Low-Gain Controller Design and Consensus of Multi-Agent Systems

A Project Report

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ASWIN KOLLI

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THESIS CERTIFICATE

This is to certify that the thesis titled **Low-Gain Controller Design and Consensus of Multi-Agent Systems**, submitted by **ASWIN KOLLI**, to the Indian Institute of Technology, Madras, for the award of the degree of **Master of Technology**, is a bona fide record of the research work done by him under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Dr. Arun D. Mahindrakar
Research Guide
Associate Professor
Dept. of Electrical Engineering
IIT-Madras, 600 036

Place: Chennai

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Chennai *Aswin Kolli*

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ABBREVIATIONS

IITM	Indian Institute of Technology, Madras
RTFM	Read the Fine Manual
ARE	Algebraic Riccati Equation

NOTATION

$\mathbb{R}$	Set of Real Numbers
$\langle a, b \rangle$	Inner product of a and b
$\lambda(A)$	Eigenvalues of matrix A
$\equiv$	Identically equal to
$\|x\|$	norm of x

CHAPTER 1

Introduction to Low gain controller Design

1.1 Introduction

The design of a low gain controller typically involves forming a set of feedback laws that are parametrized by a scalar ϵ. The controller helps to stabilize linear systems under input saturation. The input saturation can be avoided by decreasing the value of the low-gain parameter ϵ.
There are currently three methods to design low-gain controllers each of which will be discussed in the following sections.The first method proposed was the eigenstructure assignment method. This method is considerably lengthy but results in a matrix polynomial that is parametrized in terms of ϵ . The Riccati equation approach results in a direction solution but it is required to decide the value of the low-gain parameter as an input. The recent parametric Lyapunov equation approach however seems to have the best of both methods and results in a function of ϵ.

The set of feedback laws represented by a gain matrix $(F(\epsilon))$, approaches zero as the parameter ϵ tends to zero and hence the name low-gain.

CHAPTER 2

Mathematical Preliminaries

Throughout our discussion we consider the following system

$$\dot{x} = Ax + Bu, x \in \mathbb{R}^n, u \in \mathbb{R}^m$$

2.1 Asymptotically Null Controllable with Bounded Controls

The above system is said to be ANCBC if

1. The pair (A,B) is stabilizable.
2. The matrix A has it's eigenvalues in the closed left-half s-plane.

2.2 Controllable canonical form

A matrix pair (A, B) is said to be in the controllable canonical form if they are of the following form

$$A = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & \cdots & -a_1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b \end{bmatrix}$$

2.3 Matrix Equations

The following is the Riccati equation that is to be solved for X

$$A^T X + XA - XBR^{-1}B^T X + Q = 0$$

The following is the Lyapunov equation that is to be solved for X

$$AX + XA^T + Q = 0$$

CHAPTER 3

Designing a Low-Gain Controller

Low-gain was primarily conceived to avoid some of the problems encountered during high-gain feedback. Low-gain feedback has achieved several objectives that high-gain feedback failed to achieve. Some of these include the control of linear systems subject to input magnitude/rate saturation((Z.Lin, 1997) and (Z.Lin, 1998*b*)),semi-global stabilization of minimum phase input-output linearizable nonlinear systems((Z.Lin and A.Saberi, 1993)

In this chapter we discuss three different methods to design a low-gain controller

3.1 Eigenstructure assignment

Consider a system of the form

$\dot{x} = Ax + Bu, x \in \mathbb{R}^n, u \in \mathbb{R}^m$

We now design a low gain controller for the above system in three steps.

1. We find non singular matrices T_s and T_1 such that A and B are transformed into the block diagonal control canonical form, i.e the diagonal elements are of the control canonical form. Let us call the diagonal elements as A_i and B_i respectively.
2. For each A_i and B_i let $F_i \in \mathbb{R}^{1 \times n_i}$ be the state feedback gain such that $\lambda(A_i + B_iF_i(\epsilon)) = -\epsilon + \lambda(A_i) \in \mathbb{C}^-, \epsilon \in (0, 1], i = 1, 2, \dots l$

3. We now define a matrix $K(\epsilon)$ where the diagonal elements of this matrix are F_i.
$F(\epsilon) = T_1 K(\epsilon) T_s^{-1}$

$$F(\epsilon) = T_1 \begin{bmatrix} F_1(\epsilon) & 0 & \cdots & 0 & 0 \\ 0 & F_2(\epsilon) & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & F_l(\epsilon) & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} T_s^{-1}$$

The control law is $u = F(\epsilon)x$.

3.2 Algebraic Riccati equation method

The ARE based method is carried out in two steps

1. We solve the following Riccati equation for a positive definite solution $P(\epsilon)$,
$A^T P + PA - PBB^T P + Q(\epsilon) = 0, \epsilon \in (0, 1]$.
where $Q(\epsilon)$ is a positive definite matrix for all $\epsilon \in (0, 1]$ and satisfies
$lim_{\epsilon \to 0} Q(\epsilon) = 0$.

2. Now we construct the control law as
$u = F(\epsilon)x$,
where $F(\epsilon) = -B^T P(\epsilon)$.

3.3 Lyapunov equation method

The Lyapunov equation method is a very recent development((Zhou *et al.*, 2008)) that has the benefits of both Eigenstructure assignment and the Riccati equation method. We will briefly discuss the derivation of this method.

Consider the linear system

$$\dot{x} = Ax + Bu, x \in \mathbb{R}^n, u \in \mathbb{R}^m$$

We define the following cost function

$$J(u) = \int_{t=0}^{\infty} u^T(t)Ru(t)dt \text{ where } R > 0.$$

The following is a result from (Zheng, 2002)

If (A,B) is stabilizable. Then $J(u)$ is minimized with

$$u^*(t) = -R^{-1}B^T P x(t)$$

where P is the solution of the ARE

$$(A + \frac{\gamma}{2})^T P + P(A + \frac{\gamma}{2}) - PBR^{-1}B^T P = 0.$$

This is the ARE corresponding to the "Minimal energy control with guaranteed convergence rate" problem (MECGCR). The solution to this problem is $u^*(t)$ where P is the solution to the Riccati equation above. Now pre-multiply and post-multiply the above Riccati equation with P^{-1} to obtain the following Lyapunov equation where $W = P^{-1}$

$$W(A + \frac{\lambda}{2}I)^T + (A + \frac{\lambda}{2}I)W = BR^{-1}B^T, \epsilon \in (0, 1]$$

We now iterate the steps of the Lyapunov equation method.We solve for the control law as follows

1. Solve the following Lyapunov equation for $W(\epsilon)$
   $W(A + \frac{\epsilon}{2}I)^T + (A + \frac{\epsilon}{2}I)W = BB^T, \epsilon \in (0, 1]$
2. We find the matrix $P(\epsilon)$
   $$P(\epsilon) = W^{-1}(\epsilon)$$
3. Finally the control law is obtained as
   $$u = F(\epsilon)x$$
   where $F(\epsilon) = -B^T P(\epsilon)$.

3.4 An application

In this section we solve an example with the three methods explained above. We recall that in order to use a low-gain controller, the system in question must be ANCBC.

Consider the following example

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & -2 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

3.4.1 Eigenstructure assignment

1. In order to apply the method we first compute $A + BF(\epsilon)$. Let us parametrize $F(\epsilon) = [a\ b\ c\ d]$.
   Then,
   $$A + BF(\epsilon) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1+a & b & -2+c & d \end{bmatrix}$$

2. Now we compute the characteristic equation of the matrix

3. For the RHS of the equation we first compute the eigenvalues of A which are $\{-j, -j, j, j\}$

4. Now we construct a polynomial with the roots as $\lambda(A) - \epsilon$

5. Finally we equate the polynomials obtained above and compare the degree of λ to obtain the vector F.
   The obtained control law is as follows
   $u = -[\epsilon^4 + 2\epsilon^2,\ 4\epsilon^3 + 4\epsilon,\ 6\epsilon^2,\ 4\epsilon]x$

3.4.2 Lyapunov equation method

1. First we calculate $A + \frac{\epsilon}{2}I$ which is

$$\begin{bmatrix} \frac{\epsilon}{2} & 1 & 0 & 0 \\ 0 & \frac{\epsilon}{2} & 1 & 0 \\ 0 & 0 & \frac{\epsilon}{2} & 1 \\ -1 & 0 & -2 & \frac{\epsilon}{2} \end{bmatrix}$$

2. Then we solve the following equation in Mathematica

$$W(A + \tfrac{\epsilon}{2}I)^T + (A + \tfrac{\epsilon}{2}I)W = BB^T$$

   to obtain $W(\epsilon)$ as

$$\begin{bmatrix} \frac{4\left(5\epsilon^2+4\right)}{\epsilon^3\left(\epsilon^2+4\right)^3} & -\frac{2\left(5\epsilon^2+4\right)}{\epsilon^2\left(\epsilon^2+4\right)^3} & \frac{4\left(\epsilon^4-\epsilon^2-4\right)}{\epsilon^3\left(\epsilon^2+4\right)^3} & \frac{-\epsilon^4+10\epsilon^2+24}{\epsilon^2\left(\epsilon^2+4\right)^3} \\ -\frac{2\left(5\epsilon^2+4\right)}{\epsilon^2\left(\epsilon^2+4\right)^3} & \frac{2\left(3\epsilon^4+6\epsilon^2+8\right)}{\epsilon^3\left(\epsilon^2+4\right)^3} & -\frac{3\epsilon^4+6\epsilon^2+8}{\epsilon^2\left(\epsilon^2+4\right)^3} & \frac{\epsilon^6-2\epsilon^4-12\epsilon^2-16}{\epsilon^3\left(\epsilon^2+4\right)^3} \\ \frac{4\left(\epsilon^4-\epsilon^2-4\right)}{\epsilon^3\left(\epsilon^2+4\right)^3} & -\frac{3\epsilon^4+6\epsilon^2+8}{\epsilon^2\left(\epsilon^2+4\right)^3} & \frac{2\left(\epsilon^6+4\epsilon^4+10\epsilon^2+8\right)}{\epsilon^3\left(\epsilon^2+4\right)^3} & -\frac{\epsilon^6+4\epsilon^4+10\epsilon^2+8}{\epsilon^2\left(\epsilon^2+4\right)^3} \\ \frac{-\epsilon^4+10\epsilon^2+24}{\epsilon^2\left(\epsilon^2+4\right)^3} & \frac{\epsilon^6-2\epsilon^4-12\epsilon^2-16}{\epsilon^3\left(\epsilon^2+4\right)^3} & -\frac{\epsilon^6+4\epsilon^4+10\epsilon^2+8}{\epsilon^2\left(\epsilon^2+4\right)^3} & \frac{\epsilon^8+8\epsilon^6+30\epsilon^4+44\epsilon^2+16}{\epsilon^3\left(\epsilon^2+4\right)^3} \end{bmatrix}$$

3. The inverse $P(\epsilon)$ is as follows

$$\begin{bmatrix} \epsilon\left(\epsilon^6+4\epsilon^4+6\epsilon^2+4\right) & \epsilon^2\left(3\epsilon^4+8\epsilon^2+6\right) & \epsilon\left(3\epsilon^4+6\epsilon^2+4\right) & \epsilon^2\left(\epsilon^2+2\right) \\ \epsilon^2\left(3\epsilon^4+8\epsilon^2+6\right) & 2\epsilon\left(5\epsilon^4+8\epsilon^2+2\right) & \epsilon^2\left(11\epsilon^2+10\right) & 4\epsilon\left(\epsilon^2+1\right) \\ \epsilon\left(3\epsilon^4+6\epsilon^2+4\right) & \epsilon^2\left(11\epsilon^2+10\right) & 2\epsilon\left(7\epsilon^2+2\right) & 6\epsilon^2 \\ \epsilon^2\left(\epsilon^2+2\right) & 4\epsilon\left(\epsilon^2+1\right) & 6\epsilon^2 & 4\epsilon \end{bmatrix}$$

4. Finally we calculate $u = -B^TP$ as $u = -[\epsilon^4 + 2\epsilon^2,\ 4\epsilon^3 + 4\epsilon,\ 6\epsilon^2,\ 4\epsilon]x$

3.4.3 Riccati equation method

1. In order to apply the Riccati equation method we need to use a $Q(\epsilon)$ such that Q is positive definite.
2. We consider the simplest positive definite matrix $Q = \epsilon I$
3. After solving the Riccati equation for P with $\epsilon = 0.1$ we obtain $u = -B^T P = -[0.0488 \;\; 1.3062 \;\; 0.6727 \;\; 1.2022]x$
4. An interesting point to note is that, since there is no unique $Q(\epsilon)$ the control law obtained in this method is different from the previously discussed methods

CHAPTER 4

Simulations

The following are the results of applying low-gain control on the system discussed earlier.

4.1 Eigenstructure assignment and Lyapunov equation method

Let us again consider the previous example

$\dot{x} = Ax + Bu, x \in \mathbb{R}^n, u \in \mathbb{R}^m$ where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & -2 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

We recall that we derived the input as follows

$$u = -[\epsilon^4 + 2\epsilon^2,\ 4\epsilon^3 + 4\epsilon,\ 6\epsilon^2,\ 4\epsilon]x$$

for both the Eigenstructure assignment method as well as the Lyapunov equation method. On applying the above feedback to the system we observe the following results. We can observe that the required input $u(t)$ decreases with decreasing ϵ while the state values $x(t)$ increase and take much longer to stabilize. Thus the low-gain controller helps in dealing with input saturation by reducing the value of ϵ.

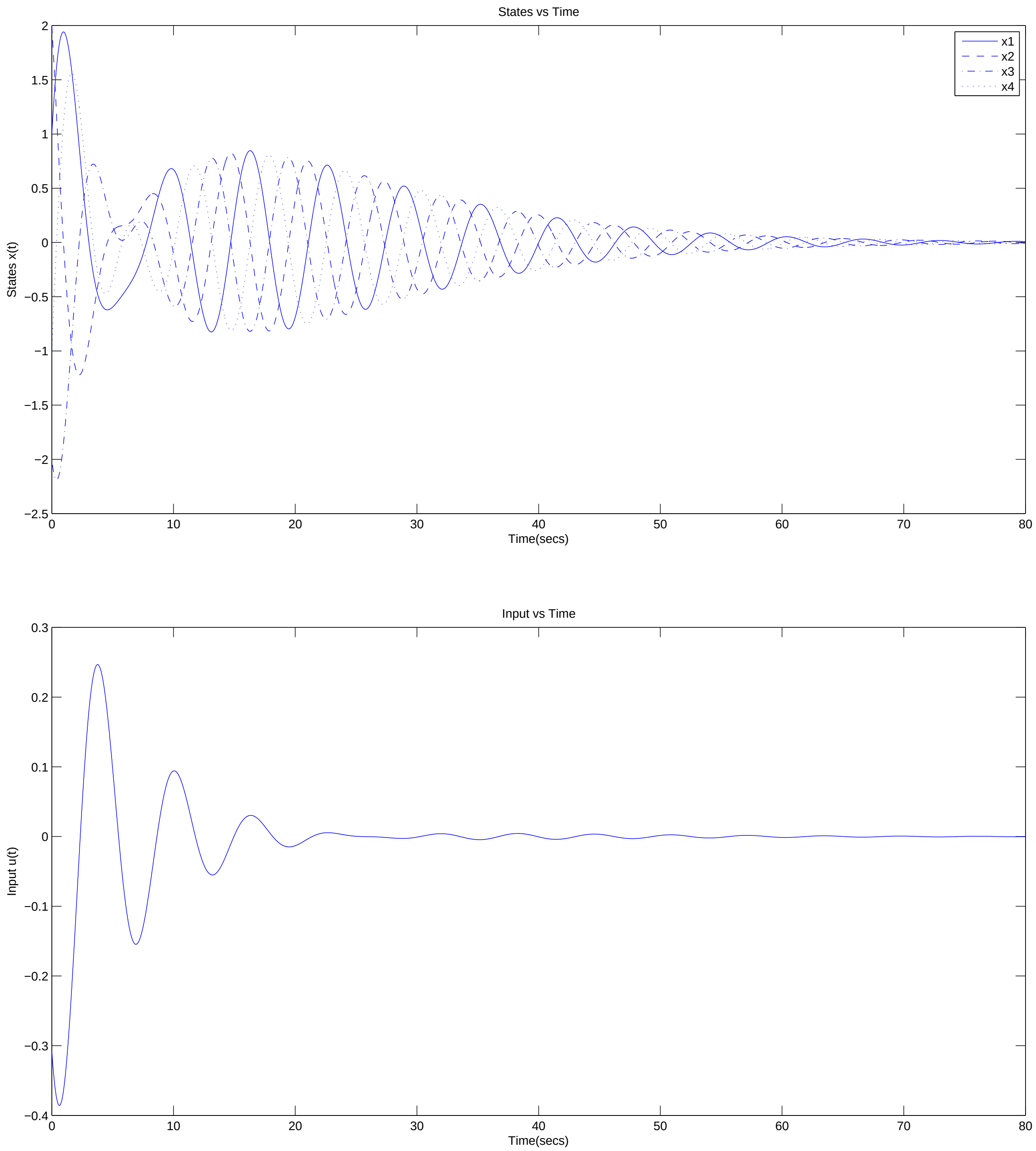

Figure 4.1: States x(t) and input u(t) for $\epsilon = 0.1$

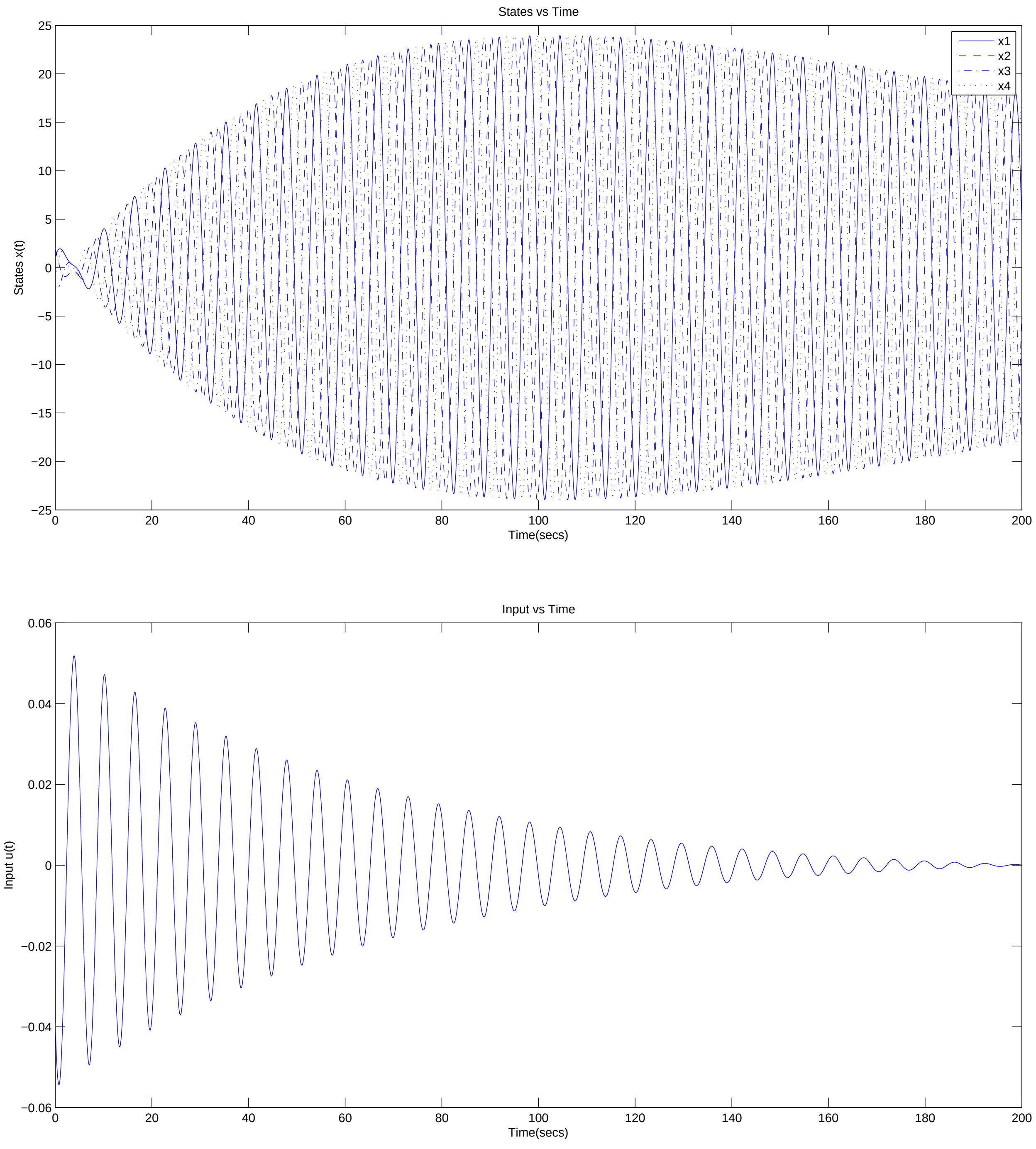

Figure 4.2: States x(t) and input u(t) for $\epsilon = 0.01$

Riccati equation Method

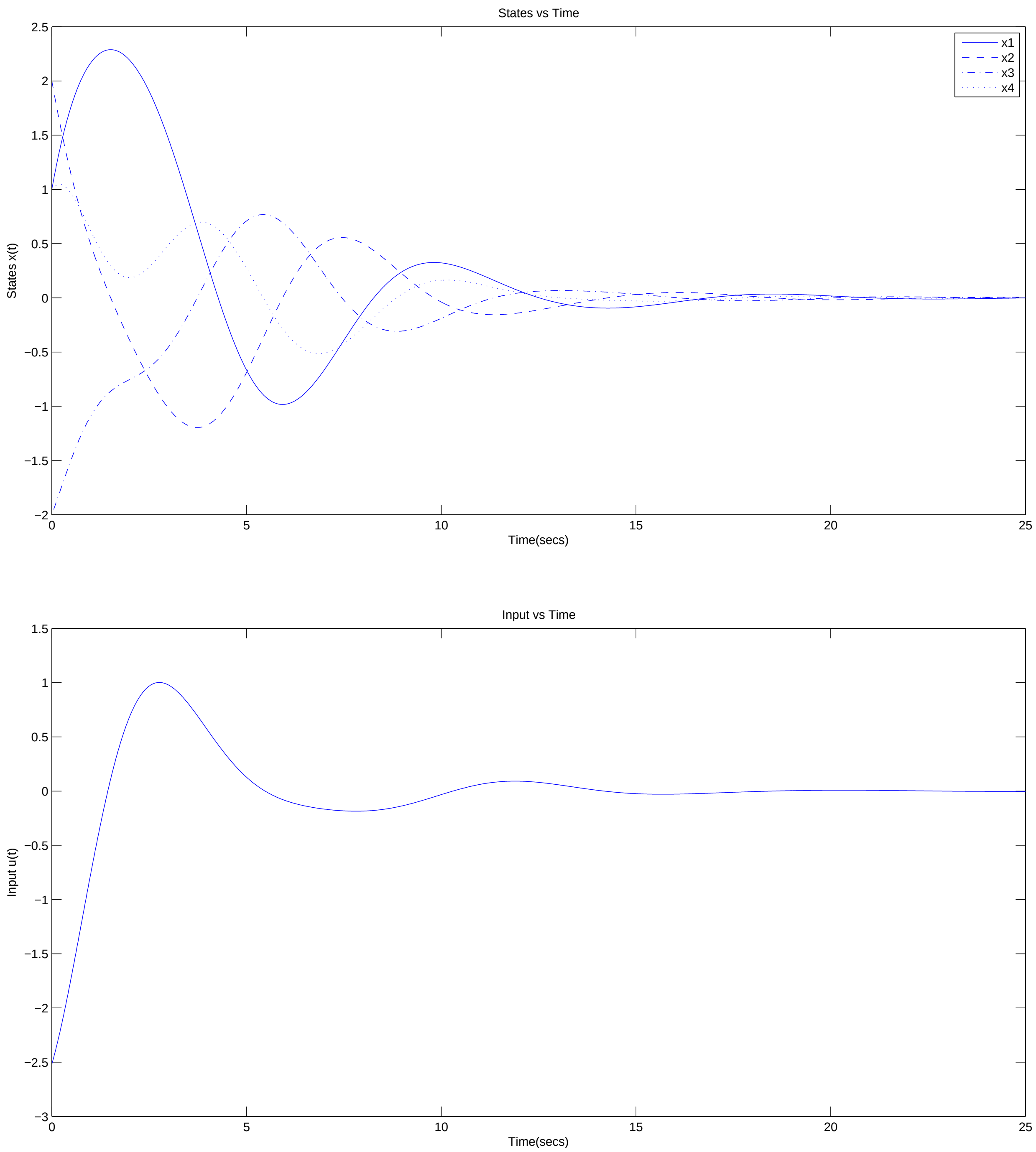

Figure 4.3: States x(t) and input u(t) for $\epsilon = 0.1$

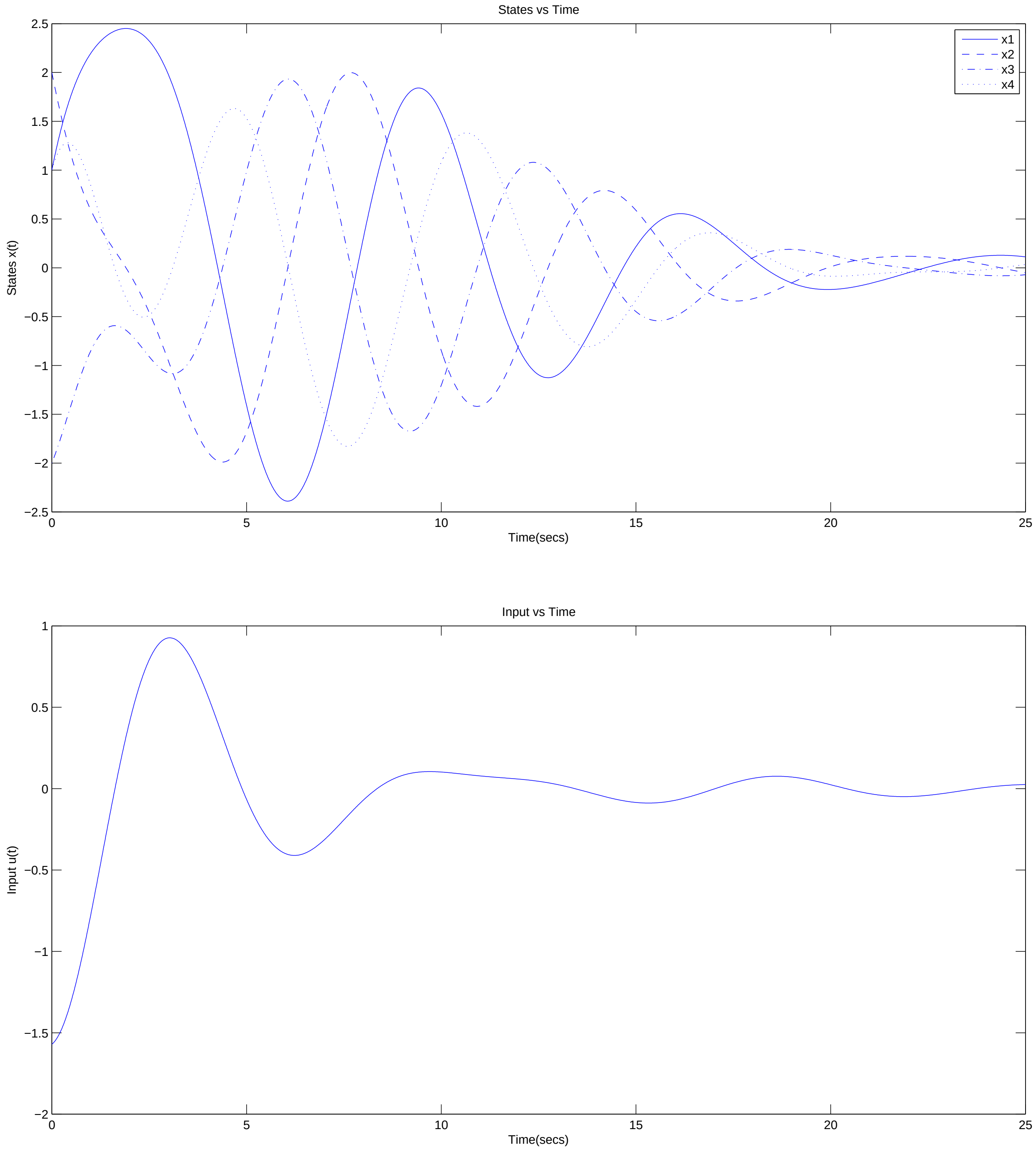

Figure 4.4: States x(t) and input u(t) for $\epsilon = 0.01$

CHAPTER 5

Consensus of Multi-Agent Systems

5.1 Introduction

Consensus of multi-agent systems primarily an agreement problem of multiple agents that are required to achieve a common goal while not violating constraints. Consensus was first introduced in (Fax and Murray, 2004) and (Olfati-Saber and Murray, 2004). There are several applications of consensus((Cao *et al.*, 2013)).The most common application we know of are military applications. Tanks or airships are required to move in formations in several situations. There are several approaches to this problem such as the leader-referenced technique or the center-referenced technique or even the neighbor-referenced technique. Each method has it's challenges, the most common of which are establishing and maintaining communication among the agents and ensuring collision avoidance.

5.2 Unified cooperative control of multiple agents on a sphere for different spherical patterns

5.2.1 Introduction

The co-operative control of agents on a sphere is achieved by the usage of state dependent repulsion coefficients to simulate the forces of attraction and repulsion between the agents. Control laws are designed to achieve one of three configurations: rendezvous, uniform deployment, formation using both first and second order models.

5.2.2 Problem description

We consider *n* agents moving in an N-dimensional Euclidean space. We wish to constrain the motion of the agents to the sphere S^{N-1} while achieving one of our 3 desired configurations. We denote the position of the agent *i* with respect to the origin as x_i. The radius of the sphere is r. Since the agent never leaves the sphere's surface we require that $x_i \equiv r$.

5.2.3 Agents on a sphere

The 3 configurations of multiple agents on a sphere are defined as follows, where x is the position vector of the agent :

1. Rendezvous
   The agents are said to rendezvous if

$\lim_{t\to\infty} \left\| x_i - x_j \right\| = 0, \lim_{t\to\infty} \dot{x}_i = 0 \;\; i,j = 1,2,\ldots n$
We define the average position vector of the agents as follows
$x := \frac{1}{n} \sum_{i=1}^{n} x_i$

2. Deployment
The agents are said to deploy if
$\lim_{t\to\infty} x = 0, \lim_{t\to\infty} \dot{x}_i = 0 \;\; i = 1,2,\ldots n$

3. Local Formation
The agents are said to deploy locally if
$\lim_{t\to\infty} \|x\| = r_o, lim_{t\to\infty} \dot{x}_i = 0 \;\; i = 1,2,\ldots n$

5.2.4 Control Laws

The goal of the following control laws is to ensure that the agents do not leave the surface of the sphere i.e $x_i \equiv r$ while also ensuring that the velocity vector is always perpendicular to the position vector, i.e $\langle \dot{x}_i, x_i \rangle \equiv 0$ In this section we will summarize the first and second order laws and briefly discuss the design.

First Order Model

$\dot{x}_i = u_i, \;\; i = 1, 2...n$

Here u_i is the control input for agent i. Since the agent is always on the surface of the sphere u_i is always perpendicular to x_i

$$\langle u_i, x_i \rangle \equiv 0$$

Second Order Model

Taking into account Lagrangian dynamics the following law is proposed:

$$\ddot{x}_i = \tau_i, \quad i = 1, 2...n$$

where we define τ_i as follows:

$$\tau_i = -\frac{\|\dot{x_i}\|^2}{r^2} x_i - k_v \dot{x}_i + u_i$$

where k_v is a damping constant. The above control law ensures that if the agent lies on the sphere initially, then it constrained to lie on the sphere for all $t > 0$, i.e.,

$$\|x\| \equiv r, \langle \dot{x}_i, x_i \rangle \equiv 0$$

Proof: Let us define $v := \frac{\|x\|^2 - r^2}{2} \in \mathbb{R}$, then we obtain $\dot{v} = \langle \dot{x}_i, x_i \rangle$

Let us now evaluate $\ddot{v}$. From the definition of τ_i, we obtain

$$\langle \ddot{x}_i, x_i \rangle = -\frac{\|\dot{x}_i\|^2 \|x_i\|^2}{r^2} - k_v \langle \dot{x}_i, x_i \rangle$$

We know that $v(0) = 0, \dot{v}(0) = 0$. Substituting these in the above expression we ascertain that $v \equiv 0$ which implies that the agent never leaves the surface of the sphere.
We can interpret the first term in τ_i as a centrifugal force, the second term as a linear damping and u_i as the force on agent i due to all other agents.

Control laws for both Models

We define the vector

$$d_{ij} = x_j - \frac{\langle x_i, x_j \rangle}{r^2} x_i.$$

This vector is analogous to distance between agents i and j. Similarly we define a force vector as follows:

$$f_{ij} = (k_a - \frac{k_r}{\|x_i - xj\|^2}) d_{ij}.$$

We now define the control input u_i as follows:

$u_i = \sum w_{ij} f_{ij}$ where $j \in Neighbours\ \ of\ \ i$ and $w_{ij} > 0$ when $j \in Neighbours\ \ of\ \ i$, else $w_{ij} = 0$ and $w_{ii} = 0$

5.2.5 Simulation results

Depending on the values of k_a and k_r we achieve either rendevouz or uniform deployment or formation (local deployment).

Rendevouz

First let us see the case of Rendevouz, this occurs only when the attraction coefficient k_a is non-zero and the repulsion coefficient k_r is zero. Below is the obtained final configuration when k_a is 1 and k_r is zero. The radius of the sphere $r = 1$. The resulting final radius of the position vector of convergence is unity as expected.

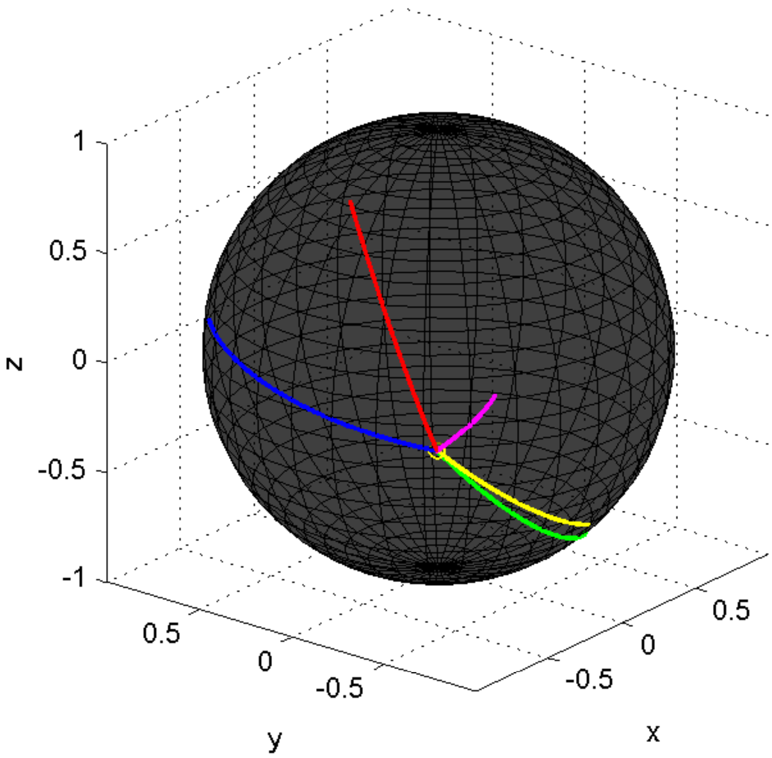

Figure 5.1: Rendevouz case for 5 agents, $k_a = 1,\ k_r = 0$

Deployment

In case of Deployment the agents repel each other and finally end up in such a way that the radius of the final average vector is zero. This happens is 2 cases. When k_a is zero and $k_r > 0$. Below is the obtained configuration for the $k_r = 1$. The obtained final radius of the average vector is 4.0901e-07 and is approximately zero.

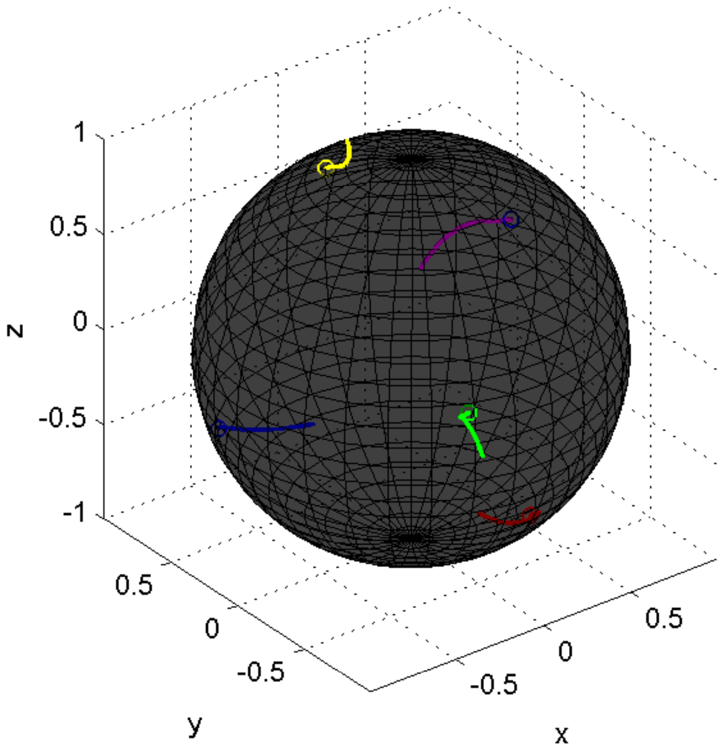

Figure 5.2: Deployment case for 5 agents, $k_a = 0,\ k_r = 1$

However there is one more case where deployment occurs. This is when $\kappa := \sqrt{k_r/k_a} \geq 2r$. Below is the case for $k_a = 1$, $k_r = 5$ The radius of the average vector is 6.133e-09.

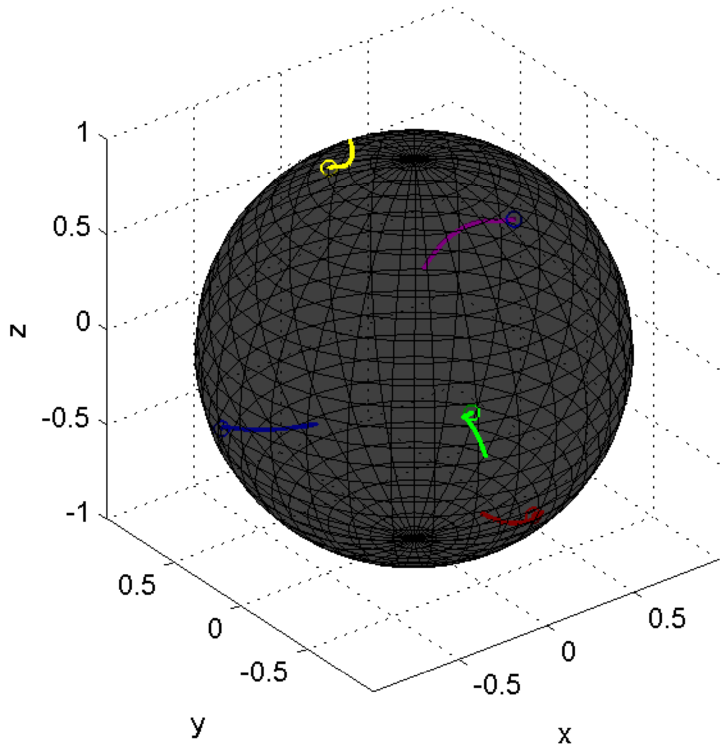

Figure 5.3: Deployment case for 5 agents, $k_a = 1,\ k_r = 5$

Local Deployment

When $\kappa < 2r$ then the agents deploy locally. The following case is for $k_a = 1$ and $k_r = 1$. The radius of the average vector is 0.7657.

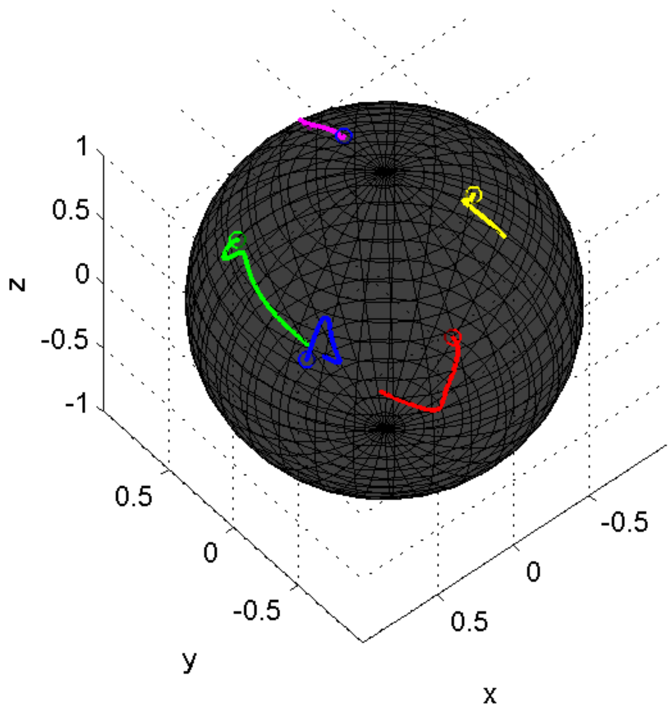

Figure 5.4: Deployment case for 5 agents, $k_a = 1,\ k_r = 1$

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