OPTICAL PHASE LOCKED LOOP

A THESIS

submitted by

G SUJAN KUMAR

for the award of the degree

of

DUAL DEGREE (B.TECH & M.TECH)



DEPARTMENT OF ELECTRICAL ENGINEERING INDIAN INSTITUTE OF TECHNOLOGY MADRAS. MAY 2014

THESIS CERTIFICATE

This is to certify that the thesis titled **OPTICAL PHASE LOCKED LOOP**, submitted

by G Sujan Kumar, to the Indian Institute of Technology, Madras, for the award of the

degree of **Dual Degree** (B.Tech & M.Tech), is a bona fide record of the research work

done by him under our supervision. The contents of this thesis, in full or in parts, have

not been submitted to any other Institute or University for the award of any degree or

diploma.

Prof. Anil Prabhakar

Research Guide Professor Dept. of Electrical Engineering IIT-Madras, 600 036

Place: Chennai

Date: 15th May 2014

Prof. S Aniruddhan

Research Guide Assistant Professor

Dept. of Electrical Engineering IIT-Madras, 600 036

ACKNOWLEDGEMENTS

I take this opportunity to thank all the people who have given me constant support and encouragement at all times. I am grateful to Prof. Anil Prabhakar and Prof. Aniruddhan for their continual support and invaluable advice throughout the project. I would also like to thank Aravind, Anish and Saikrishna for their help and guidance in the lab.

I am very grateful for being blessed with the most wonderful set of friends and classmates at IIT Madras. I am thankful to Milind, Rohan, Saurabh, Sumanth, Chandrashekar, Navneeth, Akshay, Anand, Ashish, Karthik, Prashant and Barath for all the support and friendship through my stay here. I am thankful to Rohan, Sumanth and Barath for being there for me and pushing me to work during my hardest times of my stay here. I would like to thank all the people who made my stay at IIT Madras special.

Most importantly, I would like to thank my parents and my sister for their neverending support and understanding at each and every point of my life.

ABSTRACT

KEYWORDS: Phase Locked loop; Optical Phased locked loop; Laser synchronization; Linewidth reduction; Laser locking

This thesis explores the functioning of a homodyne Optical phased locked loop. Optical phased locked loop are essentially and optical counterpart of well-known electronic phased locked loop. They were first thought of within four years of within the invention of lasers in 1960's (Enloe and Rodda, 1965). It never took off as its electronic counterparts due to difficulties in implementing them.

This thesis is an attempt to implement an optical phased locked loop with a bandwidth of few MHz. Major difficulties that limit its performance are loop delay and nonlinear FM response of the Semiconductor laser. These issues reduce the bandwidth of the loop, limiting the system to narrow linewidth lasers (Satyan, 2011). We have explored option to counter the phase drop from due to FM response by introducing a lead lag filter. Semiconductor laser is chosen as the slave laser for its various advantages that include its availability, its speed and its tunablity (which is achived by adjusting the bias current through the lasers) (Liang, 2008). A 2GHz detector and opamp, OPA657 is used for making the detector. A board has been designed and built for the implementation of Optical Phase Locked loop.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS							
Al	ABSTRACT LIST OF FIGURES						
LI							
Al	ABBREVIATIONS						
N	OTAT	ION		vii			
1	INTRODUCTION						
	1.1	Princi	ple	1			
2	ANA	ALYSIS		4			
	2.1	Loop	Noise Characterization	4			
	2.2 Practical Limitations of loop bandwidth						
		2.2.1	FM Response	5			
		2.2.2	Loop Delay	8			
3	LASER CHARECTERIZATION						
	3.1	Linew	vidth Measurement	12			
4	IMPLEMENTATION			15			
5	TES	TING	AND CONCLUSION	20			

LIST OF FIGURES

1.1	Block diagram of homodyne optical phase locked loop	3
2.1	Various noise sources and their point of entry	4
2.2	Differential errors for varies values of loop gain K where Km is the maximum allowed loop gain by stability criteria	6
2.3	Normalized phase and magnitude response of the FM response of the loop	7
2.4	Loop gain with and without lead-lag filter	8
2.5	A better suppression of differential phase error can be expected in the presence of lead-leag filter	9
2.6	Phase response of open loop transfer function in the presence of delay	10
3.1	Voltage across the laser with changing current	11
3.2	Power emitted by the laser with changing current	11
3.3	Wavelength with respect to current. Measured for one of the modes emitted by the DFB laser	12
3.4	Delayed self heterodyne mixing of the laser field	13
3.5	Setup for delayed self heterodyne mixing of the laser field	13
3.6	Spectrum at the local oscillator frequency	14
3.7	Linewidth with respect to injection current	14
4.1	Circuit and spice simulation for the transimpedance amplifier circuit that is being used	15
4.2	TIA output voltage with respect to input power	16
4.3	Circuit and spice simuation for the filter cirucit that is being used	16
4.4	Setup for testing the board performance	17
4.5	Loop gain with and without lead-lag filter	18
4.6	Layout and schematics of the OPLL Board	19
5.1	Setup for testing the performance of OPLL	20
5.2	Histogram data of the measurements of master laser	21

5.3	Histogram data of the measurements of slave laser, note that the plot representing the master laser is not to scale in y axis it is only to show its rage in the x axis	21
5.4	the slave laser at each wavelength, note that the plot representing the master laser is not to scale in y axis it is only to show its rage in the x	22
	axis	23
5.5	Wavelength measurements shown in time domain	24

ABBREVIATIONS

PLL Phase Locked Loop

OPLL Optical Phase Locked Loop

SCL Semiconductor Laser

DFB Distributed Feedback

LO Local Oscillator

PD Phase Detector

FM Frequency Modulation

TIA Transimpedance Amplifier

MZI Mach-Zehnder interferometer

RF Radio Frequency

AOM Acousto-Optic Modulator

EOM Electro-Optic Modulator

OSA Optical Spectrum Analyzer

CCO Current Controlled Oscillator

APD Avalanche Photodiode

NOTATION

 $\begin{array}{ll} \phi & \text{Instantaneous Phase} \\ \omega & \text{Average Frequency} \end{array}$

 $\phi_{m,n}$ Phase noise of master laser

 $\phi_{s,n}$ Phase noise of unlocked slave laser

 $\phi_{pd,n}$ Noise of a photodetector

 ϕ_s Phase noise of locked slave laser

 S_m Spectral density function of phase noise of master laser

 $S_{PD,n}$ Spectral density phase noise of photodetector

 $S_{s,fr}$ Spectral density function of phase noise of free running slave laser

 S_s Spectral density phase noise of slave laser

 G_{op} Open-loop transfer function Δf_m Linwidth of master laser Δf_s Linwidth of slave laser

 H_{FM} Normalized FM response of laser H_{filter} Normalized filter response of laser

 τ_D Loop Delay

 $\omega_l o$ Frequency of local oscillator

 au_c Coherence time

 $\Delta \nu$ Linewidth

INTRODUCTION

Phase Locked Loop(PLL) is a feedback system that is used to control phase and frequency of a voltage/current controlled oscillator with respect to a reference oscillator. The concept of PLL was first developed in the 1930s. The key parts in a PLL include a phase detector (PD) and a local oscillator whose frequency can be tuned by changing the input voltage or current. The system forces the local oscillator (LO) to follow the frequency/phase of an incoming reference signal (Liang, 2008).

Optical phase lock loops (OPLLs) are essentially the counterparts of PLLs in the optical domain, where a slave laser is used as the local oscillator and is phase-locked to a master laser. The role of the phase detector is played by a photo detector. In an OPLL, the most critical condition that needs to be achived for optical phase lock is, to have the summed linewidths of the master laser and the slave laser to be much smaller than the loop bandwidth (Ramos and Seeds, 1990). These condition is easily met in solid state laser and gas lasers because of their good frequency stability and narrow linewidth (Liang, 2008). These lasers are relatively bulky and expansive in contrast to DFB semiconductor lasers which are small, cheap and are easily current tuned with very high speeds. On the negative side, SCLs possess wide linewidth, which requires a high loop bandwidth and high speed feedback electronics to lock the lasers. Another serious issue is that their current frequency Modulation exhibits 90 degree phase reversal within the range 0.1-10MHz (Corrc et al., 1994), limiting the loop bandwidth in the same range.

1.1 Principle

A schematic diagram of a typical homodyne Optical Phased Locked Loop as shown in Figure 1.1. The optical signals of the master laser $Am\sin(\omega_m t + \phi_m)$ and the slave laser $As\sin(\omega_s t + \phi_s)$ are combined at a photodetector, which detects the phase and frequency

differences. The voltage output of the photodetector along with TIA will be of the form

$$v_{TIA}(t) = 2R_{PD}\sqrt{P_m P_S} \sin\left[(\omega_m - \omega_s)t + \phi_m(t) - \phi_s(t)\right]$$
(1.1)

where R_{PD} is the responsivity of the photodetector and P_m and P_s are the optical power of the master and slave laser respectively. Form henceforth $K_{PD}=2R_{PD}\sqrt{P_mP_s}$. This is filtered and finally fed back to slave laser. The laser acts as a current controlled oscillator, the derivative of the laser phase fluctuations look like

$$\frac{d\phi_s(t)}{dt} = K_{CCO} \left[v_{TIA} * f_{flt}(t) * f_{FM}(t) \right]$$
(1.2)

Where K_{CCO} is the current FM sensitivity of the slave laser and $f_{flt}(t)$ and $f_{FM}(t)$ are the impulse response of the loop filter and slave laser respectively. A steady state solution is obtained by setting all the d/dt terms to zero. Steady state solution are as $\omega_m = \omega_s$ and $\phi_e = \phi_m - \phi_s$. From henceforth $K_{DC} = K_{pd}K_fK_s$. By linearizing the system at this point and taking it into frequency domain, we can compute the open loop transfer function as below.

$$G_{op}(s) = \frac{K_{DC}H_{FM}(s)H_{filter}(s)}{s}$$
(1.3)

Closed loop transfer functions are as the following.

$$H_o(s) = \frac{G_{op}(s)}{1 + G_{op}(s)} \tag{1.4}$$

$$H_e(s) = \frac{1}{1 + G_{op}(s)} \tag{1.5}$$

In order to keep the loop stable the open loop transfer function need to satisfy the conditon where The magnitude of open loop transfer function should be less than one when its phase crosses 180° .

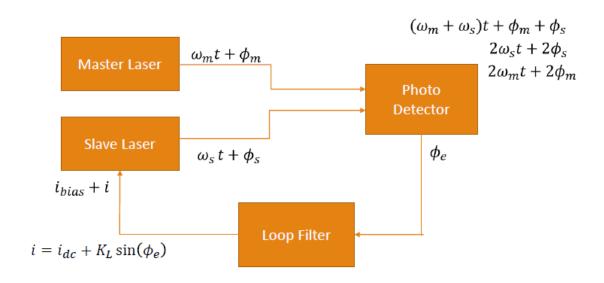


Figure 1.1: Block diagram of homodyne optical phase locked loop

ANALYSIS

2.1 Loop Noise Characterization

In order to obtain the residual noise level estimate of a laser locked under an optical phased locked loop various noise sources needs to be considered. The following are the major noise sources

- Phase noise of slave laser
- Phase noise of master laser
- Photodetector noise
- Electronics noise

Block diagram shows the various noise sources and the points of entry into the system is shown in Figure 2.1. Where $\phi_{m,n}$ and $\phi_{s,n}$ are phase noise associated with master laser and save laser. $\phi_{pd,n}$ is the phase noise due to the photodetector. Solving using

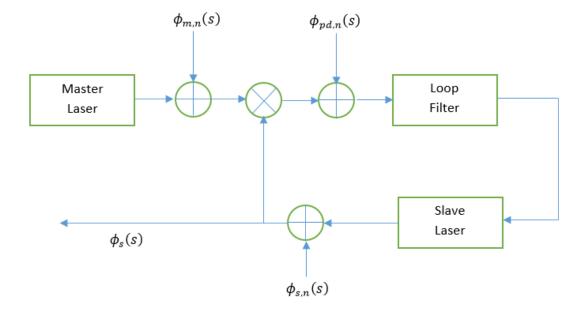


Figure 2.1: Various noise sources and their point of entry

the block diagram one can obtain the phase noise of locked slave laser and differential phase error as.

$$\phi_s(s) = (\phi_m + \phi_{m,n})H_o + \phi_{pd,n}H_o/K_{PD} + \phi_{s,n}H_e$$
 (2.1)

$$\phi_e(s) = \phi_{pd,n} H_o / K_{PD} + (\phi_m + \phi_{m,n} + \phi_{s,n}) H_e$$
 (2.2)

Corresponding spectral density functions are

$$S_s(f) = \left[S_m(f) + S_{PD,n}(f) / K_{PD}^2 \right] |H_o(f)|^2 + S_{s,fr}(f) |H_e(f)|^2$$
 (2.3)

$$S_e(f) = \left[S_m(f) + S_{s,fr}(f) \right] |H_o(f)|^2 + S_{PD,n}(f) / K_{PD}^2 |H_o(f)|^2$$
 (2.4)

Where $S_{s,fr}$, $S_{m,n}$ and $S_{s,n}$ are spectral densities functions of phase noise of free running slave laser, free running master laser and phase noise of locked slave laser. They are given by (Ramos and Seeds, 1990)

$$S_m(f) = \frac{\Delta f_m}{2\pi f^2} \tag{2.5}$$

$$S_s(f) = \frac{\Delta f_s}{2\pi f^2} \tag{2.6}$$

$$S_{PD}(f) = 2eR(P_M + P_S) \tag{2.7}$$

Here P_M and P_M are the powers of master laser and the slave lasers respectively, R is the responsivity of the photodetector and e is the electric charge of an electron. Figure 2.2 shows the differential phase noise under different loop gains. Where the spectral densities are calculated based on known linewidth of slave and master laser. The loop gain takes into account FM response and no loop filter is assumed.

2.2 Practical Limitations of loop bandwidth

2.2.1 FM Response

Here in our model we take the laser to be current controlled oscillator, with a flat frequency response. In practice, the FM response of SCLs is not uniform and exhibits different characteristics depending on the range of the modulation frequency. For a

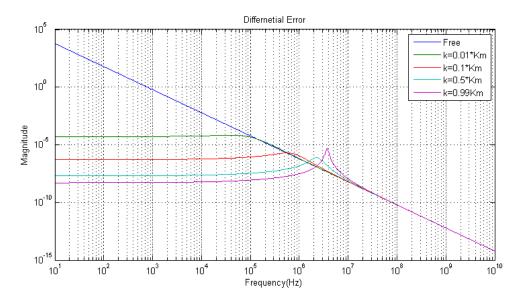


Figure 2.2: Differential errors for varies values of loop gain K where Km is the maximum allowed loop gain by stability criteria

typical single-section SCL, the low frequency (smaller than 10MHz) FM response is dominated by the thermal effect and the carrier-induced effect at high frequencies. Accurate models are available for FM response of DFB lasers, one of the most widely used model as below. Figure 2.3 shows the phase and magnitude of the FM response of a typical laser (Corrc *et al.*, 1994) where $f_c = 1.8$ MHz and b = 1.64.

$$H_{FM}^{DFB}(f) = -\frac{1}{b} \left(\frac{b - \sqrt{\frac{jf}{f_c}}}{1 + \sqrt{\frac{jf}{f_c}}} \right)$$
 (2.8)

The problem that FM response of the laser is the phase drop. In most typical lasers phase of the FM response crosses 90^0 between 0.1Mhz to 10MHz. Since magnitude of the open loop transfer function must be greater than one when the phase crosses 180^0 . In a system where

$$G_{op}(s) = \frac{K_{DC}H_{FM}(s)H_{filter}(s)}{s}$$
(2.9)

Phase of $G_{op}(s)$ drops to 180^0 when phase of the FM response drops to 90^0 . This 90^0 drop can be pushed to higher frequency by adding a phase through a lead lag filter as shown in Figure 2.4. A lead lag filter is essentially an pole zero pair such that zero comes before the pole. Its major role is to improve the phase margin of the system while not affecting its asymptotic behavior. By introducing a lead lag filter the phase of the system will be increased in the frequency band between the zero and pole. A lead

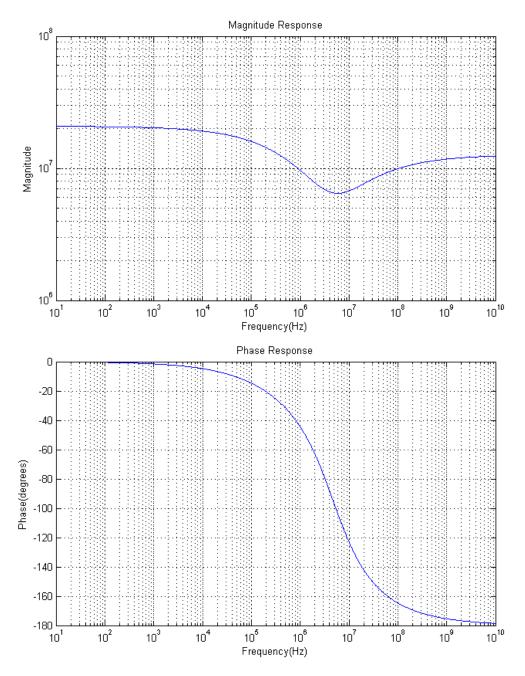


Figure 2.3: Normalized phase and magnitude response of the FM response of the loop

lag filter with $f_1 = 4 * 10^7$ and $f_2 = 4 * 10^7$ is used and differential. Figure 2.5 shows the differential phase noise suppression under the presence of a leading filter.

$$H_{filter}(s) = \frac{1 + \frac{f}{f_1}}{1 + \frac{f}{f_2}} \tag{2.10}$$

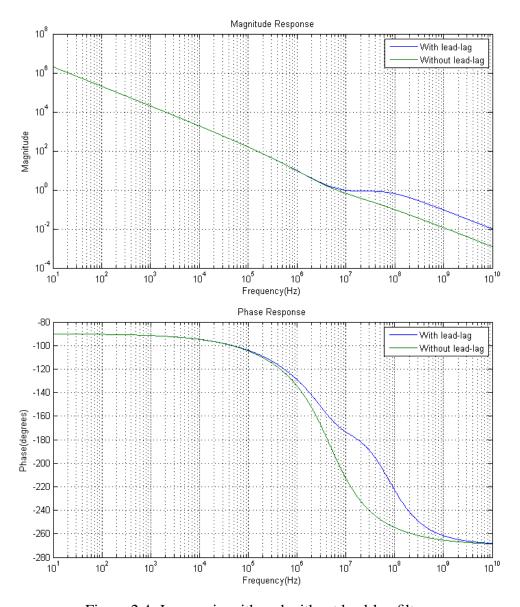


Figure 2.4: Loop gain with and without lead-lag filter

2.2.2 Loop Delay

Loop delay exists in all practical feedback control systems. In the presence of the loop delay, the phase lag increases unbounded as the frequency increases. The effects of loop

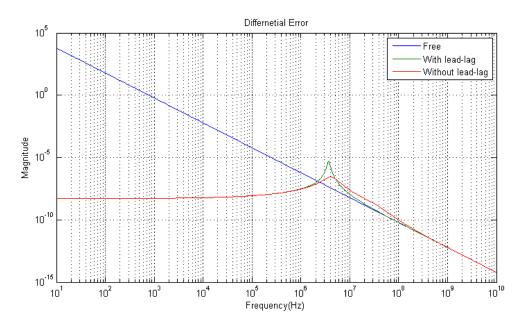


Figure 2.5: A better suppression of differential phase error can be expected in the presence of lead-leag filter

delay on OPLL had been well analyzed (Ramos and Seeds, 1990) As described earlier, the stability criterion requires the open loop gain to be restricted to less than 1 at the 180 degree phase lag frequency. Hence the loop gain and the resulting loop bandwidth will be limited. In constructing an OPLL using fiber optics using fiber optical components, the delay can be as big as a few ns. As the desired loop bandwidth in in the order of few MHz, due to the large (MHz) linewidth of SCLs, the effect of the loop delay at these frequency ranges cannot be ignored. Estimation of the overall length of the fiber in the feedback system is about 4m, a 20ns loop delay can be assumed. Figure 2.6 shows the overall phase drop of the of the open loop transfer function in presence of delay.

Open loop transfer function along with delay is as

$$G_{op}(s) = \frac{K_{DC}H_{FM}(s)H_{filter}(s)}{s}e^{s\tau_D}$$
(2.11)

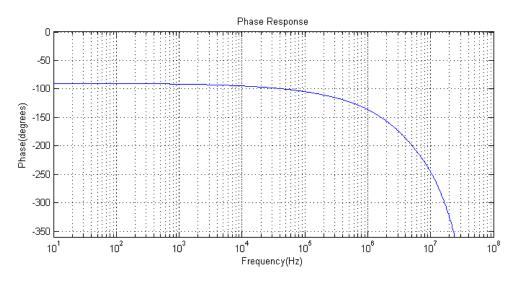


Figure 2.6: Phase response of open loop transfer function in the presence of delay

LASER CHARECTERIZATION

For the purpose of the slave laser a Distributed Feedback Laser(DFB) has been used. A DFB laser consist of a structure with periodic variation in refractive index. This grating acts like an optical filter, causing a single wavelength to be fed back to the gain region and lase. Injection current of a DFB laser was varied and voltage and power output of it was measured. The measurements are summarized in Figure 3.1 and Figure 3.2.

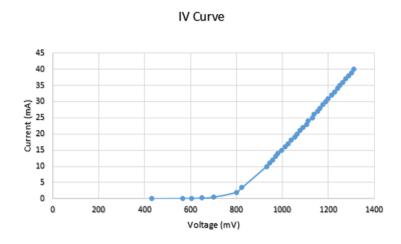


Figure 3.1: Voltage across the laser with changing current

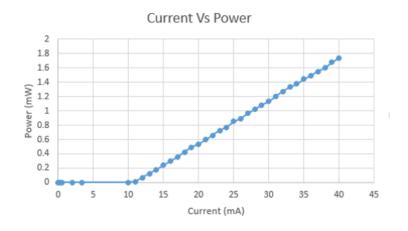


Figure 3.2: Power emitted by the laser with changing current

As mentioned earlier the wavelength of a DFB laser changes with current, the fluctuation of this wavelength is due to thermal effects. The grating expands and contracts

with current and that leads the wavelength change. The change in the center wavelength with current was been measured and summarized in Figure 3.7.

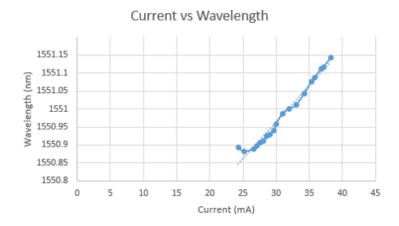


Figure 3.3: Wavelength with respect to current. Measured for one of the modes emitted by the DFB laser

3.1 Linewidth Measurement

It is the width (in frequency, wavelength, wavenumber) of the band where the power of the laser is more than half its maximum. Width of laser line due to fluctuations in the phase of the optical field. The following contribute to the laser linewidth

- Spontaneous emission which alter the phase and intensity of the lasing field.
- Carrier density fluctuation
- Fluctuations due to thermal effects

Since the fiber laser has low or narrow linewidth, its measurement with optical spectrum analyzers becomes difficult (Chen, 2006). Therefore a heterodyne method is preferred. The problem with heterodyne method is that, it required a very narrow, stable and sharp reference laser. The measurement is made by using this laser to beat the laser under test down to observable bands using a detector and electronic spectrum analyzer.

Since it is hard to find very narrow and stable reference lasers a delayed self heterodyne method can be used. A delayed self-heterodyne method can be used for linewidth measurements. The delayed self heterodyne method the incident light is split into two paths by the interferometer. The optical frequency of one arm is offset with respect to other, this is achieved by modulating the laser light through the arm. If the delay τ_d , of one of the path exceeds the coherence time, τ_c of the source, the two combining beams interfere as if they were originated from two different sources. The cartoon in Figure 3.4 clearly demonstrates the process. The setup for this measurement is as shown in

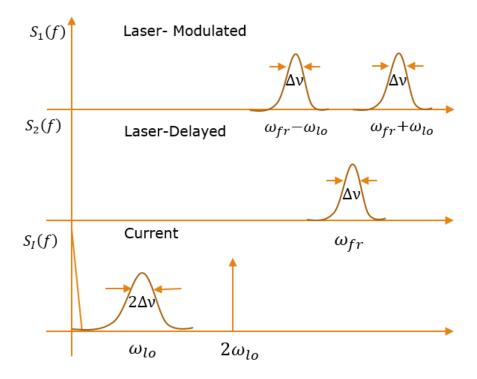


Figure 3.4: Delayed self heterodyne mixing of the laser field

Figure 3.5 The laser current measured by the photodetector is given by

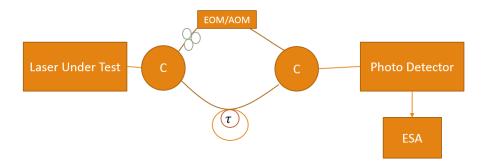


Figure 3.5: Setup for delayed self heterodyne mixing of the laser field

$$i(t) = R \left[\frac{P_L(t)}{4} + \frac{P_L(t - \tau_d)}{2} + \frac{P_L(t)}{4} \cos(2\omega_{lo}t) + 2\sqrt{P_L(t)P_L(t - \tau_d)} \cos(\omega_{lo}t + \Delta\phi(t)) \right]$$
(3.1)

In the above equation we are interested in the 4th term. Finding the 3db bandwidth of the signal measured at ω_{lo} gives us twice the linewidth we want to measure. It is twice because each of this split light interfere as light from two different source with same linewidth Linewidth of 0.83MHz is measured for master laser and for slave laser

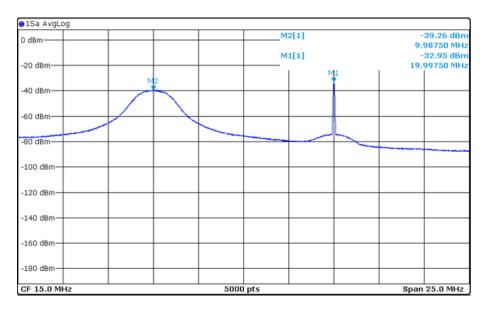


Figure 3.6: Spectrum at the local oscillator frequency

linewidth measured was between 2.4MHz-3MHz. Measured linewidth with respect to input current is as shown in Figure 3.7

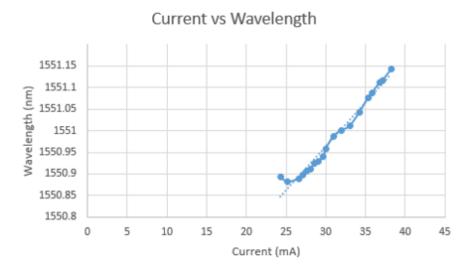


Figure 3.7: Linewidth with respect to injection current

IMPLEMENTATION

The whole feedback circuit has been implemented on a printed circuit board, and its performance has been measured.

Transimpedance Amplifier

Texas Instruments' OPA657 was used for the transimpedance amplifier. The circuit was optimized to give a flat Butterworth response. Figure 4.1 shows the circuit diagram and SPICE simulated result of this circuit. It has been designed for a gain of $5.4k\Omega$.

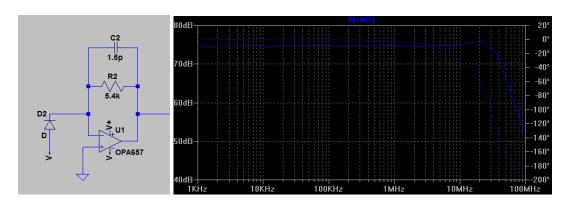


Figure 4.1: Circuit and spice simulation for the transimpedance amplifier circuit that is being used

Along with it an Avalanche Photodiode(APD), 1550nm detector with a responsivity of 0.8 and a cutuff frequency of 2GHz is being used.

Figure 4.2 are the measurement results of power vs the voltage out of the transimpidance amplifier.

Lead-lag filter

Lead lag filter has been implemented with the circuit as shown in Figure 4.3. Solving for the transfer function we get

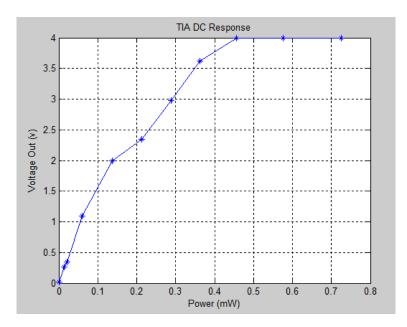


Figure 4.2: TIA output voltage with respect to input power

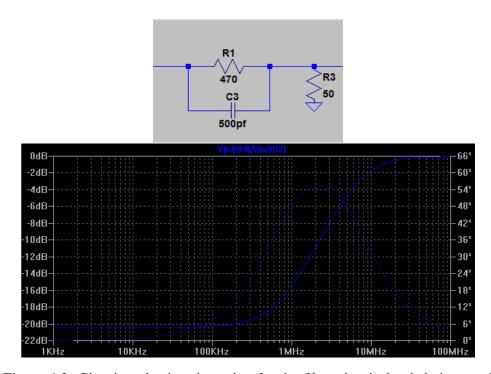


Figure 4.3: Circuit and spice simuation for the filter cirucit that is being used

$$v_{out} = \frac{v_{in}R4}{R1 + R4} \frac{1 + s\tau_1}{1 + s\tau_2} \tag{4.1}$$

$$\tau_1 = \frac{R1 * R4 * C3}{R1 + R4} \tag{4.2}$$

$$\tau_2 = R1 * C3 \tag{4.3}$$

The filter is implemented with parameters with R1=470 Ω , R4=50 Ω and C3=500pF. Figure 4.3 shows the spice simulation of the same.

The circuit made is tested under the following setup as shown in Figure 4.4. A continuous wave laser source is modulated and fed into the circuit. Measurements were made against the modulation signal. An accurate phase measurement was not possible because of the phase drop due to delay through the cables and the fiber.

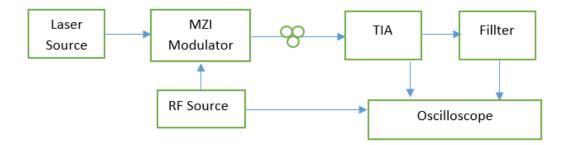


Figure 4.4: Setup for testing the board performance

Figure 4.5 has the measured magnitude and phase response of the filter. Figure 4.6 is the layout of the board designed for implementation of OPLL. A bipolar transistor is used a s driver. Bias current through the diode is controlled by adjusting resistance R4.

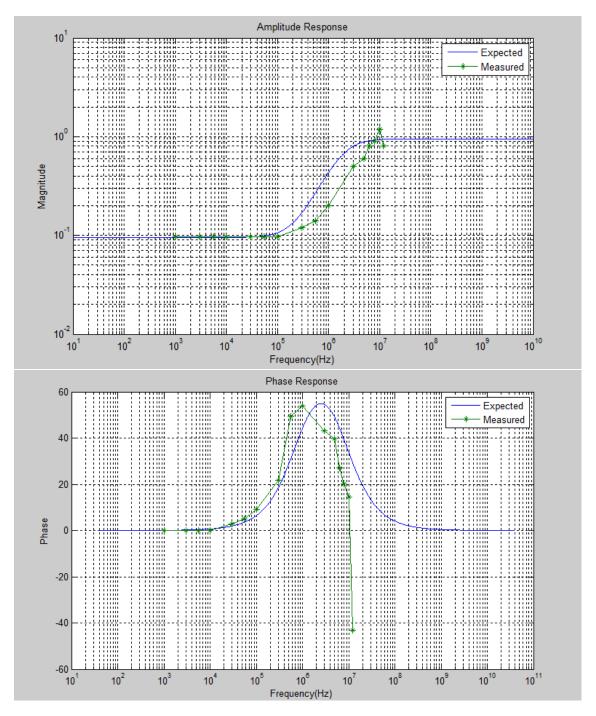


Figure 4.5: Loop gain with and without lead-lag filter

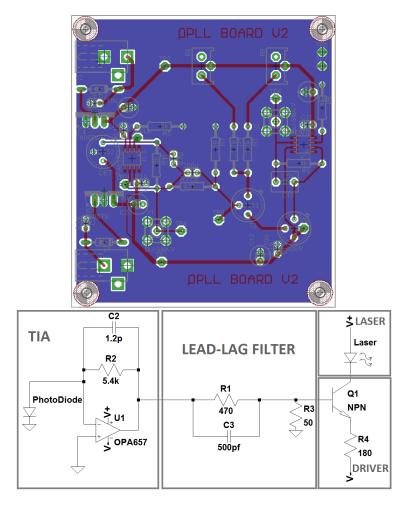


Figure 4.6: Layout and schematics of the OPLL Board

TESTING AND CONCLUSION

OPLL is tested in the setup as shown in the Figure 5.1. The light from the master and slave laser are fed into a two-by-two coupler. One of the output beam from the coupler is fed to an attenuator and to the OPLL circuit. Attenuator is used to control the loop gain. The other output of the coupler is used for monitoring the setup.

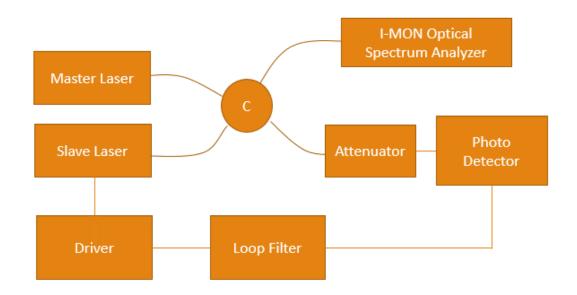


Figure 5.1: Setup for testing the performance of OPLL

Initial only the slave laser is switched on to see the wavelength at with stabilizes in, and then the master laser is switched on and set to a wavelength very close to it. Due to the drift we expect the wavelength of the slave laser to match the master laser. At this instance a locking action can be expected. Figure 5.2 shows the distribution of wavelength of the master laser measured over 100s. Figure 5.3 a similar plot is generated for the wavelength of the slave laser when under lock. It can be observed that there were multiple instances where the wavelength of the slave laser has matched that of the master laser. A closer look reveals that both the histogram data has a maximum at the same wavelength. To gain a better understanding the following have been measured, refer to Figure 5.4

• Total time spent by the slave laser at each wavelength

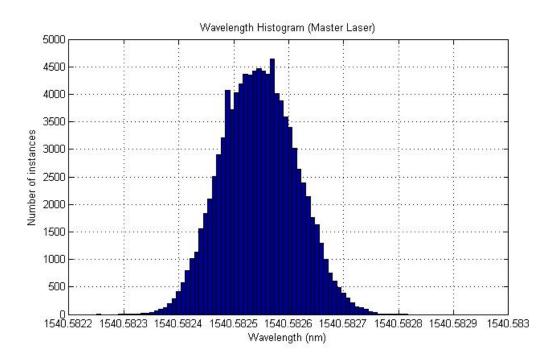


Figure 5.2: Histogram data of the measurements of master laser

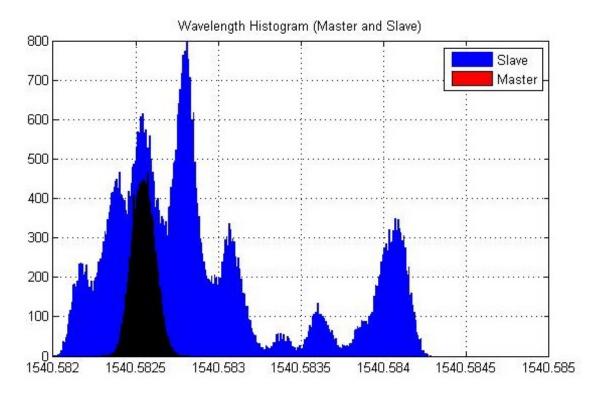


Figure 5.3: Histogram data of the measurements of slave laser, note that the plot representing the master laser is not to scale in y axis it is only to show its rage in the x axis

- Maximum time spent at one crossing by the slave laser
- Average time spent at each wavelength per crossing

In can be observed that though the total time spent is not the highest at the wavelength of the master laser the maximum time spent and the time spent per crossing is relatively higher. By this date it can be speculated that the OPLL might have locked at few instances.

Looking at the time domain measurements at of the wavelength of the slave laser in can be understood the locking might have occurred at the first 50-60s of the measurements. After which the laser drifts like it would have in the absence of the loop. Figure 5.5 also shows a comparative time domain plot of the wavelength of slave laser in the presence and absence of loop. This data shows that a relatively low drift is observed in presence of the loop.

The reason for inconsistency in locking can be attributed to the drift of the slave laser, which is in the orders of few Gigahertz while the lock range of the OPLL loop is about 5-7 MHz. A heterodyne type-II set up for the loop can help overcome this problems. Another solution is to increase the DC gain by introducing a lag-lead filter to increase the loop gain within the bandwidth limitations.

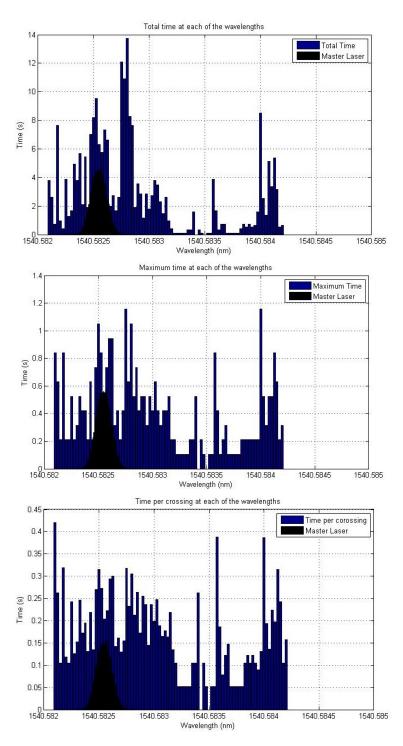


Figure 5.4: Plots showing the total, maximum and the time spent per crossing of the slave laser at each wavelength, note that the plot representing the master laser is not to scale in y axis it is only to show its rage in the x axis

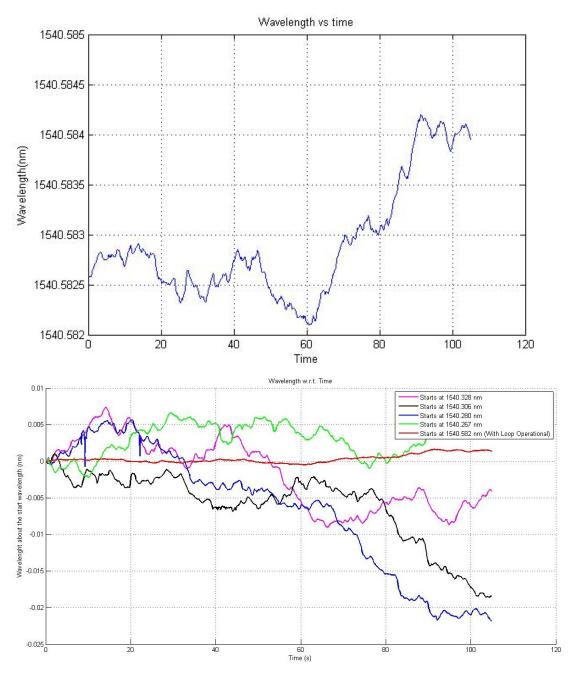


Figure 5.5: Wavelength measurements shown in time domain

REFERENCES

- 1. **Chen, X.** (2006). *Ultra-Narrow Laser Linewidth Measurement*. Ph.D. thesis, Virginia Polytechnic Institute and State University.
- 2. **Corrc, P., O. Girad**, and **I. de Faria Jr** (1994). On the thermal contribution to the fm response of dfb lasers: Theory and experiment. *Quantum Electronics, IEEE Journal of*, **30**(11), 2485–2490.
- 3. **Enloe, L.** and **J. Rodda** (1965). Laser phase-locked loop. *Proceedings of the IEEE*, **53**(2), 165–166.
- 4. **Liang, W.** (2008). Study of optical phase lock loops and the applications in coherent beam combining and coherence cloning. Ph.D. thesis, California Institute of Technology.
- 5. **Ramos, R.** and **A. Seeds** (1990). Delay, linewidth and bandwidth limitations in optical phase-locked loop design. *Electronics Letters*, **26**(6), 389–391.
- 6. **Satyan, N.** (2011). Optoelectronic control of the phase and frequency of semiconductor lasers. Ph.D. thesis, California Institute of Technology.