

Highly Reliable Matrix Parity Decoding for Low Latency Interference Limited OFDM Links

A Project Report

submitted by

ANKIT BEHURA

*in partial fulfilment of the requirements
for the award of the degree of*

**MASTER OF TECHNOLOGY
&
BACHELOR OF TECHNOLOGY**



**DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY MADRAS.**

June 2014

THESIS CERTIFICATE

This is to certify that the thesis titled **Highly Reliable Matrix Parity Decoding for Low Latency Interference Limited OFDM Links**, submitted by **Ankit Behura**, to the Indian Institute of Technology, Madras, for the award of the degree of **Masters of Technology & Bachelors of Technology**, is a bona fide record of the research work done by him under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Prof. K. Giridhar
Research Guide
Professor
Department of Electrical Engineering
IIT-Madras, 600 036

Place: Chennai
Date: June 19, 2014

Prof. Sheetal Kalyani
Research Guide
Assistant Professor
Department of Electrical Engineering
IIT-Madras, 600 036

Place: Chennai
Date: June 19, 2014

ACKNOWLEDGEMENTS

Firstly, I would like to thank Dr. K Giridhar for guiding me through this project and facilitating my learning, for always motivating and encouraging me. I have thoroughly enjoyed all the insightful discussions I have had with him, no matter what the topic was. I would also like to thank Dr. Sheetal Kalyani for always providing me a feedback of how my work was progressing, and suggesting new avenues to look for whenever I was stuck. This project has been a wonderful learning experience and I would like to thank both my guides for this opportunity.

I thank my project mate Chitra, for all the help with the project and the discussions which became a part of the work done. I would also like to thank the others involved in this project, Lakshmi, Sreenivas and Venkatesh for all the inputs on various aspects of the project, and Navinnath from CEWiT, for all the help with the simulator.

I am indebted to many of my friends and classmates for making my stay in IIT a stimulating and a fun filled experience. I wish to thank Dinesh, Sidharth, Nitish, Chinmay and Vaibhav for all the amazing times of my IIT life. I would also like to thank Pranitha, Anuja, Meghana, Prakruthi, Abraham, Jayant and Soorya for their immense support and assistance. I owe my wonderful final year to them.

Lastly, I would like to thank my parents, who have always supported me in my endeavours and have made me feel that I can achieve what I dream of. They have always inspired me to aim higher and I owe everything to them.

ABSTRACT

KEYWORDS: Error Control Coding; Matrix Parity; Interference; Joint Estimation; OFDM Systems; Chirp Detection

Wireless communication systems are ubiquitous. Over the years, tremendous amount of research has gone in to develop the various paradigms of wireless communication viz. modulation schemes, error control coding, channel estimation, data acquisition and estimation schemes, securitisation of wireless links, receiver architecture etc. However, more often than not, wireless communication links are constrained by various aspects like bandwidth, channel conditions, receiver complexity, operational conditions etc. and hence it is imperative to choose the right set of schemes for each paradigm of wireless communication addressing the specific constraints for a system. In this project we have developed a communication framework for a multiuser aeronautical system which is constrained by a severe Doppler offset, fairly high data rate requirements, asynchronous users leading to interference and low latency.

We then shift our focus mainly to the error control coding scheme and effectively using the scheme in not only error control but also in joint estimation of data. Error control coding are techniques that enable reliable delivery of digital data over unreliable communication channels. This makes them an essential element in systems that are highly error prone, just like ours. We explore various soft combining and matrix parity based methods of efficiently combining repeated copies in time and frequency. We also study the effects of modulated interference on these combining schemes and select the best for implementation.

Owing its application to defence related operation, the system is prone to insecurities like eaves dropping and jamming. Jamming can completely wipe out the signal altogether, thus destroying the link altogether. Thus we also venture into jamming signals and look at a specific type of jamming signal used frequently, called chirp signals. We explore its properties and look at some effective algorithms used to detect chirp and remove them completely.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS	i
ABSTRACT	ii
LIST OF TABLES	v
LIST OF FIGURES	vii
ABBREVIATIONS	viii
NOTATION	ix
1 INTRODUCTION	1
1.1 Motivation	2
1.2 OFDM	2
1.3 System	3
1.4 Scope of work	4
2 System Design	6
2.1 OFDM Framework	6
2.1.1 Frame Structure	6
2.1.2 Subframe Structure	7
2.1.3 Subcarrier Mapping	7
2.2 Frequency Hopping	9
2.3 Error Control Coding	10
2.4 Receiver Structure	12
2.5 Data Rate Calculations	12
3 Matrix Parity	14
3.1 Matrix Parity	14
3.2 Matrix Parity Selection Combining	16

3.3	Maximal Ratio Combining	17
3.4	Effects of Modulated Interference on the Joint Decoding Techniques	19
3.5	Results in AWGN Channel	20
4	Variations of Matrix Parity Decoding	22
4.1	Weighted Flags Algorithm	22
4.1.1	Algorithm	22
4.1.2	Performance without Interference	23
4.1.3	Performance with Interference	23
4.2	Selective MRC	25
4.2.1	Algorithm	25
4.2.2	Performance without Interference	26
4.2.3	Performance without Interference	26
5	Jamming Detection	28
5.1	Chirp Signal	28
5.2	Chirp Detection using Hough-Radon Transform	29
5.3	Chirp Detection using Discrete Polynomial Phase Transform	32
6	Conclusion	35
7	Future Work	36
A	Overview of the Simulators	37
A.1	Test Simulator for Error Control Algorithms	37
A.2	CEWiT Simulator	38

LIST OF TABLES

4.1	Weights given to each of the cases for simulation	23
-----	---	----

LIST OF FIGURES

1.1	OFDM block diagram	3
2.1	Frame Structure	7
2.2	Subframe Structure	7
2.3	Sub Bands	8
2.4	Preambles	8
2.5	Tiles	9
2.6	Hopping Pattern for one User Pair	10
2.7	Example of a $\frac{8}{15}$ Matrix Parity Check Code	11
2.8	Receiver Structure	13
3.1	Example of a Matrix Parity Check Code for data stream [1,1,1,0,0,1,1,0]	14
3.2	Example of a Matrix Parity Check Code with one bit error	15
3.3	A case where error in Matrix Parity goes undetected	16
3.4	Performance of Matrix Parity Combining	17
3.5	Performance of MRC with respect to Matrix Parity Combining in Rayleigh Channel with AWGN	18
3.6	Performance of MRC and Matrix Parity Combining in Presence of Interference at 0dB in Rayleigh Channel with AWGN	19
3.7	Performance of MRC and Matrix Parity Combining in Presence of Interference at 0dB in AWGN channel	21
4.1	Performance of Weighted Flags Algorithm with respect to MRC and Matrix Parity Combining without Interference	24
4.2	Performance of Weighted Flags Algorithm with respect to MRC and Matrix Parity Combining in presence of Interference at 0dB	24
4.3	Performance of Weighted Flags Algorithm with respect to MRC and Matrix Parity Combining in presence of Interference at 0dB upto 10% of the time	25
4.4	Performance of Selective MRC with respect to MRC and Matrix Parity Combining without interference	26
4.5	Performance of Selective MRC with respect to MRC and Matrix Parity Combining in presence of interference at 0dB	27

4.6	Performance of Selective MRC with respect to MRC and Matrix Parity Combining in presence of interference at 0dB upto 30% of the time	27
5.1	Linear Chirp in Time Domain	29
5.2	Time Frequency Representation of Linear Chirp	30
5.3	Time Frequency Representation of Linear Chirp with Amplitude . .	30
5.4	Chirps in TF plane and Hough Domain	31
A.1	Illustration of Matrix Parity Coding	37

ABBREVIATIONS

OFDM	Orthogonal Frequency Division Multiplexing
QPSK	Quadrature Phase Shift Keying
IBI	Inter Block Interference
ICI	Inter Carrier Interference
LBR	Low Bit Rate
MBR	Medium Bit Rate
HBR	High Bit Rate
FFT	Fast Fourier Transform
GCL	Generalized Chirp Like
MMSE	Minimum Mean Square Error
LMMSE	Linear Minimum Mean Square Error
MRC	Maximal Ratio Combining
ML	Maximum Likelihood
AWGN	Additive White Gaussian Noise
HT	Hough Transform
HRT	Hough Radon Transform
TF plane	Time Frequency plane
DPPT	Discrete Polynomial Phase Transform
GLRT	Generalized Likelihood Ratio Test

NOTATION

h	Channel
s	Signal
C_i	i^{th} copy of the signal
f_i	Flag weight for the i^{th} signal
n_r	Number of receiver antennas
n_c	Number of copies including repetitions in both time and frequency
S	Number of subframes in a frame
T_F	Time Duration of a frame
T_S	Time Duration of a subframe
T_{OFDM}	Time Duration of an OFDM symbol
T_{cp}	Time Duration of cyclic prefix
P	Number of preambles
B	Bandwidth

CHAPTER 1

INTRODUCTION

Wireless communication systems are ubiquitous. Over the years, tremendous amount of research has gone in to develop the various paradigms of wireless communication viz. modulation schemes, error control coding, channel estimation, data acquisition and estimation schemes, securitisation of wireless links, receiver architecture etc. However, more often than not, wireless communication links are constrained by various aspects like bandwidth, channel conditions, receiver complexity, operational conditions etc and hence it is imperative to choose the right set of schemes for each paradigm of wireless communication addressing the specific constraints for a system. In this project we have developed a communication framework for a multiuser aeronautical system which is constrained by a severe Doppler offset, fairly high data rate requirements, asynchronous users leading to interference and low latency.

We then shift our focus mainly to the error control coding scheme and effectively using the scheme in not only error control but also in joint estimation of data. Error control coding are techniques that enable reliable delivery of digital data over unreliable communication channels. This makes them an essential element in systems that are highly error prone, just like ours. We explore various soft combining and matrix parity based methods of efficiently combining repeated copies in time and frequency. We also study the effects of modulated interference on these combining schemes and select the best for implementation.

Owing its application to defence related operation, the system is prone to insecurities like eaves dropping and jamming. Jamming can completely wipe out the signal altogether, thus destroying the link altogether. Thus we also venture into jamming signals and look at a specific type of jamming signal used frequently, called chirp signals. We explore its properties and look at some effective algorithms used to detect chirp and remove them completely.

1.1 Motivation

In any communication link, coding and decoding form an integral part of the transmit and receive chain. We have multiple antennas for better reception at the receiver. These antennas give rise to diversity and enable combining of the signals received at various antennas to get a better version of the received signal. Similarly, we also have multiple copies of data by repetition of symbols in time and frequency domain. This can be viewed as data received from slightly different channels, which also allows us to apply data combining methods. The main thrust of this thesis is to explore the diversity in the different copies of data obtained and how to effectively combine data in the presence of interference.

Joint estimation is important when we have multiple copies of data. Since we are not receiving the multiple copies of symbols from different antennas, there is a lot of scope to explore the characteristics of the various copies we receive and extract meaningful data from them. The main aim of my project is to explore how we can effectively get the most useful data from various copies in the presence and absence of interference, using knowledge of the channel and the system framework.

There is a requirement for the system to be designed on an OFDM framework. The next section explains OFDM, and the coding, decoding and combining is part of the constellation mapping and equalization.

1.2 OFDM

OFDM - Orthogonal Frequency Division Multiplexing is a method of encoding digital data on multiple carrier frequencies. A large number of closely spaced orthogonal sub carrier signals are used to carry data on several parallel data streams or channels. Each sub carrier is modulated with a conventional modulation scheme at a low data rate, which in our case will be QPSK modulation.

The primary advantage of OFDM over single-carrier schemes is its ability to cope with severe channel conditions - attenuation of a certain set of frequencies or narrow-band jamming, interference, and frequency selective multi-path fading. Channel equalization is also simplified because OFDM can be viewed as using many slowly mod-

ulated narrowband signals rather than one rapidly modulated wideband signal. (Cho *et al.*, 2010; Chiueh and Tsai, 2007)

Hence we design our required system with OFDM framework and modify those aspects which are specific to the aeronautical characteristic of our model.

1.3 System

Figure 1.1 describes any conventional OFDM framework.

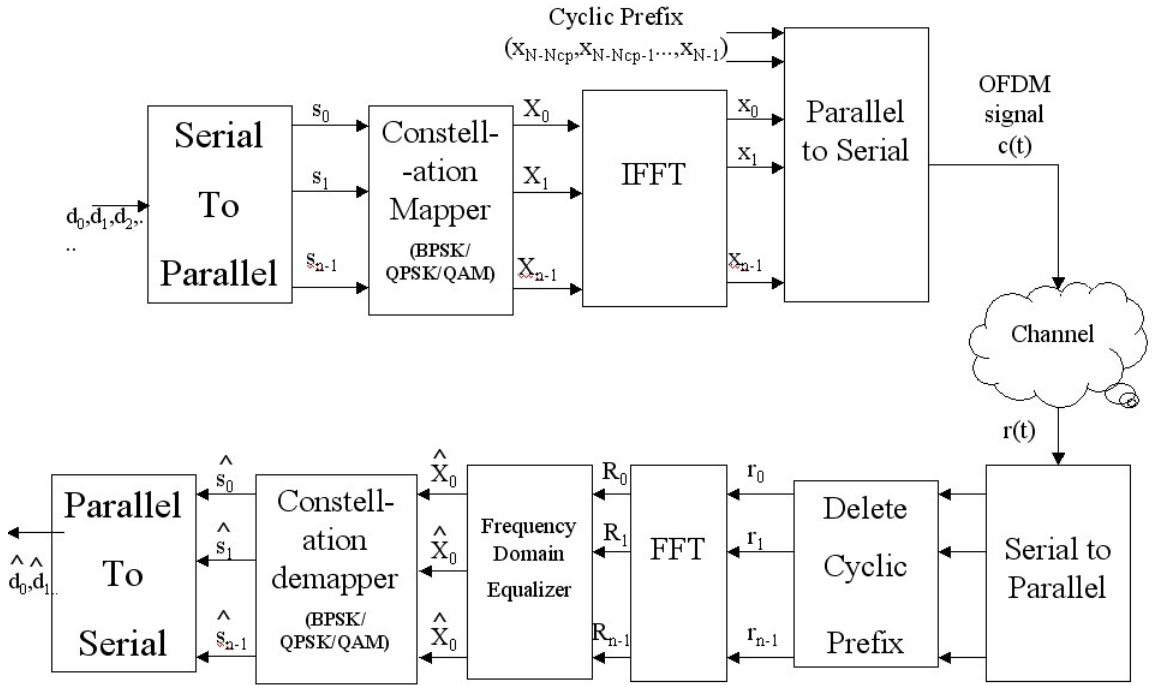


Figure 1.1: OFDM block diagram

In our work, we will specifically be looking at the receiver architecture for Equalization, Soft Combining and Decoding. The conventional OFDM structure needs to be improvised to cater to the specific requirements of the link. The main requirements imposed are:

- Need to have low latency in decoding and fast-acquisition in high Doppler environment.
- The error correction needs to be as simple as possible for lower latency and complexity in processing the data.

- Due to the presence of strong interferers, the decoded data needs to be highly reliable.
- This system is also affected by narrow band frequency jamming, to account for which the data needs to be distributed intelligently.
- The number of users in the system is variable. Correspondingly, variable data rates need to be available based on the number of users currently using the resources.

The various issues affecting the system have also been highlighted above. Another issue is that the transmission from various users is asynchronous. Considering each of these issues, we have designed an OFDM framework for the transmitter and receiver chain. This design modifies the way frequency subcarrier allocation, hopping, equalization is done. All these design aspects will be explained further in the next chapter on System Model.

1.4 Scope of work

As described in Figure 1.1, the OFDM communication model is made up of a number of blocks. These blocks have been combined into modules and we have worked on the data combining, coding and decoding part of the receiver chain. The various requirements and issues with the system have been explained in the previous section. Every issue is addressed with specific additions in the framework design, each of which is explained in the next chapter.

To test the coding, decoding and combining part of the receiver chain, we have implemented various algorithms on our coding-decoding test simulation in Matlab, for scenarios with and without interference. This simulator is explained in the Appendix.

For testing the entire system from the transmitter to the receiver with synchronization, equalization, channel estimation, coding, decoding and combining, we have worked on the simulator provided by CEWiT - Center for Wireless Technology, at the IIT Madras Research Park. This work has just been started and there is a lot of scope to continue simulations of various test cases on this simulator. We have integrated the code on combining various copies, but the test cases can be refined further and the algorithm can be improved.

Chapter 2 explains the System Model in detail, about each aspect of the OFDM framework and how we have had to modify the various aspects of conventional OFDM systems. It also explains the entire receiver architecture, coding and modulation schemes used, frame structure and subcarrier allocations.

Chapter 3 explains matrix parity and its error correction and detection properties. We then look at a simple selection combining with matrix parity and study its performance with and without interference in comparison with maximal ratio combining. Chapter 4 takes the matrix parity selection combining to the next level, by improving it with some new ideas. We look at a couple of more algorithms and their performance in the presence and absence of interference.

Chapter 5 talks about chirp signals, one of the commonly used jamming signals. We study its properties and look at a couple of novel ways of detection of chirp signals. Chapter 6 then concludes all the the work done through out the course of the thesis. Chapter 7 enlists a few future work that can be done to improve few particular modules, and hence the overall system.

The project was a collaborative effort of mine as well as another dual degree student Chitra K. R. Some other important aspects of the project like joint estimation using soft combining, Maximum Likelihood based techniques, can be found in her thesis, Chitra (2014).

CHAPTER 2

System Design

The OFDM Link that has been designed had some unique requirements. Having applications in defence aeronautical purposes, the link is subject to highly malignant channel conditions and security threats like eaves dropping and jamming, which can potentially wipe out the signal altogether. The bandwidth can be used by a number of users at the same time, without any prior notification. Thus the users are asynchronous, which in turn can lead to inter block interference (IBI). In order to achieve maximum spectral efficiency, multiple users could operate at the same frequency thus leading to Inter Carrier Interference (ICI) and IBI induced ICI.

The OFDM framework and receiver architecture developed, can deal with all the above impairments up to a certain degree. The framework is also highly scalable in terms of number of users and data rates, thus satisfying the requirement of operation in three different data rates, Low Bit Rate (LBR), Medium Bit Rate (MBR) and High Bit Rate (HBR). The following sections briefly introduce each design element of the overall system.

2.1 OFDM Framework

2.1.1 Frame Structure

The frame consists of S subframes. The first subframe consists of a wide band preamble which is used for synchronization. The subframe in the middle contains a narrowband pilot which is used for channel estimation. Rest of the subframes carry data. Each frame is T_F s long.

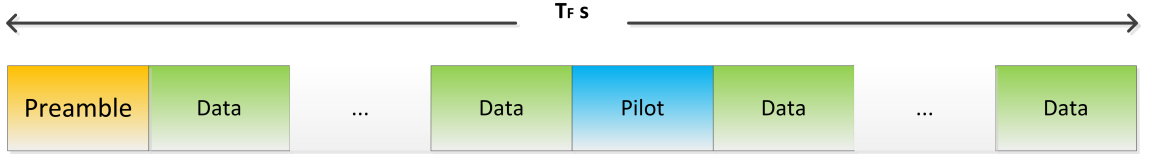


Figure 2.1: Frame Structure

2.1.2 Subframe Structure

Each subframe consists of a cyclic prefix of T_{cp} s followed by two OFDM symbols T_{OFDM} s long each, adding up to a total of $2T_{OFDM} + T_{cp}$ s. Each OFDM symbol consists of N QPSK symbols. The repetition in preamble subframe gives higher reliability for the synchronization algorithm especially for cases when there is heavy interference. Since the system is heavily affected by high Doppler offset, it is paramount that the Doppler estimation is perfect.

Repetition in data subframes can be perceived as diversity in time domain. Instead of decoding one copy of the data, an efficient joint decoding can be done on two copies instead to gain from the diversity. This of course reduces the spectral efficiency, but the design meets the data rate requirements.

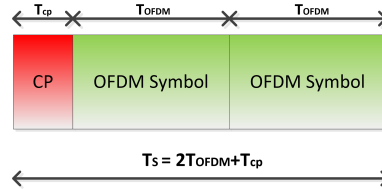


Figure 2.2: Subframe Structure

2.1.3 Subcarrier Mapping

The system is built over a B MHz band. The bandwidth is then divided into N subcarriers, enumerated from $-\frac{N}{2} + 1$ to $\frac{N}{2}$. The subcarriers are further classified into several sub groups and mapped to certain type of symbols. All these further classifications are described below.

Upper and Lower Bands

The N subcarriers are further subdivided into two sub bands, upper and lower. The upper band (UB) consists of subcarriers 1 to $\frac{N}{2}$. The lower band (LB) consists of subcarriers $-\frac{N}{2} + 1$ to -1 . Each user is given a certain number of subcarriers in UB and LB. In order to overcome jamming, the user sends the same data over both the bands.

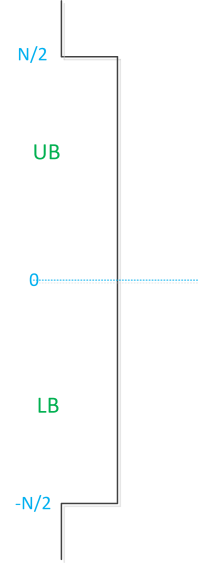


Figure 2.3: Sub Bands

Preamble Subcarriers

Certain number of equispaced subcarriers are assigned as preamble subcarriers. There are P preamble subcarrier and between every two preamble, there is a spacing of $\frac{N}{P} - 1$ subcarriers. In the preamble subframe, only these subcarriers are used to transmit GCL sequences. The preamble is used primarily for timing and frequency synchronization using a variant of the well known Schmidl Cox algorithm. It can also be used for channel estimation. In data and pilot subframes, these subcarriers carry null tones.

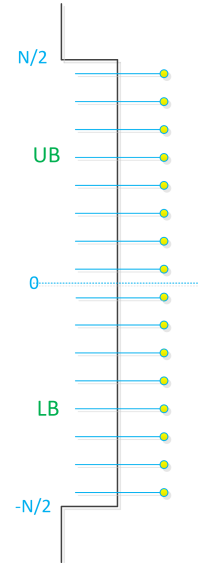


Figure 2.4: Preambles

Blocks and Tiles

The preamble subcarriers are all equispaced. Every two preamble subcarrier is separated by $\frac{N}{P} - 1$ subcarriers. This cluster of $\frac{N}{P} - 1$ subcarriers is called a block. Each block is further divided into three groups with equal number of subcarriers, called a tile. In MBR, each user is allocated one tile each from UB and LB. The assignment is done based on a fixed hopping pattern as described later. The user sends the same data over both the tiles, as mentioned earlier, in order to avoid jamming.

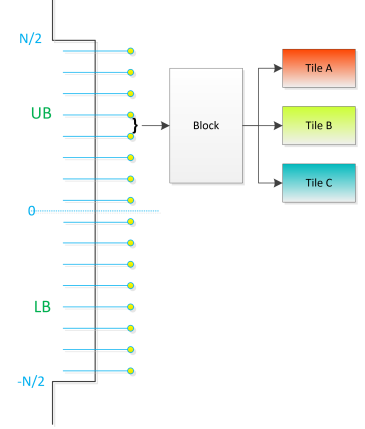


Figure 2.5: Tiles

Pilot

The tiles allocated to the user for data are allocated for pilots in the pilot subframe. Balanced QPSK is used for pilot symbols. The pilot is used for channel estimation. Since the pilot symbol is just as long as a tile, no other orthogonal sequences like PN sequences could be used.

Signalling and Future use Tones

Certain subcarriers are also left for future use. Some of them will be used for signalling whether the system is currently running at LBR or MBR or HBR. Subcarriers -1, 0 and 1 are just null tones.

2.2 Frequency Hopping

The tiles allotted to a user is altered every frame based on a prefixed periodic sequence. The number of hops per second is fairly high. Hopping is another added feature to deal with jamming. The jammer will have no information about the frequency at which the user is hopping and hence the best it can do is to make a guess.

In order to avoid inter-user interference, the users are paired and each user pair is allotted three tiles in each sub-band, chosen pseudo randomly. In every frame, the users

hop only among these tiles. The hopping pattern is illustrated below.

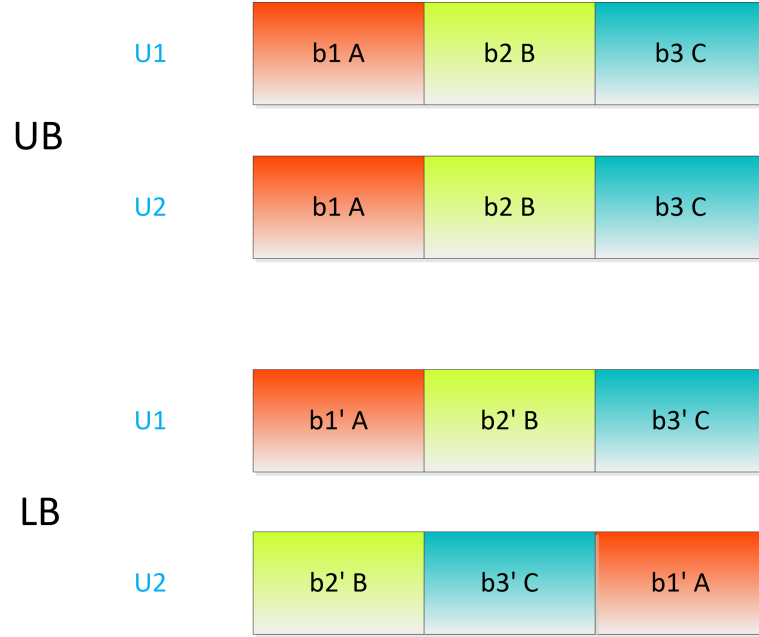


Figure 2.6: Hopping Pattern for one User Pair

As shown in the figure, both user 1 (U1) and user 2 (U2), hops amongst block 1 tile A (b1 A), block 2 tile B (b2 B) and block 3 tile C (b3 C) in the upper band. Hence if both U1 and U2 are present, they will interfere in the upper band. However in the lower band, U1 keeps hopping in the same sequence of tiles as in the upper band, but U2 is cyclically shifted by one tile and hence when U1 is on tile A, U2 is on tile B. This guarantees that when both the users are present, atleast one copy in the frequency domain is safe. This ensures maximum spectral efficiency while dealing with jamming and possibly compromising on the diversity gain by introducing interference in one of the copies.

2.3 Error Control Coding

Error control coding are techniques that enable reliable delivery of digital data over unreliable communication channels. This makes them an essential element in systems which are highly error prone. Error control coding can be broadly divided into two types, error detection and error correction codes. Error detection techniques can detect errors in the code while error correction enables reconstruction of the original data in case the received data has errors.

The general idea for achieving error detection and correction is to add some redundancy (i.e., some extra data) to a message, which receivers can use to check consistency of the delivered message, and to recover data determined to be corrupted. Error-detection and correction schemes can be either systematic or non-systematic: In a systematic scheme, the transmitter sends the original data, and attaches a fixed number of check bits (or parity data), which are derived from the data bits by some deterministic algorithm. If only error detection is required, a receiver can simply apply the same algorithm to the received data bits and compare its output with the received check bits; if the values do not match, an error has occurred at some point during the transmission. In a system that uses a non-systematic code, the original message is transformed into an encoded message that has at least as many bits as the original message.

Over the years several error control techniques have been developed. From something as simple as parity bits, Cyclic Redundancy Check (CRC) etc. to something as sophisticated as turbo codes. The system we are working on needed to have a very low latency. Thus we used something that is very simple and yet fairly powerful in both error detection and correction. Matrix Parity check code at a code rate of $\frac{8}{15}$ is used for error control.

1	1	0
1	0	1
0	1	1
1	0	1
1	0	0

Figure 2.7: Example of a $\frac{8}{15}$ Matrix Parity Check Code

The figure shown above is a $\frac{8}{15}$ matrix parity check code. Symbols in blue are actual data. Symbols in green are the parity of each row or column and are called the primary parity symbols. The one in red is the parity of all primary parity symbols and is called the secondary parity symbol. Thus there are eight data symbols and seven added parity symbols. Matrix parity has the capability of correcting one bit errors and can get detect

higher order errors. We will look into Matrix parity carefully in the next chapter.

2.4 Receiver Structure

The system currently consists of two antennas at the receiver. Both the antennas receive four copies of the same data each, repeated once in time and once in frequency. Each frame consists of 8 subframes, of which 6 are data subframes. The first subframe, the preamble is first used to estimate Doppler, and perform frequency and timing synchronization. This is then passed on to the correlators which determine the best FFT windows to be used. The module outputs two separate OFDM symbols for each subframe. FFT is performed on these subframes to convert the time domain symbols into frequency domain symbols.

The pilot subframe is then used to estimate the channel. The symbols are then jointly equalized using either LMMSE or MRC thus reducing the overall number of copies from eight to four. A joint estimation is then done using the four copies, by efficiently combining them and doing the matrix parity decoding. The decoded data can be used further to determine Figure 2.8 is a flow diagram that shows the receiver structure as described.

2.5 Data Rate Calculations

Each frame has $S - 2$ data subframes. In case of MBR, each user is effectively allocated 2 tiles. The user transmits same data over both the tiles, thus transmitting $\frac{\frac{N}{P}-1}{3}$ QPSK symbols over each subframe. Now there are $S - 2$ data subframes in every frame. So the user transmits $(S - 2)(\frac{\frac{N}{P}-1}{3})$ QPSK symbols or $2(S - 2)(\frac{\frac{N}{P}-1}{3})$ bits over a frame coded at a rate of $\frac{8}{15}$. Thus,

$$BitRate = \frac{\frac{8}{15}[CodeRate] * 2(S - 2)(\frac{\frac{N}{P}-1}{3})[BitsperFrame]}{T_{Fs}} \quad (2.1)$$

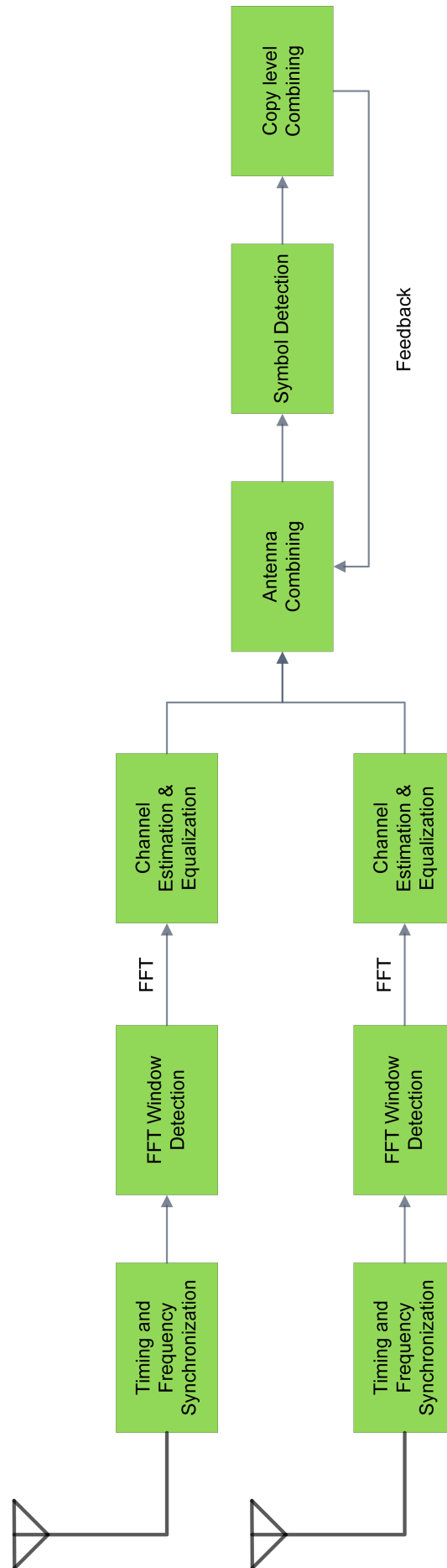


Figure 2.8: Receiver Structure

CHAPTER 3

Matrix Parity

3.1 Matrix Parity

Matrix parity is an error control code that can correct one bit errors and detect higher order errors unless one of the exceptional cases happens. It is a very simple code to implement practically. Following is an instance of matrix parity check code.

1	1	0
1	0	1
0	1	1
1	0	1
1	0	0

Figure 3.1: Example of a Matrix Parity Check Code for data stream [1,1,1,0,0,1,1,0]

Suppose there is a sequence of n symbols in one dimension. The parity of these n symbols in one dimension is the modulo 2 addition of all the n symbols.

$$paritysymbol = \sum_{i=1}^n S_i \quad (3.1)$$

In the matrix parity example shown in Figure 3.1, symbols in blue are the actual data symbols. The primary parity bits (shown in green) in the last column are the one dimensional parity of their respective rows. Similarly the primary parity symbols in the last row are the one dimensional parity symbols of their respective columns. The

Secondary parity symbol (shown in red) is the one dimensional parity of all the six primary parity symbols.

Error Correction

Matrix parity has the capability of correcting one symbol error.

1	1	0
1	1	1
0	1	1
1	0	1
1	0	0

Figure 3.2: Example of a Matrix Parity Check Code with one bit error

The figure shown above is the same matrix parity as in Figure 3.1 but with symbol in the second row and second column ([2,2]) in error. [2,2] is flipped and is shown in a chequered box. Now the primary parity symbols [2,3] and [5,2] are not satisfied, which implies that the symbol [2,2] must have been in error, thus making one bit error correction possible.

Error Detection

Single symbol errors can be corrected. If there are more number of errors, the matrix parity check code can detect that the block has errors but can not possibly predict how many or the location of the errors. There is however one possibility in which the error will go completely undetected.

In Figure 3.3, symbols [2,1], [2,2], [3,1], [3,2] are in error. However the primary and secondary parity symbols are still satisfied. Hence such 4 symbol error and any of its polymorphic variant will go undetected from matrix parity check code. However the probability of this happening is very low.

1	1	0
0	1	1
1	0	1
1	0	1
1	0	0

Figure 3.3: A case where error in Matrix Parity goes undetected

3.2 Matrix Parity Selection Combining

The receiver receives multiple copies of the same data and hence has to perform a joint estimation using all the copies to reliably decode the symbols. Matrix Parity combining is one way to do it. The receiver first performs an ML estimate individually on the received symbols. In QPSK assuming Gaussian noise, ML estimate is same as that of hard decoding. After doing symbol level decoding, the receiver performs a Matrix parity block level decoding. This decoding returns whether or not the block has errors in it or not. The basic idea of Matrix Parity Combining is to discard copies with lots of errors and keep copies with zero or comparatively lesser errors. The algorithm for Matrix Parity Combining with two copies is as follows:

- If no errors or 1 bit error which is corrected by matrix parity, keep both the copies
- If only one of the copies has more than one errors, replace the copy with the better one
- If both have more than one errors, keep the copy with least number of errors

The graph in Figure 3.4 compares matrix parity combining with just matrix parity decoding and with no matrix parity. Matrix parity combining performs best among the three.

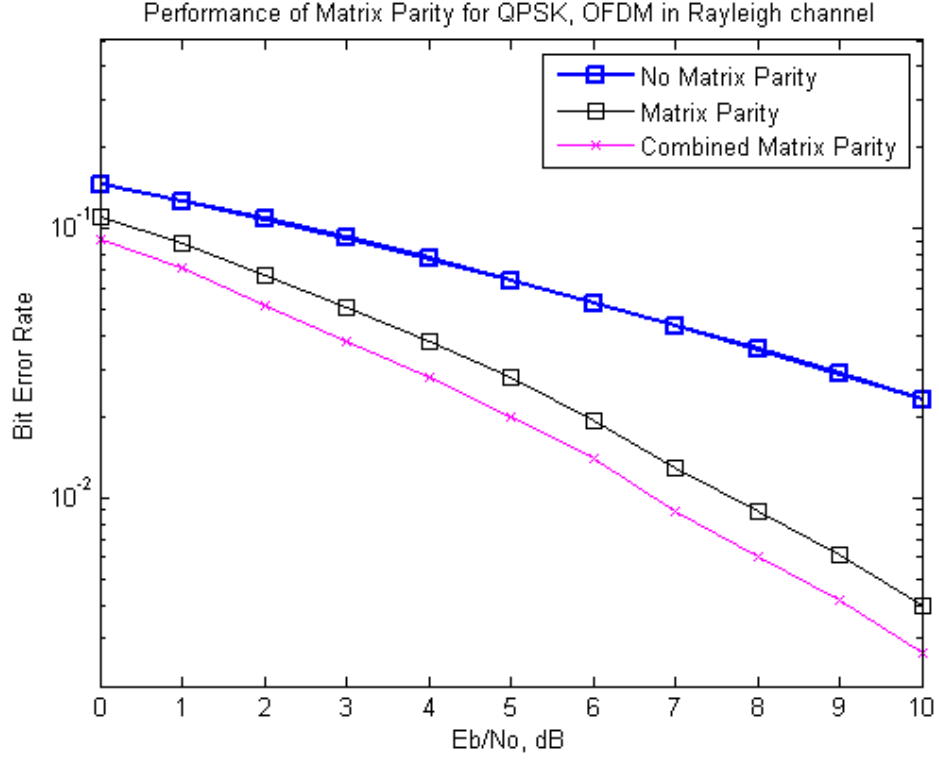


Figure 3.4: Performance of Matrix Parity Combining

3.3 Maximal Ratio Combining

Maximal Ratio Combining is the optimum combiner for data received from multiple antennas in the same system. MRC takes advantage of the receiver antenna diversity to output an optimum estimate of the received data. Let the received signal on the i^{th} of n antennas is given by:

$$y_i = h_i x + n_i \quad (3.2)$$

Where, y_i is the received symbol on the i^{th} receive antenna, h_i is the channel on the i^{th} receive antenna, x is the transmitted symbol and n_i is the noise on i^{th} receive antenna. The equalized symbol is then given by:

$$\hat{x} = \frac{h_1^* y_1 + h_2^* y_2 + \dots + h_{n_c}^* y_{n_c}}{|h_1|^2 + |h_2|^2 + \dots + |h_{n_c}|^2} \quad (3.3)$$

$$\hat{x} = \frac{\sum_{i=1}^{n_c} h_i^* y_i}{\sum_{i=1}^{n_c} |h_i|^2} \quad (3.4)$$

In the system, the multiple copies may be seen as received data coming in from different antennas. By knowing the channel or an estimate of it for all the copies, we can have an optimum estimate of all the symbols, following which a matrix parity check code is performed to correct any one bit errors.

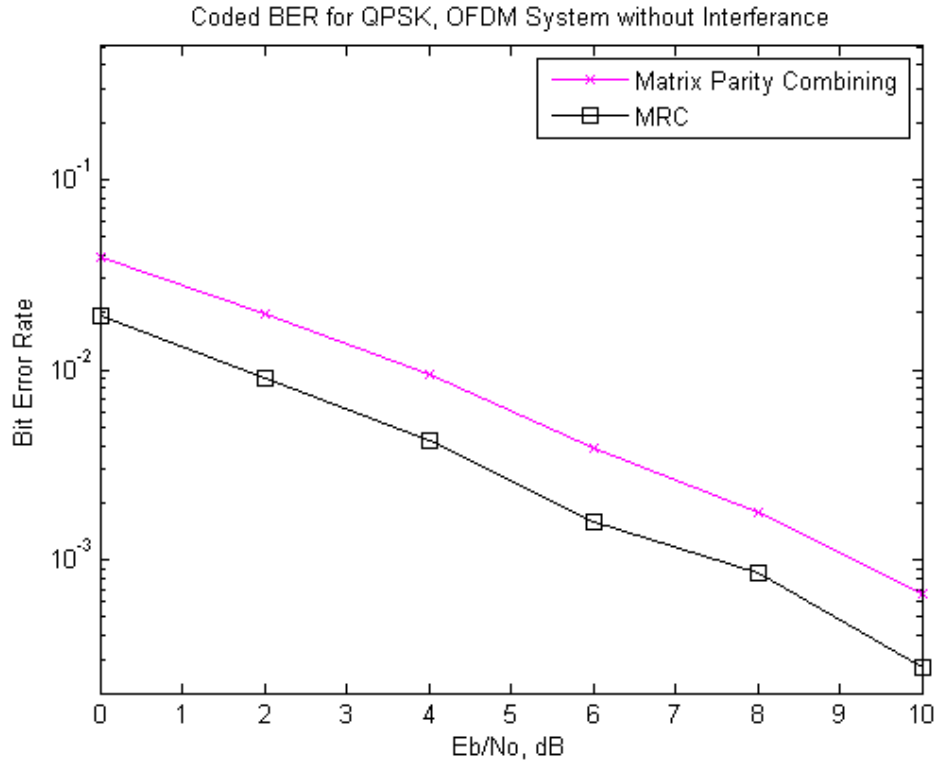


Figure 3.5: Performance of MRC with respect to Matrix Parity Combining in Rayleigh Channel with AWGN

The graph in Figure 3.5 compares the performance of MRC with respect to matrix parity combining with two copies. MRC performs better than matrix parity combining in the case shown, where there is no interference.

3.4 Effects of Modulated Interference on the Joint Decoding Techniques

As described in the previous sections, we developed two different algorithms to jointly decode the multiple copies of the data received, one by using the combined matrix parity decoding and the other by using the well known Maximal Ratio Combining (MRC) followed by doing matrix parity decoding. The results shown in the previous section were impaired only by the Rayleigh channel and AWGN noise. But, there could also be a case when both the users in an user pair are active, thus interfering with each other in one of the bands. This kind of interference is referred to as modulated interference as the interfering signal is also similarly modulated as the original signal. The following plot shows the performance of both the algorithms in the presence of interference, interfering at 0dB.

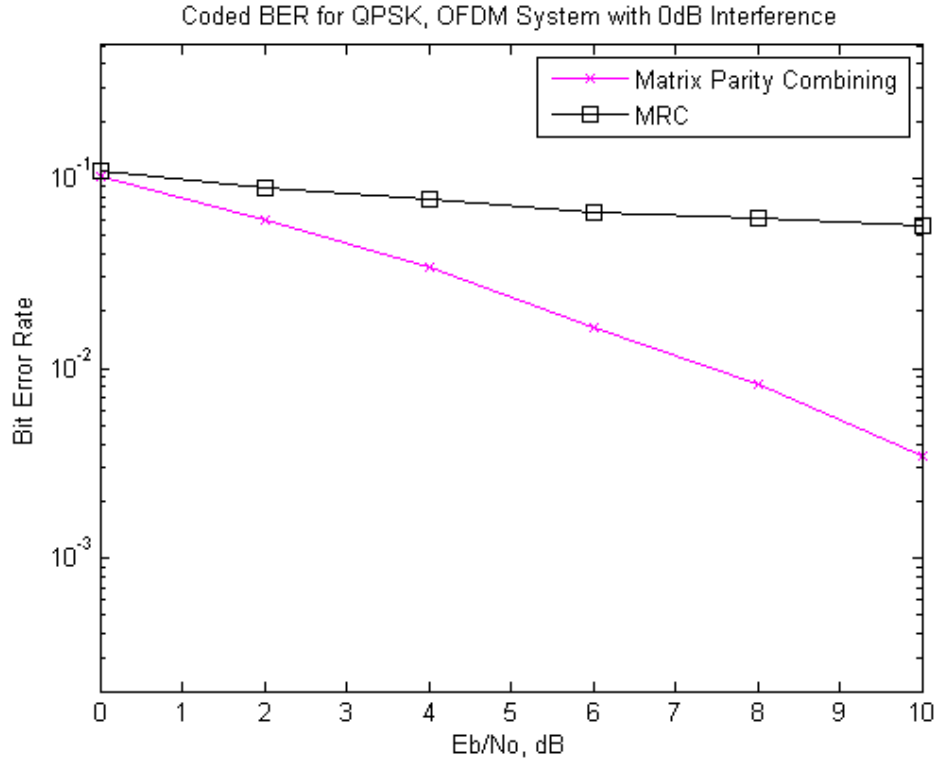


Figure 3.6: Performance of MRC and Matrix Parity Combining in Presence of Interference at 0dB in Rayleigh Channel with AWGN

As expected, the performance of both the algorithms has deteriorated with respect to the case when there was no interference shown in Figure 3.5. But, we can also observe that while in Figure 3.5, MRC performed better, in Figure 3.6, matrix parity combining

performs better. Thus in the presence of interference, matrix parity combining performs better than MRC. This can be explained by the fact that in MRC, both the copies are combined weighted by the conjugate of their respective channels and then normalized. The weights do not have any information about the interference. Now as per the system design, in case of presence of interference, we know for sure that one copy is completely corrupted. Now when this copy is combined with the relatively good copy, the whole combination gets worse.

Thus without interference, MRC works better, but in the case when there is interference, combined matrix parity works better. Thus for efficient decoding, interference detection would help as if we know the presence of interference with some degree of accuracy, we would perform matrix parity decoding on those copies, instead we could perform MRC. In the following chapter we look at a couple of improvements to these algorithms, keeping in mind the presence of interference.

3.5 Results in AWGN Channel

The previous section illustrated the performance of matrix parity combining and MRC in Rayleigh channel, which is fairly severe. For the sake of completeness, it is imperative to understand the results in benign channel conditions as well, with just AWGN. As can be expected, the performance in just AWGN channel will be much better. In the absence of interference, the BER is 0 through out, for all SNRs. In the presence of interference, the performance worsens, yet it is much better compared to Rayleigh channel, as illustrated in Figure 3.7

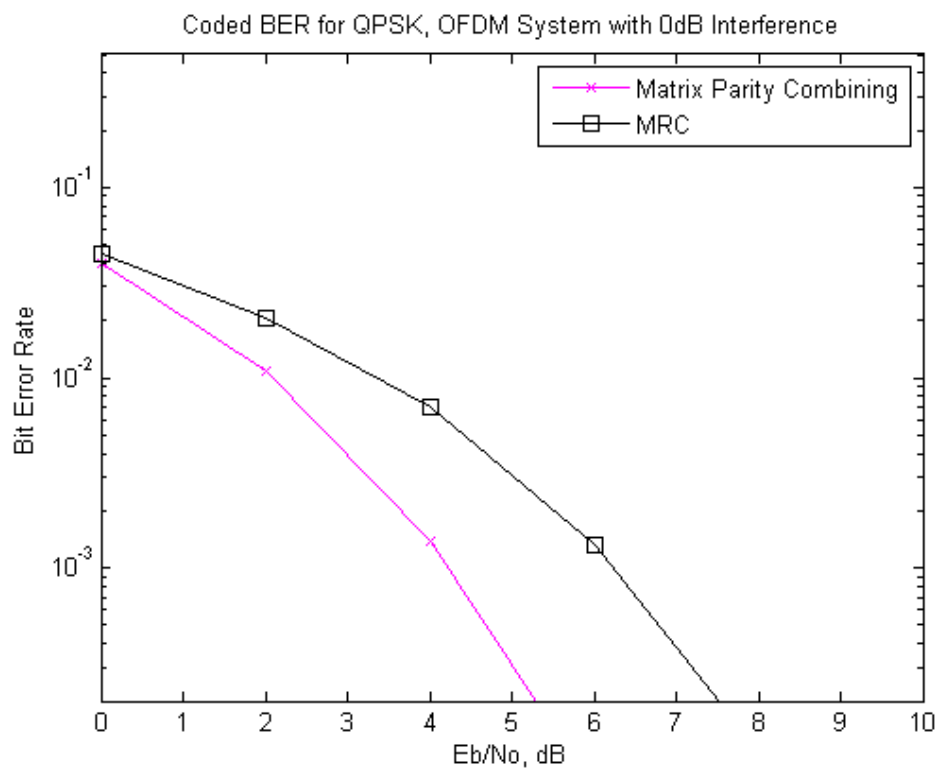


Figure 3.7: Performance of MRC and Matrix Parity Combining in Presence of Interference at 0dB in AWGN channel

CHAPTER 4

Variations of Matrix Parity Decoding

In the previous chapter we worked with two algorithms, matrix parity decoding and MRC. When the channel was impaired only by AWGN, MRC worked better as it is the optimal way of combining the two copies. But in presence of interference, matrix parity combining works better as it completely discards the copy that is heavily corrupted. In this chapter, we look at two different algorithms that focus on interference rejection to efficiently combine the two copies, and we compare their performance against the previous algorithms.

4.1 Weighted Flags Algorithm

4.1.1 Algorithm

In weighted flags method of joint decoding of the two copies, first matrix parity decoding is performed on both the copies, and based on the number of errors in the matrix parity, a weight between 0 to 1 is assigned to the matrix parity, and a weighted MRC is performed. Let the two copies be C_1 and C_2 , the two flags be f_1 and f_2 and the corresponding channels be h_1 and h_2 , then the weighted MRC for the two copies is as follows:

$$MRC_{weighted} = \frac{h_1^* f_1 C_1 + h_2^* f_2 C_2}{f_1 |h_1|^2 + f_2 |h_2|^2} \quad (4.1)$$

As we saw in the case of MRC, in the presence of interference, when the two copies are combined with equal weight, the overall combination gets messed up. Hence the rationale for doing an weighted MRC is to combine the good copy in a higher proportion and the bad copy in a lower proportion. Now the following four cases could arise while decoding the matrix parity:

1. Passes the check without any errors
2. Has 1 bit error which is corrected
3. Error in parity bits
4. Error in data bits

The weighting is done in the same order as shown above. If the matrix passes without any errors, the weight associated to it is 1 assuming the probability of exactly all the data bits flipping is very small. In case there is Error in parity bits, the probability of error in data bits is smaller and hence it is given higher priority. The weights as such is by trial and error and are shown in Table 4.1. In future, better heuristics could be studied to improve its performance.

Table 4.1: Weights given to each of the cases for simulation

Case	Weight
1	1.0
2	0.9
3	0.3
4	0.1

4.1.2 Performance without Interference

The performance of weighted flags algorithm in absence of interference is as good as MRC and hence better than matrix parity combining.

4.1.3 Performance with Interference

In the presence of interference, Matrix parity combining continues to be the best as weighted flag algorithm still combines a finite weight of the completely corrupted copy. However, unlike cellular systems in which interference is present all the time, our system has its applications in aeronautical systems, where interference can occur intermittently but will not persist for 100% of the time. Hence we do a trial and error analysis trying to figure out upto what extent of interference, can weighted flag algorithm perform better than matrix parity combining. These results indicate that the weighted flags

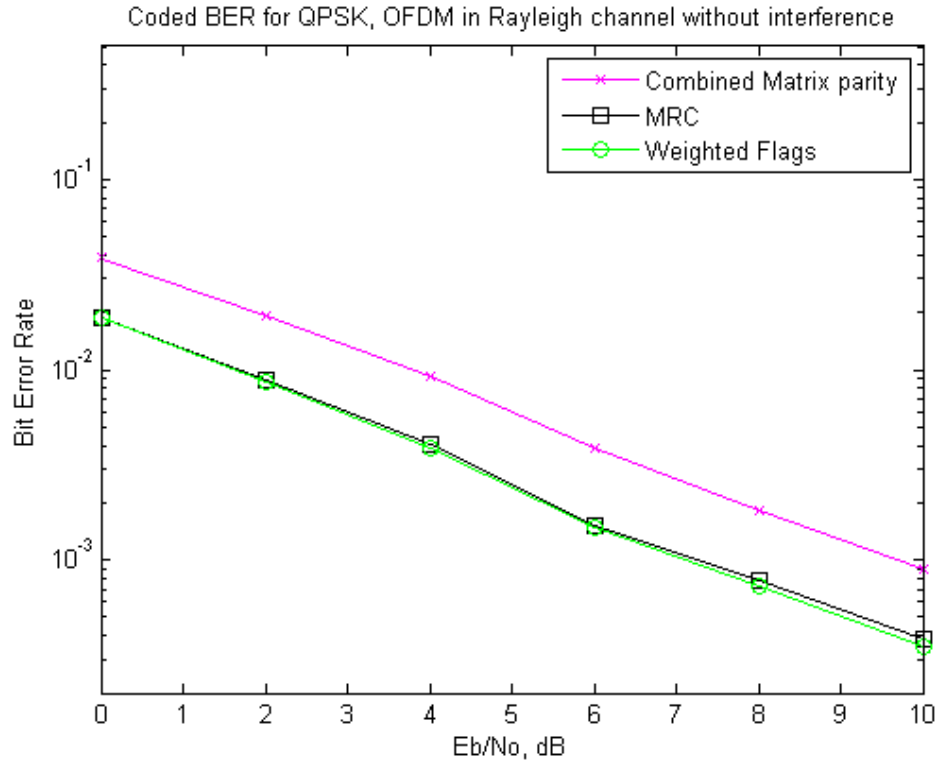


Figure 4.1: Performance of Weighted Flags Algorithm with respect to MRC and Matrix Parity Combining without Interference

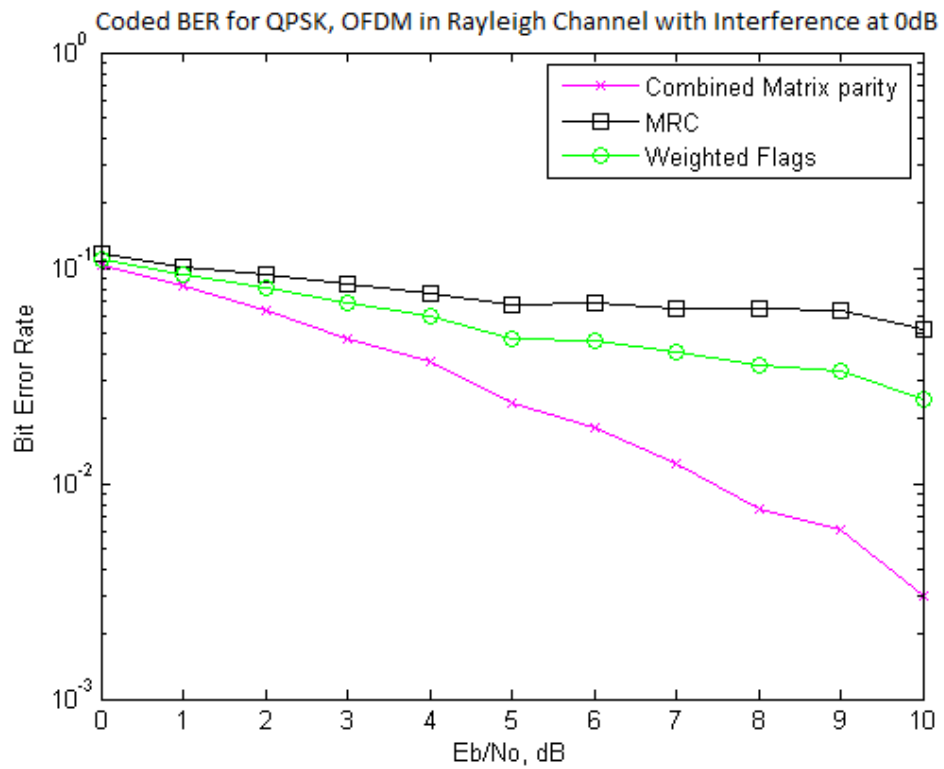


Figure 4.2: Performance of Weighted Flags Algorithm with respect to MRC and Matrix Parity Combining in presence of Interference at 0dB

algorithm performs better than matrix parity combining in cases where there is interference upto 10% of the time.

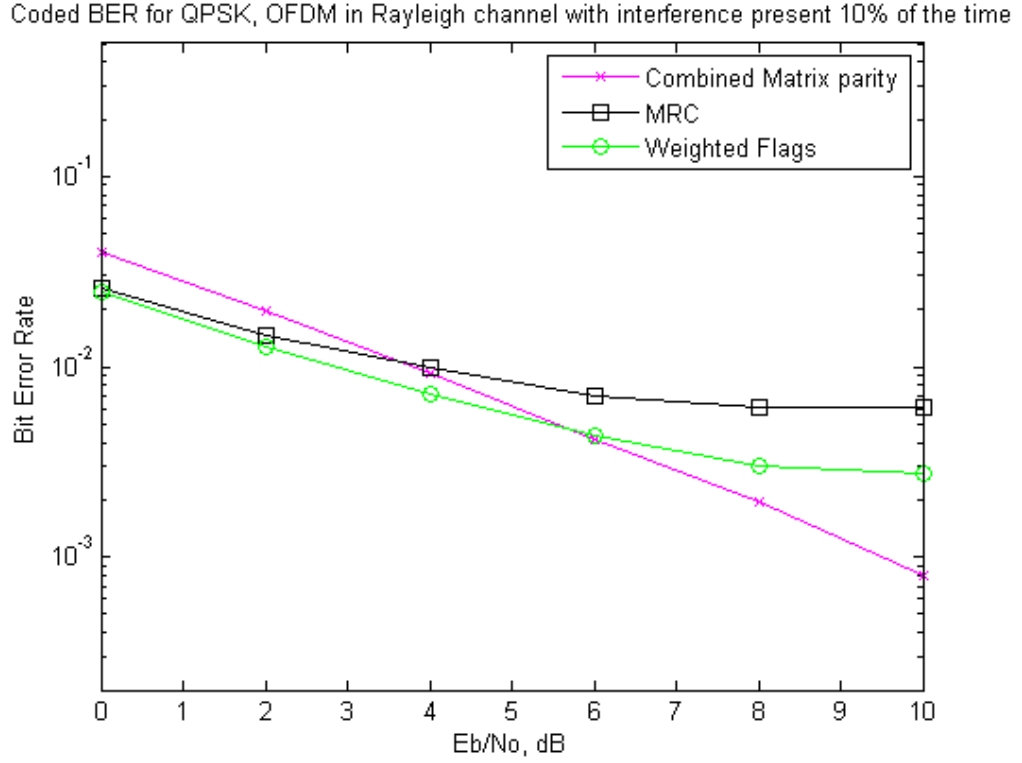


Figure 4.3: Performance of Weighted Flags Algorithm with respect to MRC and Matrix Parity Combining in presence of Interference at 0dB upto 10% of the time

4.2 Selective MRC

4.2.1 Algorithm

In the previous algorithm, the copy which was corrupted was still being accounted for with a finite weight. In this algorithm, we try and completely eliminate the corrupted copy. In this algorithm, we first perform matrix parity decoding on both the copies. If they don't have any errors (passed the matrix parity check) or has a lot of errors, then we do matrix parity combining on the two copies, i.e. we discard the bad copy. Otherwise we perform MRC on both the copies.

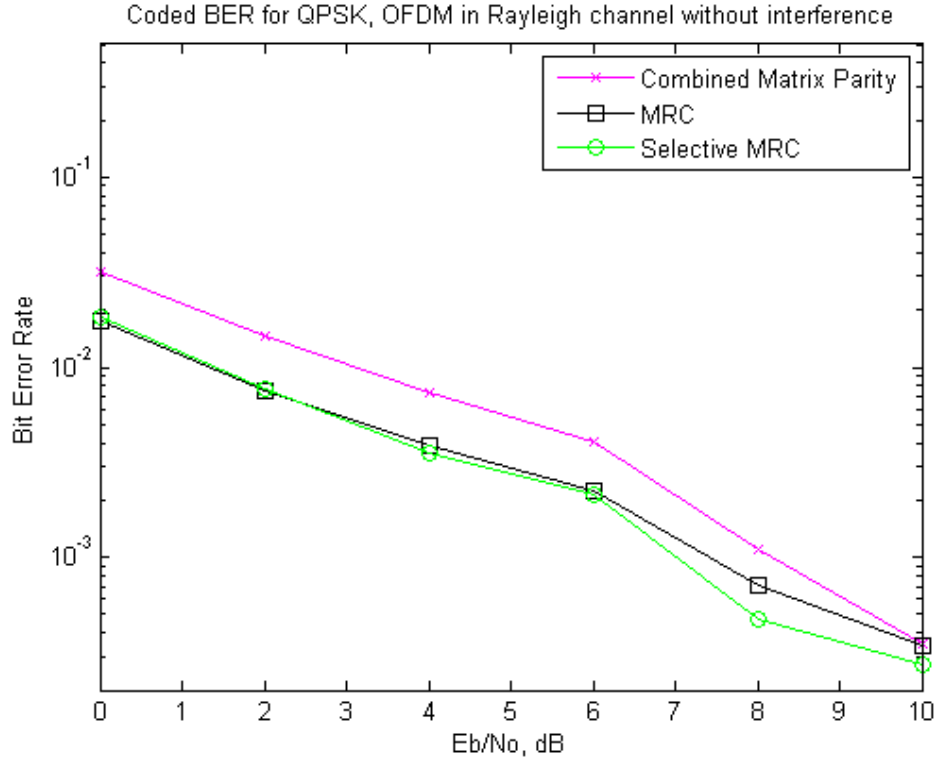


Figure 4.4: Performance of Selective MRC with respect to MRC and Matrix Parity Combining without interference

4.2.2 Performance without Interference

In absence of interference, selective interference works as good as MRC, in fact works better than MRC beyond 6dB of noise.

4.2.3 Performance without Interference

The performance of selective MRC is almost as good as that of matrix parity combining, as shown in Figure 4.5 As described in the previous section, unlike cellular systems where interference is always present, the application of our system is in aeronautical fields, where interference is intermittent. Hence by trial and error we find that selective MRC performs better than matrix parity combining, when interference is present upto 30% of the time. The figure is shown in Figure 4.6

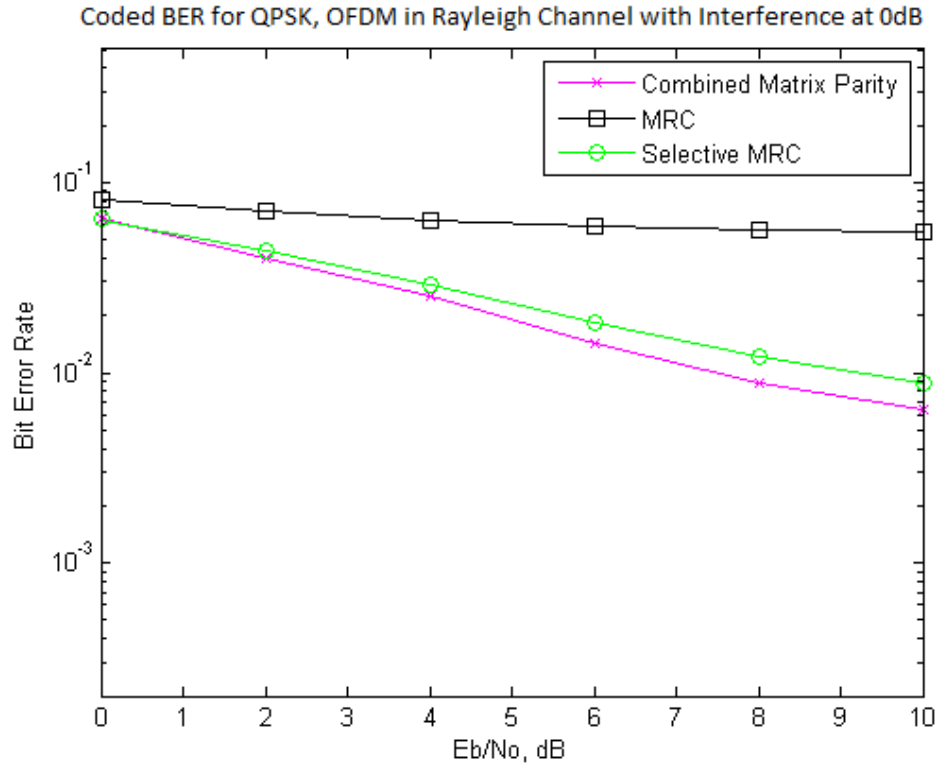


Figure 4.5: Performance of Selective MRC with respect to MRC and Matrix Parity Combining in presence of interference at 0dB

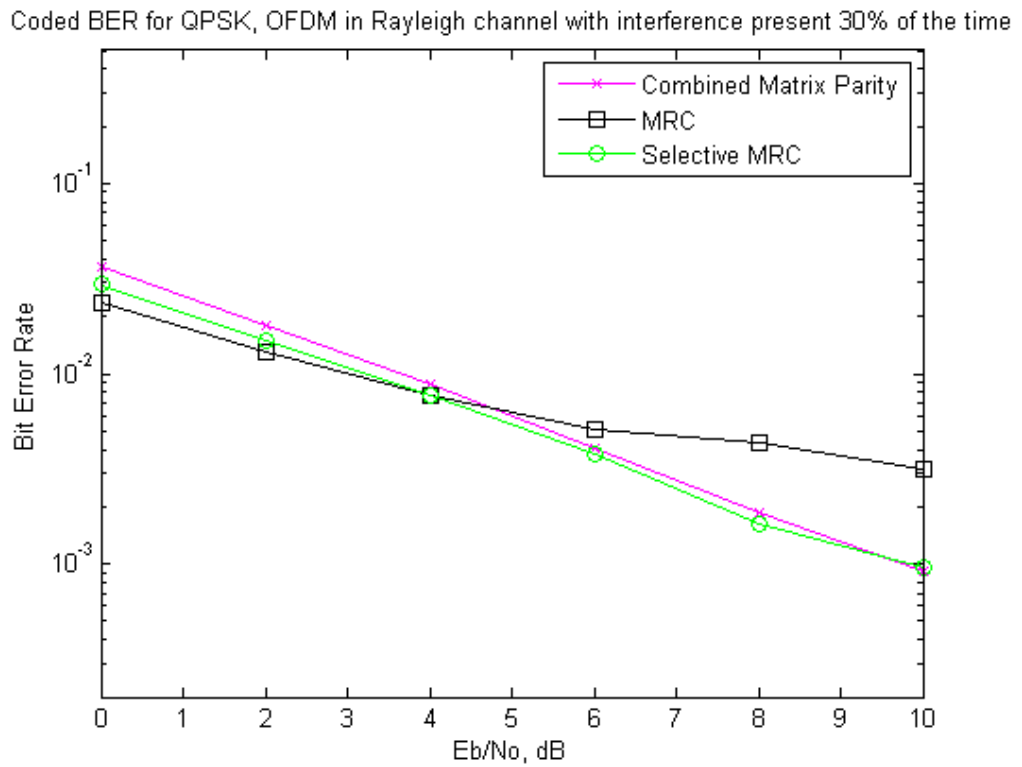


Figure 4.6: Performance of Selective MRC with respect to MRC and Matrix Parity Combining in presence of interference at 0dB upto 30% of the time

CHAPTER 5

Jamming Detection

In previous chapters we saw the effects of the presence of modulated interference, where two users having similar modulations may be using similar subcarriers and thus interfere. There could be another source of interference where, hostile sources are trying to intentionally jam the signal. The jamming signal span a huge set of frequencies and thus interfere with the actual signal. Most often, the signal is linear or parabolic chirp type signal, whose frequency is dependent on time as described in Zarifeh *et al.* (2007).

5.1 Chirp Signal

Mathematically, chirp signals are modelled as non-stationary signals with polynomial phase parameters. A polynomial phase signal $y(n)$ can be expressed as:

$$y(n) = b_0 \exp(j\phi(n)) = b_0 \exp(j \sum_{m=0}^M a_m (n\Delta)^m) \quad (5.1)$$

where $\phi(n)$ is the phase of the signal, M is the polynomial order. N is the total signal length, Δ is the sampling interval and b_0 is the signal amplitude. (Zarifeh *et al.*, 2007)

Thus linear chirp $y_l(n)$ can be expressed as follows:

$$y_l(n) = b_0 \exp(j(a_0 + a_1(n\Delta) + a_2(n\Delta)^2)) \quad (5.2)$$

The Figure 5.1 shows linear chirp in time domain. As can be seen in the figure, the frequency of the sinusoid increases with time. However it is difficult to see the linear

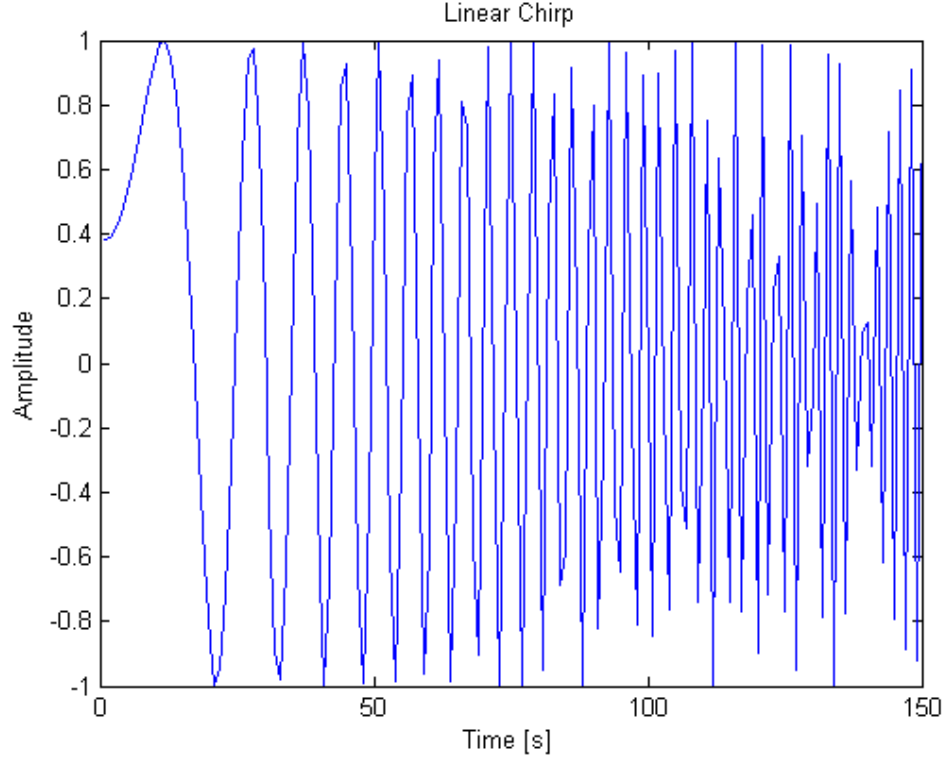


Figure 5.1: Linear Chirp in Time Domain

dependent on time. The time frequency representation of the signal in Figure 5.2 and Figure 5.3.

Parabolic and higher order chirp can be similarly represented. Usually linear and parabolic chirp signals are used for jamming. Different research works have been done in the area of interference (chirp) detection; some of the proposed methods are using: adaptive filter (Wang and Xiang-Gen, 2000), evolutionary algorithm (Dhanoa *et al.*, 2003), and maximum likelihood estimation. In the next few chapters, we take a look at some other better algorithms to detect chirp interferences.

5.2 Chirp Detection using Hough-Radon Transform

Chirp can be perceived as straight lines at arbitrary orientations in the Time Frequency (TF) plane. Thus Hough Transform (HT) may be used to determine these chirp components (Gonzalez and Wintz, 1987). Consider a point (x_i, y_i) in the image plane (image plane here refers to the TF plane). The general equation of a straight line in a slope intercept form is $y_i = mx_i + b$, where m is the slope, b is the intercept with y axis, and

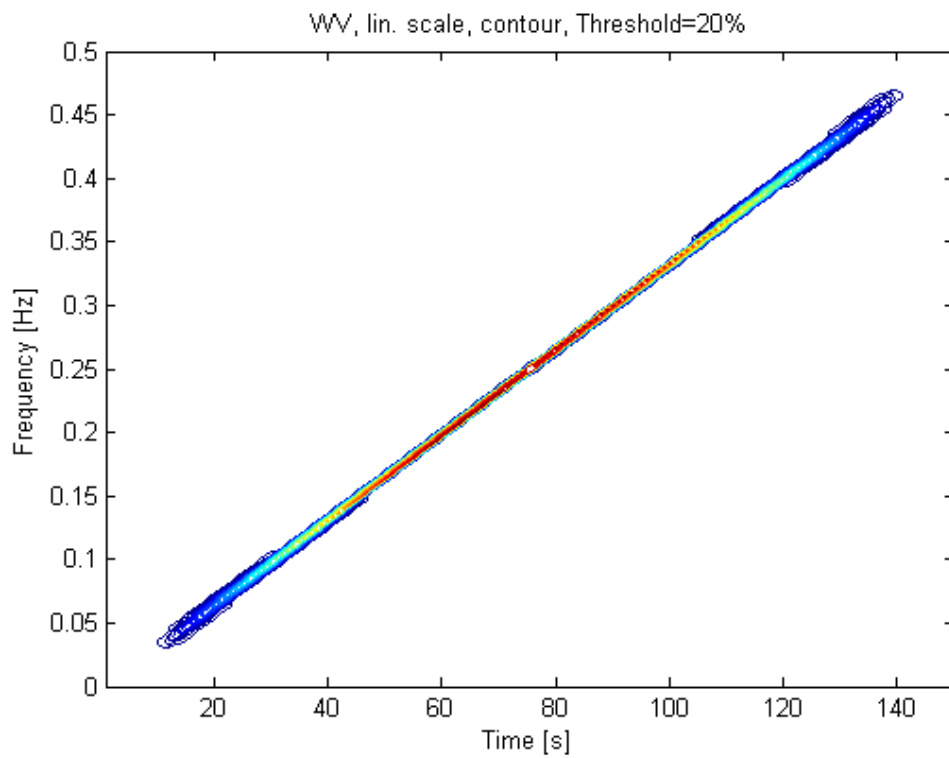


Figure 5.2: Time Frequency Representation of Linear Chirp

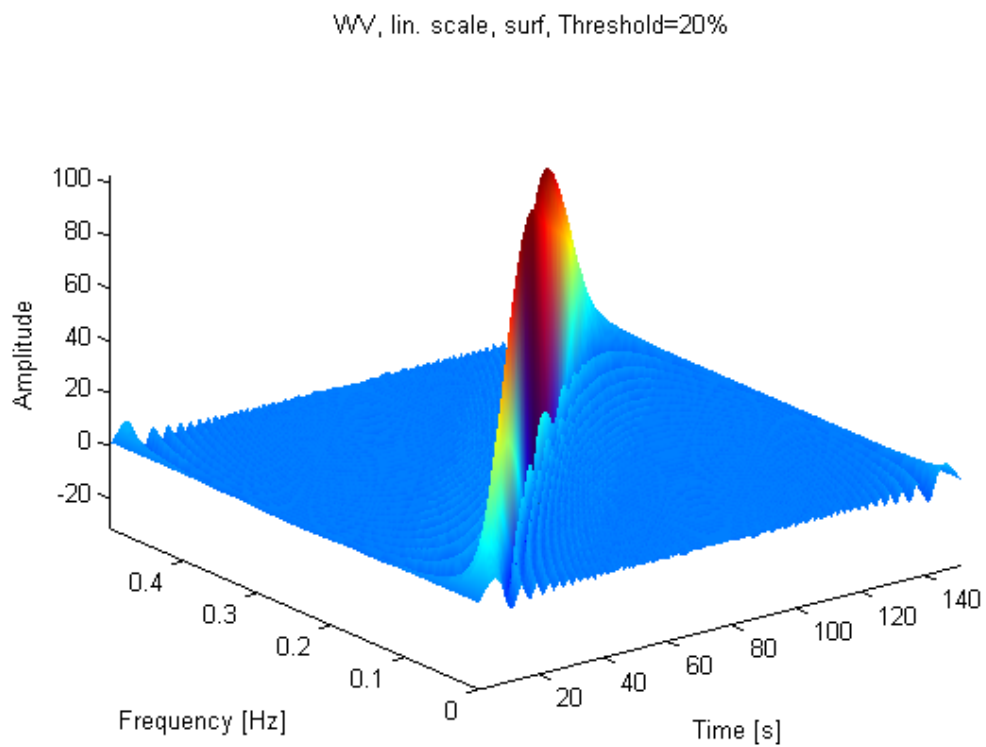


Figure 5.3: Time Frequency Representation of Linear Chirp with Amplitude

x and y axes correspond to the t and ω axes respectively. There are an infinite number of lines that pass through a point (x_i, y_i) and still satisfy the equation $y_i = mx_i + b$, for varying values of the parameters m and b . Parameterizing the TF plane into the (m, b) parameter space poses a problem because of the unbounded nature of m and b . One way to avoid this problem is to use the normal representation of a line given by

$$x \cos \theta + y \sin \theta = \rho \quad (5.3)$$

The parameter space (ρ, θ) , also known as the Hough domain, is now bounded in θ to the interval $[0, \pi]$ and in ρ by the Euclidean distance to the farthest point in the image from the centre of the image.

From (5.3), for a specific point in the TF plane (t_i, w_i) , we obtain a sinusoidal curve in the Hough domain. All of the sinusoids resulting from a mapping of a line in the TF plane have a common point of intersection in the Hough domain. Thus, linear chirps in the TF plane will correspond to high-intensity points in the Hough domain, as shown in Figure 5.4, where the two lines in TF plane correspond to two high-intensity points in the hough domain.

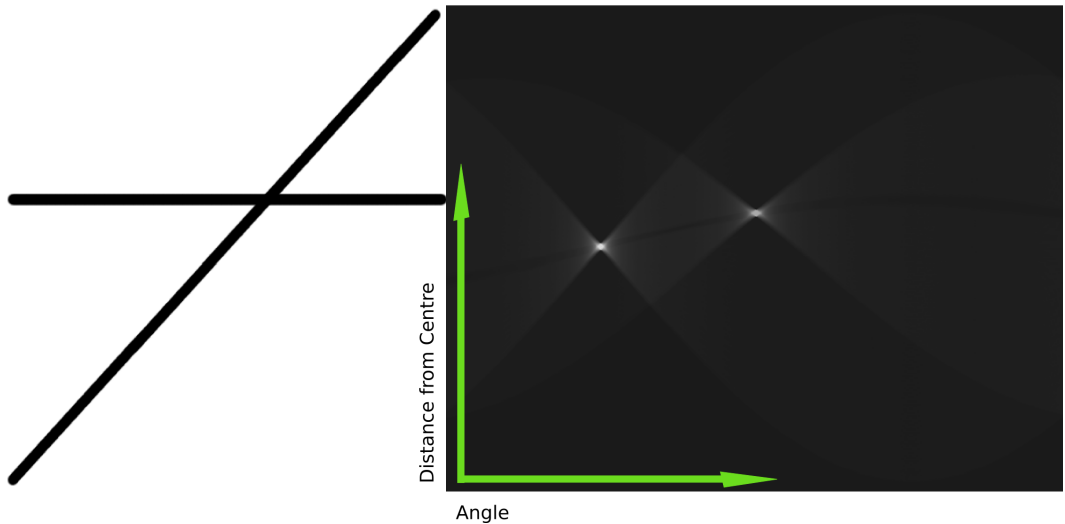


Figure 5.4: Chirps in TF plane and Hough Domain

The algorithm for Hough Transform for Chirp Detection (Krishnan and Rangayyan, 1997):

- The computational attractiveness of the HT arises from subdivision of the Hough domain into accumulator cells. The cell at coordinates (i, j) , with accumulator value $A(i, j)$, corresponds to the square associated with the parameter coordinates (θ_i, ρ_j) . Initially, the cells are set to zero.
- For every point (t_k, ω_k) in the TF plane, we let the parameter θ equal each of the allowed subdivision values on the θ axis and solve for the corresponding ρ using (5.3). The resulting ρ 's are then rounded off to the nearest allowed value on the ρ axis. If a particular θ_i value results in the solution ρ_j , the corresponding accumulator $A(i, j)$ is incremented.
- At the end of the procedure, a value of M in $A(i, j)$ corresponds to M points in the TF plane lying on the line $t \cos \theta_i + \omega \sin \theta_i = \rho_j$. It is evident that more subdivisions in the Hough domain will lead to a more accurate estimate of collinear points, but at the expense of additional computational complexity.

The main drawback of the HT is that it is usually performed on binary images, and hence may not be appropriate for gray-level images and TFDs. As energy values of chirps vary in the TF plane, they occupy different gray levels, with 255 corresponding to the highest scaled energy component. It is not appropriate to binarize the TF image, since the HT will then not be able to detect energy-varying chirp components. This drawback can be avoided by using the combined Hough Radon Transform (HRT).

With the HRT, the algorithm is exactly similar to the one discussed earlier, except that instead of counting the number of collinear points in a cell, the gray values of collinear points are added to each cell and then multiplied with the total number of collinear points in that cell. In this way, chirp components appear as high-intensity points in the HR domain, and the brightness increases with the energy of the chirp.

It can be mathematically shown that the HRT (or more generally, the RT) of a TFD provides maximum likelihood (ML) detection of a chirp signal. Wood and Barry [a] have stated that the RT of the general Wigner TFD is equivalent to the ML estimate of a chirp.

5.3 Chirp Detection using Discrete Polynomial Phase Transform

As described in (5.1), chirp can be represented as non-stationary signal with polynomial phase parameters. The DPPT is a parametric signal analysis approach for estimating

the phase parameters of a polynomial phase signal (Peleg and Friedlander, 1996, 1995). Normally, the phase parameters of a signal are determined by applying least square approximation to fit a polynomial to the phase curve. This process poses some difficulty especially when the phase is not available. The DPPT, on the other hand, is applied directly onto the signal and it works quiet well even in the presence of noise as claimed in Zarifeh *et al.* (2007).

The principle of the DPPT is as follow: when the DPPT of order M is applied to a signal with polynomial phase of order M , it produces a spectral peak. The position of this spectral peak at frequency ω_0 provides an estimation of the coefficient \hat{a}_M . After the estimation of \hat{a}_M , the order of the polynomial is reduced from M to $M - 1$ by multiplying the signal with the conjugate pair of the estimated phase. Then the coefficient \hat{a}_{M-1} will be estimated the same way by applying the DPPT of order $M - 1$ on the signal. The procedure is repeated until all of the coefficients are estimated.

The DPPT of order M of a continuous phase signal $y(n)$ is the Fourier transform of the higher order $DP_M[y(n), \tau]$ operator:

$$DPPT_M[y(n), \omega, \tau] = \sum_{(M-1)\tau}^{N-1} DP_M[y(n), \tau] \exp(-j\omega n \Delta), \quad (5.4)$$

where τ is a positive number and,

$$DP_1[y(n), \tau] := y(n), \quad (5.5)$$

$$DP_2[y(n), \tau] := y(n)y^*(n - \tau), \quad (5.6)$$

$$DP_M[y(n), \tau] := DP_2[DP_{M-1}[y(n), \tau], \tau] \quad (5.7)$$

The coefficient of a_M is estimated based on the following formula:

$$\hat{a}_M = \frac{1}{M!(\tau_M \Delta)^{M-1}} \operatorname{argmax}_{\omega} (|DPPT_M[y(n), \omega, \tau]|), \quad (5.8)$$

where $DPPT_M[y(n), \omega, \tau]$ is calculated as in (5.7). The formulas for DPPT of order one to three are shown below:

$$DPPT_1[y(n), \omega, \tau] = fft(y(n)), \quad (5.9)$$

$$DPPT_2[y(n), \omega, \tau] = fft(y(n)y^*(n - \tau)), \quad (5.10)$$

$$DPPT_3[y(n), \omega, \tau] = fft(y(n)[y^*(n - \tau)]^2 y(n - 2\tau)) \quad (5.11)$$

After the estimation of a_M , the order of the signal phase will be reduced by multiplying the signal $y(n)$ with $exp(-ja_M(n\Delta)^M)$

$$y(n)^{(M-1)} = y(n)exp(-ja_M(n\Delta)^M) \quad (5.12)$$

To determine a_{M-1} , apply the DPPT of order $M-1$ on the signal $y(n)_{(M-1)}$ from (5.9). The process is repeated until all the remaining coefficients are calculated. Coefficient a_0 and b_0 are determined by the following formulas:

$$\hat{a}_0 = phase(\sum_{n=0}^{N-1} y(n)exp(-j \sum_{m=1}^M a_m(n\Delta)^m)) \quad (5.13)$$

$$\hat{b}_0 = \frac{1}{N} \sum_{n=0}^{N-1} y(n)exp(-j \sum_{m=1}^M a_m(n\Delta)^m) \quad (5.14)$$

The final synthesized signal is:

$$\hat{y}(n) = b_0 exp(j \sum_{m=0}^M \hat{a}_m(n\Delta)^m) \quad (5.15)$$

CHAPTER 6

Conclusion

In the thesis, I have described the OFDM system design, that was designed specifically for the unique requirements for the project at hand. The design took care of the low latency and simple decoding requirements, high reliability, jamming avoidance and interference avoidance. The hopping pattern ensures that the system overcomes jamming. The preamble is used to estimate Doppler and perform timing and frequency synchronization. The pilots are used to estimate the channel and perform equalization.

The thesis then described the error control code used, namely matrix parity and described its properties. We saw the various joint estimation algorithms, effectively implemented above the matrix parity coding. Matrix parity selection combining performed the best in the presence of interference while as maximal ratio combining performed best in the absence. We then looked at Weighted flags and selective MRC algorithms which although didn't perform better than matrix parity combining in the presence of interferer at 0dB through out, but performed better in the presence of interference for a partial amount of time. Weighted flags could handle interference better then matrix parity combining upto 10% of the time where as selective MRC could handle interference better than matrix parity combining upto 30% of the time. An ML based algorithm was also developed for the project as described in Chitra (2014). A combination of selective MRC and combined matrix parity decoding seems to work the best. All the results were generated in Rayleigh channel conditions with one antenna and just two copies. The results are bound to improve when the second antenna is added and data is repeated in time as well.

We then looked at the theory of jamming detection, where we studied the chirp signal, one of the frequently used signals in jammers. We studied two algorithms to detect chirps in signals, one using the Hough Radon Transform and the other using the Discrete Polynomial Phase Transform. Hough Radon Transform is a computationally intensive algorithm and its accuracy depends on the quantization accuracy of the TF plane and the Hough domain. DPPT is a relatively simpler algorithm, which can be easily implemented in our system.

CHAPTER 7

Future Work

There is a lot of scope of improvement in various aspects of the project. The ground work has been laid for the scalability of the OFDM framework. An intelligent way of scaling it for LBR and HBR needs to be done. The hopping pattern currently is a fixed sequence. This can be made pseudo random. There are a lot of subcarriers left for future use. A valid efficient usage of these subcarriers is to be determined. Filter banks can be developed to mitigate interference at the antenna itself, before any sort of decoding is performed on the received signals.

The joint estimation algorithms can also be improved by interference detection. Various statistical tests like GLRT can be performed to determine the presence of interference. When interference is detected, the affected copies can be discarded completely. The MRC algorithm can be improved by proper estimation of interference limited channel. Better heuristics could be implemented to determine the weights in the weighted flags algorithm. We have worked on various combinations of soft combining, hard decoding and matrix parity based decoding. Other methods for decoding could also be explored.

A basic theory of jamming detection has been studied in this thesis. The algorithms needs to be implemented for our system and their performances are to be studied. Other jamming signals algorithms to for their detection and cancellation also can be explored. The CEWiT simulator is to be developed further to accommodate the advanced algorithms.

APPENDIX A

Overview of the Simulators

A.1 Test Simulator for Error Control Algorithms

The test simulator for error control algorithms was developed in MATLAB. In this simulator, the effects of Doppler offset were not considered. Hence this simulator is not focussed on synchronization and Doppler estimation. This simulator focus mainly on the algorithms for error control like MRC, matrix parity combining, weighted flag algorithm and selective MRC as described in the previous chapters. In this simulator, first required number of random QPSK data in frequency domain is generated. The I and Q channels are then encoded separately using matrix parity coding at a coding rate of $\frac{8}{15}$. Following figure illustrates an $\frac{8}{15}$ matrix parity coding.

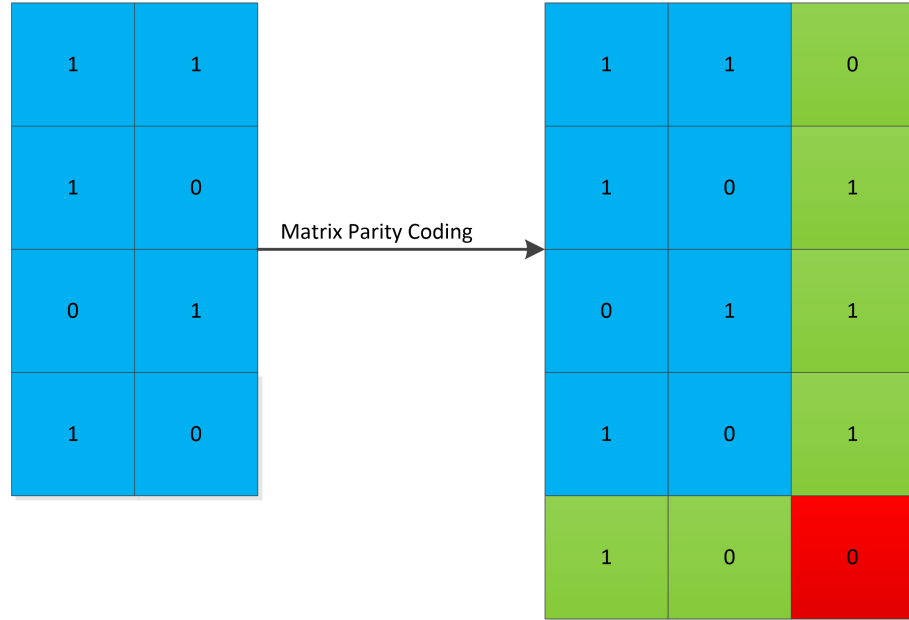


Figure A.1: Illustration of Matrix Parity Coding

After encoding, the I and Q channels are combined and thus 8 QPSK get coded into 15 QPSK symbols. When there is no interference, the user is given all the 256 subcarriers as here averaging over a large number of simulations is important. In presence of interference, the 256 subcarriers are divided into 4 bands of 64 subcarriers each (say A,

B, C, D). User 1 is allocated bands A and C, and User 2 is allocated bands A and D. This is done to closely simulated the user pair paradigm in the hopping pattern as the users interfere exactly in one band and do not interfere in the other.

After encoding, IFFT of the frequency domain signal is taken to convert it to time domain. This is then passed through a Rayleigh channel with Additive White Gaussian Noise (AWGN). If the interferer is present, the interferer is added as well. At this stage, the receiver receives the data and takes its FFT to transform it to frequency domain. We assume that the channel estimation is perfect and transform the channel as well into frequency domain. After this either we perform a zero forcing equalization or a soft MRC based on the algorithm we are using to decode.

A.2 CEWiT Simulator

This simulator is what has been provided by CEWiT. Here all the receiver side architecture has been simulated, with separate files for each process. The main configuration file, `simulationConfigFile.m` has all the configuration parameters required for the simulation. Each of the receiver blocks have been given separate files for including our algorithms - Synchronization, channel estimation and equalization, coding and decoding. The simulator can be executed by running the `main.m` file, which then shows the BER at each receiver for the specified number of iterations.

REFERENCES

1. **Chitra, K. R.** (2014). *Soft Combining for Improved Matrix Parity Decoding in Error Prone OFDM Systems*. Master's thesis, Department of Electrical Engineering, IIT-Madras, Chennai – 600036.
2. **Chiueh, T.-D.** and **P.-Y. Tsai**, *OFDM Baseband Receiver Design for Wireless Communication*. John Wiley and Sons, 2007.
3. **Cho, Y. S., J. Kim, W. Y. Yang, and C. G. Kang**, *MIMO-OFDM Wireless Communication with Matlab*. John Wiley and Sons, 2010.
4. **Dhanoa, J. S., E. J. Hughes, and R. F. Ormondroyd** (2003). Simultaneous detection and parameter estimation of multiple linear chirps. *2003 IEEE International Conference on Acoustics, Speech and Signal Processing*, **6**, VI–129–32.
5. **Gonzalez, R. C.** and **P. Wintz**, *Digital Image Processing*. Addison Wesley Inc., 1987.
6. **Krishnan, S.** and **R. M. Rangayyan** (1997). Detection of chirp and other components in the time-frequency plane using the hough and radon transforms. *1997 IEEE Pacific Rim Conference on Communications, Computers and Signal Processing*, **1**, 138–141.
7. **Peleg, S.** and **B. Friedlander** (1995). The discrete polynomial-phase transform. *IEEE Transactions on Signal Processing*, **43**(8), 1901–1914.
8. **Peleg, S.** and **B. Friedlander** (1996). Multicomponent signal analysis using the polynomial-phase transform. *IEEE Transactions on Aerospace and Electronic Systems*, **32**(1), 378–387.
9. **Wang, G.** and **X. Xiang-Gen** (2000). An adaptive filtering approach to chirp estimation and isar imaging of maneuvering targets. *IEEE 2000 International Radar Conference*, 481–486.
10. **Zarifteh, R., N. Alinier, S. Krishnan, and A. Anpalagan** (2007). Interference detection in spread spectrum communication using polynomial phase transform. *IEEE 2007 International Conference on Communications*, 2879–2984.