Analytical evaluation of Fractional Frequency Reuse for *Dynamic-TDD* in Phantom Cell eLA

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This document presents the analytical framework to characterize a Phantom cell D-TDD network employing frequency reuse techniques. We evaluate the expressions for coverage probability, rate and average energy efficiency taking the aid of stochastic geometry. Monte-Carlo simulations validate the correctness of the expressions.

Keywords: Phantom cells, Poisson point process, FFR, Dynamic TDD.

Introduction

According to recent study, mobile data traffic is expected to grow 500 times between 2010 and 2020[1]. To respond to this demand, 3GPP standards body has initiated studies on the further evolution of LTE/ LTE-A, referred as LTE-B. Enhanced Local Area (eLA) is a solution that offers high data rate to user terminals (UEs) along with high system capacity through spatial reuse of spectrum. The 3GPP has also been studying dynamic allocation of sub-frames to uplink (UL) or downlink (DL) in Time Division Duplex (TDD), called Dynamic TDD. Here, we focus on a particular architecture proposed by DoCoMo, called the Phantom Cell architecture. This has no cell-specific signaling, and hence dynamic DL/UL slot reconfiguration and dynamic DL power control can be easily realized.

Phantom Cells:

DoCoMo has proposed the Phantom Cell small cell architecture [2], [3] as a readily-implementable and robust solution yielding high system throughput together with spatial reuse. A detailed description of the architecture may be found in [6]. Some key features are summarized here:

1) Small cells handle traffic for high-throughput data sessions with UEs, but control signaling is handled by the macrocellular layer. Each macrocell controls a set of small cells through a master-slave relationship, and the small cells do not transmit any cell-specific signals (hence the name Phantom Cells). This also relaxes the backhaul and signaling requirements between the small cells and the controlling macrocells compared to a conventional RRH deployment.

2) Scales from relatively low density to very high site density (for crowded high-traffic urban areas).

3) Deployment in dedicated higher frequency bands, such as the 3.5 GHz band, for indoor and outdoor applications. There is no interference with the macro LTE network in lower frequency bands.

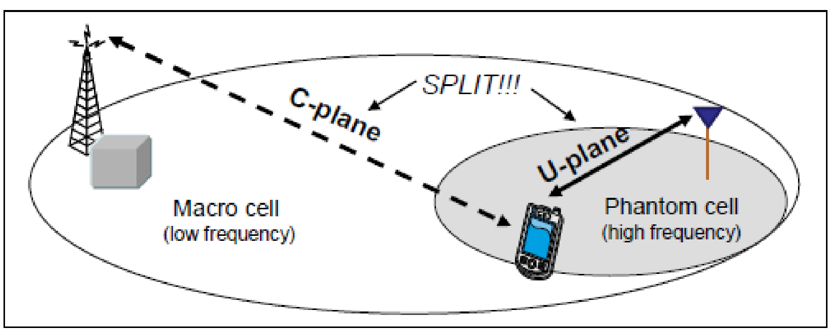

Figure 1: C-Plane and U-Plane split used in Phantom cell architecture.

Prior Work

The use of stochastic geometry to model base stations and analyzing the network is very tractable in dense areas[4]. We use various frequency reuse techniques to mitigate the interference. The most useful of these are *Fractional Frequency Reuse* techniques whose coverage in an OFDMA network has been studied[5]. Deployment of phantom cells in D-TDD setup with OLPC has also shown good downlink coverage[6]. We use the expressions from literature and analyze the network with both Dynamic time division duplexing and fractional reuse techniques arrangement.

System Model

Poisson point process:

The cellular network model consists of base stations (BSs) arranged according to some homogeneous Poisson point process (PPP) ϕ of intensity λ in the Euclidean plane[4]. Consider an independent collection of mobile users, located according to some independent stationary point process. We assume each mobile user is associated with the closest base station; namely the users in the Voronoi cell of a BS are associated with it, resulting in coverage areas that comprise a Voronoi tessellation on the plane.

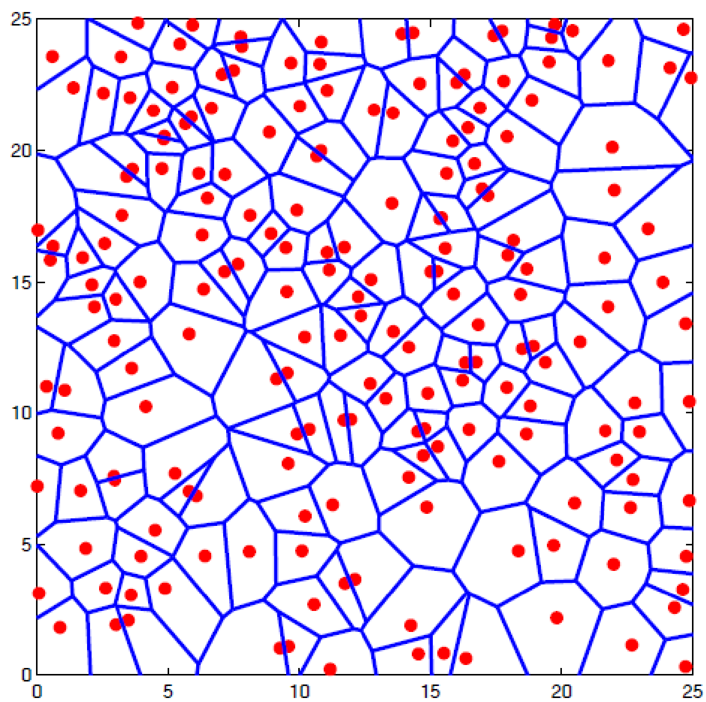

Figure 2: Poisson distribution of base stations and users, with each user associated with the nearest BS.

Nearest neighbour:

From [4] we also have, $P[R > r]$ = P[No BS closer than r] = $e^{-\lambda_b \pi r^2}$.

So, the cumulative distribution function (CDF) of R is $P[R < r] = F_R(r) = 1 - e^{-\lambda_b \pi r^2}$ and the probability density function of R can be got as

$$f_R(r) = 2\pi\lambda_b r e^{-\lambda_b \pi r^2}, r > 0 \tag{1}$$

Path loss and Fading:

We consider both the path loss and fast fading as the propagation model, where path loss exponent $\alpha > 2$, the small-scale Rayleigh fading is used to model the fast fading between the transmission points and the receiving points. The transmit power of the serving base station is given by P_y and transmit power of UE is P_u. Thus, for the calculation of the DL SINR, the received power of the desired signal at the typical UE at a distance R from its BS is given by $P_y g_y R^{-\alpha}$, where g_y is i.i.d. exponentially distributed with mean μ_b. While, for the calculation of the UL SINR, the received power of the desired signal at the typical BS a distance R from its UE is given by $P_u g_u R^{-\alpha}$, where g_u is i.i.d. exponentially distributed with mean μ_u. The noise power is assumed to be σ^2.

Interference:

When we have dynamic TDD with incomplete coordination and/or imperfect synchronization across phantom cells, there is additional interference on the DL (UL) subframes from out-of-cell UL (DL) transmissions. This arises because of possibly different sequences of UL and DL subframes in the radio frames for different phantom cells, where the start and end times of these subframes may not be exactly aligned. The set of interfering base stations is Z_i (i.e. base stations that use the same sub-band as user y) and at a distance of D_i from the parent base station. Similarly, the set of interfering UE's is Z_j and at a distance of D_j from the parent base station.

Empty cells:

In this model, we also consider the BSs that do not have any UE to serve[7]. We call these cells empty cells and will not provide any interference to the network, which means these BSs are inactive. From [8], we know that a typical BS will have a certain probability to be active, we denote it ρ_a, given by:

$$\rho_a = 1 - \left(1 + \frac{\lambda_u}{3.5\lambda_b}\right)^{-3.5} \tag{2}$$

Given ρ_a, the active BSs can be approximated as a PPP Φ_b of density $\rho_a \lambda_b$.

Implementing D-TDD :

The transmission direction of a typical cell is unaltered during one transmission time interval (TTI), which means either DL or UL and this will not change in the duration of a TTI in dynamic TDD cellular networks. In order to reflect the randomness of the cell transmission. We use a random variable p_{dir} uniformly distributed in $(0, 1)$ to determine the transmission direction of a typical cell, given by[7]:

$$\text{Transmission direction} = \begin{cases} DL, & p_{dir} \leq A. \\ UL, & p_{dir} > A. \end{cases} \tag{3}$$

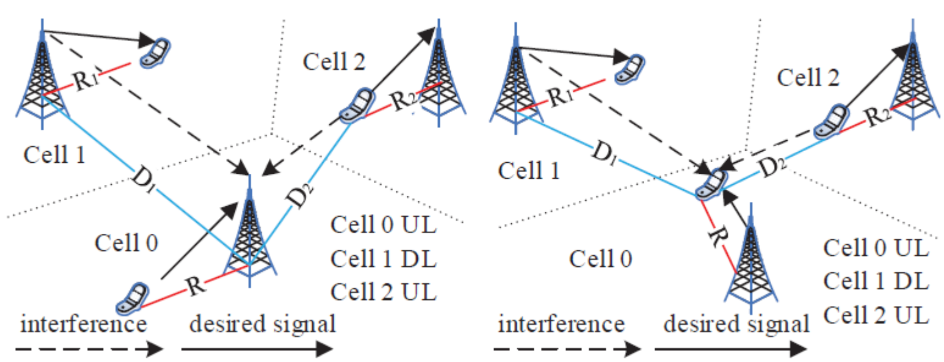

Figure 3: Model of a dynamic TDD system.

Signal to Interference Noise ratio:

Downlink SINR is given by:

$$SINR_{DL} = \frac{P_y g_y r^{-\alpha}}{\sigma^2 + I_z} \tag{4}$$

Uplink SINR is given by:

$$SINR_{UL} = \frac{P_u g_u r^{-\alpha}}{\sigma^2 + I_z} \tag{5}$$

Interference here is caused from both downlinks(z_i) and uplinks(z_j) and using [7] is:

$$I_z = I_{z_i} + I_{z_j} = \sum_{z \in i} P_z G_{z_i} D_{z_i}^{-\alpha} + \sum_{z \in j} P_z G_{z_j} D_{z_j}^{-\alpha} \tag{6}$$

Interference Model

Fractional Frequency Reuse:

The basic idea of FFR[5] is to partition the cells bandwidth so that:

(i) cell-edge users of adjacent cells do not interfere with each other and

(ii) interference received by (and created by) cellinterior users is reduced, while

(iii) using more total spectrum than conventional frequency reuse.

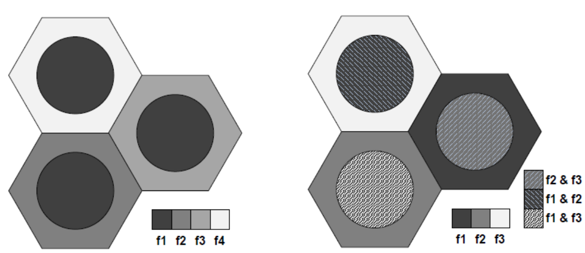

Figure 4: Strict FFR (left) and SFR (right) deployments with Δ = 3 in a standard hexagonal grid model.

We model the BS locations as a Poisson point process (PPP). One advantage of this approach is the ability to capture the non-uniform layout of modern cellular deployments due to topographic, demographic, or economic reasons [9], [10], [11] and additionally leading to more general performance.

Two common FFR deployment modes:

1) Strict FFR: Strict FFR is a modification of the traditional frequency reuse used extensively in multi-cell networks [12], [13]. Users in each cell-interior are allocated a common sub-band of frequencies while cell-edge users bandwidth is partitioned across cells based on a reuse factor of Δ. In total, Strict FFR thus requires a total of $\Delta + 1$ sub-bands. Interior users do not share any spectrum with exterior users, which reduces interference for both interior users and cell-edge users.

2) SFR: This employs the same cell-edge bandwidth partitioning strategy as Strict FFR, but the interior users are allowed to share sub-bands with edge users in other cells.

How the FFR Model works

Conventionally, a mobile tries to connect to its nearest BS and is in coverage if it is connected. With FFR, A mobile computes its SINR to the nearest BS, and if it is less than the threshold T_{FR}, then the BS chooses to transmit in a different FFR band randomly picked from Δ sub-bands reserved for the FFR users[5]. If such a shift occurs, we term the mobile as a *cell-edge user* and as an *interior user* otherwise.

In **Strict FFR**, the users who have SINR less than the reuse threshold T_{FR} on the common sub-band shared by all cells and are therefore selected by the reuse strategy to have a new sub-band allocated to them from the Δ total available sub-bands reserved for the edge users and experience new fading power and $\hat{g_y}$ and out-of-cell interference $\hat{I_z}$. Hence, we have

$$P_y = P_z = P \tag{7}$$

SFR uses power control, rather than frequency reuse for the edge users, controlled by the design parameter β. Additionally, the base stations can reuse all sub-bands, but apply β to only one of the Δ sub-bands. ie.

$$P_{edge} = \beta P \tag{8}$$

$$P_{int} = P \tag{9}$$

The interference is given by:

$$\overline{I_z} = I_{int} + I_{edge} = \left(\sum_{z \in i} P_z G_{z_i} D_{z_i}^{-\alpha} + \sum_{z \in j} P_z G_{z_j} D_{z_j}^{-\alpha} \right) + \beta \left(\sum_{z \in i} P_z G_{z_i} D_{z_i}^{-\alpha} + \sum_{z \in j} P_z G_{z_j} D_{z_j}^{-\alpha} \right) \tag{10}$$

which can be simplified as $\eta P I_z$ where I_z is from (6) and η is the effective interference power factor, consolidating the impact of interference from the higher and lower power downlinks given by:

$$\eta = \frac{\Delta - 1 + \beta}{\Delta} \tag{11}$$

Coverage Probability

We now discuss the coverage probability in dynamic TDD cellular networks. The coverage probability for a typical receiver is defined as [4]:

$$p_c(T, \lambda, \alpha) = \mathbb{P}[SINR > T] \tag{12}$$

which is equivalently

(i) the probability that a randomly chosen user can achieve a target SINR of T,

(ii) the average fraction of users who at any time achieve SINR of T, or

(iii) the average fraction of the network area that is in coverage at any time. The probability of coverage is also exactly the CCDF of SINR over the entire network, since the CDF gives $\mathbb{P}[SINR \leq T]$

Theorem 1(Strict FFR, edge user): The coverage probability of an edge user in a Strict FFR network with dynamic TDD is $\overline{F}_{FFR,e}(T) =$

$$\frac{p_c(T,\lambda_b,A,\rho_a,\alpha,\Delta) - \int_0^\infty 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{-2\pi\lambda_1\gamma_1 r^2} e^{-2\pi\lambda_2\gamma_2 r^2} e^{\frac{-\mu_b(T+T_{FR})\sigma^2 r^\alpha}{P}} dr}{1 - p_c(T_{FR},\lambda_b,A,\rho_a,\alpha)} \quad (13)$$

where,

$$p_c(T,\lambda_b,A,\rho_a,\alpha,\Delta) = \int_0^\infty 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{\frac{-\mu_b T\sigma^2 r^\alpha}{P}} e^{-2\pi\lambda_b^{dl}\int_r^\infty \frac{T}{T+(\frac{x}{r})^\alpha} x dx} e^{-2\pi\lambda_b^{ul}\int_r^\infty \frac{T}{T+(\frac{x}{r})^\alpha\frac{\mu_b}{\mu_u}} x dx} dr$$

$$p_c(T_{FR},\lambda_b,A,\rho_a,\alpha) = \int_0^\infty 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{\frac{-\mu_b T_{FR}\sigma^2 r^\alpha}{P}} e^{-2\pi\lambda_1\int_r^\infty \frac{T_{FR}}{T_{FR}+\left(\frac{x}{r}\right)^\alpha} x dx} e^{-2\pi\lambda_2\int_r^\infty \frac{T_{FR}}{T_{FR}+\left(\frac{x}{r}\right)^\alpha\frac{\mu_b}{\mu_u}} x dx} dr$$

Proof: An edge user y with SINR $< T_{FR}$ is given a FFR sub-band δ_y, where $\delta \in \{1,...,\Delta\}$ with uniform probability $\frac{1}{\Delta}$, and experiences new fading power $\hat{g_y}$ and out-of-cell interference $\hat{I_z}$, instead of g_y andI_z.

The CCDF of the edge user $\hat{F}_{FFR,e}(T)$ is conditioned on its previous SINR:

$$\Rightarrow \overline{F}_{FFR,e}(T) = \mathbb{P}\left(\frac{P\hat{g_y}r^{-\alpha}}{\sigma^2+P\hat{I_z}} > T \middle| \frac{Pg_y r^{-\alpha}}{\sigma^2+PI_z} < T_{FR}\right)$$

Using Baye's theorem:

$$= \frac{\mathbb{P}\left(\frac{P\hat{g_y}r^{-\alpha}}{\sigma^2+P\hat{I_z}} > T, \frac{Pg_y r^{-\alpha}}{\sigma^2+PI_z} < T_{FR}\right)}{\mathbb{P}\left(\frac{Pg_y r^{-\alpha}}{\sigma^2+PI_z} < T_{FR}\right)}$$

Assuming $\hat{g}_y$ and g_y to be exponentially distributed according to $exp(\mu_b)$

$$= \frac{\mathbb{E}\left[e^{-\mu_b T r^\alpha(\frac{\sigma^2}{P}+\hat{I_z})}\left(1 - e^{-\mu_b T_{FR} r^\alpha(\frac{\sigma^2}{P}+I_z)}\right)\right]}{\mathbb{E}\left[1 - e^{-\mu_b T_{FR} r^\alpha(\frac{\sigma^2}{P}+I_z)}\right]}$$

$$= \frac{\mathbb{E}\left[e^{-\mu_b T r^{\alpha}(\frac{\sigma^2}{P}+\hat{I}_z)}\right] - \mathbb{E}\left[e^{-\mu_b T r^{\alpha}(\frac{\sigma^2}{P}+\hat{I}_z)} e^{-\mu_b T_{FR} r^{\alpha}(\frac{\sigma^2}{P}+I_z)}\right]}{\mathbb{E}\left[1 - e^{-\mu_b T_{FR} r^{\alpha}(\frac{\sigma^2}{P}+I_z)}\right]}$$

$$= \frac{\mathbb{E}\left[e^{-\mu_b T r^{\alpha}(\frac{\sigma^2}{P}+\hat{I}_z)}\right] - \mathbb{E}\left[e^{\frac{-\mu_b (T+T_{FR})\sigma^2 r^{\alpha}}{P}} e^{-\mu_b r^{\alpha}(T\hat{I}_z+T_{FR}I_z)}\right]}{\mathbb{E}\left[1 - e^{-\mu_b T_{FR} r^{\alpha}(\frac{\sigma^2}{P}+I_z)}\right]} \tag{14}$$

Expanding each of the terms independently:

$$\mathbb{E}\left[e^{\frac{-\mu_b (T+T_{FR})\sigma^2 r^{\alpha}}{P}} e^{-\mu_b r^{\alpha}(T\hat{I}_z+T_{FR}I_z)}\right] = \mathbb{E}\left[e^{\frac{-\mu_b (T+T_{FR})\sigma^2 r^{\alpha}}{P}} e^{(-s_1\hat{I}_z - s_2 I_z)}\right]$$

From equation (6):

$$\Rightarrow \mathbb{E}\left[e^{-s_1\hat{I}_z - s_2 I_z}\right] = \mathbb{E}\left[e^{-s_1\left(\sum \hat{G_{z_i}} D_{z_i}^{-\alpha} + \sum \hat{G_{z_j}} D_{z_j}^{-\alpha}\right)(1\delta_z=\delta_y) - s_2\left(\sum G_{z_i} D_{z_i}^{-\alpha} + \sum G_{z_j} D_{z_j}^{-\alpha}\right)}\right]$$

Assuming $\hat{G_{z_i}}, G_{z_i}$ to be exponentially distributed according to $exp(\mu_b)$ and $\hat{G_{z_j}}, G_{z_j}$ to be exponentially distributed according to $exp(\mu_u)$:

$$\Rightarrow \mathbb{E}\left[e^{-s_1\hat{I}_z - s_2 I_z}\right] = \mathbb{E}\left[\prod \frac{\mu_b}{\mu_b + s_2 D_{z_i}^{-\alpha}}\left(1 - \frac{1}{\Delta}\left(1 - \frac{\mu_b}{\mu_b + s_1 D_{z_i}^{-\alpha}}\right)\right)\right]$$
$$\mathbb{E}\left[\prod \frac{\mu_u}{\mu_u + s_2 D_{z_j}^{-\alpha}}\left(1 - \frac{1}{\Delta}\left(1 - \frac{\mu_b}{\mu_b + s_1 D_{z_j}^{-\alpha}}\right)\right)\right] \tag{15}$$

Using PGFL of Poisson point process[14]:

$$\mathbb{E}\left[e^{-s_1\hat{I}_z - s_2 I_z}\right] = e^{-2\pi\lambda_1 r^2 \int_1^{\infty}\left[1 - \frac{1}{1+T_{FR}x^{-\alpha}}\left(1 - \frac{1}{\Delta}\left(1 - \frac{1}{1+Tx^{-\alpha}}\right)\right)\right]xdx}$$
$$e^{-2\pi\lambda_2 r^2 \int_1^{\infty}\left[1 - \frac{1}{1+T_{FR}\frac{\mu_u}{\mu_b}x^{-\alpha}}\left(1 - \frac{1}{\Delta}\left(1 - \frac{1}{1+T\frac{\mu_u}{\mu_b}x^{-\alpha}}\right)\right)\right]xdx} \tag{16}$$

Similarly the other terms can be solved,

$$\mathbb{E}\left[e^{-\mu_b T r^{\alpha}(\frac{\sigma^2}{P}+\hat{I_z})}\right] = \int_0^{\infty} 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{\frac{-\mu_b T\sigma^2 r^{\alpha}}{P}} e^{-2\pi\lambda_b^{dl}\int_r^{\infty}\frac{T}{T+(\frac{x}{r})^{\alpha}}xdx} e^{-2\pi\lambda_b^{ul}\int_r^{\infty}\frac{T}{T+(\frac{x}{r})^{\alpha}\frac{\mu_b}{\mu_u}}xdx} dr$$

(Expand $\hat{I}_z$ using (6) and decondition on r)

$$\mathbb{E}\left[e^{-\mu_b T_{FR} r^{\alpha}(\frac{\sigma^2}{P}+I_z)}\right] = \int_0^{\infty} 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{\frac{-\mu_b T_{FR}\sigma^2 r^{\alpha}}{P}} e^{-2\pi\lambda_1\int_r^{\infty}\frac{T_{FR}}{T_{FR}+(\frac{x}{r})^{\alpha}}xdx} e^{-2\pi\lambda_2\int_r^{\infty}\frac{T_{FR}}{T_{FR}+(\frac{x}{r})^{\alpha}\frac{\mu_b}{\mu_u}}xdx} dr$$

(Expand I_z using (6) and decondition on r)

Hence substituting all equations back in (14) to get the final expression:

$$\frac{\mathbb{E}[e^{-\mu_b T r^{\alpha}(\frac{\sigma^2}{P}+\hat{I_z})}] - \int_0^{\infty} 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{-2\pi\lambda_1\gamma_1 r^2} e^{-2\pi\lambda_2\gamma_2 r^2} e^{\frac{-\mu_b(T+T_{FR})\sigma^2 r^{\alpha}}{P}} dr}{\mathbb{E}\left[1 - e^{-\mu_b T_{FR} r^{\alpha}(\frac{\sigma^2}{P}+I_z)}\right]} \tag{17}$$

where,

$$\gamma_1 = \int_1^{\infty}\left[1 - \frac{1}{1+T_{FR}x^{-\alpha}}\left(1 - \frac{1}{\Delta}\left(1 - \frac{1}{1+Tx^{-\alpha}}\right)\right)\right]xdx$$

$$\gamma_2 = \int_1^{\infty}\left[1 - \frac{1}{1+T_{FR}\frac{\mu_u}{\mu_b}x^{-\alpha}}\left(1 - \frac{1}{\Delta}\left(1 - \frac{1}{1+T\frac{\mu_u}{\mu_b}x^{-\alpha}}\right)\right)\right]xdx$$

$$\lambda_b^{dl} = \frac{\rho_a\lambda_b A}{\Delta}$$

$$\lambda_b^{ul} = \frac{\rho_a\lambda_b(1-A)}{\Delta}$$

$$\lambda_1 = \rho_a\lambda_b A$$

$$\lambda_2 = \rho_a\lambda_b(1-A)$$

Theorem 2(Strict FFR, cell interior user): The coverage probability of an interior user in a Strict FFR network with dynamic TDD is $\overline{F}_{FFR,i}(T) =$

$$\frac{\mathbb{P}\left(\frac{P\hat{g_y}r^{-\alpha}}{\sigma^2+PI_z} > \max(T, T_{FR})\right)}{\mathbb{P}\left(\frac{Pg_yr^{-\alpha}}{\sigma^2+PI_z} > T_{FR}\right)} \tag{18}$$

Proof:

$$\overline{F}_{FFR,i}(T) = \mathbb{P}\left(\frac{P\hat{g_y}r^{-\alpha}}{\sigma^2+PI_z} > T \middle| \frac{Pg_yr^{-\alpha}}{\sigma^2+PI_z} > T_{FR}\right)$$

Using Baye's theorem:

$$= \frac{\mathbb{P}\left(\frac{P\hat{g_y}r^{-\alpha}}{\sigma^2+PI_z} > \max(T, T_{FR})\right)}{\mathbb{P}\left(\frac{Pg_yr^{-\alpha}}{\sigma^2+PI_z} > T_{FR}\right)}$$

$$= \frac{\mathbb{P}\left(\hat{g_y} > \frac{\max(T,T_{FR})r^{\alpha}(\sigma^2+PI_z)}{P}\right)}{\mathbb{P}\left(g_y > \frac{T_{FR}r^{\alpha}(\sigma^2+PI_z)}{P}\right)}$$

Expand I_z using (6), decondition on r:

Use $\hat{g_y}$ and $g_y \sim exp(\mu_b)$,

$$= \frac{\int_0^\infty 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{\frac{-\mu_b T_1 \sigma^2 r^\alpha}{P}} e^{-2\pi\lambda_1 \int_r^\infty \frac{T_1}{T_1+(\frac{x}{r})^\alpha} x dx} e^{-2\pi\lambda_2 \int_r^\infty \frac{T_1}{T_1+(\frac{x}{r})^\alpha \frac{\mu_b}{\mu_u}} x dx} dr}{\int_0^\infty 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{\frac{-\mu_b T_{FR} \sigma^2 r^\alpha}{P}} e^{-2\pi\lambda_1 \int_r^\infty \frac{T_{FR}}{T_{FR}+(\frac{x}{r})^\alpha} x dx} e^{-2\pi\lambda_2 \int_r^\infty \frac{T_{FR}}{T_{FR}+(\frac{x}{r})^\alpha \frac{\mu_b}{\mu_u}} x dx} dr} \tag{19}$$

where,

$$\lambda_1 = \lambda_2 = \lambda_b$$

$$T_1 = \max(T, T_{FR})$$

Theorem 3(Soft Frequency Reuse, cell edge user): The coverage probability of a cell edge user in a SFR network with dynamic TDD is $\overline{F}_{SFR,e}(T) =$

$$\frac{p_c(\beta T, \lambda_b, A, \rho_a, \alpha, \Delta, \eta) - \int_0^\infty 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{-2\pi\lambda_1\gamma_1 r^2} e^{-2\pi\lambda_2\gamma_2 r^2} e^{-\mu_b(\frac{T}{\beta}+T_{FR})\frac{\sigma^2 r^\alpha}{P}} dr}{1 - p_c(T_{FR}, \lambda_b, A, \rho_a, \alpha, \eta)} \tag{20}$$

where,

$$p_c(\beta T, \lambda_b, A, \rho_a, \alpha, \Delta, \eta) = \int_0^\infty 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{\frac{-\mu_b T\sigma^2 r^\alpha}{\beta P}} e^{-2\pi\lambda_1 \int_r^\infty \frac{T}{T+\frac{\beta}{\eta}\left(\frac{x}{r}\right)^\alpha} x dx} e^{-2\pi\lambda_2 \int_r^\infty \frac{T}{T+\frac{\beta\mu_b}{\eta\mu_u}\left(\frac{x}{r}\right)^\alpha} x dx} dr$$

$$p_c(T_{FR}, \lambda_b, A, \rho_a, \alpha, \eta) = \int_0^\infty 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{\frac{-\mu_b T_{FR}\sigma^2 r^\alpha}{P}} e^{-2\pi\lambda_1 \int_r^\infty \frac{T_{FR}}{T_{FR}+\frac{1}{\eta}\left(\frac{x}{r}\right)^\alpha} x dx} e^{-2\pi\lambda_2 \int_r^\infty \frac{T_{FR}}{T_{FR}+\frac{1}{\eta}\left(\frac{x}{r}\right)^\alpha \frac{\mu_b}{\mu_u}} x dx} dr$$

Proof: An edge user y with SINR $< T_{FR}$ is given a SFR sub-band δ, where $\delta \in \{1, ..., \Delta\}$, transmit power βP and experience new transmit power βP, fading power $\hat{g_y}$ and out-of-cell interference $\hat{I_z}$. The CCDF of the edge user $\overline{F}_{SFR,e}(T)$ is conditioned on its previous SINR:

$$\overline{F}_{SFR,e}(T) = \mathbb{P}\left(\frac{\beta P \hat{g_y} r^{-\alpha}}{\sigma^2 + \eta P \hat{I_z}} > T \middle| \frac{P g_y r^{-\alpha}}{\sigma^2 + \eta P I_z} < T_{FR}\right)$$

Using Baye's theorem and simplifying similarly as above theorems:

$$= \frac{\mathbb{E}\left[e^{-\mu_b T r^\alpha\left(\frac{\sigma^2+\eta P\hat{I_z}}{\beta P}\right)}\left(1 - e^{-\mu_b T_{FR} r^\alpha\left(\frac{\sigma^2+\eta P I_z}{P}\right)}\right)\right]}{\mathbb{E}\left[1 - e^{-\mu_b T_{FR} r^\alpha\left(\frac{\sigma^2+\eta P I_z}{P}\right)}\right]}$$

$$= \frac{\mathbb{E}\left[e^{-\mu_b T r^\alpha\left(\frac{\sigma^2+\eta P I_z}{\beta P}\right)}\right] - \mathbb{E}\left[e^{-\mu_b \sigma^2 r^\alpha(\frac{T}{\beta P}+\frac{T_{FR}}{P})} e^{-\mu_b \eta r^\alpha(\frac{T\hat{I_z}}{\beta}+T_{FR} I_z)}\right]}{1 - \mathbb{E}\left[e^{-\mu_b T_{FR} r^\alpha\left(\frac{\sigma^2+\eta P I_z}{P}\right)}\right]} \tag{21}$$

Expanding each of terms independently:

$$\mathbb{E}\left[e^{-\mu_b \sigma^2 r^\alpha (\frac{T}{\beta P}+\frac{T_{FR}}{P})} e^{-\mu_b \eta r^\alpha (\frac{T\hat{I_z}}{\beta}+T_{FR} I_z)}\right] = \mathbb{E}\left[e^{-\mu_b \sigma^2 r^\alpha (\frac{T}{\beta P}+\frac{T_{FR}}{P})} e^{(-s_1 \hat{I}_z - s_2 I_z)}\right]$$

$$\Rightarrow \mathbb{E}\left[e^{-s_1 \hat{I}_z - s_2 I_z}\right] = \mathbb{E}\left[e^{-s_1\left(\sum \hat{G_{z_i}} D_{z_i}^{-\alpha} + \sum \hat{G_{z_j}} D_{z_j}^{-\alpha}\right)(1\delta_z=\delta_y) - s_2\left(\sum G_{z_i} D_{z_i}^{-\alpha} + \sum G_{z_j} D_{z_j}^{-\alpha}\right)}\right] \quad (22)$$

Here, $\delta_z = \delta_y$ holds always and hence is equivalently setting $\Delta = 1$ in (15).

Assuming $\hat{G_{z_i}}$, G_{z_i} to be exponentially distributed according to $exp(\mu_b)$ and $\hat{G_{z_j}}$, G_{z_j} to be exponentially distributed according to $exp(\mu_u)$: $\mathbb{E}\left[e^{-\mu_b \sigma^2 r^\alpha (\frac{T}{\beta P}+\frac{T_{FR}}{P})} e^{-\mu_b \eta r^\alpha (\frac{T\hat{I_z}}{\beta}+T_{FR} I_z)}\right]$ will be expanded using $\mathcal{L}(s_1, s_2)$

$$\Rightarrow \mathcal{L}(s_1, s_2) = \mathbb{E}\left[\prod \frac{\mu_b}{\mu_b + s_2 R_z^{-\alpha}} \left(\frac{\mu_b}{\mu_b + s_1 R_z^{-\alpha}}\right)\right] \mathbb{E}\left[\prod \frac{\mu_u}{\mu_u + s_2 R_z^{-\alpha}} \left(\frac{\mu_u}{\mu_u + s_1 R_z^{-\alpha}}\right)\right]$$

Using PGFL of Poisson point process[14]:

$$\mathcal{L}(s_1, s_2) = exp\left(-2\pi\lambda_1 \int_r^\infty \left[1 - \frac{\mu_b}{\mu_b + s_2 x^{-\alpha}} \left(\frac{\mu_b}{\mu_b + s_1 x^{-\alpha}}\right)\right] x dx\right) exp\left(-2\pi\lambda_2 \int_r^\infty \left[1 - \frac{\mu_u}{\mu_u + s_2 x^{-\alpha}} \left(\frac{\mu_u}{\mu_u + s_1 x^{-\alpha}}\right)\right] x dx\right) \quad (23)$$

substituting $s_1 = \mu_b r^\alpha \eta \frac{T}{\beta}$ and $s_2 = \mu_b r^\alpha \eta T_{FR}$

$$\mathcal{L}(\mu_b r^\alpha \eta \frac{T}{\beta}, \mu_b r^\alpha \eta T_{FR}) = exp\left(-2\pi\lambda_1 r^2 \int_1^\infty \left[1 - \frac{1}{1+\eta T_{FR} x^{-\alpha}} \left(\frac{1}{1+\eta \frac{T}{\beta} x^{-\alpha}}\right)\right] x dx\right) exp\left(-2\pi\lambda_2 r^2 \int_1^\infty \left[1 - \frac{1}{1+\eta \frac{\mu_b}{\mu_u} T_{FR} x^{-\alpha}} \left(\frac{1}{1+\eta \frac{\mu_b}{\mu_u} \frac{T}{\beta} x^{-\alpha}}\right)\right] x dx\right) \quad (24)$$

Finally deconditioning on r and evaluating gives:

$$\mathbb{E}\left[e^{-\mu_b \sigma^2 r^\alpha (\frac{T}{\beta P}+\frac{T_{FR}}{P})} e^{-\mu_b \eta r^\alpha (\frac{T\hat{I_z}}{\beta}+T_{FR} I_z)}\right] = \int_0^\infty 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{-2\pi\lambda_1\gamma_1 r^2} e^{-2\pi\lambda_2\gamma_2 r^2} e^{-\mu_b (\frac{T}{\beta}+T_{FR}) \frac{\sigma^2 r^\alpha}{P}} dr \quad (25)$$

Similarly the other terms of (21) can be simplified as:

$$\mathbb{E}\left[e^{-\mu_b T r^{\alpha}\left(\frac{\sigma^2+\eta P I_z}{\beta P}\right)}\right] = \int_0^{\infty} 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{\frac{-\mu_b T \sigma^2 r^{\alpha}}{\beta P}} e^{\frac{-\mu_b T\eta}{\beta} r^{\alpha}(\sum G_{z_i} D_{z_i}^{-\alpha} + \sum G_{z_j} D_{z_j}^{-\alpha})} dr$$

(Using $\hat{g_y}, g_y \backsim exp(\mu_b)$, expanding $\hat{I_z}$ using (6) and deconditioning on r)

$$\Rightarrow \mathbb{E}\left[e^{-\mu_b T r^{\alpha}\left(\frac{\sigma^2+\eta P I_z}{\beta P}\right)}\right] = \int_0^{\infty} 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{\frac{-\mu_b T \sigma^2 r^{\alpha}}{\beta P}} e^{-2\pi\lambda_1 \int_r^{\infty} \frac{T}{T+\frac{\beta}{\eta}\left(\frac{x}{r}\right)^{\alpha}} xdx} e^{-2\pi\lambda_2 \int_r^{\infty} \frac{T}{T+\frac{\beta\mu_b}{\eta\mu_u}\left(\frac{x}{r}\right)^{\alpha}} xdx} dr$$

and similarly the denominator term becomes:

$$\mathbb{E}\left[e^{-\mu_b T_{FR} r^{\alpha}\left(\frac{\sigma^2+\eta P I_z}{P}\right)}\right] = \int_0^{\infty} 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{\frac{-\mu_b T_{FR} \sigma^2 r^{\alpha}}{P}} e^{-2\pi\lambda_1 \int_r^{\infty} \frac{T_{FR}}{T_{FR}+\frac{1}{\eta}\left(\frac{x}{r}\right)^{\alpha}} xdx} e^{-2\pi\lambda_2 \int_r^{\infty} \frac{T_{FR}}{T_{FR}+\frac{1}{\eta}\left(\frac{x}{r}\right)^{\alpha}\frac{\mu_b}{\mu_u}} xdx} dr$$

Substitute all terms back in the main equation:

$$\frac{\mathbb{E}\left[e^{-\mu_b T r^{\alpha}\left(\frac{\sigma^2+\eta P I_z}{\beta P}\right)}\right] - \int_0^{\infty} 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{-2\pi\lambda_1\gamma_1 r^2} e^{-2\pi\lambda_2\gamma_2 r^2} e^{-\mu_b(\frac{T}{\beta}+T_{FR})\frac{\sigma^2 r^{\alpha}}{P}} dr}{1-\mathbb{E}\left[e^{-\mu_b T_{FR} r^{\alpha}\left(\frac{\sigma^2+\eta P I_z}{P}\right)}\right]} \tag{26}$$

where,

$$\gamma_1 = \int_1^{\infty}\left[1-\frac{1}{1+\eta x^{-\alpha}T_{FR}}\left(\frac{1}{1+\eta\frac{T}{\beta}x^{-\alpha}}\right)\right]xdx$$

$$\gamma_2 = \int_1^{\infty}\left[1-\frac{1}{1+\eta x^{-\alpha}T_{FR}\frac{\mu_b}{\mu_u}}\left(\frac{1}{1+\eta\frac{T}{\beta}\frac{\mu_b}{\mu_u}x^{-\alpha}}\right)\right]xdx$$

$$\lambda_1 = \rho_a\lambda_b A$$

$$\lambda_2 = \rho_a\lambda_b(1-A)$$

Theorem 4(Soft Frequency Reuse, cell interior user): The coverage probability of an interior user in an SFR network with dynamic TDD is $\overline{F}_{SFR,i}(T) =$

$$\frac{\mathbb{P}\left(\frac{P\hat{g_y}r^{-\alpha}}{\sigma^2+\eta PI_z} > \max(T, T_{FR})\right)}{\mathbb{P}\left(\frac{Pg_yr^{-\alpha}}{\sigma^2+\eta PI_z} > T_{FR}\right)} \tag{27}$$

Proof:

For interior users there is no extra β power control in their transmit power, only the effective interference power factor η remains in the expressions.

$$\overline{F}_{SFR,i}(T) = \mathbb{P}\left(\frac{P\hat{g_y}r^{-\alpha}}{\sigma^2+\eta PI_z} > T \middle| \frac{Pg_yr^{-\alpha}}{\sigma^2+\eta PI_z} > T_{FR}\right)$$

Using Baye's theorem:

$$\frac{\mathbb{P}\left(\frac{P\hat{g_y}r^{-\alpha}}{\sigma^2+\eta PI_z} > \max(T, T_{FR})\right)}{\mathbb{P}\left(\frac{Pg_yr^{-\alpha}}{\sigma^2+\eta PI_z} > T_{FR}\right)}$$

$$= \frac{\mathbb{P}\left(\hat{g_y} > \frac{\max(T,T_{FR})r^{\alpha}(\sigma^2+PI_z)}{P}\right)}{\mathbb{P}\left(g_y > \frac{T_{FR}r^{\alpha}(\sigma^2+PI_z)}{P}\right)}$$

Assuming $\hat{g}_y$ and g_y to be exponentially distributed according to $exp(\mu_b)$ and deconditioning on r gives:

$$= \frac{\int_0^\infty 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{\frac{-\mu_b T_1 \sigma^2 r^\alpha}{P}} e^{-2\pi\lambda_1 \int_r^\infty \left(1-\frac{\mu_b}{\mu_b+s_1x^{-\alpha}} x dx\right)} e^{-2\pi\lambda_2 \int_r^\infty \left(1-\frac{\mu_u}{\mu_u+s_1x^{-\alpha}} x dx\right)} dr}{\int_0^\infty 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{\frac{-\mu_b T_{FR} \sigma^2 r^\alpha}{P}} e^{-2\pi\lambda_1 \int_r^\infty \left(1-\frac{\mu_b}{\mu_b+s_1x^{-\alpha}} x dx\right)} e^{-2\pi\lambda_2 \int_r^\infty \left(1-\frac{\mu_u}{\mu_u+s_1x^{-\alpha}} x dx\right)} dr}$$

substitute $s_1 = \mu_b T_1 \eta r^\alpha$ and $s_2 = \mu_b T_{FR} r^\alpha \eta$ in the above equation to finally obtain:

$$= \frac{\int_0^\infty 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{\frac{-\mu_b T_1 \sigma^2 r^\alpha}{P}} e^{-2\pi\lambda_1 \int_r^\infty \left(1-\frac{\mu_b}{\mu_b+s_1x^{-\alpha}} x dx\right)} e^{-2\pi\lambda_2 \int_r^\infty \left(1-\frac{\mu_u}{\mu_u+s_1x^{-\alpha}} x dx\right)} dr}{\int_0^\infty 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{\frac{-\mu_b T_{FR} \sigma^2 r^\alpha}{P}} e^{-2\pi\lambda_1 \int_r^\infty \left(1-\frac{\mu_b}{\mu_b+s_1x^{-\alpha}} x dx\right)} e^{-2\pi\lambda_2 \int_r^\infty \left(1-\frac{\mu_u}{\mu_u+s_1x^{-\alpha}} x dx\right)} dr} \tag{28}$$

where,

$$\lambda_1 = \lambda_2 = \lambda_b$$

$$T_1 = \max(T, T_{FR})$$

Average Achievable Rate

We define the average data rate for a typical BS or UE where adaptive modulation coding (AMC) is used so each BS or UE can achieve Shannon bound for their instantaneous SINR[4].

$$\overline{\tau}(\lambda,\alpha)=\mathbb{E}\left[\ln(1+SINR)\right] \tag{29}$$

Theorem 5(Strict FFR, cell edge user): The rate achieved by a cell edge user in a Strict FFR network with dynamic TDD is $\overline{\tau}_{FFR}(T_{FR},\lambda,\alpha,\Delta)=$

$$\int_{t>0}\frac{\mathbb{A}-\mathbb{B}}{1-\mathbb{C}}dt \tag{30}$$

Proof: The achievable rate is:

$$=\int_0^{\infty}e^{-\pi\lambda_b r^2}2\pi\lambda_b r\mathbb{E}\left[\ln\left(1+\frac{P\hat{g}_y r^{-\alpha}}{\sigma^2+P\hat{I}_z}\right)\right]dr \tag{31}$$

$$\Rightarrow\overline{\tau}_{FFR}(T_{FR},\lambda,\alpha,\Delta)=\int_0^{\infty}e^{-\pi\lambda_b r^2}\int_{t>0}2\pi\lambda_b r\mathbb{P}\left[(*)\right]drdt \tag{32}$$

where,

$$\mathbb{P}\left[(*)\right]=\mathbb{P}\left(\ln\left(1+\frac{P\hat{g}_y r^{-\alpha}}{\sigma^2+P\hat{I}_z}\right)>t\middle|\frac{Pg_y r^{-\alpha}}{\sigma^2+PI_z}<T_{FR}\right)$$
$$=\frac{\mathbb{P}\left[\ln\left(1+\frac{P\hat{g}_y r^{-\alpha}}{\sigma^2+P\hat{I}_z}\right)>t,\frac{Pg_y r^{-\alpha}}{\sigma^2+PI_z}<T_{FR}\right]}{\mathbb{P}\left(\frac{Pg_y r^{-\alpha}}{\sigma^2+PI_z}<T_{FR}\right)}$$

Following our regular procedure of simplification we get:

$$\mathbb{A}=\int_0^{\infty}2\pi\lambda_b r e^{-\pi\lambda_b r^2}e^{-\mu_b\frac{\sigma^2\theta}{P}r^{\alpha}}e^{-2\pi\lambda_{bac}^{dl}\int_r^{\infty}\frac{\theta}{\theta+(\frac{x}{r})^{\alpha}}xdx}e^{-2\pi\lambda_{bac}^{ul}\int_r^{\infty}\frac{\theta}{\theta+(\frac{x}{r})^{\alpha}\frac{\mu_b}{\mu_u}}xdx}dr \tag{33}$$

$$\mathbb{B} = \int_0^\infty 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{-2\pi\lambda_1\gamma_1 r^2} e^{-2\pi\lambda_2\gamma_2 r^2} e^{-\mu_b(\theta+T_{FR})\frac{\sigma^2 r^\alpha}{P}} dr \tag{34}$$

$$\mathbb{C} = \int_0^\infty 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{-\mu_b\frac{\sigma^2 T_{FR}}{P} r^\alpha} e^{-2\pi\lambda_1\int_r^\infty \frac{T_{FR}}{T_{FR}+(\frac{x}{r})^\alpha} xdx} e^{-2\pi\lambda_2\int_r^\infty \frac{T_{FR}}{T_{FR}+(\frac{x}{r})^\alpha\frac{\mu_b}{\mu_u}} xdx} dr \tag{35}$$

where,

$$\gamma_1 = \int_1^\infty \left[1 - \frac{1}{1+T_{FR}x^{-\alpha}}\left(1-\frac{1}{\Delta}\left(1-\frac{1}{1+\theta x^{-\alpha}}\right)\right)\right] xdx$$

$$\gamma_2 = \int_1^\infty \left[1 - \frac{1}{1+T_{FR}\frac{\mu_u}{\mu_b}x^{-\alpha}}\left(1-\frac{1}{\Delta}\left(1-\frac{1}{1+\theta\frac{\mu_u}{\mu_b}x^{-\alpha}}\right)\right)\right] xdx$$

$$\lambda_{bac}^{dl} = \frac{\rho_a\lambda_b A}{\Delta}$$

$$\lambda_{bac}^{ul} = \frac{\rho_a\lambda_b(1-A)}{\Delta}$$

$$\theta = e^t - 1$$

$$\lambda_1 = \rho_a\lambda_b A$$

$$\lambda_2 = \rho_a\lambda_b(1-A)$$

The numerator of our final rate expression is :

$$\Rightarrow \int_0^\infty 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{-\mu_b\frac{\sigma^2\theta}{P} r^\alpha} e^{-2\pi\lambda_{bac}^{dl}\int_r^\infty \frac{\theta}{\theta+(\frac{x}{r})^\alpha} xdx} e^{-2\pi\lambda_{bac}^{ul}\int_r^\infty \frac{\theta}{\theta+(\frac{x}{r})^\alpha\frac{\mu_b}{\mu_u}} xdx} dr$$
$$- \int_0^\infty 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{-2\pi\lambda_1\gamma_1 r^2} e^{-2\pi\lambda_2\gamma_2 r^2} e^{-\mu_b\left(e^t-1+T_{FR}\right)\frac{\sigma^2 r^\alpha}{P}} dr \tag{36}$$

and the denominator is :

$$1 - \int_0^\infty 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{-\mu_b\frac{\sigma^2 T_{FR}}{P} r^\alpha} e^{-2\pi\lambda_1\int_r^\infty \frac{T_{FR}}{T_{FR}+(\frac{x}{r})^\alpha} xdx} e^{-2\pi\lambda_2\int_r^\infty \frac{T_{FR}}{T_{FR}+(\frac{x}{r})^\alpha\frac{\mu_b}{\mu_u}} xdx} \tag{37}$$

Our final analytical closed form is obtained by integrating $\frac{\mathbb{A}-\mathbb{B}}{1-\mathbb{C}}$ over time

$$\Rightarrow \overline{\tau}_{FFR} = \int_{t>0} \frac{\mathbb{A}-\mathbb{B}}{1-\mathbb{C}} dt \tag{38}$$

Theorem 6(Soft Frequency Reuse, cell edge user): The coverage probability of a cell edge user in a SFR network with dynamic TDD is $\overline{\tau}_{SFR}(T_{FR}, \lambda, \alpha, \eta, \beta) =$

$$\int_{t>0} \frac{\mathbb{P} - \mathbb{Q}}{1 - \mathbb{R}} dt \tag{39}$$

Proof: The achievable rate is:

$$= \int_0^\infty e^{-\pi\lambda_b r^2} 2\pi\lambda_b r \mathbb{E}\left[\ln\left(1 + \frac{\beta P \hat{g_y} r^{-\alpha}}{\sigma^2 + P\eta \hat{I}_z}\right)\right] dr \tag{40}$$

$$\Rightarrow \overline{\tau}_{SFR}(T_{FR}, \lambda, \alpha, \eta, \beta) = \int_0^\infty e^{-\pi\lambda_b r^2} \int_{t>0} 2\pi\lambda_b r \mathbb{P}\left[(*)\right] dr dt \tag{41}$$

where,

$$\mathbb{P}\left[(*)\right] = \mathbb{P}\left(\ln\left(1 + \frac{\beta P \hat{g_y} r^{-\alpha}}{\sigma^2 + P\eta \hat{I}_z}\right) > t \middle| \frac{\beta P g_y r^{-\alpha}}{\sigma^2 + P I_z} < T_{FR}\right)$$

$$= \frac{\mathbb{P}\left[\ln\left(1 + \frac{\beta P \hat{g_y} r^{-\alpha}}{\sigma^2 + P\eta \hat{I}_z}\right) > t, \frac{\beta P g_y r^{-\alpha}}{\sigma^2 + P I_z} < T_{FR}\right]}{\mathbb{P}\left(\frac{\beta P g_y r^{-\alpha}}{\sigma^2 + P I_z} < T_{FR}\right)}$$

Following our regular procedure of simplification we get:

$$\mathbb{P} = \int_0^\infty 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{-\mu_b \frac{\sigma^2\theta}{\beta P} r^\alpha} e^{-2\pi\lambda_1 \int_r^\infty \frac{\theta}{\theta + \frac{\beta}{\eta}(\frac{x}{r})^\alpha} x dx} e^{-2\pi\lambda_2 \int_r^\infty \frac{\theta}{\theta + \frac{\beta}{\eta}(\frac{x}{r})^\alpha \frac{\mu_b}{\mu_u}} x dx} dr \tag{42}$$

$$\mathbb{Q} = \int_0^\infty 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{-2\pi\lambda_1\gamma_1 r^2} e^{-2\pi\lambda_2\gamma_2 r^2} e^{-\mu_b\left(\frac{\theta}{\beta} + T_{FR}\right)\frac{\sigma^2 r^\alpha}{P}} dr \tag{43}$$

$$\mathbb{R} = \int_0^\infty 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{-\mu_b \frac{\sigma^2 T_{FR}}{P} r^\alpha} e^{-2\pi\lambda_1 \int_r^\infty \frac{T_{FR}}{T_{FR} + \frac{1}{\eta}(\frac{x}{r})^\alpha} x dx} e^{-2\pi\lambda_2 \int_r^\infty \frac{T_{FR}}{T_{FR} + \frac{1}{\eta}(\frac{x}{r})^\alpha \frac{\mu_b}{\mu_u}} x dx} dr \tag{44}$$

where,

$$\gamma_1 = \int_1^{\infty} \left[1 - \frac{1}{1+T_{FR}x^{-\alpha}}\left(1-\frac{1}{\Delta}\left(1-\frac{1}{1+\theta x^{-\alpha}}\right)\right)\right] x dx$$

$$\gamma_2 = \int_1^{\infty} \left[1 - \frac{1}{1+T_{FR}\frac{\mu_u}{\mu_b}x^{-\alpha}}\left(1-\frac{1}{\Delta}\left(1-\frac{1}{1+\theta \frac{\mu_u}{\mu_b} x^{-\alpha}}\right)\right)\right] x dx$$

$$\theta = e^t - 1$$

$$\lambda_1 = \rho_a \lambda_b A$$

$$\lambda_2 = \rho_a \lambda_b (1-A)$$

The numerator of our final rate expression is :

$$\Rightarrow \int_0^{\infty} 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{-\mu_b \frac{\sigma^2\theta}{\beta P} r^{\alpha}} e^{-2\pi\lambda_1 \int_r^{\infty} \frac{\theta}{\theta+\frac{\beta}{\eta}\left(\frac{x}{r}\right)^{\alpha}} x dx} e^{-2\pi\lambda_2 \int_r^{\infty} \frac{\theta}{\theta+\frac{\beta}{\eta}\left(\frac{x}{r}\right)^{\alpha}\frac{\mu_b}{\mu_u}} x dx} dr$$
$$- \int_0^{\infty} 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{-2\pi\lambda_1\gamma_1 r^2} e^{-2\pi\lambda_2\gamma_2 r^2} e^{-\mu_b\left(\frac{\theta}{\beta}+T_{FR}\right)\frac{\sigma^2 r^{\alpha}}{P}} dr \quad (45)$$

and the denominator is :

$$1 - \int_0^{\infty} 2\pi\lambda_b r e^{-\pi\lambda_b r^2} e^{-\mu_b \frac{\sigma^2 T_{FR}}{P} r^{\alpha}} e^{-2\pi\lambda_1 \int_r^{\infty} \frac{T_{FR}}{T_{FR}+\frac{1}{\eta}\left(\frac{x}{r}\right)^{\alpha}} x dx} e^{-2\pi\lambda_2 \int_r^{\infty} \frac{T_{FR}}{T_{FR}+\frac{1}{\eta}\left(\frac{x}{r}\right)^{\alpha}\frac{\mu_b}{\mu_u}} x dx} dr \quad (46)$$

Our final analytical closed form is obtained by integrating $\frac{\mathbb{P}-\mathbb{Q}}{1-\mathbb{R}}$ over time

$$\Rightarrow \overline{\tau}_{SFR} = \int_{t>0} \frac{\mathbb{P}-\mathbb{Q}}{1-\mathbb{R}} dt \quad (47)$$

Average Energy Efficiency

Energy efficiency is defined as the number of information bits that can be reliably conveyed over the channel per-unit energy consumption. We have average energy efficiency of a typical BS or UE by[7]:

$$\text{AEE} = \mathbb{E}\left[\frac{Rate}{Power}\right] = \mathbb{E}\left[\frac{\ln(1+SINR)}{P}\right] \tag{48}$$

where P is the power consumption of BS or UE.

We do not exclusively derive the expressions for this as it is very apparent from the previous derivations of rate expression.

Conclusions

In this document we have investigated an analytically tractable model of coverage, rate and energy efficiency for dynamic TDD cellular networks that employ fractional frequency reuse techniques. The presented work is to study this model based on stochastic geometry in dynamic TDD cellular networks. Based on a stochastic geometry, we first derive the expressions for the coverage probability of a phantom cell network (which is a homogeneous network as macro's do not interfere with these) and then extend it for the average achievable rate.

Future Work

A instinctive extension of this work is to cover the cellular uplink. Comparison of the performance of these frequency techniques with the traditional frequency reuse to see the benefits and trade-offs for uplink and downlink will give us an insight to use the appropriate reuse technique for a model. A more cumbersome but utile model is to consider *open loop power control* (OLPC) being applied to users and base stations(phantom cells) in the D-TDD network and similarly derive the analytical expressions.

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