ANALYSIS OF PF AND MLWDF SCHEDULING ALGORITHMS IN WIRELESS COMMUNICATIONS

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CERTIFICATE

This is to certify that the report titled **ANALYSIS OF PF AND MLWDF SCHEDULING ALGORITHMS IN WIRELESS COMMUNICATIONS**, submitted by **P SURYA PRASAD REDDY, EE08B079** to the Indian Institute of Technology Madras, for the award of the degrees of **Bachelor of Technology** and **Master of Technology**, is a bona fide record of the project work done by him under my supervision. The contents of this report, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

Analytical expressions for average delay, throughput and packet drop rate are derived for Proportionally Fair Scheduling (PFS) algorithm for the cases of both IID and Non IID Rayleigh channels for the case of data transfer rate linear with SNR of the wireless channel and only under IID Rayleigh channels for the case of data transfer rate logarithmic with SNR of the wireless channel. We studied the influence of packet drop rate on average delay and throughput of PF scheduling. Similarly for Modified Largest Weighted Delay First (MLWDF) scheduling algorithm analytical expressions for average delay, throughput and packet drop rate are derived for the case of IID Rayleigh channel and also analyzed the influence of packet drop rate on average delay and throughput in MLWDF scheduling.We also analyzed the tradeoff between delay parameters, throughput and packet drop for the above scheduling algorithms. The above relations have been derived using a Markov chain state model in which the states represent delay of the users.

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CHAPTER 1

Introduction and related work

The increasing demand for the data services both real time and non real time has drawn attention to the importance of scheduling algorithms which utilizes the constrained wireless resources to the maximum extent possible as well as meeting Quality of Service (QoS) requirements. Various scheduling algorithms have been proposed in [1]–[3]to meet the diversified interests of various organizations. In the discussion below we assume time is slotted and the base station schedules the user at each time slot for down link data transfer. Each user ends his data transfer at the end of this given slot. Apart from this channels vary in both in slow and fast time scale. With the case of fast fading the channel varies asynchronously from good to bad in a matter of milli seconds, similarly slow fading channels causes the users with bad channel conditions demand more air resources than the users with good channel conditions. A good scheduling policy must take advantage of this and adapt to scheduling the users when they are in good state. The important QoS requirements include multiple real time users be supported simultaneously with good quality of service for all users, namely packet delays not exceeding thresholds with high probability simultaneously the mixture of real time and non real time users , with real time users receiving the desired QoS and non real time users having the maximum possible throughput without compromising the QoS of real time users. Different algorithms provide different significance to above criteria while scheduling. It is in our best interest to be able to understand the trade off to select the most appropriate scheduling algorithm to meet our desires.To analyze the tradeoff between delay and throughput first we require the analytical expressions for delay and throughput. In this paper we attempt to derive analytical expressions for the average delay and throughput for the popular Proportionally Fair (PF) and Modified Largest Weighted Delay First (MLWDF) scheduling algorithms. Ideal requirements include no packet drop, but in real time situations packets are dropped when the TTL (Time To Live) of the packet expires. High packet drop rate worsens the effective utilization of the channel, hence the packet drop rate should also be monitored. This brings packet drop rate parameter into the picture. The objective is now modified to select the scheduling algorithm with the desired trade off between delay, throughput as well as packet drop rate.

So far the majority of the works [4], [5] in PF concentrated over throughput gain.Analysis of delay in PF scheduling is done in [6]–[8]. MLWDF scheduling algorithm was analysed in [9], [10].The focus of the work is to analyze the behavior of PF scheduling algorithm's various aspects analytically such as average delay, no of slots required to schedule $X\%$ of packets, packet drop rate and throughput gain for various scenarios. We derived the analytical expressions for the above which are verified by comparing them with the simulated results. We also derived analytical expressions for MLWDF scheduling algorithm for the case of iid Rayleigh channels with no packet drop and rate is linear with SNR. We observed the trade off of better throughput gain of PFS with the better delay performance in MLWDFS along with some interesting relations. Our work is organized below.

In section II , we explain the commonly used scheduling algorithms in wireless communication and a brief explanation of its functionality. In section III, we derived the expressions for throughput gain, packet drop rate and delay metrics for PF scheduling under various conditions. In section IV, similar to section III we derived the expressions for the case of iid Rayleigh channels with no packet drop and rate is linear with SNR. The differences between PFS and MLWDF is analyzed in section V.

CHAPTER 2

Scheduling Algorithms

Consider a single cellular system where there are N active users, each with an infinite buffer of data and no data rate constraints. Let $r_i(t)$ denote the transmission rate of user i at slot t if scheduled and is assumed that $\{r_i(t), t \geq 0\}$ is a stationary and ergodic process. Similar to [11], it is assumed that rate prediction is perfect. $\tau_i(t)$ is a stationary process which denotes the received channel power of user i at time slot t on a sub carrier in the OFDM symbol, then the SNR of the same sub carrier seen by the user i at time t can be defined as $ga_i\tau_i(t)$ where $g = \frac{E_s}{\sigma_T^2}$,$E_s$ is the transmitted symbol energy and σ_T^2 is the thermal noise variance and a_i denotes the path loss and shadowing effects which captures the slow scale fading characteristics. Let $W_i(t)$ denote the delay of user i at time t and it is assumed to be stationary. If the user i is scheduled at time t, then $W_i(t+1) = 0$.Some of the commonly used scheduling algorithms have been described from [12].

2.1 Round Robin Scheduling

Round Robin (RR) Schedules the users in a cyclic fashion i.e. all the users experience the same amount of time to get scheduled. One can describe the RR scheduling policy as selecting the user $i(t)$ at slot t according to

$$i(t) = arg_{j=1...N} W_i(t) = N - 1$$

All the users experience the same delay and it is unable to exploit the channel variations. So though it is very simple it is commonly not used in applications requiring high performance.

2.2 Max Rate Scheduling

Max Rate scheduling provides the system with the maximum possible throughput it can achieve. The Max Rate scheduling policy selects the user $i(t)$ at slot t according to

$$i(t) = \arg \max_{j=1,\ldots,n} r_j(t)$$

It schedules the user which supports the maximum rate at the time of scheduling decision. Though the system can achieve maximum possible throughput it almost does not provide any service to the users with bad channel conditions. It is not a good algorithm considering the QoS requirements imposed on the scheduling algorithms.

2.3 Proportionally Fair Scheduling

Proportional Fair scheduling tries to maximize the wireless channel bandwidth effective utilization as well as balancing the throughput's of the users according to their channel conditions. It was designed for delay insensitive users. PFS selects the user $i(t)$ at slot t according to

$$i(t) = \arg \max_{j=1,\ldots,n} \frac{r_j(t)}{T_j(t)}$$

where $r_j(t)$ is the data rate that can be supported by user i at time t. The throughput $T_j(t)$ is typically evaluated through an exponentially smoothed average :

$$T_j(t) = \begin{cases} \left(1 - \frac{1}{t_c}\right) T_j(t-1) + \frac{1}{t_c} r_j(t), & j = i(t) \\ \left(1 - \frac{1}{t_c}\right) T_j(t-1), & j \neq i(t) \end{cases}$$

where t_c is the time-window length over which one wants to regulate the fairness. Pf provides the better QoS service since the user with bad channel conditions have lower denominator which enhances the decision metric until it is scheduled. In spite of the above it still does not cater to the meet the delay constraints which is very important for real time users.

2.4 Modified Largest Weighted Delay First Scheduling

MLWDFS tries to provide the better delay QoS as well as maximizing the throughput possible. MLWDFS selects the user $i(t)$ at slot t according to

$$i(t) = \arg \max_{j=1,\ldots,n} \frac{r_j(t)W_j(t)}{T_j(t)}$$

where $r_i(t)$ is the data rate that can be supported by user i at time t and $W_i(t)$ is the HOL delay of user i at time t.The throughput $T_j(t)$ is typically evaluated through an exponentially smoothed average as described in section 2.3.

2.5 Exponential Rule Scheduling

Exponential Rule scheduling selects the user $i(t)$ at slot t according to

$$i(t) = \arg \max_{j=1,\ldots,n} \frac{r_j(t)}{T_j(t)} exp\left(\frac{W_i(t) - \overline{W}(t)}{1 + \overline{W}(t)}\right)$$

where $r_i(t)$ is the data rate that can be supported by user i at time t, $W_i(t)$ is the HOL delay of user i at time t and $\overline{W}(t) = \frac{1}{N}\sum_{i=1..N} W_i(t)$ is the average delay of the system at time t.The throughput $T_j(t)$ is typically evaluated through an exponentially smoothed average as described in section 2.3.

CHAPTER 3

System Model

The wireless channel of the sub carrier $\mathbf{h}_i(t)$ in time domain at slot t of the i^{th} user comprises of L taps and has unit energy. The analysis below is done for Rayleigh fading .For Rayleigh fading, the received channel power $\tau_i(t)$ seen on a sub carrier for the i^{th} user has a pdf given by

$$f_{\tau_i(t)}(x) = \lambda e^{-\lambda x} \tag{3.1}$$

We define $Y(t) = [W_1(t)...W_N(t)]$ is a $1-N$ system state vector whose entries denote the delays of users $1...N$ at time t and is stationary. Let Z_i be the r.v that denotes the delay at which the user is scheduled(renewed is also used interchangeably). Since $W_i(t)$ and $Y_i(t)$ are assumed to be stationary, we denote them by W_i and Y. We need to distinguish the difference between $P_W(W_i = n)$ and $P_Z(Z_i = n)$, the former represents the probability of user i having delay n at time t while the latter represents the probability of renewal at delay n .To simplify our analysis we assume independence between delay states as explained in (3.2) and the system model as in Figure 1 .

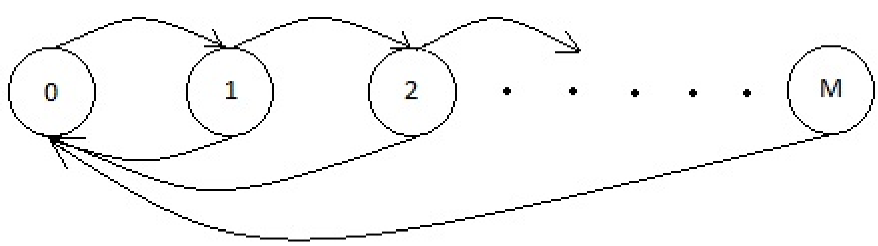

Fig. 3.1 Model depicting the state flow of each user

$$P(Y=[m_1,m_2,...,m_N]) = \prod_{i=1}^{N} P_W(W_i=m_i) \tag{3.2}$$

The pmf of the renewal delay of the model as given in [13]

$$P_Z(Z_i=n) = P^i_{0,1}P^i_{1,2}...P^i_{n-1,n}P^i_{n,0} \tag{3.3}$$

where $P^i_{n,n+1}$ denotes the transition probability of user i in delay state n not being scheduled while $P^i_{n,o}$ denotes the probability of user i in delay state n being scheduled.The above equation intuitively implies that probability of renewal at a particular delay is the probability of being present in that delay state and then being scheduled.The pmf of the delay state of user i as given in [13]

$$P_W(W_i=n) = P_W(W_i=0)*P^i_{0,1}P^i_{1,2}...P^i_{n-1,n} \tag{3.4}$$

for $i=1,2...$ From (3.3) and(3.4) , the transition probability is

$$P^i_{n,o} = \frac{P(W_i=0)*P_Z(Z_i=n)}{P_W(W_i=n)}$$

We derive the expression for transition probabilities by expectation over the conditional transitional probabilities i.e. we are averaging over all the possible scenarios of being scheduled.

$$P^i_{n,0} = E_{\tau_i}[E_{Y_i'}[P^i_{n,0/Y,\tau_i}]] \tag{3.5}$$

$P^i_{n,0/Y,\tau_i}$ denotes the probability of scheduling of user i given the current system state and received channel power of user i. The conditional expectation of transition probability on received channel power is evaluated by summing over all the possible system delay states $Y_i^{'} = [m_1...m_N]$ where m_j is varied from 0 to ∞, $\forall j=1,..,N$ and $j \neq i$ and $m_i = n$

$$E_Y[P^i_{n,0/Y,\tau_i}] = \sum_{Y_i^{'}} P^i_{n,0/Y,\tau_i} * P(Y=y) \tag{3.6}$$

To simplify the above expression, as stated above we assume the independence of the delay states between the users and(3.6) simplifies to

$$E_Y[P^{i}_{n,0/Y,\tau_i}] = P_W(W_i = n) \sum_{Y_i'} P^{i}_{n,0/Y,\tau_i} * \prod_{j=1, j\neq i}^{N} P(W_j = m_j) \tag{3.7}$$

Similarly we try to derive the throughput expression by expectation over the rate conditioned on being scheduled over all the possible scenarios of being scheduled.

$$T_i = E_{\tau_i}[r(\tau_i) * E_Y[P^{i}_{n,0/Y,\tau_i}]] \tag{3.8}$$

where $r(\tau_i)$ denotes the data rate as a function of received channel power. We have to note the difference between (3.5) and (3.8) , the former is averaged over conditional transitional probabilities for all the states Y_i' while the latter is averaged over conditional rates over the states Y .

CHAPTER 4

PF Scheduling

PFS selects the user $i(t)$ at slot t according to

$$i(t) = \arg\max_{j=1,\ldots,n} \frac{r_j(t)}{T_j(t)}$$

where $r_j(t)$ is the data rate that can be supported by user i at time t. The throughput $T_j(t)$ is typically evaluated through an exponentially smoothed average :

$$T_j(t) = \begin{cases} \left(1 - \frac{1}{t_c}\right) T_j(t-1) + \frac{1}{t_c} r_j(t), & j = i(t) \\ \left(1 - \frac{1}{t_c}\right) T_j(t-1), & j \neq i(t) \end{cases}$$

where t_c is the time-window length over which one wants to regulate the fairness. The PFS is asymptotically fair in the sense that all users receive the same fraction of the time slots in a homogeneous system [11], [14]. When $t_c \to \infty$, the average throughput's are stationary for the Rayleigh channel where the rate is linear with received SNR [11] i.e. $T_i(t) = T_i(t+1) = T_i$. But we assume that the throughput's are stationary even for the case of rate logarithmic with received SNR as verified by simulations. Based on this ground work for PFS we now try to analyze the delay parameters, throughput gain and packet drop rate for various scenarios below.

4.1 IID Rayleigh Channels, Rate is linear with SNR

The date rate supported by user i at time t is given by $r_i(t) = log(1 + ga_i\tau_i(t))$, where $ga_i\tau_i(t)$ is the SNR of user i at time t. We begin by making the assumption,rate is linear with SNR i.e $r_i(t) = ga_i\tau_i(t)$ and also with all the users having the same iid Rayleigh channel conditions.

4.1.1 Delay

The transition probability is state independent in this case and is assumed to be different constant for different users. Let p_i denote the transition probability of user i being scheduled.

$$P^i_{n,0} = p_i \tag{4.1}$$

Form the above eqn, plugging into (3.3) and (3.4) ,we get the pmf of renewal and delay states to be geometric distribution's

$$P_Z(Z_i = n) = (1 - p_i)^n p_i \tag{4.2}$$

$$P_W(W_i = n) = P_W(W_i = 0) * (1 - p_i)^n \tag{4.3}$$

By normalizing, i.e $\sum_{n=0}^{\infty} P_W(W_i = n) = 1$, (4.3) simplifies to

$$P_W(W_i = n) \quad = \quad (1 - p_i)^n p_i \tag{4.4}$$

To proceed further we need to evaluate the parameter p_i.Since we are considering iid channel conditions all the users experience the same shadow and path losses i.e. $a_i = a$, $\forall i = 1,...,N$. We now try to simplify (4.1) the with the linear assumption between rate and SNR and the stationary property of throughput's T_i and received channel power τ_i. The user i with received channel power τ_i is scheduled if

$$\frac{\tau_i}{T_i} > \frac{\tau_j}{T_j}$$

$\forall j = 1,...,N$and $j \neq i$. We already stated that asymptotically average throughput's T_i are stationary. But since it is iid channel conditions , each user experiences the same decisions and hence the same stationary average throughput implies $T_i = T_j$ $\forall i, j = 1,...,N$.The above scheduling rule simplifies to $\tau_i > \tau_j$ similar to Max Rate scheduling rule.The scheduling decision is delay independent and hence (3.6) simplifies to

$$E_Y[P^{i}_{n,0/Y,\tau_i}] = \sum_{Y_i'} P^{i}_{n,0/Y,\tau_i} * \prod_{j=1}^{N} P(W_j = m_j) \tag{4.5}$$

$$= P^{i}_{n,0/\tau_i} \tag{4.6}$$

$P^{i}_{n,0/\tau_i}$denotes the probability of scheduling given the channel strength of user i. for $j = 1...N$. The conditional transitional probability is now

$$P^{i}_{n,0/\tau_i} = \prod_{j=1, j\neq i}^{N} F_{\tau_j(t)}(\tau_i) \tag{4.7}$$

$$= \left(F_{\tau_i(t)}(\tau_i)\right)^{N-1} \tag{4.8}$$

where $F_{\tau_i(t)}(x)$ and $f_{\tau_i(t)}(x)$ are the cdf and pdf of the received channel power of user i. The probability of scheduling of user i is the probability that the remaining users have received channel power less than his own. The next step is obvious as all the users have same channel conditions and hence same $F_{\tau_i(t)}(x)$and $f_{\tau_i(t)}(x)$. (3.5) can be written after simplification of (4.6) as

$$\begin{aligned} p_i &= E_{\tau_i}[P^{i}_{n,0/\tau_i}] \\ &= \int_0^\infty f_{\tau_i(t)}(x) P^{i}_{n,0/\tau_i} dx \\ &= \int_0^\infty f_{\tau_i(t)}(x) \left(F_{\tau_i(t)}(\tau_i)\right)^{N-1} dx \end{aligned}$$

Since it is a Rayleigh channel, from (3.1) $f_{\tau_i(t)}(x) = \lambda_i e^{-\lambda_i x}$ and $F_{\tau_i(t)}(x) = 1 - e^{-\lambda_i x}$ and we get

$$\begin{aligned} p_i &= \int_0^\infty \lambda_i e^{-\lambda_i x} \left(1 - e^{-\lambda_i x}\right)^{N-1} dx \\ &= \sum_{r=0}^{N-1} \frac{(-1)^r \binom{N-1}{r}}{(r+1)} = \frac{1}{N} \end{aligned}$$

The above result makes sense as the users experience same iid channel conditions and the decision is delay independent for PFS, every one has equal probability of being scheduled at each slot. For the infinite window length, the average delay experience by

No of Users N	Simulated μ	Calculated μ
10	9.0007	9
25	24.0015	24
40	38.9992	39

Table 4.1 The table of calculated and simulated average delays for various no of users$N$

each user is

$$\begin{aligned}\mu_i &= \sum_{n=0}^{\infty} n * P_Z(Z_i = n) \\ &= \sum_{n=0}^{\infty} n * (1-p_i)^n * p_i \\ &= \frac{1}{p_i} - 1 \\ &= N - 1\end{aligned}$$

The simulated and calculated average delays of users under the iid Rayleigh channel conditions and rate linear with SNR assumption for infinite window is tabulated in Table 4.1 for various values of no of users N.As you can see that for iid channel conditions average delay of the users in PF behaves like the delay of Round Robin scheduling.

In practical situations, people do not consider average delay as a reliable parameter instead they want to know, no of slots M'required to schedule $X\%$ of packets as a parameter in analysis i.e. they want to ensure that the packets delays are with in the threshold limits with high probability.Let us recall the definition of r.v Z which is the probability of renewal at a particular delay .Hence no of slots M' required to scheduled $X\%$of packets can be be thought of as probability of renewal at delay less than M' to be $\frac{X}{100}$.

$$\begin{aligned}\frac{X}{100} &= \sum_{n=0}^{M'} P_Z(Z_i = n) \qquad (4.9) \\ &= \sum_{n=0}^{M'} p_i(1-p_i)^n \\ &= 1-(1-p_i)^{M'+1} \\ M' &= \frac{log(1-\frac{X}{100})}{log(1-p_i)} - 1\end{aligned}$$

No of Users N	M' Simulated $X = 50\%$	M' Calculated $X = 50\%$	M' Simulated $X = 75\%$	M' Calculated $X = 75\%$	M' simulated $X = 95\%$	M' Calculated $X = 95\%$
10	6	6	13	13	28	28
25	17	17	34	33	73	73
40	27	27	54	54	118	117

Table 4.2 The table of calculated and simulated M' for various no of users N and X

We consider the cases of $X\%$ to be 50%,75% and 95% . So we tabulated the simulated and calculated M' for various no of users in Table 4.2 and $M'$is rounded off to integer greater than M'.We observe the results for the case of 10 user system. The no of slots required to schedule X% of packets increases with X. We also notice that for the increase of $X\%$ from 50% to 75% , the no of slots increased by 6, but from 75% to 95% the no of slots increases by 15. For the same $X\%$, the value of M'increases with no of users.

4.1.2 Throughput

As stated above as $t_c \to \infty$ the average throughput's are stationary.Since we are now considering iid channel conditions, each user's throughput converges to the same value.$T_i = T_j, \forall i, j = 1, ..., N$. From (3.8) and (4.5)we evaluate throughput as

$$T_i = E_{\tau_i}[r(\tau_i) * P^{i}_{n,0/\tau_i}]$$

From (4.7) we simplify the above expression to

$$\begin{aligned} T_i &= \int_0^\infty r(x) * \lambda_i e^{-\lambda_i x}(1 - e^{-\lambda_i x})^{N-1} dx \\ &= g \int_0^\infty x * \lambda_i e^{-\lambda_i x}(1 - e^{-\lambda_i x})^{N-1} dx \\ &= \frac{g}{\lambda_i} \sum_{r=0}^{N-1} \frac{(-1)^r \binom{N-1}{r}}{(r+1)^2} \end{aligned}$$

One is often interested to calculate the gain of the scheduler with respect to the Round robin scheduler. For the RR scheduler each user gets scheduled once in N slots. Hence the average throughput of RR scheduler is $T_i^{RR} = \frac{1}{N} E[r(\tau_i)] = \frac{g}{N * \lambda_i}$. so the scheduling

No of Users$N$	G Simulated	G Calculated
10	2.9296	2.9290
25	3.8166	3.8160
40	4.3000	4.2785

Table 4.3 The table of calculated and simulated scheduling throughput gain for various no of users N

throughput gain of the scheduler G is given by

$$G = \frac{\sum_{i=1}^{N} T_i}{\sum_{i=1}^{N} T_i^{RR}}$$
$$= N * \sum_{r=0}^{N-1} \frac{(-1)^r \binom{N-1}{r}}{(r+1)^2}$$

The simulated and calculated gains of the iid Rayleigh channel with infinite window and linear assumption is tabulated in Table 4.3 for different no of users. One can notice that the throughput gain is directly proportional to no of users N, but the rate of increase decrease with N.

4.2 Packets dropped after maximum Delay,IID Rayleigh Channels, Rate is linear with SNR

We now analyze how the delay and throughput changes for the case of finite maximum delay setting M beyond which the packet is dropped by the scheduler. We still consider the iid Rayleigh channels with the assumption that the rate is linear with SNR $r_i(t) = ga_i\tau_i(t)$.

4.2.1 Delay

Since the scheduling decision is delay independent and does not depend on maximum delay setting, we have the same transition probabilities, $P_{n,0}^{i} = p_i = \frac{1}{N}$, for $n = 0, \ldots, M$ and $i = 1, \ldots, N$. So we now have to derive the expressions for average delay, before we proceed further we have to re examine the pmf of renewal delay as the infinite delay

No of Users$N$	μ Simulated $M=N$	μ Calculated $M=N$	μ Simulated $M=1.5*N$	μ Calculated $M=1.5*N$	μ Simulated $M=2*N$	μ Calculated $M=2*N$
10	3.9718	3.9694	5.3657	5.3608	6.3979	6.4199
25	10.2256	10.2458	14.0797	14.0354	16.7918	16.7349
40	16.4934	16.5176	22.4745	22.4468	27.1193	27.0419

Table 4.4 The table of calculated and simulated average delays for various no of users N and maximum delay settings M

states in previous cases reduced to finite length M.

$$P_Z(Z_i = n) = k(1-p_i)^n p_i$$

The constant k is evaluated by normalizing $\sum_{n=0}^{M} P_Z(Z_i = n) = 1$ which results in $k = \frac{1}{1-(1-p_i)^{M+1}}$. The average delay is evaluated as

$$\begin{aligned}
\mu_i &= \sum_{n=0}^{M} n * P_Z(Z_i = n) \\
&= \sum_{n=0}^{M} n * k(1-p_i)^n * p_i \\
&= k\left(\frac{(1-p_i)(1-M*(1-p_i)^{M+1}+(M+1)*(1-p_i)^{M})}{p_i}\right)
\end{aligned}$$

The simulated and calculated average delays of the iid Rayleigh channel with finite M and linear assumption is tabulated in Table 4.4 for different no of users N and maximum delay setting M. The average delays in this scenario decreased compared with the same case considered in section 4.1. This is because the packets which waited long for scheduling in the previous case are now dropped hence the the reduction in average delay but finite packet drop rate. As you increase M, some of the packets with greater delay dropped in the previous case are now scheduled which causes the increase in average delay.

As discussed above we are interested to know the no of slots M'required to schedule X%of packets rather than average delay. We use the same derivation as in 4.9

No of Users$N$	M' Simulated $M=1.5N$ $X=50\%$	M' Calculated $M=1.5N$ $X=50\%$	M' Simulated $M=2N$ $X=50\%$	M' Calculated $M=2N$ $X=50\%$	M' Simulated $M=1.5N$ $X=75\%$	M' Calculated $M=1.5N$ $X=75\%$	M' Simulated $M=2N$ $X=75\%$	M' Calculated $M=2N$ $X=75\%$
10	5	4	5	5	8	8	10	10
25	12	12	14	14	22	22	26	26
40	19	19	22	22	35	35	41	41

Table 4.5 The table of calculated and simulated M' for various no of users and maximum delay settings

$$\begin{aligned}\frac{X}{100} &= \sum_{n=0}^{M'} P_Z(Z_i = n) \\ &= \sum_{n=0}^{M'} k * p_i(1-p_i)^n \\ &= k\left(1-(1-p_i)^{M'+1}\right) \\ M' &= \frac{log(1-\frac{X}{k*100})}{log(1-p_i)} - 1 \end{aligned} \tag{4.10}$$

The simulated and calculated M' of the iid Rayleigh channel with finite window length M and linear assumption is tabulated in Table 4.5 for different no of users and maximum delay setting M and M' rounded off to integer greater than M'. We consider the case of $X\%$ to be 50% and 95%.Similar to the average delay, no of slots M' decreased for the same case in previous section because the longer delayed packets are dropped, thus the packets which are considered are the packets that are scheduled with delay less than M. hence the decrease in M'.

4.2.2 Packet Drop

In the previous subsection we considered the case of no packet loss, but in reality packets are dropped beyond certain waiting time. Higher packet drop rates lessens the effective utilization of the bandwidth which in turns degrades system performance. So the packet drop rate must also be constrained for good QoS. Let P_d^i denote the packet drop probability of the user i which we define as ratio of no of packets dropped by user i (which would have have been scheduled if not dropped) to the sum of no of packets renewed for the user i and no of packets dropped by the same user. The comprehensive

No of Users$N$	P_d Simulated $M=N$	P_d Calculated $M=N$	P_d Simulated $M=1.5N$	P_d Calculated $M=1.5N$	P_d Simulated $M=2N$	P_d Calculated $M=2N$
10	0.3137	0.3138	0.1856	0.1853	0.1103	0.1094
25	0.3463	0.3460	0.2028	0.2035	0.1241	0.1247
40	0.3544	0.3542	0.2132	0.2134	0.1279	0.1286

Table 4.6 The table of calculated and simulated packet drop probabilities for various no of users and maximum delay settings

derivation of the expression for packet drop probability is derived in Appendix to be

$$\frac{P_d^i}{1-P_d^i} = \frac{N-1-\mu_i}{M+1} \tag{4.11}$$

Note that for no packet drop case $\mu_i = N-1$ and hence $P_d = 0$ is verified.The simulated and calculated packet drop probability P_d^i of the iid Rayleigh channel with finite window length M and linear assumption is tabulated in Table 4.6 for different no of users and maximum delay setting M. The packet drop rate decreases with M because some of the packets which are dropped in lesser Mcase are now scheduled which caused the reduction in packet drop rate.

4.2.3 Throughput

The expression to evaluate the average throughput for PF scheduling for a Rayleigh channel is derived similar to the section 4.1.3 with the same linear assumption of the rate except window length is finite.

$$T_i = E_{\tau_i}[r(\tau_i) * P^i_{n,0/\tau_i}]$$

Since $P_{n,0/\tau_i}$is same as in section 4.1 , the expression for throughput and scheduling gain are same i.e.

$$T_i = \frac{g}{\lambda_i} \sum_{r=0}^{N-1} \frac{(-1)^r \binom{N-1}{r}}{(r+1)^2}$$

No of Users	Gain Simulated	Gain Calculated
10	$2.9439(M=10)$	$2.9290(M=10)$
25	$3.8298(M=25)$	$3.8160(M=25)$
40	$4.2822(M=40)$	$4.2785(M=40)$

Table 4.7 The calculated and simulated scheduling gain for different no of users and maximum delay settings

$$G = N * \sum_{r=0}^{N-1} \frac{(-1)^r \binom{N-1}{r}}{(r+1)^2}$$

The simulated and calculated scheduling gain G of the iid Rayleigh channel with finite window and linear assumption is tabulated in Table 4.7 for different no of users and maximum delay setting M. It is exactly like the case where no packets are dropped as scheduling here is independent of delay.

4.3 IID Rayleigh Channels, Rate is logarithmic with SNR

In the previous two sections we simplified the analysis with the data rate to be linear to SNR of the user i. Now we try to derive for the generalized logarithmic rate, the data rate supported by user i at time t given his received channel power $\tau_i(t)$ as described in previous sections is $r_i(t) = log(1 + ga_i\tau_i(t))$ .Since we are considering iid channel conditions all the users experience the same shadow and path losses i.e. $a_i = a, \forall i = 1, ..., N$.

4.3.1 Delay

To proceed further in the analysis of delay we need to evaluate the parameter p_i which is evaluated as $p_i = E_{\tau_i}[P^i_{n,0/\tau_i}]$. The user i with received channel power τ_i is scheduled if

$$\frac{r(\tau_i)}{T_i} > \frac{r(\tau_j)}{T_j}$$

for all $j = 1, ..., N$and $j \neq i$. We already stated that asymptotically average throughput'sT_i are stationary. But since it is iid channel conditions , each user experiences the same

No of Users N	μSimulated	μCalculated
10	9	9
25	23.9990	24
40	38.9984	39

Table 4.8 The table of calculated and simulated average delays for various no of users

No of Users N	M' Simulated $X=50\%$	M' Calculated $X=50\%$	M' Simulated $X=75\%$	M' Calculated $X=75\%$	M' simulated $X=95\%$	M' Calculated $X=95\%$
10	6	6	13	13	28	28
25	17	17	34	33	73	73
40	27	27	54	54	117	117

Table 4.9 The table of calculated and simulated M' for various no of users and maximum delay settings

decisions and hence the same stationary average throughput implies $T_i = T_j \ \forall i, j = 1,...,N$.The above scheduling rule simplifies to $log(1+ga\tau_i) > log(1+ga\tau_j) \implies \tau_i > \tau_j$. The scheduling decision principle is same as in section 4.1 and hence $P_{n,0/\tau_i}$ is same in both cases which implies same $p_i = \frac{1}{N}$. The expressions for average delay.

$$\mu_i = N-1$$

The simulated and calculated average delays of users under the iid Rayleigh channel conditions and logarithmic rate assumption for infinite window is tabulated in Table 4.8 for various values of no of users N. Even though the rate is logarithmic with SNR, the scheduling decision essentially simplified to that of rate linear with SNR, hence the same p_i causing to have same average delay and similar results in M'.

To know no of slots M'required to schedule $X\%$ of packets we need parameter p_i, but it is stated above that it is unchanged and hence the same expression,

$$M' = \frac{log(1-\frac{X}{100})}{log(1-p_i)} - 1$$

We consider the cases of $X\%$ to be 50%,75% and 95% . So we tabulated the simulated and calculated M' for various no of users and M'in Table 4.9 is rounded off to integer greater than M'.

4.3.2 Throughput

As stated above as $t_c \to \infty$ the average throughput's are stationary.Since we are now considering iid channel conditions, each user's throughput converges to the same value.$T_i = T_j, \forall i, j = 1, ..., N$. From (3.8) and (4.5)we evaluate throughput as

$$T_i = E_{\tau_i}[r(\tau_i) * P^{i}_{n,0/\tau_i}]$$

From (4.7)we simplify the above expression to

$$\begin{aligned} T_i &= \int_0^\infty r(x) * \lambda_i e^{-\lambda_i x}(1 - e^{-\lambda_i x})^{N-1} dx \\ &= \int_0^\infty log(1 + gax) * \lambda_i e^{-\lambda_i x}(1 - e^{-\lambda_i x})^{N-1} dx \\ &= \frac{\sum_{r=0}^{\infty}(-1)^{r+1} * \binom{N-1}{r} * Ei\left(-\frac{r\lambda_i}{ag}\right) * e^{\frac{(r+1)\lambda_i}{ag}}}{r+1} \end{aligned}$$

where $Ei(x)$ is the exponential integral represented by $Ei(x) = \int_{-\infty}^{x} \frac{e^t}{t} dt$. The above integral was simplified using mathematica. The scheduling throughput gain of the scheduler G is given by

$$\begin{aligned} G &= \frac{\sum_{i=1}^{N} T_i}{\sum_{i=1}^{N} T_i^{RR}} \\ &= \frac{N}{e^{\frac{\lambda_i}{ag}} Ei\left(-\frac{r\lambda_i}{ag}\right)} * \frac{\sum_{r=0}^{\infty}(-1)^{r+1} * \binom{N-1}{r} * Ei\left(-\frac{r\lambda_i}{ag}\right) * e^{\frac{(r+1)\lambda_i}{ag}}}{r+1} \end{aligned}$$

The simulated and calculated gains of the iid Rayleigh channel with infinite window and linear assumption is tabulated Table 4.10 for different no of users with the value of $g = 10$ and $a = 1$. Compared with the case of rate linear with SNR, the throughput gain for 10 user system is higher in this scenario but is lesser for the case of 40 users system.

No of Users$N$	G Simulated	G Calculated
10	3.3170	3.3301
25	3.6143	3.6173
40	3.7516	3.7381

Table 4.10 The table of calculated and simulated scheduling gain for various no of users

4.4 Packets dropped after maximum Delay,IID Rayleigh Channels, Rate is logarithmic with SNR

We have a finite window length M i.e maximum delay in this case beyond which the packet is dropped by the scheduler as its TTL expires. We still consider the iid Rayleigh channels with the data rate supported by user i at time t given his received channel power $\tau_i(t)$ as described in previous sections is $r_i(t) = log(1+g\tau_i(t))$.

4.4.1 Delay

Since the scheduling decision is delay independent and also independent of maximum delay setting M and we have shown in previous section even with the logarithmic rate consideration the scheduling decision still remains same as that of linear assumption and therefore has same $P_{n,0/\tau_i}$ as that of finite case and hence same $P^i_{n,0} = p_i = \frac{1}{N}$, for $n = 0,...,M$ and $i = 1,...,N$. The expressions for average delay remains as that of average delay in section 4.2 for the reason given above .Average delay is evaluated as

$$\mu_i = k\left(\frac{(1-p_i)(1-M*(1-p_i)^{M+1}+(M+1)*(1-p_i)^M)}{p_i}\right)$$

The simulated and calculated average delays of the iid Rayleigh channel with finite window and with logarithmic rate is tabulated Table 4.11 for different no of users and maximum delay setting M. The average delays in this scenario decreased compared with the same case considered in section 4.2. similar to the case of rate linear with SNR.

As discussed above even the expression to evaluate the no of slots M'required to schedule $X\%$of packets rather than average delay remains the same as that of (4.10)

No of Users$N$	μ Simulated $M=N$	μ Calculated $M=N$	μ Simulated $M=1.5*N$	μ Calculated $M=1.5*N$	μ Simulated $M=2*N$	μ Calculated $M=2*N$
10	3.9650	3.9694	5.3655	5.3608	6.4467	6.4199
25	10.2256	10.2458	14.06	14.0354	16.7125	16.7349
40	16.5166	16.5176	22.4727	22.4468	27.1426	27.0419

Table 4.11 The table of calculated and simulated average delays for various no of users and Maximum Delay Settings

No of Users$N$	M' Simulated $M=1.5N$ $X=50\%$	M' Calculated $M=1.5N$ $X=50\%$	M' Simulated $M=2N$ $X=50\%$	M' Calculated $M=2N$ $X=50\%$	M' Simulated $M=1.5N$ $X=75\%$	M' Calculated $M=1.5N$ $X=75\%$	M' simulated $M=1.5N$ $X=75\%$	M' Calculated $M=1.5N$ $X=75\%$
10	4	4	5	5	9	8	10	10
25	12	12	14	14	22	22	26	26
40	19	19	22	22	35	35	42	41

Table 4.12 The table of calculated and simulated M' for various no of users and Maximum Delay Settings

$$M' = \frac{log(1-\frac{X}{k*100})}{log(1-p_i)} - 1$$

The simulated and calculated M' of the iid Rayleigh channel with finite window and logarithmic is tabulated Table 4.12 for different no of users and maximum delay setting M and M' rounded off to integer greater than M'. We consider the case of $X\%$ to be 50% and 95%. The observations in change in $M'$are similar to that of explained in section 4.2.

4.4.2 Packet Drop

As noted above , even with the logarithmic rate the transitional probabilities and scheduling decision are unaltered and hence this is similar to the result in (4.11) of section 4.2

$$\frac{P_d^i}{1-P_d^i} = \frac{N-1-\mu_i}{M+1}$$

Note that for infinite window $\mu = N-1$ and hence $P_d = 0$ is verified.The simulated and calculated packet drop probability P_d^i of the iid Rayleigh channel with finite window

No of Users$N$	P_d Simulated $M = N$	P_d Calculated $M = N$	P_d Simulated $M = 1.5*N$	P_d Calculated $M = 1.5*N$	P_d Simulated $M = 2*N$	P_d Calculated $M = 2*N$
10	0.3160	0.3138	0.1851	0.1853	0.1084	0.1094
25	0.3458	0.3460	0.2031	0.2035	0.1250	0.1247
40	0.3542	0.3542	0.2132	0.2134	0.1277	0.1286

Table 4.13 The table of calculated and simulated packet drop probabilities for various no of users and Maximum Delay Settings

and logarithmic rate is tabulated Table 4.13 for different no of users and maximum delay setting M.

4.4.3 Throughput

The expression to evaluate the average throughput for PF scheduling for a Rayleigh channel is derived similar to the section 4.3.3 with the same logarithmic rate except window length is finite.

$$T_i = E_{\tau_i}[r(\tau_i) * P^{i}_{n,0/\tau_i}]$$

Since $P_{n,0/\tau_i}$is same as in section 4.3 , the expression for throughput and scheduling gain are same i.e.

$$T_i = \frac{\sum_{r=0}^{\infty} (-1)^{r+1} * \binom{N-1}{r} * Ei\left(-\frac{r\lambda_i}{ag}\right) * e^{\frac{(r+1)\lambda_i}{g}}}{r+1}$$

$$G = \frac{N}{e^{\frac{\lambda_i}{ag}} Ei\left(-\frac{r\lambda_i}{ag}\right)} * \frac{\sum_{r=0}^{\infty} (-1)^{r+1} * \binom{N-1}{r} * Ei\left(-\frac{r\lambda_i}{ag}\right) * e^{\frac{(r+1)\lambda_i}{ag}}}{r+1}$$

where $Ei(x)$ is the exponential integral represented by $Ei(x) = \int_{-\infty}^{x} \frac{e^t}{t} dt$.The simulated and calculated scheduling gain G of the iid Rayleigh channel with finite window and linear assumption is tabulated in Table 4.14 for different no of users and maximum delay setting M. Since the scheduling decision is independent of maximum delay setting M,

No of Users	Gain Simulated	Gain Calculated
10	3.3212	3.3301
25	3.6208	3.6173
40	3.7412	3.7381

Table 4.14 The calculated and simulated scheduling gain for different no of users and maximum delay settings

the throughput gains are exactly similar to the case with no maximum delay setting.

4.5 Non IID large scale fading, Rate is linear with SNR

We consider same small scale fading i.e. $\lambda_i = \lambda_j, \forall i, j = 1, ..., N$ and different large scale fading .

4.5.1 Delay

To proceed further in the analysis of delay we need to evaluate the parameter p_i which is evaluated as $p_i = E_{\tau_i}[P^{i}_{n,0/\tau_i}]$. The user i with received channel power τ_i is scheduled if

$$\frac{r(\tau_i)}{T_i} > \frac{r(\tau_j)}{T_j}$$

for all $j = 1, ..., N$and $j \neq i$. We already stated that asymptotically average throughput'sT_i are stationary. Using [15], it is shown in Appendix it is shown that for a Rayleigh channel with rate linear with SNR assumption

$$\frac{T_i}{T_j} = \frac{a_i}{a_j} \tag{4.12}$$

$\forall i = 1, ..., N$. With the linear assumption, the scheduling decision for user i becomes $\frac{a_i \tau_i}{T_i} > \frac{a_j \tau_j}{T_j}$ $\forall j = 1, ..., N, j \neq i$. Using (4.12) the scheduling decision simplifies to $\tau_j < \tau_i$ which is the same scheduling rules above for iid channel conditions. The conditional

User	Avg Channel Rate	μ Simulated $N=10$	μ Calculated $N=10$	Avg Channel Rate	μ Simulated $N=25$	μ Calculated $N=25$	Avg Channel Rate	μ Simulated $N=40$	μ Calculated $N=40$
1	0.2368	8.9746	9	0.1224	24.0544	24	0.0851	38.9069	39
2	23.3218	8.9897	9	76.1566	24.0242	24	135.5026	39.0196	39

Table 4.15 The table of calculated and simulated average delays for various no of users

transitional probability on τ_i is now evaluated as

$$P^i_{n,0/\tau_i} = \prod_{j=1, j\neq i}^{N} F_{\tau_i(t)}(\tau_i) \tag{4.13}$$

$$= \left(1 - e^{-\lambda_i x}\right)^{N-1} \tag{4.14}$$

$$\begin{aligned} p_i &= E_{\tau_i}[P^i_{n,0/\tau_i}] \\ &= \int_0^\infty f_{\tau_i(t)}(x) P^i_{n,0/\tau_i} dx \\ &= \int_0^\infty \lambda_i e^{-\lambda_i x}\left(1 - e^{-\lambda_i x}\right)^{N-1} dx \\ &= \frac{1}{N} \end{aligned}$$

For the infinite window length, the parameter p_i is same as that for iid channels, hence the average delay is same as that of of iid case.

$$\mu_i = N - 1$$

The simulated and calculated average delays for two users with different average channel strengths i.e. under the non iid Rayleigh channel conditions with infinite window and linear assumption is tabulated in Table 4.15 for various values of no of users N. Even in this case the average delay is similar to that of iid channel conditions under similar settings.

To know no of slots M'required to schedule $X\%$ of packets we need parameter p_i, but it

User	Avg Channel Rate	M' Simulated $N=10$ $X=50\%$	M' Calculated $N=10$ $X=50\%$	M' Simulated $N=10$ $X=75\%$	M' Calculated $N=10$ $X=75\%$	M' Simulated $N=10$ $X=95\%$	M' Simulated $N=10$ $X=95\%$
1	0.2942	6	6	13	13	28	28
2	29.4065	6	6	13	13	28	28

Table 4.16 The table of calculated and simulated M' of two users with different average channel strengths for a system of 10 users

User	Avg Channel Rate	M' Simulated $N=25$ $X=50\%$	M' Calculated $N=25$ $X=50\%$	M' Simulated $N=25$ $X=75\%$	M' Calculated $N=25$ $X=75\%$	M' Simulated $N=25$ $X=95\%$	M' Simulated $N=25$ $X=95\%$
1	0.1519	17	17	33	33	75	73
2	95.8810	16	17	33	33	73	73

Table 4.17 The table of calculated and simulated M' of two users with different average channel strengths for a system of 25 users

is stated above that it is unchanged and hence the same expression,

$$M' = \frac{log(1-\frac{X}{100})}{log(1-p_i)} - 1$$

We consider the cases of $X\%$ to be 50%,75% and 95% . The simulated and calculated values of M' for two users with different average channel strengths i.e. under the non iid Rayleigh channel conditions with infinite window and linear assumption is tabulated in Table 4.16 and Table 4.17 for various values of no of users N. $M'$is rounded off to integer greater than M'. Since p_i is unchanged to that of iid channel conditions, hence the same results as that of iid channel conditions case.

4.5.2 Throughput

As stated above as $t_c \to \infty$ the average throughput's are stationary. From (3.8) and (4.5)we evaluate throughput with linear assumption of rate as

$$T_i = E_{\tau_i}[r(\tau_i) * P^i_{n,0/\tau_i}]$$

No of Users$N$	G Simulated	G Calculated
10	2.9350	2.9290
25	3.8424	3.8160
40	4.2683	4.2785

Table 4.18 The table of calculated and simulated scheduling gain for various no of users

From (4.13)we simplify the above expression to

$$\begin{aligned} T_i &= \int_0^\infty r(x) * \lambda_i e^{-\lambda_i x}(1-e^{-\lambda_i x})^{N-1} dx \\ &= \int_0^\infty g a_i x * \lambda_i e^{-\lambda_i x}(1-e^{-\lambda_i x})^{N-1} dx \\ &= \frac{g a_i}{\lambda_i} \sum_{r=0}^{N-1} \frac{(-1)^r \binom{N-1}{r}}{(r+1)^2} \end{aligned}$$

The scheduling throughput gain of the scheduler G is given by

$$\begin{aligned} G &= \frac{\sum_{i=1}^{N} T_i}{\sum_{i=1}^{N} T_i^{RR}} \\ &= N * \sum_{r=0}^{N-1} \frac{(-1)^r \binom{N-1}{r}}{(r+1)^2} \end{aligned}$$

Note that despite the non iid channel conditions the scheduling gain remains the same as that of iid channel conditions with rate linear with SNR assumption. The large scale parameters have no effect on the scheduling gain and delay performance. The simulated and calculated gains of the non iid Rayleigh channel with infinite window and linear assumption is tabulated Table 4.18 for different no of users. The scheduling decision is unchanged compared to that of iid channel conditions , hence the same results.We can say that the throughput gain and average delay, no of slots $M'$are unaffected by non iid large scale fading.

User	Avg Channel Rate	μ Simulated $N=10$ $M=N$	μ Calculated $N=10$ $M=N$	Avg Channel Rate	μ Simulated $N=25$ $M=1.5N$	μ Calculated $N=25$ $M=1.5N$	Avg Channel Rate	μ Simulated $N=40$ $M=2*N$	μ Calculated $N=40$ $M=2*N$
1	0.2924	3.9578	3.9694	0.1522	14.1073	14.0354	0.1083	27.0645	27.0419
2	29.3277	3.9462	3.9694	96.4285	14.5543	14.0354	172.0611	26.7392	27.0419

Table 4.19 The table of calculated and simulated average delays for various no of users

4.6 Packets dropped after maximum Delay, Non IID large scale fading, Rate is linear with SNR

We consider same small scale fading i.e. $\lambda_i = \lambda_j, \forall i, j = 1, ..., N$ and different large scale fading.

4.6.1 Delay

Since the scheduling decision is delay independent and independent of maximum delay setting M,we have the same transition probabilities, $P^i_{n,0} = p_i = \frac{1}{N}$, for $n = 0, ..., M$ and $i = 1, ..., N$ as in section 4.5.1.since the parameter $p_i$is same as that of iid channel case with finite window , the expression for average delay and parameter $M'$are same as they only depend on p_i. The average delay is evaluated as

$$\mu_i = k\left(\frac{(1-p_i)(1-M*(1-p_i)^{M+1}+(M+1)*(1-p_i)^M)}{p_i}\right)$$

The simulated and calculated average delays for two users with different average channel strengths i.e. under the non iid Rayleigh channel conditions with finite window and linear assumption is tabulated in Table 4.19 for various values of no of users N. The observations with maximum delay setting are similar to that of section 4.2. Interestingly the value of p_i is also unchanged hence the same results under similar settings.

The expression for nos of slots required $M'$to schedule $X\%$ of packets is

$$M' = \frac{log(1-\frac{X}{k*100})}{log(1-p_i)} - 1$$

We consider the cases of $X\%$ to be 50%,75% and 95% . The simulated and calculated

User	Avg Channel Rate	M' Simulated $N = 10$ $X = 50\% M = N$	M' Calculated $N = 10 X = 50\% M = N$	M' Simulated $N = 10$ $X = 75\% M = 1.5*N$	M' Calculated $N = 10 X = 75\% M = 1.5*N$	M' Simulated $N = 10 X = 95\% M = 2*N$	M' Calculated $N = 10 X = 95\% M = 2*N$
1	0.2911	3	3	8	8	18	17
2	29.2118	3	3	8	8	17	17

Table 4.20 The table of calculated and simulated M' of two users with different average channel strengths for a system of 10 users

User	Avg Channel Rate	M' Simulated $N = 25$ $X = 50\% M = N$	M' Calculated $N = 25 X = 50\% M = N$	M' Simulated $N = 25$ $X = 75\% M = 1.5*N$	M' Calculated $N = 25 X = 75\% M = 1.5*N$	M' Simulated $N = 25 X = 95\% M = 2*N$	M' Calculated $N = 25 X = 95\% M = 2*N$
1	0.1531	9	9	26	26	44	43
2	95.4865	9	9	25	26	43	43

Table 4.21 the table of calculated and simulated M' of two users with different average channel strengths for a system of 25 users

values of M' for two users with different average channel strengths i.e. under the non iid Rayleigh channel conditions with finite window and linear assumption is tabulated in Table 4.20 and Table 4.21 for various values of no of users N. $M'$is rounded off to integer greater than M'.The observations with maximum delay setting are similar to that of section 4.2. Interestingly the value of p_i is also unchanged hence the same results under similar settings.

4.6.2 Packet Drop

In the previous subsection we considered the infinite window meaning no packet loss, but in reality packets are dropped beyond certain waiting time. The packet drop probability is shown Appendix A as

$$\frac{P_d^i}{1-P_d^i} = \frac{N-1-\mu_i}{M+1} \tag{4.15}$$

The simulated and calculated average delays for two users with different average channel strengths i.e. under the non iid Rayleigh channel conditions with finite window and linear assumption is tabulated in Table 4.22 for various values of no of users N.The observations with maximum delay setting are similar to that of section 4.2. Interestingly the value of p_i is also unchanged hence the same results under similar settings.

Note that for infinite window $\mu = N-1$ and hence $P_d = 0$ is verified.

User	Avg Channel Rate	P_d^i Simulated $N=10$ $M=N$	P_d^i Calculated $N=10$ $M=N$	Avg Channel Rate	P_d^i Simulated $N=25$ $M=1.5N$	P_d^i Calculated $N=25$ $M=1.5N$	Avg Channel Rate	P_d^i Simulated $N=40$ $M=2N$	P_d^i Calculated $N=40$ $M=2N$
1	0.2943	0.3138	0.3342	0.1534	0.1966	0.2030	0.1065	0.1278	0.1281
2	29.1376	0.3195	0.3342	95.0688	0.2018	0.2030	170.0511	0.1295	0.1281

Table 4.22 The table of calculated and simulated packet drops for various no of users and maximum delay settings

No of Users	Gain Simulated	Gain Calculated
10	$2.9439(M=10)$	$2.9290(M=10)$
25	$3.8298(M=25)$	$3.8160(M=25)$
40	$4.2822(M=40)$	$4.2785(M=40)$

Table 4.23 The calculated and simulated scheduling gain for different no of users and maximum delay settings

4.6.3 Throughput

The expression to evaluate the average throughput for PF scheduling for a Rayleigh channel is derived similar to the section 4.5.3 with the same linear assumption of the rate except window length is finite.

$$T_i = E_{\tau_i}[r(\tau_i) * P^i_{n,0/\tau_i}]$$

Since $P_{n,0/\tau_i}$ is same as in section 4.1 , the expression for throughput and scheduling gain are same i.e.

$$T_i = \frac{ga_i}{\lambda_i} \sum_{r=0}^{N-1} \frac{(-1)^r \binom{N-1}{r}}{(r+1)^2}$$

$$G = N * \sum_{r=0}^{N-1} \frac{(-1)^r \binom{N-1}{r}}{(r+1)^2}$$

The simulated and calculated scheduling gain G of the non iid Rayleigh channel with finite window and linear assumption is tabulated in Table 4.23 for different no of users and maximum delay setting M. The scheduling decision is still unchanged and hence the same results under same settings.We can say that the throughput gain and average delay, no of slots $M'$are unaffected by non iid large scale fading.

4.7 Non IID small scale fading , Rate is linear with SNR

We consider same large scale fading and different small scale fading i.e. $a_i = a_j, \forall i, j = 1, ..., N$.

4.7.1 Delay

To proceed further in the analysis of delay we need to evaluate the parameter p_i which is evaluated as $p_i = E_{\tau_i}[P^i_{n,0/\tau_i}]$. The user i with received channel power τ_i is scheduled if

$$\frac{r(\tau_i)}{T_i} > \frac{r(\tau_j)}{T_j}$$

for all $j = 1, ..., N$and $j \neq i$. We already stated that asymptotically average throughput'sT_i are stationary. Using [15], it is shown in Appendix it is shown that for a Rayleigh channel with rate linear with SNR assumption

$$\frac{T_i}{T_j} = \frac{\lambda_j}{\lambda_i} = k_{ij} \tag{4.16}$$

$\forall i = 1, .., N$. With the linear assumption, the scheduling decision for user i becomes $\frac{\tau_i}{T_i} > \frac{\tau_j}{T_j}$ $\forall j = 1, ..., N, j \neq i$. Using (4.16) the scheduling decision simplifies to $k_{ij}\tau_j < \tau_i$ which is the same scheduling rule as above for iid channel conditions. The conditional transitional probability on τ_i is now evaluated as

$$P^i_{n,0/\tau_i} = \prod_{j=1, j\neq i}^{N} F_{\tau_i(t)}(\frac{\tau_i}{k_{ij}}) \tag{4.17}$$

$$= \left(1 - e^{-\lambda_i x}\right)^{N-1} \tag{4.18}$$

User	Avg Channel Rate	μ Simulated $N=10$	μ Calculated $N=10$	Avg Channel Rate	μ Simulated $N=25$	μ Calculated $N=25$	Avg Channel Rate	μ Simulated $N=40$	μ Calculated $N=40$
1	0.2368	8.9746	9	0.1224	24.0544	24	0.0851	38.9069	39
2	23.3218	8.9897	9	76.1566	24.0242	24	135.5026	39.0196	39

Table 4.24 The table of calculated and simulated average delays for various no of users

$$\begin{aligned} p_i &= E_{\tau_i}[P^i_{n,0/\tau_i}] \\ &= \int_0^\infty f_{\tau_i(t)}(x)P^i_{n,0/\tau_i}dx \\ &= \int_0^\infty \lambda_i e^{-\lambda_i x}\left(1-e^{-\lambda_i x}\right)^{N-1} dx \\ &= \frac{1}{N} \end{aligned}$$

For the infinite window length, the parameter p_i is same as that for iid channels, hence the average delay is same as that of of iid case.

$$\mu_i = N-1$$

The simulated and calculated average delays for two users with different average channel strengths i.e. under the non iid Rayleigh channel conditions with infinite window and linear assumption is tabulated in Table 4.24 for various values of no of users $N.p_i$is still the same and hence the expressions still remain unchanged.

To know no of slots M'required to schedule $X\%$ of packets we need parameter p_i, but it is stated above that it is unchanged and hence the same expression,

$$M' = \frac{log(1-\frac{X}{100})}{log(1-p_i)} - 1$$

We consider the cases of $X\%$ to be 50%,75% and 95% . The simulated and calculated values of M' for two users with different average channel strengths i.e. under the non iid Rayleigh channel conditions with infinite window and linear assumption is tabulated in Table 4.25 and Table 4.26 for various values of no of users N. $M'$is rounded off to integer greater than M'.

User	Avg Channel Rate	M' Simulated $N = 10$ $X = 50\%$	M' Calculated $N = 10$ $X = 50\%$	M' Simulated $N = 10$ $X = 75\%$	M' Calculated $N = 10$ $X = 75\%$	M' Simulated $N = 10$ $X = 95\%$	M' Calculated $N = 10$ $X = 95\%$
1	0.2942	6	6	13	13	28	28
2	29.4065	6	6	13	13	28	28

Table 4.25 The table of calculated and simulated M' of two users with different average channel strengths for a system of 10 users

User	Avg Channel Rate	M' Simulated $N = 25$ $X = 50\%$	M' Calculated $N = 25$ $X = 50\%$	M' Simulated $N = 25$ $X = 75\%$	M' Calculated $N = 25$ $X = 75\%$	M' Simulated $N = 25$ $X = 95\%$	M' Calculated $N = 25$ $X = 95\%$
1	0.1519	17	17	33	33	75	73
2	95.8810	16	17	33	33	73	73

Table 4.26 The table of calculated and simulated M' of two users with different average channel strengths for a system of 25 users

4.7.2 Throughput

As stated above as $t_c \to \infty$ the average throughput's are stationary. From (3.8) and (4.5)we evaluate throughput with linear assumption of rate as

$$T_i = E_{\tau_i}[r(\tau_i) * P^{i}_{n,0/\tau_i}]$$

From (4.17)we simplify the above expression to

$$\begin{aligned} T_i &= \int_0^\infty r(x) * \lambda_i e^{-\lambda_i x}(1 - e^{-\lambda_i x})^{N-1} dx \\ &= \int_0^\infty gax * \lambda_i e^{-\lambda_i x}(1 - e^{-\lambda_i x})^{N-1} dx \\ &= \frac{ga}{\lambda_i} \sum_{r=0}^{N-1} \frac{(-1)^r \binom{N-1}{r}}{(r+1)^2} \end{aligned}$$

One is often interested to calculate the gain of the scheduler with respect to the Round robin scheduler. For the RR scheduler each user gets scheduled once in N slots. Hence the average throughput of RR scheduler is $T_i^{RR} = \frac{1}{N} E[r(\tau_i)] = \frac{ga}{N * \lambda_i}$. so the scheduling throughput gain of the scheduler G is given by

$$\begin{aligned} G &= \frac{\sum_{i=1}^{N} T_i}{\sum_{i=1}^{N} T_i^{RR}} \\ &= N * \sum_{r=0}^{N-1} \frac{(-1)^r \binom{N-1}{r}}{(r+1)^2} \end{aligned}$$

No of Users$N$	G Simulated	G Calculated
10	2.9350	2.9290
25	3.8424	3.8160
40	4.2683	4.2785

Table 4.27 The table of calculated and simulated scheduling gain for various no of users

Note that despite the non iid channel conditions the scheduling gain remains the same as that of iid channel conditions with rate linear with SNR assumption. The large scale parameters have no effect on the scheduling gain and delay performance. The simulated and calculated gains of the non iid Rayleigh channel with infinite window and linear assumption is tabulated Table 4.27 for different no of users. The expression is evaluated under these conditions to that of iid conditions. We can say that the throughput gain and average delay, no of slots M' required to schedule X% are unaffected by non iid small scale fading.

4.8 Packets dropped after maximum Delay,Non IID small scale fading , Rate is linear with SNR

We consider same large scale fading and different small scale fading i.e. $a_i = a_j, \forall i, j = 1, ..., N$.

4.8.1 Delay

Since the scheduling decision is delay independent and independent of maximum delay setting M,we have the same transition probabilities, $P^i_{n,0} = p_i = \frac{1}{N}$, for $n = 0, ..., M$ and $i = 1, ..., N$ as in section 4.5.1.since the parameter $p_i$is same as that of iid channel case with finite window , the expression for average delay and parameter $M'$are same as they only depend on p_i. The average delay is evaluated as

$$\mu_i = k\left(\frac{(1-p_i)(1-M*(1-p_i)^{M+1}+(M+1)*(1-p_i)^{M})}{p_i}\right)$$

User	Avg Channel Rate	μ Simulated $N=10$ $M=N$	μ Calculated $N=10$ $M=N$	Avg Channel Rate	μ Simulated $N=25$ $M=1.5N$	μ Calculated $N=25$ $M=1.5N$	Avg Channel Rate	μ Simulated $N=40$ $M=2N$	μ Calculated $N=40$ $M=2N$
1	0.2924	3.9578	3.9694	0.1522	14.1073	14.0354	0.1083	27.0645	27.0419
2	29.3277	3.9462	3.9694	96.4285	14.5543	14.0354	172.0611	26.7392	27.0419

Table 4.28 The table of calculated and simulated average delays for various no of users

User	Avg Channel Rate	M' Simulated $N=10$ $X=50\%$ $M=N$	M' Calculated $N=10$ $X=50\%$ $M=N$	M' Simulated $N=10$ $X=75\%$ $M=1.5N$	M' Calculated $N=10$ $X=75\%$ $M=1.5N$	M' Simulated $N=10$ $X=95\%$ $M=2N$	M' Calculated $N=10$ $X=95\%$ $M=2N$
1	0.2911	3	3	8	8	18	17
2	29.2118	3	3	8	8	17	17

Table 4.29 The table of calculated and simulated M' of two users with different average channel strengths for a system of 10 users

The simulated and calculated average delays for two users with different average channel strengths i.e. under the non iid Rayleigh channel conditions with finite window and linear assumption is tabulated in Table 4.28 for various values of no of users N.

The expression for nos of slots required $M'$to schedule $X\%$ of packets is

$$M' = \frac{log(1 - \frac{X}{k*100})}{log(1 - p_i)} - 1$$

We consider the cases of $X\%$ to be 50%,75% and 95% . The simulated and calculated values of M' for two users with different average channel strengths i.e. under the non iid Rayleigh channel conditions with finite window and linear assumption is tabulated in Table 4.29 and Table 4.30 for various values of no of users N. $M'$is rounded off to integer greater than M'.

User	Avg Channel Rate	M' Simulated $N=25$ $X=50\%$ $M=N$	M' Calculated $N=25$ $X=50\%$ $M=N$	M' Simulated $N=25$ $X=75\%$ $M=1.5N$	M' Calculated $N=25$ $X=75\%$ $M=1.5N$	M' Simulated $N=25$ $X=95\%$ $M=2N$	M' Calculated $N=25$ $X=95\%$ $M=2N$
1	0.1531	9	9	26	26	44	43
2	95.4865	9	9	25	26	43	43

Table 4.30 The table of calculated and simulated M' of two users with different average channel strengths for a system of 25 users

User	Avg Channel Rate	P_d^i Simulated $N = 10$ $M = N$	P_d^i Calculated $N = 10$ $M = N$	Avg Channel Rate	P_d^i Simulated $N = 25$ $M = 1.5N$	P_d^i Calculated $N = 25$ $M = 1.5N$	Avg Channel Rate	P_d^i Simulated $N = 40$ $M = 2N$	P_d^i Calculated $N = 40$ $M = 2N$
1	0.2943	0.3138	0.3342	0.1534	0.1966	0.2030	0.1065	0.1278	0.1281
2	29.1376	0.3195	0.3342	95.0688	0.2018	0.2030	170.0511	0.1295	0.1281

Table 4.31 The table of calculated and simulated packet drops for various no of users and maximum delay settings

4.8.2 Packet Drop

In the previous subsection we considered the infinite window meaning no packet loss, but in reality packets are dropped beyond certain waiting time. The packet drop probability is shown Appendix A as

$$\frac{P_d^i}{1-P_d^i} = \frac{N-1-\mu_i}{M+1} \tag{4.19}$$

The simulated and calculated average delays for two users with different average channel strengths i.e. under the non iid Rayleigh channel conditions with finite window and linear assumption is tabulated in Table 4.31 for various values of no of users N.

Note that for infinite window $\mu = N-1$ and hence $P_d = 0$ is verified.

4.8.3 Throughput

The expression to evaluate the average throughput for PF scheduling for a Rayleigh channel is derived similar to the section 4.5.3 with the same linear assumption of the rate except window length is finite.

$$T_i = E_{\tau_i}[r(\tau_i) * P^i_{n,0/\tau_i}]$$

Since $P_{n,0/\tau_i}$ is same as in section 4.1 , the expression for throughput and scheduling gain are same i.e.

$$T_i = \frac{ga_i}{\lambda_i} \sum_{r=0}^{N-1} \frac{(-1)^r \binom{N-1}{r}}{(r+1)^2}$$

No of Users	Gain Simulated	Gain Calculated
10	$2.9439(M=10)$	$2.9290(M=10)$
25	$3.8298(M=25)$	$3.8160(M=25)$
40	$4.2822(M=40)$	$4.2785(M=40)$

Table 4.32 The calculated and simulated scheduling gain for different no of users and maximum delay settings

$$G = N * \sum_{r=0}^{N-1} \frac{(-1)^r \binom{N-1}{r}}{(r+1)^2}$$

The simulated and calculated scheduling gain G of the non iid Rayleigh channel with finite window and linear assumption is tabulated in Table 4.32 for different no of users and maximum delay setting M.The throughput gain expression is evaluated to that of iid channel conditions with maximum delay settings.Further it is same as that of with no maximum delay setting.We can say that the throughput gain and average delay, no of slots M' required to schedule X% are unaffected by non iid small scale fading.

4.9 Non IID in small scale fading and large scale fading, Rate is logarithmic with SNR

We start the analysis of non iid channel case with the generalized logarithmic rate i.e the data rate supported by user i at time t given his received channel power $a_i\tau_i(t)$ as described in previous sections is $r_i(t) = log(1 + ga_i\tau_i(t))$.

4.9.1 Delay

To proceed further in the analysis of delay we need to evaluate the parameter p_i which is evaluated as $p_i = E_{\tau_i}[P^i_{n,0/\tau_i}]$. The user i with received channel power τ_i is scheduled if

$$\frac{r(\tau_i)}{T_i} > \frac{r(\tau_j)}{T_j}$$

for all $j = 1,...,N$and $j \neq i$. We already stated that asymptotically average throughput'sT_i are stationary. Using [15], it is shown in Appendix it is shown that for a Rayleigh channel with logarithmic rate

$$\frac{T_i}{T_j} = r_{i,j} == \frac{\int_0^\infty \lambda_2 e^{-\lambda_2 x}\left(log(1+a_1 g^{ar}x^{ar})e^{-\lambda_1 g^{ar-1}x^{ar}} - e^{\frac{\lambda_1}{ga_1}}Ei[-\lambda_1 g^{ar-1}x^{ar} - \frac{\lambda_1}{ga_1}]\right)dx}{\int_0^\infty \lambda_1 e^{-\lambda_1 x}\left(log(1+ag^{\frac{1-ar+r}{r}}x^{\frac{1}{ar}})e^{-\lambda_2 g^{\frac{1-ar}{r}}x^{\frac{1}{ar}}} - e^{\frac{\lambda_1}{ga_1}}Ei[-\lambda_2 g^{\frac{1-ar}{r}}x^{\frac{1}{ar}} - \frac{\lambda_{21}}{ga_2}]\right)dx} \tag{4.20}$$

$\forall i,j = 1,...,N$. Though the above expression can not be simplified to closed form expression, we can numerically evaluate it using mathematica. With the logarithmic rate, the scheduling decision for user i becomes $\frac{log(1+ga_i\tau_i)}{T_i} > \frac{log(1+ga_j\tau_j)}{T_j}$ $\forall j = 1,...,N, j \neq i$. We can approximate this to $log(g\tau_i) > r_{i,j}log(g\tau_j)$ which is simplified to $\tau_j < g^{\frac{1}{r_{i,j}-1}}\tau_i^{\frac{1}{r_{i,j}}} = k_{i,j}\tau_i^{\frac{1}{r_{i,j}}}$. The conditional transitional probability on τ_i is now evaluated as

$$P^i_{n,0/\tau_i} = \prod_{j=1,j\neq i}^{N} F_{\tau_j(t)}(k_{ij}\tau_i^{\frac{1}{r_{i,j}}}) \tag{4.21}$$

$$= \prod_{j=1,j\neq i}^{N}(1-e^{\lambda_j\tau_i^{\frac{1}{r_{i,j}}}}) \tag{4.22}$$

$$\begin{aligned} p_i &= E_{\tau_i}[P^i_{n,0/\tau_i}] \\ &= \int_0^\infty f_{\tau_i(t)}(x)P^i_{n,0/\tau_i}dx \\ &= \int_0^\infty \lambda_i e^{-\lambda_i x}\prod_{j=1\longrightarrow N,j\neq i}(1-e^{\lambda_j\tau_i^{\frac{1}{r_{i,j}}}})dx \end{aligned}$$

For the case of no packet drop rate, the average delay is evaluated as

$$\mu_i = \frac{1}{p_i} - 1$$

To know no of slots M'required to schedule $X\%$ of packets we need parameter p_i, the analysis still remains the same hence the same expression,

$$M' = \frac{log(1-\frac{X}{100})}{log(1-p_i)} - 1$$

4.9.2 Throughput

As stated above as $t_c \to \infty$ the average throughput's are stationary. From (3.8) and (4.5) we evaluate throughput with linear assumption of rate as

$$T_i = E_{\tau_i}[r(\tau_i) * P^{i}_{n,0/\tau_i}]$$

From (4.21) we simplify the above expression to

$$\begin{aligned} T_i &= \int_0^\infty r(x) * \lambda_i e^{-\lambda_i x} \prod_{j=1, j\neq i}^{N} (1 - e^{\lambda_j \tau_i^{\frac{1}{r_{i,j}}}}) dx \\ &= \int_0^\infty log(1 + ga_i x) * \lambda_i e^{-\lambda_i x} \prod_{j=1, j\neq i}^{N} (1 - e^{\lambda_j \tau_i^{\frac{1}{r_{i,j}}}}) dx \end{aligned}$$

The scheduling throughput gain of the scheduler G given by

$$G = \frac{\sum_{i=1}^{N} T_i}{\sum_{i=1}^{N} T_i^{RR}}$$

CHAPTER 5

MLWDF Scheduling

Major drawback of the PF scheduling is that it assumes that users are delay insensitive, but recently the delay has become the important factor for real time applications like video conferencing, video streaming etc... There is no way to control delay in PFS given a system of users N. To address this Modified Largest Weighted Delay First (MLWDF) scheduling algorithm has been proposed to address delay issue. MLWDFS selects the user $i(t)$ at slot t according to

$$i(t) = \arg \max_{j=1,\ldots,n} \frac{r_j(t)W_j(t)}{T_j(t)}$$

where $r_i(t)$ is the data rate that can be supported by user i at time t and $W_i(t)$ is the HOL delay of user i at time t.The throughput $T_j(t)$ is typically evaluated through an exponentially smoothed average :

$$T_j(t) = \begin{cases} \left(1 - \frac{1}{t_c}\right) T_j(t-1) + \frac{1}{t_c} r_j(t), & j = i(t) \\ \left(1 - \frac{1}{t_c}\right) T_j(t-1), & j \neq i(t) \end{cases}$$

where t_c is the time-window length over which one wants to regulate the fairness. When $t_c \to \infty$,we assume the average throughput's are stationary (no proof).Unlike PFs the transitional probability is delay dependent, so the analysis becomes complicated. To better understand the problem, we carried out the simulations of the MLWDF scheduling for a single cellular system with 10 users under iid channel conditions with infinite window length and rate linear with SNR assumption.Figure 2 shows the pmf of the scheduling delay for one user.

It follows from the above, that we can assume the pmf of renewal delay to be of Poisson distribution. The pmf of Scheduling Delay is

$$P_Z(Z_i = n) = \frac{e^{-\lambda_i} \lambda_i^n}{n!}$$

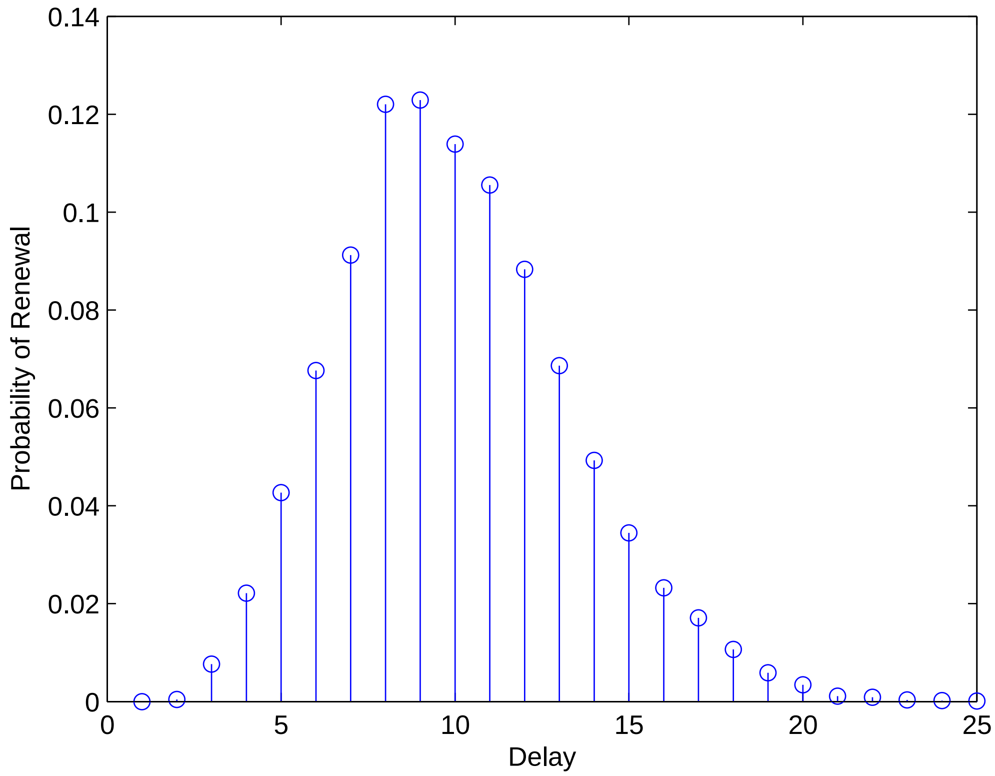

Fig. 5.1 Plot of pmf of Renewal delay of users for MLWDF scheduling

$\forall n = 0 \longrightarrow \infty, i = 1 \longrightarrow N$.From Appendix it is shown that for iid channel conditions

$$\begin{aligned} P_W(W_i = n) &= \sum_{j=n}^{\infty} \frac{1}{N} P(Z_i = j) \\ &= \frac{1}{N} \frac{\gamma(n+1, \lambda)}{\Gamma(n+1)} \end{aligned}$$

where $\gamma(n, \lambda)$ is an incomplete gamma function defined by$\gamma(n, \lambda) = \int_0^{\infty} e^{-t} t^{n-1} dt$ and $\Gamma(n)$ is the gamma function defined as $\Gamma(n) = (n-1)!$.Based on this ground work for MLWDFS we now try to analyze the delay parameters, throughput gain and packet drop rate.

5.1 IID Rayleigh Channels, Rate is linear with SNR, No packet drop rate

We consider the case of no packet is being dropped by the scheduler. We still consider the iid Rayleigh channels with the linear assumption of the data rate $r_i(t) = g a_i \tau_i(t)$.

No of Users$N$	μ_i Simulated	μ_i Calculated
10	9.001	9
25	24.008	24

Table 5.1 The table of simulated and estimated average delays for various no of users and Maximum Delay Settings

5.1.1 Delay

To proceed further to calculate average delay μ_i and no of slots M'required to schedule $X\%$ of packets of user i,we need the parameter λ_i or $\mu_i$which is the parameter of the Poisson distribution and the average delay. The average delay of user i is calculated as

$$\mu_i = \sum_{n=0}^{\infty} n * P(Z_i = n)$$
$$= \lambda_i$$

But we know that for iid channel conditions , average delay is $N-1$ and hence $\mu_i = \lambda_i = N-1,\ \forall i = 1, ..., N$.The simulated and calculated average delay of user i ,μ_i of the iid Rayleigh channel with infinite window and rate linear with SNR is tabulated for different no of users N and maximum delay setting M in Table 5.1. Since it is iid channel conditions with no maximum delay setting, it is similar to that of round robin scheduler.

We now shift our focus to the calculation of no of slots M'required to schedule $X\%$ of packets .Let us recall the definition of r.vZ which is the probability of renewal at a particular delay of the total renewals.Hence no of slots M' required to scheduled $X\%$of packets is simplified above as

$$\frac{X}{100} = \sum_{n=0}^{M'} P_Z(Z_i = n) \tag{5.1}$$

From the above equation reduces to

$$\frac{X}{100} = \sum_{n=0}^{M'} \frac{e^{-\lambda_i} \lambda_i^n}{n!}$$

We consider the cases of $X\%$ to be 50%,75% and 95% . So we tabulated the simulate and calculated M' for various no of users and $M'$is rounded off to integer greater than

No of Users$N$	M' Simulated $X=50\%$	M' Calculated $X=50\%$	M' Simulated $X=75\%$	M' Calculated $X=75\%$	M' simulated $X=95\%$	M' Calculated $X=95\%$
10	9	9	11	11	15	14
25	24	24	29	27	34	32
40	39	39	46	43	58	55

Table 5.2 The table of calculated and simulated M' for various no of users and Maximum Delay Settings

M'in Table 5.2.We observe the results for the case of 10 user system. The no of slots required to schedule X% of packets increases with X. We also notice that for the increase of $X\%$ from 50% to 75% , the no of slots increased by 2, but from 75% to 95% the no of slots increases by 4. For the same $X\%$, the value of M'increases with no of users.

5.1.2 Throughput

As stated above as $t_c \to \infty$ the average throughput's are stationary.Since we are now considering iid channel conditions, each user's throughput converges to the same value.$T_i = T_j, \forall i, j = 1, ..., N$. The user i with received channel power τ_i is scheduled if

$$\frac{W_i(t) * r(\tau_i(t))}{T_i} > \frac{W_j(t) * r(\tau_j(t))}{T_j}$$

for all $j = 1, ..., N$and $j \neq i$. With the above relation,linear assumption of rate and stationary property of received channel power as well as delay state process, the scheduling decision for user i becomes $m_i \tau_i > m_j \tau_j \ \forall j = 1, ..., N, j \neq i$. the scheduling decision simplifies to $\tau_j < \tau_i \frac{m_i}{m_j}$. The conditional transitional probability on delay state Y in which user i is in delay state n, i.e. $m_i = n$ and received channel power τ_i is now evaluated as

$$P^i_{n,0/\tau_i} = \prod_{j=1, j\neq i}^{N} F_{\tau_i(t)}(\tau_i \frac{n}{m_j}) \tag{5.2}$$

$$= \prod_{j=1, j\neq i}^{N} \left(1 - e^{-\lambda_i \tau_i \frac{n}{m_j}}\right) \tag{5.3}$$

From (3.8) and (4.5) we evaluate throughput as

$$T_i = E_{\tau_i}[r(\tau_i) * E_Y[P^{i}_{n,0/Y,\tau_i}]]$$

where the summation is over all the possible system delay states $Y = [m_1 ... m_N]$ where m_j is varied from 0 to M for all $j = 1, ..., N$ .To move ahead we need to evaluate

$$E_Y[P^i_{n,0/Y,\tau_i}] = \sum_Y P(Y = y) * P^i_{n,0/Y,\tau_i}$$

From (3.2) it simplifies to

$$\begin{aligned} E_Y[P^i_{n,0/Y,\tau_i}] &= \sum_Y P_W(W_i = n) \prod_{j=1, j\neq i}^{N} P_W(W_j = m_j) * P^i_{n,0/Y,\tau_i} \\ &= \sum_{n=0}^{M} P_W(W_i = n) \sum_{Y_i'} \prod_{j=1, j\neq i}^{N} P_W(W_j = m_j) * P^i_{n,0/Y,\tau_i} \end{aligned}$$

where $Y_i^{'} = [m_1 ... m_N]$ where m_j is varied from 0 to M for all $j = 1, ..., N$ except for $j = i$ and $m_i = n$. It is further simplified by (5.3) to

$$E_Y[P^i_{n,0/Y,\tau_i}] = \sum_{n=0}^{\infty} P_W(W_i = n) \sum_{Y'} \prod_{j=1, j\neq i}^{N} P_W(W_j = m_j) * \prod_{j=1, j\neq i}^{N} \left(1 - e^{-\lambda_i \tau_i \frac{n}{m_j}}\right)$$

The summation over Y' is simplified to

$$E_Y[P^i_{n,0/Y,\tau_i}] = \sum_{n=0}^{\infty} P_W(W_i = n) * g_j(n\tau_i)^{N-1} \tag{5.4}$$

where

$$\begin{aligned} g_j(n\tau_i) &= \sum_{m_j=0}^{\infty} (1 - e^{-\lambda_i \tau_i \frac{n}{m_j}}) * P(W_j = m_j) \\ &= \sum_{m=0}^{\infty} (1 - a_i^{\frac{1}{m}}) * P(W_j = m_j) \end{aligned}$$

where $a_i = e^{-\lambda_i x_i}$ and $a < 1$.

$$\begin{aligned} g_j(n\tau_i) =& 1 - \sum_{m=0}^{\infty} a_i^{\frac{1}{m}} * P(W_j = m) \\ =& 1 - \sum_{m=0}^{\infty} l_m(m_i x_i) \end{aligned} \tag{5.5}$$

where $l_m(x) = a_i^{\frac{1}{m}} * P(W_j = m_j)$. and by ratio test the series $\sum_{m=0}^{\infty} l_m(m_i x_i)$ converges. So we can approximate the summation to finite no of terms say M. So the expression simplifies to

$$g_j(n\tau_i) = 1 - \sum_{m=0}^{M} l_m(m_i x_i)$$

So the throughput is derived by using multinomial expansion, we get

$$T_i = ga \int_{x=0}^{\infty} x * \lambda_i e^{-\lambda_i x} * \sum_{n=0}^{M} P_W(W_i = n) * g_i(nx)^{N-1} dx$$

Since the summation converges we can swap integration and union we get

$$T_i = ga \sum_{n=0}^{M} P(W_i = n) * \int_0^{\infty} x * \lambda_i e^{-\lambda_i x} * g_i(nx)^{N-1} dx$$

Using multinomial expansion, it is further simplified to

$$T_i = ga \sum_{n=0}^{M} P(W = n) * \qquad (5.6)$$

$$\sum_{k_o + \ldots + k_M = N-1} \int_0^{\infty} \lambda_i * (-1)^{k_0} * c(k_0 \ldots k_M) * \prod_{m=1}^{M} \left(P(W = m)^{k_m} * x * e^{-\lambda_i x (1 + \frac{n \sum_{i=1}^{M} k_i}{m})} \right) dx$$

$$= \frac{ga}{\lambda_i} \sum_{n=0}^{M} P(W = n) * \sum_{k_o + \ldots + k_M = N-1} \left(\frac{(-1)^{k_0} c(k_0 \ldots k_M)}{(1 + n \sum_{m=0}^{M} \frac{k_m}{m})^2} * \prod_{m=1}^{M} P(W_i = m)^{k_m} \right) \qquad (5.7)$$

One is often interested to calculate the gain of the scheduler with respect to the Round robin scheduler. For the RR scheduler each user gets scheduled once in N slots. Hence the average throughput of RR scheduler is $T_i^{RR} = \frac{1}{N} E[r(\tau_i)] = \frac{g}{N * \lambda_i}$. so the scheduling throughput gain of the scheduler G is given by

$$G = \frac{\sum_{i=1}^{N} T_i}{\sum_{i=1}^{N} T_i^{RR}}$$

$$= N * \sum_{n=0}^{M} P(W = n) * \sum_{k_o + \ldots + k_M = N-1} \left(\frac{(-1)^{k_0} c(k_0 \ldots k_M)}{(1 + n \sum_{m=0}^{M} \frac{k_m}{m})^2} * \prod_{m=1}^{M} P(W_i = m)^{k_m} \right)$$

The simulated throughput gains of user i ,μ_i of the iid Rayleigh channel with finite window and rate linear with SNR is tabulated for different no of users N in Table 5.3.

No of Users	Gain Simulated	Gain Calculated
10	2.3419	2.51(M=20)
25	3.0773	3.140(M=40)

Table 5.3 Table showing the simulated and calculated gains for mlwdf scheduling

The through put similar to that of PF increases with no of users but the gain is less that that of PF scheduling.

5.2 Packets dropped after finite time,IID Rayleigh channels, rate linear with SNR

We have finite window length M, the maximum delay beyond which the packet is dropped by the scheduler as its TTL expires. We still consider the iid Rayleigh channels with the linear assumption of the data rate $r_i(t) = g\tau_i(t)$.To understand the problem for finite window case, we carried out the simulations of the MLWDF scheduling for a single cellular system with 10 users under iid channel conditions with finite window length and rate linear with SNR assumption.Figure 3 shows the pmf of the scheduling delay for one user

From the figure , we can assume for the finite case, the pmf of renewal delay to be of truncated Poisson distribution.The pmf of Scheduling Delay is

$$P_Z(Z_i = n) = k_i * \frac{e^{-\lambda_i}\lambda_i^n}{n!} \tag{5.8}$$

$\forall n = 0 \longrightarrow M, i = 1 \longrightarrow N$ and k_i is a normalizing constant derived by $\sum_{n=0}^{\infty} P_Z(Z_i = n) = 1$, we get $k_i = \frac{1}{\sum_{n=0}^{\infty} \frac{e^{-\lambda_i}\lambda_i^n}{n!}}$. The following relation between the pmf for W_i and Z_i is derived in appendix.

$$P_W(W_i = n) = \frac{1}{N}\left(\sum_{m=n}^{M} P_Z(Z_i = m) + \frac{P_d^i}{1 - P_d^i}\right)$$

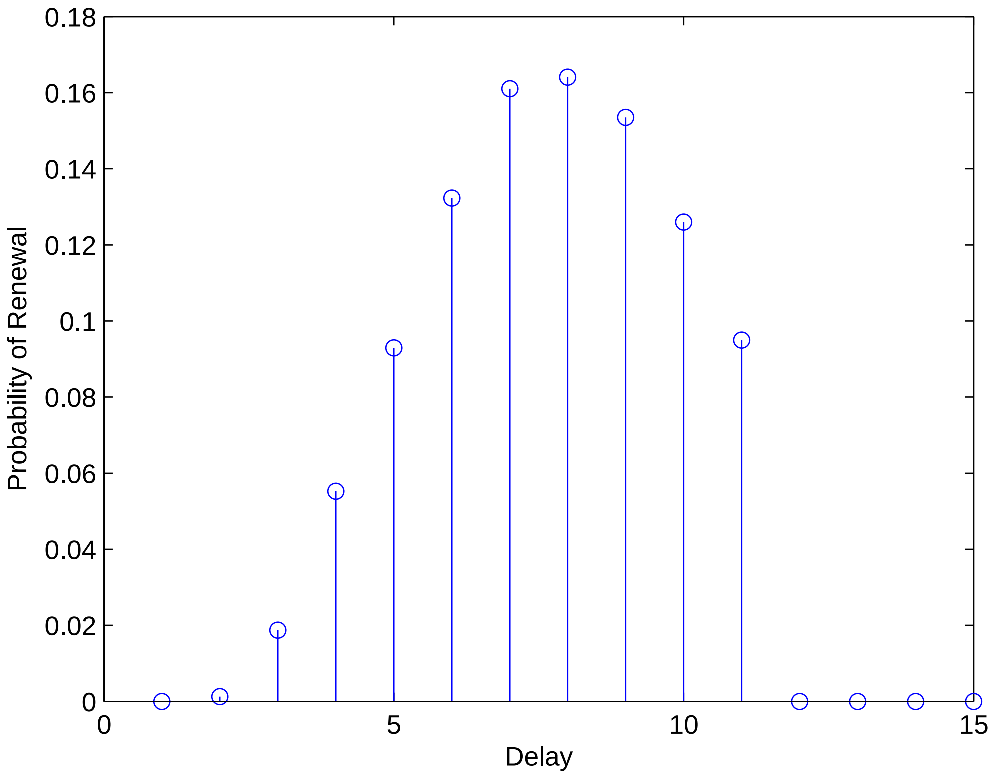

Fig. 5.2 Plot of pmf of Renewal delay of users for MLWDF scheduling for finite window length

No of Users$N$	P_dSimulated $M = N$	P_dSimulated $M = 1.5N$	P_dSimulated $M = 2N$
25	0.1946	0.0267	0.00072
40	0.1951	0.0250	0.00034

Table 5.4 The Table of simulated packet drop probabilities for various no of users and Maximum Delay Settings

5.2.1 Packet Drop

The packet drop rate of user i for iid channel conditions as a function of average delay $\mu_i$is derived in appendix to be $\frac{P_d^i}{1-P_d^i} = \frac{N-1-\mu_i}{M+1}$. We do not have another relation between P_d or $\mu_i$or λ_i. Hence the analysis was not unable to move forward from here. So we end the MLWDF section only with the results from simulation.The simulated packet drop probability P_d^i of the iid Rayleigh channel with finite window and rate linear with SNR is tabulated below for different no of users and maximum delay setting M.

No of Users$N$	μ_i Simulated $M = N$	μ_i Simulated $M = 1.5 * N$	μ_i Simulated$M = 2 * N$
25	17.6991	22.9314	23.9634
40	29.0615	37.4378	38.9734

Table 5.5 The table of simulated and estimated average delays for various no of users and Maximum Delay Settings

No of Users$N$	M' Simulated $M = 1.5N$ $X = 50\%$	M' Simulated $M = 2N$ $X = 50\%$	M' Simulated $M = 1.5N$ $X = 75\%$	M' Simulated $M = 2N$ $X = 75\%$
25	23	24	28	29
40	37	39	45	46

Table 5.6 The table of calculated and simulated M' for various no of users and Maximum Delay Settings

5.2.2 Delay

The simulated average delay of user i ,μ_i of the iid Rayleigh channel with finite window and rate linear with SNR is tabulated below for different no of users N and maximum delay setting M.The average delays in this scenario decreased compared with the same case considered in section 5.1. This is because the packets which waited long for scheduling case in the previous case are now dropped hence the the reduction in average delay but finite packet drop rate. As you increase M, some of the packets with greater delay dropped in the previous case are now scheduled which causes the increase in average delay with increase in M.

We consider the cases of $X\%$ to be 50%,75% and 95% . So we tabulated the simulate and calculated M' for various no of users and $M'$is rounded off to integer greater than M'.The average delays in this scenario decreased compared with the same case considered in section 5.1. This is because the packets which waited long for scheduling case in the previous case are now dropped hence the the reduction in average delay but finite packet drop rate. As you increase M, some of the packets with greater delay dropped in the previous case are now scheduled which causes the increase in average delay.

No of Users	Gain Simulated
10	2.3829 (M=10)
25	3.078 (M=40)
40	3.4437 (M=80)

Table 5.7 Table showing the simulated and calculated gains for mlwdf scheduling

5.2.3 Throughput

The simulated throughput gains of user i ,μ_i of the iid Rayleigh channel with finite window and rate linear with SNR is tabulated below for different no of users N and maximum delay setting M.in Table 5.7 The throughput here is supposed to be increased compared to the above case, here there is increase but it is very negligible.

CHAPTER 6

Conclusions

In all the cases of PFS with no maximum delay setting M and rate linear with SNR, the throughput gain, average delay and no of slots M'required to schedule $X\%$ of packets are unaffected by both non iid large scale and small scale fading.Similarly in all the cases of PFS with maximum delay setting M and rate linear with SNR, the throughput gain, average delay and no of slots M'required to schedule $X\%$ of packets are unaffected by both non iid large scale and small scale fading. further in PFS all the cases of rate linear with SNR, the expression for throughput gain is independent of maximum delay setting M.We now compare of various trade offs between PFS and MLWDFS.

The average delay is same in MLWDF and PF under iid channel condition with no maximum delay setting and rate linear with SNR. Hence it is proof of the fact that average delay is not the relaible indicator. How ever we observe that the no fo slots required to schedule 95%packets in 25 users system is 73 in PFS but 34 in MLWDFS. The better performance in MLWDFs is because it considers delay in scheduling. the trade off in better delay performance is in throuput gain which is 2.9296 in PFS and 2.3419 in MLWDFS for 10 users system.

Interestingly the average delay in MLWDF is less than in PF under iid channel condition with same maximum delay setting and rate linear with SNR. In the 25 users system with maximum delay $M = 1.5N$,the average delay is 14.0797 in MLWDFS and is 22.9314 in PFS. Similarly the performance in no of slots required to schedule $X\%$ packets is worse in MLWDF than in PF under iid channel condition with same maximum delay setting and rate linear with SNR. In the 40 users system with maximum delay $M = 2N$ and $X = 50\%$,the M' is 39 in MLWDFS and is 22 in PFS.This better performance in delay does not come without a tradeoff. In the 25 users system with maximum delay $M = N$,the packet drop is 0.1946 in MLWDFS and is 0.3463 in PFS which is almost double. These are the interesting tradeoffs between throughput gain average delay and packet drop rate. One can choose the algorithm which is beneficial depending upon their working conditions.

Generically we can say for the non real time users PFs is preferred over MLWDFS but for mixture of real time and non real time users, MLWDF is the more apt one.

APPENDIX A

Let us consider a cell with only 2 users. We need to evaluate the ratio of throughput of theses users given that these two users experience different channel conditions. We assume Rayleigh channel with rate linear with SNR. From [15] , the relation between ratio of throughput's is evaluated as

$$\frac{T_1}{T_2} = r = \frac{P(X_1 > rX_2)E\left[X_1/X_1 > rX_2\right]}{P(X_2 > r^{-1}X_1)E\left[X_2/X_2 > r^{-1}X_1\right]} \tag{A.1}$$

where X_1 and X_2denote the data rates of two users. For rate linear with SNR assumption $X_i = g * a_i\tau_i(t).a_i$ is the distance dependent component of the received channel power (Large scale fading) while $\tau_i(t)$ is the random component which is modeled here as Rayleigh fading (Small scale fading). The scheduling rule $X_1 > \frac{T_1}{T_2}X_2$ simplifies to $\tau_1(t) > \frac{a_2T_1}{a_1T_2}\tau_2(t) = ar\tau_2(t)$, where $a = \frac{a_2}{a_1}$.Since $\tau_i(t)$ is assumed to be stationary ,(A.1)modifies to

$$\frac{T_1}{T_2} = r = \frac{P(\tau_1 > ar\tau_2)E\left[ga_1\tau_1/\tau_1 > ar\tau_2\right]}{P(\tau_2 > (ar)^{-1}\tau_1)E\left[ga_2\tau_2/\tau_2 > (ar)^{-1}\tau_1\right]} \tag{A.2}$$

The above expression simplifies to

$$\frac{T_1}{T_2} = r = \frac{a_1}{a_2}\frac{\int_0^\infty f_{\tau_2(t)}(x)\int_{arx}^\infty y * f_{\tau_1(t)}(y)dydx}{\int_0^\infty f_{\tau_1(t)}(x)\int_{\frac{x}{ar}}^\infty y * f_{\tau_2(t)}(y)dydx}$$

$$ar = \frac{\int_0^\infty f_{\tau_2(t)}(x)\int_{arx}^\infty y * f_{\tau_1(t)}(y)dydx}{\int_0^\infty f_{\tau_1(t)}(x)\int_{\frac{x}{ar}}^\infty y * f_{\tau_2(t)}(y)dydx}$$

where $f_{\tau_1(t)}(x) = \lambda e^{-\lambda_1 x}$ and $f_{\tau_2(t)}(x) = \lambda_2 e^{-\lambda_2 x}$ are pdfs of randomly varying component of received channel power.The above equation simplifies to

$$ar = \frac{\lambda_2}{\lambda_1 ar} * \frac{\frac{ar}{\lambda_1 ar+\lambda_2} + \frac{1}{\lambda_1}}{\frac{1}{\lambda_1 ar+\lambda_2} + \frac{1}{\lambda_2}}$$

Of the solutions of the above equation, the only real solution is $ar = \frac{\lambda_2}{\lambda_1}$. So for same small scale channel fade conditions, i.e $\lambda_1 = \lambda_2$. The throughput ratio simplifies to ratio of large scale fading components as

$$\frac{T_1}{T_2} = \frac{a_1}{a_2}$$

Similarly for same large scale channel fade conditions, i.e $a_1 = a_2$. The throughput ratio simplifies to

$$\frac{T_1}{T_2} = \frac{\lambda_2}{\lambda_1}$$

We can extend this analogy given any number of users N, Consider only the slots these two users are scheduled ,the ratio of throughput's of these two users is still same as which reduces to

$$\frac{T_i}{T_j} = \quad r_{i,j} = a_{i,j}^{-1} \quad \frac{\lambda_j}{\lambda_i}$$

$\forall i, j = 1, ..., N$.We now move on to logarithmic rate case. To evaluate the ratio of throughput of theses users given that these two users experience different channel conditions. We assume Rayleigh channel with logarithmic rate. The scheduling rule here simplifies to $log(1 + g\tau_1(t)) > \frac{a_2 T_1}{a_1 T_2} log(1 + g\tau_2(t))$,with assumption $log(1 + gx) = log(gx)$ which further simplifies to $\tau_1(t) > g^{ar-1}\tau_2^{ar}(t)$. With this (A.2) ,simplifies to

$$\frac{T_1}{T_2} = r = \frac{P(\tau_1 > g^{ar-1}\tau_2^{r})E\left[log(1 + ga_1\tau_1)/\tau_1 > g^{ar-1}\tau_2^{ar}\right]}{P(\tau_2 > g^{\frac{1-ar}{r}}\tau_1^{\frac{1}{ar}})E\left[log(1 + ga_2\tau_2)/\tau_2 > g^{\frac{1-ar}{r}}\tau_1^{\frac{1}{ar}}\right]}$$

$$\frac{T_1}{T_2} = \quad r = \quad \frac{\int_0^\infty f_{\tau_2(t)}(x)\int_{g^{ar-1}x^{ar}}^\infty log(1 + ga_1y) * f_{\tau_1(t)}(y)dydx}{\int_0^\infty f_{\tau_1(t)}(x)\int_{g^{\frac{1-ar}{r}}x^{\frac{1}{ar}}}^\infty log(1 + ga_2y) * f_{\tau_2(t)}(y)dydx}$$

After substituting for pdfs of Rayleigh channel it appears as

$$\frac{T_1}{T_2} = r = \frac{\int_0^\infty \lambda_2 e^{-\lambda_2 x} \int_{g^{ar-1}x^{ar}}^\infty log(1+ga_1 y) * \lambda_1 e^{-\lambda_1 x} dydx}{\int_0^\infty \lambda_1 e^{-\lambda_1 x} \int_{g^{\frac{1-ar}{r}} x^{\frac{1}{ar}}}^\infty log(1+ga_2 y) * \lambda_2 e^{-\lambda_2 y} dydx}$$

$$= \frac{\int_0^\infty \lambda_2 e^{-\lambda_2 x} \left(log(1+a_1 g^{ar} x^{ar}) e^{-\lambda_1 g^{ar-1} x^{ar}} - e^{\frac{\lambda_1}{ga_1}} Ei[-\lambda_1 g^{ar-1} x^{ar} - \frac{\lambda_1}{ga_1}] \right) dx}{\int_0^\infty \lambda_1 e^{-\lambda_1 x} \left(log(1+ag^{\frac{1-ar+r}{r}} x^{\frac{1}{ar}}) e^{-\lambda_2 g^{\frac{1-ar}{r}} x^{\frac{1}{ar}}} - e^{\frac{\lambda_1}{ga_1}} Ei[-\lambda_2 g^{\frac{1-ar}{r}} x^{\frac{1}{ar}} - \frac{\lambda_{21}}{ga_2}] \right) dx}$$

We consider the system of N users in a cell with each user having a maximum delay constraint of M slots beyond which the packets are dropped. All the users are assumed to be having non iid channel conditions. Below are the terms that are used for derivation for packet drop rate expression.

$K-$No of slots for which the users are scheduled.

Q_i-No of renewals of user i of total renewals of K

$n^i_{0,} n^i_1, ..., n^i_M$ are the no of renewals occurred at Delays 0....M of user i

$w^i_{0,} w^i_1, ..., w^i_M$ are the no of times the user is the delay states 0....M of user i

n^i_d- the no of packets dropped of user i

μ_i-average Delay of user i

We now derive the expression for packet drop in terms of the parameter μ.The total no of slots for which users are scheduled is total no of renewals of all users as any one user is scheduled at each slot

$$K = \sum_{i=0}^{N} Q_i \tag{A.3}$$

No of renewals of a user i is sum of all its renewals i.e.

$$Q_i = \sum_{j=0}^{M} n^i_j \tag{A.4}$$

Packet drop Probability of user i is defined as follows

$$P^i_d = \frac{n^i_d}{Q_i + n^i_d} \tag{A.5}$$

The scheduling probability of the user at delay state i is given by

$$P(Z = i) = \frac{n_i}{Q}$$

The average delay is given by

$$\begin{aligned}\mu_i &= \sum_{i=0}^{M} i * P_Z(Z = i) \\ &= \sum_{i=0}^{M} i * \frac{n_i}{Q_i}\end{aligned}$$

At each slot user will be in any of delay states $0....M$ and hence the total no of slots can be written as

$$K = \sum_{j=0}^{M} w_j^i \tag{A.6}$$

No of times the user is in a state j implies sum of no of renewals at states greater than and equal to state j plus the no of packets dropped i.e..

$$w_j^i = \sum_{k=j}^{M} n_k^i + n_d^i \tag{A.7}$$

We recall the definition of pmf of Delay states $P_W(W_i = n)$, as the probability of being present in that delay state of user i which can be translated to

$$P_W(W_i = n) = \frac{w_n^i}{K}$$

From (A.7) and (A.3), the above equation simplifies to

$$P_W(W_i = n) = \frac{\sum_{k=j}^{M} n_k^i + n_d^i}{\sum_{i=0}^{N} Q_i} \tag{A.8}$$

Substituting back into (A.6)we get

$$
\begin{aligned}
K &= \sum_{j=0}^{M} \left(\sum_{k=j}^{M} n_k^i + n_d^i \right) \\
\sum_{i=1}^{N} Q_i &= \sum_{j=0}^{M} (j+1) * n_j^i + (M+1) * n_d^i \\
\frac{\sum_{i=1}^{N} Q_i}{Q_i} &= \frac{\sum_{j=0}^{M} j * n_j^i}{Q_i} + \frac{\sum_{j=0}^{M} n_j^i}{Q_i} + \frac{(M+1) * n_d^i}{Q_i}
\end{aligned}
$$

For iid channel conditions$Q_i = Q \forall i = 1 \longrightarrow N$, and (A.4),(A.5) from the above equation reduces to

$$
\begin{aligned}
N &= \mu_i + 1 + (M+1) * \frac{P_d^i}{1 - P_d^i} \\
\frac{P_d^i}{1 - P_d^i} &= \frac{N - 1 - \mu_i}{M+1} \qquad \text{(A.9)}
\end{aligned}
$$

For iid channel conditions , (A.8) and with same above simplification it reduces to

$$
\begin{aligned}
P_W(W_i = n) &= \frac{\sum_{k=j}^{M} n_k^i + n_d^i}{N * Q} \\
&= \frac{1}{N} \left(\frac{\sum_{k=j}^{M} n_k^i}{Q} + \frac{n_d^i}{Q} \right) \\
&= \frac{1}{N} \left(\sum_{m=n}^{M} P_Z(Z_i = m) + \frac{P_d^i}{1 - P_d^i} \right)
\end{aligned}
$$

For PF in [16]it was proved for rate linear with SNR assumption even under non iid conditions, every one gets the same no of slots to be served which implies$Q_i = Q \forall i = 1 \longrightarrow N$, which simplifies to

$$
\frac{P_d^i}{1 - P_d^i} = \frac{N - 1 - \mu_i}{M+1}
$$

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