), (5.5) and (5.6). The figures represent the change in the states upon occurrence of a renewal, same valued CQI, OFF→ON transition and fade boundary respectively.

We have thus built an algorithm, which upon observing CQI data up to time t provides a set of probable states of the system with the probabilities of each state. We will

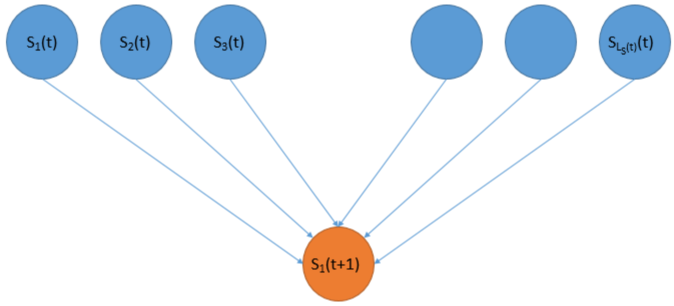


Figure 5.3: States collapsing on renewal

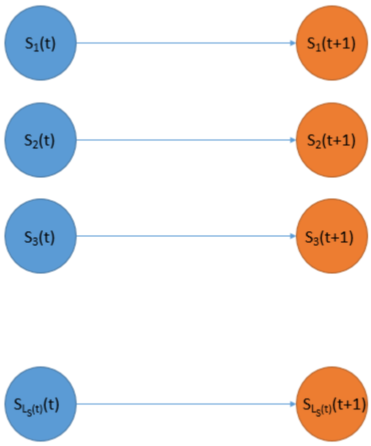


Figure 5.4: States shuffling upon no change in CQI

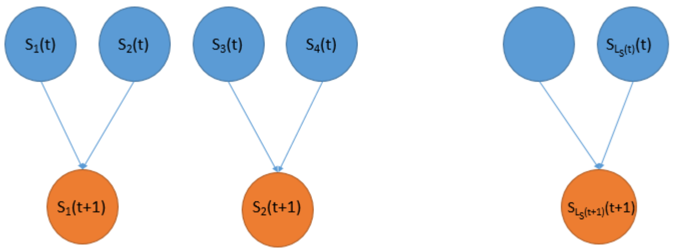


Figure 5.5: States correcting upon $0 \to 1$ transition

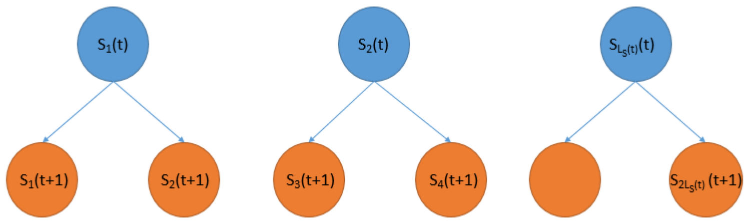


Figure 5.6: States expanding at fade boundaries

now make use of this information to compute the conditional outage probability.

5.3.3 Multi-user scenario

In the last 2 sections we saw the method to keep track of the set of probable states when there is only one user in the system. Now, we will show how to extend the proposed algorithm to extend to the case with multiple users in the system. As assumed initially, lets assume there are K users with appropriate delays. Say at time $t + \delta_{min}$, where $\delta_{min} = \min_{1 \le i \le K} \delta_i$, we receive the set of SINRs $\{\gamma_1(t_1), \gamma_2(t_2), \ldots, \gamma_K(t_K)\}$, t_i being the times as appropriated by the delays of the users. Notice that the set of states currently being updated is that corresponding to time $t+1$. Lets assume that we have $\mathcal{S}(t)$.

Without loss of generality, we will assume that the $\delta_i \le \delta_j,\ \forall 1 \le i \le j \le K$. Also, let $\delta_i = \delta_{min},\ \forall i \le m_1$ for some $m_1 \le K$. Then, $t_i = t+1,\ \forall i \le m_1$. We will first assume that we receive just these m_1 values and show how the update expressions are modified. Note that the observations of renewal, Off→On transition, fade boundaries and no change in SINR will be consistent across all users almost surely. It is trivially evident that if these users observe a renewal within a fading block, then the update expressions remain the same as was described earlier. It can further be proved that, within a fading block, multiple observations for slot $t+1$ do not change the probabilities of the states. This is shown in the following lemma. The following lemma describes the set of actions to be adopted based on the observations.

Lemma 6. *For a system with set of probable states $\mathcal{S}(t)$, given a set of observations $\{\gamma_1(t+1), \gamma_2(t+1), \ldots, \gamma_{m_1}(t+1)\}$, the state probabilities are updated as governed by the following set of equations. If $kM + 2 \le t \le (k+1)M$, then the update expressions given in (5.20) and (5.22) hold true for the case of multiple simultaneous observations as well.*

Proof. We now have $p_i(t+1) = \mathbb{P}(S_i(t+1)|\{\gamma_1(t+1), \ldots \gamma_{m_1}(t+1)\})$. Say $\gamma_i(t+1) = \gamma_i(t),\ \forall i \le m_1$. Then, we have

$$\begin{aligned} p_i(t+1) &= \mathbb{P}(S_i(t+1)|\{\gamma_1(t+1), \ldots \gamma_{m_1}(t+1)\}) \\ &= \mathbb{P}(S_i(\tau(t))|\{\underline{\gamma_i}(\tau(t)), \{\gamma_i(j) = \gamma, \tau(t) \le j \le t\}, \forall i \le m_1\}) \\ &= \frac{q_i}{\sum_{k=1}^{L_S(t)} q_k} \end{aligned} \tag{5.28}$$

where (5.28) follows from an application of Bayes' theorem with

$$q_i = \psi_i(t)\Pi_{j=\tau(t)}^{t}\mathbb{P}(\omega_i(j+1)|z_i(j), y_i(j))$$

as was described in Lemma 4. Thus we can see that the update probabilities of states are not affected by multiple simultaneous observations in this case.

Using a similar argument as was presented in Lemma 4, we can also show that the update criterion for the case of multiple simultaneous observations of the Off→On observation also remains unchanged. □

Having studied the case of in block observations, we shall look at the fade boundaries now. Say we have such multiple simultaneous observations of a fade boundary. We know that at a fade boundary we still have the set of states expanding by a factor of 2. Owing to the independence of fading gains across users, we can easily see that the update expressions are now modified as follows.

$$\begin{aligned} p_{2i-1}(t+1) &= \frac{f_0\mathbb{P}(S_i(t), 0|\underline{\gamma}(t))}{\sum_{j\in\{0,P\}} f_j(\sum_{k=1}^{L_S(t)}\mathbb{P}(S_k(t), j|\underline{\gamma}(t)))} \\ p_{2i}(t+1) &= \frac{f_P\mathbb{P}(S_i(t), 0|\underline{\gamma}(t))}{\sum_{j\in\{0,P\}} f_j(\sum_{k=1}^{L_S(t)}\mathbb{P}(S_k(t), j|\underline{\gamma}(t)))} \end{aligned}$$

where, $f_j = \Pi_{i=1}^{m_1} f(\gamma_i(t+1)|P(t+1) = j), \ \forall j \in \{0, P\}$. The terms in the product can yet again be computed using (5.12)

Thus, in essence we know how to update the state space for multiple simultaneous observations. We will now expand the domain to the more generic case of arrivals with differing delays. Lets split the set of users, ordered according to delays as assumed earlier, into classes such that all users of a specified class have the same information delay. Let these classes be $D_1, D_2, \ldots, D_m$, where $\delta_i < \delta_j, \ \forall i \in D_{k_1}, j \in D_{k_2}, k_1 < k_2$. Let the size of class D_i be m_i. Thus the class D_1 corresponds to the users with minimal delay. We will use the update expressions highlighted above to first compute the state probabilities.

Let us assume that we have exactly one other class of arrivals for convenience. First of all, we will show that only observations of the 2^{nd} class that correspond to fade boundaries alter the state probabilities. Let the observation slot of class D_i be t_i. Thus at each step, we update the state for time t_1.

Lemma 7. *The state probabilities are conditionally independent of all observations of class D_2 that do not correspond to fade boundaries given the observations of class D_1.*

Proof. Let $kM + 2 \leq t \leq (k+1)M$ for some k. Starting from the definition of the state probabilities, we have,

$$\begin{aligned} p_i(t_1) &= \mathbb{P}(S_i(t_1)|\underline{\gamma_1}(t_1), \underline{\gamma_2}(t_2)) \\ &= \frac{\mathbb{P}(S_i(t_1), \gamma_2(t_2)|\underline{\gamma_1}(t_1), \underline{\gamma_2}(t_2-1))}{\mathbb{P}(\gamma_2(t_2)|\underline{\gamma_1}(t_1), \underline{\gamma_1}(t_2-1))} \qquad (5.29) \\ &= \mathbb{P}(S_i(t_1)|\underline{\gamma_1}(t_1), \underline{\gamma_2}(t_2-1))\mathbb{P}(\gamma_2(t_2)|\underline{\gamma_1}(t_1), \underline{\gamma_2}(t_2-1), S_i(t_1)) \qquad (5.30) \\ &= \mathbb{P}(S_i(t_1)|\underline{\gamma_1}(t_1), \underline{\gamma_2}(t_2-1)) \end{aligned}$$

where (5.29) follows from the fact that given $\underline{\gamma_1}(t_1)$, we know $\underline{\gamma_1}(t_2)$. Within a fading block, given $\underline{\gamma_1}(t_2)$, we can deterministically compute $\gamma_2(t_2)$ and thus the conditional probability is 1. The same reason further results in (5.30). Thus we have shown that the observations of class D_2 of time slots within a fading block do not affect the state probabilities. □

This result can of course be extended to the case of multiple delay classes too. Thus the classes other than the minimum delay class affect the state probabilities only at the fade boundaries. Further, lets assume that t_1 belongs to the $(k+1)^{th}$ fading block for some $k \in \mathbb{Z}^+$. Now, if $\tau(t+1) > kM + 1$, then we know that the process has observed either a renewal or an Off→On transition. Either way, it is evident that the state of the interferer for the interval $[kM + 1, \tau(t_1)]$ is known. Thus an observation made in this interval by another class, say D_2 does not contribute to the probabilities of the states. Hence, it is clear that the observations of other classes are significant only at fade boundaries that are the most recent change points. We will now show how the state probabilities are affected by class D_2 observations that lie on a fade boundary that are current change points.

Lemma 8. *Let $kM+1 < t_1 \leq (k+1)M$ and $t_2 = \tau(t_1) = kM+1$. If the initial stage of the algorithm gives the states probabilities $p_i(t_1)$, then the update corresponding to* *the class D_2 observations can be made as*

$$p_i(t_1) \leftarrow \frac{f_0(1+\eta)p_i(t_1)}{f_0+\eta f_P}, \textit{if i is odd} \tag{5.31}$$

$$p_i(t_1) \leftarrow \frac{f_P(1+\eta)p_i(t_1)}{f_0+\eta f_P}, \textit{if i is even} \tag{5.32}$$

where, as was used earlier, $f(\gamma_i(t_2)|P') = f(\gamma_i(t_2)|P_{Int}(t_2) = P'),\ \forall P' \in \{0, P\}$. *Similarly,* $f_0 = \Pi_{k\in D_2} f(\gamma_k(t_2)|0)$ *and* $f_P = \Pi_{k\in D_2} f(\gamma_k(t_2)|P)$ *The factor,* η*, is given by*

$$\eta = \frac{\sum_{\{i:i \textit{ is even, } i\leq L_S(t_1)\}} p_i(t_1)}{\sum_{\{i:i \textit{ is odd, } i\leq L_S(t_1)\}} p_i(t_1)} \tag{5.33}$$

Further, the change point probabilities are updated as

$$\psi_i \leftarrow \frac{f_0\psi_i}{\sum_{k \textit{ odd}} f_0\psi_k + \sum_{k \textit{ even}} f_P\psi_k}, \textit{if i is odd}$$

$$\psi_i \leftarrow \frac{f_P\psi_i}{\sum_{k \textit{ odd}} f_0\psi_k + \sum_{k \textit{ even}} f_P\psi_k}, \textit{if i is even}$$

Proof. We will first start with the state probabilities. We know that the expressions for the state probabilities are governed by (5.28). From the definition of state probability, we get the following. Letγ_1 and γ_2 be SINRs representative of the entire class of arrivals. We know that in the case of multiple simultaneous observations, the joint pdfs just scale as the product of the individual observations. Thus the translation is fairly trivial.

$$\begin{aligned} p_i(t_1) &= \mathbb{P}(S_i(t_1)|\underline{\gamma_1}(t_1), \underline{\gamma_2}(t_2)) \\ &= \mathbb{P}(S_i(t_2)|\underline{\gamma_1}(t_2), \underline{\gamma_2}(t_2), \{\gamma_1(j) = \gamma_1(t_2), \forall t_2 \leq j \leq t_1\}) \\ &= \frac{\psi_i(t_1)(\Pi_{j=\tau(t)}^{t}\mathbb{P}(\omega_i(j+1)|z_i(j), y_i(j)))f(\gamma_2(t_2)|S_i(t_2))}{\sum_{k=1}^{L_S(t)}\psi_k(t_1)(\Pi_{k=\tau(t)}^{t}\mathbb{P}(\omega_k(j+1)|z_k(j), y_k(j)))f(\gamma_2(t_2)|S_k(t_2))} \end{aligned}$$

The above shown derivations follow simply from Bayes' theorem and the independence of fading across users. Further, we make use of the fact that given γ_1, γ_2 can be deterministically obtained. For the case of multiple class 2 observations, we just take a product of the conditional pdfs in the expression. Using simple algebra, we can derive the update expressions mentioned in the lemma.

For the update of the change point probability, the proof proceeds on similar lines and utilizes conditional independence and Bayes' rule to result in the expression stated in the lemma. Due to the similarity of the proof, it is omitted here. □

Having obtained these rules for the case with 2 classes of observations, it is easy to extend to the case of multiple classes. We just have to sequentially update the state probabilities using the rules mentioned above. The sequential flow of the algorithm is represented in Figure 5.7 for a system with 3 delay classes. As is shown, the algorithm is first performed on the class of minimum delay users and the probabilities are updated at fade boundaries based on the observations of the other classes. Having described the

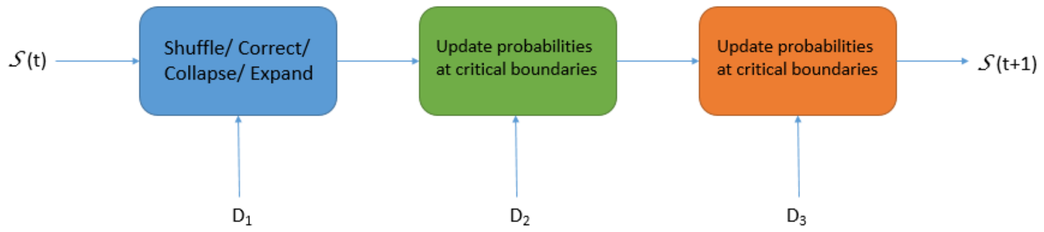


Figure 5.7: Flow of outage computation algorithm for a system with 3 delay classes

way to take into account the observations of all users, we will now elaborate the method to compute the outage probability for each user.

5.3.4 Computing Conditional Outage Probability

We shall now describe the method to obtain the outage probability. The outage probability is computed on a user by user basis and thus for convenience, we will drop the sub scripts and work with a single user system. Consider the k^{th} fading block, i.e, $(k-1)M < t \leq kM$, wherein the channel fades are some constants $g_0(k)$ and $g_1(k)$. If the user observes 2 different values of SINR in the block, then we can trivially compute the channel gains corresponding to the block. However, if we have observed just one value of SINR, γ, then we could be in one of two possible scenarios. If the interferer is in the Off state, then we know that $g_0(k) = \frac{\gamma N_0}{P}$. On the other hand, if the interferer is in the On state, then, we only know that $g_0(k)P = \gamma(N_0 + g_1 P)$. Since we have a system with a delay, we could also be in a state wherein we have observed no value of the SINR corresponding to the current block. Note that the states used are the ones corresponding to the minimum delay as they are the most recent and thus the most accurate estimates.

The following is quite evident owing to law of total probability

$$\mathbb{P}_{out}(r) = \sum_{i=1}^{L_S(t-\delta_{min})} \mathbb{P}(C(t) < r|S_i(t-\delta_{min}), \bar{\gamma}(t))p_i(t-\delta_{min}) \tag{5.34}$$

Here we abuse notation slightly for convenience when we condition on the state $S_i(t-\delta)$. We will assume that such a conditioning implies that the path of the interference process followed that represented by the state S_i. Any subsequent use of such conditioning also implies the same. In the event of knowing both fading gains of the channel, the problem simplifies to the case of the static network outage and $\mathbb{P}(C(t) < r|S_i(t-\delta))$ can be computed exactly as is given in (5.5). Further, we know that

$$F_C(r|S_i, \bar{\gamma}(t)) = \sum_{P' \in \{0,P\}} F_{CP}(r|P', S_i, \bar{\gamma}(t))\mathbb{P}(P_{Int}(t) = P'|S_i) \tag{5.35}$$

where $F_C(r|S_i, \bar{\gamma}(t)) = \mathbb{P}(C(t) < r|S_i(t-\delta), \bar{\gamma}(t))$, $F_{CP}(r|P', S_i, \bar{\gamma}(t)) = \mathbb{P}(C(t) < r|P_{Int}(t) = P', S_i(t-\delta), \bar{\gamma}(t))$ and $\mathbb{P}(P_{Int}(t) = P'|S_i) = \mathbb{P}(P_{Int}(t) = P'|S_i(t-\delta))$.

If we observe only one value, γ, of SINR corresponding to the block, i.e, $kM < t-\delta < t \leq (k+1)M$, then the following lemma provides the corresponding outage probability.

Lemma 9. *If $(k-1)M < t-\delta < t \leq kM$ and the base station has exactly one SINR value γ corresponding to this k^{th} fading block, then the conditional outage probabilities are given by*

$$F_{CP}(r|0, S_i, \bar{\gamma}(t)) = \begin{cases} 1 - e^{-\frac{1}{\alpha_1}\left(\frac{g(r)}{\gamma}-1\right)} & \text{, if } \omega_i(t-\delta) = 1,\ g(r) > \gamma \\ 0 & \text{, o/w} \end{cases} \tag{5.36}$$

and

$$F_{CP}(r|P, S_i, \bar{\gamma}(t)) = \begin{cases} e^{-\frac{1}{\alpha_1}\left(\frac{\gamma}{g(r)}-1\right)} & \text{, if } \omega_i(t-\delta) = 0 \\ 0 & \text{, o/w} \end{cases} \tag{5.37}$$

where, $g(r) = 2^r - 1$.

Proof. If $\omega_i(t-\delta) = 0$, then we know that $g_0 = \frac{\gamma N_0}{P}$. Then, if $P_{Int}(t) = 0$, $\gamma(t) = \gamma,\ w.p.1$. Thus there is no outage if $P_{Int}(t) = 0$ and $\omega_i(t-\delta) = 0$. For the other case, we have

$$\begin{aligned} F_{CP}(r|P, S_i, \bar{\gamma}(t)) &= \mathbb{P}(C(t) < r | P(t) = P, S_i(t-\delta), \bar{\gamma}(t)) \\ &= \mathbb{P}\Big(g_1 > (\frac{\gamma}{g(r)} - 1)\frac{N_0}{P}\Big) \\ &= e^{-\frac{1}{\alpha_1}\left(\frac{\gamma}{g(r)}-1\right)} \end{aligned} \tag{5.38}$$

Now, if $\omega_i(t-\delta) = 1$, then we know that $g_0 P = \gamma(N_0 + g_1 P)$. Thus if $P_{Int}(t) = 1$, $\gamma(t) = \gamma,\ w.p.1$. Thus the conditional outage probability in this case is 0. On the other hand, we have

$$\begin{aligned} F_{CP}(r|0, S_i, \bar{\gamma}(t)) &= \mathbb{P}(C(t) < r | P_{Int}(t) = 0, \bar{\gamma}(t)) \\ &= \mathbb{P}\Big(\gamma\big(1 + \frac{g_1 P}{N_0}\big) < g(r)\Big) \\ &= \begin{cases} 1 - e^{-\frac{1}{\alpha_1}\left(\frac{g(r)}{\gamma}-1\right)} & \text{, if } g(r) > \gamma \\ 0 & \text{, o/w} \end{cases} \end{aligned}$$

This gives us the conditional outage probabilities. □

When the base station does not have any information regarding the SINR values of the current fading block, i.e, $t-\delta \le (k-1)M < t \le kM$, the scenario simplifies to the case discussed with the memoryless channel environment earlier. Thus the conditional outage probability can be computed using (5.12).

Having obtained the conditional outage probabilities, we can substitute back in the law of total probability expression to compute the outage probability for the system.

5.3.5 Memory Overhead for Computation

An issue with the algorithm described above is probably the number of states that one needs to keep track of. This number increases exponentially with the number of blocks for which the change points of the renewal process occur only at the fade boundaries.

This adds to the storage overhead, apart from storing the information about the SINR. Thus we will show that under slow fading, the probability that the system will need to use a large amount of memory decays exponentially, thereby validating computational efficiency of the proposed scheme.

Theorem 2. *Under the assumptions of slow fading and interferer load assumed in Chapter 2, the probability that renewals of the ON/OFF process occur only at the fading boundaries for at least N blocks decays exponentially.*

Proof. Let us assume, without loss of generality that we have a renewal at some time t and

$$\Delta t = \min_{\{k:kM \geq t\}} (kM - t)$$

Let $\mathbb{P}(\Delta t, N) = \mathbb{P}$(for at least N fading blocks starting Δt away, renewals, if any, occur only at fade boundaries). Thus we can see that

$$\begin{aligned} \mathbb{P}(\Delta t, N) &= \mathbb{P}_X(\Delta t)\mathbb{P}(0, N) + \mathbb{P}_X(\Delta t + M)\mathbb{P}(0, N-1) + \dots \\ &\quad + \mathbb{P}_X(\Delta t + (N-1)M)\mathbb{P}(0, 1) + \mathbb{P}(X \geq \Delta t + NM) \end{aligned} \tag{5.39}$$

Writing out a similar recurrence relation, we get

$$\begin{aligned} \mathbb{P}(0, N) &= \mathbb{P}_X(M)\mathbb{P}(0, N-1) + \mathbb{P}_X(2M)\mathbb{P}(0, N-2) + \dots \\ &\quad + \mathbb{P}_X((N-1)M)\mathbb{P}(0, 1) + \mathbb{P}(X \geq NM) \end{aligned} \tag{5.40}$$

Owing to the assumptions made on slow fading and light tailed renewal periods, we know that $\exists p, \alpha \leq 1$, such that $\mathbb{P}_X(iM) \leq \alpha^{i-1}p$. Further, from the Chernoff bound of X, we know that $\exists s > 0$, such that $\mathbb{P}(X \geq x) \leq \mathbb{E}[e^{sX}]e^{-sx} = \beta e^{-sx}$. Thus using these in (5.40), we get

$$\begin{aligned} \mathbb{P}(0, N) &\leq p\mathbb{P}(0, N-1) + \alpha p\mathbb{P}(0, N-2) + \\ &\quad \cdots + \alpha^{N-2}\mathbb{P}(0, 1) + \beta\theta^N \end{aligned} \tag{5.41}$$

where $\theta = e^{-sM} < 1$. Thus, recursively solving the above inequality, we get

$$\mathbb{P}(0, N) \leq a_1\phi^N + a_2\theta^N \tag{5.42}$$

where $\phi = \alpha + p$, $a_1 = \frac{p\beta\theta}{(\phi-\theta)\phi}$ and $a_2 = \beta\big(1 - \frac{p}{(\phi-\theta)}\big)$. Under slow fading conditions, we can assume that M is such that $\phi < 1$.

Further, we know that, for any t, $\exists q \leq 1$ such that, $\mathbb{P}_X(t + iM) \leq \alpha^i q$. Using this and (5.42) in (5.39), we get

$$\mathbb{P}(\Delta t, N) \leq a_\alpha\alpha^N + a_\phi\phi^N + a_\theta\theta^N \tag{5.43}$$

where $a_\alpha = \hat{a}_1 + \hat{a}_2$, $a_\phi = -\hat{a}_1$ and $a_\theta = \hat{\beta} - \hat{a}_2$. Here, $\hat{a}_1 = \frac{qa_1\phi}{(\alpha-\phi)}$, $\hat{a}_2 = \frac{qa_2\theta}{(\alpha-\theta)}$ and $\hat{\beta} = \beta e^{-s\Delta t}$.

Thus, the above inequality shows that the probability of having renewals only at fading boundaries for at least N blocks decays exponentially. □

Now, the number of states in the system doubles at every fading boundary and decreases only upon observation of either a renewal or an Off to On transition. Thus the event that I will have at least 2^N states in the system, is equivalent to having at least N fading blocks which have transitions only at the boundaries. Further we know that the probability of such an event is upper bounded by the event of having renewals only at fading boundaries for at least N blocks. We need $N_b = O(N)$ bits to store 2^N states. We have essentially proven here that $\mathbb{P}(N_b \geq \beta N) \leq \mathbb{P}(\Delta t, N)$, where $\beta < \infty$ is the number of bits needed to store the information for one state. Thus, an $O(N)$ bit state memory overflows with exponentially low probability. Hence we can conclude that the proposed algorithm will work with an arbitrarily low error probability while using $O(N)$ bits of memory.

In the scenario of a memory overflow in the storage of the states, the most ideal action to take is to get rid of states that have the least probability. For instance if I have 2^{N-1} states in the system with limited storage, then, while computing the next set of 2^N states, the half the number of states are discarded, the least probable states being discarded first.

5.3.6 Discussions

The above theorem also signifies the importance of slow fading to the efficient functioning of the proposed algorithm. As fading becomes faster, the space and consequently the time complexity of the algorithm also worsens.

In fact, the state expansion at the fade boundaries can be decreased from doubling in size. At every fade boundary, as of now, we have assumed that each state splits into two possible cases. However, all state transitions of $1 \to 0$ correspond to that of a renewal. Hence, all states that were in state 1 can in fact coalesce into one common 0 state and combine the appropriate probabilities. That is, we can represent $\{S_{2i-1}(t+1) : \omega_i(t) = 1\}$ when $t = kM$, for some $k \in \mathbb{Z}^+$, by one common state $S_0(t+1)$ whose probability is the sum of probabilities of all coalesced states. Using this, we can prove that the set of states evolves as $O(k^2)$ over k fading blocks. This in turn reduces the number of bits for storage. However, for analytical ease, we shall avoid its usage.

The task of outage probability computation, as we have seen above, hinges strongly on the occurrences of renewals and tracking the same. Though the task of detecting change points in the Interference process by observing a sequence of SINR measurements does seem to point to the use of "Sequential Change Point Detection" (Wald *et al.* (1945), Lorden *et al.* (1971)), we cannot use the theory to good effect in this scenario. The primary downside of this idea is that the error probability of a change point detection is only bounded and no exact expression is known. Also, we may not have enough samples to detect a change thereby raising the possibility of wrong hypothesis estimates that can percolate through the process to increase the error.

The scenario of having more than one interferer causes an exponential increase in the set of probable states thereby increasing the space and computational complexity of the algorithms. However, the proposed algorithm can be used for the multiple interferer case if the assumption of "Single strong interferer" is allowed, i.e, each user faces significant interference from one interferer while the rest of the interference is regarded as noise. In this case, we will group users, not just based on the delay profile but also based on the interferer. For a class of users having the same strong interferer, the algorithm can be applied.

This section thus provides the method to compute the conditional outage probability that is critical to the scheduling algorithm. We will now describe some insights into the proportional fair scheme and some possible areas of future work.

CHAPTER 6

$\alpha-$fair Scheduling

In this chapter, we will discuss some on going work in the realm of $\alpha-$fair scheduling in the presence of time-varying interference and information delay. As was discussed earlier, the objective in this scheduling routine is to maximize a specific non-decreasing, concave utility function. Thus the objective is to show that the scheduling decisions that are made, converge at an asymptotically stable equilibrium point that maximizes the utility function. For convenience, in this section we will assume that the discrete time slots are represented by n.

The scheduling is hinged primarily on the average throughput, $\theta(n)$, offered to users and the utility function $U(.)$. The average throughput is given by

$$\theta_i(n) = \frac{1}{n}\sum_{\tau=1}^{n} r_i(\tau)\mathbb{I}_{r_i(\tau)\leq C_i(\tau)} \tag{6.1}$$

Using simple algebra, we can derive :

$$\theta_i(n+1) = \theta_i(n) + a(n)(r_i(n+1)\mathbb{I}_{r_i(n+1)\leq C_i(n+1)-\theta_i(n)}) \tag{6.2}$$

where $a(n) = \frac{1}{n+1}$. The optimality of proportional fair schemes are usually proved using the theory of Stochastic Approximation (Borkar (2008)). Thus, we can rewrite the above equation as

$$\theta_i(n+1) = \theta_i(n) + a(n)(h_i(\theta_i(n)) + \epsilon(n) + M_i(n)) \tag{6.3}$$

where

$$\begin{aligned} h_i(\theta_i(n)) &= \mathbb{E}[r_i(n+1)\mathbb{I}_{r_i(n+1)\leq C_i(n+1)}] - \theta_i(n) \\ \epsilon(n) &= \mathbb{E}_n[r_i(n+1)\mathbb{I}_{r_i(n+1)\leq C_i(n+1)}] - \mathbb{E}[r_i(n+1)\mathbb{I}_{r_i(n+1)\leq C_i(n+1)}] \\ M_i(n) &= \mathbb{E}_n[r_i(n+1)\mathbb{I}_{r_i(n+1)\leq C_i(n+1)}] - r_i(n+1)\mathbb{I}_{r_i(n+1)\leq C_i(n+1)} \end{aligned}$$

Here, $\mathbb{E}_n[x] = \mathbb{E}[x|\mathscr{F}_n]$, where $\mathscr{F}_n = \sigma(\theta(m), M(m), m \leq n)$. We can observe that $M(n)$ forms a Martingale Difference sequence, since we have

$$\mathbb{E}_n[M(n+1)] = 0, n \geq 0 \tag{6.4}$$

We can further prove that this sequence is also square-integrable.

The sequence of steps given by $a(n)$ also satisfies $\sum_{n=1}^{\infty} a(n) = \infty$ and $\sum_{n=1}^{\infty} a^2(n) < \infty$. Let us assume for this section that we have a maximum rate of transmission, R_{max}. Then it is evident that the average throughput of any user is bounded by this. Thus, the iterates, $\theta(n)$ are also bounded over time. Further, $-R_{max} \leq \epsilon(n) \leq R_{max}$. Thus we know for sure that the random sequence is deterministically $O(1)$. If we assume that the mapping h is Lipschitz continuous, then from (Borkar, 2008, Section 2.2), we know that the limiting ODE that tracks the process described here is given by

$$\frac{d\theta(t)}{dt} = h(\theta(t)) \tag{6.5}$$

This result states that the discrete process asymptotically approaches the limit points of the above mentioned ODE. However, the result holds only under the assumptions made above and thus have to be studied more closely. Further, we need to prove that the stable equilibrium point that the process converges to, i.e, the limit point of the ODE, is unique (path independent) and maximizes the utility function. We are currently working to this end using results from convex analysis and differential equations.

Intuitive extension of the gradient algorithm that is optimal in the case of accurate channel information seems to suggest that the parameter to decide scheduling would in this case be maximizing the gradient goodput product. While this claim is yet to be theoretically proven, we can expect the algorithm to be optimal in the proportional fair domain. Current work aims at designing this optimal algorithm and theoretically proving its optimality.

CHAPTER 7

Conclusion

In this dissertation, we introduced and studied the problem of downlink resource allocation using delayed information in a scenario where the UEs face time-varying interference from neighboring cells. We exploit the statistics of the time-varying interference to compute a conditional outage probability expression, given the unreliable CQI available at the base-station. We then combined the outage probability expression with a Lyapunov stability framework to obtain a throughput optimal resource allocation policy for the scenario considered. We also stated the on-going work in the area of $\alpha-$fair scheduling and the results that we aim to achieve in the domain.

As an extension to this study, one could consider the case of having more than one interferer. In such a case the outage computation becomes more complex and might require stronger assumptions on the interference processes to enable the computation. One could also take the scenario of having randomized delays. While we have conveniently assumed here that the delays are constant and known, these constraints could be relaxed. This is expected of a practical system where the delays are not always a constant.

With the target of making the system more practical, one could also try to work with a limited feedback system. In our model, we have assumed that all users report their CQI data at every change point. Relaxing this assumption would again make the system more practical. We have also made a strong assumption of not having power modulation in the system. We can relax this to a case of having a finite number of power levels. In that case, the ON/OFF interference process is replaced by a regenerative stochastic process. Such small changes show that the field does hold potential for substantial work in the future.

In essence, this work shows how the max-weight Tassiulas and Ephremides (1993) family of resource allocation policies can be adapted to mitigate time-varying interference. Although we have only studied throughput optimal resource allocation in detail, we remark that our characterization of the outage probability also enables us to suitably modify other widely studied resource allocation policies, to combat time-varying interference.

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