

# **Wideband Spectrum Sensing for Cognitive Radio**

A Project Report

Submitted by

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# THESIS CERTIFICATE

This is to certify that the thesis titled “**Wideband Spectrum Sensing for Cognitive Radio**”, submitted by **Ranagam Rajesh Kumar (EE09B072)**, to the Indian Institute of Technology Madras, Chennai for the award of the degree of **Master of Technology**, is a bonafide record of the research work done by him under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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## **ABSTRACT**

**KEY WORDS:** Cognitive Radio, Primary users, Secondary users, Spectral holes, Spectrum sensing, Opportunistic throughput, Interference to primary users

In wireless communications, we have spectrum scarcity problem because of increase in demand for RF spectrum with increase of wireless services. Cognitive radio has become a promising solution to overcome spectrum scarcity problem by allocating unused spectrum to secondary users. To do so, it has to sense spectrum reliably using spectrum sensing algorithms. In this thesis, various aspects of spectrum sensing and challenges associated with spectrum sensing are presented. Various traditional narrowband and Nyquist wideband spectrum sensing algorithms are discussed along with advantages and disadvantages of each algorithm. The main objective of this thesis is to implement energy detection based sensing and multiband joint detection which exploits more spectral opportunities.

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## **ABBREVIATIONS**

<b>CR</b>	Cognitive Radio
<b>RF</b>	Radio Frequency
<b>SDR</b>	Software Defined Radio
<b>KB</b>	Knowledge Base
<b>DSA</b>	Dynamic Spectrum Access
<b>PSD</b>	Power Spectral Density
<b>SNR</b>	Signal to Noise Ratio
<b>AWGN</b>	Additive White Gaussian Noise
<b>QPSK</b>	Quadrature Phase Shift keying
<b>GSM</b>	Global System for Mobile communications
<b>FFT</b>	Fast Fourier Transform

# 1. Introduction

## 1.1 Overview

In wireless communication, as the wireless services are increasing the demand for RF spectrum is also increasing which leads to spectrum scarcity. The current policies and allocation schemes are not able to meet the requirements of increasing of demand for spectrum. The spectrum usage is also low in time and space dimensions which leads to spectrum inefficiency. In time domain, a particular band may not be used all the time and in frequency domain, all frequency bands of our interested bandwidth may not be used at a particular instant. The unused spectrum bands can be allocated to opportunistic or secondary users. So, Federal Communications Commission (FCC) has made a new policy that allows the secondary users to access the spectrum bands that are not being used by their licensed or primary users. To do so, the spectrum needs to be sensed and detected for white spaces (unused spectrum bands) in order to allocate them for secondary users to maximize the spectrum efficiency and achievable data rate.

Cognitive Radio (CR) is the solution for spectrum scarcity due to its ability to sense, learn and adapt to its environment. Since CR helps secondary users in detecting white spaces in licensed spectrum to use them, the important requirement of CR is that they should find spectral opportunities effectively without causing destructive interference to primary users. Here comes the need of spectrum sensing algorithms. There are a few challenges in spectrum sensing such as hardware requirement, hidden primary user problem, difficulty in detecting spread spectrum, selection of sensing duration and frequency, decision fusion in cooperative sensing and security issues in CR network which are explained in Chapter 3.

There are two types of spectrum sensing algorithms. They are Narrowband and wideband spectrum sensing algorithms. Narrowband spectrum sensing algorithms focus on narrower bands and make decisions for the whole spectrum by binary hypothesis tests. They can detect at most one opportunity at a time. But for requirement of future cognitive radios, more opportunities are to be exploited over wider bandwidths. This is supported by Shannon's formula that, the maximum achievable rate is directly proportional to bandwidth. So, it requires wideband sensing. Hence the objective is to maximize the achievable throughput by exploiting more spectral opportunities over wider bandwidth with constraint on interference to primary users.

## **1.2 Organization of thesis**

Background of CR, CR architecture and some security issues in CR network are explained in Chapter 2. In security issues, we will study possible threats and attacks to cognitive radio, followed by ways to mitigate such attacks.

In Chapter 3, important aspects of spectrum sensing and challenges in spectrum sensing are discussed. After that, overview of narrowband and Nyquist wideband sensing algorithms along with advantages and disadvantages of each algorithm is given.

In Chapter 4 and Chapter 5, Energy Detection based sensing and Multiband joint detection algorithms are presented respectively along with simulation results.

In Chapter 6, summary and conclusions of thesis are given followed by future scope of the project.

## **2. Background**

### **2.1 Cognitive Radio (CR)**

There is no agreement in formal definition of CR. The definition that is adopted by FCC is that “Cognitive radio is a software radio or system that senses its operational electromagnetic environment and can dynamically and autonomously adjust its radio operating parameters to modify system operation such as maximizing throughput, mitigate interference, facilitate interoperability and access secondary markets.” [2]. CR is able to sense, learn, adapt to environment and thus be aware of availability of spectrum. It is associated with secondary users as they need to sense the spectrum for opportunities.

Over the last decade, it has become a promising solution for spectrum scarcity problem by exploiting spectral opportunities in time, frequency and space domains. Hence, its main objective is to find spectrum holes or white spaces in order to allow secondary users to access the unused spectrum. In finding spectrum holes, it should make sure that it shouldn't create any harmful interference to primary users.

While sensing the spectrum, if it detects the primary user, CR has to withdraw from spectrum so as to minimize interference to primary user. There will be tradeoff between minimum interference to primary user and maximum achievable throughput. Minimizing interference to primary users will prevent secondary users to access spectrum, leading to lower utilization of spectrum and lower throughput. Maximizing throughput will cause interference to primary users as the secondary users access the spectrum when the primary user needs it. By selecting the sensing duration and frequency carefully, CR can sense reliably and efficiently.

### **2.2 Cognitive Radio Architecture**

The CR architecture is explained in [1]. Cognitive Radio is extension of Software Defined Radio (SDR). It consists mainly of four parts. They are:

1. SDR
2. Knowledge base (KB)
3. Reasoning Engine
4. Learning Engine

Knowledge base, Reasoning engine and Learning engine together are called as Cognitive Engine (shown in figure 2.1). SDR and Cognitive engine are connected by APP programming interface.

### 2.2.1 Software Defined Radio

SDR is a highly configurable communication device which is capable of synthesizing many communication waveforms by making processing graphs of different radio components.

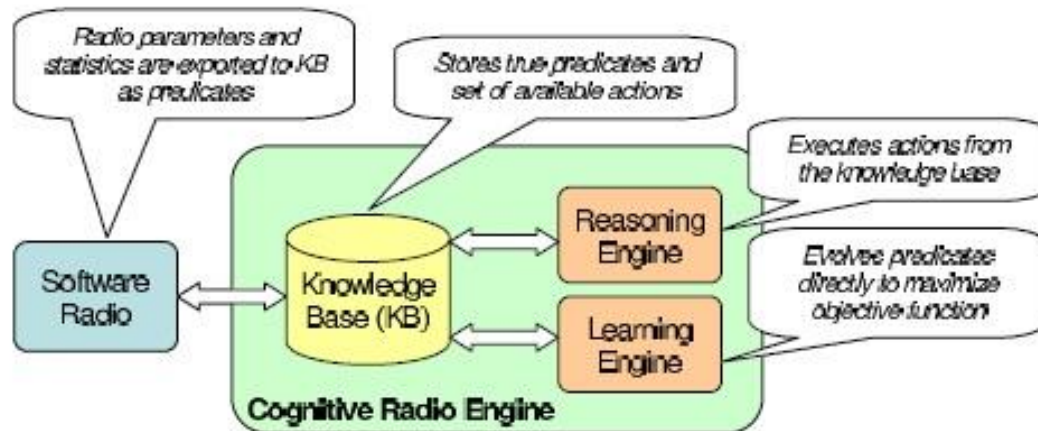


Figure 2.1: Cognitive Radio Architecture (Ref. 1)

SDR has a large number of components. They are:

- **Adjustable front end:** allow SDR to tune over different frequency ranges
- **Amplifier :** allows communication at different power levels
- **Modem Components:** can implement different type of modulations with different rates
- **Many varieties of sensors:** These sensors take digitized RF energy and give quantitative results. For example, energy detectors can measure the received power in order to know whether the channel is occupied or not. Specific waveform sensors can identify the exact type of signal that is occupying the channel.

### 2.2.2 Cognitive Radio Engine

Knowledge Base (KB) is a set of logical expression that represents the state of Cognitive Radio. It stores true predicates and available actions. Reasoning engine is set of logical rules and executes the available actions from KB. Reasoning engine is provided with the set of actions and conditions under which those actions are executable. Learning engine is able to learn adaptively

from its environment. Whenever the radio reaches to new state which is optimal, then learning engine allows radio to add to its memory in KB and use in next operation.

## 2.3 Security issues in Cognitive Radio Networks

In this section we will see some possible threats and attacks to CR network and discuss the ways to mitigate those attacks [1].

### 2.3.1 Threats to Cognitive Radios

Cognitive Radio selects optimal and secure means of communications as it is able to sense, learn and adapt. However, by putting artificial intelligence engines in charge of these radios, they can be provided false sensory inputs. These false sensory inputs affect their beliefs and behavior.



Figure 2.2: Relationship between sensory input, beliefs and behavior in cognitive radio (Ref. 1)

In wireless communications, there can be attackers who can spoof, inject, remove and manipulate data. There are three types of attacks - sensory manipulation attacks, belief manipulation attacks and self-propagating behavior (it leads to Cognitive Radio viruses). All these attacks affect cognitive radio's behavior such that it acts either sub-optimally or even in some cases maliciously.

#### Sensory manipulation attacks

Usually, radio sensors of CR take digitized RF and extract useful statistics from it. In sensory manipulation attacks, an attacker can alter or modify those sensory inputs which cause faulty statistics to appear in the knowledge base. Once if the attacker understands how the statistics are calculated using the sensory input, they can manipulate them and create problems to the CR system.

### **Belief manipulation attacks**

These types of attacks are also due to false sensory input. Here, the attacker causes the sub-optimality by manipulating data. Hence, learning engine will change CR's beliefs. These effects will be long-term since learning engine uses this in its future operations. For example [1], when CR tries to use higher data rate, the attacker can force it to use lower data rate by introducing jamming signal. This effect causes that CR use lower data rate whenever it tries to use higher data rate. This effect will be long-term.

### **Self-Propagating behavior**

In these types of attacks, a state or sequence of states on one radio will be induced to its neighbor radio and likewise will be induced on all radios in that network which results in cognitive radio virus. This is most powerful attack. These types of attacks can spread cognitive radio virus even between the radios that never have direct interaction.

## **2.3.2 Classes of attacks**

### **Dynamic Spectrum Access Attacks**

One such attack is Primary User Emulation (PUE) attack and it is usually appeared in DSA applications. These attacks apply to spectrum sensing algorithms. In such attacks, the attacker makes secondary user to believe that the primary user is active, causing spectrum inefficiency. Depending on Spectrum sensing Algorithms used, the effect of these attacks might be long-term or short-term.

### **Objective function attacks**

These types of attacks apply to learning algorithms which use objective function. There is large number of radio parameters like center frequency, bandwidth, power, modulation type, coding rate and channel access protocol over which cognitive radio has control to maximize its objective function. Objective function is a combination of its multi goals. Three of such goals are low power, high rates and secure communication. Depending on application, the weightages for these goals will be different. In learning phase, they try different sets of inputs (radio parameters), measures observed statistics and evaluate objective function to see which inputs gives optimality condition. The attacker manipulates the radio parameters to lead to sub-optimality.



For example [1], if a radio tries to use more secured level  $S_2$ , an attacker tries to decrease the rate from  $r_2$  to  $r_1$  with  $r_1 < r_2$ . As a consequence, the objective function decreases and hence, the system uses less secured level  $S_1$  rather than  $S_2$ .

### **Malicious Behavior attacks**

This attack is extension of objective function attack. Here, the radio will become unknowingly malicious.

For example [1], consider DSA case where the secondary user has an opportunity to use channel when it detects the primary user is not active by its sensing algorithms. They have an objective function:

$$f = w_1R - w_2I ; \text{ where } R \text{ is throughput and } I \text{ is Interference}$$

Here, the objective is to maximize  $R$  and minimize  $I$  to maximize  $f$ . But an attacker can artificially decrease  $R$  when the channel is free by using jamming signal which can't be detected. So, CR learnt that it can achieve optimum ' $f$ ' when the primary user is active. So CR will become unknowingly a jammer to secondary users.

### **2.3.3 Attack mitigation**

#### **Robust Sensory Input**

By improving sensory input, the attacks can be mitigated. Such sensors can also help to find the signals that are trying to corrupt the radio beliefs. In a network of cognitive radios, combining sensory data from different radios helps to improve performance. Besides these, consider the sensory input always as noisy since even without presence of the attacker, statistics can be incorrect.

#### **Mitigation in Individual Radios**

We need some intuition and common sense to mitigate attacks against individual radios. There are some strategies to prevent attacker.

They are:

1. Constantly reevaluate radio beliefs.
2. Constantly update the feedback loop.
3. If continuously learning radios are not desired, then conduct learning phase in controlled environment.

4. Outside auditing must be done to make sure that no attacker is present during learning phase.
5. Invalidate learned actions that were known as sub-optimal (that is violating optimal conditions).

### **Mitigation in networks**

In a network, there is a control channel between independent CRs. Each CR wants to maximize their own performance and possibly entire system performance depending on cooperation from other CRs. There are many techniques to optimize. One of them is PSO (Particle Swarm Optimization) [1]. In PSO, each Cognitive radio is a particle and has its own hypothesis about how to behave best in a particular situation. However, the behavior it selects is not completely depending on its own hypothesis but weighted average of all CRs' hypotheses in the network. The application of PSO is to mitigate PUE attack in DSA environment.

### 3. Spectrum Sensing

#### 3.1 Spectrum Sensing

The cognitive radio maximizes the achievable throughput by allocating the unused spectrum bands to secondary users. In order to do it, the spectrum needs to be sensed for spectrum holes. So spectrum sensing is a task of obtaining awareness of spectrum usage and primary users' occupancy in the spectrum. Traditionally, it is understood as measuring radio frequency energy over the spectrum but it is general term that describes obtaining awareness of spectrum usage in multiple dimensions such as time, frequency, space, code and angle of arrival [2]. A frequency band may not be in use for all the time and at a particular time, all the frequency bands may not be in use. So time dimension is also as important as frequency. So we need to explore in all dimensions for more spectral opportunities. The conventional sensing deals with only time, frequency and geographical space dimensions.

Apart from time, frequency and space, there are many dimensions like code and angle which are not explored well. By considering them also for spectrum sensing, more opportunities can be created. For example, if we know the code used by primary user or angle of primary user's transmission, both primary user and secondary user share the same channel at same time in same geographical area. So we need to explore the other dimensions as well in order to find more opportunities. But exploring those dimensions imposes new challenges in spectrum sensing.

Dimension	What needs to be sensed?
Time	Opportunity of a specific band in time
Frequency	Opportunity in frequency
space	Geographical location and distance of primary users
code	Spread code, Time Hopping and Frequency Hopping used by primary users
angle	Directions of Primary users' transmissions

Table 3.1: Dimensions and opportunities that need to be sensed in spectrum sensing

By introducing these new dimensions to radio space, it can be defined as, “a theoretical hyperspace occupied by radio signals, which has dimension like location, time, frequency, angle,

code and possibly others” [2]. All these dimensions along with what needs to be sensed in those dimensions are summarized in Table 3.1.

## 3.2 Challenges in spectrum sensing

In this section, we will see a few challenges associated with spectrum sensing [2].

### 3.2.1 Hardware Requirement

Spectrum sensing applications require high sampling rate, high resolution analog to digital converters and high speed digital signal processors. In order to sense in wider bandwidths for more spectral opportunities, more number of RF components like antennas and power amplifiers are needed.

Sensing can be done by either single radio or dual radio architecture. One between those two architectures needs to be selected for spectrum sensing based on the requirement. In single radio, data transmission happens after detection of free channel but in dual radio, one chain of radios will be assigned for sensing and the other will be assigned for data transmission. Advantages of single radio are its simplicity and low cost but it loses accuracy of sensing because of limit on sensing duration. Advantages of dual radio are that sensing and transmission happens parallel, accuracy of sensing will be high as one chain of radios dedicated for sensing but they are complicated to implement, consume more power and expensive.

### 3.2.2 Hidden Primary user problem

Figure 3.1 shows the operating regions of primary user and Cognitive Radio in dashed circles.

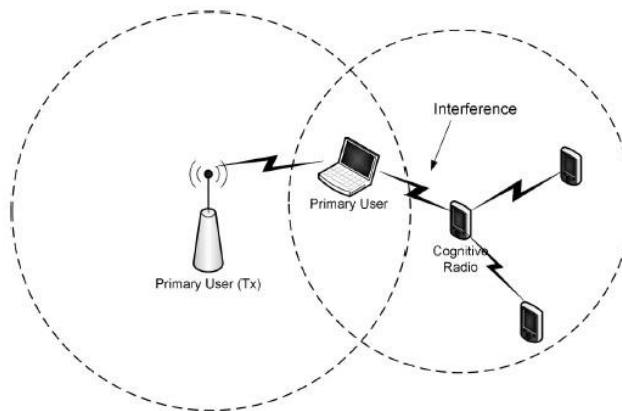


Figure 3.1 Illustration of hidden primary user problem (Ref. 2)

Here, as CR is not able to detect primary signal because of its location and tries to sense for free channel, it creates harmful interference to primary user receiver. Cooperative sensing helps in overcoming this hidden primary user problem.

### **3.2.3 Difficulty in detection of spread spectrum**

It is difficult to detect a primary signal using spread spectrum since its power is distributed over the bandwidth that is wider than actual bandwidth. This can be partially overcome by knowing the hopping pattern and synchronizing perfectly to the signal. However, conventional sensing doesn't deal with code dimension.

### **3.2.4 Selection of sensing duration and frequency**

Sensing duration and frequency are design parameters that should be chosen carefully to avoid interference. Let us assume that CR is operating in a free channel. After a while, the primary user claims for his channel. If CR detects the primary user, the CR has to withdraw from the channel in order to avoid creating interference to primary user. So CR has to detect the presence of primary user as quick as possible. The duration in which it has to detect is sensing duration.

Sensing frequency is that how frequently CR has to perform spectrum sensing. The optimum value of sensing frequency depends on CR's capacity and temporal characteristics of primary signal. For example [2], in case of TV channels, the signal rarely changes unless an existing station stops its broadcasting or a new channel starts broadcasting. In such scenarios, sensing frequency can be relaxed. It means if primary signals are slow to change, sensing frequency can be relaxed.

The guard intervals between transmitted symbols can be replaced by quiet periods and spectrum sensing is performed in those periods so that the useful bandwidth won't be wasted. But a channel that is being used by secondary user for data transmission can't be used for spectrum sensing. So the secondary user's data transmission should be interrupted to perform spectrum sensing which reduces spectrum efficiency. To mitigate this problem, an alternate method called Dynamic Frequency Hopping (DFH) is followed in [2]. DFH method is based on assumption of having more than one channel for transmission. During the operation on working channel, the desired channel will be sensed. If channel is available, then channel switching takes place and it will become new working channel.

### 3.2.5 Decision fusion in cooperative sensing

Cooperative sensing is used for efficient spectrum sensing. But here the problem is that how to combine decisions of all individual radios in the network. There are many combining rules such as Equal Gain Combining (EGC), Selection Combining (SC), likelihood ratio combining, OR rule, AND rule, M-out-of-N rule and weighted average. Depending on requirement, suitable combining rule has to be selected. Generally we follow weighted average combining rule.

### 3.2.6 Security issues

As we saw in chapter 2, there can be threats and attacks to cognitive radios from malicious users who can spoof, remove and modify the primary user data. So identifying and protecting from such attacks is a difficult task.

## 3.3 Spectrum Sensing Algorithms

To perform spectrum sensing efficiently, suitable sensing algorithms are needed. There are two types of algorithms. They are Narrowband spectrum sensing and wideband sensing algorithms. In this section, overview of each algorithm with advantages and disadvantages is given.

### 3.3.1 Narrowband sensing algorithms

These algorithms focus on narrowband and find spectral opportunity based on binary hypothesis. Three of such algorithms are energy detection, matched filtering and cyclostationarity based sensing. Overview of these three algorithms is given in this section.

Hypotheses:  $H_0$  – Primary user is absent;  $H_1$  – Primary user is present;

#### 3.3.1.1 Energy detection based sensing

Energy detection [2] is the most common method of spectrum sensing because it is simple and robust. It doesn't need any prior knowledge of primary signal.

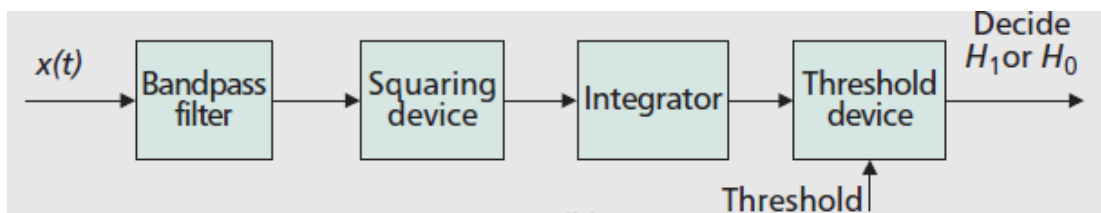


Figure 3.2: Block diagram of Energy detection based sensing (Ref. 4)

As shown in Figure 3.2, first the signal is passed through bandpass filter and then squaring device followed by integrator. The output of the integrator is energy of the signal which will be compared against the threshold to make decision on presence of primary user. The threshold depends on false alarm probability and noise variance.

**Advantages:**

1. Doesn't require prior knowledge of primary user's signal
2. Low computational and implementation complexity

**Disadvantages:**

1. Poor performance for low SNRs
2. Cannot differentiate primary and secondary users, and users from interference since it just takes whatever signal comes and find energy

**3.3.1.2 Matched filtering based sensing**

Matched filtering based sensing [2] is an optimal method when the transmitted signal's features like modulation, pulse shaping and bandwidth are known since it will do exact counter parts to detect primary users. Block diagram for matched-filtering based sensing is shown in Figure 3.3.

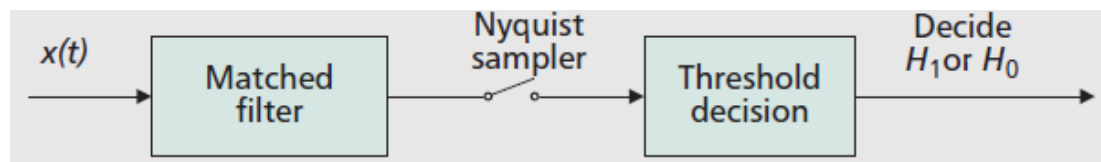


Figure 3.3: Block diagram of matched-filtering based sensing (Ref. 4)

**Advantages:**

1. Optimal method in case of prior knowledge of primary signals
2. Low computational cost

**Disadvantages:**

1. Require prior knowledge of primary user's signal
2. High implementation complexity since it needs receivers for all types of signals
3. Large power consumption since it has to be run for many various receiver algorithms

### 3.3.1.3 Cyclostationarity-Based Sensing

Cyclostationarity based feature detection [2], [3] is a method for detecting primary user by exploiting cyclostationarity features of received signal. Cyclostationarity features are caused by periodicity in signal. Here, instead of PSD, cyclic correlation function will be found. Then cyclic frequency will be detected to decide on presence of primary signal.

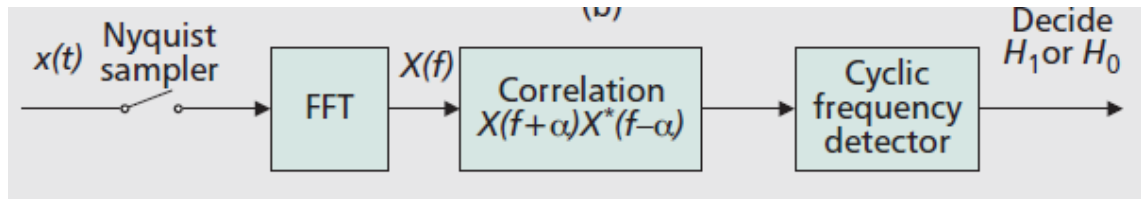


Figure 3.4: Block diagram of cyclostationarity based sensing (Ref. 4)

#### Advantages:

1. Can differentiate interference from users and different users
2. Valid in low SNR region

#### Disadvantages:

1. Requires partial prior information
2. High computational cost

### 3.3.2 Nyquist Wideband Sensing Algorithms

Unlike narrowband sensing, this type of algorithms focus on wider band widths and find more than one opportunity. Narrowband techniques can't be applied directly to wideband because it makes binary decision for whole spectrum and is not able to identify opportunities in individual bands within the wideband. The wideband will be divided into individual bands and then binary hypothesis tests can be applied for each band independently. These algorithms can be classified into two types: Nyquist wideband sensing and sub-Nyquist wideband sensing algorithms. In former type, signals are processed using sampling rate at or above Nyquist rate whereas in latter type, signals are processed using sampling rate lower than Nyquist rate. In this thesis, we will see only Nyquist wideband sensing algorithms.



### 3.3.2.1 Multiband joint detection

The Multiband joint detection algorithm [4], [5] is proposed to overcome the limitations of narrowband sensing. It can sense primary signal over all sub-bands in a wideband. As shown in Figure 3.5, the received signal is sampled using Nyquist rate, and then passed through serial to parallel converter to convert serial data into parallel data. Then, FFT is applied to convert into frequency domain. After that the wideband will be divided into individual sub-bands. Then each band will be compared independently with its threshold to make a decision.

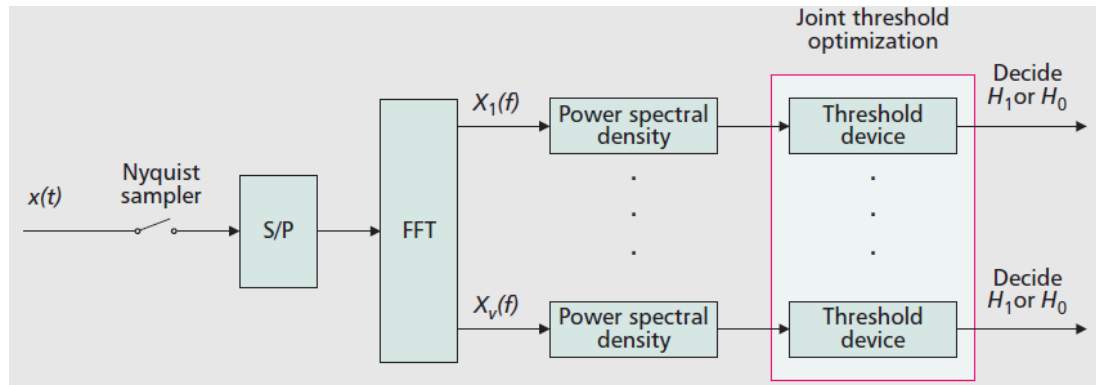


Figure 3.5: Block diagram of Multiband joint detection (Ref. 4)

#### Advantages:

1. simple structure
2. low computational and implementation complexity

#### Disadvantages:

1. High sampling rate
2. Energy cost

### 3.3.2.2 Wavelet-based spectrum sensing

In this algorithm, the PSD of wideband spectrum is modeled as a train of consecutive bands [4], [6]. Here, PSD is smooth in each sub-band and shows irregularities at the edges of the sub-bands. Using the fact that PSD shows irregularities at the edges, we are able to identify the edges and so to find number of bands and location of bands. As shown in Figure 3.6, the signal is sampled with Nyquist rate and then FFT is applied to convert into frequency domain. Then PSD of the signal will be found and then wavelet transform which is known as solution for edge detection

problem is applied. Then local maxima of output of wavelet transform block will be found to get the information of edges. After finding edges, we will find number of sub-bands and location of each band. After that we will find PSD level in each band and decide whether intended band is free or not using binary hypothesis tests.

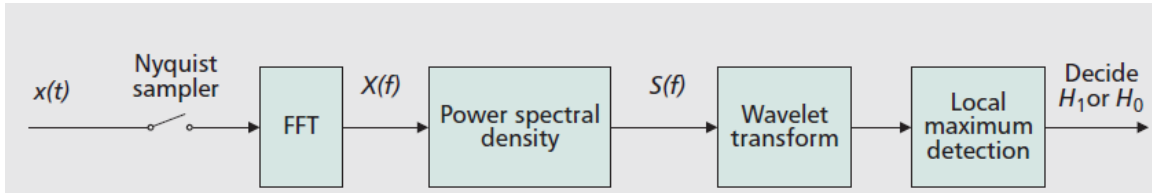


Figure 3.6: Block diagram of wavelet based spectrum sensing (Ref. 4)

#### Advantages:

1. simple structure
2. low computational complexity

#### Disadvantages:

1. High sampling rate
2. Energy cost

#### 3.3.2.3 Sweep-tune detection

The sweep-tune detection [4] is proposed to relax the Nyquist sampling rate. This algorithm uses super-heterodyne (frequency mixing) techniques that “sweep” across the frequency range of interest. Here by mixing the received signal with sine wave produced by Local Oscillator (LO), we can bring down the requirement of Nyquist sampling rate as signal is down converted. As shown in Figure 3.7, first received signal is mixed with output of LO and then pass through bandpass filter followed by sampling. Then binary hypothesis test are applied to make decisions.

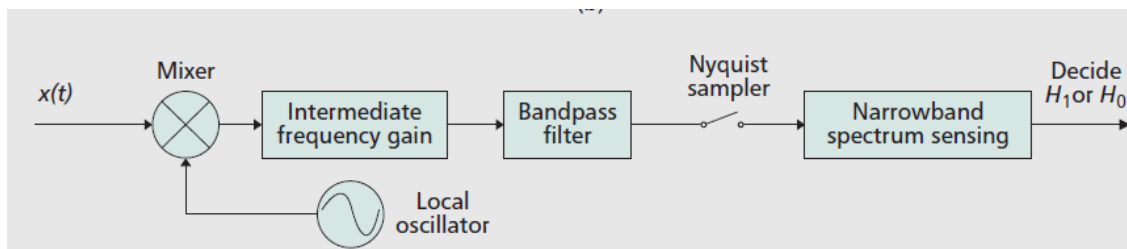


Figure 3.7: Block diagram of sweep-tune detection (Ref.4)

**Advantages:**

1. Relaxes the requirement of high sampling rate

**Disadvantages:**

1. Slow approach
2. Inflexible algorithm
3. High implementation complexity

**3.3.2.4 Filter bank algorithm**

Filter bank algorithm [4] is another algorithm to relax Nyquist sampling rate. As shown in Figure 3.8, a bank of prototype filters (with different shifted central frequencies) is used to process the wideband signal. The baseband can be processed by a prototype filter, and other bands can be processed through shifted versions of the prototype filter. In each band, the signal was down-converted to baseband and then low-pass filtered. The corresponding portion of spectrum is estimated and then narrowband sensing techniques can be applied to decide.

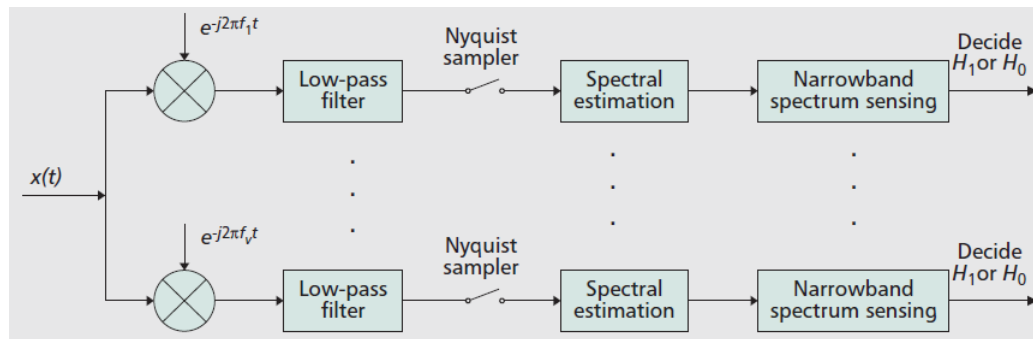


Figure 3.8: Block diagram of filter bank algorithm (Ref. 4)

**Advantages:**

1. Low sampling rate

**Disadvantage:**

1. High implementation complexity
2. Require large number of RF components because of parallel structure of filter bank.

In the following chapters, energy detection based sensing and multiband joint detection algorithms are implemented because of their simplicity and ability to determine the spectrum availability information quickly.

## 4. Energy Detection Based Sensing

Energy detection is the most common way of spectrum sensing because of its low computational and implementation complexities. It doesn't need any prior knowledge of primary signal. The signal is detected by comparing the output of energy detector with threshold which depends on noise variance.

### 4.1 System Model

Let the received signal be in the following form:

$$r(n) = s(n) + v(n)$$

Where  $s(n)$  - primary signal samples,  $v(n)$  - noise samples,  $n=1, 2, 3, \dots N$  and  $N$  is the total number of samples.

The noise is assumed to be Additive white Gaussian noise with variance  $\sigma_v^2$ . The primary signal samples can also be modeled as Gaussian random process with variance  $\sigma_s^2$ .

$$\text{under } H_0: r(n) = v(n)$$

$$\text{under } H_1: r(n) = s(n) + v(n)$$

### 4.2 Algorithm

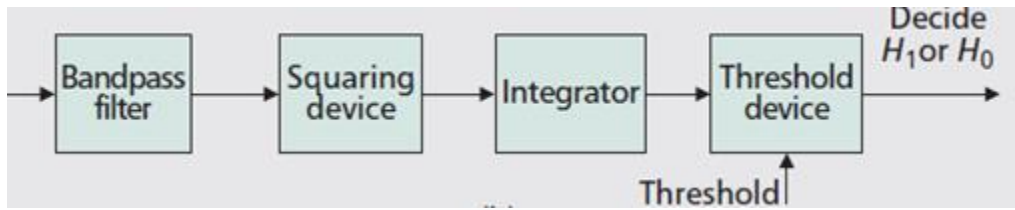


Figure 4.1: Block diagram of energy detection based sensing (Ref. 4)

The decision metric for energy detector can be written as:

$$\Delta = \frac{1}{N} \sum_{n=1}^N |r(n)|^2$$

This metric will be compared with threshold ( $\gamma$ ) to make decision. The threshold can be found by fixing the false alarm probability.

$\Delta \geq \gamma \Rightarrow$  Primary signal is present

$\Delta < \gamma \Rightarrow$  Primary signal is absent

### 4.3 Calculating Threshold

Since  $s(n)$  and  $v(n)$  are Gaussian random variables, decision metric has chi-square distribution with  $2N$  degrees of freedom under both  $H_0$  and  $H_1$ . By central limit theorem for large  $N$ , the decision metric can be approximated further as  $\mathcal{N}(m, \sigma^2)$  (i.e., Gaussian with mean  $m$  and variance  $\sigma^2$ ). The derivations are given in Appendix A.

$$H_0 : \Delta \sim \mathcal{N}\left(\sigma_v^2, \frac{(\sigma_v^2)^2}{N}\right)$$

$$H_1 : \Delta \sim \mathcal{N}\left(\sigma_s^2 + \sigma_v^2, \frac{(\sigma_s^2 + \sigma_v^2)^2}{N}\right)$$

Performance of energy detector is measured by a resulting pair of probability of detection ( $P_d$ ) and probability of false alarm ( $P_{fa}$ ). These probabilities can be formulated using above approximations of  $\Delta$ .

$$P_d = P[\Delta \geq \gamma | H_1] = Q\left(\frac{\sqrt{N}}{\sigma_s^2 + \sigma_v^2}(\gamma - (\sigma_s^2 + \sigma_v^2))\right)$$

$$P_{fa} = P[\Delta \geq \gamma | H_0] = Q\left(\frac{\sqrt{N}}{\sigma_v^2}(\gamma - \sigma_v^2)\right)$$

The above expressions for  $P_d$  and  $P_{fa}$  are derived in Appendix B. Therefore threshold  $\gamma$  is:

$$\gamma = \sigma_v^2 \left(1 + \frac{Q^{-1}(P_{fa})}{\sqrt{N}}\right)$$

Probability of misdetection  $P_{md}$  is equal to  $1 - P_d$ . By fixing  $P_{fa} = 0.05$ , threshold  $\gamma$  can be found using above formula.

#### 4.4 Performance of energy detector

The probability of detection is plotted against SNR. Figure 4.2 shows the performance of energy detector for QPSK signal with  $N=1136$ . The practical performance of an energy detector is matched with theoretical performance.

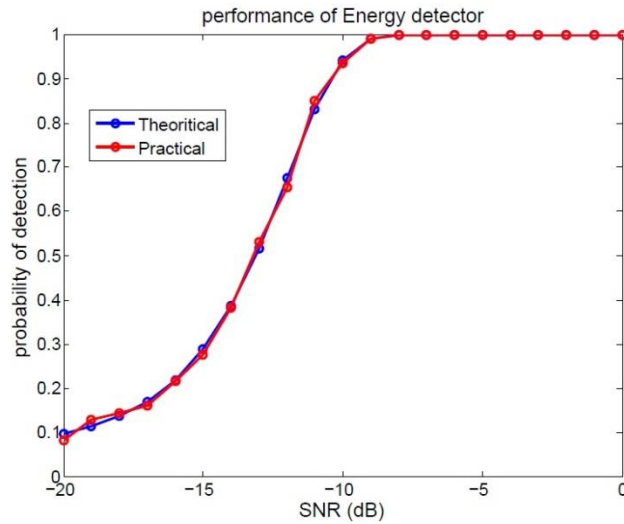


Figure 4.2: Performance of energy detector for AWGN case for QPSK signal

Figure 4.3 shows the performance of energy detector for GSM signal with  $N=1136$  and is matched with theoretical performance.

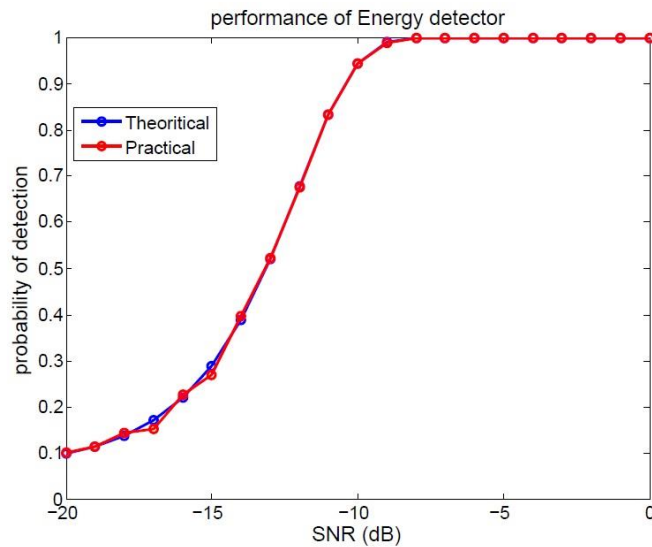


Figure 4.3: Performance of energy detector for AWGN case for GSM signal

Figures 4.4 and 4.5 show the performance of energy detector for QPSK and GSM signals under fading respectively. Under fading the performance get worse.

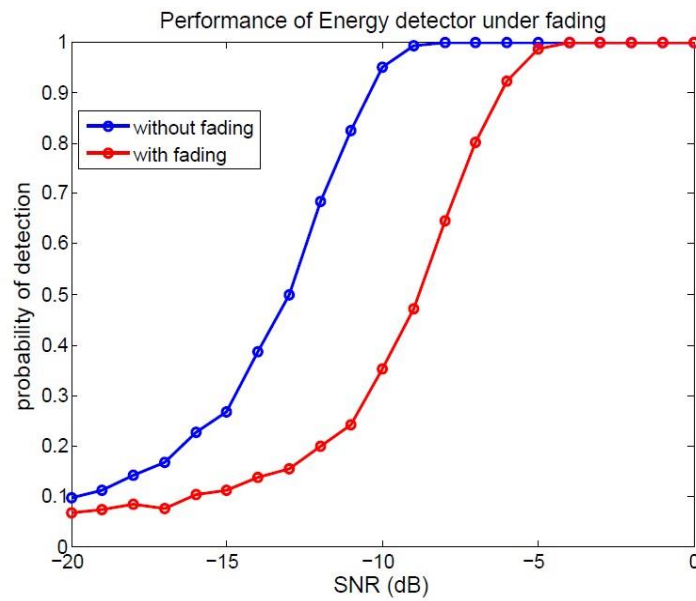


Figure 4.4: Performance under fading for QPSK signal

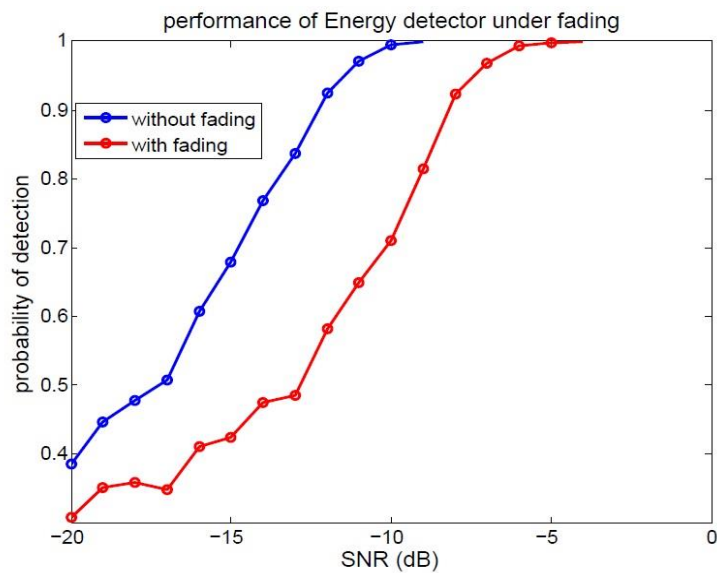


Figure 4.5: Performance under fading for GSM signal



Figure 4.6 shows the performance for QPSK with different number of samples. Larger the samples better the performance.

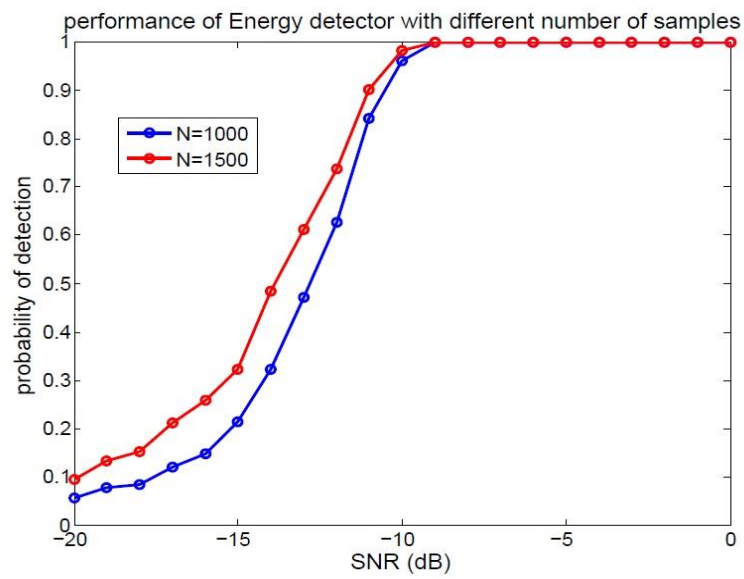


Figure 4.6: Performance for QPSK signal with different number of samples

## 5. Multiband Joint Detection

Multiband joint detection [5] is Nyquist wideband sensing. It helps in overcoming the limitation of narrowband sensing that is finding fewer opportunities. But it uses sampling rate equal to Nyquist rate to process the signal. This algorithm uses energy detection based sensing as a building block. After dividing the wideband into sub-bands, energy detector is applied independently on all sub-bands.

### 5.1 System model

Let the received signal be in the following form:

$$r(n) = s(n) + v(n)$$

Where  $s(n)$  - primary signal samples and  $v(n)$  - noise samples.

The noise is assumed to be Additive white Gaussian noise with variance  $\sigma_v^2$ . The primary signal samples can also be modeled as Gaussian random process.

Assume there are K bands in our interested wider bandwidth. Each band is needed to be sensed for spectral opportunity. Consider sub-band k:

$$H_{0,k} : r_k(n) = v_k(n) \leftrightarrow R_k = V_k$$

$$H_{1,k} : r_k(n) = s_k(n) + v_k(n) \leftrightarrow R_k = S_k + V_k$$

Where  $r_k(n)$  is the portion of the received signal in subband k and  $s_k(n)$  is portion of primary signal in subband k. Here,  $k=1, 2, 3, \dots, K$  and  $n=1, 2, 3, \dots, N$ .

## 5.2 Algorithm

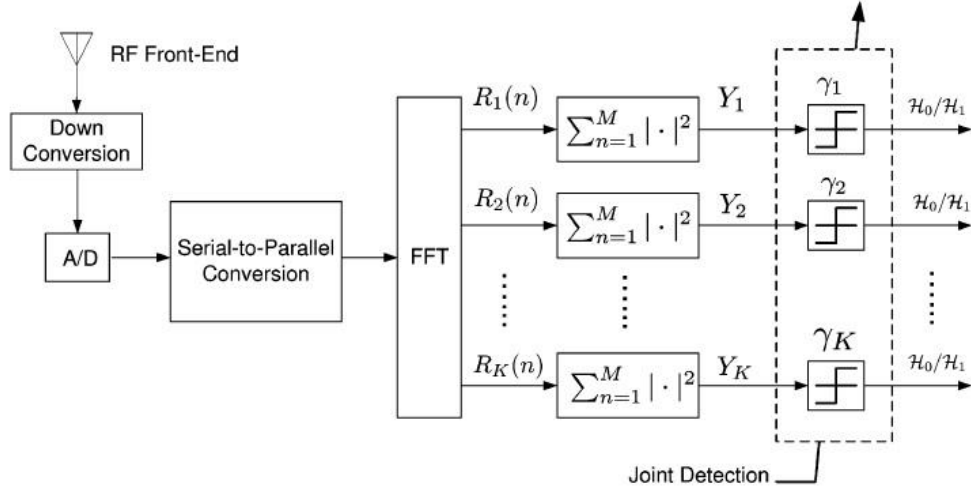


Figure 5.1: Block diagram of Multiband joint detection (Ref. 5)

The decision metric for sub-band k:

$$Y_k = \frac{1}{N} \sum_{n=1}^N |R_k(n)|^2 \quad \text{where } k = 1, 2, 3, \dots, K.$$

The decision metric  $Y_k$  will be compared with threshold  $\gamma_k$  to make decision.

$$Y_k \geq \gamma_k \Rightarrow \text{Primary signal is present in subband } k$$

$$Y_k < \gamma_k \Rightarrow \text{Primary signal is absent in subband } k$$

Here,  $\gamma_k$  is threshold for sub-band k. Then the threshold vector is defined as  $\gamma = [\gamma_1, \gamma_2, \dots, \gamma_k, \dots, \gamma_K]$ .

## 5.3 Calculating optimal threshold vector

By central limit theorem, for large N,  $Y_k$  can be approximated as  $\mathcal{N}(m, \sigma^2)$  that is Gaussian with mean m and variance  $\sigma^2$ .

$$H_{0,k} : Y_k \sim \mathcal{N}\left(\sigma_v^2, \frac{(\sigma_v^2)^2}{N}\right)$$

$$H_{1,k} : Y_k \sim \mathcal{N} \left( \sigma_{s_k}^2 + \sigma_v^2, \frac{(\sigma_{s_k}^2 + \sigma_v^2)^2}{N} \right)$$

The probability of detection ( $P_d$ ) and probability of false alarm ( $P_{fa}$ ) in sub-band  $k$  can be formulated using above approximation of  $Y_k$  as follows:

$$P_d^{(k)}(\gamma_k) = Q \left( \frac{\sqrt{N}}{\sigma_{s_k}^2 + \sigma_v^2} (\gamma - (\sigma_{s_k}^2 + \sigma_v^2)) \right)$$

$$P_{fa}^{(k)}(\gamma_k) = Q \left( \frac{\sqrt{N}}{\sigma_v^2} (\gamma - \sigma_v^2) \right), \quad \text{Where } k = 1, 2, \dots, K$$

Then probability vectors are defined as:

$$P_d(\gamma) = [P_d^{(1)}(\gamma_1), P_d^{(2)}(\gamma_2), P_d^{(3)}(\gamma_3), \dots, P_d^{(K)}(\gamma_K)]^T$$

$$P_{fa}(\gamma) = [P_{fa}^{(1)}(\gamma_1), P_{fa}^{(2)}(\gamma_2), P_{fa}^{(3)}(\gamma_3), \dots, P_{fa}^{(K)}(\gamma_K)]^T$$

$$P_{md} = 1 - P_d$$

In energy detection, threshold was found by fixing  $P_{fa}$  to 5%. But here the threshold vector will be found by solving optimization problem. The choice of  $\gamma_k$  leads to tradeoff between  $P_{fa}(\gamma)$  and  $P_{md}(\gamma)$ . The lower threshold will result in a smaller misdetection probability but larger false alarm probability and vice versa. False alarm probability prevent secondary users to access spectrum and misdetection probability measures the interference of secondary users to primary users. So our objective is to find optimal threshold vector  $\{\gamma_k\}_{k=1}^K$  for  $K$  sub-bands in order to maximize aggregate opportunistic throughput subject to constrain on interference for every primary user.

$$\max_{\gamma} R(\gamma) \quad (P1)$$

$$s.t \quad c_k P_{md}^{(k)}(\gamma_k) \leq \varepsilon, \quad k = 1, 2, 3, \dots, K$$

$$P_{md}(\gamma) \leq \alpha \quad (1)$$

$$P_{fa}(\gamma) \leq \beta \quad (2)$$

Where  $R(\gamma)$  is aggregate throughput; equal to  $\sum_{k=1}^K r_k \left(1 - P_{fa}^{(k)}(\gamma_k)\right)$  and  $r_k$  is available throughput of sub-band  $k$ .  $c_k$  is interference cost to primary user  $k$ . Constraint (1) limits the interference in each sub-band with  $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_K]^T$  and constraint (2) ensures that minimum achievable opportunistic throughput of each band with spectrum utilization given by  $[1 - \beta_1, 1 - \beta_2, \dots, 1 - \beta_K]^T$ .

Our optimization problem (P1) is non-convex but we can convert it to convex by using Q function's monotonically non-increasing property. Since  $P_{md}$  and  $P_{fa}$  are Q functions of  $\gamma$ , constraints (1) and (2) can be rewritten as:

$$\gamma_k \leq \gamma_{max,k} = (\sigma_{s_k}^2 + \sigma_v^2) \left(1 + \frac{Q^{-1}(1-\alpha_k)}{\sqrt{N}}\right) \quad (3)$$

$$\gamma_k \geq \gamma_{min,k} = \sigma_v^2 \left(1 + \frac{Q^{-1}(\beta_k)}{\sqrt{N}}\right) \quad (4)$$

Still our problem is non-convex. But we can convert it into convex problem. If  $0 < \alpha_k \leq \frac{1}{2}$ , then  $P_{fa}^{(k)}(\gamma_k) \leq \frac{1}{2} \Rightarrow P_{fa}^{(k)}(\gamma_k)$  is convex. Similarly if  $0 < \beta_k \leq \frac{1}{2}$ , then  $P_{md}^{(k)}(\gamma_k) \leq \frac{1}{2} \Rightarrow P_{md}^{(k)}(\gamma_k)$  is convex.

$$\max_{\gamma} R(\gamma) \quad (P2)$$

$$s.t \quad c_k P_{md}^{(k)}(\gamma_k) \leq \varepsilon, \quad k = 1, 2, 3, \dots, K$$

$$\gamma_{min,k} \leq \gamma_k \leq \gamma_{max,k}$$

$$0 < \alpha_k \leq \frac{1}{2} \text{ and } 0 < \beta_k \leq \frac{1}{2}, \quad \text{where } k = 1, 2, \dots, K$$

(P2) is convex problem and by solving it we will get optimal threshold vector  $(\gamma)$ .

We also can get optimized threshold vector by solving the below optimization problem:

$$\min_{\gamma} (c^T P_{md}(\gamma)) \quad (P3)$$

$$s. t \quad r^T (1 - P_{fa}(\gamma)) \geq \delta$$

$$P_{md}(\gamma) \leq \alpha$$

$$P_{fa}(\gamma) \leq \beta$$

In (P3), we are minimizing the aggregate interference to primary user. And  $\delta$  is the minimum required opportunistic throughput.

We can convert this also into convex as we did earlier. So we can find threshold vector  $\gamma$  by solving either (P2) or (P3).

#### 5.4 Performance comparison with uniform threshold method

Aggregate opportunistic throughput against aggregate interference to primary communication system is showed in Figure 5.2. This is a result of problem (P2). It can be observed that multiband joint detection can achieve higher aggregate opportunistic throughput compared to uniform threshold method.

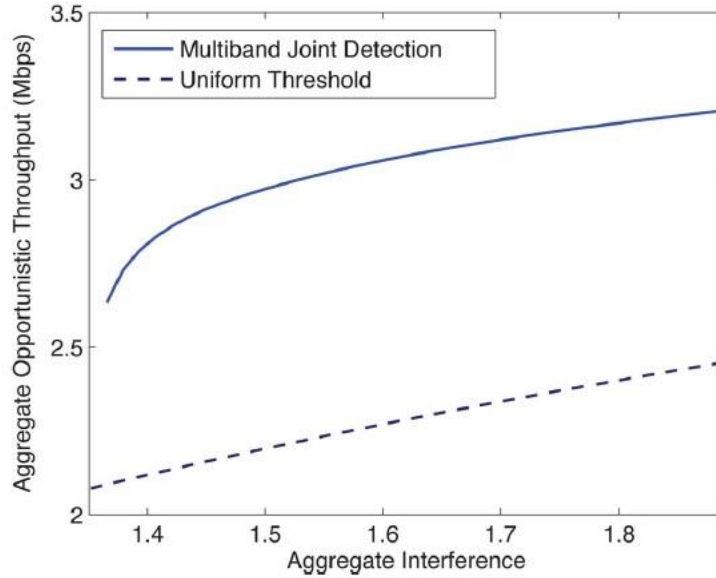


Figure 5.2 Aggregate opportunistic throughput vs Aggregate interference

Solving problem (P3) results Figure 5.3 which shows the aggregate interference to primary communication system against aggregate opportunistic throughput. It can be observed that multiband joint detection will cause less interference to primary communication system compared to uniform threshold method.

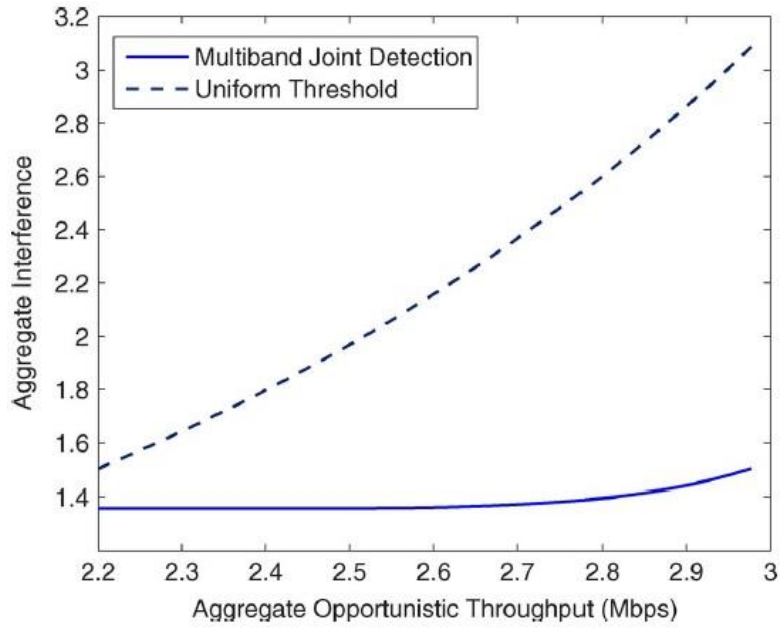


Figure 5.3 Aggregate interference vs Aggregate opportunistic throughput

## **6. Summary and Conclusions**

### **6.1 Summary**

In Chapter 2, a brief background on CR and its architecture was given. Then the security issues in CR network were discussed in detail and then ways and strategies to mitigate the attacks were listed out. In chapter 3, many aspects of spectrum sensing and challenges in spectrum sensing were explained. In spectrum sensing, code and angle dimensions also have been studied as they are also important dimensions to explore for more opportunities. About challenges, we have seen the challenges that CR might face in spectrum sensing and possible solutions to overcome those challenges are given. After that, various narrowband and Nyquist wideband sensing algorithms have been studied. Advantages, disadvantages and challenges of each algorithm were discussed. Then in Chapter 4, energy detection based sensing was implemented for QPSK and GSM signals. In this algorithm, threshold was calculated by fixing false alarm probability to 5%. Then performance of energy detector was plotted for AWGN case and fading case. For QPSK signal, performance was also plotted for different number of samples. But it is limited to only one narrow band and finds one opportunity at a time by making decision on whole spectrum. To overcome over this limitation of energy detection based sensing, in Chapter 5 multiband joint detection was implemented which exploits more opportunities considering all sub-bands in a wideband and applying energy detection on all  $K$  sub-bands individually. In multiband the threshold vector was calculated by solving either optimization problem with objective of maximizing aggregate opportunistic throughput and constraint on interference to each primary user or optimization problem with objective of minimizing aggregate interference to primary communication system and constrain on opportunistic throughput. Then performance of multiband joint detection was compared with uniform threshold method.

### **6.2 Conclusions**

- We have realized the need for exploration in code and angle dimensions for more opportunities.
- Among all the narrowband algorithms, energy detection is preferred to use in practice or implement due to its simplicity and robust nature.
- Energy detection is able to determine spectrum availability information quickly.



- The energy detection based sensing is optimal in case of no prior knowledge of primary signal, except local noise statistics.
- The performance of energy detector matches with theoretical performance in AWGN case.
- Under fading, the performance of energy detector slightly gets worse.
- For larger  $N$  (number of samples), the performance of energy detector is better
- Multiband joint detection is implemented to overcome the limitations of energy detection based sensing.
- Using multiband joint detection, the cognitive radio is able to sense all the bands at same time rather than considering one at a time and can find more opportunities.
- The multiband joint detection with optimized threshold can achieve higher opportunistic throughput with less interference to primary communication system compared to uniform threshold method.

### **6.3 Further scope of project**

The multiband joint detection uses sampling rate equal to Nyquist rate, which is unaffordable to acquire wideband signal. Building such sampling hardware is a challenge. With current technologies, high rate ADC's with high resolution and reasonable power consumption are difficult to implement. Even if it comes true, the digital signal processing of sampled data will be more expensive. So sampling rate should be reduced from Nyquist rate to sub-Nyquist rate. In practice, there may also be limitation on sensing duration. Hence we may not be able to measure all the measurements. So to overcome over these challenges, there are suitable sub-Nyquist wideband sensing algorithms such as Compressive sensing [7] and Multi-channel sensing. In these algorithms, signals are acquired using sampling rate lower than Nyquist rate and few measurements by which unique representation of the signal can be found based on the sparseness of the signal in some domain. As wideband spectrum is sparse in its frequency domain because of its low spectrum utilization, the sub-Nyquist algorithms are suitable to perform wideband spectrum sensing using sub Nyquist rates.

## APPENDIX A

### Approximation of decision metric

Received signal:  $r(n) = s(n) + v(n)$

Decision metric:  $\Delta = \frac{1}{N} \sum_{n=1}^N |r(n)|^2$

Under  $H_0$ :

$$r(n) = v(n)$$

$$\Rightarrow \Delta = \frac{1}{N} \sum_{n=1}^N |v(n)|^2$$

$$mean(\Delta) = E[\Delta]$$

$$= E \left[ \frac{1}{N} \sum_{n=1}^N |v(n)|^2 \right]$$

$$= \frac{1}{N} \sum_{n=1}^N (E[|v(n)|^2])$$

$$= \frac{1}{N} (N \sigma_v^2)$$

$$\Rightarrow mean(\Delta) = \sigma_v^2$$

$$var(\Delta) = var \left[ \frac{1}{N} \sum_{n=1}^N |v(n)|^2 \right]$$

$$= \frac{1}{N^2} [\sum_{n=1}^N var(|v(n)|^2) + covar()]$$

$$= \frac{1}{N^2} [N(\sigma_v^2)^2 + 0]$$

$$\Rightarrow var(\Delta) = \frac{(\sigma_v^2)^2}{N}$$

So, by central limit theorem, for large N,  $\Delta$  can be approximated as  $\mathcal{N} \left( \sigma_v^2, \frac{(\sigma_v^2)^2}{N} \right)$ .

**Under  $H_1$ :**

$$r(n) = s(n) + v(n)$$

$$\Rightarrow \Delta = \frac{1}{N} \sum_{n=1}^N |(s(n) + v(n))|^2$$

$$\begin{aligned} \text{mean}(\Delta) &= E \left[ \frac{1}{N} \sum_{n=1}^N |(s(n) + v(n))|^2 \right] \\ &= \frac{1}{N} \sum_{n=1}^N E(s^2(n) + v^2(n) + 2s(n)v(n)) \end{aligned}$$

Since s and v are independent and have zero mean

$$\Rightarrow \text{mean}(\Delta) = \sigma_s^2 + \sigma_v^2$$

$$\begin{aligned} \text{var}(\Delta) &= \text{var}\left(\frac{1}{N} \sum_{n=1}^N |s(n) + v(n)|^2\right) \\ &= \frac{1}{N^2} \left( \sum_{n=1}^N \text{var}\left((s(n) + v(n))^2\right) + \text{covar}() \right) \end{aligned}$$

Since s and v are independent,  $\text{covar}() = 0$

$$\begin{aligned} &= \frac{1}{N^2} \left( \sum_{n=1}^N \text{var}(s^2(n) + v^2(n) + 2s(n)v(n)) + 0 \right) \\ &= \frac{1}{N^2} \left( N((\sigma_s^2)^2 + (\sigma_v^2)^2 + 2\sigma_s^2\sigma_v^2) \right) \\ \Rightarrow \text{var}(\Delta) &= \frac{(\sigma_s^2 + \sigma_v^2)^2}{N} \end{aligned}$$

So by central limit theorem, for large N,  $\Delta$  can be approximated as  $\mathcal{N}\left(\sigma_s^2 + \sigma_v^2, \frac{(\sigma_s^2 + \sigma_v^2)^2}{N}\right)$

## APPENDIX B

### Deriving Expressions for $P_d$ and $P_{fa}$

#### Probability of detection $P_d$

$$P_d = P[\Delta \geq \gamma | H_1]$$

We know that  $\Delta \sim \mathcal{N}\left(\sigma_s^2 + \sigma_v^2, \frac{(\sigma_s^2 + \sigma_v^2)^2}{N}\right)$  under  $H_1$  that is Gaussian with  $m = \sigma_s^2 + \sigma_v^2$  and

$$\sigma^2 = \frac{(\sigma_s^2 + \sigma_v^2)^2}{N}$$

$$\Rightarrow P_d = \int_{\gamma}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\Delta-m)^2}{2\sigma^2}} d\Delta$$

$$\text{Let } z = \frac{\Delta-m}{\sigma}$$

$$\Rightarrow P_d = \int_{\frac{\gamma-m}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$$

$$= Q\left(\frac{\gamma-m}{\sigma}\right)$$

Substitute m and  $\sigma$

$$\Rightarrow P_d = Q\left(\frac{\sqrt{N}}{\sigma_s^2 + \sigma_v^2} (\gamma - (\sigma_s^2 + \sigma_v^2))\right)$$

#### Probability of false alarm $P_{fa}$

$$P_{fa} = P[\Delta \geq \gamma | H_0]$$

We know that  $\Delta \sim \mathcal{N}\left(\sigma_v^2, \frac{(\sigma_v^2)^2}{N}\right)$  under  $H_0$  that is Gaussian with  $m = \sigma_v^2$  and  $\sigma^2 = \frac{(\sigma_v^2)^2}{N}$

$$\Rightarrow P_d = \int_{\gamma}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\Delta-m)^2}{2\sigma^2}} d\Delta$$

$$\text{Let } z = \frac{\Delta - m}{\sigma}$$

$$\Rightarrow P_d = \int_{\frac{\gamma - m}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$$

$$= Q\left(\frac{\gamma - m}{\sigma}\right)$$

Substitute m and  $\sigma$

$$\Rightarrow P_d = Q\left(\frac{\sqrt{N}}{\sigma_v^2}(\gamma - \sigma_v^2)\right)$$

**Probability of misdetection:**

$$P_{md} = P[\Delta < \gamma | H_1]$$

$$P_{md} = 1 - P_d$$

$$\Rightarrow P_{md} = Q\left(\frac{\sqrt{N}}{\sigma_s^2 + \sigma_v^2}((\sigma_s^2 + \sigma_v^2) - \gamma)\right)$$

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