Use of biopotential sensors in human computer interface applications

A Project Report

submitted by

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THESIS CERTIFICATE

This is to certify that the thesis titled Use of biopotential sensors in human-computer

interface applications, submitted by Jobin Jacob Kavalam, to the Indian Institute

of Technology, Madras, for the award of the degree of Bachelor of Technology and

Master of Technology, is a bona fide record of the research work done by him under

our supervision. The contents of this thesis, in full or in parts, have not been submitted

Place: Chennai

to any other Institute or University for the award of any degree or diploma.

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Abstract

Keywords: Bio-potential sensors, assistive technology, human computer interfaces

Human computer interfaces in a very crucial way determine the accessibility of various technology to the end user, in particular, to a disabled user. Hence, designing and building new modes of human-computer interaction is a major thrust area within the realm of assistive technology.

This work explores the use of bio-potentials, or electrical signals picked up from the human body, in building such interfaces. Bio-potentials provide a host of options ranging from direct decoding of brain intentions to detection of gestures. However, a key engineering challenge is the reliable acquisition of these bio-electric signals and their subsequent processing.

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Introduction

The use of biopotential sensors in assistive technology is motivated by the fact the conventional interfaces such as the mouse and the keyboard require the fine motor control of the hand. Such control is often missing in individuals with disabilities such as cerebral palsy.

Hence we identified two modes of interaction that are feasible during biopotential sensors. One was the Brain Computer Interface based on the Electroencephalogram and the second one - an eye tracker - based on the electrooculogram.

A prototype of the eye tracking system has been built with good preliminary results.

However, with respect to the brain computer interface, we focused on building the instrumentation for acquiring the EEG signals. This effort led to the formulation of a set of guidelines that are critical in the design and construction of a reliable EEG acquisition system.

Prior Art

Non-biopotential based

Head mouse and Eye gaze tracking

System tracks a IR reflective spot worn by the user on forehead or on a cap. Cursor movement is achieved by moving the head and click commands are based on holding steady position for a sufficient dwell time. Uses a camera fixed at a specific distance in front of the eye. Enables cursor control by determining the point of the gaze.

Advantages: Often faster than many other approaches.

Disadvantages: Control requires a lot of attention from the user. Can be quite strenuous. In cursor control applications, the method of signaling a mouse click is based on dwell times. This approach can be very tricky for the user. Camera and processing can make the system more expensive. Very difficult to use for CP patients who doesn't have the fine motor control required.

Bio-potential based

EEG based BCI.

Researchers have successfully used the possible voluntary control of Mu-rhythms. A trained user can interact through imagined movements.

Advantages: Accessible to patients who are even completely paralyzed. This is because no level of motor control is assumed and the control signals are directly picked up from the brain.

Disadvantages: EMG contamination adversely affects. This is particularly strong in patients with cerebral palsy etc. for whom involuntary muscle can frequently occur.

Secondly, much neurofeedback training is required before sufficient voluntary control over the Murhythms can be achieved.

EEG/EMG based cursor control

Facial gestures such as Left/Right Jaw movement, Full Jaw Clench, Eyebrows Up etc. are used to signal cursor movements and clicks. The EEG is used as an enable switch to engage/disengage gesture detection.

Advantages: Since, based on the EMG signals, requires almost no training and is accessible to most users.

More robust in real use compared to pure EEG/gaze tracking systems as false detections are more difficult in this approach.

Disadvantages: Quite slow for a cursor control like application. Moving the cursor from one corner to the Centre of a typical screen can take roughly 15s.

More detailed survey can be found in Appendix A.

EEG or the Brain waves

EEG or the brain waves is a manifestation of the electrical activity within the brain. Unlike the familiar ECG which has a unique shape and signature, the EEG signal is essentially a superposition of the various neuronal firings that occur all the time inside the brain. The exact signal varies a lot from one location on the scalp to the other. The origin of the signal can be traced back to the currents that are generated when a large number of neurons in a particular region fire in unison.

Hence neuroscientists look at the EEG as composed of various rhythms or periodic oscillations. Depending on the frequency of oscillation, its classified as alpha, beta, delta etc. They have also been map relative strength of these rhythms to certain gross mental states - such as relaxed, active - and in certain cases to specific mental activities such as imagined movements etc. Another feature of the EEG that is often exploited in building Brain computer interfaces are the event evoked potentials. These are certain patterns that are picked from particular sites of the brain when the subject responds to a stimulus. The visually evoked potential is an example. It is an oscillation of corresponding frequency picked up from the occipital region of the brain when the subject is exposed to a flickering source of light.

Brain Computer Interface

However the major challenge associated in implementing a Brain Computer Interface or (BCI) is that these brain waves are extremely low amplitude signals. The alpha and beta rhythms are of the order of a few microvolts, while the event related potentials are even smaller and show up only when the EEG signal is averaged over several cycles.

Evaluation of the Neurosky Mindwave for BCI Application

As a separate study we also evaluated a commercial Brainwave device. Called the Neurosky Mindwave Mobile this device provides the attention level of the user using proprietary processing algorithms on the EEG data. We built a simple application in which the attention numbers were logged for a duration of 5 minutes. Tests were conducted in which the subject performed different activities.

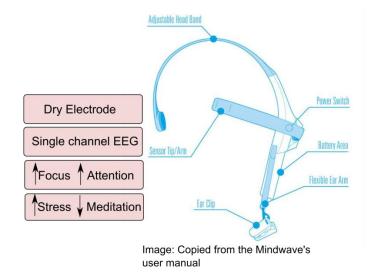


Figure 1 Neurosky Mindwave Mobile

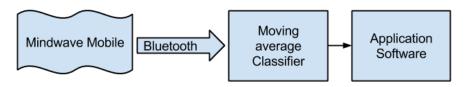


Figure 2 Block diagram of a simple BCI system built around the Neurosky device

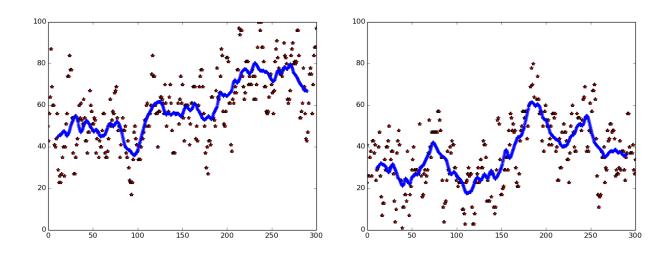


Figure 3 Attention vs. Time profiles obtained for two different activities. Left: Subject stays relaxed for 5 minutes. Right:

Subject is solving arithmetic problems.

Key components of a Brain Computer Interface (BCI)

Although the Neurosky mindwave shows appreciable differences in the averaged recording over a long time, the variability from sample-to-sample is high and hence makes its use difficult for a real-time BCI application.

As a result, there is the need to develop a more reliable EEG data acquisition system with higher signal quality.

The key components of a BCI are,

- 1. Electrodes
- 2. Electrode mounting assembly
- 3. Analog Front End
- 4. ADC and Digital Signal Processing
- 5. Classification algorithm
- 6. Application software

As discussed above our work mostly focused on the data acquisition part including electrodes and the analog front end.

Development of Dry Electrode for EEG

In typical clinical practice wet electrodes are often used owing to requirements on high signal quality. This involves the use of a conductive paste. Moreover, scalp preparation is carried out. These factors make the use inconvenient. Hence there is a need for dry electrodes that have sufficient signal quality, but also can be used in the case of a haired scalp.

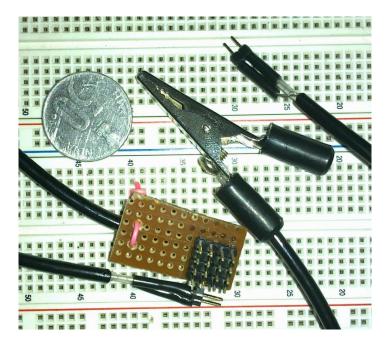


Figure 4 Dry electrode pair developed. One contact is formed by an array of pins connected to a shielded wire lead. The other is a crocodile clip based electrode.

Testing of the dry electrode

In order to test the electrode it was hooked onto an Analog Front End circuit (as described in a later section). As shown in figure below, there is appreciable pickup of test signals by the electrode under

test conditions. However, when the electrode was attached to an actual subject, much noise levels were identified.

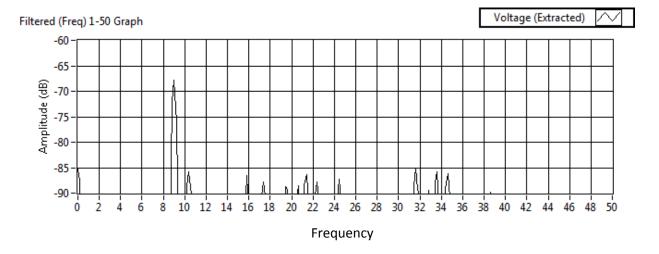


Figure 5 FFT of the acquired signal. The dry electrode pair was stimulated from a function generator producing 9 Hz / 50 uV sine wave.

A methodology for developing the data acquisition system

Data acquisition is hence a key step in enabling the Brain Computer Interface. For the same reason in this project we gave emphasis to getting the data acquisition right. Drawing from the lessons learnt, especially with the shortcomings of the dry electrodes, I propose a methodology that is useful for anyone thinking of building a brain computer interface.

The guiding principle is the identification of the noise sources and taking the proper precaution.

The work "Scalp electrode impedance, infection risk, and EEG data quality" by Feree et. al. was found very insightful.

The two important indicators of signal quality that we have identified are the skin-electrode impedance and the 50Hz noise level in the acquired signal. These two parameters should be carefully monitored during the development of the system and potentially even during use.

Skin Electrode Impedance

The skin electrode impedance is the impedance found at the electrode contact and the skin. If the biopotential being measured was to be modelled as a voltage source embedded in the brain, this impedance would form the bulk of its source impedance. In traditional clinial practice it was considered important for signal quality to bring this impedance down to 5 kOhm or less. To this end, scalp preparation involving skin abrasion was followed. This practice is now considered unnecessary. Nevertheless, it is important to ensure that impedance levels are low enough. For the same reason most EEG measurement systems have built in impedance measurement function. This can be a very useful diagnostic tool that can be built into the hardware right from the beginning.

Besides the skin condition of the user, which is really beyond the control of the designer, the skin electrode impedance is determined by two major factors - one the use of a conductive gel/paste and two the mounting or how well the electrode is fixed. While the use of conductive paste is an incovenience, few working systems which doesn't use the paste has been demonstrated. On the other hand, the skin-electrode should be carefully monitored while developing a dry electrode system. Secondly, the mounting or the assembly that holds the electrode in place is perhaps equally important as the electrode itself, as it is important to ensure adequate pressure between the electrode and the skin to maintain proper electrical contact.

Grounding

Besides the impedance another important indicator of signal quality is the amount of 50Hz noise in the acquired signal. In practice this is intimately related to the grounding technique used.

In the isolated bias configuration, sometimes known as the driven right leg, the bias on the subject is set by feeding back the acquired signal. No connection to the earth is made.

Today it is considered the best practice to use a common isolated electrode in contrast to the direct ground connection. As demonstrated by Ferrer this isolated bias connection improves noise immunity of the system by orders of magnitude. This also has the added advantage of ensuring the safety of the subject. In fact, this is the natural configuration when the instrument is battery operated.

Integrated Analog Front Ends such as the Texas Instruments ADS1299 has this feature built in.

Other factors

Another factor to look out for includes large wire loops that can pick up magnetically coupled noise.

EOG based Eye tracking system

"Eye gestures" as control signals

The human eye acts as an electric dipole with the retina and the cornea as the two charge centers. Hence different positions of the eye as shown in Figure 6 lead to small but measurable shifts in the voltages measured from the face.

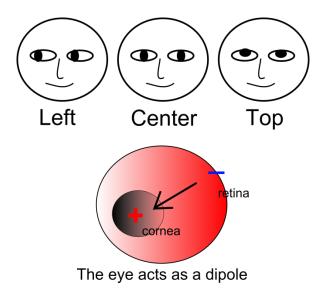


Figure 6 Different orientations of the eye cause the dipole to correspondingly move. This leads to small but measurable shifts in the voltages measured from the face

The objective is to detect a set of six such eye gestures – Eye looking left, right, up, down as well as blinking of eye and twitching of the eye brow – in real time and map them to control actions on a computer.

Example applications

Below are some ideas for applications that such an interface will enable.

- A basic menu where the selection automatically moves from one item to the next. To select the current item the user performs an eye brow gesture.
- A file browser based on the above menu concept. Eye brow gesture opens the corresponding file/folder.
- Game 'Lost in Space' where up/down/left/right gestures fires thrusters in the corresponding direction. The objective is to navigate the space ship to the nearest worm hole.
- Accessibility features to help web browsing. Panning/Zooming of the web page controlled through eye gestures.

System block diagram



Figure 7 System level diagram of the Eye gesture based interface.

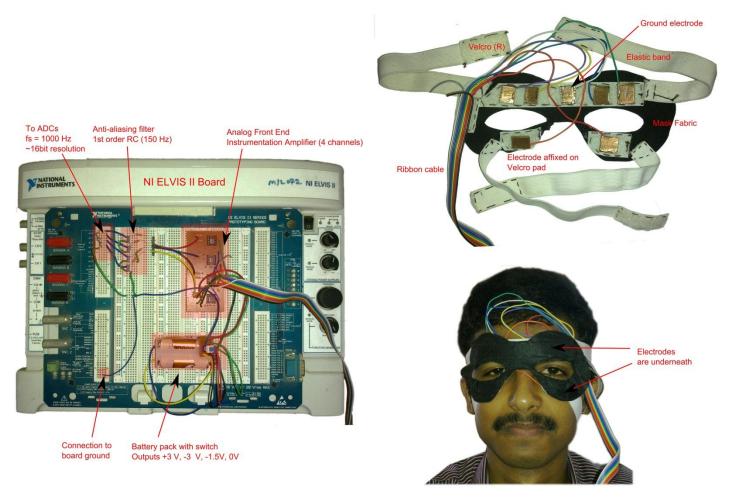


Figure 8 Various hardware components of the eye gesture tracking system

Electrode

Electrode placement

For the present prototype we chose the electrode positions shown in Figure 9.

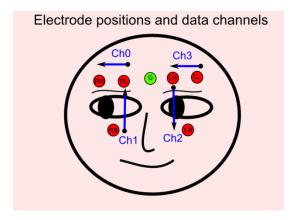
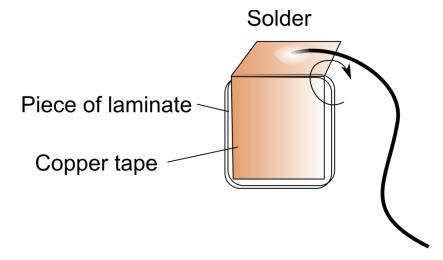


Figure 9 Placement of electrode on the face. Three electrodes around either eye form two pairs of differential channels. Thus there are a total of four channels of signals. The green position denotes the ground electrode that sets the common bias.

Construction of electrodes

The guidelines discussed in the section "A methodology for developing data acquisition system "A methodology for developing data acquisition system" were considered. However, the EoG signal being considerably larger in amplitude compared to the EEG signal significantly reduced the chances of failure.



The electrode forms a key component of the system as a proper contact is important to ensure good signal quality.

We used copper tape as the contact surface. It was supported on a piece of laminate using adhesive. The tape was bent along the edge of the rectangular laminate piece. A multi-stranded conductor was directly soldered to the tape.

Electrode mounting assembly - Mask

As discussed in the section ""A methodology for developing data acquisition system", the mounting of the electrodes play a crucial role in ensuring reliable measurement. It also impacts the user experience. Hence the following factors were considered while coming up with a design for the electrode mounting assembly:

- 1. Easy to wear/remove
- 2. Ensure contact with adequate pressure
- 3. Ensure there is no electrode slipping

Two alternative designs were considered. However, the mask in Figure 10 was chosen based on ease of fabrication, adaptability to different faces, and general ease of use. Another design considered was inspired by headsets of music players etc. (Figure 11)

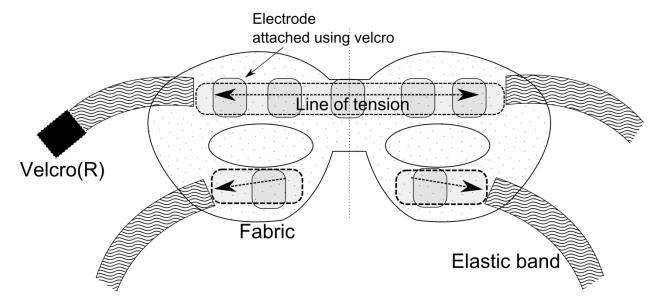


Figure 10 Electrodes were mounted on a mask

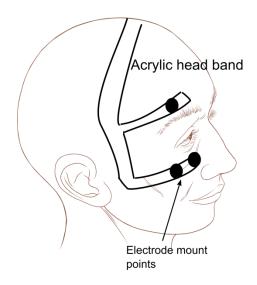


Figure 11 A design that was considered for electrode mounting

Construction of mask

A piece of fabric was cut out in the outline of the mask. The openings for the eye were also made. Elastic bands were prepared by stitching Velcro to their one end. These were then stitched to the mask at the other end. The attachment of the electrodes to the mask was also made possible using Velcro strips stitched on the mask surface.

Analog Front End

Figure 12 shows an instrumentation amplifier (INA 118) followed by an opamp buffer for scaling the output down to the input range of the ADC. The INA is the amplifier of choice owing to the following factors:

- Differential input Voltage measurement is by definition differential.
- Very high input impedance As a result the IR drop across the input impedance is kept very low, allowing the system to work even in the presence of a relatively high skin-electrode impedance.
- High IMRR (Insulation Mode Rejection Ratio) The ability to reject the common mode signals give much immunity against power line noise

The gain of the INA was set to about 10.

The circuit was powered using a battery pack consisting of AA cells. This provided intermediate voltage levels that was used to reference the ADCs (see the 'tied to uC GND').

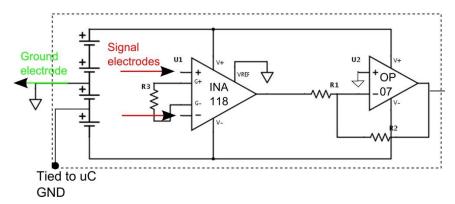


Figure 12 Circuit diagram of the analog front end

Data acquisition using Elvis board

As shown in Figure 8 the National Instrument's ELVIS board was used to sample and digitize the amplified signal. The grounding scheme used is shown in Figure 13. This is not strictly an isolated common configuration as prescribed in section "A methodology for developing data acquisition system" as the bias electrode is connected to the earth. Hence, this connection may not be the optimal for more sensitive measurements such as EEG, in which case, the use of the ELVIS board has to be done with caution.

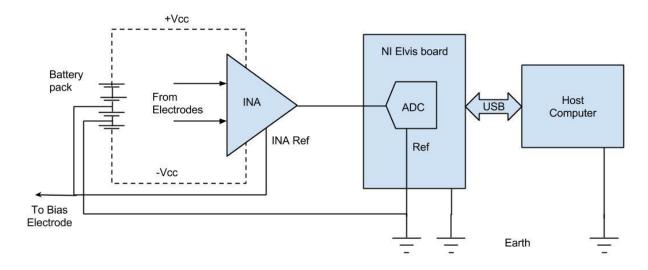


Figure 13 Grounding scheme

Detection Algorithm

The detection of gestures is based on observation of patterns in the voltage time series obtained from the electrodes.

For example, assume the voltage vs. time plots of differential voltages measured from the face were observed to be as shown in Figure 14.

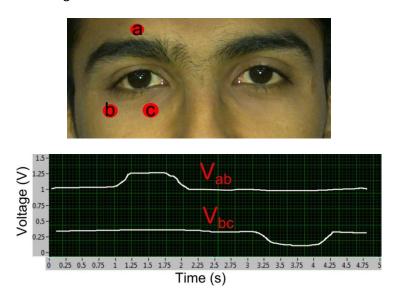


Figure 14 Illustration of how various channels respond to eye gestures

According to the dipole principle discussed previously, Vab will respond to vertical movements of the eye while Vbc to the horizontal movements. Hence, the given time series can be interpreted as resulting from a vertical movement followed by a horizontal movement. The polarity of the deflection can be used to determine the exact direction of the movement.

It is this decision making process that the detection algorithm aims to automate. The following flow chart captures the working of the algorithm.

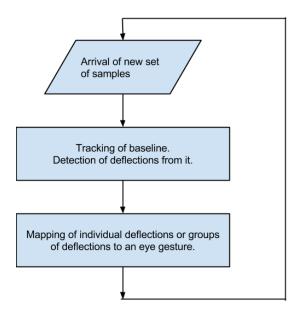


Figure 15 Detection algorithm

The tracking of baseline is important because there can be slow shifts in the baseline with time. Figure 16 shows the essential logic used in detecting various gestures. The blue curve represents the original time series with the green showing the trajectory of the baseline tracker. The stars in the second graph denote the start and end of an event as detected by the algorithm.

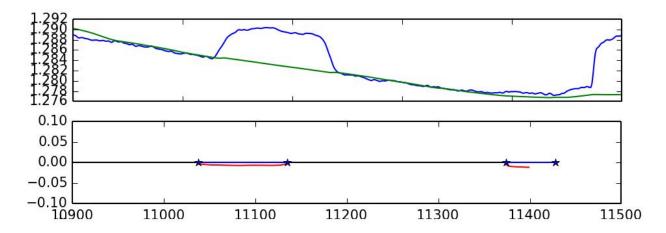


Figure 16 (Above) Blue is the acquired voltage in volts. X axis is the sample number. Green is the path followed by the baseline tracker. Second plot marks the regions where the error between blue and green exceeds the threshold.

The overall algorithm can be expressed in more precise terms as shown in Figure 17.

```
Let Y = y_1, y_2, ..., y_n be a buffer of n samples. a_1, a_0 corresponds to the coefficients of a linear fit to Y such that, \{y_i' := a_0 + a_1 \times i\} has the least sum of squared errors. Using these coefficients we can make the prediction, y_{(n+1)}' = a_0 + a_1 \times (n+1) The error is defined as the difference of this prediction from the actual, e = y_{(n+1)} - y_{(n+1)}' if |e| > e_{thresh} then y \leftarrow \{y_2, y_3, ..., y_n, y_{(n+1)}'\} else y \leftarrow \{y_2, y_3, ..., y_n, y_{(n+1)}\}
```

Figure 17 Baseline tracking and the Detection algorithm

For each channel the baseline tracking is performed and deflections about the baseline are detected. Corresponding to each deflection (such as the bump in Figure 16) a new 'FEvent' ('Feature Event') is created. Besides, the time stamps corresponding to the beginning and ending of a deflection, an FEvent is also characterized by the maximum and sum of the error values in that interval. Such events observed across the channels are aggregated and processed by the following algorithm to detect gestures.

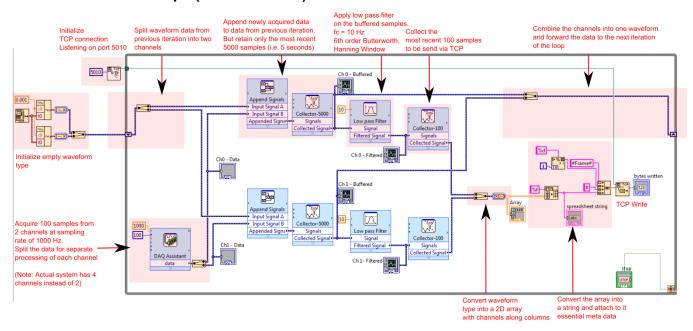
Based on these feature values, to each set of overlapping events (taken one or more at a time) a confidence measure is assigned that denotes the probability that a particular gesture has occurred.

To ensure that a prediction with low confidence is not released prematurely, a time limit, called mat_n (maturity time) is set before which the prediction cannot be released. This gives ample time for other events with higher confidence scores to get considered over and above the current prediction.

Software

While using the device as a computer interface, the availability of a capable computer means that the embedded software only needs to forward the raw data onto the host computer – the bulk of the processing can then be done on the host. The overall processing flow is captured in Figure 19 while Figure 18 shows the server/client setup used to make the acquired data available to the detection algorithms.

The Server Loop (LabVIEW)



Client (Python)

Figure 18 A data server is setup using LabVIEW and a client written in python connects to it over TCP

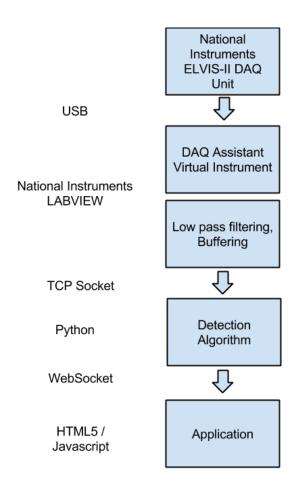


Figure 19 The components of the software system

Testing

A demo application was built with the interface as shown in Figure 20.

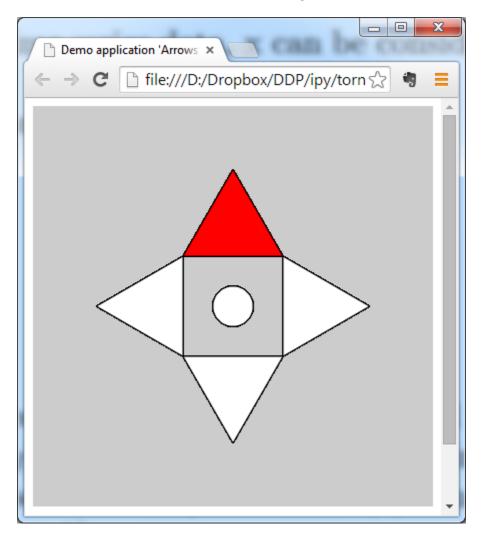


Figure 20 Demo application that responds to the eye gestures. This figure shows the response to an 'eye up' gesture.

Conclusion

To conclude the following are the key contributions of this work.

- A working prototype of an eye-gesture based computer interface
- Successful application of biopotential measurements around the eye in assistive technology
- Formulation of a set of guidelines for design and construction of similar biopotential instrumentations including EEG
- Evaluation of a commercial BCI device, the Neurosky Mindwave

Appendix A: A survey of existing 'bio' based Interfaces

In recent months, there has been growing interest in novel computer interfaces among the global hacker community and also with product companies and startups. As one group put it, 'Effortless interaction' methods have attained much attention and many are looking to biopotentials as a promising candidate technology. Other alternatives include computer vision based solutions.

Based on Biopotential sensors

EEG

The OpenBCI initiative that started in late 2013 aims to build open source hardware/software for EEG based Brain Computer Interfaces. They have already come out with a board for data acquisition and has demonstrated some popular BCI paradigms such as SSVEP etc. eeghacker.blogspot.com is an associated blog that could be a good starting point for building BCIs.

The OpenEEG is another well known, but outdated effort to build EEG hardware. The basic principles of EEG data acquisition are well presented in their website and design documents. However, given the availability of integrated solutions for biomedical applications from chip manufactures, their designs can be considered outdated.

Other commercially available hardware include the Neurosky, Emotiv devices. Of all the Neurosky is the most convenient to use of all. But, extracting useful information from the acquired signal proved difficult. The author has since heard similar experience from another undergraduate student who worked with the device. The Emotiv headset has the disadvantage of not usable on an unshaven head.

EOG

On the eeghacker blog, the use of the OpenBCI board for acquiring EOG signals was demonstrated. This is in line with the authors experience that no modification other than changes to the electrode positions are required to use a working EEG instrumentation for acquiring other kinds of signals.

EMG

Myo is an EMG armband being worked on by Thalmic Labs, a startup. They claim the arm band can detect numerous hand gestures.

Not based on bio-potentials

Some sensors work on non-electrical quantities such as vibration etc. Some such interfaces are documented here.

Vibration based

"Skinput" is an example of a device that uses the vibrations picked up by a contact microphone (a piezo device) from the skin. It was demonstrated as a viable interface by Microsoft Research in which the user taps specific parts of the arm to signal different control actions.

References

Thomas C Ferree, Phan Luu, Gerald S Russell, Don M Tucker, Scalp electrode impedance, infection risk, and EEG data quality, Clinical Neurophysiology, Volume 112, Issue 3, March 2001, Pages 536-544, ISSN 1388-2457, http://dx.doi.org/10.1016/S1388-2457(00)00533-2.

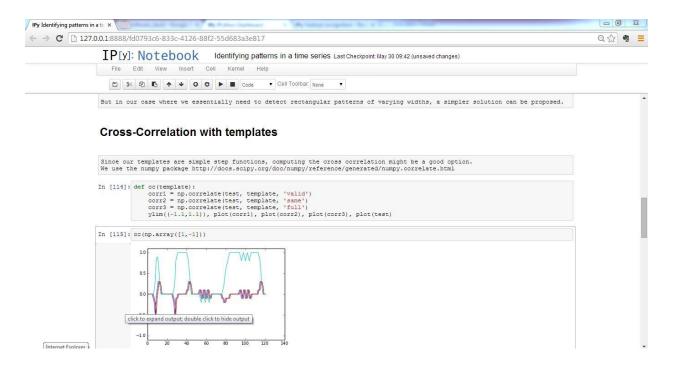
(http://www.sciencedirect.com/science/article/pii/S1388245700005332)

Keywords: Electroencephalography; Scalp abrasion; Infection risk; Electrode impedance; Signal quality; 60 Hz Noise

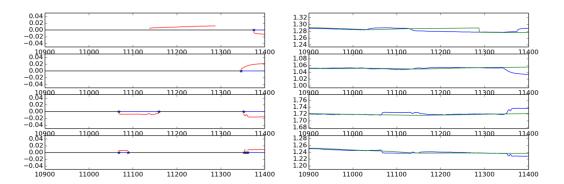
Appendix B: Tools used for data visualization

This section briefly describes the tools used for analyzing data and to aid the development of the algorithm.

1. IPython Notebook – This browser based interactive python environment was very useful in testing out snippets of python code as well as investigating various segments of the acquired time series data (using the pyplot plotting tools)



2. Matplotlib



3. Dygraph Javascript plotting library - dygraphs.com – was used to interactively look at patterns in the recorded traces



Appendix C: Incremental baseline tracking

The following algorithm Figure 21 was used to incremental compute the linear fit parameters.

```
if y = a_0 + a_1 x then a_1 = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}
a_0 = \frac{\sum y - a_1 \sum x}{\sum x}
Now in the case of windowed time series data, x can be considered as the range
of integers 1 ... n
In that case, the above equations become,
a_1 = \frac{n\Sigma(jy) - \Sigma j\Sigma y}{n\Sigma j^2 - (\Sigma j)^2}
a_0 = \frac{\sum y - a_1 \sum j}{2}
where,
\begin{split} \Sigma j &= 0 + 1 + \ldots + n - 1 \\ \Sigma j^2 &= 0^2 + 1^2 + \ldots + (n-1)^2 \end{split}
are constants, while,
\Sigma y, \Sigma y^2 can be incrementally computed in constant time and O(n) memory.
The only remaining term, \Sigma(jy) may seem to involve O(n) time.
But, as we see below, it can also be accomplished in O(1) time
For example (index of y starts at 1),
(\Sigma jy)_1 = 0y_1 + 1y_2 + 2y_3 + 3y_4
(\Sigma jy)_2 = 0y_2 + 1y_3 + 2y_4 + 3y_5
then.
(\Sigma jy)_2 - (\Sigma jy)_1 = -y_2 - y_3 - y_4 + 3y_5
i.e. (\Sigma jy)_2 = (\Sigma jy)_1 - (\Sigma y)_1 + y_1 + 3y_5
in general,
(\Sigma jy)_{k+1} = (\Sigma jy)_k - (\Sigma y)_k + y_k + (n-1)y_{k+n}
```

Figure 21 Algorithm used to incrementally compute the linear fit parameters