Electricity Price Forecasting using Mean Reverting Model

A Project Report Submitted in partial fulfilment of the requirements for the award of the degree of

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CERTIFICATE

This is to certify that the project report entitled "**Electricity Price Forecasting using Mean Reverting Model**" submitted by **Mr. Ranjit Nair (EE10B032)** is a Bonafide record of work carried out by him at Department of Electrical Engineering, Indian Institute of Technology Madras, in partial fulfilment for the award of degree of **BACHELOR OF TECHNOLOGY** in Electrical Engineering. The contents of this report have not been submitted and will not be submitted to any other Institute or University for the award of any degree or diploma.

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In the end I would like to thank the reviewers who have painstakingly gone through the thesis and helped me making this B.Tech. Project a success.

Contents

List of Abbreviations

ANN	Artificial Neural Network
AR	Autoregressive
ARIMA	Autoregressive Integrated Moving Average
DAM	Day Ahead Market
DR	Dynamic Regression
GARCH	General Auto Regressive Conditional Heteroscedastic
ISO	Independent System Operator
MA	Moving Average
MAPE	Mean Absolute Percentage Error
MCP	Market Clearing Price
MLP	Multi-Layer Perceptron
MLR	Multiple Linear Regression
PBUC	Price-based Unit Commitment
SMP	System Marginal Price
SVR	Support Vector Regression
UC	Unit Commitment

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Abstract

During the end of 20^{th} century, electricity markets around the globe have undergone massive restructuring. Earlier the task of generation, transmission and distribution was all handled by one entity but after deregulation these three areas have developed into separate markets with multiple players in each segment. With the advent number of generating companies and distribution companies, the electricity market has become a competitive market with electricity being traded like any other commodity like stocks, bonds etc. over a power exchange. Like any other traded commodity, price forecasting remains at the heart of research in the area of electricity prices.

Neural Networks seem to be the most accurate models in this area of electricity price forecasting but works very well only in the case of presence of good quality of predictive indicators. Since Indian electricity exchange (unlike foreign exchanges) doesn't provide such data, focus of the work has been to develop a fairly good forecasting technique by just using the price and the demand data so that it works well in the Indian scenario.

After the comprehensive review of the forecasting techniques used to predict the electricity prices, the mean reverting model proposed by Danielle et al. (2010) is examined due to the strong economic rationale underlying the model. This model is then subsequently extended by incorporating demand data to improve the accuracy of the forecast. Both the models are compared and finally results are presented in the end.

Chapter 1: Introduction

Just around a few decades back, electricity power sector was believed to be a natural monopoly sector in nature, that is, the most efficient way to operate in this sector was to control all three vertical operations of generation, transmission and distribution. In essence, before deregulation, electric power industry was heavily dominated by utilities that used to control each and every activity in the area. All this changed when deregulation of the power sector was first experimented in the Latin America in the late 80s. This experiment in the Latin America changed the power sector forever and for good, and slowly but steadily, all the countries around the world have been rapidly de-regularizing their power sector. In today's deregulated electricity market, the party that benefits the most is the end user, as they get an option to choose between various service providers, that is, electricity suppliers. This chapter aims to present the restructuring that this sector has undergone over the past few decades and how those changes have benefited the society as a whole. We'll also discuss the history and the process of deregulation of the power sector in India in detail. We'll then discuss how the **MCP** is determined. The concept of bidding in the electricity market and the issue of strategic bidding will be briefly touched upon as well. We'll then finally move on to the discussion of some important issues in a deregulated market such as electricity price forecasting, price volatility and uncertainty in price.

1.1 Motivation for Research

The electricity power industry worldwide has undergone a number of fundamental and structural changes after the early 80s. Before that, all the activities of the power sector such as generation, transmission and distribution, used to be controlled by large utility firms. In such a system there generally used to be only one service provider and the customers were forced to pay the price determined by the utilities. Unlike the regulated market that was heavily monopolistic in nature, deregulation leads to separation of generation, transmission and each area transforms into an independent industry. The rationale or the driving force behind the deregulation of a heavily monopolized industry was that competition will lead to more efficient utilization of resources. After the early attempts to deregulate the power sector in countries like Brazil, Australia, U.K. and other Scandinavian countries, pretty much every country in the world have already restructured, currently restructuring or plan to restructure their electricity market [1].

Unlike the earlier monopolistic scenario, in a deregulated market, a number of market players participate in the day to day market activities of the market and hence to know the supply/demand relationship ahead of the time is very crucial for all the market participants. Form this it follows that the supply/demand relationship and hence the electricity price must be pre-estimated before the real time price determination in order to maximize the profit. Hence in the current era of deregulated electricity market the need to precisely and accurately predict the electricity prices ahead of time has become a hot research issue in the area.

Another reason why electricity price forecasting plays such a key role in the new deregulated power industry, other than being useful for structuring optimal bidding strategies and correctly pricing the bilateral contracts, is that price of the electricity is one of the most important factors that's considered before deciding to set up a new generation unit as it (the price) determines the profitability of the new project. The reason electricity price forecasting is an important research area is because the electricity market is extremely volatile. Although some amount of volatility is embedded in each every commodity market but the degree of volatility mainly because the electricity supply and demand has to be balanced real time and storage of electricity, unlike other commodities, is very costly. Besides, there are a number of factors that influence the price of electricity and hence play a crucial role in determining the price volatility. Some of these factors include, among others, sudden changes in the weather, unexpected generation and transmission issues, changes in fuel price, availability of generation units, hydro generation production [2].

On the other hand, electricity price forecasts are uncertain because a number of determinants of price are not known well in advance, such as weather conditions, which affect the demand, and the rainfall in future, which determines the amount and availability of hydro-electric power generation. In [3], it’s been said uncertainty that the embedded uncertainty of price forecasting is relatively higher than the load forecasting as it is more complex in the sense that it requires forecasts of both the supply and the demand.

As a result of this very uncertain and volatile nature of electricity prices coupled with the fact that it's a commodity that the consumers need in their daily life makes the task of accurately forecasting the prices all the more important to all the market participants. Price forecasting can mean different things to different people but for an **ISO**, it is equivalent to determining the **MCP** which is not forecasting in the true sense as the **ISO** can numerically calculate the prices after receiving the bid/offers from the market players [4]. On the contrary, for a **GENCO** forecasting prices means predicting the **MCP** before submitting their bids without knowing the prices of its market opponents based on a very limited set of data such as forecasted weather, forecasted load and historical data.

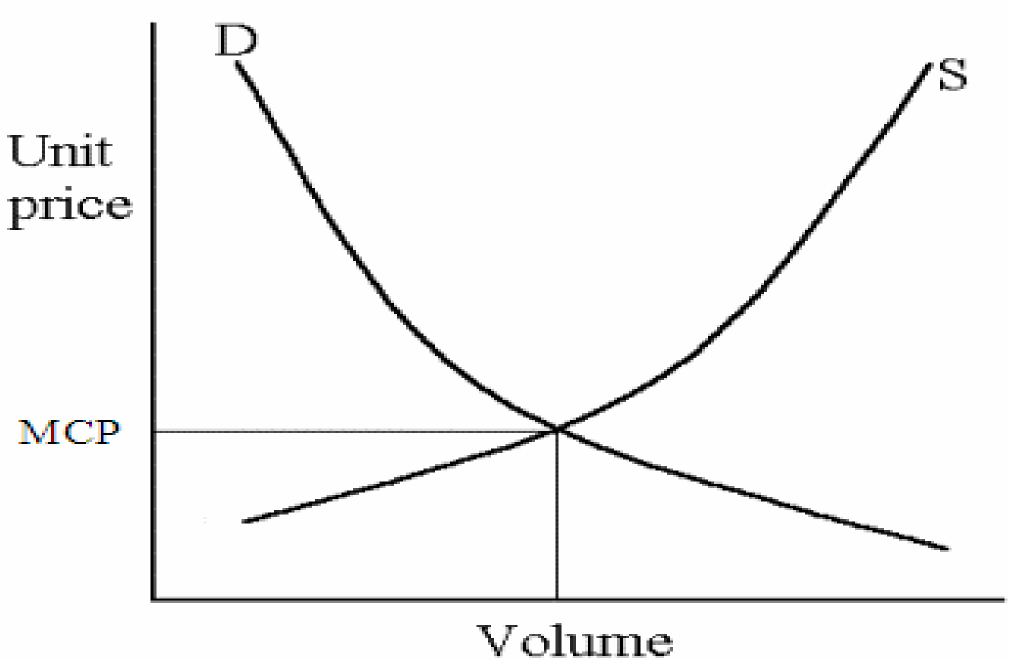

Figure 1.1: Calculating MCP Source 5

In a pool market, where buyers and sellers quote their bids and offers to a clearing house, the market players should submit their bid/offer quotes as close to the MCP. Otherwise, if the bid quote of the buyer is too low then it may not be able to purchase the require amount of electricity. On the other hand, if the offer quote of the seller is too low then it may not be able to sell all the electricity it produced [5]. Hence, in general, the market participants should submit quotes as close to the **MCP** as possible. The **MCP** for an electricity market can be determined by following the steps mentioned below. After all the market players submit their respective bid/offer quotes, all buy and sell orders are aggregated into two separate curves, a demand curve and a supply curve. The **MCP** is then determined by the intersection of the demand and the supply curves. In figure 1, for example, the **MCP** is the intersection of the aggregate supply curve and the aggregate demand curve.

Given the above complex nature of the new deregulated electricity market, this thesis addresses the core issue of electricity price forecasting and develops a model that is built specifically for developing countries like India where predictor data such as accurate weather (both actual and forecasted) data is either not available at many places or in case it's available, then it's very unreliable. Hence we'll discuss two models, one purely autoregressive mean-reverting model and the other an auto-regressive mean reverting model with demand as an exogenous variable. The issue of strategic bidding also will be discussed in the next chapter and we'll end by first describing, studying, comparing and analysing the two forecasting models and finally end by summarizing and concluding the results obtained.

1.2 Objective of the Work and Thesis Outline

Objective of this thesis is to develop a computationally efficient model that takes very minimal number of inputs, only price and the demand, that works well for the Indian electricity market.

In chapter 2, we'll study the issue of strategic bidding and how having an accurate price forecast plays a crucial role in developing an effective bidding strategy.

In chapter 3, we'll do a comprehensive review of existing literature on the electricity price forecasting.

In chapter 4, we'll describe the Autoregressive Mean Reverting model proposed by [6] for electricity price forecasting and then subsequently suggest an improvement in the model by proposing a new model that incorporates the demand data.

In chapter 5, we'll compare the two models and present the results.

Chapter 2: Strategic Bidding

2.1 Introduction

In the previous chapter, the various aspects of the new deregulated electricity markets were discussed in detail and the importance of accurate and precise price forecasts was also briefly touched upon. Price forecast is an important input tool in the decision making process of various market players for their day to day activities. For example, for **GENCO's** to maximize their profit, they need to have a goof estimate of the next day's price so that they can organize their generation scheduling and submit strategic bid accordingly. Different previous strategic bidding approaches along with the generation scheduling problem and **PBUC** is discussed in this chapter.

2.2 Price Based Unit Commitment

GENCOs are required to compete for supplying energy at the cheapest possible price and are required to provide the market operator with a number of quotes like offer price, ramp rates, minimum up and down time etc. The market operator then uses this information to conduct a price based unit commitment to optimize dispatch and settle the market and hence schedule the generation. The following figure shows the day to day market activities of a **GENCO** in a bilateral setup with an option of participating in the spot market. The figure below is also applicable to **GENCOs** in the pool market with the slight difference that the bilateral unit commitments have to be internalized while placing the bids [7].

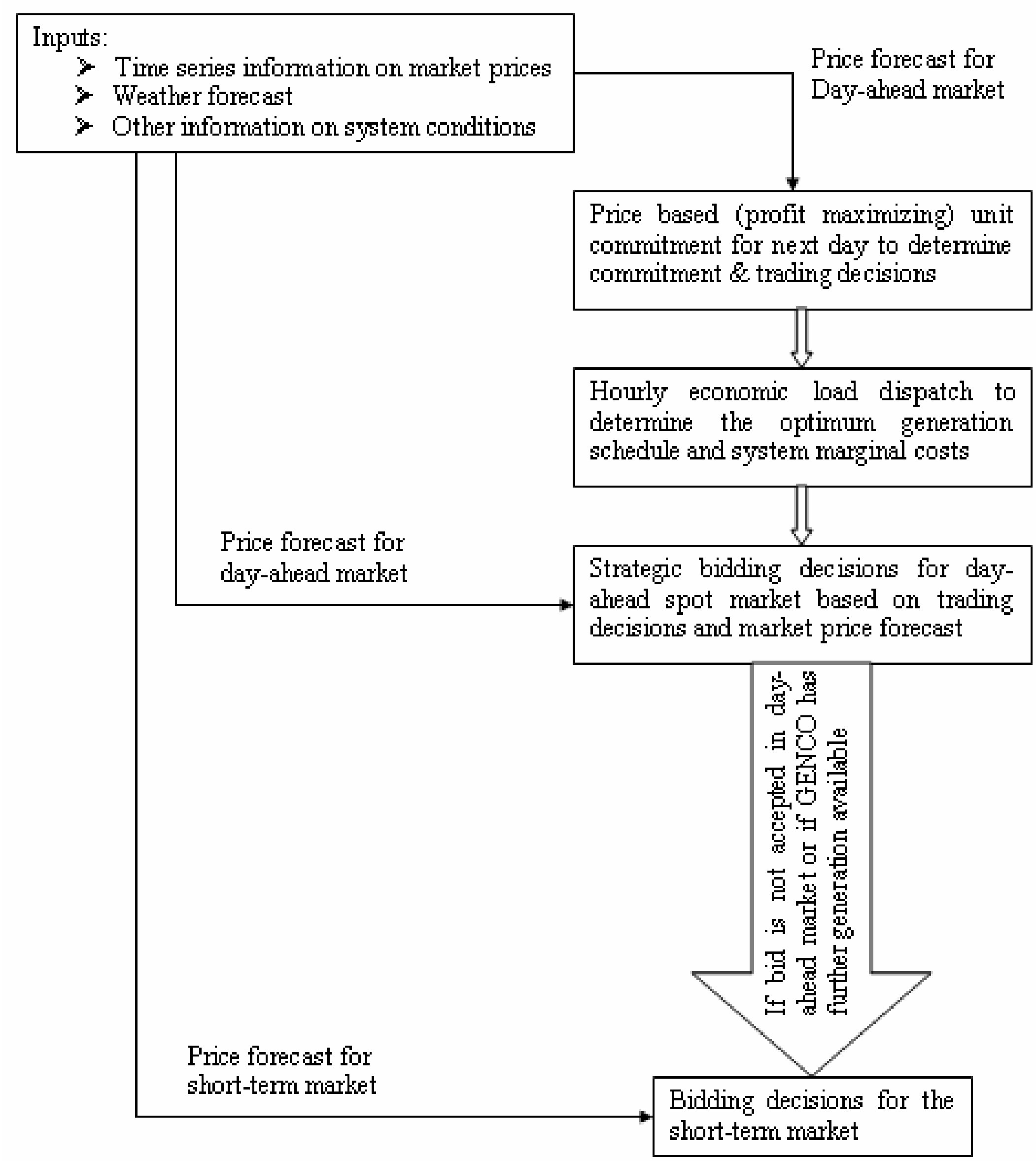

Figure 2.1: Short-term operational activities of GENCO's Source 7

Earlier when the utilities were vertically integrated, the unit commitment was simply scheduling of available generating units that run in such a way that total production cost of the electricity is kept at the minimum while satisfying all system and unit constraints. In the new deregulated market, however, unit commitment has a completely different objective of maximizing the operating profit. Hence, the optimization of all available generating units so as to maximize the profit is called as unit commitment. This simply yields to a strategy of trying to satisfy the demand with the cheapest available units and only when the demand is not met by cheap units the expensive units are operated, given adequate price. Once the day-ahead electricity price forecasts are available, the **GENCOs** can arrive at a generation schedule that helps them to maximize the profit based on the electricity price forecasts and the available generation units and their characteristics [7].

The objective function of the **GENCO** operating in the above mentioned way is given in [7] as:

$$Profit = Revenue\ from[\ spot\ market\ sale + biletaral\ market\ sale\ ]$$
$$- Cost\ for[\ spot\ market\ buy + startup\ costs + shutdown\ costs$$
$$+ unit\ operating\ costs\ ]$$

Mathematically this translates into:

$$Profit = \sum_{k} [\ \rho_{Mk} * PSell_k + BC_k * CP_k - \rho_{Mk} * PBuy_k$$
$$- \sum_{i=1}^{NG} W_{i,k} * CMin_i + PG_{i,k} * GCst_i + UST_{i,k} * ST_i + USD_{i,k} * SD_i$$

Where, K is the time index, $\rho_m$is the spot price, PSell and PBuy are decision variables denoting the quantity of power to be sold and purchased respectively from the spot electricity market. BC is bilateral contract power at contract price CP, $Cmin$ is the the generation cost at the minimum generation limit of unit $PMin$, $GCst$ is the generation cost beyond the minimum limit, ST and SD are unit start up and shutdown cost respectively. W, USD and UST are binary status variables denoting the unit status, unit shut down status and unit start up status respectively.

GENCOs have to take a number of complex issues into account while optimizing its generation to maximize profit and scheduling its generating units in order to meet its obligations the next trading day. These complex issues arise mainly due to the fact that the electricity prices in a deregulated market are extremely uncertain and that there are different operating constraints that the **GENCOs** have to satisfy while meeting its obligations. These constraints can be mathematically stated as follows:

1. Unit Generation Limits:

$$P_i^{min} \leq P_{i,k} * W_{i,k} \leq P_i^{max}$$

2. Unit Minimum ON/OFF Durations:

$$\sum_{n=1}^{MUT} V_{th,k-n+1} \leq 1; \qquad \forall \; k \geq MUT$$

$$\sum_{m=1}^{MDT} V_{th,k-m+1} \leq 1; \qquad \forall \; k \geq MDT$$

3. Unit Ramping Constraints:

$$P_{i,k} \leq RUP_i * P_{i,k-1} \qquad as\ unit\ ramps\ up$$

$$P_{i,k} \geq RDN_i * P_{i,k-1} \qquad as\ unit\ ramps\ down$$

4. Must-run:

$$W_{i,k} = 1; \qquad \forall \; i \in MR; k$$

Different methods to solve the unit constraint scheduling problems are discussed in [8]. Once the next day's unit constraint and buy and sell decisions are made using apt methodology, the subsequent stage is to formulate a optimum bidding strategy that maximizes some objective function (usually profit).

2.3 Strategic Bidding

Due to various reasons, the current market structure of the electricity power industry resembles and imperfect competitive industry. One of the main reasons for this is that consumers do not have full freedom when it comes to choosing their service provider sue to issues such as network transmission constraints. Since the electricity power industry is imperfect in nature, it gives the utility some degree of freedom to quote offer prices at a higher rate than their marginal cost of production in order to maximize their gain. This practice of quoting a price different from that of the marginal cost of production by the utility with the intention of maximizing or increasing the profit is called strategic bidding; and formulating an optimal bidding strategy (for profit maximization) is very crucial for a **GENCO** [9].

The bidding strategy of a **GENCO** depends upon or is affected by a number of factors such as load forecast, weather forecast, constraints on unit operation, availability of hydroelectricity, historical **MCP**. All this information together combined with a precise and accurate next day forecast of electricity prices form the input parameters required to formulate an optimal bidding strategy. Coming up with successful bidding strategies is nothing but understanding the complex interaction between technical aspects of unit operations, market uncertainties as well as the economic interests of the **GENCO**.

Over the years, a number of bidding strategies have been proposed by the researchers. All these bidding models/strategies can be essentially grouped into three categories. The first is relatively straightforward and it essentially involves generating an accurate forecast of the day-ahead **MCPs** and then the offer price is quoted slightly below the estimated the **MCP** in order to sell all the generated electricity at the maximum possible price. Game theory based approach are another popular way of modelling bidding strategies. The game theoretic approach towards formulating an optimal bidding strategy hinges on the fact that market players react to the opponents move/strategies in order to maximize their own profit [10]. In [11], the optimal bidding strategy was developed using a game theoretic approach by modelling the competition among the market participants of the electricity pool market as a non-cooperative game. Under this framework, it was assumed that all the market participants had only partial knowledge of the game and hence a game of complete is formulated with incomplete information and the game was finally solved using the well-known Nash Equilibrium. The third category of bidding strategy formulation methodology is based on estimation the competitors bidding behaviour by using various data points such as their past bidding strategies. Most of the methods for estimating the bidding behaviour of the opponents are probability analysis, fuzzy sets and neural network.

Irrespective of the method employed, the underlying objective behind the implementation of strategic bidding remains the same, which is, maximizing profit. Hence, once the price forecasts for the next day are estimated and the generation scheduling is arrived at in accordance with it, the **GENCO** can start formulating its optimal bidding strategy using day-ahead price forecasts and the generation schedule as the inputs and subsequently submit its offer quote.

Chapter 3: Comprehensive Review of Price Forecasting

3.1 Introduction

We saw in detail as to how the electricity power industry that used to be vertically integrated and monopolistic in nature, where all the three tasks of generation, transmission and distribution were undertaken by the same entity, slowly and gradually transformed into a competitive market. This phenomenal structural transformation of the power industry provided a new window of opportunity to a number of people like economists, researchers, commodity traders etc. Economists for example are interested in assessing the overall impact of the new and evolving industry on the entire economy of the country. Researchers in this area work towards solving problems in the new industry, for example, electricity price forecasting. Commodity traders wish to develop models that mimic the commodity prices in order to understand the commodity price behaviour better and then use these models to price complex and exotic commodity derivatives.

Electricity price forecasting has been increasingly becoming more and more important to each and every market player, but this wasn't the case before the deregulation. Before deregulation, the price forecasting was only done by the utilities and price forecasting to them meant computing the operating cost of the utility and then forecasting the prices of the components that made up the operating cost [3]. These utilities then used to set tariff based on the total average cost of all the operation combined by adding an appropriate profit margin to the cost and since there was no open market to trade electricity like today, the consumers had no choice but to pay up if they wished to purchase the required amount of electricity.

We've already said that each and every market participant needs an accurate electricity price forecasts for different reasons. For example, load distribution companies need to know how much their consumers will consume and at what time and then they'll try to meet those needs need at the lowest possible price by entering into a number of short term and long term contracts and how the price variations across different regions affect the demand on transmission network will be something that a transmission company will be interested in. Similarly, large industrial customers for whom electricity price is an important input cost for production, will be interested in measuring their market risk exposure to electricity price and will seek to hedge their risks through derivative contracts, time of use rates etc.

Since there are so many interested parties in forecasting electricity prices and there are multiple application for it, this area has attracted a number of researchers, scientists, modellers across the world in the previous 20-25 years. Since it's a very new area of research, quite a few forecasting methods have only been published very recently. These forecasting techniques mainly follow either one of the two main approaches. The first approach to electricity price forecasting is simulation based which require a lot of system based information and are generally used only by the market operator and the power utilities and since it requires very deep insight into the system operation these techniques are not very practical for the market players. The second approach relies on constructing a mathematical model and then forecasting prices using the historical market data [2, 12]. In this thesis, `the time-series analysis based approach has been chosen to forecast electricity prices for the Indian market. In this chapter the various different approaches to solve the problem of forecasting the electricity prices have been covered in detail. Before we delve into various approaches we'll briefly describe what a time-series is and what its various applications are.

A time-series is nothing but a chronologically ordered set of data points that are uniformly spaced in time such as daily stock market closing price or hourly temperature. Time series occurs naturally in a number of fields and their analysis has a wide range of application in areas such as social network analysis, process control, stock market, economic forecasting, population studies, marketing etc. A time-series might be analysed due to a number of objectives but one of the important application of time series analysis is to predict the future value of the time series by using the historical or the observed value of the time-series. This is quite important in the analysis of industrial and economic time series [13]. The time-series models can be again divided into two types. First being the linear time-series models such as autoregressive, moving average, **ARIMA**, linear regression and dynamic regression models. Second being the non-linear time series models such as **GARCH**, **ANN** and **SVM** models.

3.2 Linear Time Series Model

3.2.1 Autoregressive Models

One of the most important and effective approaches in the field of statistical time series modelling are the Autoregressive (**AR**) processes and the Moving Average (**MA**) processes. Autoregressive process is one in which the future state or the value of the response variable say Z_t, is dependent on its past values Z_{t-1}, Z_{t-2},….,Z_{t-p}. Mathematically an **AR** process can be represented as follows:

$$Z_t = C + \Phi_1 Z_{t-1} + \Phi_2 Z_{t-2} + \cdots + \Phi_p Z_{t-p} + \varepsilon_t$$

Where: Φ_1, Φ_2, Φ_3,….,Φ_p are the coefficients and ε_t is the forecast error and C is a constant.

This is an **AR** model of degree p as the variable Z is dependent on its p lagged values. The above formula can be rewritten concisely as:

$$Z_t = C + \sum_{i=1}^{p} \Phi_i Z_i + \varepsilon_t$$

3.2.2 Moving Average Models

Moving Average (**MA**) is also one of the most popular techniques used in time series analysis. The new value is found by taking the average of previous values. As the time series progresses, the new observation or information is added to the average and the oldest observation is discarded and as a result, the average moves and thereby giving the process the name "*moving average*." Mathematically, **MA** model is defined as:

$$Z_t = \mu + \varepsilon_t + \sum_{i=1}^{p} \Phi_i \varepsilon_{t-i}$$

Where: Φ_1, Φ_2, Φ_3,….,Φ_p are the model parameters and ε_t is the forecast error and μ is the series average.

3.2.3 ARIMA

On integrating **AR** and **MA** models we get Autoregressive Moving Average (**ARMA**) models which are among the most useful time-series models that are used to understand a time series data or for forecasting a time series. As said earlier, these models are formed by the combination of **AR** and **MA** models. An ARMA (*p, q*) can be mathematically expressed as

$$Z_t = C + \sum_{i=1}^{p} \Phi_i Z_i + \varepsilon_t + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}$$

Where: C, Φ_i and θ_j are the model parameters and ε_t is the forecast error.

Most naturally occurring models are non-stationary in the nature and the model described above is a stationary model. So in order to fit a non-stationary time series to a stationary model like the one above, the concerned time series can be differenced in order to make it stationary. Another way to tackle the problem of non-stationarity is to integrate the **ARMA** (p, q) process. By integrating the **ARMA** (p, q) process to the d_{th} order we get a model that can explain the non-stationary time-series. This model is called **ARIMA** (p, d, q) model where d is a positive integer [14]. **ARIMA** (p, d, q) models have been very extensively used to forecast loads but quite recently have also been used to forecast the electricity prices. Contreras *et al.* [15] have proposed **ARIMA** models to predict day-ahead electricity market prices for the Spanish and Californian markets. These models were tested both in the Spanish and the Californian markets and Average Mean Weekly Errors (**MWE**) of around 5% in the stable period of the Californian market and10% in the Spanish market are reported.

3.2.4 Regression Analysis

One of the problems that commonly occur in science and engineering is exploring the relationship between two or more variables. One of the statistical techniques that work well for such kind of problem is regression analysis and has wide applications in areas such as process control or forecasting. In regression analysis, the dependent variable is modelled in terms of the independent variables, constant and the error term in a regression equation. The performance of the regression models depends heavily upon the estimate of the model parameters [16]. Now we'll see some different kind of regression models in the following sections.

3.2.4.1 Simple Linear Regression Models

A simple linear regression model considers a single independent variable Y and a single predictor variable X. For each given value of X, Y is a random variable but the expected value of Y is assumed to be linearly dependent on dependent variable X. The model is mathematically represented as:

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

Where, the y intercept β_0 and the slope β_1 are the regression coefficients and ε_i is the random error.

3.2.4.2 Multiple Linear Regression Models

As mentioned in the previous section, if the response variable is dependent only on one independent variable then the regression is called simple linear regression but is the response variable is affected by more than one independent variable then the regression model is called multiple linear regression model. In general, a dependent variable Y may depend on k independent variables. The mathematical form of multiple regression model is:

$$Y = \beta_0 + \sum_{i=1}^{k} \beta_i X_i + \varepsilon$$

This method basically tries to best fit the observed data on a straight line determined by the model parameters. The coefficients can be estimated by applying the least squares method. In the present thesis, the multiple regression approach is used to estimate the parameters of the mean-reverting model used to predict electricity prices in the Indian power market.

3.2.4.3 Dynamic Regression Models

The previous two regression approaches, that is linear and multiple regression, are generally called traditional regressions. Traditional regressions might not fit the data very well if the concerned data has irregular or cyclical components and in such cases the dynamic regression is a good alternative option [17]. The relationship between the dependent variable and the independent variables is expressed in terms of a constant C, a transfer function f and a disturbance term N_t as follows:

$$Y_t = C + f\left(X_{1,t}, \dots, X_{n,t}\right) + N_t$$

The paper [2] uses the **DR** model to forecast the day-ahead electricity prices in the Ontario market with a weekly **MAPE** of around 15%. Considering the very volatile nature of the Ontario markets, this result is termed acceptable.

3.3 Non-Linear Time Series Models

3.3.1 GARCH Models

GARCH stands for Generalized Auto Regressive Conditional Heteroskedastic, and are very widely used to model time series in the field of engineering and statistics. Methods like **ARMA**, **ARIMA** etc. assume homoskedasticity, that is, the variance of the time series remains constant throughout. **GARCH** addresses this problem of homoskedasticity very well and hence can be used for predicting electricity prices as electricity price variance is known to be strongly dependent on its own past variance [18].

The general **GARCH** for is mathematically represented as:

$$Z_t = C + \sum_{i=1}^{p} \alpha_i Z_{t-i} + \sum_{I=1}^{Q} \beta_i \varepsilon_{t-i}^2$$

Where ε_t is a white noise and $\varepsilon_t^2 = v_t^2 Z_t$ and $Var(v_t) = 1$

GARCH models have been used in [19] for forecasting electricity prices in the Spanish and Californian markets with a forecast error of about 9%. The authors further claim that the **GARCH** models outperform **ARIMA** models in the presence of volatility and price spikes.

3.3.2 Artificial Neural Network (ANN) Models

In earlier days, Artificial Neural Network (ANN) based models have been found to be the most popular for load forecasting applications [14]. Likewise, ANNs appeared to be among the first techniques to emerge in the price prediction concept as well.

To recall the basics, ANN works in the same way as a human brain does. It is composed of highly interconnected neurons working in unison to accomplish a specific task. Neural networks learn by example and relate a set of input variables to a set of output variables . ANNs can be classified by their architecture, processing and training. Whereas, architecture describes the neural connections, processing describes how networks produce output for every input and weight. On the other hand, training algorism describes how ANN adapts its weight for every training vector [5]. A neural network architecture consists of three parts; input layer, hidden layers and output layers. The multilayer perceptron feedforward network trained by backpropagation is the most popular class in ANN studies and is discussed in [20] in detail. This class of network consists of an input layer, a number of hidden layers and an output layer as shown in figure 3 below.

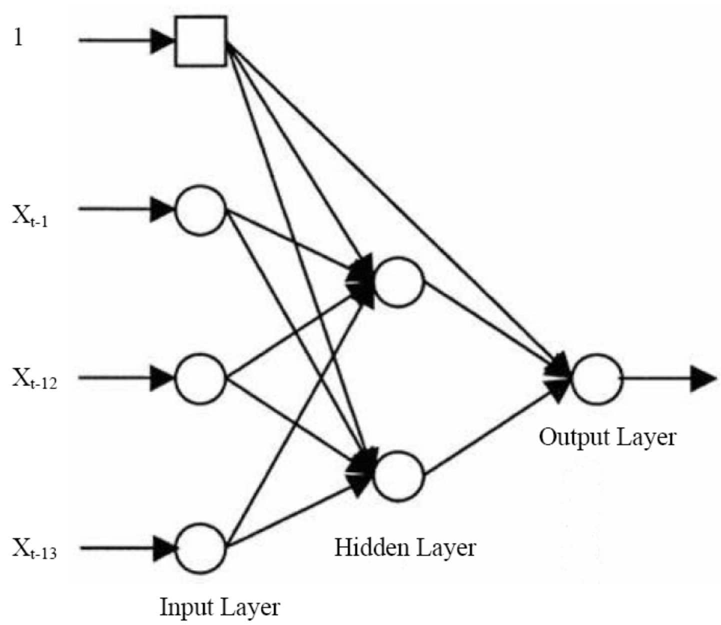

Figure 3.1: Multilayer Perceptron Source: 20

Returning back to the discussion of ANN in electricity price forecasting, a paper presented by [21] was one of the earlier price prediction methods that used Artificial Neural Networks to predict System Marginal Price (SMP) at each settlement period on the next scheduling day in the UK Pool. A daily average MAPE of 10.39% (for Friday being the minimum error) and 18.06% (for Thursday being the maximum error) is reported.

3.3.3 Support Vector Machines (SVM) Models

SVM is among the latest price forecast reports [12]. To deal with nonlinear price forecast problems which cannot be well captured by linear time series approaches, SVM is claimed to have good performance. SVM is based on the framework of statistical learning theory and it aims to minimize the structural risk, instead of the usual empirical risk, by minimizing an upper bound of the generalization error. Being suitable especially for solving problems of small sample size, SVM has already been used for classification, regression and time series predictions. In [12] a flexible C_i Support Vector Regression (SVR) model for price forecast is proposed. SVR is to map the input data x into a higher dimensional feature space through a nonlinear mapping Φ and then a linear regression problem is obtained and solved in this feature space.

Results achieved by this model for the Spanish market of the year 2000 are compared with results found using ARIMA models proposed by [15] and it is concluded that this model gives a better result; having a yearly average MAPE of 8.85% for SVR and about 10.57% for the ARIMA model

Chapter 4: Model Development Using Mean Reverting Stochastic Process

In [6], the authors propose a mean reverting process for electricity price forecasting that uses only price information to make the forecast. Their rationale for suggesting a mean reverting process is that say due to sudden fault or outage the supply reduces and demand remains the same, the prices will sky rocket but most market practitioners will not expect the prices to sustain at such a high level and will expect them to come down.

Mathematically the process of mean reversion is captured as follows:

$$S_{t+1} - S_t = \alpha(S^* - S_t) + \sigma\varepsilon_t$$

Where:

$S_{t+1} - S_t$ is the expected price change from t to t+1;

$\alpha(S^* - S_t)$ is the mean reversion component;

$\sigma\varepsilon_t$ is the random component;

S* is the mean reversion level or long run equilibrium price;

S_t is the spot price;

α is the mean reversion rate;

σ is the volatility;

ε is the random shock to price from t to t+ I;

Now in Indian electricity markets, the electricity is traded for every 15 minute block. That means in each day there are a total of 96 different electricity prices that one has to predict. In the concerned paper, the authors apply the above model separately to each trading block. So the above model becomes:

$$P_{ti} = P_{(t-1)i} + \alpha_{ti}\left(P_{ti}^{*} - P_{(t-1)i}\right) + \delta_{ti}\varepsilon_{ti}$$

Where:

P_{ti}^{*} is the long run equilibrium price;

$P_{(t-1)i}$ is the spot price in the i_{th} block;

α_{ti} is the mean reversion rate;

δ_{ti} is the volatility;

ε_{ti} is the random price shock from t-1 to 1.

The parameters can be found by following the below mentioned steps in excel.

1. For each trading block i, prices varying with days during a given time interval preceding the planning period are considered.
2. Then absolute daily changes are computed.
3. Linear regression is run treating price changes as the dependent variables and the price level as the independent variable. As the output of the linear regression, you get intercept, slope and residual standard deviation.

The mean reversion rate will be nothing but the negative of the slope and the long run equilibrium price will be intercept divided by the mean reversion rate and the residual standard deviation divided by the long run equilibrium price is the percentage volatility. Let's call this model Autoregressive Mean Reverting (**ARMR**) model.

This thesis proposes a slight modification to the above forecast model by incorporating the demand data to improve the accuracy of the forecast as there seems to be a very high correlation between demand and the prices, around 70% to 80%. Demand directly influences the prices and just like the prices, the demand too experiences mean reversion and hence we try to exploit the mean reversion characteristic of the demand to improve the accuracy of the forecast.

Just like in the previous case, each one of the 96 trading block is dealt with separately. The model that this paper proposes is:

$$P_{ti} = P_{(t-1)i} + \alpha_{ti}\left(P^*_{ti} - P_{(t-1)i}\right) + \beta_{ti}(D^*_{ti} - D_{(t-1)i}) + \delta_{ti}\varepsilon_{ti}$$

Where:

$P_{ti} - P_{(t-1)i}$ is the expected price change from t to t+1;

$\alpha_{ti}\left(P^*_{ti} - P_{(t-1)i}\right)$ is the mean reversion component from price;

$\beta_{ti}(D^*_{ti} - D_{(t-1)i})$ is the mean reversion component from demand;

$\delta_{ti}\varepsilon_{ti}$ is the random component;

P^*_{ti} is the mean reversion level or long run equilibrium price;

D^*_{ti} is the mean reversion level or long run equilibrium demand;

$P_{(t-1)i}$ is the spot price;

$D_{(t-1)i}$ is the spot demand;

α_{ti} is the mean reversion rate for price;

β_{ti} is the mean reversion rate for demand;

δ_{ti} is the volatility;

ε_{ti} is the random shock to price from t to t+ I;

In the above equation, we need to estimate four parameters α_{ti}, β_{ti}, P^*_{ti}, D^*_{ti} in order to forecast the price change. We can rewrite the above equation so as to reduce the number of parameters to be estimated and hence increasing the degree of freedom and hence improved accuracy in forecasting. We can rewrite the above equation as:

$$P_{ti} - P_{(t-1)i} = \alpha_{ti}P^*_{ti} + \beta_{ti}D^*_{ti} - \alpha_{ti}P_{(t-1)i} - \beta_{ti}D_{(t-1)i} + \delta_{ti}\varepsilon_{ti}$$

Let us say that $C = \alpha_{ti}P^*_{ti} + \beta_{ti}D^*_{ti}$

The above equation becomes:

$$P_{ti} - P_{(t-1)i} = C - \alpha_{ti}P_{(t-1)i} - \beta_{ti}D_{(t-1)i} + \delta_{ti}\varepsilon_{ti}$$

Now the complex mean reversion model turns into a simple two variable linear regression problem and be regressing the change in price in a particular trading block on the past data of demand and price we'll obtain the estimates for $C, \alpha_{ti}, \beta_{ti}, \delta_{ti}$. By substituting these values in the previous equation along with the demand data and the price data we'll obtain the expected change in the price and by adding that to the today's price, we'll obtain the day ahead electricity price forecast.

Let's call this model Autoregressive Mean Reverting Model with Exogenous Variable (**ARMRX**) model. In the next section we'll discuss the relative performance of the two models on the Indian electricity market. Data for the simulation has been obtained by the www.iexindia.com for the A1 pool.

Chapter 5: Results and Discussions

The two models mentioned in the previous section are finally applied from 10th February 2014 to 16th February 2014 (1 Week) and the results are then compared.

The price data for the concerned week is as follows:

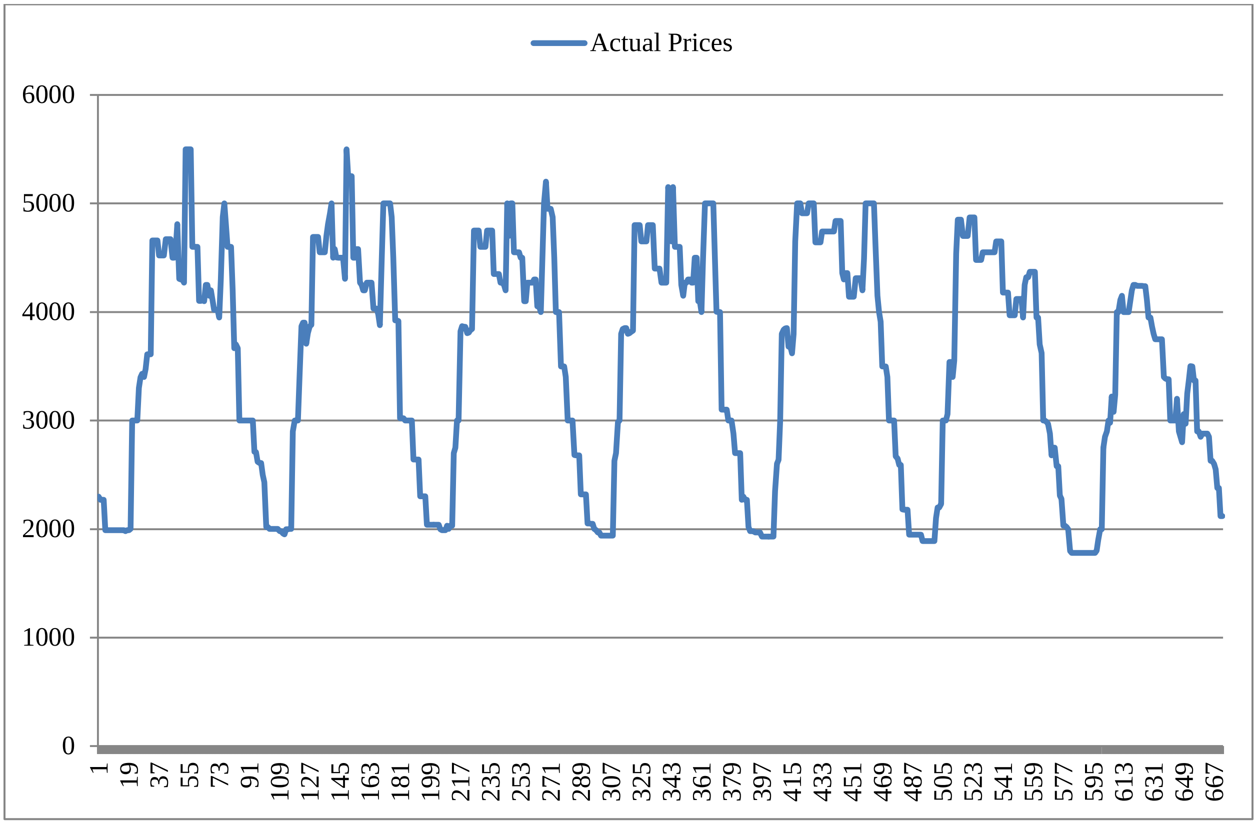

Figure 4: Price data for the second week of February 2014

The unit on the Y-axis is Rs/MWh and on the X-axis are the trading slot numbers, a total of 672 slots.

Following is the price forecast that was obtained by applying the **ARMR** model.

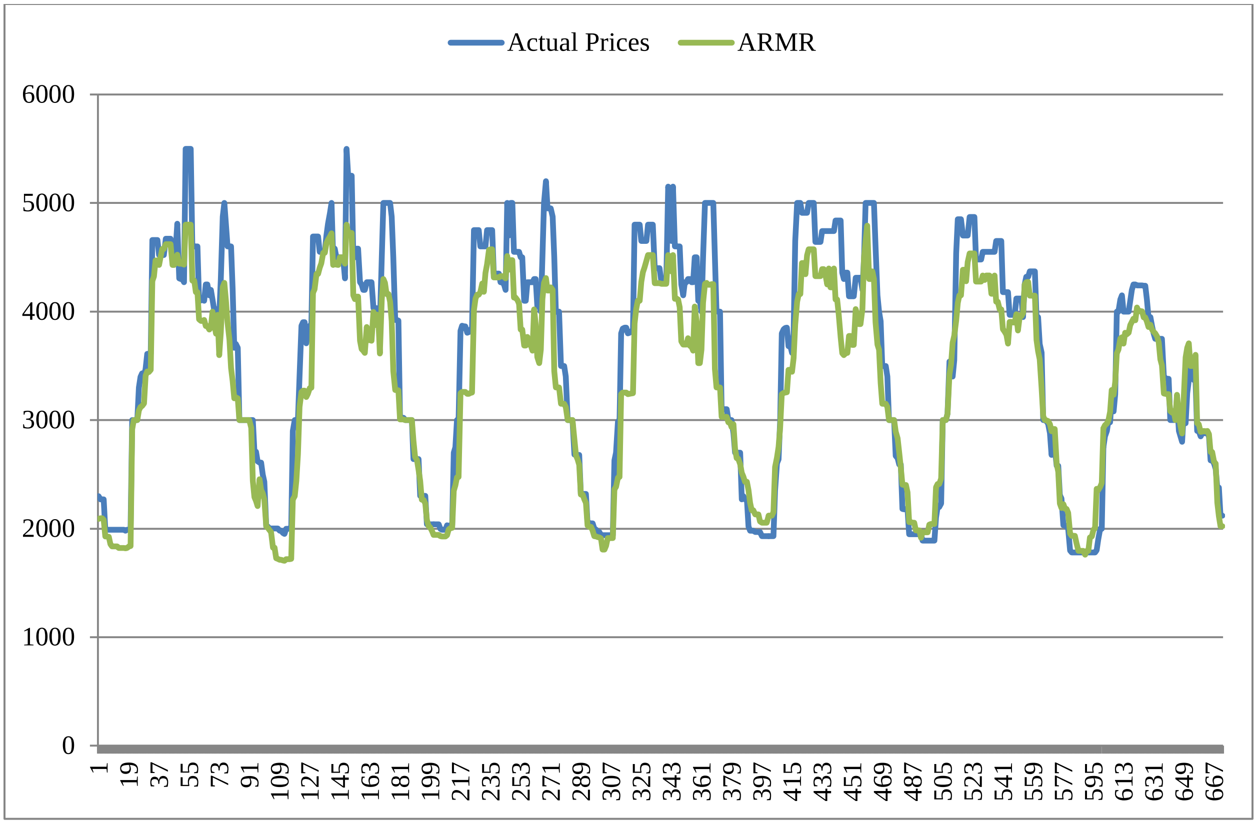

Figure 5: Actual Price vs Forecasted Price by ARMR Model

From the comparison it appears clear that we are able either to capture the shape or to predict prices for most of the hours making a very small error. The graph below plots the absolute error of the **ARMR** model.

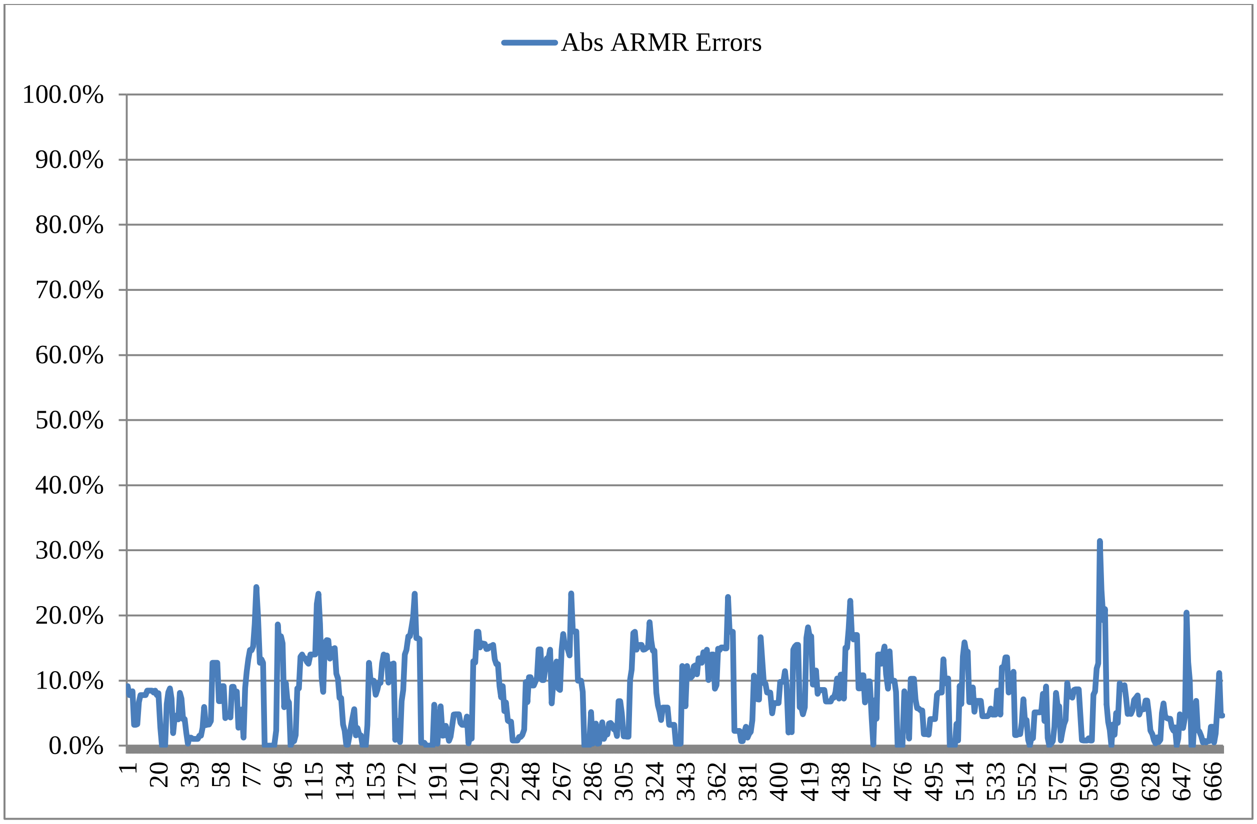

Figure 6: Absolute Percentage Errors of ARMR Model

As we can see above, **ARMR** does a fairly good job in predicting most of the prices with reasonable error but it fails to capture the price spikes very well. **ARMRX** model does a marginally better job in predicting the electricity prices but does a significantly good job in forecasting the prices.

The graph below shows the forecast obtained by applying the **ARMRX** model.

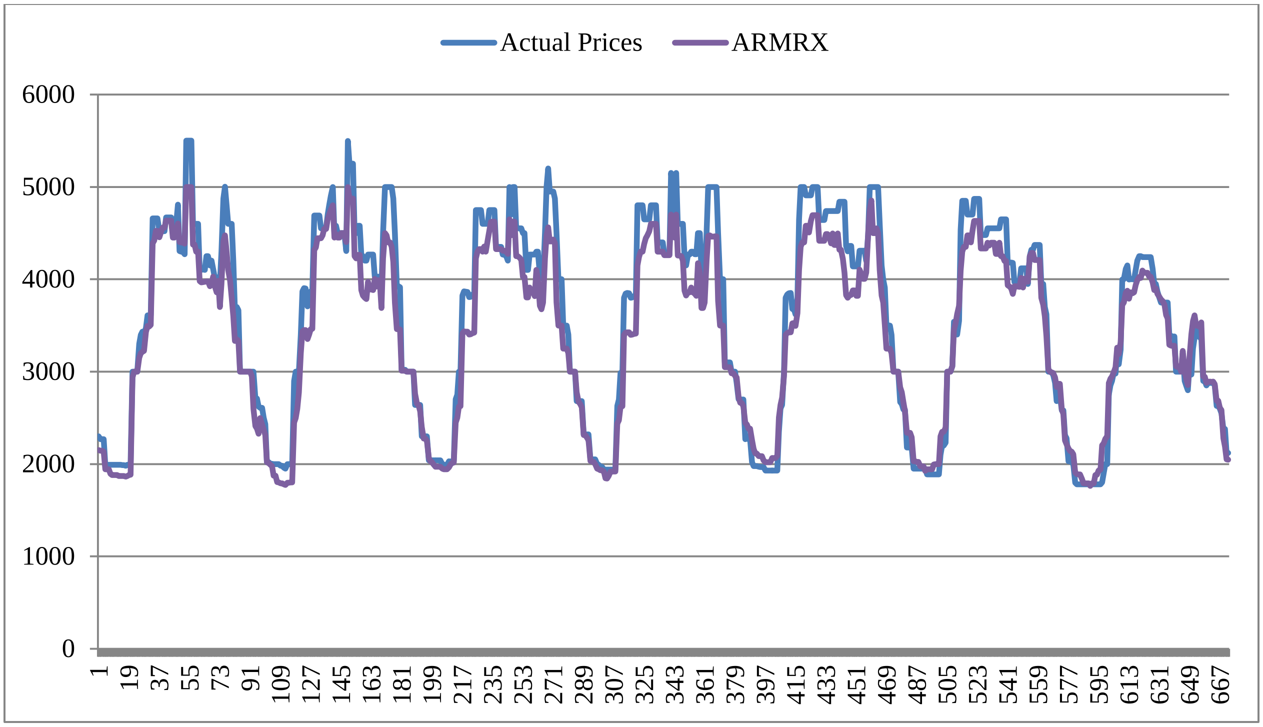

Figure 7: Actual Price vs Forecasted Price of ARMRX Model

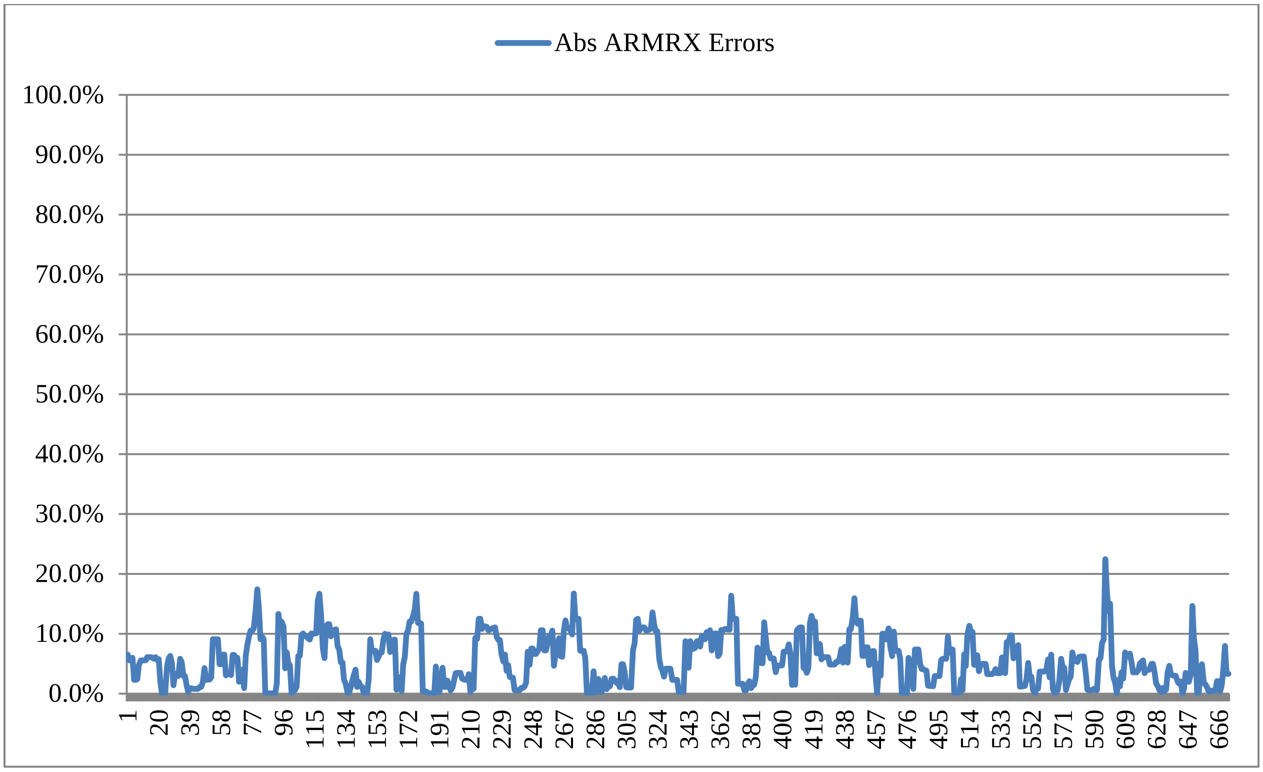

Figure 8: Absolute Percentage Error of the ARMRX Model

From the above graphs we can see that the **ARMRX** does a significantly better job when it comes to predicting the price spikes this is mainly due to its efficient extraction of information from the demand data. The Figure 9 plots absolute errors of the **ARMR** model vs **ARMRX** model.

The following table summarises the performance of the two models.

	ARMR Model	ARMRX Model
MAPE	7.56%	5.40%
Standard Deviation of Errors	7.86%	5.61%
Maximum Error	31.4%	22.5%
Minimum Error	-24.4%	-17.4%

Table 5.1: Comparison of ARMR Model and ARMRX Model

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